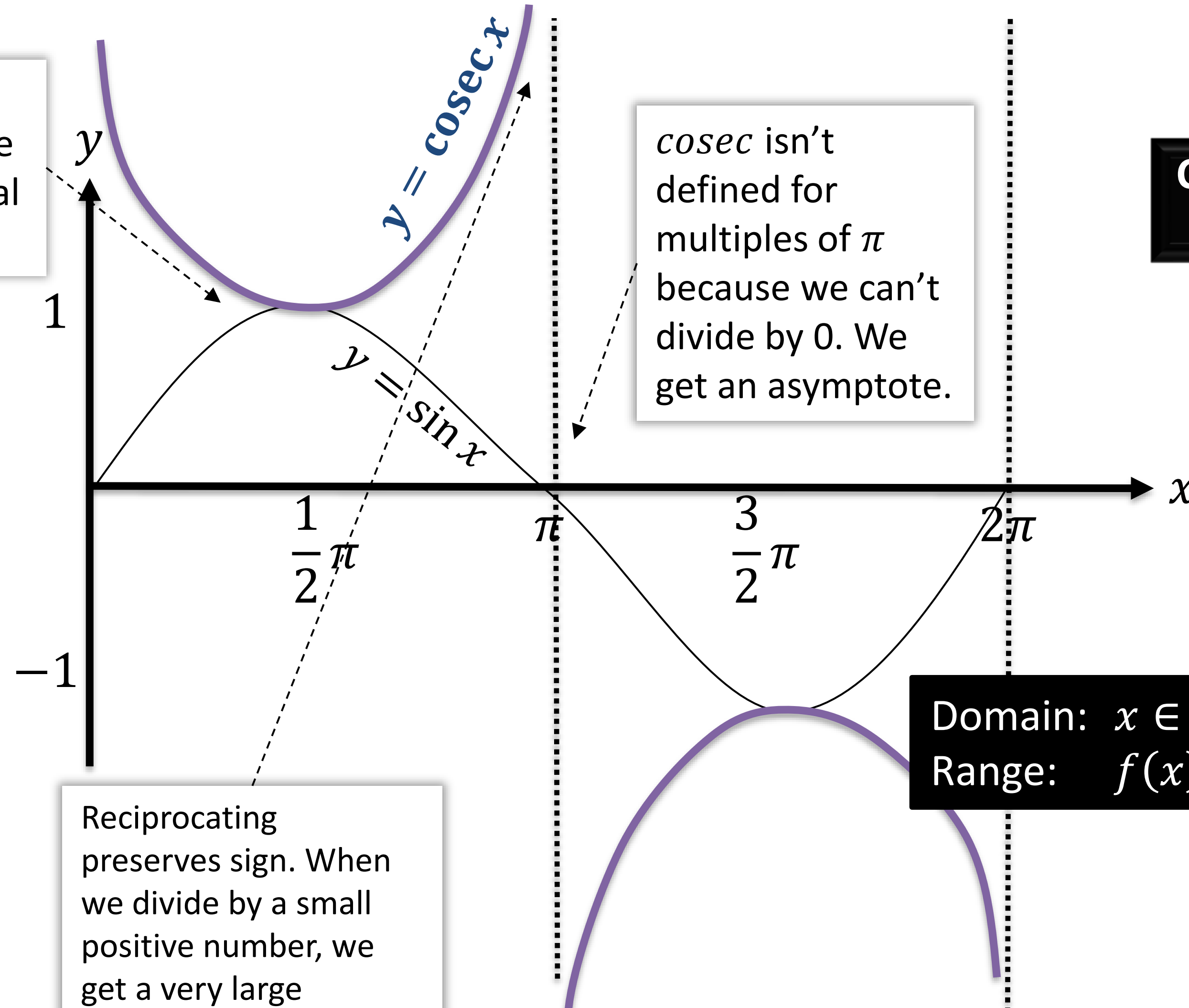
P2 Chapter 6: CoAngle Trigonometry

Co-angle Graphs

Sketches

To draw a graph of $y = cosec\ x$, start with a graph of $y = \sin x$, then consider what happens when we reciprocate each y value.

It touches here because the reciprocal of 1 is 1.



