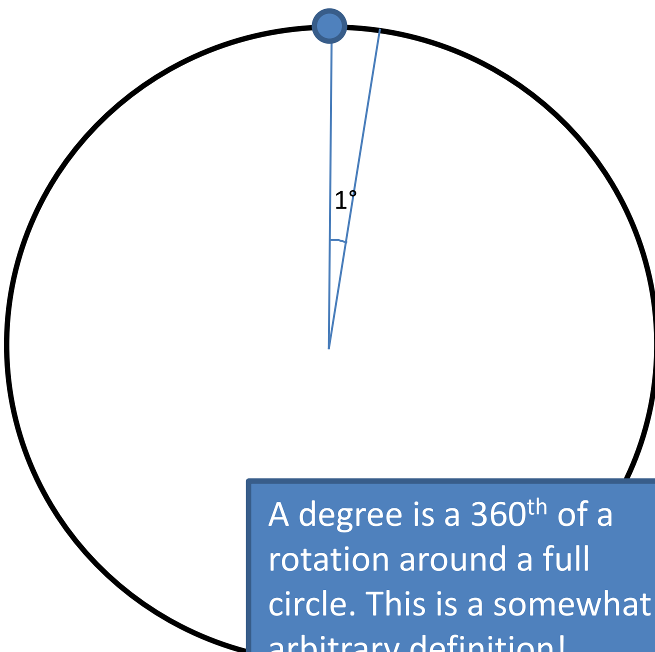
P2 Chapter 5: Radians

Radian Measure

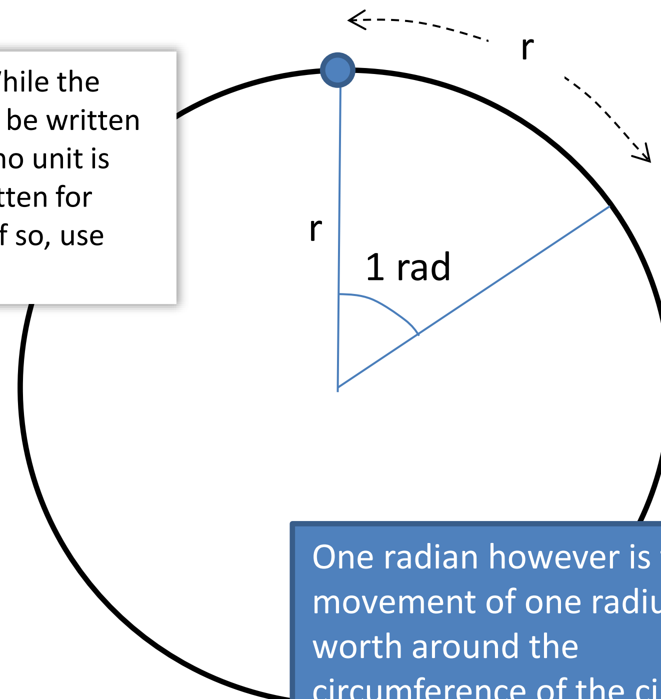
Radians

So far you've used **degrees** as the unit to measure angles.
But outside geometry, mathematicians pretty much always use **radians**.



A degree is a 360th of a rotation around a full circle. This is a somewhat arbitrary definition!

Unit Note: While the unit "°" must be written for degrees, no unit is generally written for radians (but if so, use "rad").



One radian however is the movement of one radius' worth around the circumference of the circle.

Click to Start Degree Animation

Click to Start Radian Animation

Thinking about how many radii around the circumference we can go: $\mathbf{360° = 2\pi}$ **rad**

Converting between radians and degrees

?

$$180^\circ = \pi$$

?

