P1 Chapter 5: Linear Graphs

Linear Modelling

What's the point of straight line equations?

We saw in Chapter 2 that lots of things in real life have a 'quadratic' relationship, e.g. vertical height with time. Lots of real life variables have a 'linear' relationship, i.e. **there is a fixed increase/decrease in one variable each time the other variable goes up by 1 unit.**

Examples

Car sales made and take home pay.

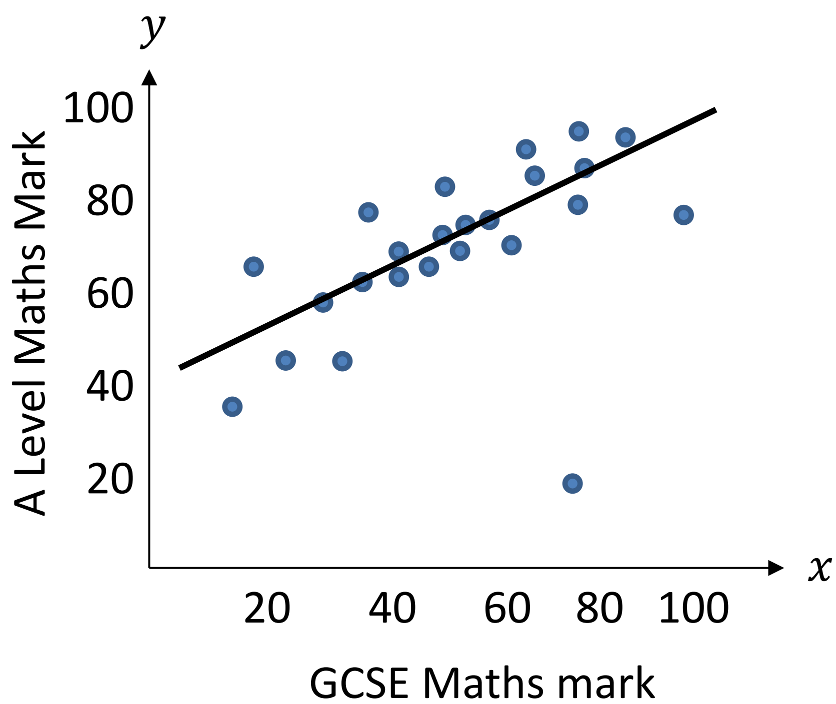
The relationship between Celsius and Fahrenheit.

Temperature and altitude (in a particular location)

(And a pure maths one:) The nth term of an arithmetic series.

3, 5, 8, 11, 14, ...

Modelling



A mathematical model is an attempt to model a real-life situation based on mathematical concepts.

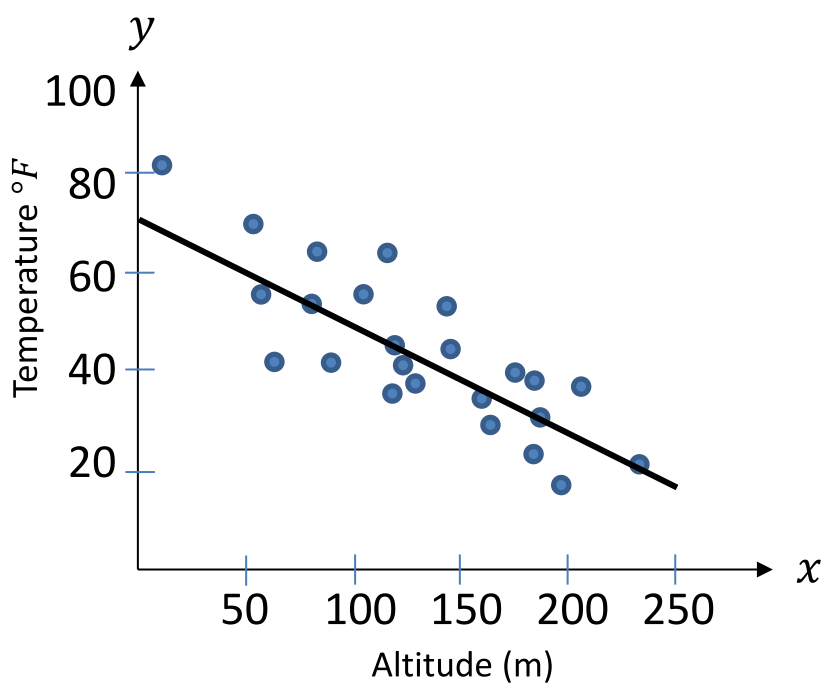
For this example, **our model might be a linear model with equation $\boldsymbol{y = mx + c}$** where x is a student's GCSE mark and y is the predicted A Level mark.

