Subspace Homework Solutions

DS5020

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Basis

Express $v = (1, -2, 5)$ in $\mathbf{R}^3$ as a linear combination of the

vectors

$$u_1 = (1,1,1), \qquad u_2 = (1,2,3), \qquad u_3 = (2,-1,1)$$

Subspace

Let $V = \mathbf{R}^3$. Show that W is not a subspace of V, where

(a) $W = \{(a, b, c) : a \geq 0\}$, (b) $W = \{(a, b, c) : a^2 + b^2 + c^2 \geq 1\}$.

Subspace

Prove that the intersection of two subspaces of space V is also a subspace of V.

Vector Span

Find conditions on a, b, c so that $v = (a, b, c)$ in $\mathbf{R}^3$ belongs to

$W = \text{span}(u_1, u_2, u_3)$, where

$$u_1 = (1,2,0), \;\; u_2 = (-1,1,2), \;\; u_3 = (3,0,-4)$$

Linear Dependence

Determine whether or not u and v are linearly dependent

where

(a) $u = (1, 2), v = (3, -5)$,

(b) $u = (1, -3), v = (-2, 6)$,

(c) $u = (1, 2, -3), v = (4, 5, -6)$

(d) $u = (2, 4, -8), v = (3, 6, -12)$

Linear Dependence

Determine whether or not u and v are linearly dependent,

where

(a) $u = 2t^2 + 4t - 3, v = 4t^2 + 8t - 6$,

(b) $u = 2t^2 - 3t + 4, v = 4t^2 - 3t + 2$,

(c) $u = \begin{bmatrix} 1 & 3 & -4 \\ 5 & 0 & -1 \end{bmatrix}, v = \begin{bmatrix} -4 & -12 & 16 \\ -20 & 0 & 4 \end{bmatrix}$,

(d) $u = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}, v = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

Linear Dependence

Determine whether or not each of the following lists of vec-

tors in $\mathbf{R}^3$ is linearly dependent:

(a) $u_1 = (1, 2, 5), u_2 = (1, 3, 1), u_3 = (2, 5, 7), u_4 = (3, 1, 4)$,

(b) $u = (1, 2, 5), v = (2, 5, 1), w = (1, 5, 2)$,

(c) $u = (1, 2, 3), v = (0, 0, 0), w = (1, 5, 6)$.

Basis

Determine whether or not each of the following form a basis of $\mathbf{R}^3$:

(a) $(1, 1, 1), (1, 0, 1)$;

(b) $(1, 2, 3), (1, 3, 5), (1, 0, 1), (2, 3, 0)$;

(c) $(1, 1, 1), (1, 2, 3), (2, -1, 1)$;

(d) $(1, 1, 2), (1, 2, 5), (5, 3, 4)$.

Basis

Determine whether $(1, 1, 1, 1), (1, 2, 3, 2), (2, 5, 6, 4), (2, 6, 8, 5)$ form a basis of $\mathbf{R}^4$. If not, find the dimension of the subspace they span.

Basis and Dimension

Find a basis and dimension of the subspace W of $\mathbf{R}^3$ where

(a) $W = \{(a, b, c) : a + b + c = 0\}$,

(b) $W = \{(a, b, c) : (a = b = c)\}$
