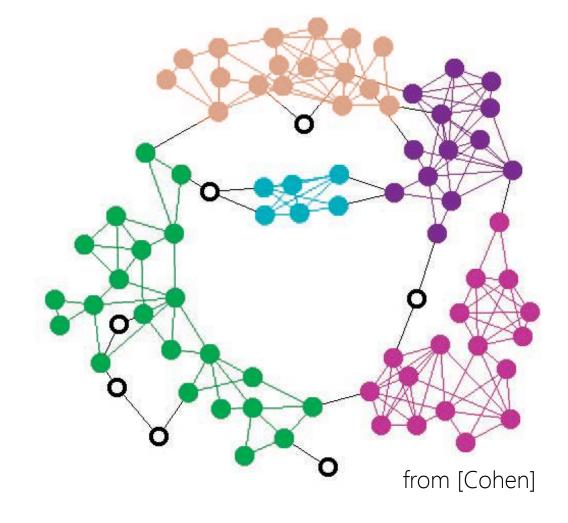


Problem



Finding highly connected sub-graphs

- Why is this important?
 - Social media graphs: groups of friends/family/co-workers
 - Website interliking
- Why is this difficult?
 - Possible solution set size: $2^{|V|}$
 - → Exponential runtime for naive approach
 - Often millions of vertices
 - \rightarrow \otimes



The Data



Wikipedia

- Directed graph of English Wikipedia page interlinks from 2007
- ~1.9 million vertices, ~40 million edges, 1 GB size on disc

• Bidirectional version with 3.4 million edges, 54 MB

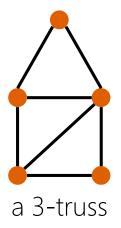


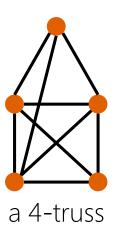
k-Trusses



k-Truss

- **Definition**: a maximal subgraph so that every edge is part of at least k-2 triangles
- Indicates a high density and high connectivity between its nodes
- Can be seen as a relaxation of the clique problem (= fully connected subgraph)

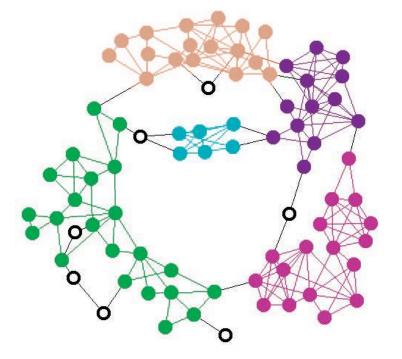




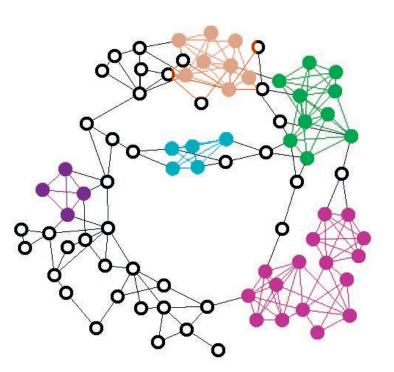
k-Trusses



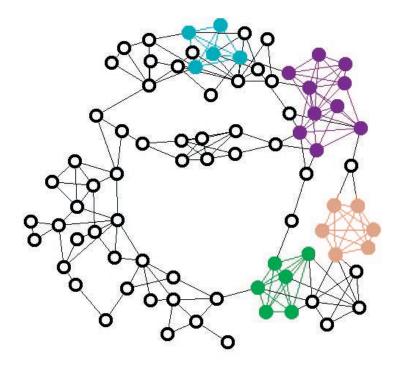
k = 3



k = 4



k = 5

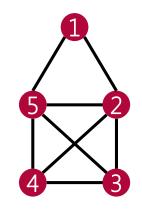


from [Cohen]



