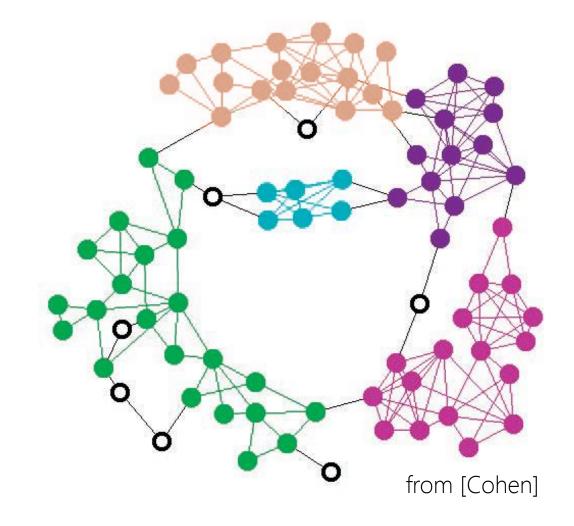


Problem



Finding highly connected sub-graphs

- Why is this important?
 - Social media graphs: groups of friends/family/co-workers
 - Website interliking
- Why is this difficult?
 - Possible solution set size: $2^{|V|}$
 - → Exponential run time for naive approach
 - Often millions of vertices
 - \rightarrow \otimes



The Data



Wikipedia

- Directed graph of English Wikipedia page interlinks from 2007
- ~1.9 million vertices, ~40 million edges, 1 GB size on disc

Twitter

- Directed graph of anonymous Twitter follower/following data from 2009
- ~41 million vertices, ~1.6 billion edges, 25.5 GB size on disc



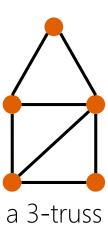


k-Trusses



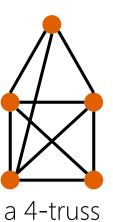
k-Truss

- **Definition**: a maximal subgraph so that every edge is part of at least k-2 triangles
- Indicates a high connectivity between its nodes
- Can be seen as a relaxation of the clique problem (= fully connected subgraph)



How?

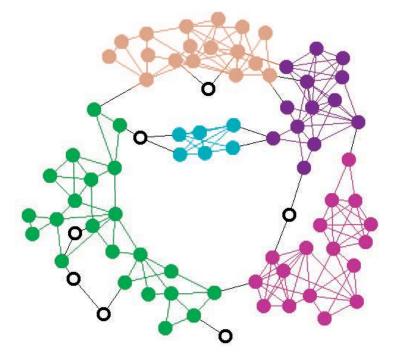
- Find all triangles in the graph
- Recursively remove all edges that are in <k-2 triangles
- Return sets of nodes that are still connected



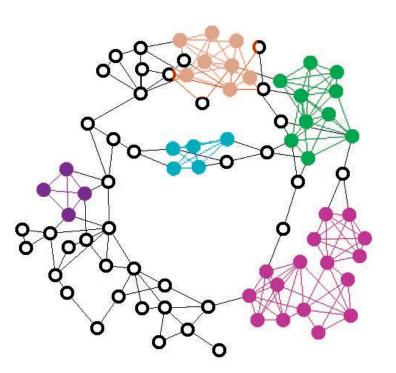
k-Trusses



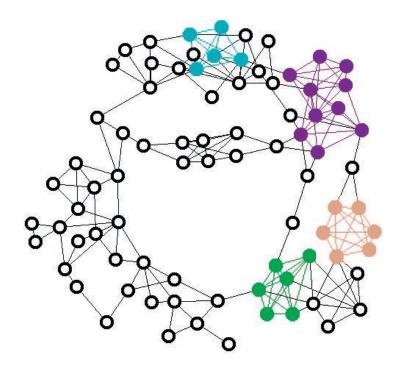




k = 4



k = 5



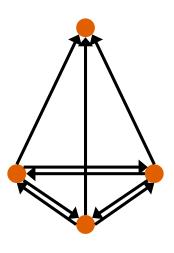
from [Cohen]

Direction?



Two possibilities

- Any direction
 - Accept trusses where either direction of an edge exists
- Both directions
 - Accept trusses only when both directions of an edge exist

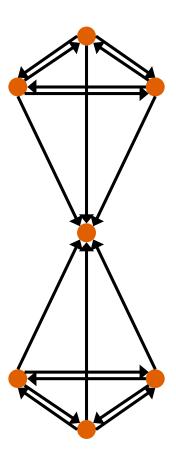


Direction?