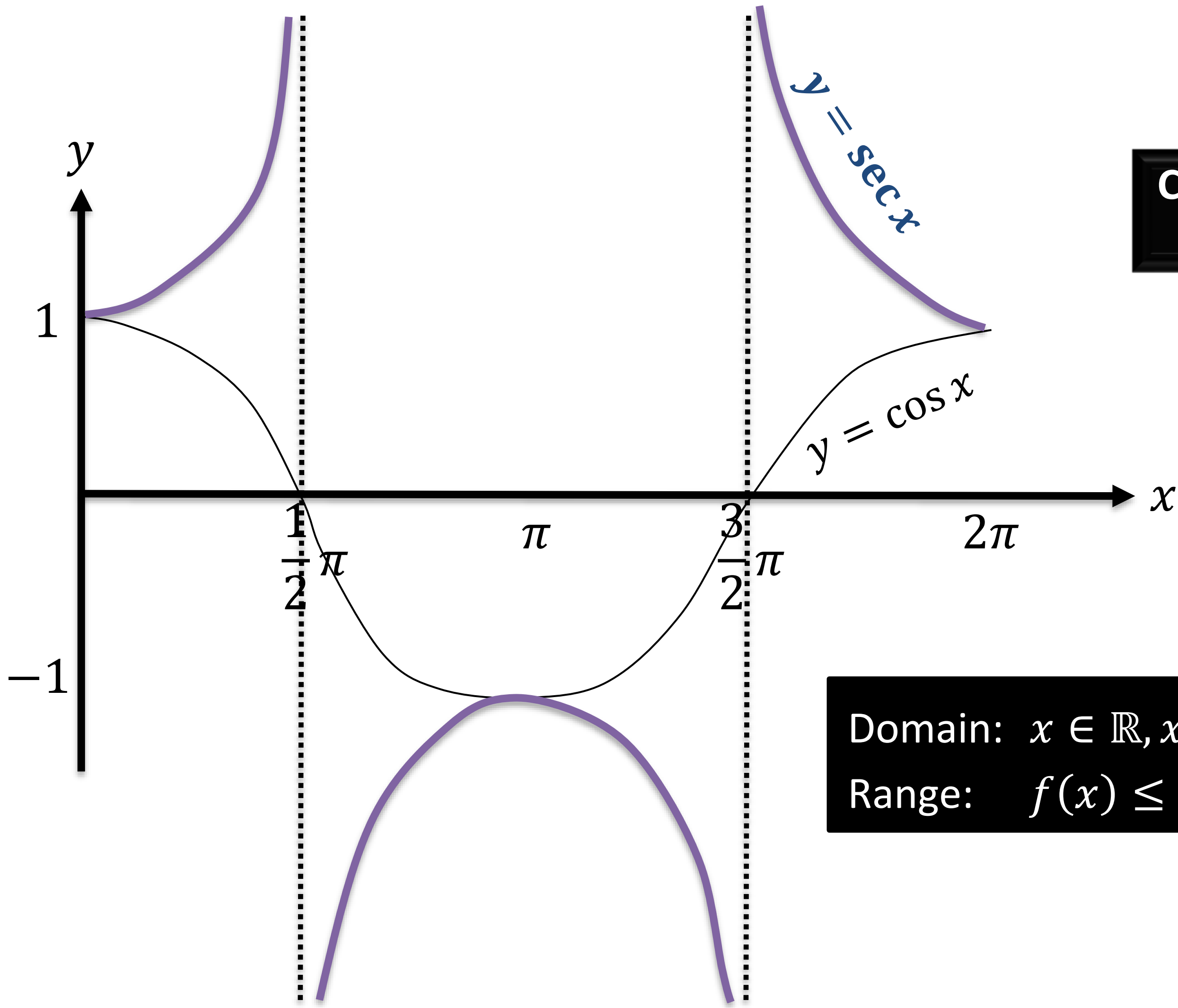
$cosec$ isn't defined for multiples of π because we can't divide by 0. We get an asymptote.

Click to Frosketch $y = cosec\ x$

Reciprocating preserves sign. When we divide by a small positive number, we get a very large positive number.

Domain: $x \in \mathbb{R}, x \neq \pm\pi, \pm 2\pi, \ldots$
Range: $f(x) \leq 1$ or $f(x) \geq 1$

