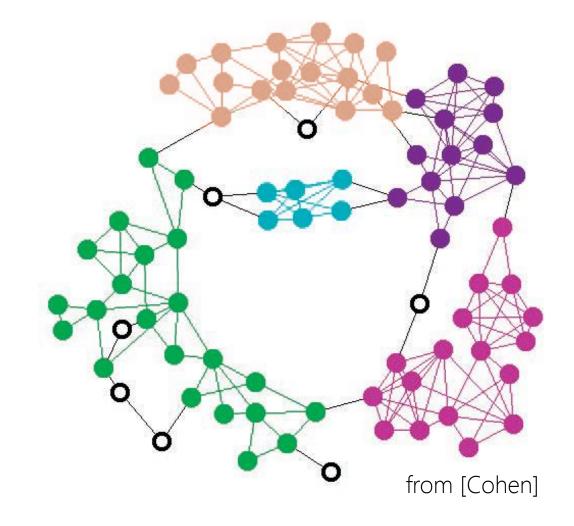


## Problem



### Finding highly connected sub-graphs

- Why is this important?
  - Social media graphs: groups of friends/family/co-workers
  - Website interliking
- Why is this difficult?
  - Possible solution set size:  $2^{|V|}$
  - → Exponential run time for naive approach
  - Often millions of vertices
  - $\rightarrow$   $\otimes$



### The Data



### Wikipedia

- Directed graph of English Wikipedia page interlinks from 2007
- ~1.9 million vertices, ~40 million edges, 1 GB size on disc

### **Twitter**

- Directed graph of anonymous Twitter follower/following data from 2009
- ~41 million vertices, ~1.6 billion edges, 25.5 GB size on disc



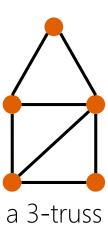


### k-Trusses



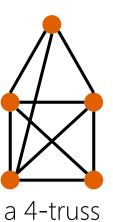
#### k-Truss

- **Definition**: a maximal subgraph so that every edge is part of at least k-2 triangles
- Indicates a high connectivity between its nodes
- Can be seen as a relaxation of the clique problem (= fully connected subgraph)



#### How?

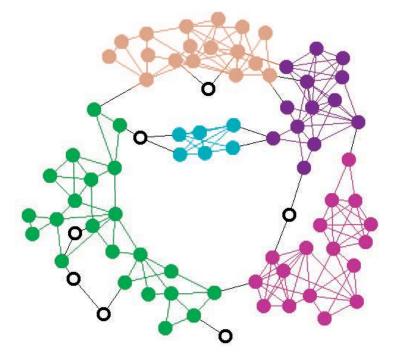
- Find all triangles in the graph
- Recursively remove all edges that are in <k-2 triangles
- Return sets of nodes that are still connected



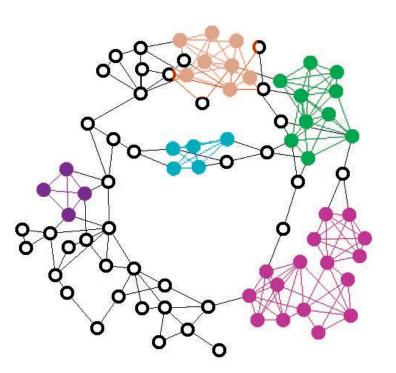
## k-Trusses



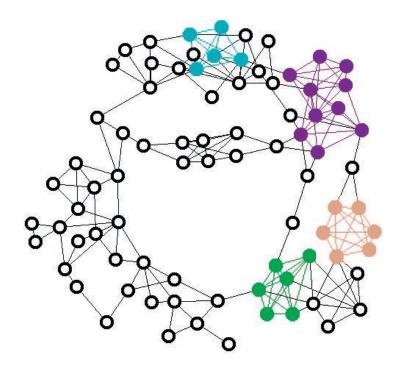




k = 4



k = 5



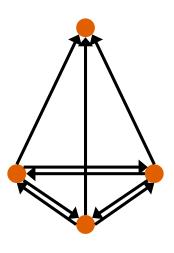
from [Cohen]

