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# P2 Chapter 1: Algebra Techniques

## Algebraic Fractions

# Multiplying/Dividing Algebraic Fractions

As you saw at GCSE level, multiplying algebraic fractions is no different to multiply numeric fractions.

You may however need to **cancel common factors, by factorising where possible**.

$$\frac{a}{b} \times \frac{c}{a} = \frac{\mathbf{ac}}{\mathbf{ab}} = \frac{\mathbf{c}}{\mathbf{b}}$$

$$\frac{x+1}{2} \times \frac{3}{x^2-1} = \frac{\mathbf{3(x+1)}}{\mathbf{2(x+1)(x-1)}} = \frac{\mathbf{3}}{\mathbf{2(x-1)}}$$

To divide by a fraction, **multiply by the reciprocal of the second fraction**.

$$\frac{p}{q} \div \frac{r}{q} = \frac{\mathbf{p}}{\mathbf{q}} \times \frac{\mathbf{q}}{\mathbf{r}} = \frac{\mathbf{p}}{\mathbf{r}}$$

$$\frac{x+2}{x+4} \div \frac{3x+6}{x^2-16} = \frac{x+2}{x+4} \times \frac{(x+4)(x-4)}{\mathbf{3(x+2)}} = \frac{x-4}{\mathbf{3}}$$

# Test Your Understanding

$$\frac{x+3}{5} \times \frac{10}{x^2-9}$$

=

?

$$\frac{x^2+x}{y} \div \frac{x^2-x-2}{y^2}$$

=

?

“Crimes against Mathematics”:

$$\frac{x^2 + \cancel{y}}{2\cancel{y}} = \frac{x^2}{2}$$

?

# Test Your Understanding

$$\frac{x+3}{5} \times \frac{10}{x^2-9} = \frac{x+3}{5} \times \frac{10}{(x+3)(x-3)} = \frac{2}{x-3}$$

$$\frac{x^2+x}{y} \div \frac{x^2-x-2}{y^2} = \frac{x(x+1)}{y} \times \frac{y^2}{(x+1)(x-2)} = \frac{xy}{x-2}$$

“Crimes against Mathematics”:

$$\frac{x^2 + \cancel{y}}{2\cancel{y}} = \frac{x^2}{2}$$

Cancelling in fractions only occurs by dividing. But the student has subtracted  $y$  in the numerator.

# Exercise 1.2

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# Homework Exercise

1 Simplify:

**a**  $\frac{a}{d} \times \frac{a}{c}$

**b**  $\frac{a^2}{c} \times \frac{c}{a}$

**c**  $\frac{2}{x} \times \frac{x}{4}$

**d**  $\frac{3}{x} \div \frac{6}{x}$

**e**  $\frac{4}{xy} \div \frac{x}{y}$

**f**  $\frac{2r^2}{5} \div \frac{4}{r^3}$

2 Simplify:

**a**  $(x+2) \times \frac{1}{x^2-4}$

**b**  $\frac{1}{a^2+6a+9} \times \frac{a^2-9}{2}$

**c**  $\frac{x^2-3x}{y^2+y} \times \frac{y+1}{x}$

**d**  $\frac{y}{y+3} \div \frac{y^2}{y^2+4y+3}$

**e**  $\frac{x^2}{3} \div \frac{2x^3-6x^2}{x^2-3x}$

**f**  $\frac{4x^2-25}{4x-10} \div \frac{2x+5}{8}$

**g**  $\frac{x+3}{x^2+10x+25} \times \frac{x^2+5x}{x^2+3x}$

**h**  $\frac{3y^2+4y-4}{10} \div \frac{3y+6}{15}$

**i**  $\frac{x^2+2xy+y^2}{2} \times \frac{4}{(x-y)^2}$

3 Show that  $\frac{x^2-64}{x^2-36} \div \frac{64-x^2}{x^2-36} = -1$

(4 marks)

4 Show that  $\frac{2x^2-11x-40}{x^2-4x-32} \times \frac{x^2+8x+16}{6x^2-3x-45} \div \frac{8x^2+20x-48}{10x^2-45x+45} = \frac{a}{b}$  and find the values of the constants  $a$  and  $b$ , where  $a$  and  $b$  are integers.

