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LECC Quiz DS5020

1. Find $||u||$, where: (a) $u = (3, -12, -4)$, (b) $u = (2, -3, 8, -7)$.

First find $||u||^2 = u \cdot u$ by squaring the entries and adding.
Then $||u|| = \sqrt{||u||^2}$.

a. $u = (3, -12, -4)$

$$||u||^2 = u \cdot u$$
$$||u||^2 = (3)^2 + (-12)^2 + (-4)^2$$
$$= 9 + 144 + 16$$
$$= 169$$
$$||u|| = \sqrt{||u||^2}$$
$$= \sqrt{169}$$
$$= 13$$

b. $u = (2, -3, 8, -7)$

$$||u||^2 = u \cdot u$$
$$= (2)^2 + (-3)^2 + (8)^2 + (-7)^2$$
$$= 4 + 9 + 64 + 49$$

$$\|v\|^2 = 126$$

$$\|v\| = \sqrt{\|v\|^2}$$

$$\|v\| = \sqrt{126} \,_{\|}$$

2. Gaussian Elimination

$$\begin{aligned} x_1 - 3x_2 + 2x_3 - x_4 + 2x_5 &= 2 \\ 3x_1 - 9x_2 + 7x_3 - x_4 + 3x_5 &= 7 \\ 2x_1 - 6x_2 + 7x_3 + 4x_4 - 5x_5 &= 7 \end{aligned} \rightarrow (1)$$

$$R_3 \rightarrow R_3 - 2(R_1)$$
$$R_2 \rightarrow R_2 - 3(R_1)$$

$$\begin{aligned} x_1 - 3x_2 + 2x_3 - x_4 + 2x_5 &= 2 \\ 0 + 0 + x_3 + 2x_4 - 3x_5 &= 1 \\ 0 + 0 + 3x_3 + 6x_4 - 9x_5 &= 3 \end{aligned} \iff \left[\begin{array}{ccccc|c} 1 & -3 & 2 & -1 & 2 & 2 \\ 0 & 0 & 1 & 2 & -3 & 1 \\ 0 & 0 & 3 & 6 & -9 & 3 \end{array}\right]$$

$$R_3 \rightarrow R_3 - 3(R_2)$$

$$\left[\begin{array}{ccccc|c} 1 & -3 & 2 & -1 & 2 & 2 \\ 0 & 0 & 1 & 2 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_3 + 2x_4 - 3x_5 = 1$$

$$x_3 = 1 + 3x_5 - 2x_4$$

Sub x_3 in (1)

$$x_1 - 3x_2 + 2(1 + 3x_5 - 2x_4) - x_4 + 2x_5 = 2$$

$$x_1 - 3x_2 + 2 + 6x_5 - 4x_4 - x_4 + 2x_5 = 2$$

$$x_1 - 3x_2 - 5x_4 + 8x_5 = 0$$

$$x_1 = 3x_2 + 5x_4 - 8x_5$$

$$x_3 = 1 + 3x_5 - 2x_4$$

and, x_2, x_4, x_5 are free variable

3. Augmented Matrix

Solve the following system using its augmented matrix M:

$$\begin{aligned} x_1 + 2x_2 - 3x_3 - 2x_4 + 4x_5 &= 1 \\ 2x_1 + 5x_2 - 8x_3 - x_4 + 6x_5 &= 4 \\ x_1 + 4x_2 - 7x_3 + 5x_4 + 2x_5 &= 8 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -3 & -2 & 4 & 1 \\ 2 & 5 & -8 & -1 & 6 & 4 \\ 1 & 4 & -7 & 5 & 2 & 8 \end{array}\right]$$

$$R_2 \to R_2 - 2(R_1)$$

$$R_3 \to R_3 - R_1$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -3 & -2 & 4 & 1 \\ 0 & 1 & -2 & 3 & -2 & 2 \\ 0 & 2 & -4 & 7 & -2 & 7 \end{array}\right]$$

$$R_3 \to R_3 - 2(R_2)$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -3 & -2 & 4 & 1 \\ 0 & 1 & -2 & 3 & -2 & 2 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{array}\right]$$

$$R_1 \longrightarrow R_1 - 2(R_2)$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & -8 & 8 & -3 \\ 0 & 1 & -2 & 3 & -2 & 2 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{array}\right]$$

$$R_1 \rightarrow R_1 + 8(R_3)$$

$$R_2 \rightarrow R_2 - 3(R_3)$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 24 & 21 \\ 0 & 1 & -2 & 0 & -8 & -7 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{array}\right]$$

$$x_1 = 21 - x_3 - 24x_5$$

$$x_2 = 2x_3 + 8x_5 - 7$$

$$x_4 = 3 - 2x_5$$

x_3 & x_5 are free variables.