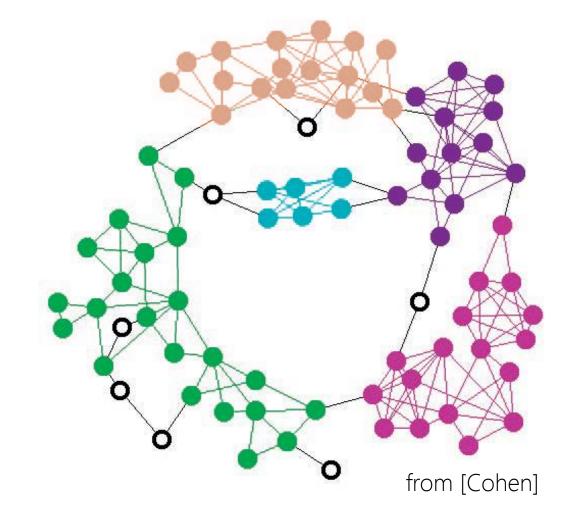


### Problem



#### Finding highly connected sub-graphs

- Why is this important?
  - Social media graphs: groups of friends/family/co-workers
  - Website interliking
- Why is this difficult?
  - Possible solution set size:  $2^{|V|}$
  - → Exponential run time for naive approach
  - Often millions of vertices
  - $\rightarrow$   $\otimes$



### The Data



#### Wikipedia

- Directed graph of English Wikipedia page interlinks from 2007
- ~1.9 million vertices, ~40 million edges, 1 GB size on disc

• Bidirectional version with 3.4 million edges, 54 MB

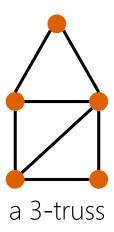


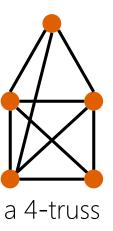
#### k-Trusses



#### k-Truss

- Definition: a maximal subgraph so that every edge is part of at least k-2 triangles
- Indicates a high density and high connectivity between its nodes
- Can be seen as a relaxation of the clique problem (= fully connected subgraph)

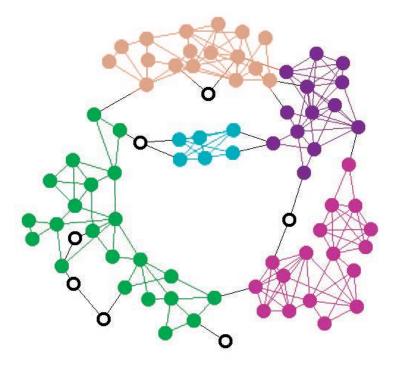




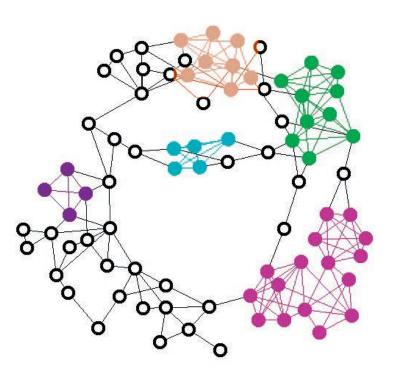
## k-Trusses



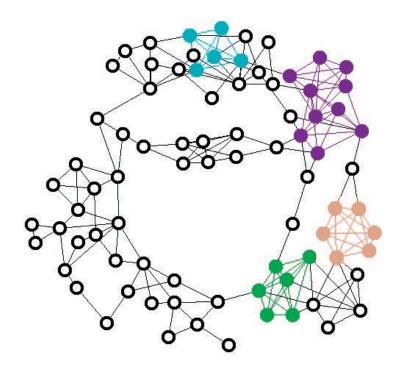




$$k = 4$$



k = 5

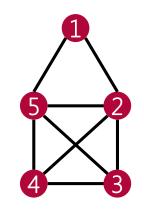


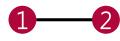
from [Cohen]