## Direction?



### Two possibilities

- Any direction
  - Accept trusses where either direction of an edge exists
- Both directions
  - Accept trusses only when both directions of an edge exist

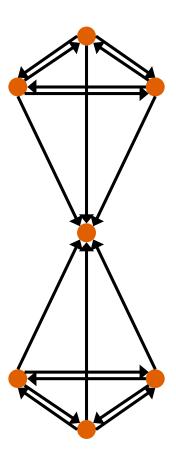


## Direction?



### Two possibilities

- Any direction
  - Accept trusses where either direction of an edge exists
- Both directions
  - Accept trusses only when both directions of an edge exist



### Direction?

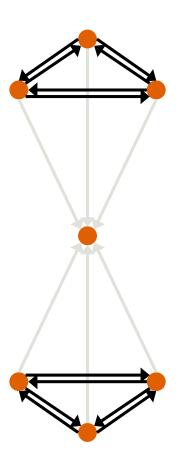


### Two possibilities

- Any direction
  - Accept trusses where either direction of an edge exists
- Both directions
  - Accept trusses only when both directions of an edge exist

#### **Decision**

- Bidirectional edges only
- Only nodes that actually interact with one another should form a truss
- → Can use pre-processing step to create graph with bidirectional edges only



## Finding the maximum Truss



- 1. Create new graph from bidirectional edges only
- 2. Set k = arbitrary value, subraphs = (fullGraph)
- 3. Find all k-trusses for each subgraph *after [Cohen]*
- 4. If none exist:
- 5. Reduce k, go to 3.
- 6. Set subgraphs =(truss1, truss2, ...)
- 7. Increase k, go to 3.

Abort if k has already been seen before

(Increase or reduce k according to a binary search strategy)

## Evaluation – starting k

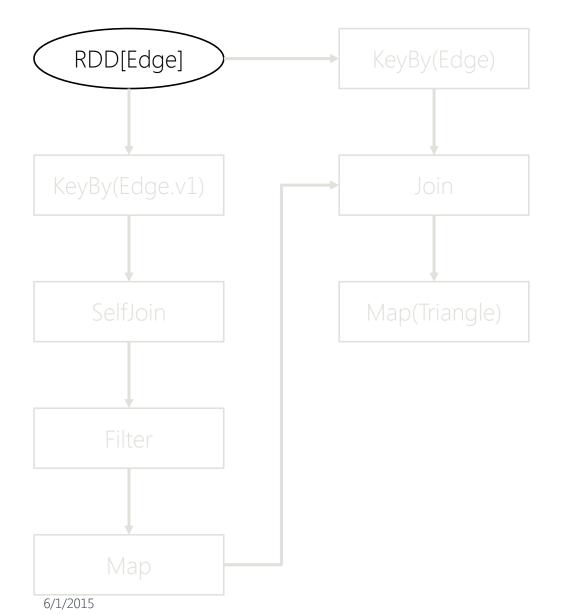


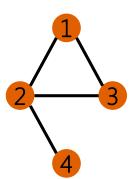
#### Real maxTrussSize = 28

- k = 10 17 minutes
  - k values tried: 10, 20, 40, 30, 25, 27, 28, 29, 28
- k = 20 11 minutes
  - k values tried: 20, 40, 30, 25, 27, 28, 29, 28
- k = 28 10 minutes
  - k values tried: 28, 56, 42, 35, 31, 29, 28
- k = 40 20 minutes
  - k values tried: 40, 21, 30, 25, 27, 28, 29, 28