Two possibilities

- Any direction
 - Accept trusses where either direction of an edge exists
- Both directions
 - Accept trusses only when both directions of an edge exist



Direction?

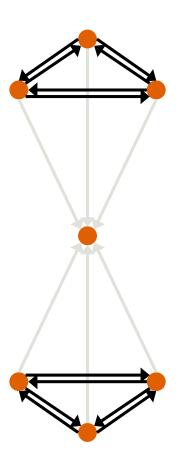


Two possibilities

- Any direction
 - Accept trusses where either direction of an edge exists
- Both directions
 - Accept trusses only when both directions of an edge exist

Decision

- Bidirectional edges only
- Only nodes that actually interact with one another should form a truss
- → Can use pre-processing step to create graph with bidirectional edges only



Finding the maximum Truss



- 1. Create new graph from bidirectional edges only
- 2. Set k = arbitrary value, subraphs = (fullGraph)
- 3. Find all k-trusses for each subgraph *after [Cohen]*
- 4. If none exist:
- 5. Reduce k, go to 3.
- 6. Set subgraphs =(truss1, truss2, ...)
- 7. Increase k, go to 3.

Abort if k has already been seen before

(Increase or reduce k according to a binary search strategy)

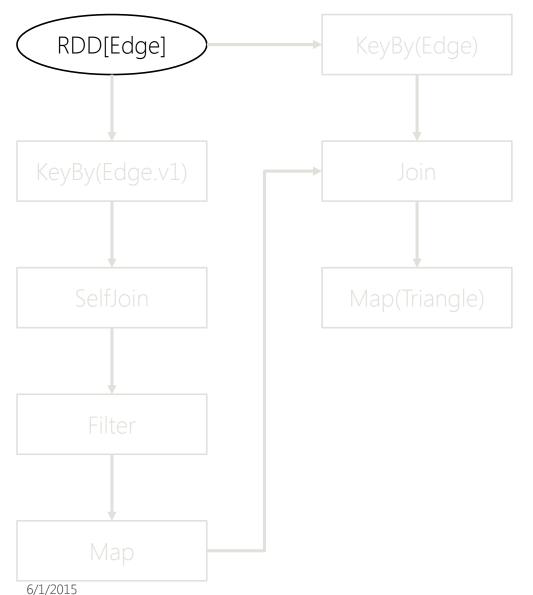
Evaluation – starting k

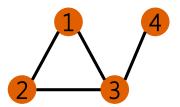


Real maxTrussSize = 28

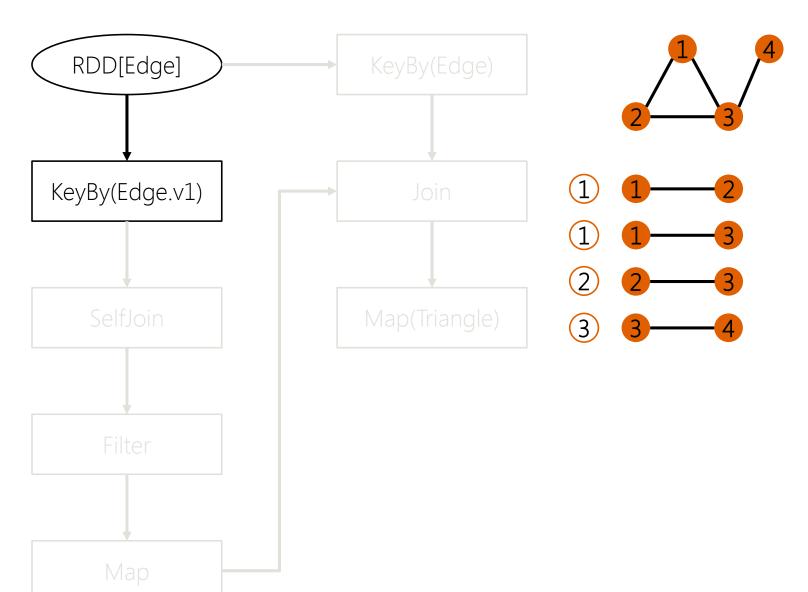
- k = 10 17 minutes
 - k values tried: 10, 20, 40, 30, 25, 27, 28, 29, 28
- k = 20 11 minutes
 - k values tried: 20, 40, 30, 25, 27, 28, 29, 28
- k = 28 10 minutes
 - k values tried: 28, 56, 42, 35, 31, 29
- k = 40 20 minutes
 - k values tried: 40, 21, 30, 25, 27, 28, 29, 28