$$90^\circ = \boxed{?}$$

$$\frac{\pi}{3} = \boxed{?}$$

$$45^\circ = \boxed{?}$$

$$\frac{\pi}{6} = \boxed{?}$$

$$135^\circ = \boxed{?}$$

$$\frac{3}{2}\pi = \boxed{?}$$

$$72^\circ = \boxed{?}$$

$$\frac{5\pi}{6} = \boxed{?}$$

Converting between radians and degrees

$$\begin{tikzcd} 180^\circ \arrow[r, phantom, "="] \arrow[r, bend left=60, "\div 180 \text{ and } \times \pi"] & \pi \arrow[l, bend left=60, "\div \pi \text{ and } \times 180"] \end{tikzcd}$$

$$90^\circ = \boldsymbol{\frac{\pi}{2}}$$

$$\frac{\pi}{3} = \mathbf{60^\circ}$$

$$45^\circ = \boldsymbol{\frac{\pi}{4}}$$

$$\frac{\pi}{6} = \mathbf{30^\circ}$$

$$135^\circ = \boldsymbol{\frac{3}{4}\pi}$$

$$\frac{3}{2}\pi = \mathbf{270^\circ}$$

$$72^\circ = \boldsymbol{\frac{2}{5}\pi}$$

$$\frac{5\pi}{6} = \mathbf{150^\circ}$$

Be able to convert common angles in your head…

$45° =$?

$30° =$?

$60° =$?

$135° =$?

$270° =$?

$90° =$?

$120° =$?