Sketches



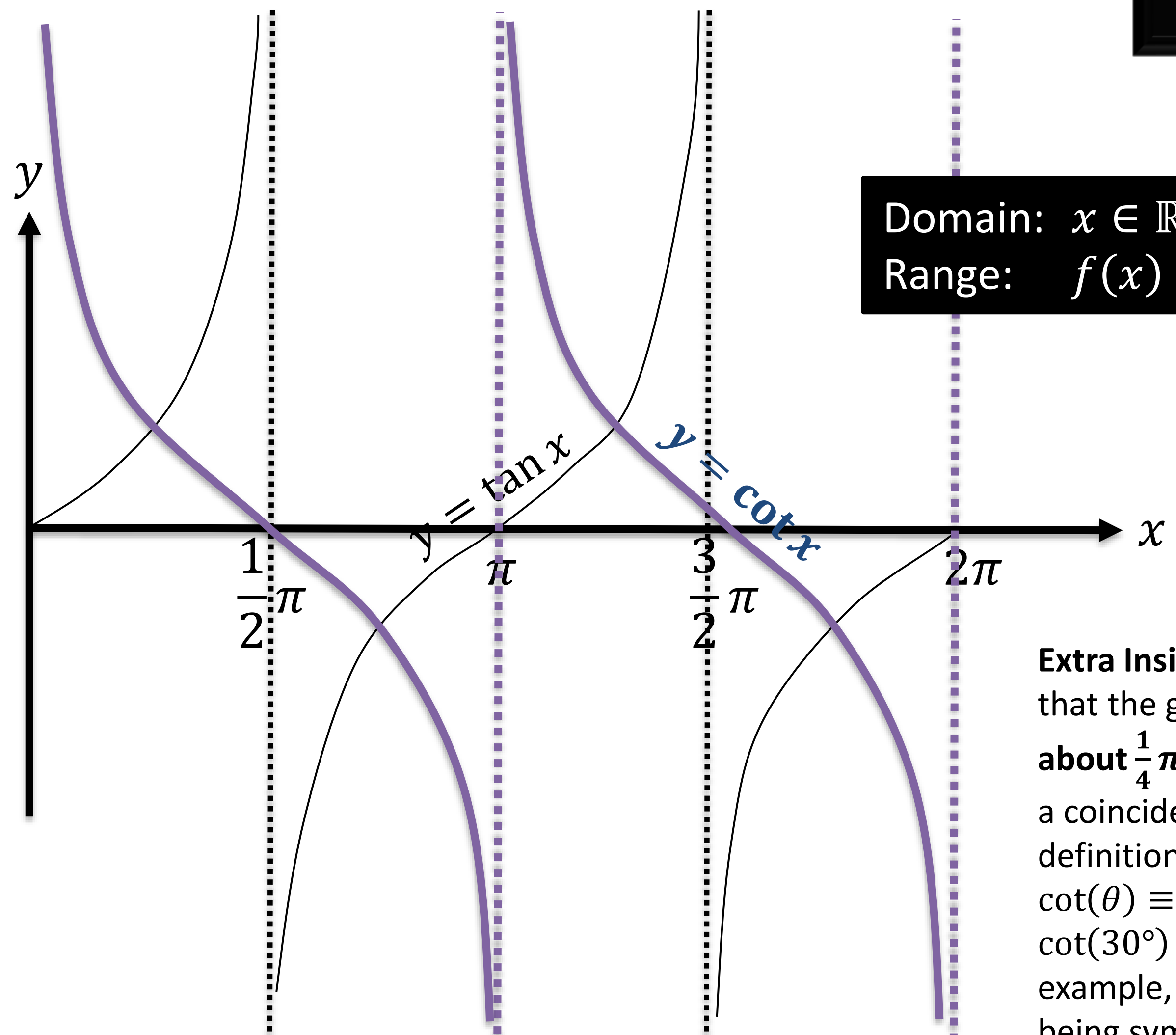
Click to Frosketch $y = sec\ x$

Domain: $x \in \mathbb{R}, x \neq \pm\frac{1}{2}\pi, \pm\frac{3}{2}\pi, \ldots$

Range: $f(x) \leq 1\ or\ f(x) \geq 1$

Sketches

Click to Brosketch $y = cot\ x$



Domain: $x \in \mathbb{R}, x \neq \pi, 2\pi, \ldots$
Range: $f(x) \in \mathbb{R}$

Extra Insight: We might spot that the graph is **symmetrical about** $\boldsymbol{\frac{1}{4}\pi, \frac{3}{4}\pi}$, etc. This is not a coincidence: the 'proper' definition of $\cot(\theta) \equiv \tan(90° - \theta)$, so $\cot(30°) = \tan(60°)$ for example, with $30°$ and $60°$ being symmetrical about $45°$.

Example

[Textbook]

a) Sketch the graph of $y = 4cosec\ x, \quad -\pi \leq x \leq \pi$.

b) On the same axes, sketch the line $y = x$.

c) State the number of solutions to the equation $4cosec\ x - x = 0, \ -\pi \leq x \leq \pi$

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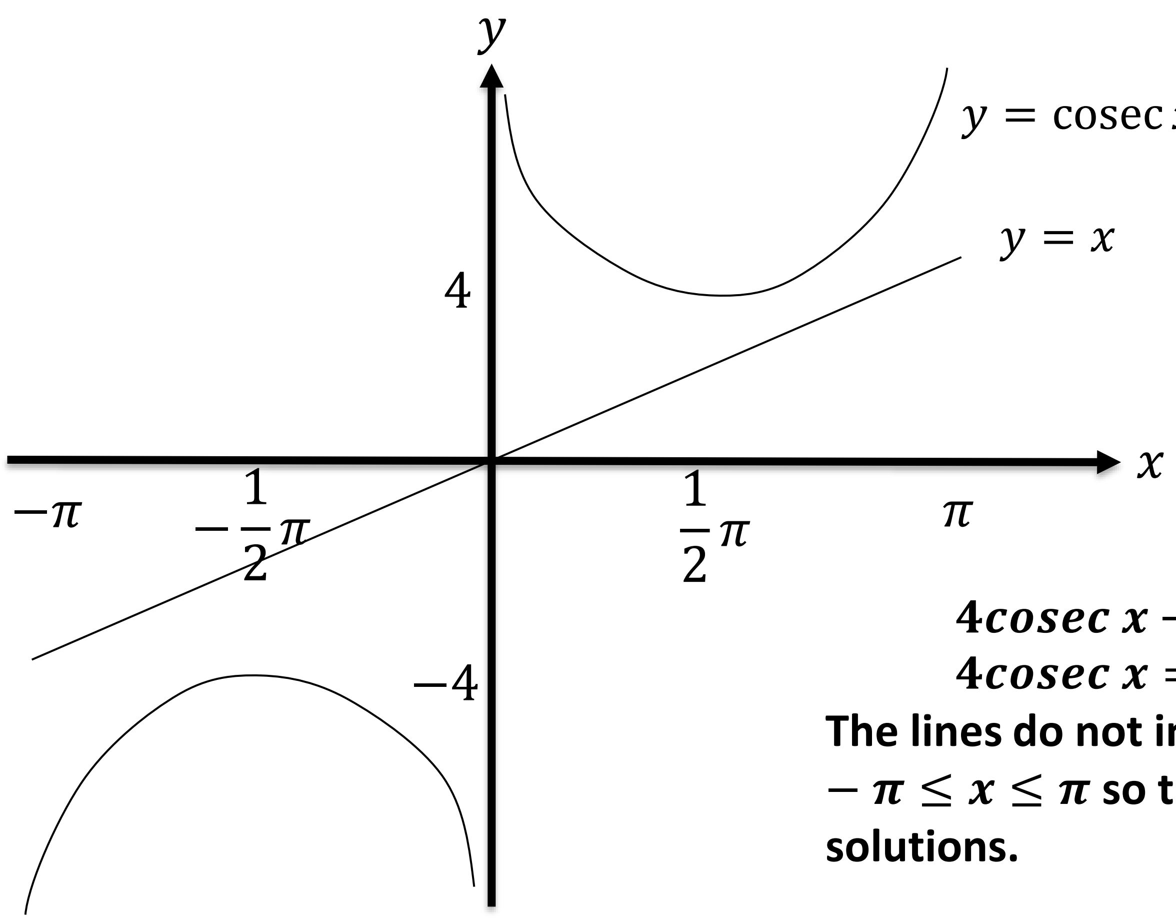

$$\boldsymbol{4cosec\ x - x = 0}$$
$$\boldsymbol{4cosec\ x = x}$$

The lines do not intersect for $-\pi \leq x \leq \pi$ so there are no solutions.