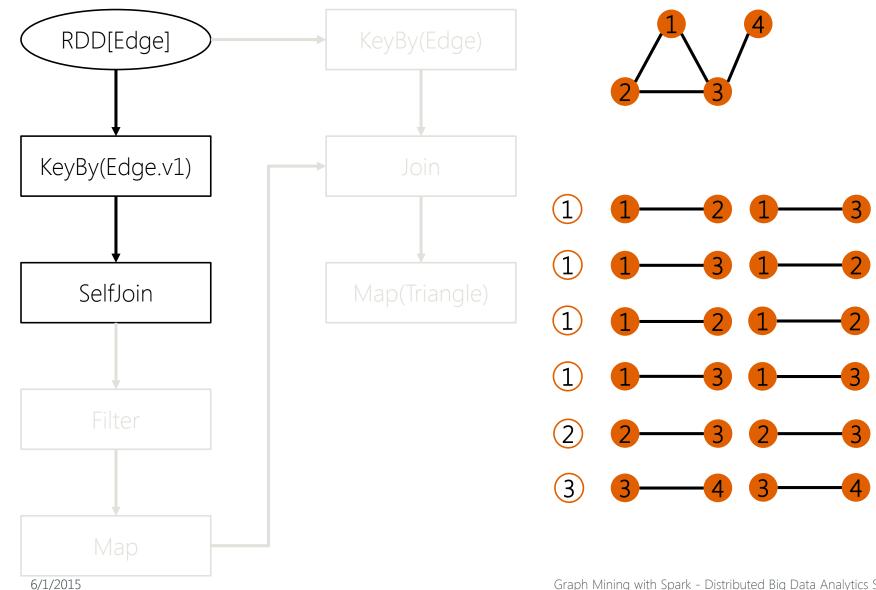




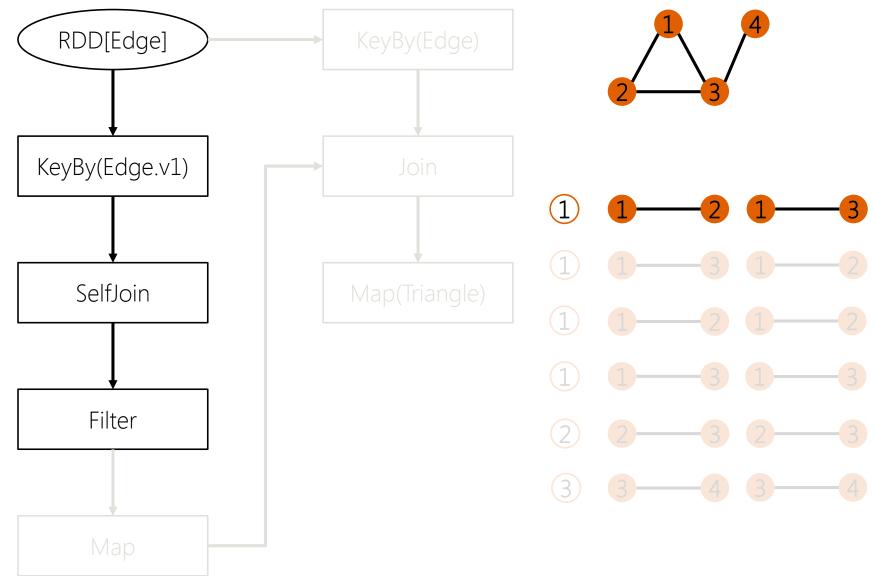


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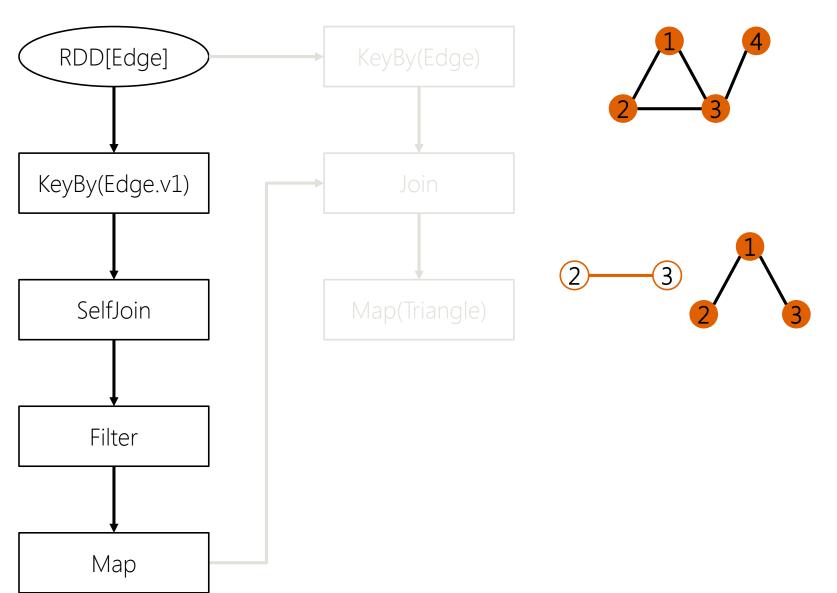






