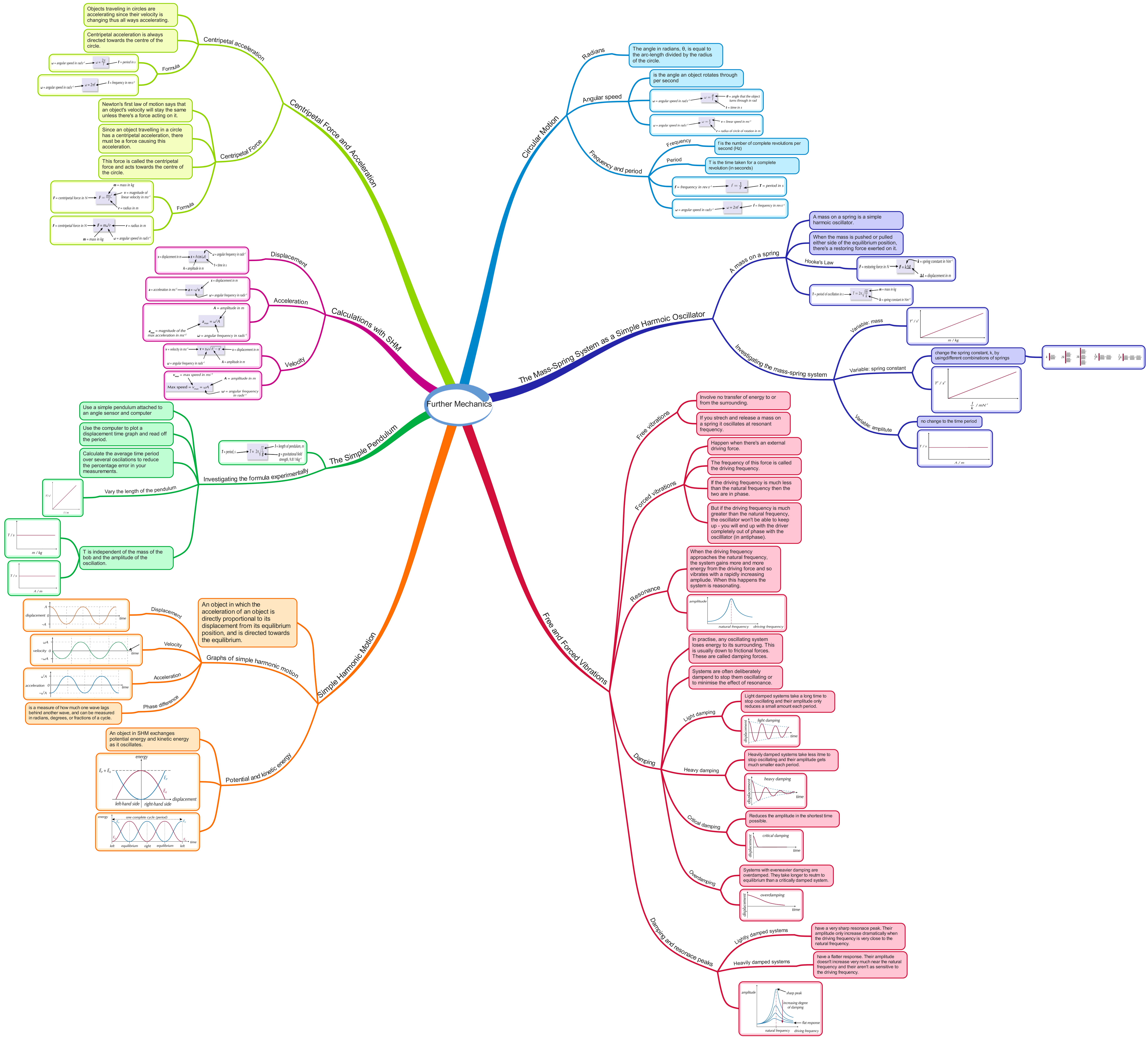
Further Mechanics
Centripetal Force and Acceleration
Centripetal acceleration
Objects traveling in circles are accelerating since their velocity is changing thus all ways accelerating.
Centripetal acceleration is always directed towards the centre of the circle.
Formula
ω = angular speed in rad s⁻¹ → $\omega = \frac{2\pi}{T}$ ← T = period in s
ω = angular speed in rad s⁻¹ → $\omega = 2\pi f$ ← f = frequency in rev s⁻¹
Centripetal Force
Newton's first law of motion says that an object's velocity will stay the same unless there's a force acting on it.
Since an object travelling in a circle has a centripetal acceleration, there must be a force causing this acceleration.
This force is called the centripetal force and acts towards the centre of the circle.