Test Your Understanding

Sketch $y = -1 + \sec 2x$ in the interval $0 \leq x < 360°$.

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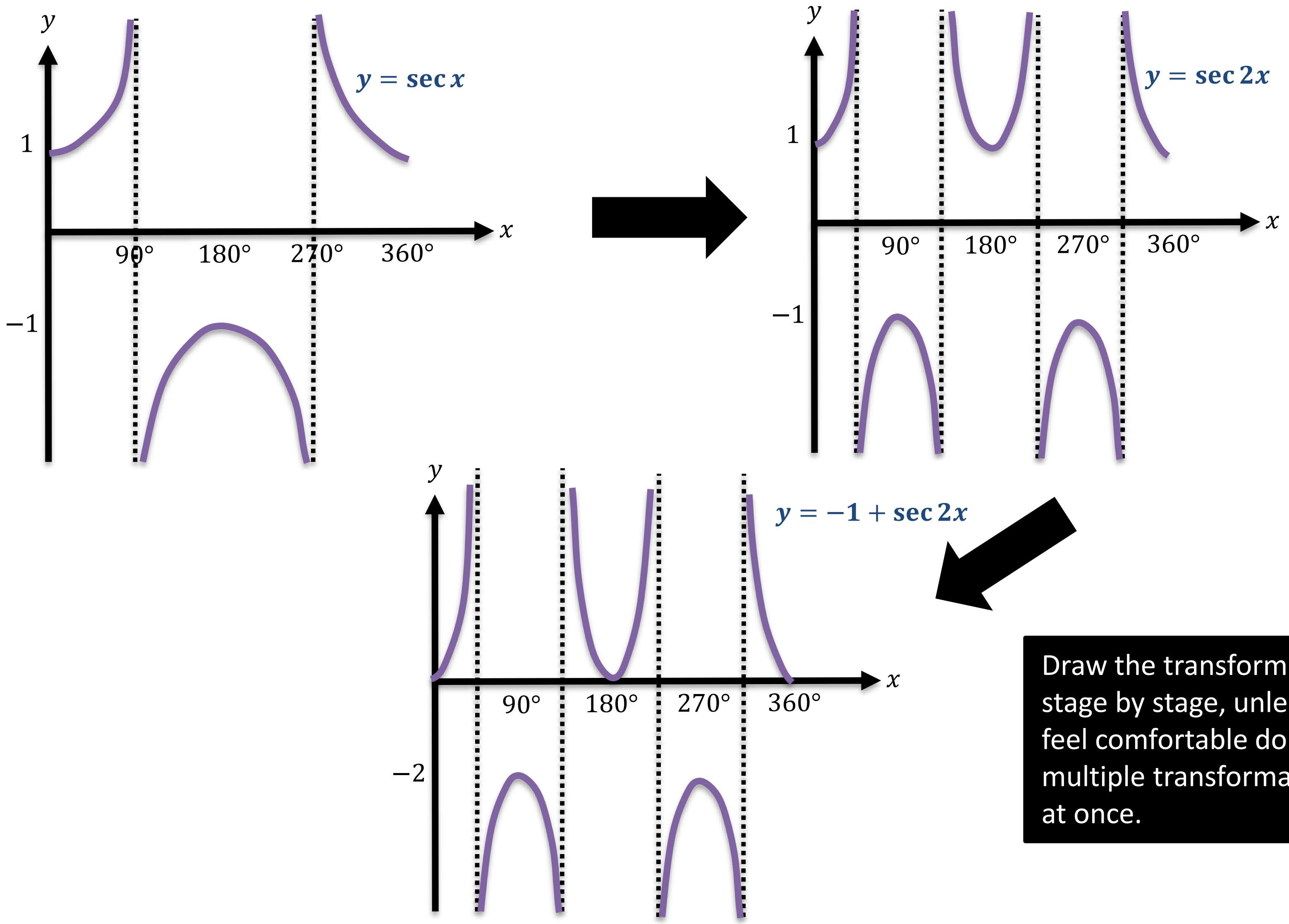

Draw the transformations stage by stage, unless you feel comfortable doing multiple transformations at once.

Exercise 6.2

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Homework Exercise

1 Sketch, in the interval $-540° \leqslant \theta \leqslant 540°$, the graphs of:

a $y = \sec\theta$ **b** $y = \operatorname{cosec}\theta$ **c** $y = \cot\theta$

2 **a** Sketch, on the same set of axes, in the interval $-\pi \leqslant x \leqslant \pi$, the graphs of $y = \cot x$ and $y = -x$.

b Deduce the number of solutions of the equation $\cot x + x = 0$ in the interval $-\pi \leqslant x \leqslant \pi$.

3 **a** Sketch, on the same set of axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $y = \sec\theta$ and $y = -\cos\theta$.

b Explain how your graphs show that $\sec\theta = -\cos\theta$ has no solutions.

4 **a** Sketch, on the same set of axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $y = \cot\theta$ and $y = \sin 2\theta$.

b Deduce the number of solutions of the equation $\cot\theta = \sin 2\theta$ in the interval $0 \leqslant \theta \leqslant 360°$.