Such a linear model can be drawn as a line of best fit.
The data obviously doesn't fit this line exactly. This chosen model may only partially fit the data (and the further the points are away from the line, the less suitable this model is).
We might decide another model, e.g. $y = ax^2 + bx + c$, is more appropriate.

But if we choose a well-known model such as a linear one, then we can use **established mathematical theory** in useful ways:

- We need to choose the most appropriate 'parameters' m and c so the model best matches the data. You will learn in S1 there are existing techniques to do this.
- We can then predict a student's A Level mark based on their maths mark.

Example



The temperature y at different points on a mountain is recorded at different altitudes x.
Suppose we were to use a linear model $y = mx + c$.

---

a Determine m and c (you can assume the line goes through $(0,70)$ and $(250,20)$.

?

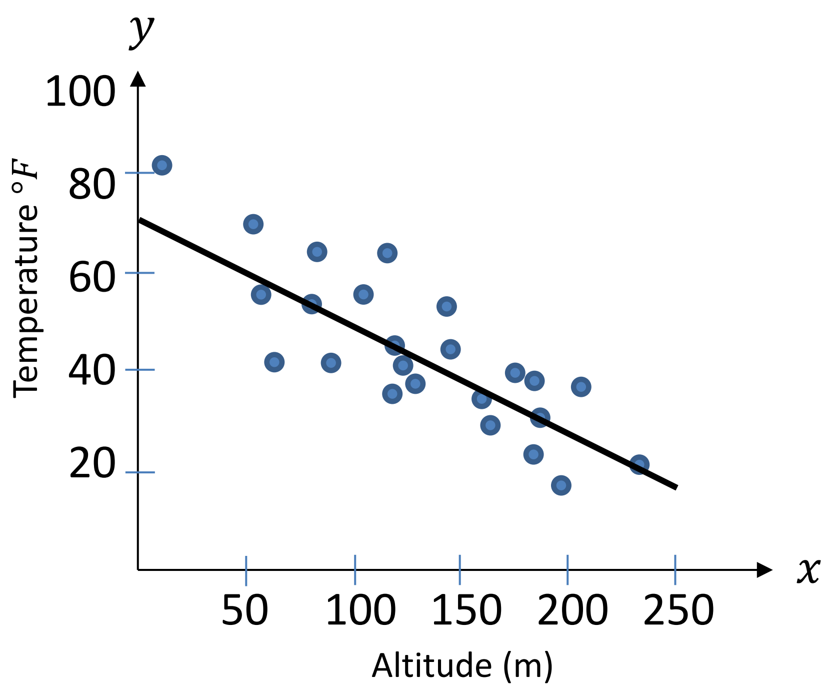
b **<u>Interpret</u>** the meaning of m and c **<u>in this context</u>**.

?

c Predict at what altitude the temperature reaches $0°F$.

?

Example



The temperature y at different points on a mountain is recorded at different altitudes x. Suppose we were to use a linear model $y = mx + c$.

---

a Determine m and c (you can assume the line goes through $(0,70)$ and $(250,20)$).

$$\boldsymbol{m = \frac{-50}{250} = -0.2 \qquad c = 70}$$

b **<u>Interpret</u>** the meaning of m and c **<u>in this context</u>**.

The temperature at sea level is $70°F$. The temperature goes down $0.2°F$ for each extra metre in altitude.

c Predict at what altitude the temperature reaches $0°F$.

$$\boldsymbol{0 = -0.2x + 70 \quad \rightarrow \quad x = 350°C}$$

Evaluating if a linear model is most suitable

Choosing a model for our data usually involves making **simplifying assumptions**. For a linear model we are assuming:

The y value goes up the same amount for each unit increase in x.

