LECC Homework
DS 5020

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1. Vector Norm

a. $\vec{u} = (3, -4)$

$$\|\vec{u}\| = \sqrt{(3)^2 + (-4)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5$$

$$\hat{u} = \frac{\vec{u}}{\|\vec{u}\|} = \left(\frac{3}{5}, \frac{-4}{5}\right)$$

b. $\vec{v} = (4, -2, -3, 8)$

$$\|\vec{v}\| = \sqrt{(4)^2 + (-2)^2 + (-3)^2 + (8)^2}$$
$$= \sqrt{16 + 4 + 9 + 64}$$
$$= \sqrt{93}$$

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|} = \left(\frac{4}{\sqrt{93}}, \frac{-2}{\sqrt{93}}, \frac{-3}{\sqrt{93}}, \frac{8}{\sqrt{93}}\right)$$

c. $\vec{\omega} = \left(\frac{1}{2}, \frac{2}{3}, \frac{-1}{4}\right)$

$$\|\vec{\omega}\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{4}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{4}{9} + \frac{1}{16}}$$

$$= \sqrt{\frac{36 + 64 + 9}{144}} = \sqrt{\frac{109}{144}} = \frac{\sqrt{109}}{12}$$

$$\hat{\omega} = \frac{\vec{\omega}}{\|\vec{\omega}\|} = \left(\frac{12}{2\sqrt{109}}, \frac{24}{3\sqrt{109}}, \frac{-12}{4\sqrt{109}}\right) \Leftrightarrow \left(\frac{6}{\sqrt{109}}, \frac{8}{\sqrt{109}}, \frac{-3}{\sqrt{109}}\right)$$

2. Pivot and Free Variables

Determine the pivot and free variables in each of the following systems:

a. $2x_1 - 3x_2 - 6x_3 - 5x_4 + 2x_5 = 7$

$x_3 + 3x_4 - 7x_5 = 6$

$x_4 - 2x_5 = 1$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \end{array}$$
$$\begin{bmatrix} 2 & -3 & -6 & -5 & +2 & 7 \\ 0 & 0 & 1 & 3 & -7 & 6 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{bmatrix}$$

$\Updownarrow (R_1 \to R_1/2)$

$$\begin{bmatrix} 1 & -3/2 & -3 & -5/2 & 1 & 7/2 \\ 0 & 0 & 1 & 3 & -7 & 6 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{bmatrix}$$

~~Lets assume varia~~

Pivot Variables: x_1, x_3, x_4

Free Variables: x_2, x_5

b. $2x - 6y + 7z = 1$
$4y + 3z = 8$
$2z = 4$

$$\begin{bmatrix} 2 & -6 & 7 & 1 \\ 0 & 4 & 3 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$\Updownarrow \begin{cases} R_1 \to R_1/2 \\ R_2 \to R_2/4 \\ R_3 \to R_3/2 \end{cases}$$

$$\begin{bmatrix} 1 & -3 & 7/2 & 1/2 \\ 0 & 1 & 3/4 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Pivot Variables : x, y, z

Free Variables :

c. $x + 2y - 3z = 2$
$2x + 3y + z = 4$
$3x + 4y + 5z = 8$

$$\iff \begin{bmatrix} 1 & 2 & -3 & 2 \\ 2 & 3 & 1 & 4 \\ 3 & 4 & 5 & 8 \end{bmatrix}$$

$(R_2 \to R_2 - 2(R_1))$

$$\begin{bmatrix} 1 & 2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 3 & 4 & 5 & 8 \end{bmatrix}$$

$(R_2 \to -R_2)$ & $(R_3 \to R_3 - 3(R_1))$

$$\begin{bmatrix} 1 & 2 & -3 & 2 \\ 0 & 1 & -7 & 0 \\ 0 & -2 & 14 & 2 \end{bmatrix}$$

$$[R_3 \to R_3 + 2(R_2)]$$

$$\begin{bmatrix} 1 & 2 & -3 & 2 \\ 0 & 1 & -7 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Pivot Variables : x, y

Free Variables : z

3. Augmented Matrix

Solve each of the following systems using its augmented matrix M.

a. $x + 2y - z = 3$
$x + 3y + z = 5$
$3x + 8y + 4z = 17$

→ The augmented matrix is :