5 **a** Sketch on separate axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $y = \tan\theta$ and $y = \cot(\theta + 90°)$.

b Hence, state a relationship between $\tan\theta$ and $\cot(\theta + 90°)$.

Homework Exercise

6 **a** Describe the relationships between the graphs of:

i $y = \tan\left(\theta + \frac{\pi}{2}\right)$ and $y = \tan\theta$

ii $y = \cot(-\theta)$ and $y = \cot\theta$

iii $y = \operatorname{cosec}\left(\theta + \frac{\pi}{4}\right)$ and $y = \operatorname{cosec}\theta$

iv $y = \sec\left(\theta - \frac{\pi}{4}\right)$ and $y = \sec\theta$

b By considering the graphs of $y = \tan\left(\theta + \frac{\pi}{2}\right)$, $y = \cot(-\theta)$, $y = \operatorname{cosec}\left(\theta + \frac{\pi}{4}\right)$ and $y = \sec\left(\theta - \frac{\pi}{4}\right)$, state which pairs of functions are equal.

7 Sketch on separate axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of:

a $y = \sec 2\theta$

b $y = -\operatorname{cosec}\theta$

c $y = 1 + \sec\theta$

d $y = \operatorname{cosec}(\theta - 30°)$

e $y = 2\sec(\theta - 60°)$

f $y = \operatorname{cosec}(2\theta + 60°)$

g $y = -\cot(2\theta)$

h $y = 1 - 2\sec\theta$

In each case show the coordinates of any maximum and minimum points, and of any points at which the curve meets the axes.

8 Write down the periods of the following functions. Give your answers in terms of π.

a $\sec 3\theta$

b $\operatorname{cosec}\frac{1}{2}\theta$

c $2\cot\theta$

d $\sec(-\theta)$

Homework Exercise

9 **a** Sketch, in the interval $-2\pi \leqslant x \leqslant 2\pi$, the graph of $y = 3 + 5\operatorname{cosec} x$. **(3 marks)**

b Hence deduce the range of values of k for which the equation $3 + 5\operatorname{cosec} x = k$ has no solutions. **(2 marks)**

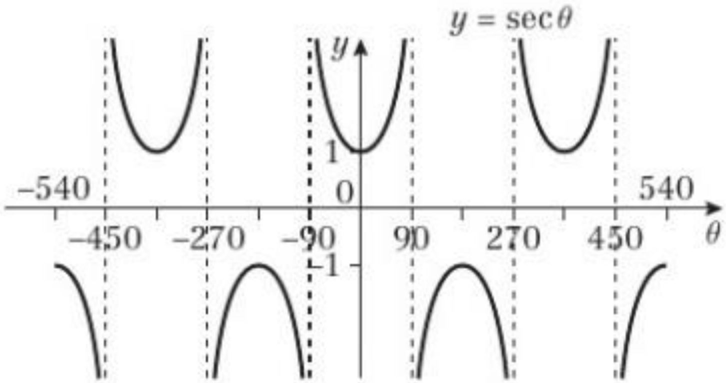
10 **a** Sketch the graph of $y = 1 + 2\sec\theta$ in the interval $-\pi \leqslant \theta \leqslant 2\pi$. **(3 marks)**

b Write down the θ-coordinates of points at which the gradient is zero. **(2 marks)**

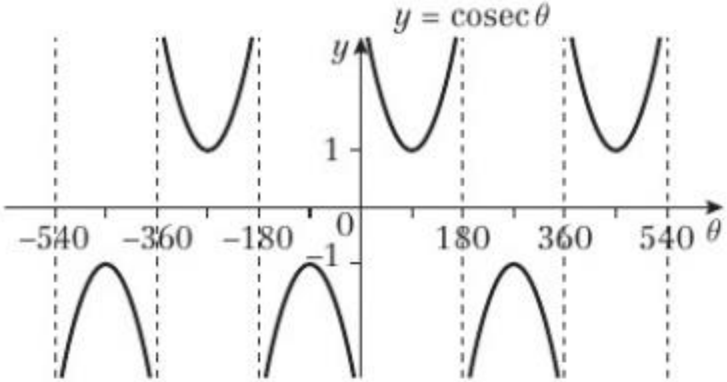
c Deduce the maximum and minimum values of $\dfrac{1}{1 + 2\sec\theta}$, and give the smallest positive values of θ at which they occur. **(4 marks)**