Read graph

Find 4-truss





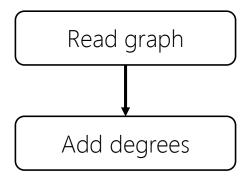




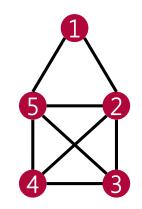








Find 4-truss



2 1 2 4

4 2 --- 5 4

2 1 5

3 3 4 3

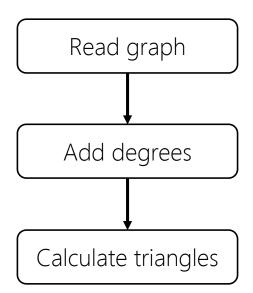
4 **2-3** 3

3 3 4

4 2 4 3

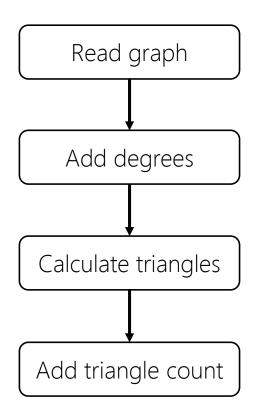
3 4 --- 5 4

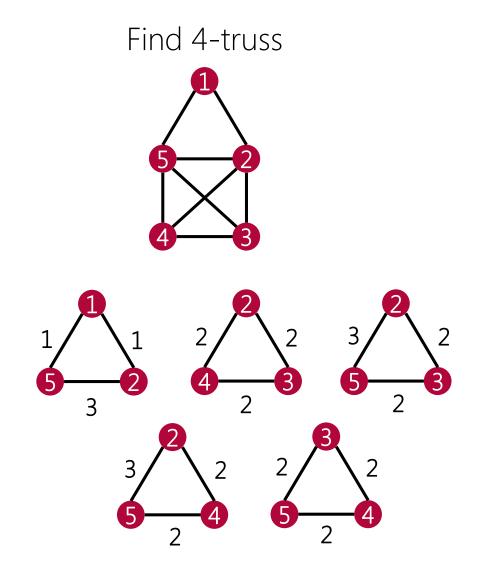




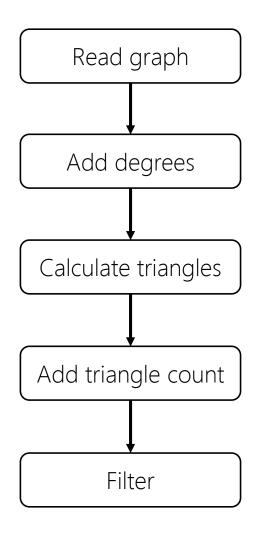
Find 4-truss

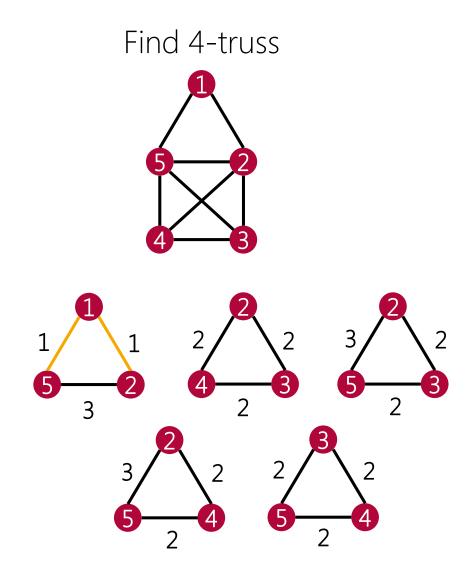




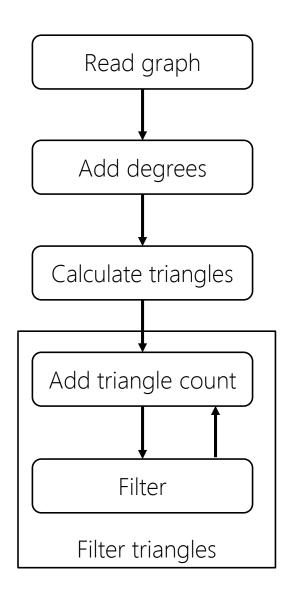


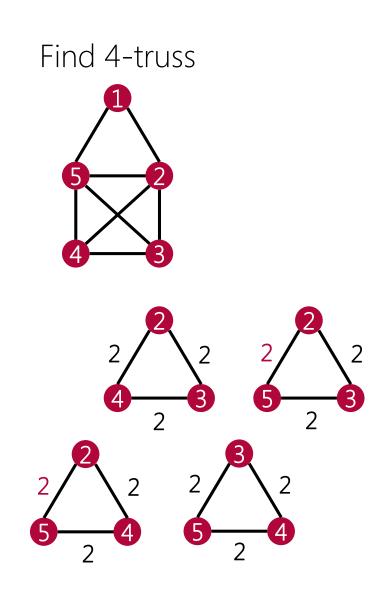




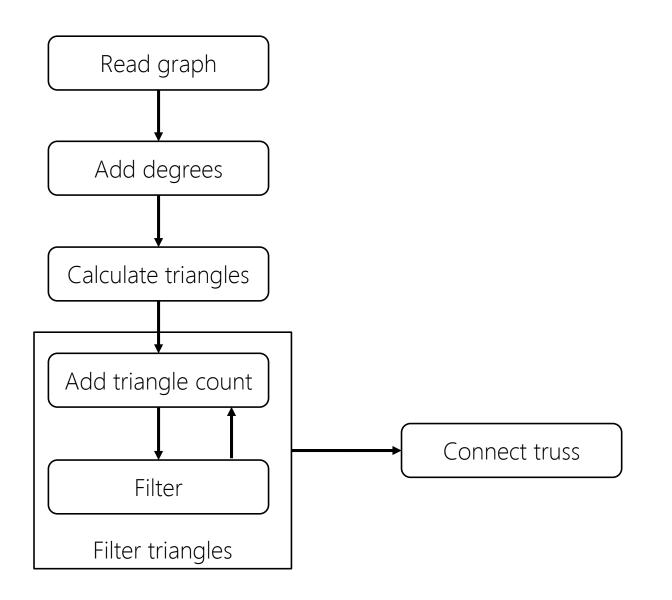




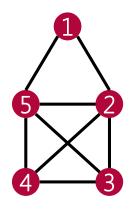


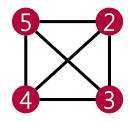






Find 4-truss





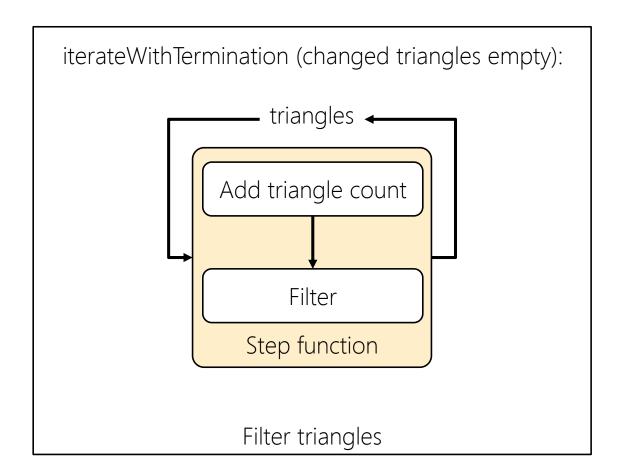
Implementation – Filter Triangles



Spark

while (number of triangles has changed): Add triangle count Filter Filter triangles

Flink





Finding the most dense truss in a graph

- Find a k-truss with user-defined k
- If at least one truss was found
 - set new k' > k
 - Search for new k'-trusses in the found trusses
- Else
 - Set new k' < k
 - Search for new k'-trusses in previous graph
- Repeat until a truss is found at k but none at k+1
- k is increased and decreased according to a binary search strategy



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True Max Truss at k = 28Initial k = 20

k = 20 Truss found

k = 40 No truss found

k = 30 No truss found

k = 25 Truss found

k = 27 Truss found

k = 28 Truss found

k = 29 No truss found

Done. k = 28



Finding the most dense truss in a graph – Flink issues

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- Else
 - Set new k' < k
 - Search for new k'-trusses in previous graph
- Repeat until a truss is found at k but none at k+1 Flink Iteration?
- k is increased and decreased according to a binary search strategy



Finding the most dense truss in a graph – Flink issues

- Find a k-truss with user-defined k
- If at least one truss was found
 - set new k' > k
 - Search for new k'-trusses in the found trusses
- Else
 - Set new k' < k
 - Search for new k'-trusses in previous graph
- Repeat until a truss is found at k but none at k+1 Flink Iteration? → No nested iterations!
 - → while loop
- k is increased and decreased according to a binary search strategy