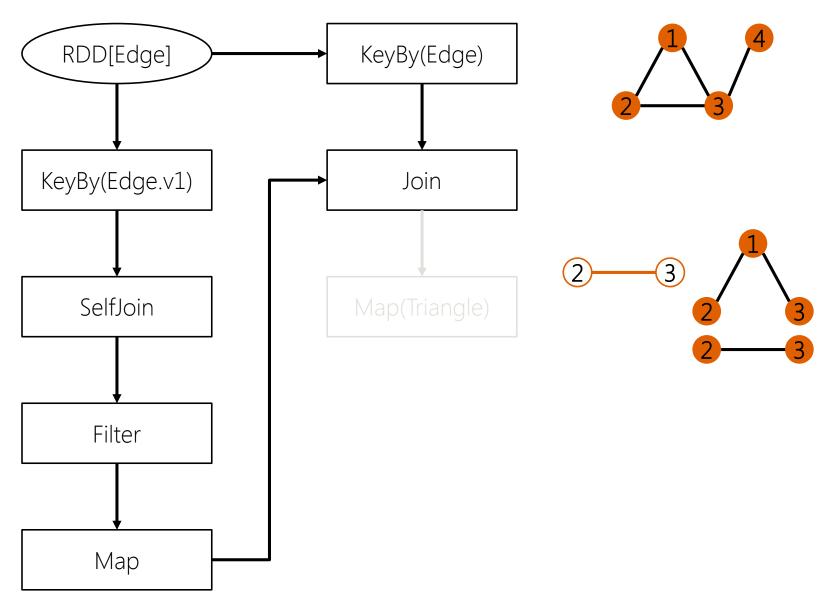
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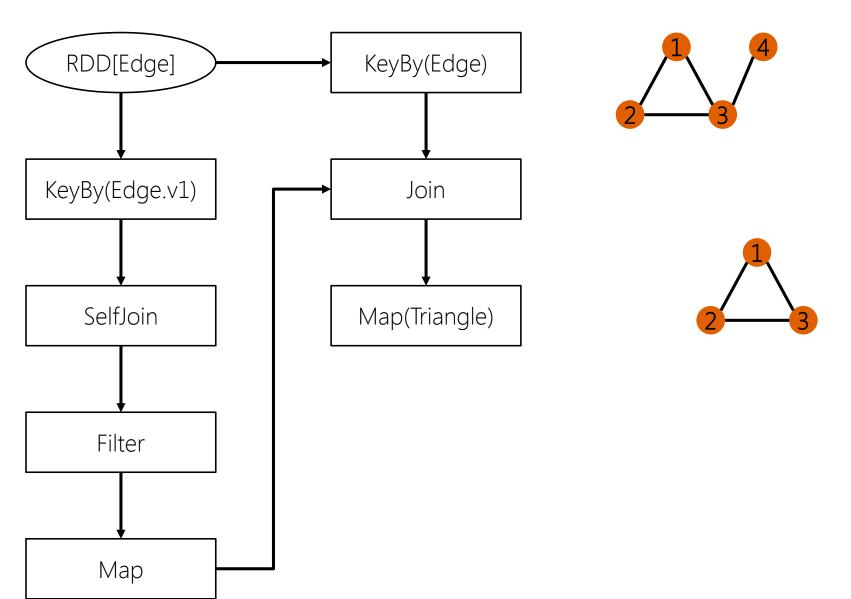