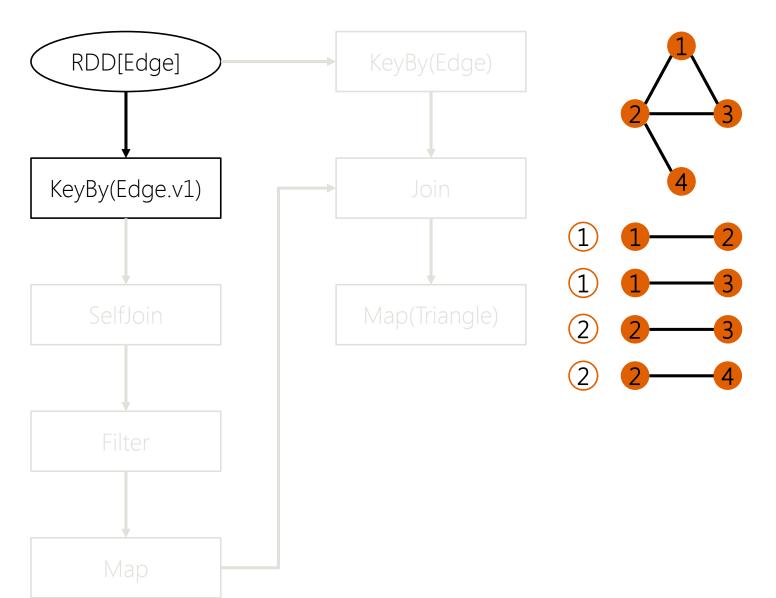
Using 10 machines, 20 cores, 4GB RAM







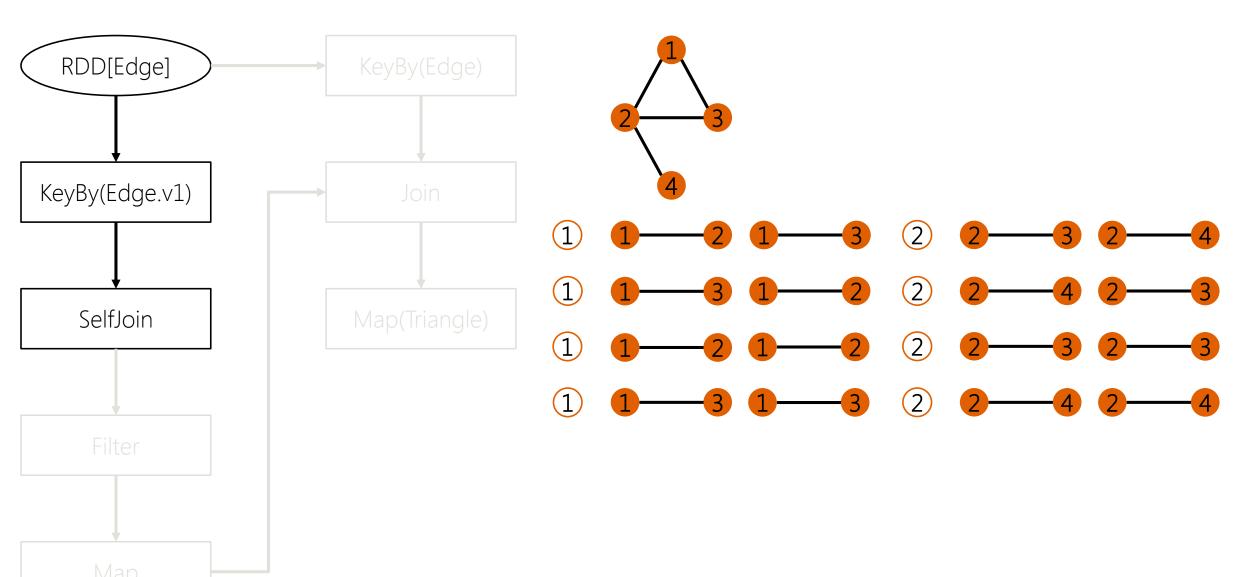