Formula
F = centripetal force in N → $F = \frac{mv^2}{r}$; m = mass in kg; v = magnitude of linear velocity in ms⁻¹; r = radius in m
F = centripetal force in N → $F = m\omega^2 r$; r = radius in m; m = mass in kg; ω = angular speed in rad s⁻¹
Calculations with SHM
Displacement
x = displacement in m → $x = A\cos(\omega t)$; ω = angular frequency in rads⁻¹; t = time in s; A = amplitude in m
Acceleration
a = acceleration in ms⁻² → $a = -\omega^2 x$; x = displacement in m; ω = angular frequency in rads⁻¹
$a_{max} = \omega^2 A$; A = amplitude in m; a_{max} = magnitude of the max acceleration in ms⁻²; ω = angular frequency in rads⁻¹
Velocity
v = velocity in ms⁻¹ → $v = \pm\omega\sqrt{A^2 - x^2}$; x = displacement in m; ω = angular frequency in rads⁻¹; A = amplitude in m
Max speed = $v_{max} = \omega A$; v_{max} = max speed in ms⁻¹; A = amplitude in m; ω = angular frequency in rads⁻¹
The Simple Pendulum
T = period, s → $T = 2\pi\sqrt{\frac{l}{g}}$; l = length of pendulum, m; g = gravitational field strength, 9.81 Nkg⁻¹
Investigating the formula experimentally
Use a simple pendulum attached to an angle sensor and computer
Use the computer to plot a displacement time graph and read off the period.
Calculate the average time period over several oscillations to reduce the percentage error in your measurements.
Vary the length of the pendulum
T²/ s²
l / m
T is independent of the mass of the bob and the amplitude of the osciliation.
T / s
m / kg
T / s
A / m
Simple Harmonic Motion
An object in which the acceleration of an object is directly proportional to its displacement from its equilibrium position, and is directed towards the equilibrium.
Graphs of simple harmonic motion
Displacement
A
displacement 0
–A
time
Velocity
ωA
velocity 0
–ωA
time
Acceleration
ω²A
acceleration 0
–ω²A
time
Phase difference
is a measure of how much one wave lags behind another wave, and can be measured in radians, degrees, or fractions of a cycle.
Potential and kinetic energy
An object in SHM exchanges potential energy and kinetic energy as it oscillates.
energy
$E_P + E_K$
E_P
E_K
displacement
left-hand side
right-hand side
energy
one complete cycle (period)
E_P
E_K
time
left
equilibrium
right
equilibrium
left
Circular Motion
Radians
The angle in radians, θ, is equal to the arc-length divided by the radius of the circle.
Angular speed
is the angle an object rotates through per second
ω = angular speed in rad s⁻¹ → $\omega = \frac{\theta}{t}$; θ = angle that the object turns through in rad; t = time in s
ω = angular speed in rad s⁻¹ → $\omega = \frac{v}{r}$; v = linear speed in ms⁻¹; r = radius of circle of rotation in m
Frequency and period
Frequency
f is the number of complete revolutions per second (Hz)
Period
T is the time taken for a complete revolution (in seconds)
f = frequency in rev s⁻¹ → $f = \frac{1}{T}$ ← T = period in s
ω = angular speed in rad s⁻¹ → $\omega = 2\pi f$ ← f = frequency in rev s⁻¹
The Mass-Spring System as a Simple Harmonic Oscillator
A mass on a spring
A mass on a spring is a simple harmoic oscillator.
When the mass is pushed or pulled either side of the equilibrium position, there's a restoring force exerted on it.
Hooke's Law
F = restoring force in N → $F = k\Delta L$; k = spring constant in Nm⁻¹; ΔL = displacement in m
T = period of oscillation in s → $T = 2\pi\sqrt{\frac{m}{k}}$; m = mass in kg; k = spring constant in Nm⁻¹
Investigating the mass-spring system
Variable: mass
T² / s²
m / kg
Variable: spring constant
change the spring constant, k, by usingdifferent combinations of springs
k
2k
3k
$\frac{1}{2}k$
$\frac{1}{3}k$
T² / s²
$\frac{1}{k}$ / mN⁻¹
Variable: amplitude
no change to the time period
T / s
A / m
Free and Forced Vibrations
Free vibrations
Involve no transfer of energy to or from the surrounding.
If you strech and release a mass on a spring it oscillates at resonant frequency.
Forced vibrations
Happen when there's an external driving force.
The frequency of this force is called the driving frequency.
If the driving frequency is much less than the natural frequency then the two are in phase.
But if the driving frequency is much greater than the natural frequency, the oscillator won't be able to keep up - you will end up with the driver completely out of phase with the oscilllator (in antiphase).
Resonance
When the driving frequency approaches the natural frequency, the system gains more and more energy from the driving force and so vibrates with a rapidly increasing ampliude. When this happens the system is reasonating.
amplitude
natural frequency
driving frequency
Damping
In practise, any oscillating system loses energy to its surrounding. This is usually down to frictional forces. These are called damping forces.
Systems are often deliberately dampend to stop them oscillating or to minimise the effect of resonance.
Light damping
Light damped systems take a long time to stop oscillating and their amplitude only reduces a small amount each period.
light damping
displacement
time
Heavy damping
Heavily damped systems take less itme to stop oscillating and their amplitude gets much smaller each period.
heavy damping
displacement
time
Critical damping
Reduces the amplitude in the shortest time possible.
critical damping
displacement
time
Overdamping
Systems with eveneavier damping are overdamped. They take longer to reutrn to equilibrium than a critically damped system.
overdamping
displacement
time
Damping and resonace peaks
Lightly damped systems
have a very sharp resonace peak. Their amplitude only increase dramatically when the driving frequency is very close to the natural frequency.
Heavily damped systems
have a flatter response. Their amplitude doesn't increase very much near the natural frequency and their aren't as sensitive to the driving frequency.
amplitude
sharp peak
increasing degree of damping
flat response
natural frequency
driving frequency