Finding the most dense truss in a graph – Flink issues

- Find a k-truss with user-defined k
- If at least one truss was found requires count → Data Sink
 - set new k' > k
 - Search for new k'-trusses in the found trusses
- Else
 - Set new k' < k
 - Search for new k'-trusses in previous graph
- Repeat until a truss is found at k but none at k+1
- k is increased and decreased according to a binary search strategy



Finding the most dense truss in a graph – Flink issues

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- k is increased and decreased according to a binary search strategy

Solution 1:

- Accept that we lose previous result
- Recalculate with new k on initial graph

Solution 2:

- Write previous result to disk
- Read at the start of every new iteration



Finding the most dense truss in a graph – Flink issues

- Find a k-truss with user-defined k
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Solution 1:

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- Recalculate with new k on initial graph

Solution 2:

- Write previous result to disk
- Read at the start of every new iteration

Speedup factor = \sim 3

• k is increased and decreased according to a binary search strategy

Evaluation – Conditions

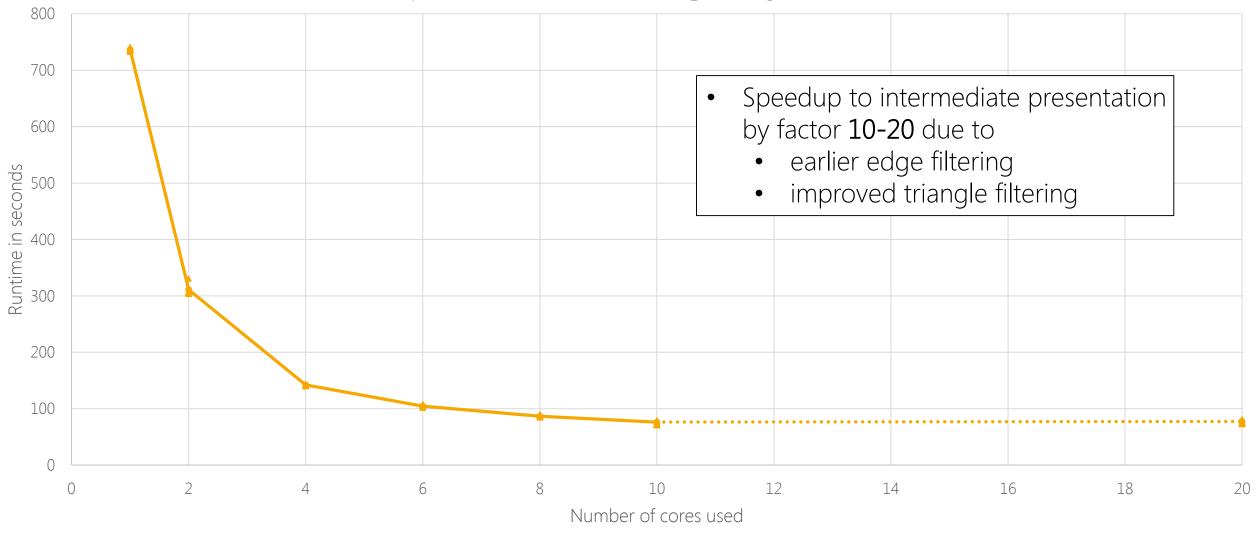


HPI IS chair cluster

- Master: 1 Dell PowerEdge R310
 - 4(8)x2.66 GHz
 - 8 GB DDR3 RAM
- Slaves: 10 Dell OptiPlex 780
 - 2x2.6 GHz
 - using 4 GB DDR3 RAM
- Using only 1 core per slave
- k value of 20
- Bidirectional Wikipedia data set
- Averaged 5 measurements (Unless otherwise noted)