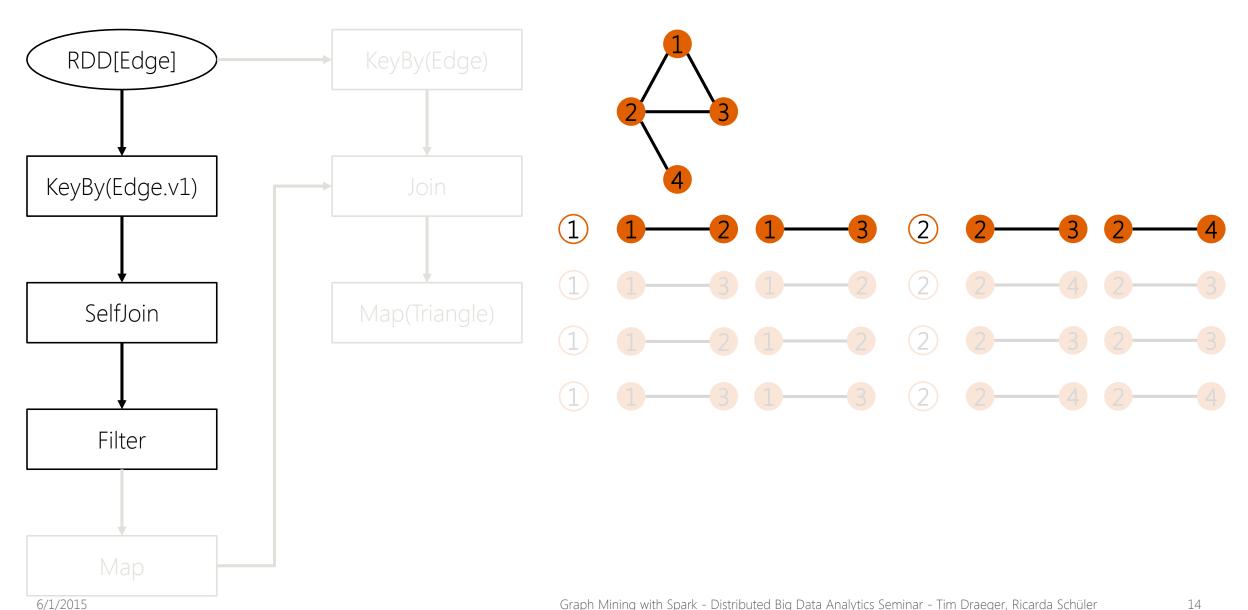
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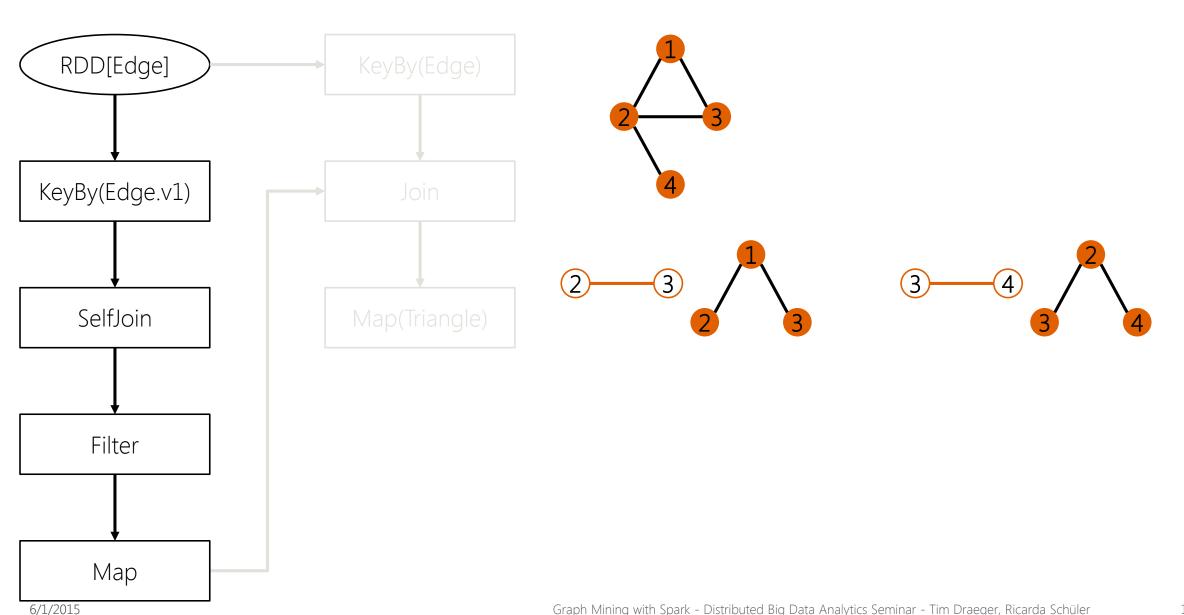