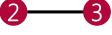
Read graph

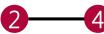
Find 4-truss





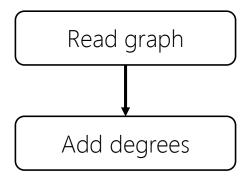




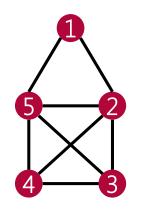








Find 4-truss



2 1 4

4 2 5 4

2 1 6 4

3 **3 4** 3

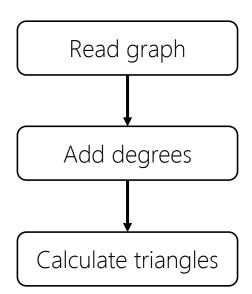
4 **2 3** 3

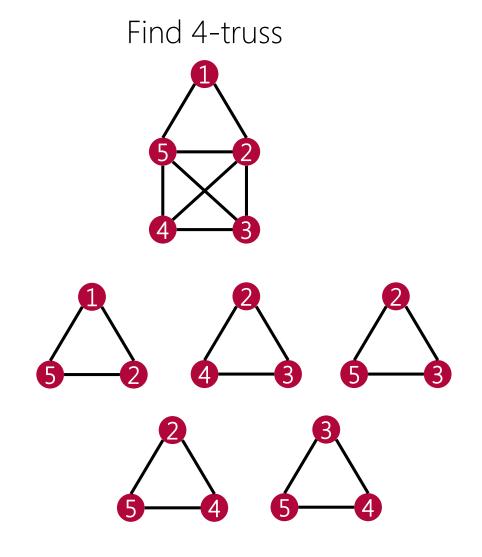
3 3 5

4 2 4 3

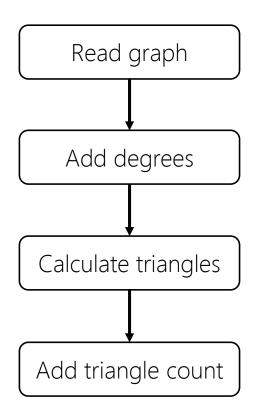
3 4 --- 5 4

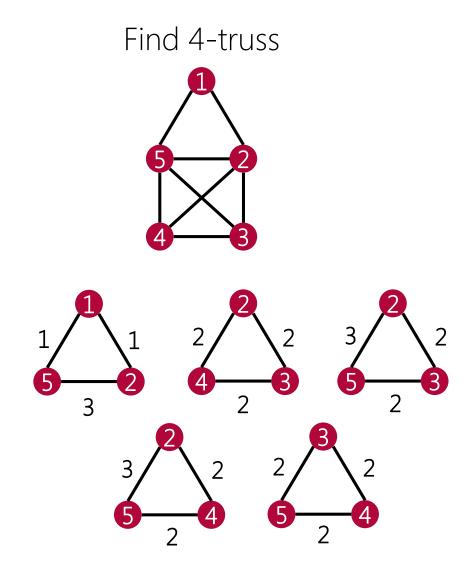




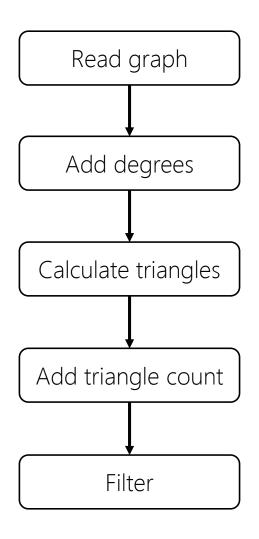


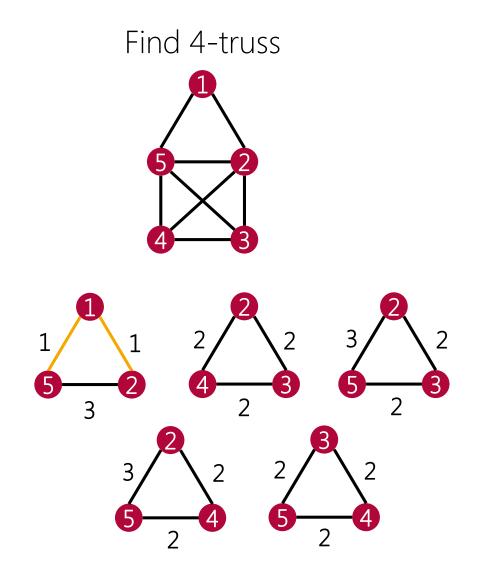




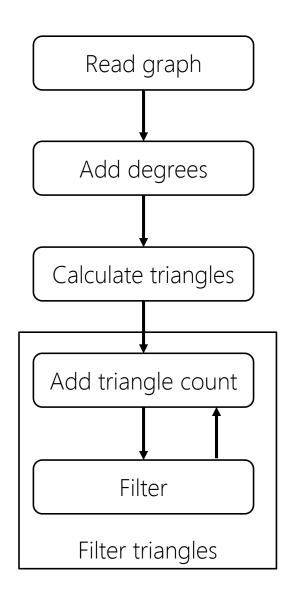


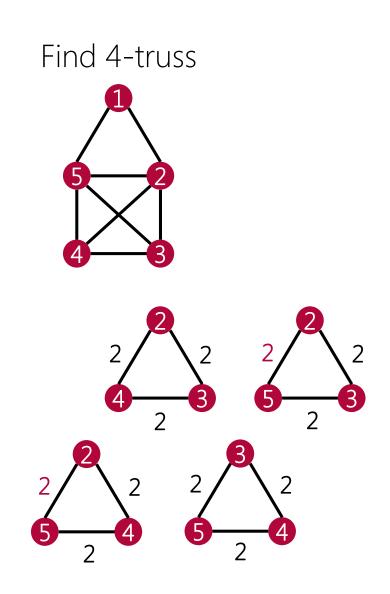




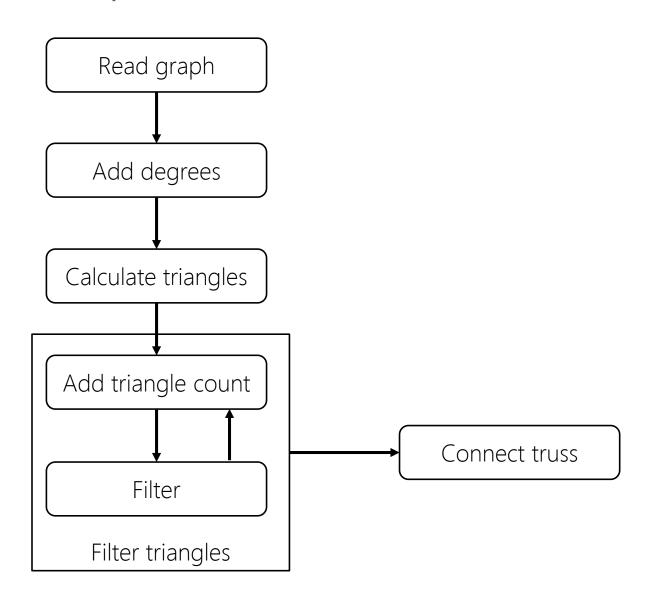




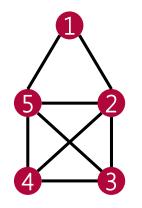


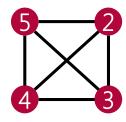






Find 4-truss





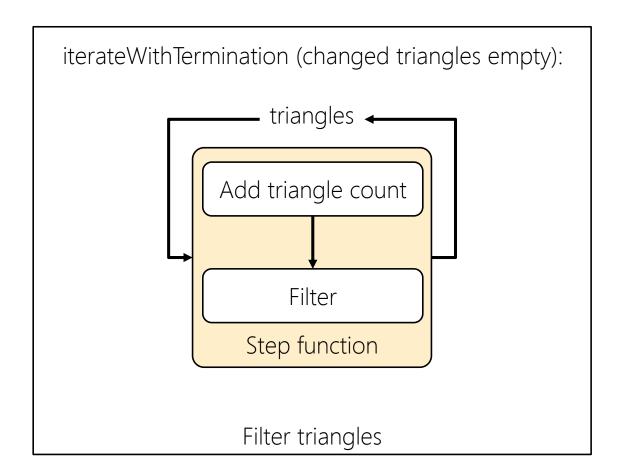
# Implementation – Filter Triangles



#### Spark

while (number of triangles has changed): Add triangle count Filter Filter triangles

#### Flink





#### Finding the most dense truss in a graph

- Find a k-truss with user-defined k
- If at least one truss was found
  - set new k' > k
  - Search for new k'-trusses in the found trusses
- Else
  - Set new k' < k
  - Search for new k'-trusses in previous graph
- Repeat until a truss is found at k but none at k+1
- k is increased and decreased according to a binary search strategy



#### Finding the most dense truss in a graph – Flink issues

- Find a k-truss with user-defined k
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- Else
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  - Search for new k'-trusses in previous graph
- Repeat until a truss is found at k but none at k+1 Flink Iteration?
- k is increased and decreased according to a binary search strategy



