Mech1 Chapter 7: Force and Motion

Chapter Practice

Key Points

1 **Newton's first law** of motion states that an object at rest will stay at rest and that an object moving with constant velocity will continue to move with constant velocity unless an unbalanced force acts on the object.

2 A **resultant** force acting on an object will cause the object to **accelerate in the same direction** as the resultant force.

3 You can find the **resultant** of two or more forces given as vectors by adding the vectors.

4 **Newton's second law** of motion states that the force needed to accelerate a particle is equal to the product of the mass of the particle and the acceleration produced: $F = ma$.

5 $W = mg$

6 You can use $\mathbf{F} = m\mathbf{a}$ to solve problems involving vector forces acting on particles.

7 You can solve problems involving connected particles by considering the particles separately or, if they are moving in the same straight line, as a single particle.

8 **Newton's third law** states that for every action there is an equal and opposite reaction.

Chapter Exercises

1 A motorcycle of mass 200 kg is moving along a level road. The motorcycle's engine provides a forward thrust of 1000 N. The total resistance is modelled as a constant force of magnitude 600 N.

a Modelling the motorcycle as a particle, draw a force diagram to show the forces acting on the motorcycle.

b Calculate the acceleration of the motorcycle.

2 A man of mass 86 kg is standing in a lift which is moving upwards with constant acceleration $2\,\mathrm{m\,s^{-2}}$. Find the magnitude and direction of the force that the man is exerting on the floor of the lift.

3 A car of mass 800 kg is travelling along a straight horizontal road. A constant retarding force of F N reduces the speed of the car from $18\,\mathrm{m\,s^{-1}}$ to $12\,\mathrm{m\,s^{-1}}$ in 2.4 s. Calculate:

a the value of F

b the distance moved by the car in these 2.4 s.

4 A block of mass 0.8 kg is pushed along a rough horizontal floor by a constant horizontal force of magnitude 7 N. The speed of the block increases from $2\,\mathrm{m\,s^{-1}}$ to $4\,\mathrm{m\,s^{-1}}$ in a distance of 4.8 m. Calculate:

a the magnitude of the acceleration of the block **(3 marks)**

b the magnitude of the frictional force between the block and the floor. **(3 marks)**

5 A car of mass 1200 kg is moving along a level road. The car's engine provides a constant driving force. The motion of the car is opposed by a constant resistance.

Given that car is accelerating at $2\,\mathrm{m\,s^{-2}}$, and that the magnitude of the driving force is three times the magnitude of the resistance force, show that the magnitude of the driving force is 3600 N.

Chapter Exercises

6 Forces of $(3\mathbf{i} + 2\mathbf{j})$ N and $(4\mathbf{i} - \mathbf{j})$ N act on a particle of mass 0.25 kg. Find the acceleration of the particle.

7 Forces of $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ N, $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ N and $\begin{pmatrix} a \\ -2b \end{pmatrix}$ N act on a particle of mass 2 kg causing it to accelerate at $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ m s^{-2}. Find the values of a and b.

8 A sled of mass 2 kg is initially at rest on a horizontal plane. It is acted upon by a force of $(2\mathbf{i} + 4\mathbf{j})$ N for 3 seconds. Giving your answers in surd form,

a find the magnitude of acceleration

b find the distance travelled in the 3 seconds.

9 In this question **i** and **j** represent the unit vectors east and north respectively.

The forces $(3a\mathbf{i} + 4b\mathbf{j})$ N, $(5b\mathbf{i} + 2a\mathbf{j})$ N and $(-15\mathbf{i} - 18\mathbf{j})$ N act on a particle of mass 2 kg which is in equilibrium.

a Find the values of a and b.

b The force $(-15\mathbf{i} - 18\mathbf{j})$ N is removed. Work out:

i the magnitude and direction of the resulting acceleration of the particle

ii the distance travelled by the particle in the first 3 seconds of its motion.