Homework Answers

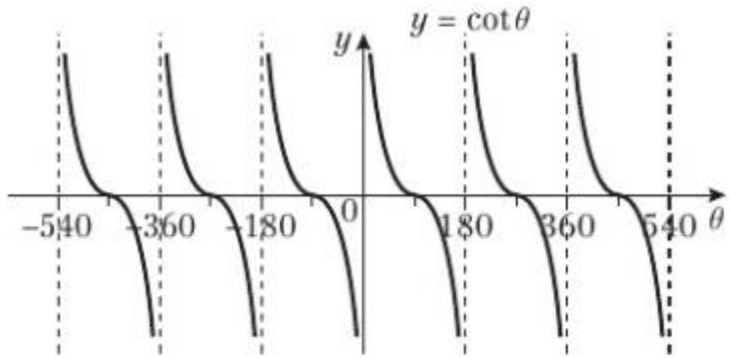
1 **a**



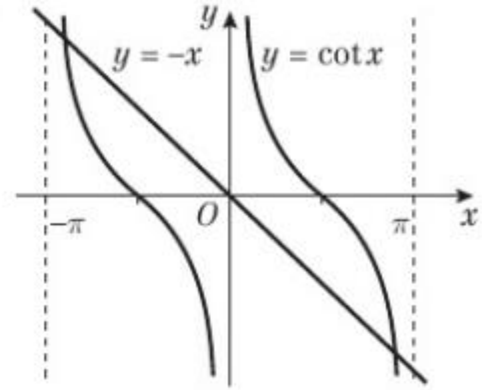
b



c

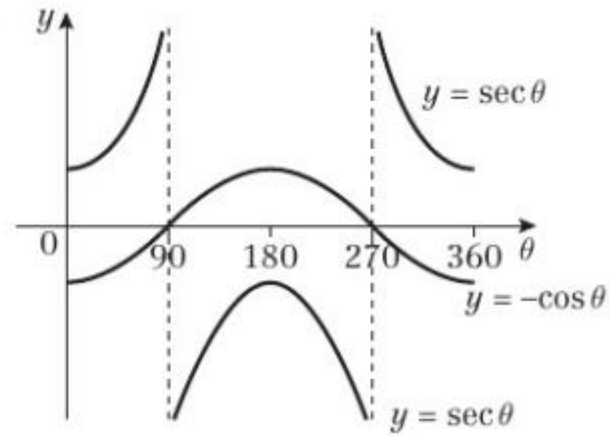


2 **a**



b 2 solutions

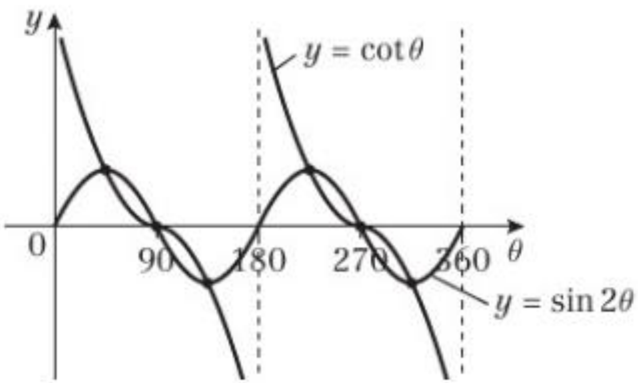
3 **a**



b The solutions of $\sec\theta = -\cos\theta$ are the θ values of the points of intersection of $y = \sec\theta$ and $y = -\cos\theta$. As they do not meet, there are no solutions.

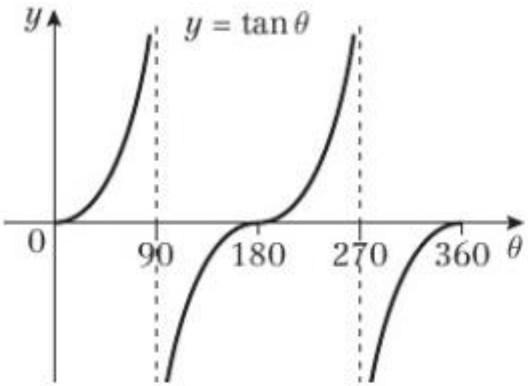
Homework Answers

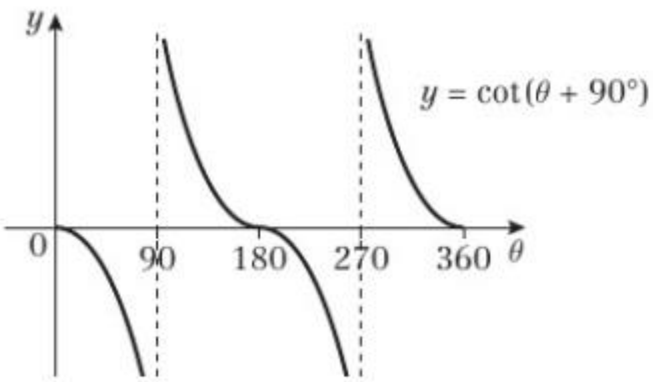
4 **a**



b 6

5 **a**





5 **b** $\cot(90° + \theta) = -\tan\theta$

6 **a** **i** The graph of $y = \tan\left(\theta + \frac{\pi}{2}\right)$ is the same as that of $y = \tan\theta$ translated by $\frac{\pi}{2}$ to the left.

ii The graph of $y = \cot(-\theta)$ is the same as that of $y = \cot\theta$ reflected in the y-axis.

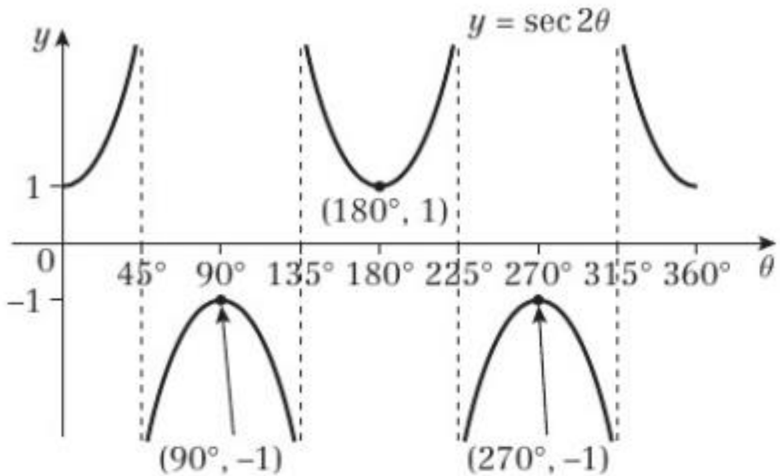
iii The graph of $y = \operatorname{cosec}\left(\theta + \frac{\pi}{4}\right)$ is the same as that of $y = \operatorname{cosec}\theta$ translated by $\frac{\pi}{4}$ to the left.

iv The graph of $y = \sec\left(\theta - \frac{\pi}{4}\right)$ is the same as that of $y = \sec\theta$ translated by $\frac{\pi}{4}$ to the right.