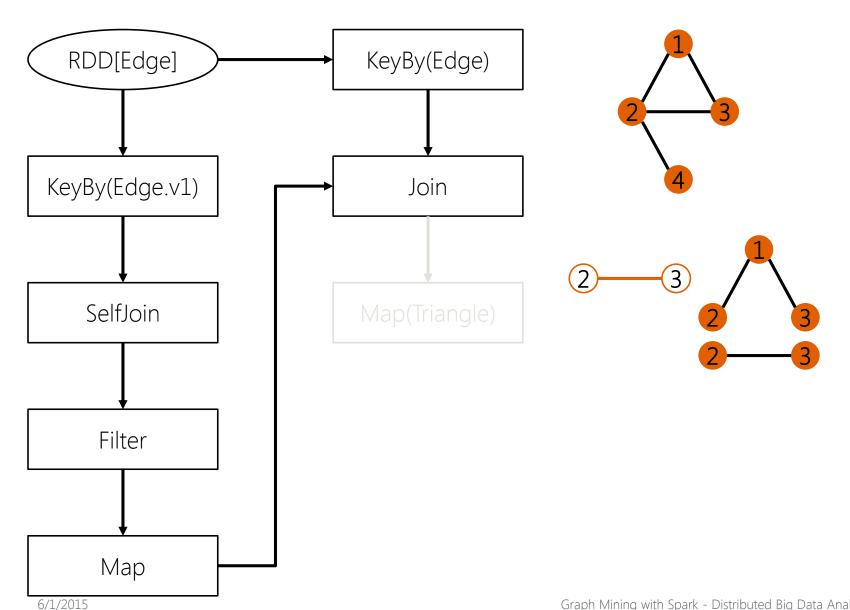




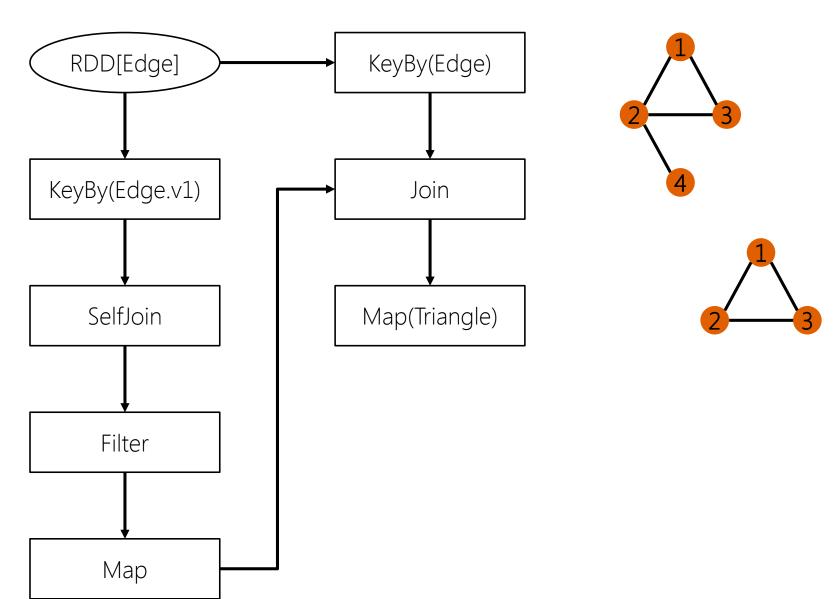






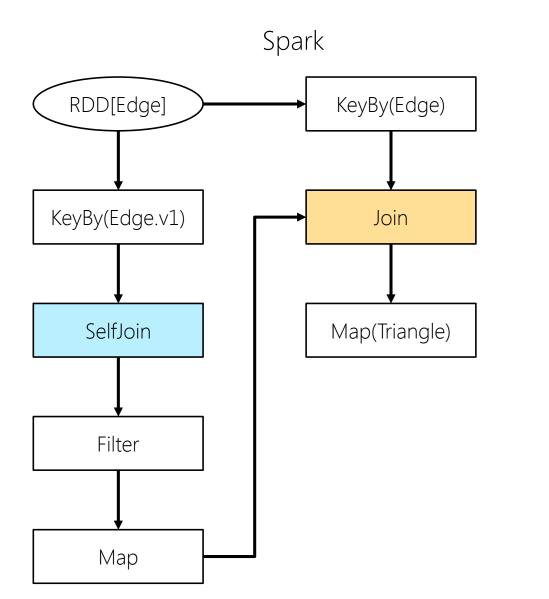


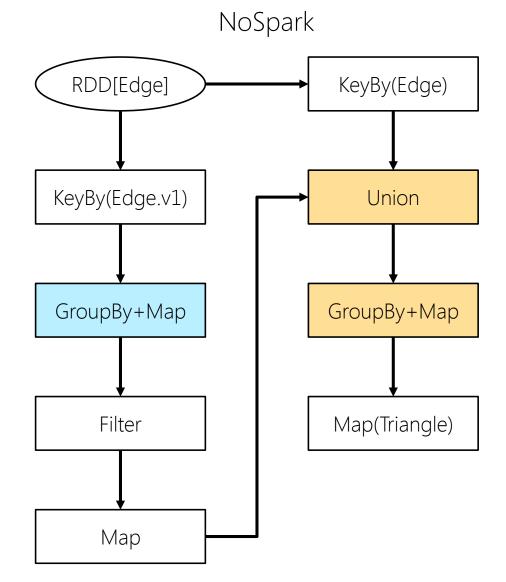




# Evaluation – Triangle Generation







## Evaluation – Triangle Generation



### By number of cores used

- 1 core x 5 machines
  - Spark 14 minutes
  - NoSpark 12 minutes
- 1 core x 10 machines
  - Spark 6.8 minutes
  - NoSpark 5.7 minutes
- 2 cores x 10 machines
  - Spark 6.5 minutes
  - NoSpark 4.1 minutes

using 4GB RAM

## Conclusions



#### Distributed Calculation

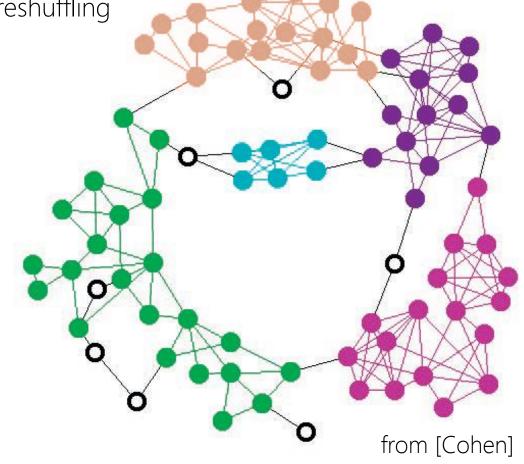
• Great scaling with distribution over multiple machines

• Current Spark-specific implementation seems limited by reshuffling

• Choice of starting k is very influencial on run time

#### Future Work

- Try larger data sets
- Distributed clique calculation
- Usage of directed graphs instead



### References



Image on title slide: http://polkadotimpressions.com/2013/01/18/facebook-graph-search-3/

[Bron, Kerbosh]: Bron, Coen, and Joep Kerbosch. 'Algorithm 457: finding all cliques of an undirected graph.' *Communications of the ACM* 16, no. 9 (1973): 575-577.

[Cohen]: Jonathan Cohen, 'Graph Twiddling in a MapReduce World'. in *Computing in Science and Engineering* 11(4): 29-41 (2009)