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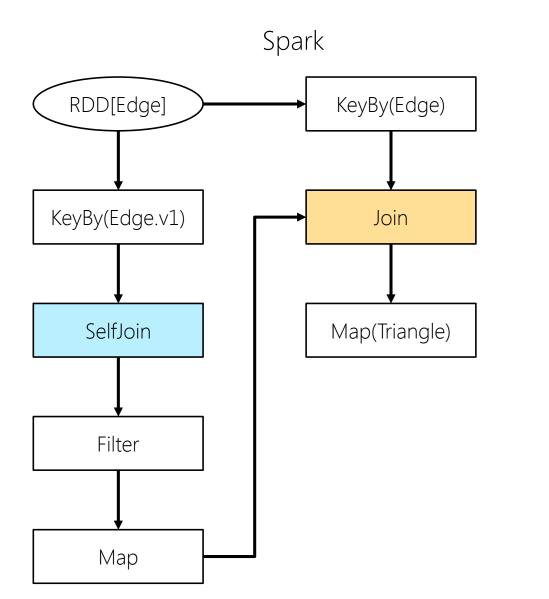


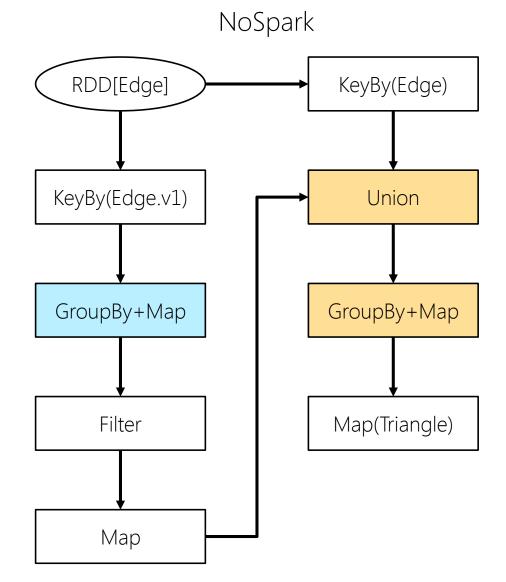




Evaluation – Triangle Generation







Evaluation – Triangle Generation



By number of cores used

- 1 core x 5 machines
 - Spark 14 minutes
 - NoSpark 12 minutes
- 1 core x 10 machines
 - Spark 6.8 minutes
 - NoSpark 5.7 minutes
- 2 cores x 10 machines
 - Spark 6.5 minutes
 - NoSpark 4.1 minutes

Conclusions



Distributed Calculation

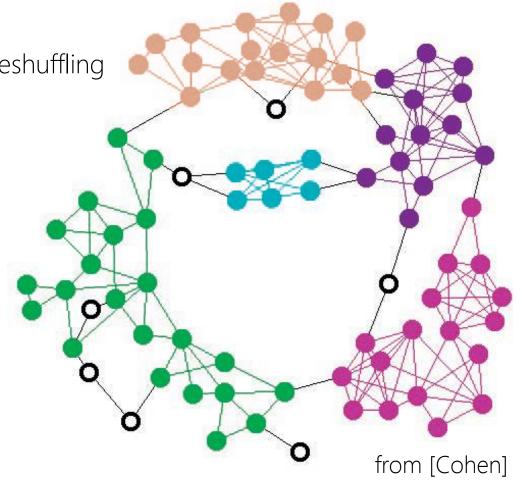
• Choice of starting k is very influencial on run time

• Great scaling with distribution over multiple machines

• Current Spark-specific implementation seems limited by reshuffling

Future Work

- Try larger data sets
- Distributed clique calculation
- Usage of directed graphs instead



References



Image on title slide: http://polkadotimpressions.com/2013/01/18/facebook-graph-search-3/

[Bron, Kerbosh]: Bron, Coen, and Joep Kerbosch. 'Algorithm 457: finding all cliques of an undirected graph.' *Communications of the ACM* 16, no. 9 (1973): 575-577.

[Cohen]: Jonathan Cohen, 'Graph Twiddling in a MapReduce World'. in *Computing in Science and Engineering* 11(4): 29-41 (2009)