The current population of Bickerstonia is 26000. This year (2017) the population increased by 150. Matt decides to model the population P based on the years t after 2017 by the linear model:

$$P = mt + c$$

Why might this not be a suitable model?

?

Evaluating if a linear model is most suitable

Choosing a model for our data usually involves making **simplifying assumptions**. For a linear model we are assuming:

The y value goes up the same amount for each unit increase in x.

The current population of Bickerstonia is 26000. This year (2017) the population increased by 150. Matt decides to model the population P based on the years t after 2017 by the linear model:

$$P = mt + c$$

Why might this not be a suitable model?

The population is unlikely to increase by the same amount each year.

i.e. the assumption of a linear model is violated.

Note: Population tends to grow/decline by a percentage each year. Therefore a better model would be an **exponential** function $P = ab^t$ where a, b are constants. The a would be the initial population and the b the factor increase each year, e.g. $b = 1.1$ would be a 10% annual growth.

Exercise 5.5

Pearson Pure Mathematics Year 1/AS
Page 42

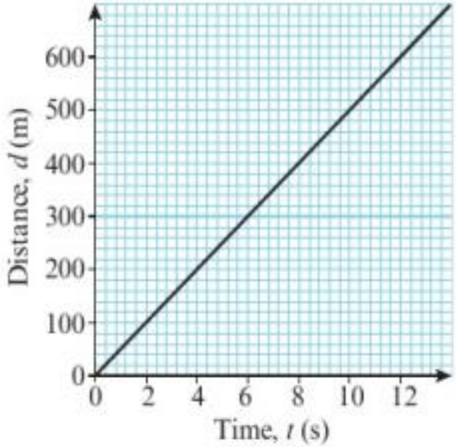
Homework Exercise

1 For each graph

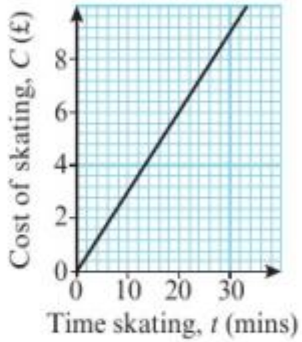
i calculate the gradient, k, of the line

ii write a direct proportion equation connecting the two variables.

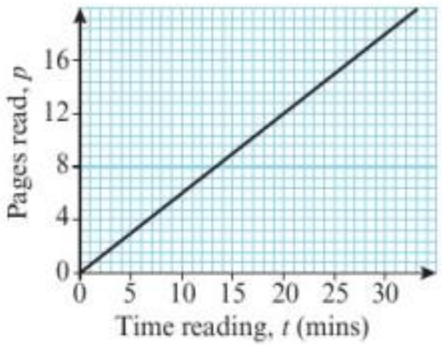
a



b



c



2 Draw a graph to determine whether a linear model would be appropriate for each set of data.

a

v	p
0	0
15	2
25	6
40	12
60	25
80	50

b

x	y
0	70
5	82.5
10	95
15	107.5
25	132.5
40	170

c

w	l
3.1	45
3.4	47
3.6	50
3.9	51
4.5	51
4.7	53

Hint A linear model can be appropriate even if all the points do not lie exactly in a straight line. In these cases, the points should lie close to a straight line.

Homework Exercise

3 The cost of electricity, E, in pounds and the number of kilowatt hours, h, are shown in the table.

kilowatt hours, *h*	0	15	40	60	80	110
cost of electricity, *E*	45	46.8	49.8	52.2	54.6	58.2

a Draw a graph of the data. **(3 marks)**

b Explain how you know a linear model would be appropriate. **(1 mark)**

c Deduce an equation in the form $E = ah + b$. **(2 marks)**

d Interpret the meaning of the coefficients a and b. **(2 marks)**

e Use the model to find the cost of 65 kilowatt hours. **(1 mark)**

4 A racing car accelerates from rest to 90 m/s in 10 seconds. The table shows the total distance travelled by the racing car in each of the first 10 seconds.

time, *t* seconds	0	1	2	3	4	5	6	7	8	9	10
distance, *d* m	0	4.5	18	40.5	72	112.5	162	220.5	288	364.5	450

a Draw a graph of the data.

b Explain how you know a linear model would not be appropriate.

