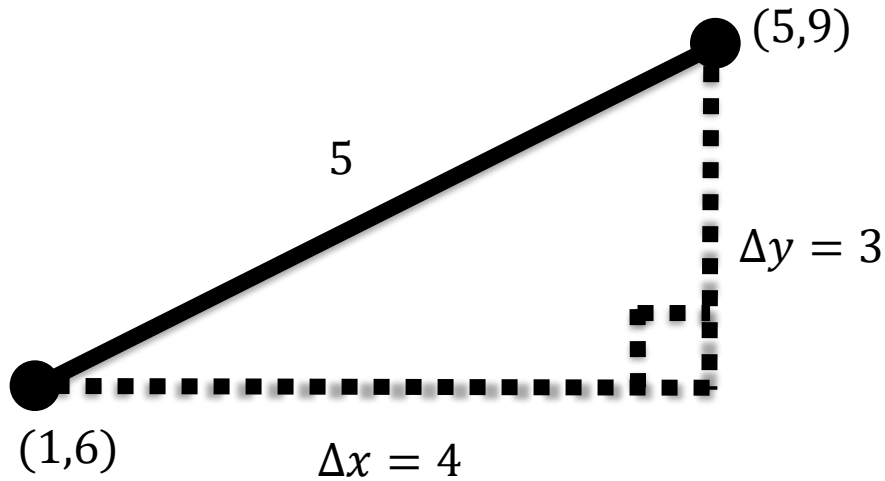

P1 Chapter 5: Linear Graphs

Length and Area

Distances between points

Recall: Δ (said 'delta') means "change in".



How could we find the **distance** between these two points?

Form a right-angled triangle using the change in x and change in y , then use *Pythagoras*.



Distance between two points:

$$\sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Examples

Distance between:

$(3,4)$ and $(5,7)$

$(5,1)$ and $(6,-3)$

$(0,-2)$ and $(-1,3)$

?
?
?

Fro Note: Unlike with gradient, we don't care if the difference is positive or negative (it's being squared to make it positive anyway!)

Quickfire Questions:

Distance between:

$(1,10)$ and $(4,14)$

$(3,-1)$ and $(0,1)$

$(-4,-2)$ and $(-12,4)$

?
?
?

Examples

Distance between:

(3,4) and (5,7)


$$\sqrt{2^2 + 3^2} = \sqrt{13}$$

(5,1) and (6, -3)

$$\sqrt{1^2 + 4^2} = \sqrt{17}$$

(0, -2) and (-1,3)

$$\sqrt{1^2 + 5^2} = \sqrt{26}$$



Fro Note: Unlike with gradient, we don't care if the difference is positive or negative (it's being squared to make it positive anyway!)

Quickfire Questions:

Distance between:

(1,10) and (4,14)

$$\sqrt{3^2 + 4^2} = 5$$

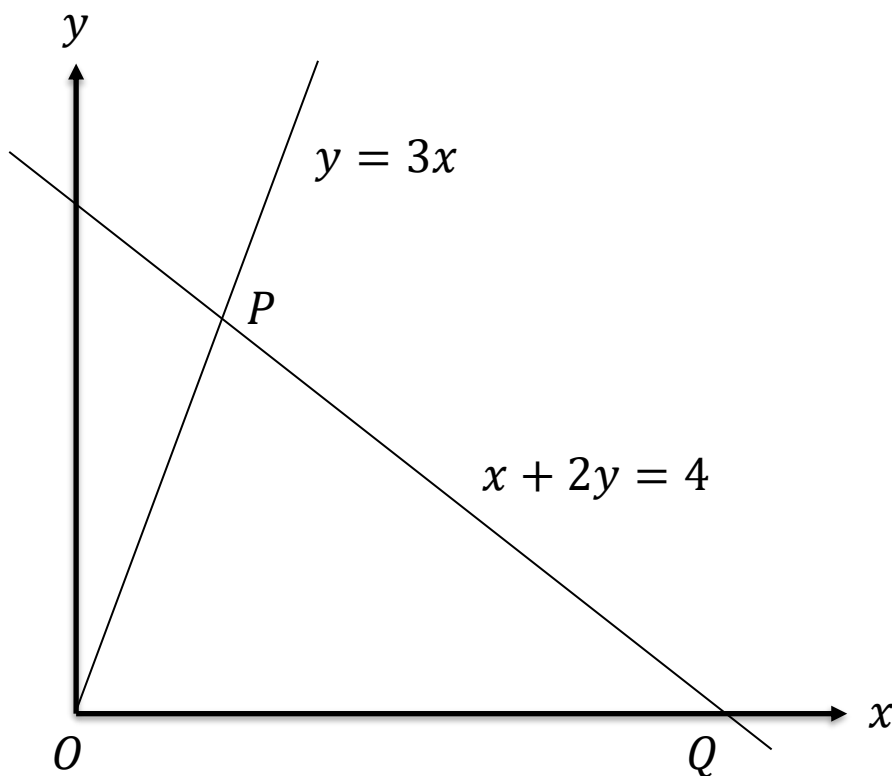
(3, -1) and (0,1)