(4 marks)

# Homework Exercise

5 a Simplify fully  $\frac{x^2 + 2x - 24}{2x^2 + 10x} \times \frac{x^2 - 3x}{x^2 + 3x - 18}$  (3 marks)

**Hint** Simplify and then solve the logarithmic equation.

← Year 1, Section 14.6

b Given that

$\ln((x^2 + 2x - 24)(x^2 - 3x)) = 2 + \ln((2x^2 + 10x)(x^2 + 3x - 18))$  find  $x$  in terms of  $e$ . (4 marks)

6  $f(x) = \frac{2x^2 - 3x - 2}{6x - 8} \div \frac{x - 2}{3x^2 + 14x - 24}$

**Hint** Differentiate each term separately.

← Year 1, Section 12.5

a Show that  $f(x) = \frac{2x^2 + 13x + 6}{2}$  (4 marks)

b Hence differentiate  $f(x)$  and find  $f'(4)$ . (3 marks)

# Homework Exercise

1 Write as a single fraction:

a  $\frac{1}{3} + \frac{1}{4}$

b  $\frac{3}{4} - \frac{2}{5}$

c  $\frac{1}{p} + \frac{1}{q}$

d  $\frac{3}{4x} + \frac{1}{8x}$

e  $\frac{3}{x^2} - \frac{1}{x}$

f  $\frac{a}{5b} - \frac{3}{2b}$

2 Write as a single fraction:

a  $\frac{3}{x} - \frac{2}{x+1}$

b  $\frac{2}{x-1} - \frac{3}{x+2}$

c  $\frac{4}{2x+1} + \frac{2}{x-1}$

d  $\frac{1}{3}(x+2) - \frac{1}{2}(x+3)$

e  $\frac{3x}{(x+4)^2} - \frac{1}{x+4}$

f  $\frac{5}{2(x+3)} + \frac{4}{3(x-1)}$

3 Write as a single fraction:

a  $\frac{2}{x^2+2x+1} + \frac{1}{x+1}$

b  $\frac{7}{x^2-4} + \frac{3}{x+2}$

c  $\frac{2}{x^2+6x+9} - \frac{3}{x^2+4x+3}$

d  $\frac{2}{y^2-x^2} + \frac{3}{y-x}$

e  $\frac{3}{x^2+3x+2} - \frac{1}{x^2+4x+4}$

f  $\frac{x+2}{x^2-x-12} - \frac{x+1}{x^2+5x+6}$

4 Express  $\frac{6x+1}{x^2+2x-15} - \frac{4}{x-3}$  as a single fraction in its simplest form. (4 marks)

5 Express each of the following as a fraction in its simplest form.

a  $\frac{3}{x} + \frac{2}{x+1} + \frac{1}{x+2}$

b  $\frac{4}{3x} - \frac{2}{x-2} + \frac{1}{2x+1}$

c  $\frac{3}{x-1} + \frac{2}{x+1} + \frac{4}{x-3}$

6 Express  $\frac{4(2x-1)}{36x^2-1} + \frac{7}{6x-1}$  as a single fraction in its simplest form. (4 marks)

7  $g(x) = x + \frac{6}{x+2} + \frac{36}{x^2-2x-8}$ ,  $x \in \mathbb{R}$ ,  $x \neq -2$ ,  $x \neq 4$

a Show that  $g(x) = \frac{x^3 - 2x^2 - 2x + 12}{(x+2)(x-4)}$  (4 marks)

b Using algebraic long division, or otherwise, further show that  $g(x) = \frac{x^2 - 4x + 6}{x-4}$  (4 marks)



# Homework Answers

1   a  $\frac{a^2}{cd}$    b  $a$    c  $\frac{1}{2}$    d  $\frac{1}{2}$    e  $\frac{4}{x^2}$    f  $\frac{r^5}{10}$