5 A website designer charges a flat fee and then a daily rate in order to design new websites for companies.

Company A's new website takes 6 days and they are charged £7100.

Company B's new website take 13 days and they are charged £9550.

Hint Let $(d_1, C_1) = (6, 7100)$ and $(d_2, C_2) = (13, 9550)$.

a Write an equation linking days, d and website cost, C in the form $C = ad + b$. **(3 marks)**

b Interpret the values of a and b. **(2 marks)**

c The web designer charges a third company £13 400. Calculate the number of days the designer spent working on the website. **(1 mark)**

Homework Exercise

6 The average August temperature in Exeter is 20 °C or 68 °F. The average January temperature in the same place is 9 °C or 48.2 °F.

a Write an equation linking Fahrenheit F and Celsius C in the form $F = aC + b$. **(3 marks)**

b Interpret the values of a and b. **(2 marks)**

c The highest temperature recorded in the UK was 101.3 °F. Calculate this temperature in Celsius. **(1 mark)**

d For what value is the temperature in Fahrenheit the same as the temperature in Celsius? **(3 marks)**

7 In 2004, in a city, there were 17 500 homes with internet connections. A service provider predicts that each year an additional 750 homes will get internet connections.

a Write a linear model for the number of homes n with internet connections t years after 2004.

b Write down one assumption made by this model.

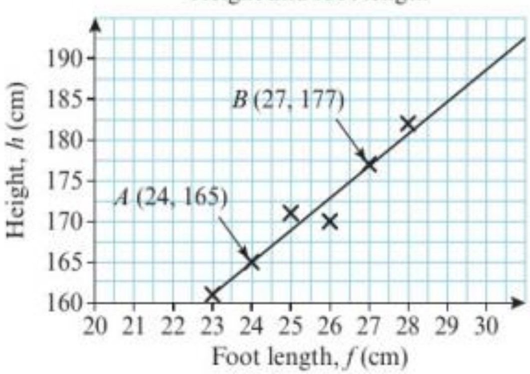
8 The scatter graph shows the height h and foot length f of 8 students. A line of best fit is drawn on the scatter graph.

a Explain why the data can be approximated to a linear model. **(1 mark)**

b Use points A and B on the scatter graph to write a linear equation in the form $h = af + b$. **(3 mark)**

c Calculate the expected height of a person with a foot length of 26.5 cm. **(1 mark)**

Height and foot length



9 The price P of a good and the quantity Q of a good are linked.

The demand for a new pair of trainers can be modelled using the equation $P = -\frac{3}{4}Q + 35$.

The supply of the trainers can be modelled using the equation $P = \frac{2}{3}Q + 1$.

a Draw a sketch showing the demand and supply lines on the same pair of axes.

The equilibrium point is the point where the supply and demand lines meet.

b Find the values of P and Q at the equilibrium point.

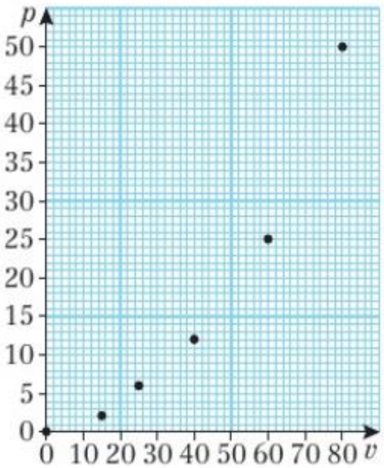
Homework Answers

1 **a** **i** $k = 50$ **ii** $d = 50t$

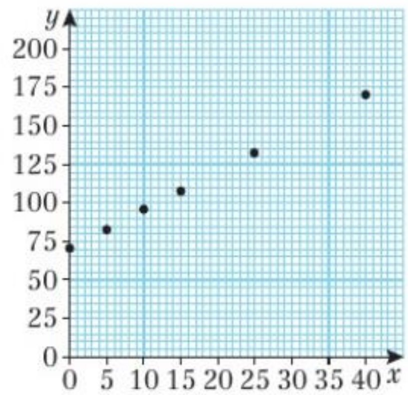
b **i** $k = 0.3$ or £0.30 **ii** $C = 0.3t$