#### Finding the most dense truss in a graph – Flink issues

- Find a k-truss with user-defined k
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  - Search for new k'-trusses in previous graph
- Repeat until a truss is found at k but none at k+1 Flink Iteration? → No nested iterations!
  - → while loop
- k is increased and decreased according to a binary search strategy



#### Finding the most dense truss in a graph – Flink issues

- Find a k-truss with user-defined k
- If at least one truss was found requires count → Data Sink
  - set new k' > k
  - Search for new k'-trusses in the found trusses
- Else
  - Set new k' < k
  - Search for new k'-trusses in previous graph
- Repeat until a truss is found at k but none at k+1
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#### Finding the most dense truss in a graph – Flink issues

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- Repeat until a truss is found at k but none at k+1
- k is increased and decreased according to a binary search strategy

#### Solution 1:

- Accept that we lose previous result
- Recalculate with new k on initial graph

#### Solution 2:

- Write previous result to disk
- Read at the start of every new iteration



#### Finding the most dense truss in a graph – Flink issues

- Find a k-truss with user-defined k
- If at least one truss was found requires count → Data Sink
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#### Solution 1:

- Accept that we lose previous result
- Recalculate with new k on initial graph

#### Solution 2:

- Write previous result to disk
- Read at the start of every new iteration

Speedup factor =  $\sim$ 3

• k is increased and decreased according to a binary search strategy

### Evaluation – Conditions

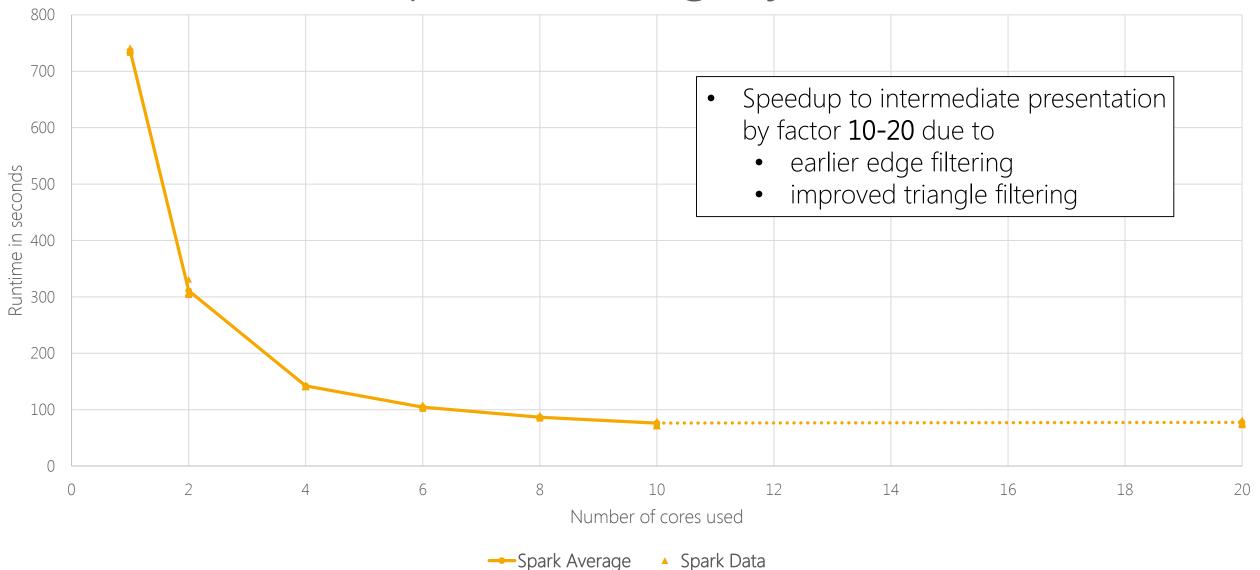


#### HPI IS chair cluster

- Master: 1 Dell PowerEdge R310
  - 4(8)x2.66 GHz
  - 8 GB DDR3 RAM
- Slaves: 10 Dell OptiPlex 780
  - 2x2.6 GHz
  - using 4 GB DDR3 RAM
- Using only 1 core per slave
- k value of 20
- Bidirectional Wikipedia data set
- Averaged 5 measurements (Unless otherwise noted)