2   a  $\frac{1}{x-2}$    b  $\frac{a-3}{2(a+3)}$    c  $\frac{x-3}{y}$    d  $\frac{y+1}{y}$   
e  $\frac{x}{6}$    f  $4$    g  $\frac{1}{x+5}$    h  $\frac{3y-2}{2}$    i  $\frac{2(x+y)^2}{(x-y)^2}$

3 All factors cancel exactly except  $\frac{x-8}{8-x} = \frac{x-8}{-(x-8)} = -1$

4  $a = 5, b = 12$

5   a  $\frac{x-4}{2x+10}$    b  $x = \frac{10e^2 + 4}{1 - 2e^2}$

6   a  $\frac{2x^2 - 3x - 2}{6x - 8} \div \frac{x - 2}{3x^2 + 14x - 24} = \frac{2x^2 - 3x - 2}{6x - 8}$   
 $\times \frac{3x^2 + 14x - 24}{x - 2} = \frac{(2x + 1)(x - 2)}{2(3x - 4)} \times \frac{(3x - 4)(x + 6)}{x - 2}$   
 $= \frac{(2x + 1)(x + 6)}{2} = \frac{2x^2 + 13x + 6}{2}$

b  $f'(x) = 2x + \frac{13}{2}; f'(4) = \frac{29}{2}$

# Homework Answers

$$1 \quad \mathbf{a} \quad \frac{7}{12} \quad \mathbf{b} \quad \frac{7}{20} \quad \mathbf{c} \quad \frac{p+q}{pq} \quad \mathbf{d} \quad \frac{7}{8x} \quad \mathbf{e} \quad \frac{3-x}{x^2} \quad \mathbf{f} \quad \frac{2a-15}{10b}$$

$$2 \quad \mathbf{a} \quad \frac{x+3}{x(x+1)} \quad \mathbf{b} \quad \frac{-x+7}{(x-1)(x+2)} \quad \mathbf{c} \quad \frac{8x-2}{(2x+1)(x-1)}$$

$$\mathbf{d} \quad \frac{-x-5}{6} \quad \mathbf{e} \quad \frac{2x-4}{(x+4)^2} \quad \mathbf{f} \quad \frac{23x+9}{6(x+3)(x-1)}$$

$$3 \quad \mathbf{a} \quad \frac{x+3}{(x+1)^2} \quad \mathbf{b} \quad \frac{3x+1}{(x-2)(x+2)} \quad \mathbf{c} \quad \frac{-x-7}{(x+1)(x+3)^2}$$

$$\mathbf{d} \quad \frac{3x+3y+2}{(y-x)(y+x)} \quad \mathbf{e} \quad \frac{2x+5}{(x+2)^2(x+1)} \quad \mathbf{f} \quad \frac{7x+8}{(x+2)(x+3)(x-4)}$$

$$4 \quad \frac{2x-19}{(x+5)(x-3)}$$

$$5 \quad \mathbf{a} \quad \frac{6x^2+14x+6}{x(x+1)(x+2)} \quad \mathbf{b} \quad \frac{-x^2-24x-8}{3x(x-2)(2x+1)}$$

$$\mathbf{c} \quad \frac{9x^2-14x-7}{(x-1)(x+1)(x-3)}$$

$$6 \quad \frac{50x+3}{(6x+1)(6x-1)}$$

$$\begin{aligned} 7 \quad \mathbf{a} \quad & x + \frac{6}{x+2} + \frac{36}{x^2-2x-8} \\ &= \frac{x(x+2)(x-4)}{(x+2)(x-4)} + \frac{6(x-4)}{(x+2)(x-4)} + \frac{36}{(x+2)(x-4)} \\ &= \frac{x^3-2x^2-2x+12}{(x+2)(x-4)} \end{aligned}$$

$$\mathbf{b} \quad \text{Divide } x^3-2x^2-2x+12 \text{ by } (x+2) \text{ to give } x^2-4x+6$$