c **i** $k = \frac{3}{5}$ **ii** $p = \frac{3}{5}t$

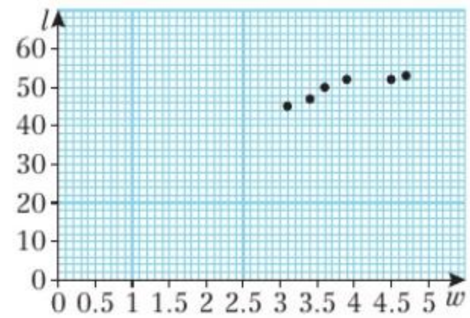
2 **a** not linear



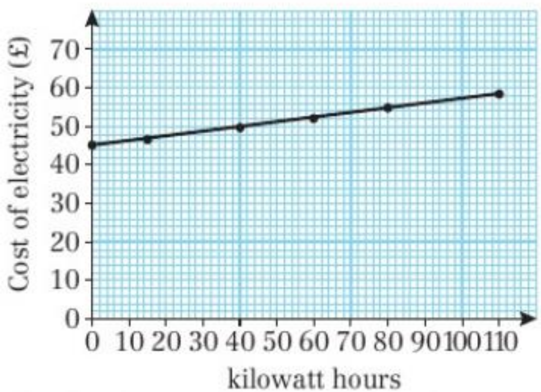
b linear



c not linear



3 **a**



b The data forms a straight line, so a linear model is appropriate.

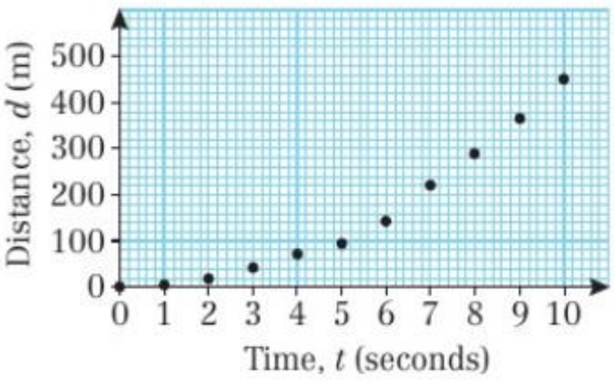
c $E = 0.12h + 45$

d $a = £0.12$ = cost of 1 kilowatt hour of electricity, $b = £45$ = fixed electricity costs (per month or per quarter)

e £52.80

Homework Answers

4 **a**



b The data does not follow a straight line. There is a definite curve to the points on the graph.

5 **a** $C = 350d + 5000$

b $a = 350$ = daily fee charged by the website designer.
$b = 5000$ = initial cost charged by the website designer.

c 24 days

6 **a** $F = 1.8C + 32$ or $F = \frac{9}{5}C + 32$

b $a = 1.8$ = increase in Fahrenheit temperature when the Celsius temperature increases by 1°C.
$b = 32$ temperature in Fahrenheit when temperature in Celsius is 0°.

c 38.5°C

d −40°C

7 **a** $n = 750t + 17\,500$

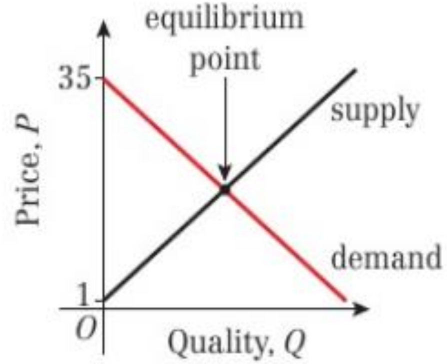
b The increase in the number of homes receiving the internet will be the same each year.

8 **a** All the points lie close to the straight line shown.

b $h = 4f + 69$

c 175 cm

9 **a**



b $Q = 24$, $P = 17$