$$\sqrt{3^2 + 2^2} = \sqrt{13}$$

(-4, -2) and (-12,4)

$$\sqrt{8^2 + 6^2} = 10$$

Area of Shapes



The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point P .

a) Determine the coordinates of P .

(We did this in a previous lesson)

Just solve two equations simultaneously.

$$x + 2(3x) = 4$$

$$7x = 4$$

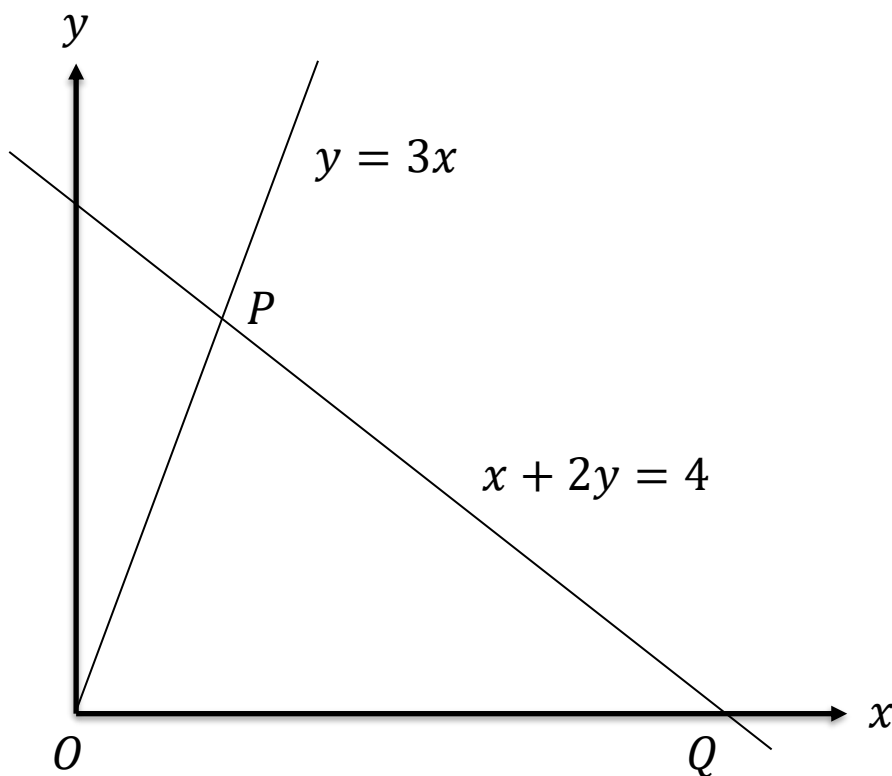
$$x = \frac{4}{7}$$

$$y = 3\left(\frac{4}{7}\right) = \frac{12}{7}$$

b) The line $x + 2y = 4$ intersects the x -axis at the point Q . Determine the area of the triangle OPQ .

?

