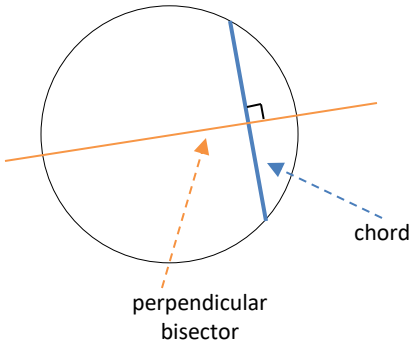
P1 Chapter 6: Circles

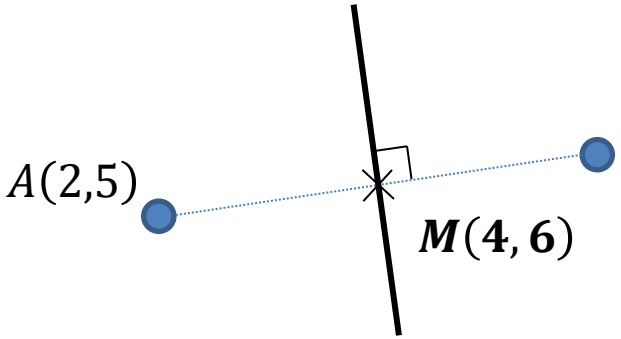
Perpendicular Bisectors

Midpoints and Perpendicular Bisectors

Later in the chapter you will need to find the perpendicular bisector of a chord of a circle.



What two properties does a perpendicular bisector of two points A and B have?



Just find the mean of the x values and the mean of the y values.

1. It passes through the midpoint of AB.
2. It is perpendicular to AB.

Equation?

$$\boldsymbol{m_{AB} = \frac{2}{4} = \frac{1}{2}}$$

$$\boldsymbol{m_{\perp} = -2}$$

$$\boldsymbol{y - 6 = -2(x - 4)}$$

Test Your Understanding

Find the perpendicular bisector of the line AB where A and B have the coordinates:

a) $A(4,7), B(10,17)$

b) $A(x_1, y_1), B(x_2, y_2)$

a

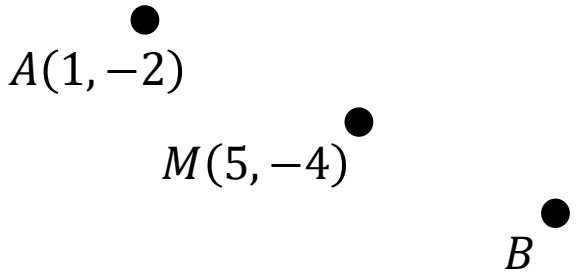
?

b

?

Fro Note: Do <u>not</u> try to memorise this!

A line segment AB is the diameter of a circle with centre $(5, -4)$. If A has coordinates $(1, -2)$, what are the coordinates of B?



?

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a

$M(7,12)$

$$m_{AB} = \frac{10}{6} = \frac{5}{3} \quad \therefore m_{\perp} = -\frac{3}{5}$$

$$y - 12 = -\frac{3}{5}(x - 7)$$

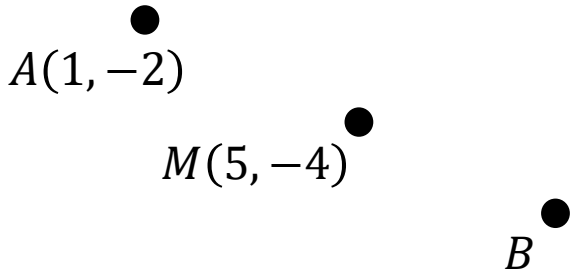
b

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_{\perp} = \frac{x_1 - x_2}{y_2 - y_1}$$

$$y - \frac{y_1 + y_2}{2} = \frac{x_1 - x_2}{y_2 - y_1}\left(x - \frac{x_1 + x_2}{2}\right)$$

Note: No need to memorise!



We could use the formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, but it is easy to see from the diagram that whatever movement there is from A to M, we can continue to get to B.

$$5 + 4 = 9$$

$$-4 - 2 = -6$$

$$B(9, -6)$$

Exercise 6.1

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Homework Exercise

1 Find the midpoint of the line segment joining each pair of points:

a $(4, 2), (6, 8)$ **b** $(0, 6), (12, 2)$ **c** $(2, 2), (-4, 6)$

d $(-6, 4), (6, -4)$ **e** $(7, -4), (-3, 6)$ **f** $(-5, -5), (-11, 8)$

g $(6a, 4b), (2a, -4b)$ **h** $(-4u, 0), (3u, -2v)$ **i** $(a + b, 2a - b), (3a - b, -b)$

j $(4\sqrt{2}, 1)\ (2\sqrt{2}, 7)$ **k** $(\sqrt{2} - \sqrt{3}, 3\sqrt{2} + 4\sqrt{3}), (3\sqrt{2} + \sqrt{3}, -\sqrt{2} + 2\sqrt{3})$

2 The line segment AB has endpoints $A(-2, 5)$ and $B(a, b)$. The midpoint of AB is $M(4, 3)$. Find the values of a and b.

3 The line segment PQ is a diameter of a circle, where P and Q are $(-4, 6)$ and $(7, 8)$ respectively. Find the coordinates of the centre of the circle.

4 The line segment RS is a diameter of a circle, where R and S are $\left(\frac{4a}{5}, -\frac{3b}{4}\right)$ and $\left(\frac{2a}{5}, \frac{5b}{4}\right)$ respectively. Find the coordinates of the centre of the circle.

Problem-solving

Your answer will be in terms of a and b.

5 The line segment AB is a diameter of a circle, where A and B are $(-3, -4)$ and $(6, 10)$ respectively.

a Find the coordinates of the centre of the circle.

b Show the centre of the circle lies on the line $y = 2x$.