Evaluation – Spark scaling by #cores



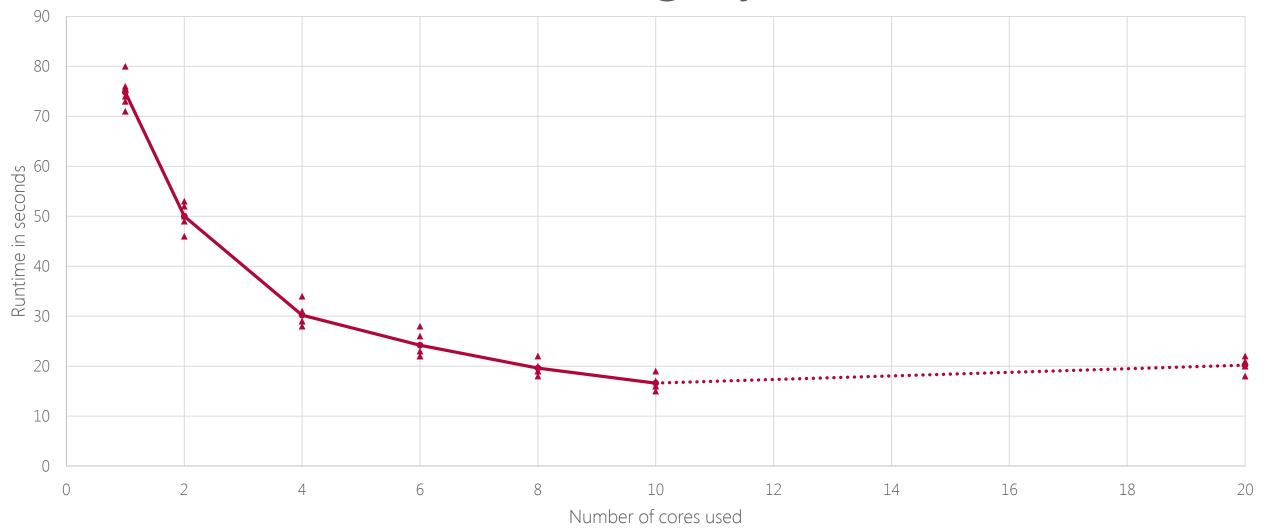


→ Spark Average

Spark Data

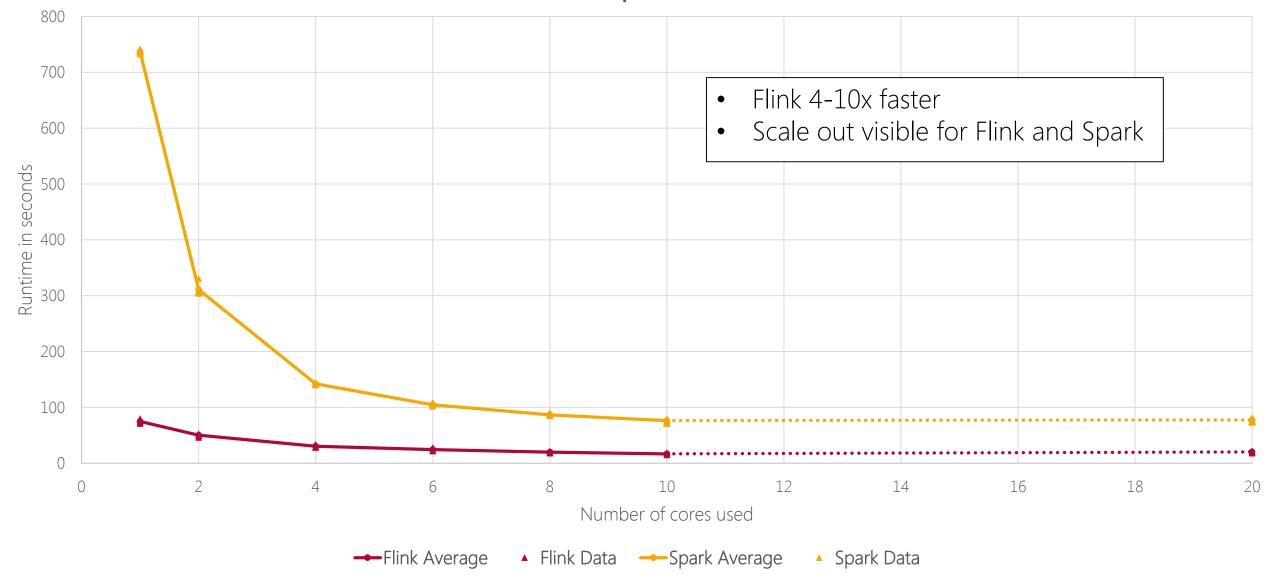
Evaluation – Flink scaling by #cores





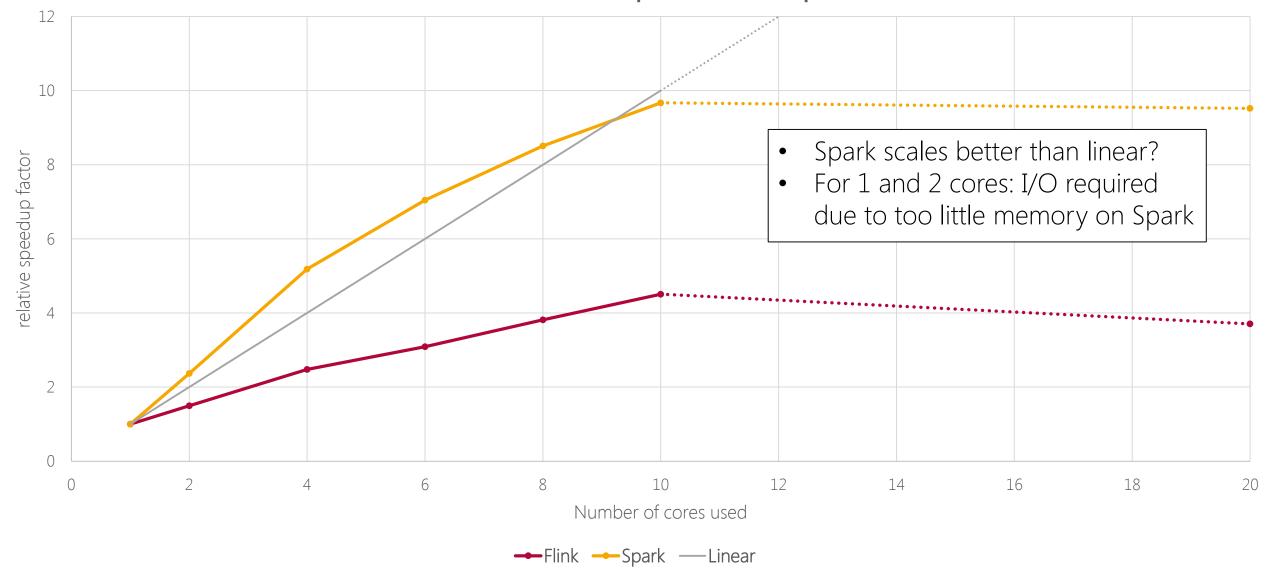
Evaluation – Flink vs Spark





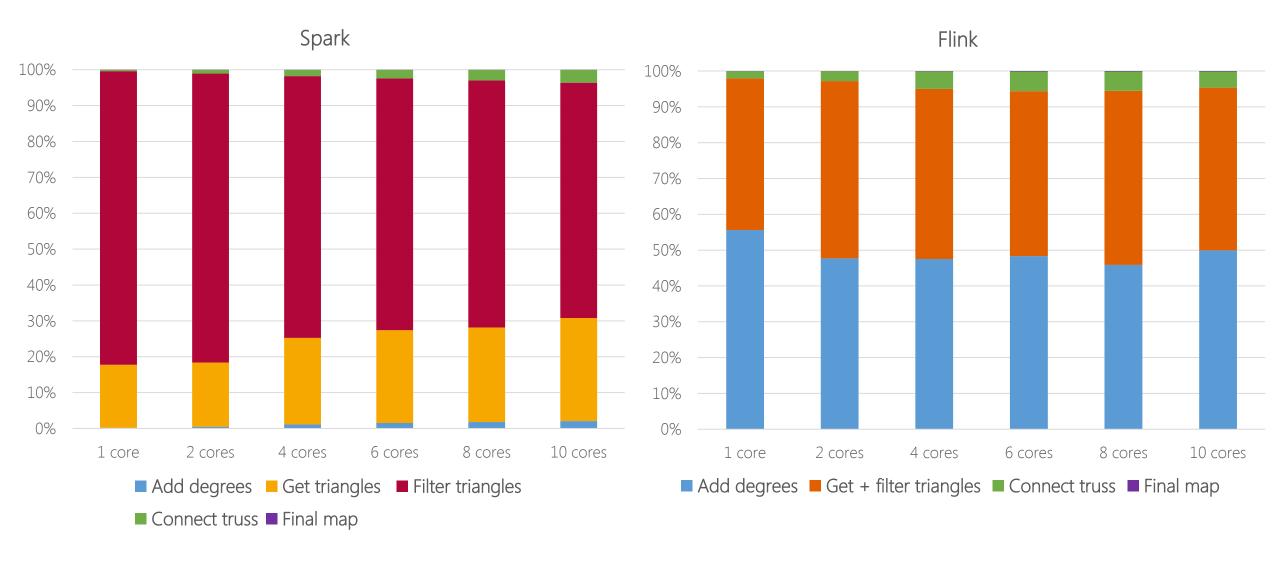
Evaluation – Relative Speedup