Area of Shapes



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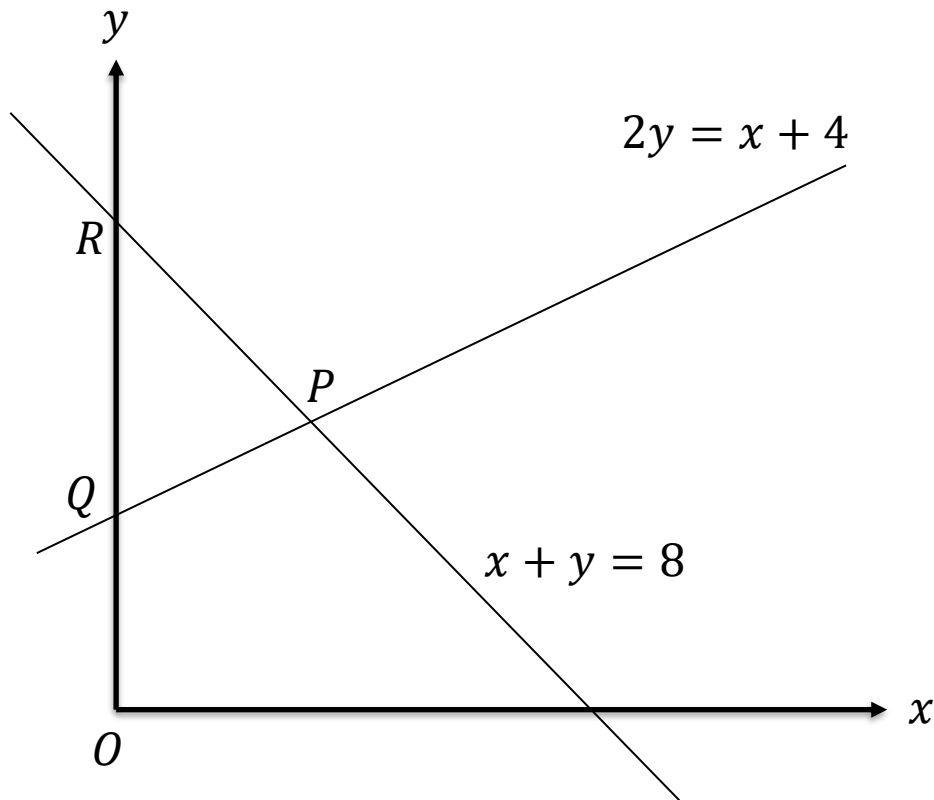
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b) The line $x + 2y = 4$ intersects the x -axis at the point Q . Determine the area of the triangle OPQ .

When $y = 0$, $x = 4$

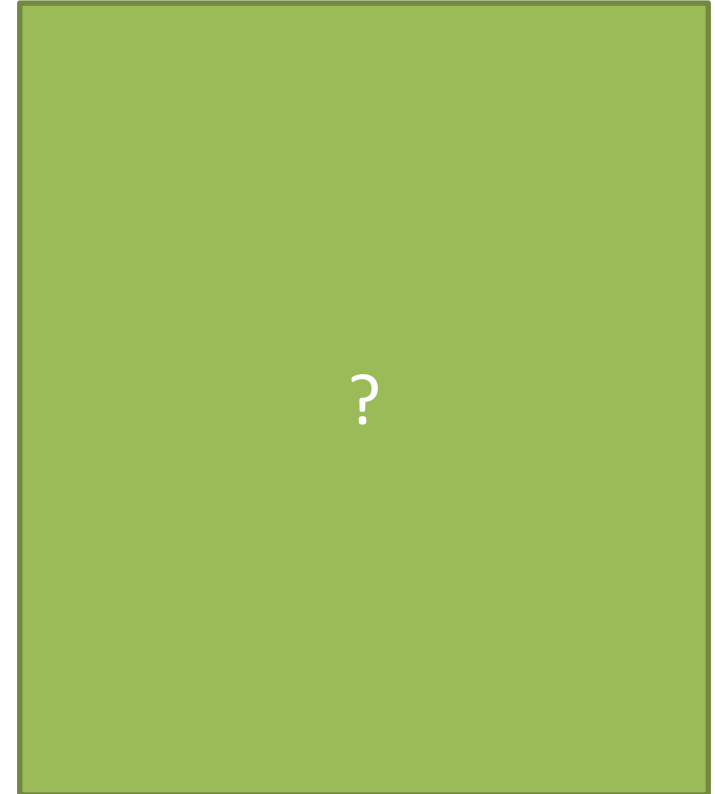
$$\text{Area} = \frac{1}{2} \times 4 \times \frac{12}{7} = \frac{24}{7}$$

Further Example

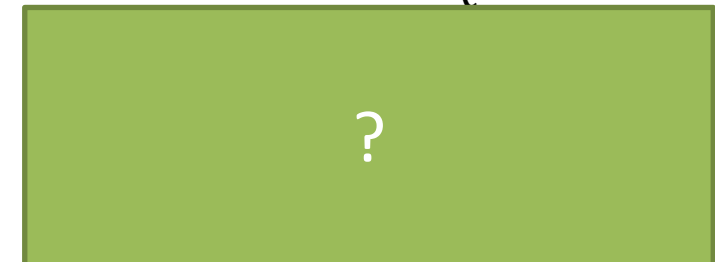


Fro Tip: When finding areas of triangles in exam questions, one line is often vertical or horizontal. You should generally choose this to be the 'base' of your triangle.