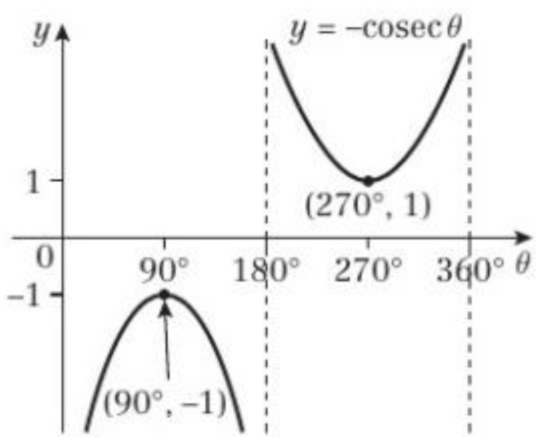
b $\tan\left(\theta + \frac{\pi}{2}\right) = \cot(-\theta)$; $\operatorname{cosec}\left(\theta + \frac{\pi}{4}\right) = \sec\left(\theta - \frac{\pi}{4}\right)$

Homework Answers

7 a



b



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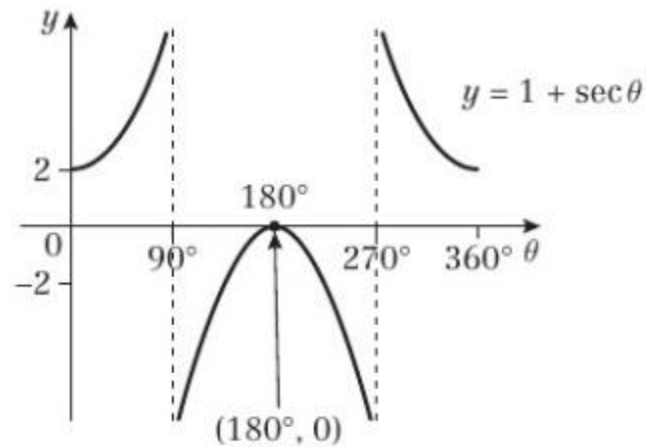

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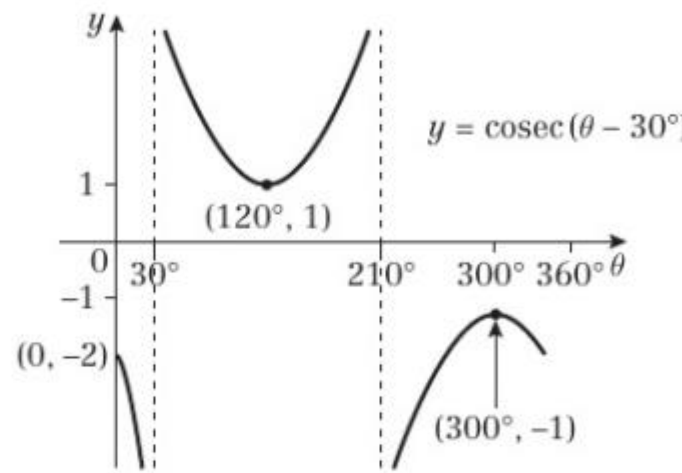
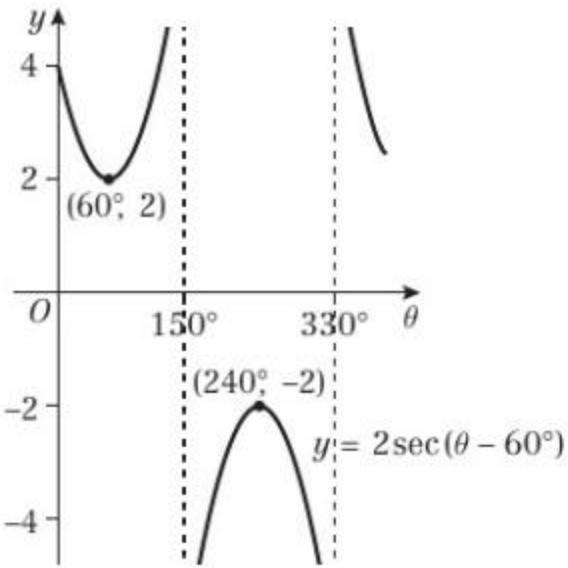


Homework Answers

7 e



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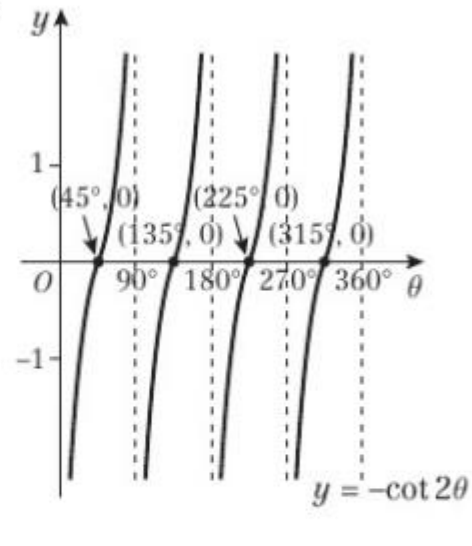