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 1 & 3 & 1 & 5 \\ 3 & 8 & 4 & 17 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 2 \\ 3 & 8 & 4 & 17 \end{array}\right]$$

$$R_3 \to R_3 - 3(R_1)$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 7 & 8 \end{array}\right]$$

$$R_3 \to R_3 - 2(R_2)$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 3 & 4 \end{array}\right]$$

$$R_3 \to R_3/3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 4/3 \end{array}\right]$$

$$R_1 \to R_1 - 2(R_2)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 4/3 \end{array}\right]$$

$$R_1 \to R_1 + 5(R_3)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 17/3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 4/3 \end{array}\right]$$

$R_2 \to R_2 - 2(R_3)$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 17/3 \\ 0 & 1 & 0 & -2/3 \\ 0 & 0 & 1 & 4/3 \end{array}\right]$$

$\therefore \quad x = \frac{17}{3}, \; y = \frac{-2}{3}, \; z = \frac{4}{3}$

b. $x - 2y + 4z = 2$
$2x - 3y + 5z = 3$
$3x - 4y + 6z = 7$

→ The augmented matrix is:

$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & 2 \\ 2 & -3 & 5 & 3 \\ 3 & -4 & 6 & 7 \end{array}\right]$$

$R_2 \to R_2 - 2(R_1)$
$R_3 \to R_3 - 3(R_1)$

$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 2 & -6 & 1 \end{array}\right]$$

$R_3 \to R_3 - 2(R_2)$

$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

$\therefore$ The system is inconsistent. ($\because 0x + 0y + 0z = 3$)

c. $x + y + 3z = 1$
$2x + 3y - z = 3$
$5x + 7y + z = 7$

→ The augmented matrix is:

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 2 & 3 & -1 & 3 \\ 5 & 7 & 1 & 7 \end{array}\right]$$

$R_2 \to R_2 - 2(R_1)$
$R_3 \to R_3 - 5(R_1)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 1 & -7 & 1 \\ 0 & 2 & -14 & 2 \end{array}\right]$$

$R_3 \to R_3 - 2(R_2)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 1 & -7 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 10 & 0 \\ 0 & 1 & -7 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x = -10z$
$y = 1 + 7z$

4. Row Canonical Form

· Reduce each of the following matrices to row canonical form:

a. $A = \begin{bmatrix} 2 & 2 & -1 & 6 & 4 \\ 4 & 4 & 1 & 10 & 13 \\ 8 & 8 & -1 & 26 & 23 \end{bmatrix}$

$R_1 \to R_1/2$

$$\begin{bmatrix} 1 & 1 & -1/2 & 3 & 2 \\ 4 & 4 & 1 & 10 & 13 \\ 8 & 8 & -1 & 26 & 23 \end{bmatrix}$$

$R_2 \to R_2 - 4(R_1)$
$R_3 \to R_3 - 8(R_1)$

$$\begin{bmatrix} 1 & 1 & -1/2 & 3 & 2 \\ 0 & 0 & 3 & -2 & 5 \\ 0 & 0 & 3 & 2 & 7 \end{bmatrix}$$

$R_2 \to R_2/3$

$$\begin{bmatrix} 1 & 1 & -1/2 & 3 & 2 \\ 0 & 0 & 1 & -2/3 & 5/3 \\ 0 & 0 & 3 & 2 & 7 \end{bmatrix}$$

$R_1 \to R_1 + \frac{(R_2)}{2}$

$$\begin{bmatrix} 1 & 1 & 0 & 8/3 & 17/6 \\ 0 & 0 & 1 & -2/3 & 5/3 \\ 0 & 0 & 3 & 2 & 7 \end{bmatrix}$$

$$R_3 = R_3 - 3(R_2)$$

$$\begin{bmatrix} 1 & 1 & 0 & 8/3 & 17/6 \\ 0 & 0 & 1 & -2/3 & 5/3 \\ 0 & 0 & 0 & 4 & 2 \end{bmatrix}$$

$$R_3 \to \frac{R_3}{4}$$

$$\begin{bmatrix} 1 & 1 & 0 & 8/3 & 17/6 \\ 0 & 0 & 1 & -2/3 & 5/3 \\ 0 & 0 & 0 & 1 & 1/2 \end{bmatrix}$$

$$R_1 \to R_1 - \frac{8}{3}(R_3)$$

$$R_2 \to R_2 + \frac{2}{3}(R_3)$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 3/2 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1/2 \end{bmatrix}$$