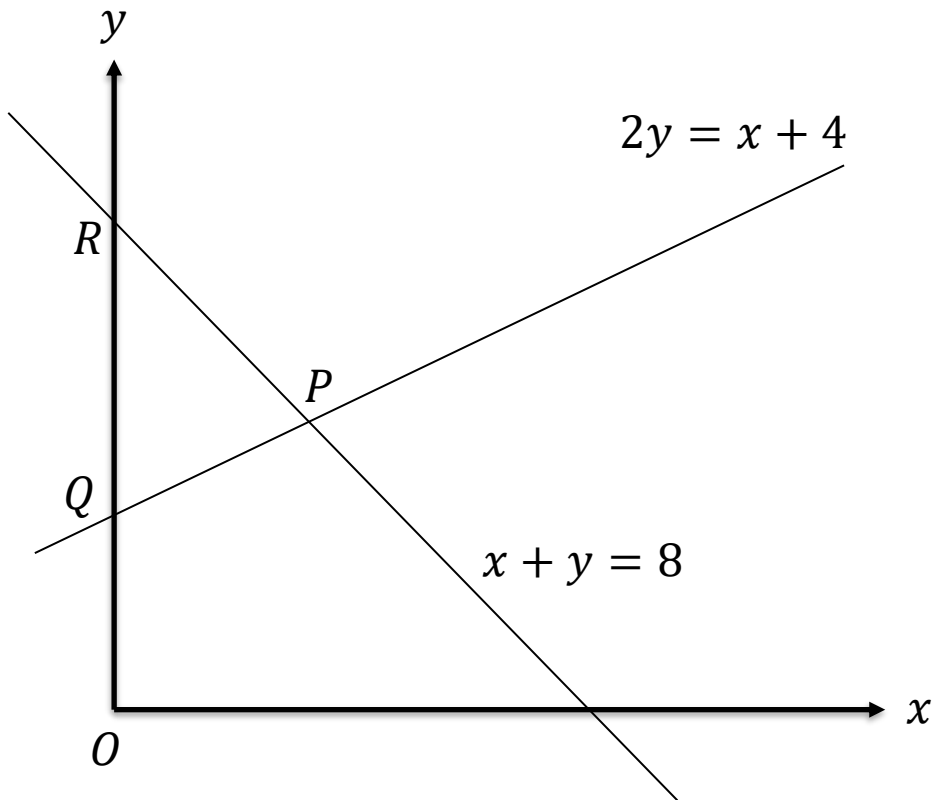
a Determine the length of PQ .



b Determine the area PQR .



Further Example



Fro Tip: When finding areas of triangles in exam questions, one line is often vertical or horizontal. You should generally choose this to be the 'base' of your triangle.

a Determine the length of PQ .

$$\begin{aligned}x &= 8 - y \\2y &= 8 - y + 4 \\3y &= 12 \\y &= 4 \rightarrow x = 4 \\P(4, 4)\end{aligned}$$

When $x = 0$

$$\begin{aligned}2y &= 4 \\y &= 2 \\Q(0, 2)\end{aligned}$$

Distance PQ :