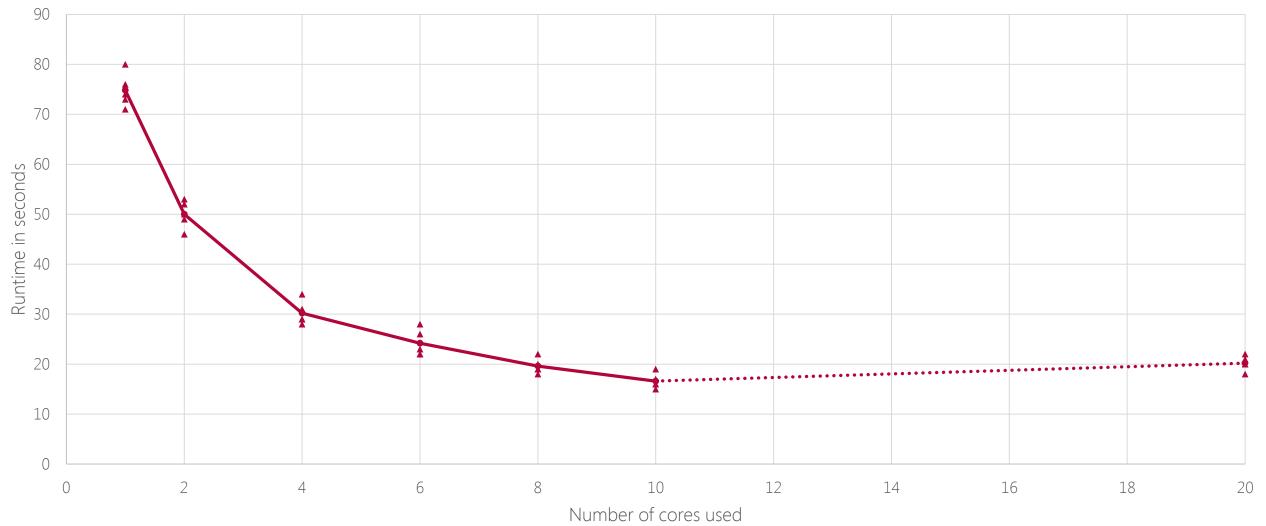
# Evaluation – Spark scaling by #cores





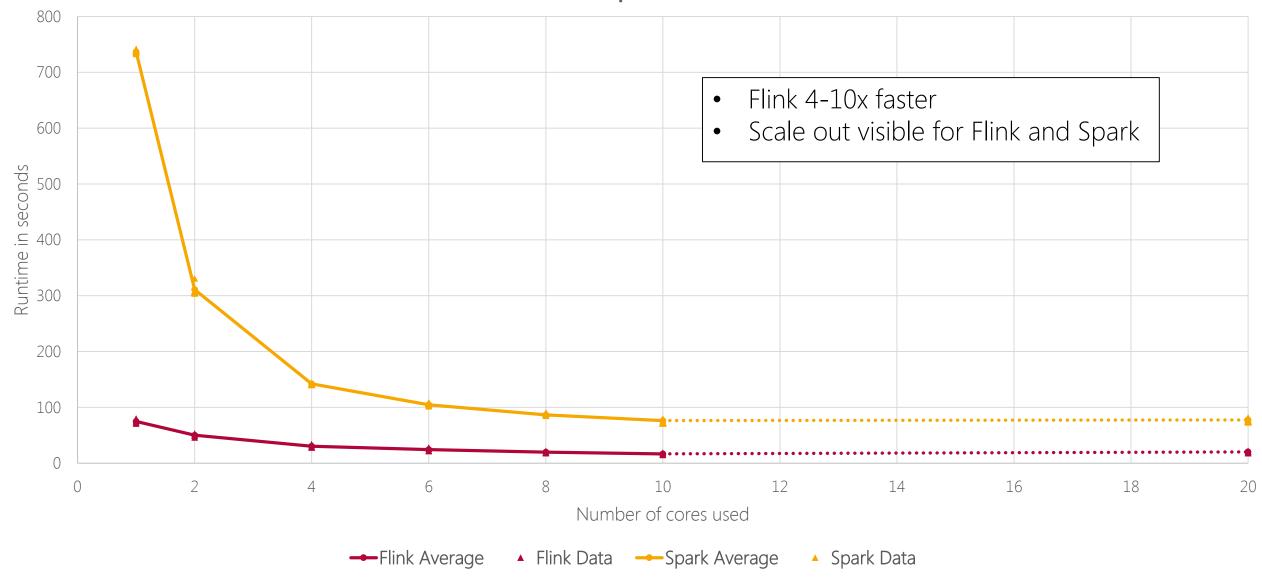
# Evaluation – Flink scaling by #cores





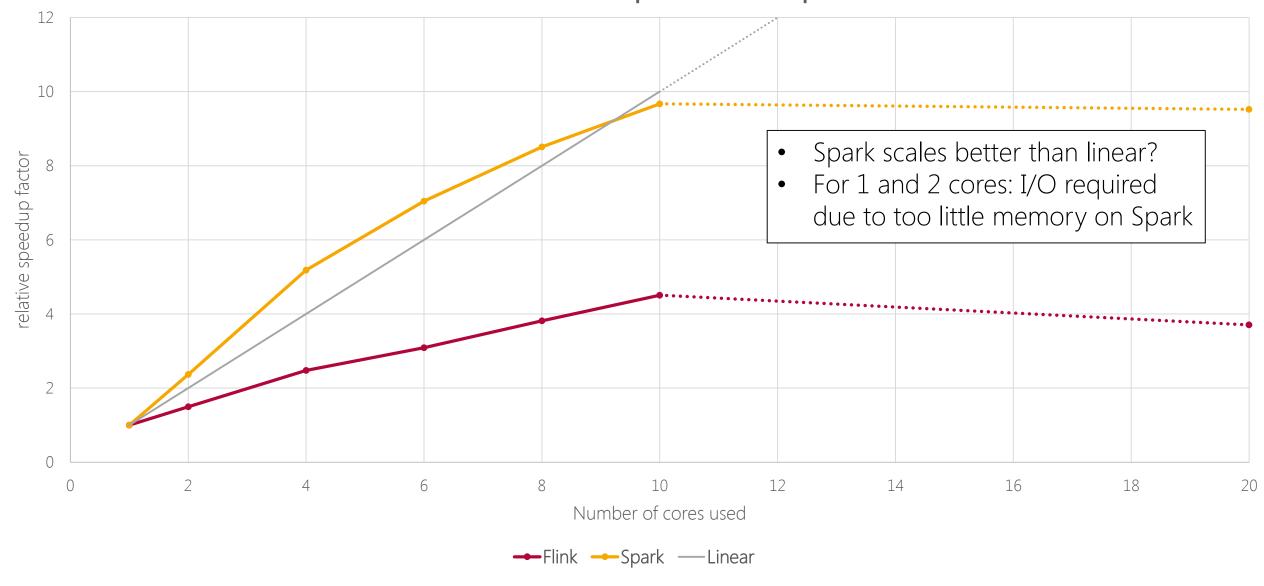
# Evaluation – Flink vs Spark





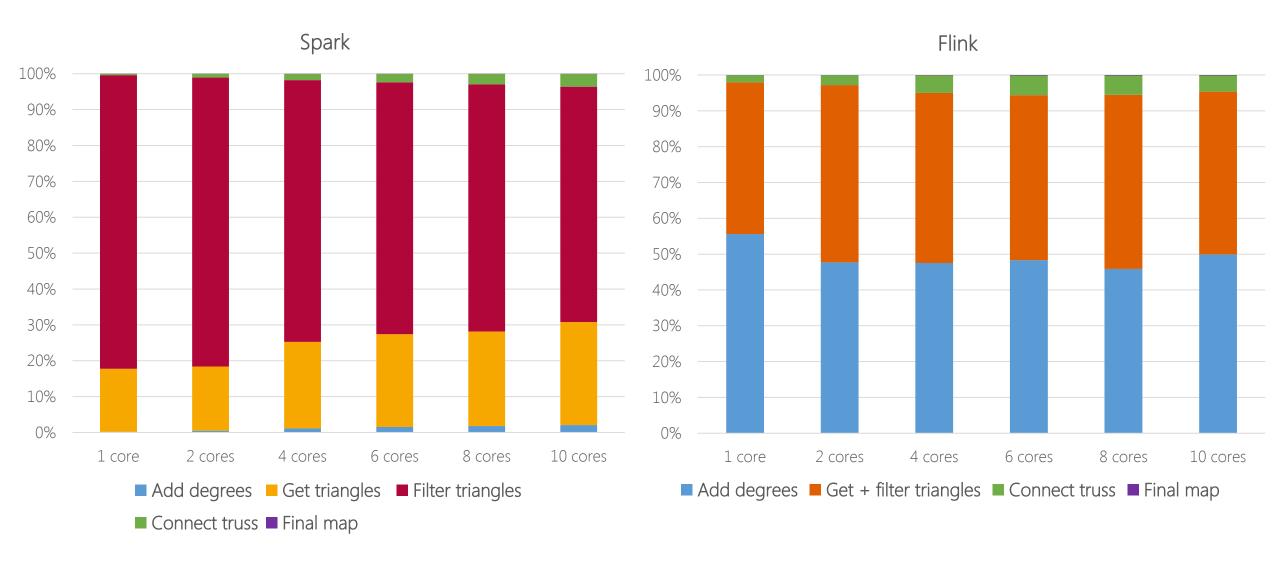
# Evaluation – Relative Speedup





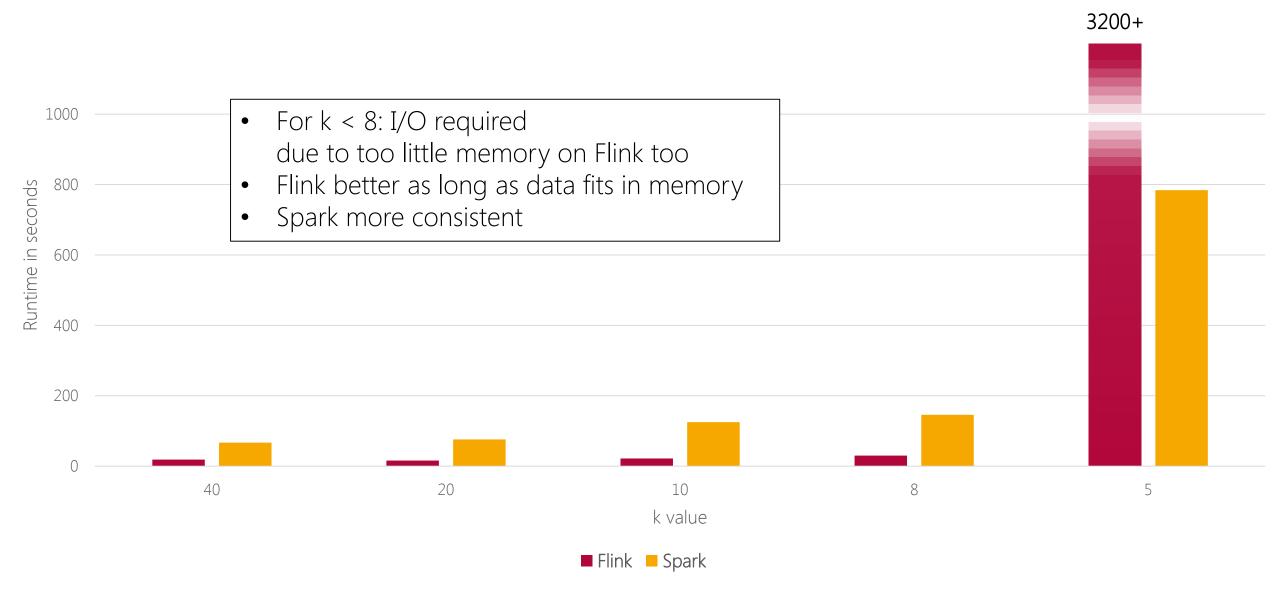
# Evaluation – Program Parts





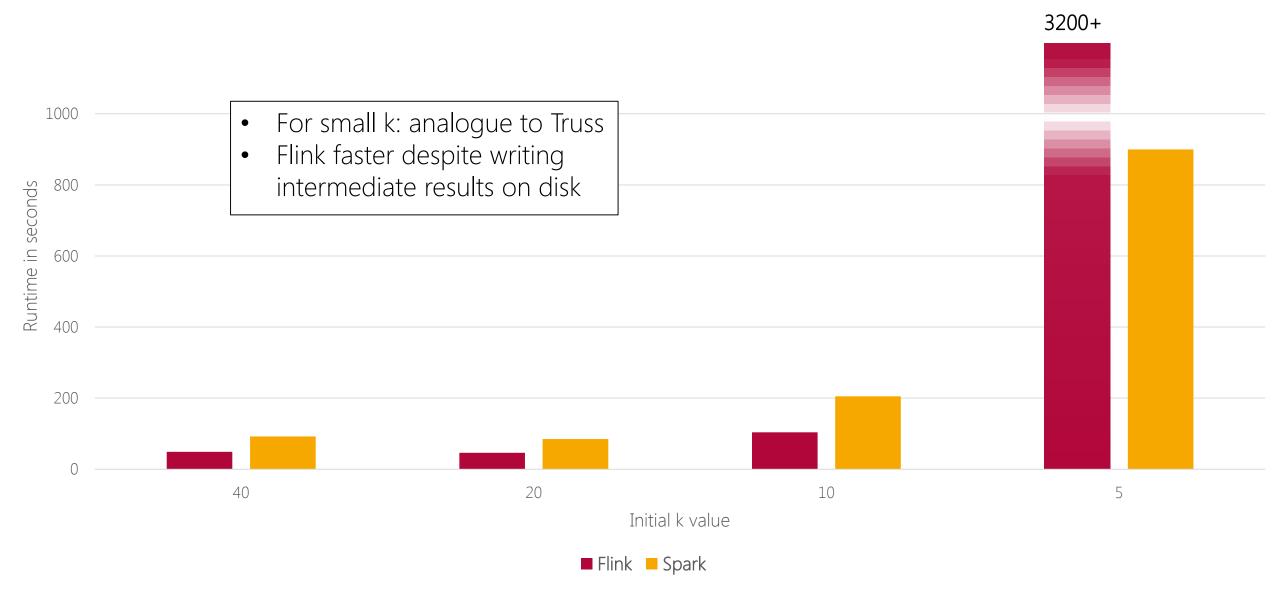
### Evaluation — Truss





### Evaluation – Maximal Truss





### Conclusions



#### Distributed Calculation

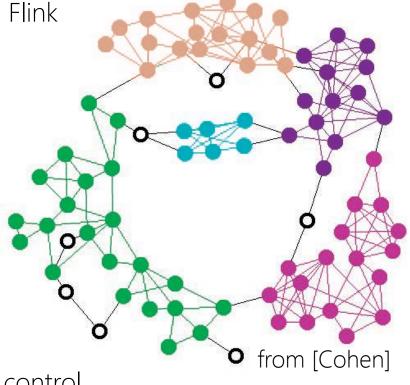
• Great scaling with distribution over multiple machines for Spark and Flink

#### • Flink

- Can hold more data in main memory due to different serialization
- As long as the data fits in main memory, Flink is also faster
- Speedup due to more efficient iterations
- Could be improved even more by handling nested iterations

#### Spark

- Can deal much better with full main memory due to improved user control
- More consistent performance



#### References



Image on title slide: http://polkadotimpressions.com/2013/01/18/facebook-graph-search-3/

[Bron, Kerbosh]: Bron, Coen, and Joep Kerbosch. 'Algorithm 457: finding all cliques of an undirected graph.' *Communications of the ACM* 16, no. 9 (1973): 575-577.