6 The line segment JK is a diameter of a circle, where J and K are $\left(\frac{3}{4}, \frac{4}{3}\right)$ and $\left(-\frac{1}{2}, 2\right)$ respectively. The centre of the circle lies on the line segment with equation $y = 8x + b$. Find the value of b.

Homework Exercise

7 The line segment AB is a diameter of a circle, where A and B are $(0, -2)$ and $(6, -5)$ respectively. Show that the centre of the circle lies on the line $x - 2y - 10 = 0$.

8 The line segment FG is a diameter of the circle centre $(6, 1)$. Given F is $(2, -3)$, find the coordinates of G.

9 The line segment CD is a diameter of the circle centre $(-2a, 5a)$. Given D has coordinates $(3a, -7a)$, find the coordinates of C.

10 The points $M(3, p)$ and $N(q, 4)$ lie on the circle centre $(5, 6)$. The line segment MN is a diameter of the circle. Find the values of p and q.

Problem-solving

Use the formula for finding the midpoint:

$$\left(\frac{3+q}{2}, \frac{p+4}{2}\right) = (5, 6)$$

11 The points $V(-4, 2a)$ and $W(3b, -4)$ lie on the circle centre $(b, 2a)$. The line segment VW is a diameter of the circle. Find the values of a and b.

Challenge

A triangle has vertices at $A(3, 5)$, $B(7, 11)$ and $C(p, q)$. The midpoint of side BC is $M(8, 5)$.

a Find the values of p and q.

b Find the equation of the straight line joining the midpoint of AB to the point M.

c Show that the line in part **b** is parallel to the line AC.

Homework Answers

1 **a** $(5, 5)$ **b** $(6, 4)$ **c** $(-1, 4)$ **d** $(0, 0)$

e $(2, 1)$ **f** $(-8, \frac{3}{2})$ **g** $(4a, 0)$ **h** $\left(-\frac{u}{2}, -v\right)$

i $(2a, a - b)$ **j** $(3\sqrt{2}, 4)$ **k** $(2\sqrt{2}, \sqrt{2} + 3\sqrt{3})$

2 $a = 10, b = 1$

3 $(\frac{3}{2}, 7)$

4 $\left(\frac{3a}{5}, \frac{b}{4}\right)$

5 **a** $(\frac{3}{2}, 3)$ or $(1.5, 3)$ **b** $y = 2x$, $3 = 2 \times 1.5$

6 **a** $\left(\frac{1}{8}, \frac{5}{3}\right)$ **b** $\frac{2}{3}$

7 Centre is $\left(3, -\frac{7}{2}\right)$. $3 - 2\left(-\frac{7}{2}\right) - 10 = 0$

8 $(10, 5)$

9 $(-7a, 17a)$

10 $p = 8, q = 7$

11 $a = -2, b = 4$

Challenge

a $p = 9, q = -1$

b $y = -x + 13$

c AC: $y = -x + 8$. Lines have the same slope, so they are parallel.

Homework Exercise

1 Find the perpendicular bisector of the line segment joining each pair of points:

a $A(-5, 8)$ and $B(7, 2)$ **b** $C(-4, 7)$ and $D(2, 25)$ **c** $E(3, -3)$ and $F(13, -7)$

d $P(-4, 7)$ and $Q(-4, -1)$ **e** $S(4, 11)$ and $T(-5, -1)$ **f** $X(13, 11)$ and $Y(5, 11)$

2 The line FG is a diameter of the circle centre C, where F and G are $(-2, 5)$ and $(2, 9)$ respectively. The line l passes through C and is perpendicular to FG. Find the equation of l. **(7 marks)**

3 The line JK is a diameter of the circle centre P, where J and K are $(0, -3)$ and $(4, -5)$ respectively. The line l passes through P and is perpendicular to JK. Find the equation of l. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

4 Points A, B, C and D have coordinates $A(-4, -9)$, $B(6, -3)$, $C(11, 5)$ and $D(-1, 9)$.

a Find the equation of the perpendicular bisector of line segment AB.

b Find the equation of the perpendicular bisector of line segment CD.

c Find the coordinates of the point of intersection of the two perpendicular bisectors.

5 Point X has coordinates $(7, -2)$ and point Y has coordinates $(4, q)$. The perpendicular bisector of XY has equation $y = 4x + b$. Find the value of q and the value of b.

Problem-solving

It is often easier to find unknown values in the order they are given in the question. Find q first, then find b.

Challenge

Triangle PQR has vertices at $P(6, 9)$, $Q(3, -3)$ and $R(-9, 3)$.

a Find the perpendicular bisectors of each side of the triangle.

b Show that all three perpendicular bisectors meet at a single point, and find the coordinates of that point.

Links This point of intersection is called the **circumcentre** of the triangle. → Section 6.5

Homework Answers

1 **a** $y = 2x + 3$ **b** $y = -\frac{1}{3}x + \frac{47}{3}$ **c** $y = \frac{5}{2}x - 25$

d $y = 3$ **e** $y = -\frac{3}{4}x + \frac{37}{8}$ **f** $x = 9$

2 $y = -x + 7$

3 $2x - y - 8 = 0$

4 **a** $y = -\frac{5}{3}x - \frac{13}{3}$ **b** $y = 3x - 8$ **c** $\left(\frac{11}{14}, \frac{79}{14}\right)$

5 $q = -\frac{5}{4}$, $b = -\frac{189}{8}$

Challenge

a PR: $y = -\frac{5}{2}x + \frac{9}{4}$

PQ: $y = -\frac{1}{4}x + \frac{33}{8}$

RQ: $y = 2x + 6$

b $\left(-\frac{5}{6}, \frac{13}{3}\right)$