Chapter Exercises

10 A car is towing a trailer along a straight horizontal road by means of a horizontal tow-rope. The mass of the car is 1400 kg. The mass of the trailer is 700 kg. The car and the trailer are modelled as particles and the tow-rope as a light inextensible string. The resistances to motion of the car and the trailer are assumed to be constant and of magnitude 630 N and 280 N respectively. The driving force on the car, due to its engine, is 2380 N. Find:

a the acceleration of the car **(3 marks)**

b the tension in the tow-rope. **(3 marks)**

When the car and trailer are moving at $12\,\text{m}\,\text{s}^{-1}$, the tow-rope breaks. Assuming that the driving force on the car and the resistances to motion are unchanged:

c find the distance moved by the car in the first 4 s after the tow-rope breaks. **(6 marks)**

d State how you have used the modelling assumption that the tow-rope is inextensible. **(1 mark)**

11 A train of mass 2500 kg pushes a carriage of mass 1100 kg along a straight horizontal track. The engine is connected to the carriage by a shunt which is parallel to the direction of motion of the coupling. The horizontal resistances to motion of the train and the carriage have magnitudes R N and 500 N respectively. The engine of the train produces a constant horizontal driving force of magnitude 8000 N that causes the train and carriage to accelerate at $1.75\,\text{m}\,\text{s}^{-2}$.

a Show that the resistance to motion R acting on the train is 1200 N. **(2 marks)**

b Find the magnitude of the compression in the shunt. **(3 marks)**

The train must stop at the next station so the driver reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the train of magnitude 2000 N causing the engine and carriage to decelerate.

c Given that the resistances to motion are unchanged, find the magnitude of the thrust in the shunt. Give your answer correct to 3 s.f. **(7 marks)**

12 Particles P and Q of masses $2m$ kg and m kg respectively are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. They both hang at a distance of 2 m above horizontal ground. The system is released from rest.

a Find the magnitude of the acceleration of the system.

b Find the speed of P as it hits the ground.

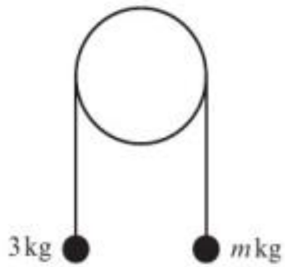
Given that particle Q does not reach the pulley:

c find the greatest height that Q reaches above the ground.

d State how you have used in your calculation:

i the fact that the string is inextensible **ii** the fact that the pulley is smooth.

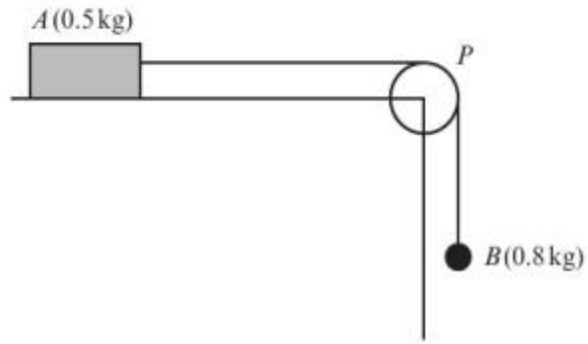
13 Two particles have masses 3 kg and m kg, where $m < 3$. They are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The particles are held in position with the string taut and the hanging parts of the string vertical, as shown. The particles are then released from rest. The initial acceleration of each particle has magnitude $\frac{3}{7}g$. Find:



a the tension in the string immediately after the particles are released **(3 marks)**

b the value of m. **(3 marks)**

Chapter Exercises



14 A block of wood A of mass 0.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a ball B of mass 0.8 kg which hangs freely below the pulley, as shown in the figure. The resistance to motion of A from the rough table has a constant magnitude F N. The system is released from rest with the string taut. After release, B descends a distance of 0.4 m in 0.5 s. Modelling A and B as particles, calculate:

a the acceleration of B **(3 marks)**

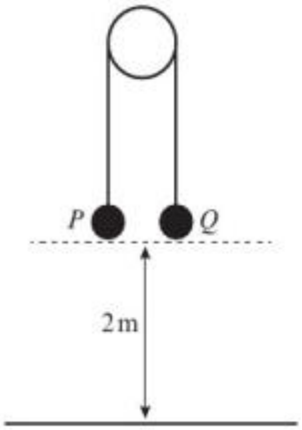
b the tension in the string **(4 marks)**

c the value of F. **(3 marks)**

d State how in your calculations you have used the information that the string is inextensible. **(1 mark)**