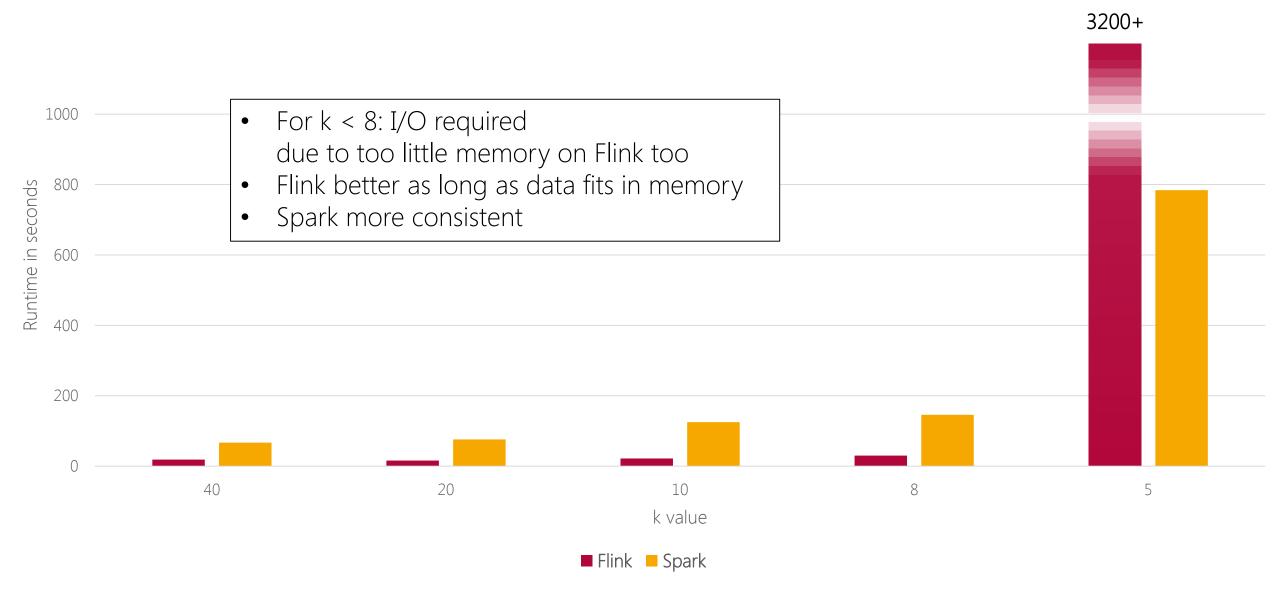
Evaluation – Program Parts





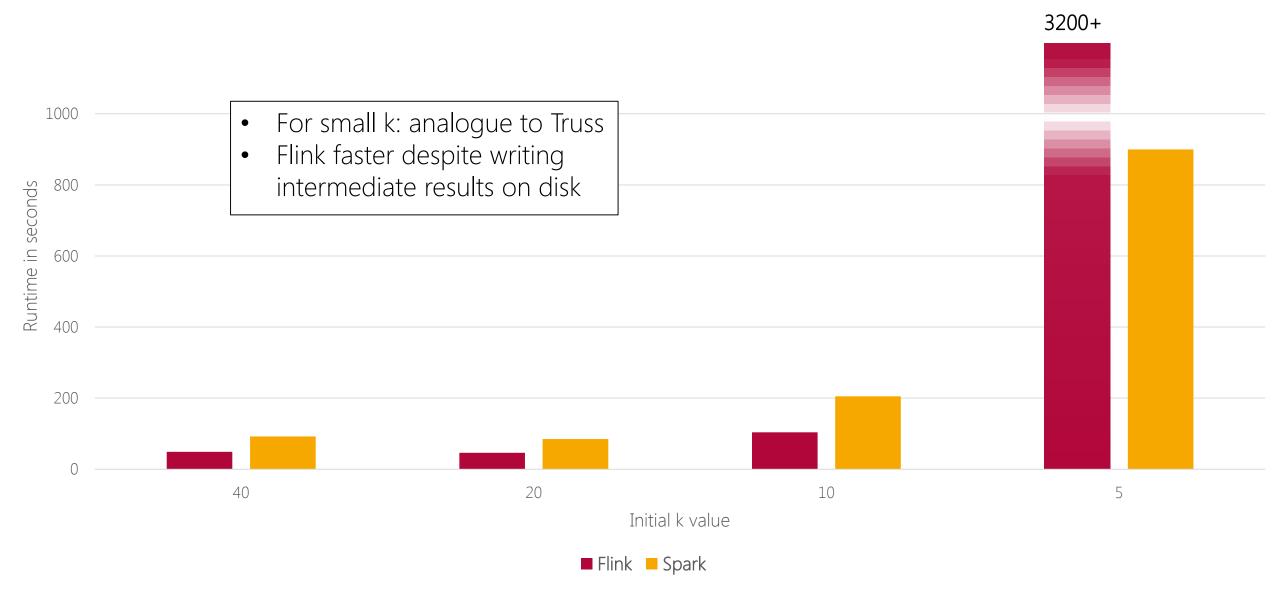
Evaluation — Truss





Evaluation – Maximal Truss





Conclusions



Distributed Calculation

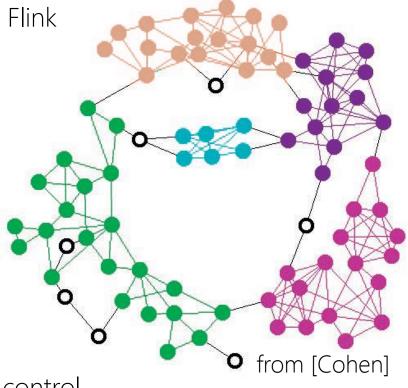
• Great scaling with distribution over multiple machines for Spark and Flink