$$\sqrt{4^2 + 2^2} = \sqrt{20}$$

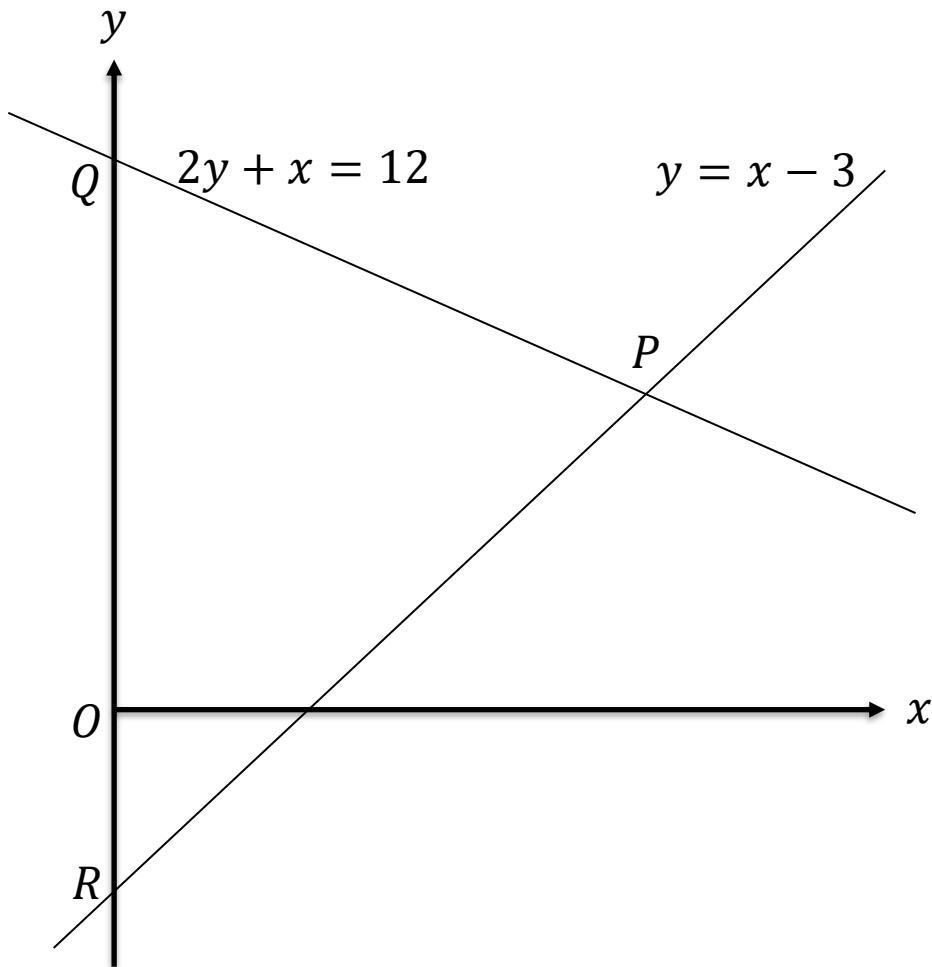
b Determine the area PQR .

$$R(0, 8)$$

Use RQ as the base:

$$\therefore \text{Area} = \frac{1}{2} \times 6 \times 4 = 12$$

Test Your Understanding



a) Determine the coordinate of P .

?

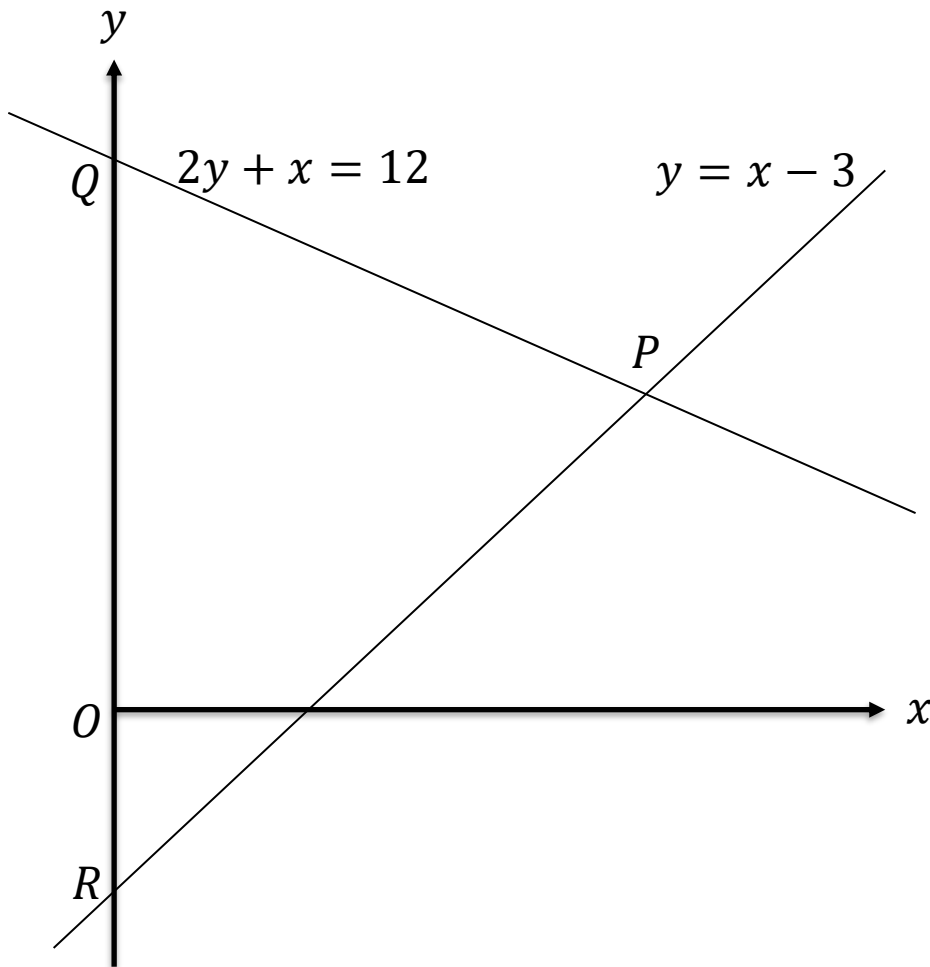
b) Determine the area of PQR .

