DS5020 Midterm Exam - Linear Algebra

Northeastern University

Directions: You will have 24 hours to complete this exam. Please choose the correct answer for each question. You may use computers, calculators, books and notes. Do not collaborate with anyone else.

This is test version: 74.

Problem 1

Let $\mathbf{u} = [\ 1,\ 1,\ 3\ ]^T$ and $\mathbf{v} = [\ 5,\ 5,\ 4\ ]^T$. The value of $3\mathbf{u} - 4\mathbf{v}$ is given by

1. $[\ 1,\ -8,\ -3\ ]^T$
2. $[\ -17,\ -17,\ -7\ ]^T$
3. $[\ 8,\ -11,\ -9\ ]^T$
4. $[\ -8,\ -5,\ 2\ ]^T$

Problem 2

What is the inner product between vector $\mathbf{v} = [\ 1,\ -1,\ 1,\ -1\ ]^T$ and $\mathbf{u} = [\ 28,\ 14,\ 7,\ 41\ ]^T$? Is it:

1. -20
2. 2
3. 32
4. -4

Problem 3

What is the vector projection of $\mathbf{u} = [\ 1,\ 14,\ 13\ ]^T$ onto $\mathbf{v} = [\ 3,\ 5,\ 8\ ]^T$? Is it:

1. 1.1326531 $\mathbf{v}$
2. 1.8061224 $\mathbf{v}$
3. 1.1632653 $\mathbf{v}$
4. 1.3061224 $\mathbf{v}$

Problem 4

Find all $\mathbf{v} = [a, b]^T$ for which $\mathbf{v}$ is unit norm and orthogonal to $\mathbf{u} = [\ 17,\ 7\ ]^T$. Is it:

1. a = -1.3333333 b, $b^2 =$ 1 / 2.7777778
2. a = -0.4117647 b, $b^2 =$ 1 / 1.1695502
3. a = -2 b, $b^2 =$ 1 / 5
4. a = -1.3 b, $b^2 =$ 1 / 2.69

Problem 5

Consider a hyperplane which contains the vector $[2, 2, 2]^T$ and has a normal vector $[1, -1, 1]^T$. Which of the following vectors are in the hyperplane?

1. [1, 9, 7]^T
2. [8, 8, 7]^T
3. [10, 9, 7]^T
4. [2, 3, 3]^T

Problem 6

An eigenvector of matrix $M = \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}$ is:

1. [1, 10]^T
2. [2, 6]^T
3. [1, 1]^T
4. [5, 11]^T

Problem 7

A function of a real-vector argument is defined by $f([x\ y]^T) = 2xy$. The gradient of f at the point $[x\ y]^T =$ [2, 18]^T is

1. [36, 4]^T
2. [34, 22]^T
3. [26, 24]^T
4. [30, 40]^T