[Cohen]: Jonathan Cohen, 'Graph Twiddling in a MapReduce World'. in *Computing in Science and Engineering* 11(4): 29-41 (2009)



#### Finding the most dense truss in a graph

- Find a k-truss with user-defined k
- If at least one truss was found
  - set new k' > k
  - Search for new k'-trusses in the found trusses
- If no trusses were found
  - Set new k' < k
  - Search for new k'-trusses in previous graph
- Repeat until a truss is found at k but none at k+1
- k is increased and decreased according to a binary search strategy

True Max Truss at k = 28Initial k = 20

k = 20 Truss found

k = 40 No truss found

k = 30 No truss found

k = 25 Truss found

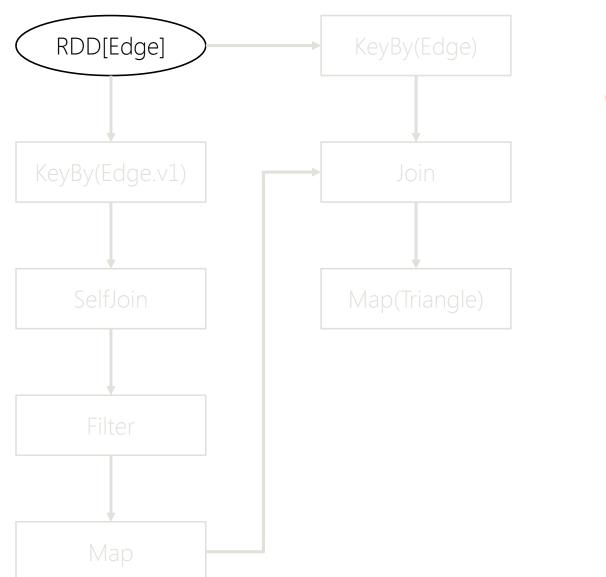
k = 27 Truss found

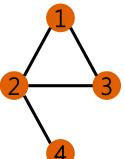
k = 28 Truss found

k = 29 No truss found

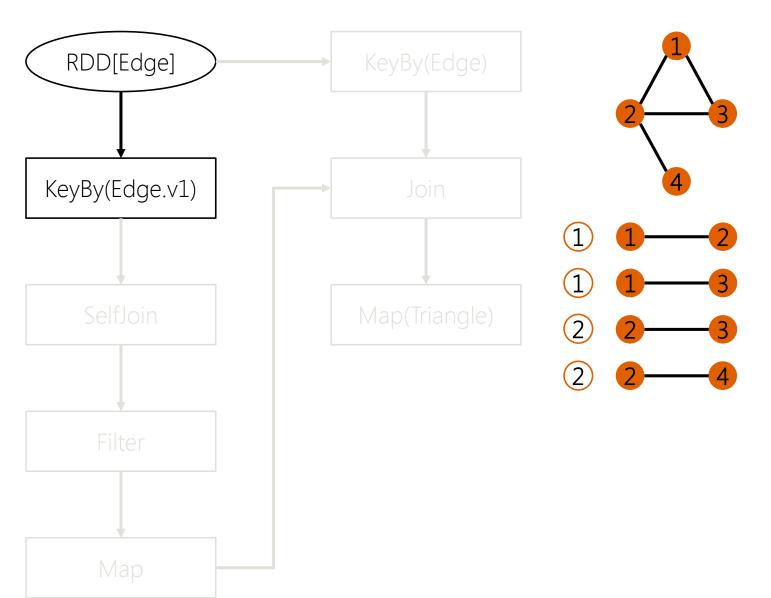
Done. k = 28





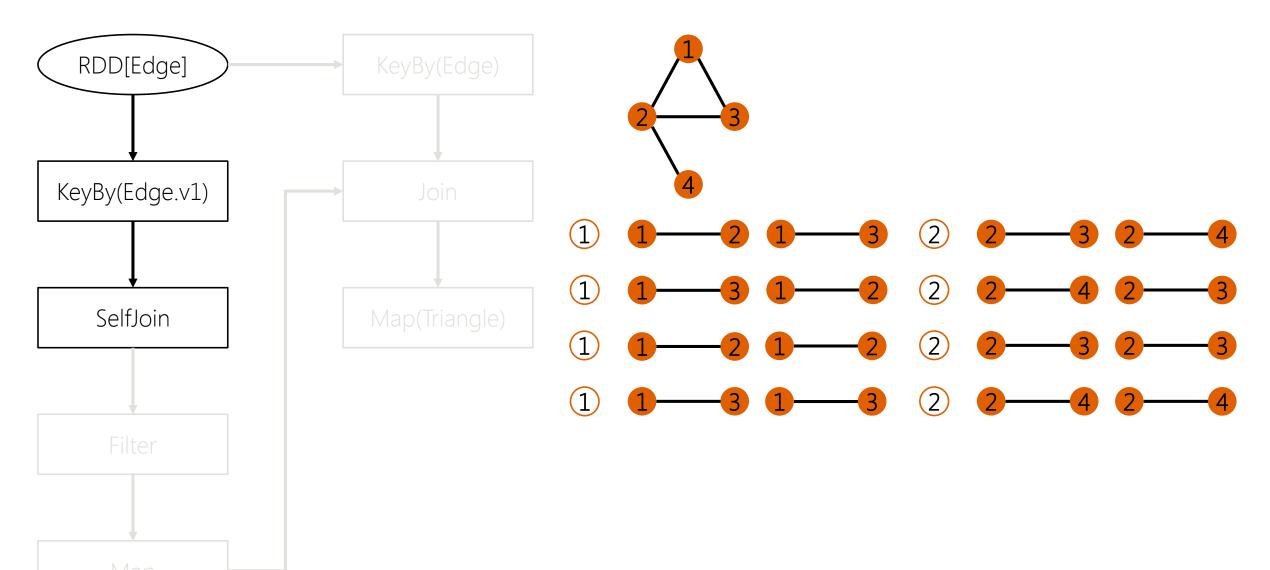




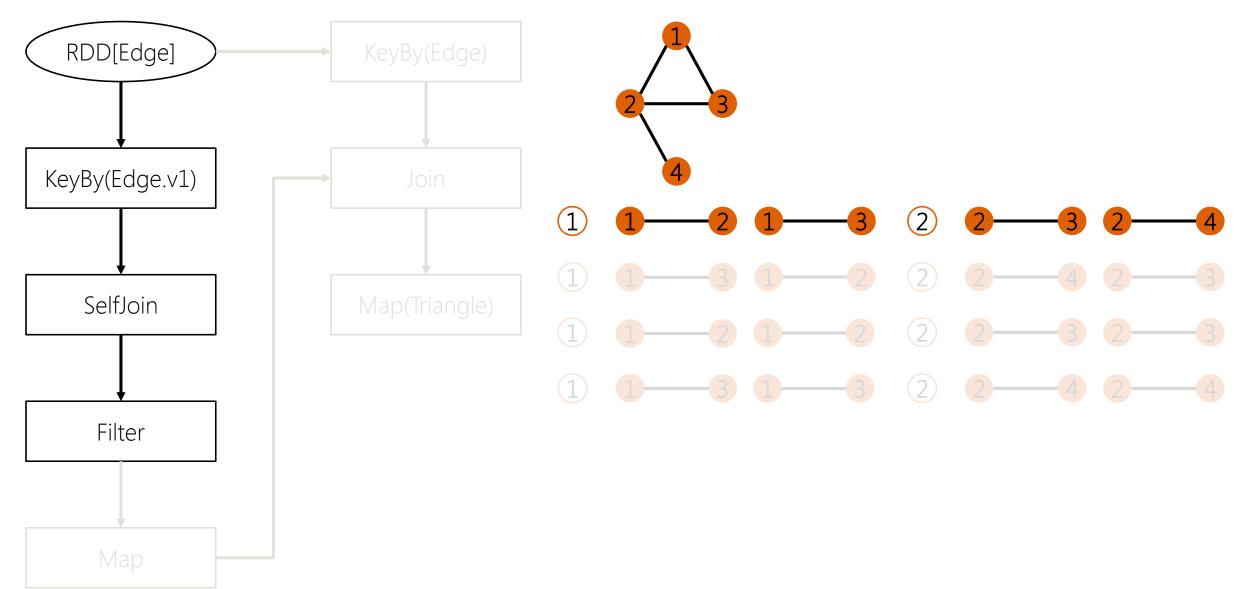


7/12/2015

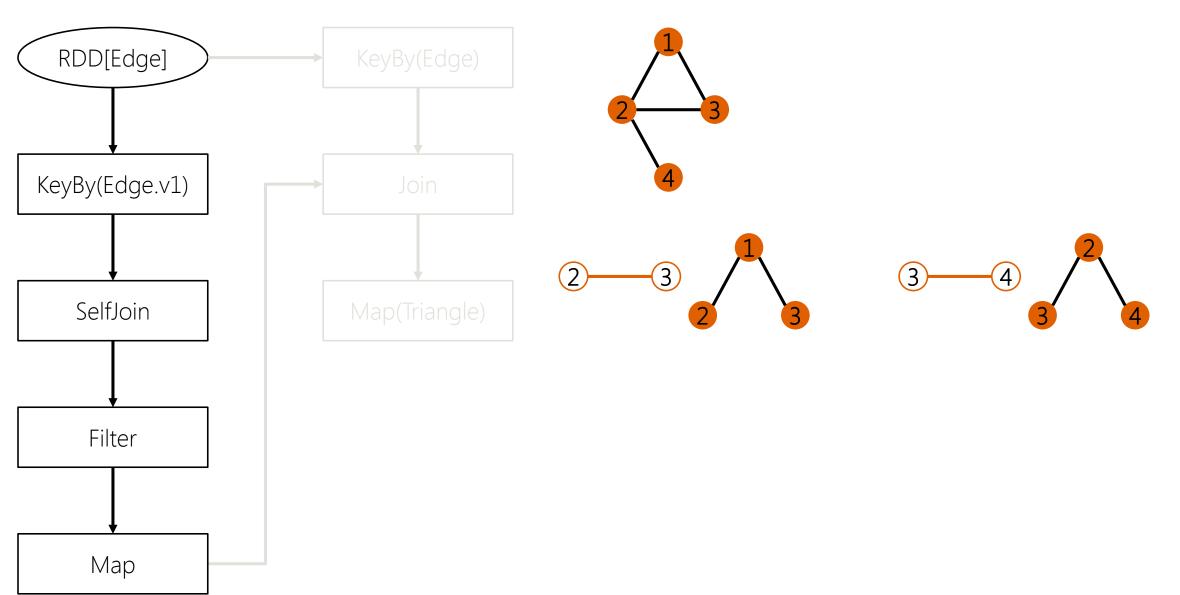




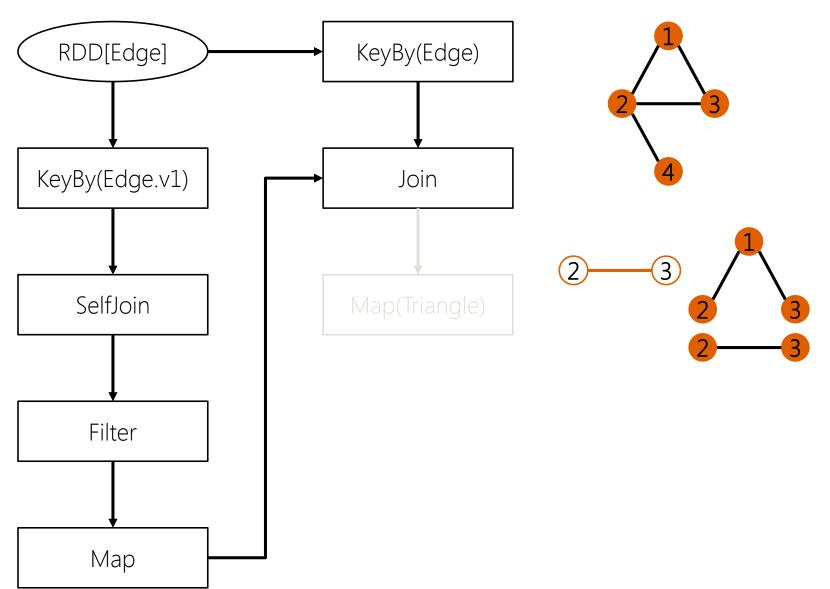