Chapter Exercises

15 Two particles P and Q have masses 0.5 kg and 0.4 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 2 m above the floor, as shown. The particles are released from rest and in the subsequent motion Q does not reach the pulley.



a **i** Write down an equation of motion for P. **(2 marks)**

ii Write down an equation of motion for Q. **(2 marks)**

b Find the tension in the string immediately after the particles are released. **(2 marks)**

c Find the acceleration of A immediately after the particles are released. **(2 marks)**

When the particles have been moving for 0.2 s, the string breaks.

d Find the further time that elapses until Q hits the floor. **(9 marks)**

Challenge

In this question **i** and **j** are the unit vectors east and north respectively.

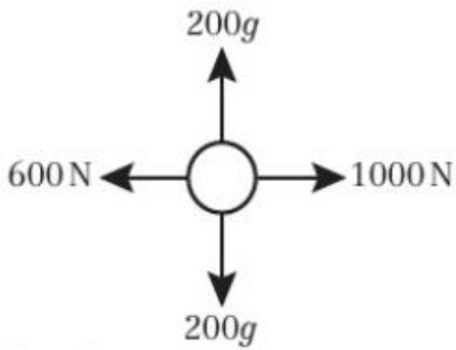
Two boats start from rest at different points on the south bank of a river. The current in the river provides a constant force of magnitude 3**i** N on both boats.

The motor on boat A provides a thrust of (−7**i** + 2**j**) N and the motor on boat B provides a thrust of (k**i** + **j**) N.

Given that the boats are accelerating in perpendicular directions, find the value of k.

Chapter Answers

1 a



b $2\,\text{m s}^{-2}$

2 1000 N (2 s.f.) vertically downwards

3 a 2000 N **b** 36 m

4 a $1.25\,\text{ms}^{-2}$ **b** 6 N

5 Res(→) $3R - R = 1200 \times 2 \Rightarrow R = 1200$
Driving force = $3R$ = 3600 N

6 $(28\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-2}$

7 $a = 1$, $b = -3$

8 a $\sqrt{5}\,\text{m s}^{-2}$ **b** $\dfrac{9\sqrt{5}}{2}$ m

9 a $a = -15$, $b = 12$
b **i** $11.7\,\text{m s}^{-2}$ (3 s.f.) on a bearing of 039.8° (3 s.f.)
ii 52.7 m (3 s.f.)

10 a $0.7\,\text{ms}^{-2}$ **b** 770 N **c** 58 m
d Inextensible ⇒ tension the same throughout, and the acceleration of the car and the trailer is the same.

11 a R(→) $8000 - 500 - R = 3600 \times 1.75$, $R = 1200$ N
b 2425 N **c** 630 N (2 s.f.)

12 a $\frac{1}{3}g\,\text{m s}^{-2}$ **b** $3.6\,\text{m s}^{-1}$ **c** $2\frac{2}{3}$ m
d **i** Acceleration both masses equal.
ii Same tension in string either side of the pulley.

13 a $\frac{12}{7}g$ N **b** $m = 1.2$

14 a $3.2\,\text{m s}^{-2}$ **b** 5.3 N (2 s.f.) **c** $F = 3.7$ (2 s.f.)
d The information that the string is inextensible has been use in part **c** when the acceleration of A has been taken to be equal to the acceleration of B.

15 a **i** $0.5g - T = 0.5a$ **ii** $T - 0.4g = 0.4a$
b $\frac{4}{9}g$ N **c** $\frac{1}{9}g\,\text{m s}^{-2}$ **d** 0.66 s (2 s.f.)

Challenge
$k = -\frac{5}{2}$