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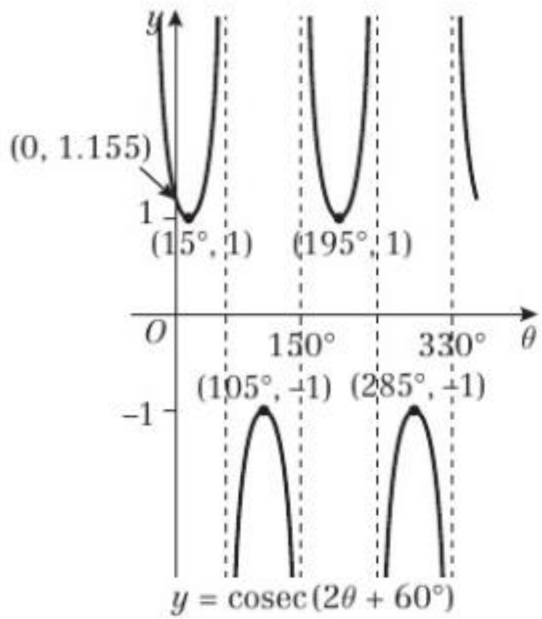

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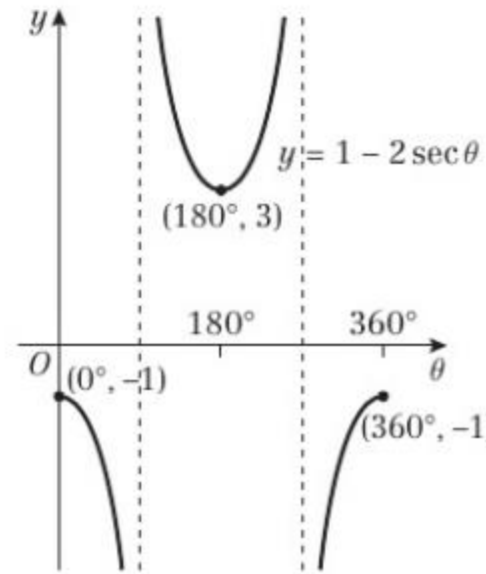
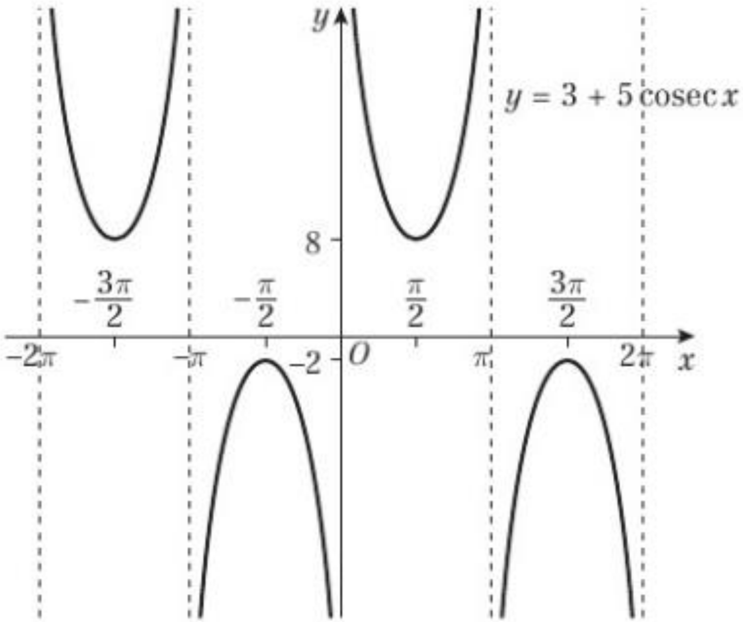


Homework Answers

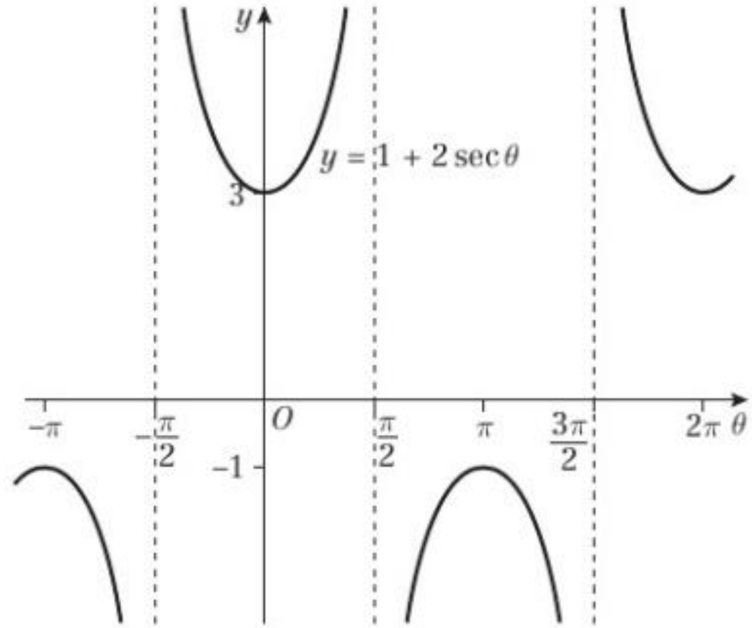
8 **a** $\frac{2\pi}{3}$ **b** 4π **c** π **d** 2π

9 **a**



b $-2 < k < 8$

10 **a**



b $\theta = -\pi, 0, \pi, 2\pi$

c Max $= \frac{1}{3}$, first occurs at $\theta = 2\pi$

Min $= -1$, first occurs at $\theta = \pi$