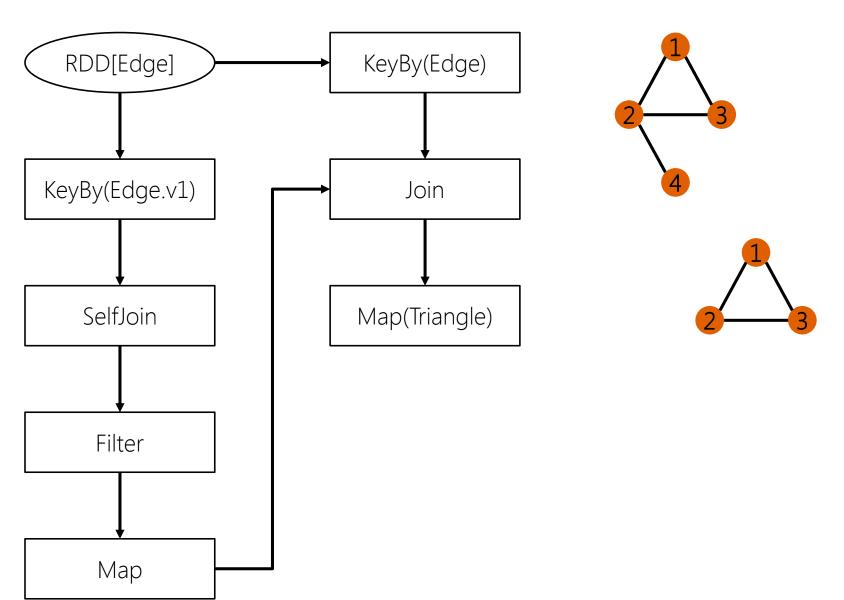






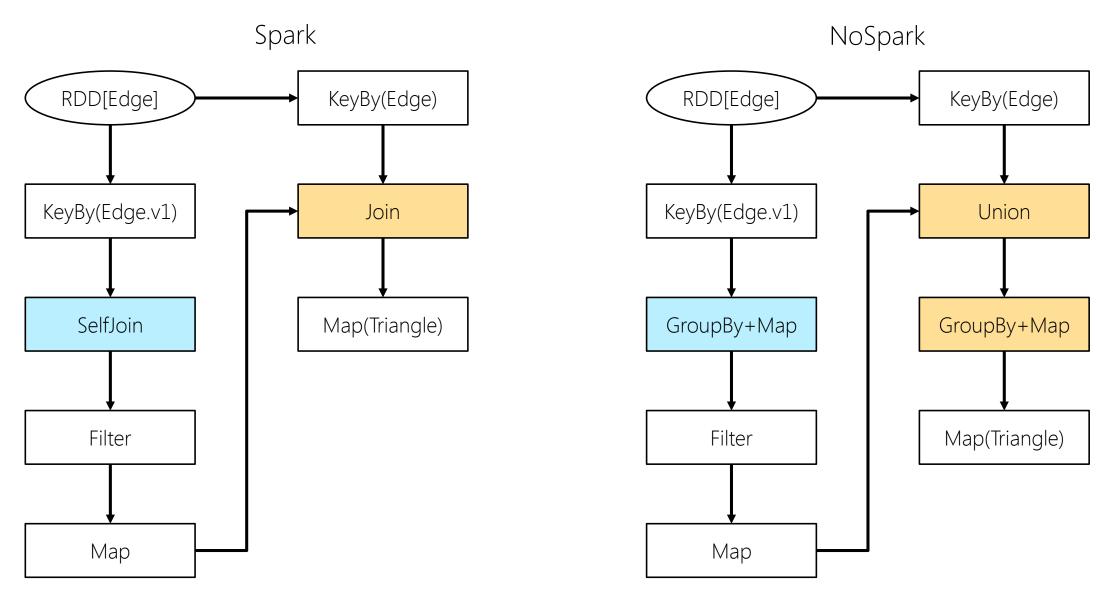






# Evaluation – Triangle Generation





## Evaluation – Triangle Generation



#### By number of cores used

- 1 core x 5 machines
  - Spark 14 minutes
  - NoSpark 12 minutes
- 1 core x 10 machines
  - Spark 6.8 minutes
  - NoSpark 5.7 minutes
- 2 cores x 10 machines
  - Spark 6.5 minutes
  - NoSpark 4.1 minutes

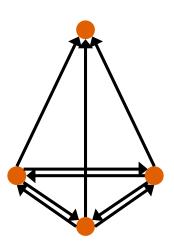
using 4GB RAM

### Direction?



#### Two possibilities

- Any direction
  - Accept trusses where either direction of an edge exists
- Both directions
  - Accept trusses only when both directions of an edge exist

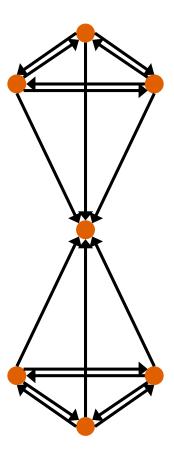


### Direction?



#### Two possibilities

- Any direction
  - Accept trusses where either direction of an edge exists
- Both directions
  - Accept trusses only when both directions of an edge exist



#### Direction?

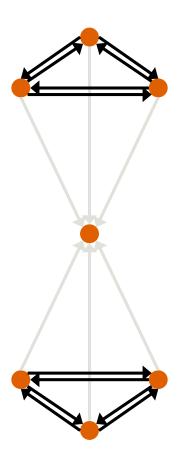


#### Two possibilities

- Any direction
  - Accept trusses where either direction of an edge exists
- Both directions
  - Accept trusses only when both directions of an edge exist

#### **Decision**

- Bidirectional edges only
- Only nodes that actually interact with one another should form a truss
- → Can use pre-processing step to create graph with bidirectional edges only



## Finding the maximum Truss



- 1. Create new graph from bidirectional edges only
- 2. Set k = arbitrary value, subraphs = (fullGraph)
- 3. Find all k-trusses for each subgraph *after [Cohen]*
- 4. If none exist:
- 5. Reduce k, go to 3.
- 6. Set subgraphs = (truss1, truss2, ...)
- 7. Increase k, go to 3.

Abort if k has already been seen before

(Increase or reduce k according to a binary search strategy)

# Evaluation – starting k



#### Real maxTrussSize = 28

- k = 10 17 minutes
  - k values tried: 10, 20, 40, 30, 25, 27, 28, 29, 28
- k = 20 11 minutes
  - k values tried: 20, 40, 30, 25, 27, 28, 29, 28
- k = 28 10 minutes
  - k values tried: 28, 56, 42, 35, 31, 29, 28
- k = 40 20 minutes
  - k values tried: 40, 21, 30, 25, 27, 28, 29, 28

Using 10 machines, 20 cores, 4GB RAM