• Flink

- Can hold more data in main memory due to different serialization
- As long as the data fits in main memory, Flink is also faster
- Speedup due to more efficient iterations
- Could be improved even more by handling nested iterations

Spark

- Can deal much better with full main memory due to improved user control
- More consistent and thus predictable performance



References

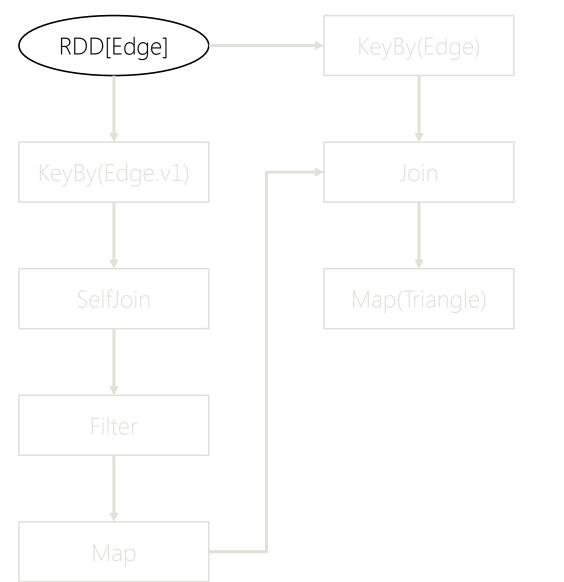


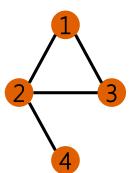
Image on title slide: http://polkadotimpressions.com/2013/01/18/facebook-graph-search-3/

[Bron, Kerbosh]: Bron, Coen, and Joep Kerbosch. 'Algorithm 457: finding all cliques of an undirected graph.' *Communications of the ACM* 16, no. 9 (1973): 575-577.

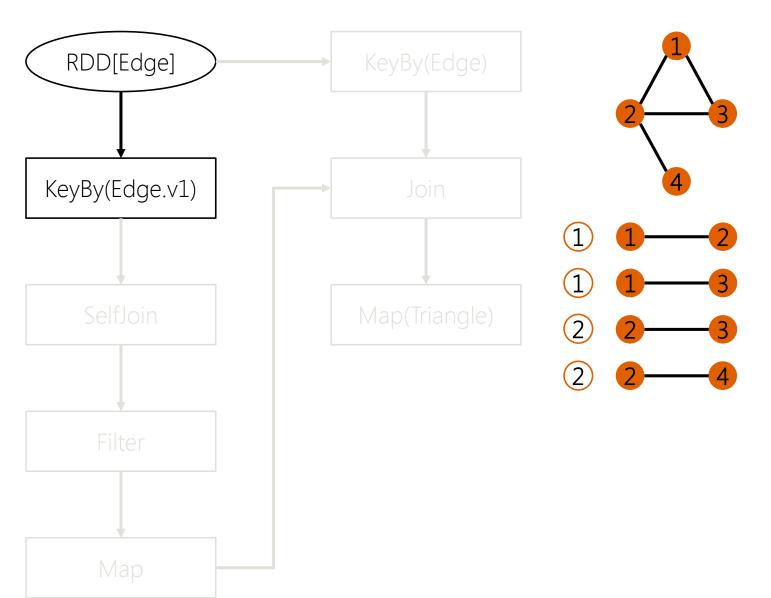
[Cohen]: Jonathan Cohen, 'Graph Twiddling in a MapReduce World'. in *Computing in Science and Engineering* 11(4): 29-41 (2009)