?

c) Determine the length PQ .

?

Test Your Understanding



a) Determine the coordinate of P .

$$2(x - 3) + x = 12$$

$$x = 6 \rightarrow y = 3$$

b) Determine the area of PQR .

$$R(0, -3)$$

$$Q(0, 6)$$

$$Area = \frac{1}{2} \times 9 \times 6 = 27$$

c) Determine the length PQ .

$$P(6, 3), \quad Q(0, 6)$$

$$Dist = \sqrt{6^2 + 3^2} = \sqrt{45}$$

Exercise 5.4

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Extension Problems

1

[MAT 2001 1C]

The shortest distance from the origin to the line $3x + 4y = 25$ is what?

?

Exercise 5.4

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Extension Problems

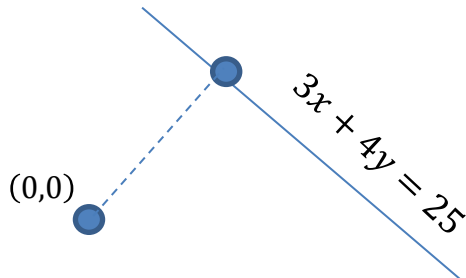
1 [MAT 2001 1C]

The shortest distance from the origin to the line $3x + 4y = 25$ is what?

Gradient of $3x + 4y = 25$ is $-\frac{3}{4}$. Therefore gradient of line going through origin and closest point is $\frac{4}{3}$ (see diagram). This line therefore has equation $y = \frac{4}{3}x$

Solving two equations simultaneously, we get point of intersection $A(3, 4)$.

Therefore distance $OA = \sqrt{3^2 + 4^2} = 5$.



Homework Exercise

1 Find the distance between these pairs of points:

a $(0, 1), (6, 9)$

b $(4, -6), (9, 6)$

c $(3, 1), (-1, 4)$

d $(3, 5), (4, 7)$

e $(0, -4), (5, 5)$

f $(-2, -7), (5, 1)$

2 Consider the points $A(-3, 5)$, $B(-2, -2)$ and $C(3, -7)$. Determine whether the line joining the points A and B is congruent to the line joining the points B and C .

Hint Two line segments are congruent if they are the same length.

3 Consider the points $P(11, -8)$, $Q(4, -3)$ and $R(7, 5)$. Show that the line segment joining the points P and Q is not congruent to the line joining the points Q and R .

4 The distance between the points $(-1, 13)$ and $(x, 9)$ is $\sqrt{65}$. Find two possible values of x .

Problem-solving

Use the distance formula to formulate a quadratic equation in x .

5 The distance between the points $(2, y)$ and $(5, 7)$ is $3\sqrt{10}$. Find two possible values of y .

6 a Show that the straight line l_1 with equation $y = 2x + 4$ is parallel to the straight line l_2 with equation $6x - 3y - 9 = 0$.

b Find the equation of the straight line l_3 that is perpendicular to l_1 and passes through the point $(3, 10)$.

c Find the point of intersection of the lines l_2 and l_3 .

d Find the shortest distance between lines l_1 and l_2 .

Problem-solving The shortest distance between two parallel lines is the perpendicular distance between them.

Homework Exercise

- 7 A point P lies on the line with equation $y = 4 - 3x$. The point P is a distance $\sqrt{34}$ from the origin. Find the two possible positions of point P . (5 marks)

- 8 The vertices of a triangle are $A(2, 7)$, $B(5, -6)$ and $C(8, -6)$.

Notation

Scalene triangles have three sides of different lengths.