b. $B = \begin{bmatrix} 5 & -9 & 6 \\ 0 & 2 & 3 \\ 0 & 0 & 7 \end{bmatrix}$

$$R_1 \to \frac{R_1}{5}$$

$$\begin{bmatrix} 1 & -9/5 & 6/5 \\ 0 & 2 & 3 \\ 0 & 0 & 7 \end{bmatrix}$$

$$R_2 \to \frac{R_2}{2}$$

$$\begin{bmatrix} 1 & -9/5 & 6/5 \\ 0 & 1 & 3/2 \\ 0 & 0 & 7 \end{bmatrix}$$

$$R_1 \to R_1 + \frac{9}{5}(R_2)$$

$$\begin{bmatrix} 1 & 0 & 39/10 \\ 0 & 1 & 3/2 \\ 0 & 0 & 7 \end{bmatrix}$$

$$R_3 \to \frac{R_3}{7}$$

$$\begin{bmatrix} 1 & 0 & 39/10 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \to R_1 - \frac{39}{10}(R_3)$$

$$R_2 \to R_2 - \frac{3}{2}(R_3)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Gaussian Elimination

· Solve each of the following systems:

a. $x + 2y - 4z = -4$
$2x + 5y - 9z = -10$
$3x - 2y + 3z = 11$

$$\begin{bmatrix} 1 & 2 & -4 \\ 2 & 5 & -9 \\ 3 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -10 \\ 11 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -4 \\ 2 & 5 & -9 & -10 \\ 3 & -2 & 3 & 11 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2(R_1)$
$R_3 \rightarrow R_3 - 3(R_1)$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -4 \\ 0 & 1 & -1 & -2 \\ 0 & -8 & 15 & 23 \end{array}\right]$$

$R_1 \rightarrow R_1 - 2(R_2)$
$R_3 \rightarrow R_3 + 8(R_2)$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 7 & 7 \end{array}\right]$$

$R_3 \rightarrow \frac{R_3}{7}$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 \to R_1 + 2(R_3)$
$R_2 \to R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

b. $x + 2y + (-3z) = -1$
$-3x + y - 2z = -7$
$5x + 3y - 4z = 2$

$$\begin{bmatrix} 1 & 2 & -3 \\ -3 & 1 & -2 \\ 5 & 3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -7 \\ 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ -3 & 1 & -2 & -7 \\ 5 & 3 & -4 & 2 \end{array}\right]$$

$R_2 \to R_2 + 3(R_1)$
$R_3 \to R_3 - 5(R_1)$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 7 & -11 & -10 \\ 0 & -7 & 11 & 7 \end{array}\right]$$

$$R_2 \to \frac{R_2}{7}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 1 & -11/7 & -10/7 \\ 0 & -7 & 11 & 7 \end{array}\right]$$

$$R_1 \to R_1 - 2(R_2)$$
$$R_3 \to R_3 + 7(R_2)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/7 & 13/7 \\ 0 & 1 & -\frac{11}{7} & -10/7 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

$\therefore$ The system is inconsistent

C. $x + 2y - 3z = 1$
$2x + 5y - 8z = 4$
$3x + 8y - 13z = 7$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 2 & 5 & -8 & 4 \\ 3 & 8 & -13 & 7 \end{array}\right]$$

$$R_2 \to R_2 - 2(R_1)$$

$$R_3 \to R_3 - 3(R_1)$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 2 & -4 & 4 \end{array}\right]$$

$$R_1 \rightarrow R_1 - 2(R_2)$$
$$R_3 \rightarrow R_3 - 2(R_2)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + z = -3 \quad \& \quad y - 2z = 2$$

$$x = -(3 + z)$$
$$y = 2(1 + z)$$

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6. Hyperplane 1

· Find an equation of the plane H in R^3 that contains $P(1,-3,-4)$ and is parallel to the plane H' determined by the equation $3x-6y+5z=2$.

Ans: $\vec{n} = (\underset{a}{3}, \underset{b}{-6}, \underset{c}{5})$, $P(\underset{x_0}{1}, \underset{y_0}{-3}, \underset{z_0}{-4})$

Equation of plane:

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$3(x-1)-6(y+3)+5(z+4)=0$$

$$3x-3-6y-18+5z+20=0$$

$$3x-6y+5z=1$$

7. Hyperplane 2.

· Find an equation of the hyperplane H in R^4 that passes through $P(3,-4,1,-2)$ and is normal to $v=[2,5,-6,-3]$.

Ans. $\vec{v} = (2,5,-6,-3)$, $P(3,-4,1,-2)$

Equation of hyperplane:

$$2(x-3)+5(y+4)-6(z-1)-3(w+2)=0$$

$$2x-6+5y+20-6z+6-3w-6=0$$

$$2x+5y-6z-3w=-14$$