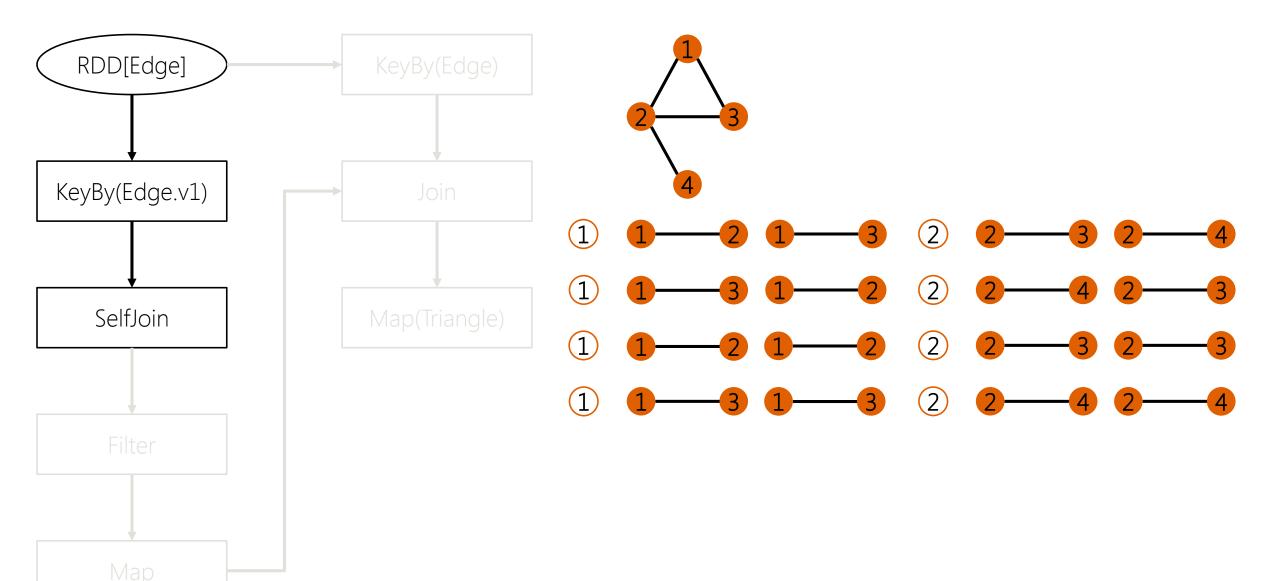




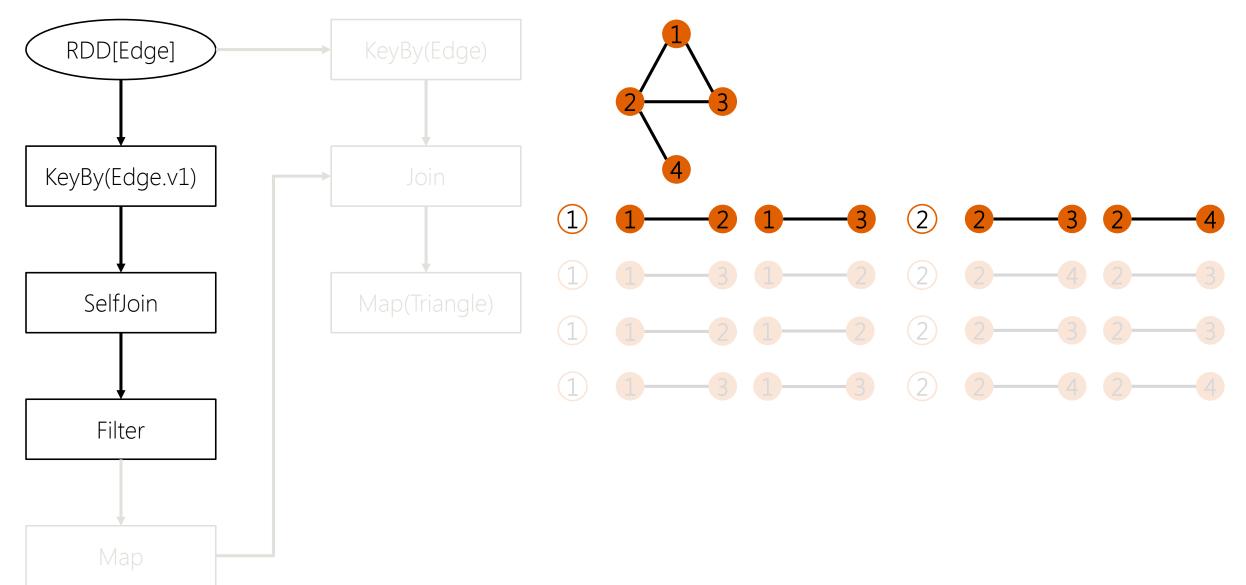


7/13/2015

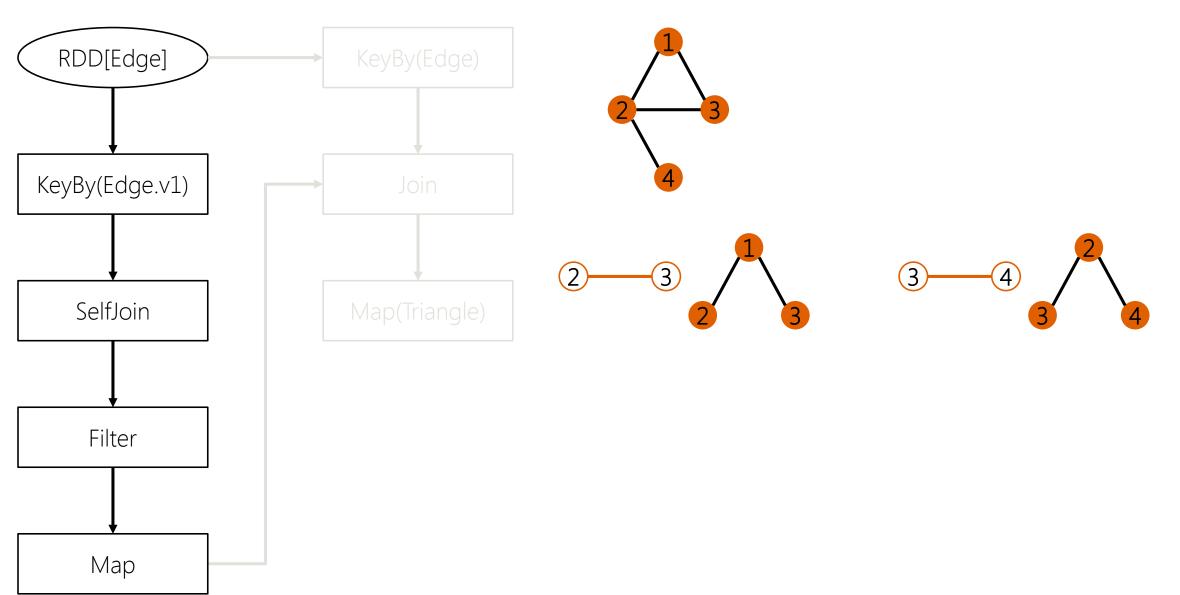




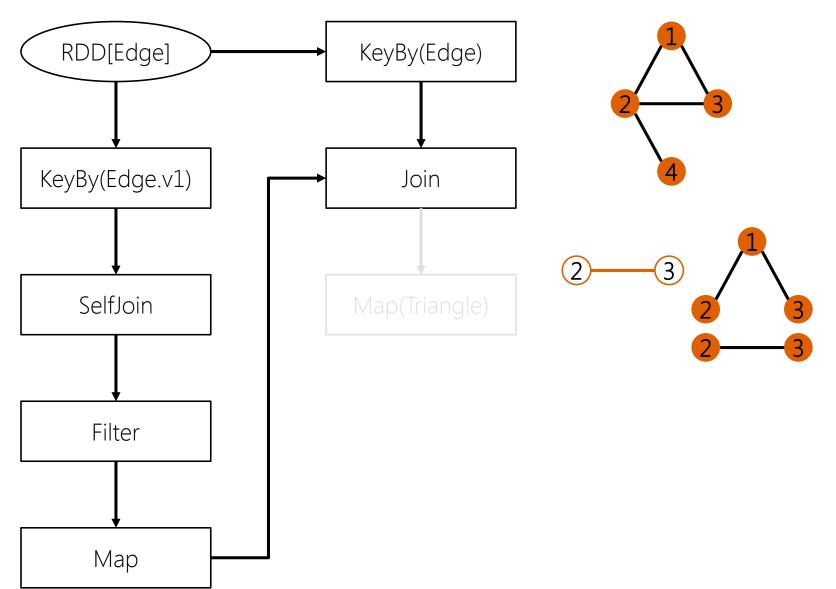




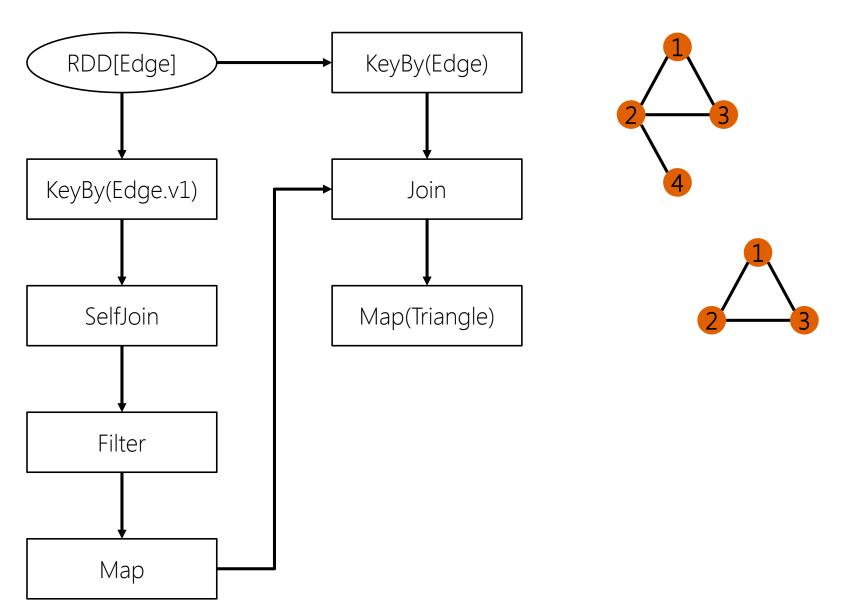






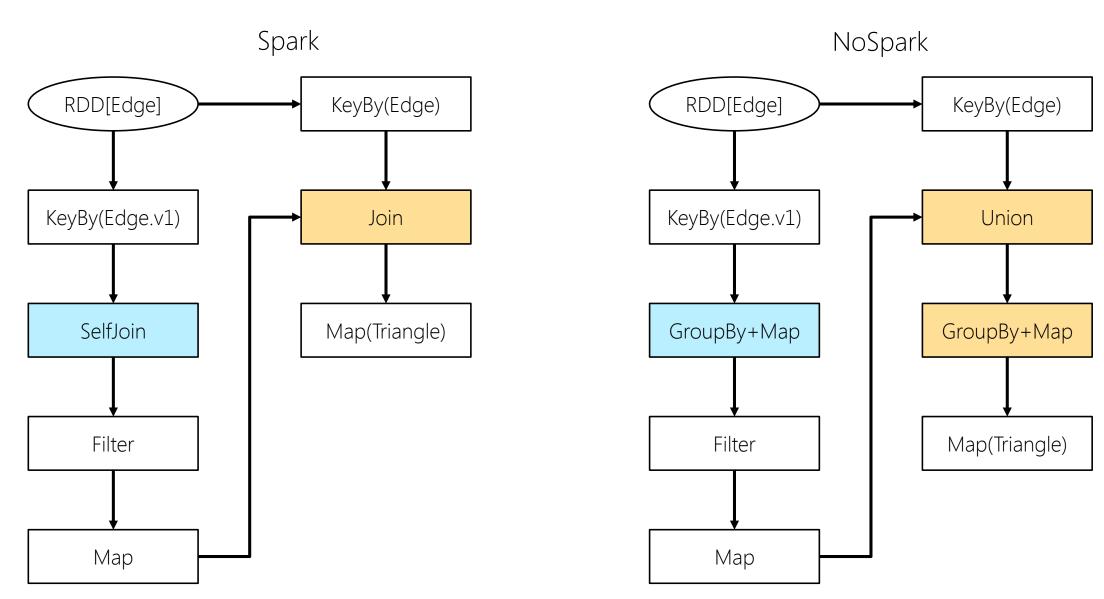






Evaluation – Triangle Generation





Evaluation – Triangle Generation



By number of cores used

- 1 core x 5 machines
 - Spark 14 minutes
 - NoSpark 12 minutes
- 1 core x 10 machines
 - Spark 6.8 minutes
 - NoSpark 5.7 minutes
- 2 cores x 10 machines
 - Spark 6.5 minutes
 - NoSpark 4.1 minutes

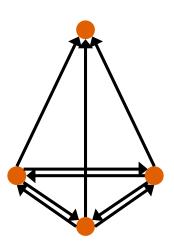
using 4GB RAM

Direction?



Two possibilities

- Any direction
 - Accept trusses where either direction of an edge exists
- Both directions
 - Accept trusses only when both directions of an edge exist

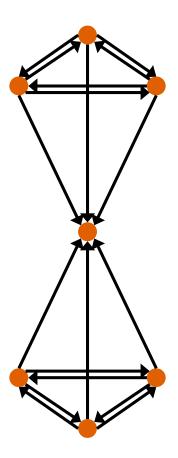


Direction?



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Direction?

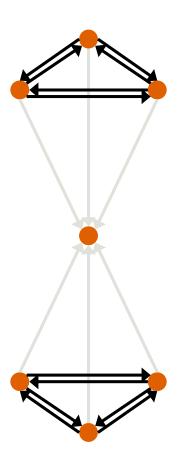


Two possibilities

- Any direction
 - Accept trusses where either direction of an edge exists
- Both directions
 - Accept trusses only when both directions of an edge exist

Decision

- Bidirectional edges only
- Only nodes that actually interact with one another should form a truss
- → Can use pre-processing step to create graph with bidirectional edges only



Finding the maximum Truss



- 1. Create new graph from bidirectional edges only
- 2. Set k = arbitrary value, subraphs = (fullGraph)
- 3. Find all k-trusses for each subgraph *after* [Cohen]
- If none exist:
- 5. Reduce k, go to 3.
- 6. Set subgraphs =(truss1, truss2, ...)
- 7. Increase k, go to 3.

Abort if k has already been seen before

(Increase or reduce k according to a binary search strategy)

Evaluation – starting k



Real maxTrussSize = 28

- k = 10 17 minutes
 - k values tried: 10, 20, 40, 30, 25, 27, 28, 29, 28
- k = 20 11 minutes
 - k values tried: 20, 40, 30, 25, 27, 28, 29, 28
- k = 28 10 minutes
 - k values tried: 28, 56, 42, 35, 31, 29, 28
- k = 40 20 minutes
 - k values tried: 40, 21, 30, 25, 27, 28, 29, 28

Using 10 machines, 20 cores, 4GB RAM