- a Show that the triangle is a scalene triangle.
b Find the area of the triangle ABC .

Problem-solving

Draw a sketch and label the points A , B and C . Find the length of the base and the height of the triangle.

- 9 The straight line l_1 has equation $y = 7x - 3$. The straight line l_2 has equation $4x + 3y - 41 = 0$. The lines intersect at the point A .

- a Work out the coordinates of A .

The straight line l_2 crosses the x -axis at the point B .

- b Work out the coordinates of B .
c Work out the area of triangle AOB .

- 10 The straight line l_1 has equation $4x - 5y - 10 = 0$ and intersects the x -axis at point A . The straight line l_2 has equation $4x - 2y + 20 = 0$ and intersects the x -axis at the point B .

- a Work out the coordinates of A .
b Work out the coordinates of B .

The straight lines l_1 and l_2 intersect at the point C .

- c Work out the coordinates of C .
d Work out the area of triangle ABC .

Homework Exercise

11 The points $R(5, -2)$ and $S(9, 0)$ lie on the straight line l_1 as shown.

a Work out an equation for straight line l_1 . (2 marks)

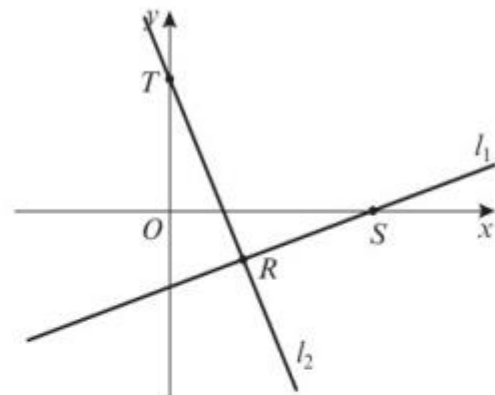
The straight line l_2 is perpendicular to l_1 and passes through the point R .

b Work out an equation for straight line l_2 . (2 marks)

c Write down the coordinates of T . (1 mark)

d Work out the lengths of RS and TR leaving your answer in the form $k\sqrt{5}$. (2 marks)

e Work out the area of $\triangle RST$. (2 marks)



12 The straight line l_1 passes through the point $(-4, 14)$ and has gradient $-\frac{1}{4}$

a Find an equation for l_1 in the form $ax + by + c = 0$, where a , b and c are integers. (3 marks)

b Write down the coordinates of A , the point where straight line l_1 crosses the y -axis. (1 mark)

The straight line l_2 passes through the origin and has gradient 3. The lines l_1 and l_2 intersect at the point B .

c Calculate the coordinates of B . (2 marks)

d Calculate the exact area of $\triangle OAB$. (2 marks)

Homework Answers

- 1 a 10 b 13 c 5 d $\sqrt{5}$
e $\sqrt{106}$ f $\sqrt{113}$
- 2 Distance between A and B = $\sqrt{50}$ and distance between B and C = $\sqrt{50}$ so the lines are congruent.
- 3 Distance between P and Q = $\sqrt{74}$ and distance between Q and R = $\sqrt{73}$ so the lines are not congruent.
- 4 $x = -8$ or $x = 6$
- 5 $y = -2$ or $y = 16$
- 6 a Both lines have gradient 2.
b $y = -\frac{1}{2}x + \frac{23}{2}$ or $x + 2y - 23 = 0$
c $(\frac{29}{5}, \frac{43}{5})$
d $\frac{7\sqrt{5}}{5}$
- 7 $P(-\frac{3}{5}, \frac{29}{5})$ or $P(3, -5)$
- 8 a $AB = \sqrt{178}$, $BC = 3$ and $AC = \sqrt{205}$. All sides are different lengths, therefore the triangle is a scalene triangle.
b $\frac{39}{2}$ or 19.5

- 9 a $A(2, 11)$
b $B(\frac{41}{4}, 0)$
c $\frac{451}{8}$
- 10 a $(\frac{5}{2}, 0)$
b $(-5, 0)$
c $(-10, -10)$
d $\frac{75}{2}$
- 11 a $y = \frac{1}{2}x - \frac{9}{2}$
b $y = -2x + 8$
c $T(0, 8)$
d $RS = 2\sqrt{5}$ and $TR = 5\sqrt{5}$
e 25
- 12 a $x + 4y - 52 = 0$
b $A(0, 13)$
c $B(4, 12)$
d 26