Be able to convert common angles in your head…

$$45° = \frac{\pi}{4}$$

$$30° = \frac{\pi}{6}$$

$$60° = \frac{\pi}{3}$$

$$135° = \frac{3\pi}{4}$$

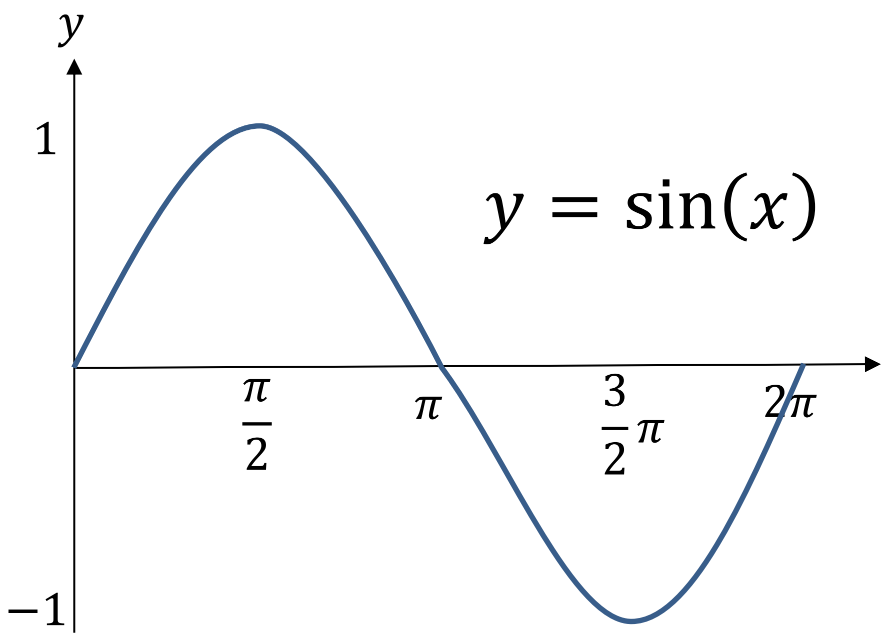
$$270° = \frac{3\pi}{2}$$

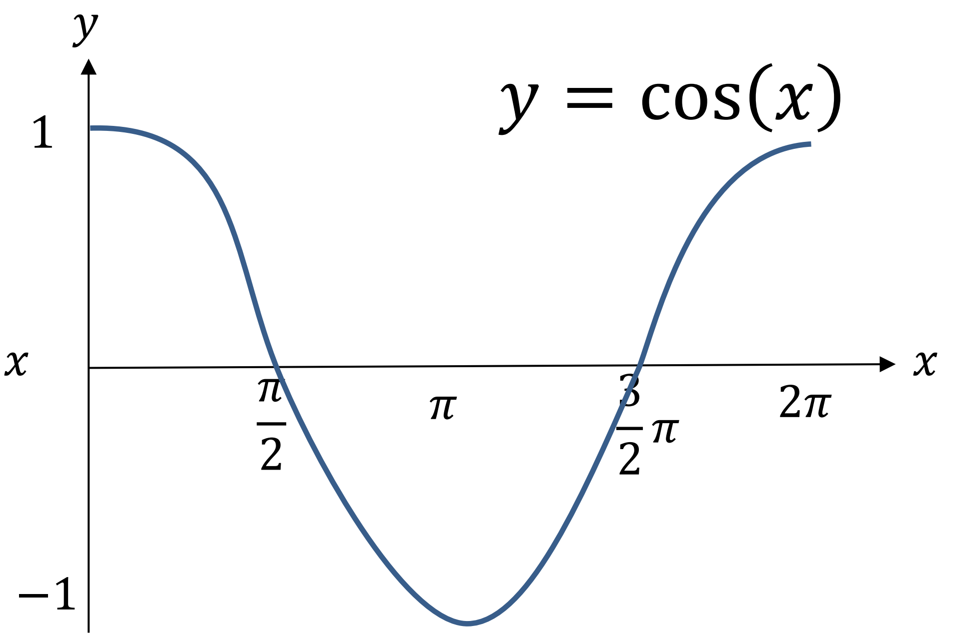
$$90° = \frac{\pi}{2}$$

$$120° = \frac{2\pi}{3}$$

Graph Sketching with Radians

We can replace the values $90°, 180°, 270°, 360°$ on the x-axis with their equivalent value in radians.





Test Your Understanding

Sketch the graph of $y = \cos\left(x + \frac{\pi}{2}\right)$ for $0 \leq x < 2\pi$.

?

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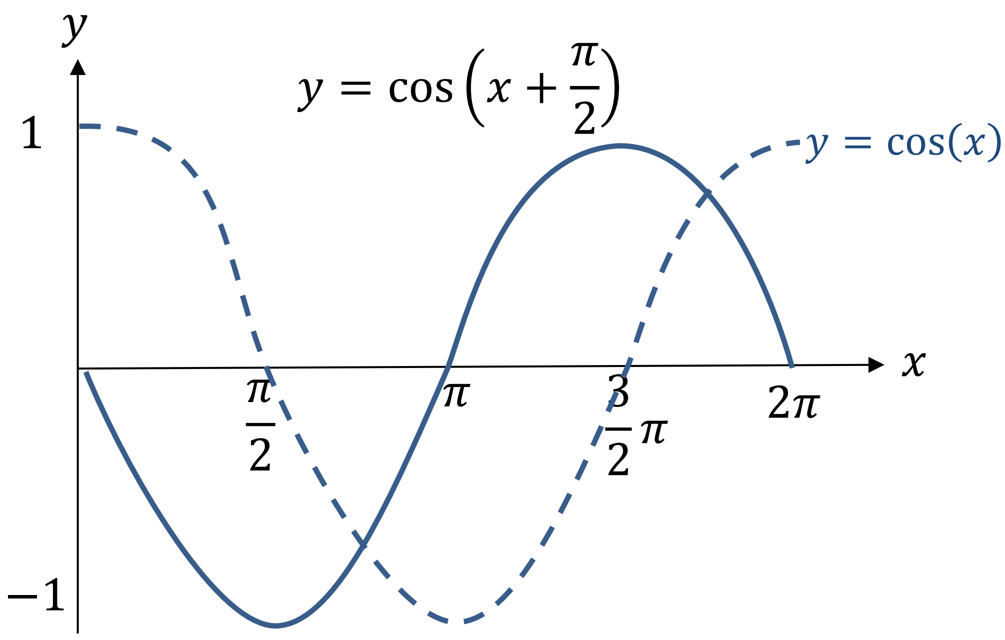

sin, cos, tan of angles in radians

Reminder of laws from Year 1:

- $\sin(x) = \sin(180 - x)$
- $\cos(x) = \cos(360 - x)$
- sin, cos repeat every $360°$ but tan every $180°$

- $\sin(x) = \sin(\pi - x)$
- $\cos(x) = \cos(2\pi - x)$
- sin, cos repeat every 2π but tan every π

To find sin/cos/tan of a '**common**' angle in radians without using a calculator, it is easiest to just **convert to degrees first**.

$\cos\left(\frac{4\pi}{3}\right) =$?

$\sin\left(-\frac{7\pi}{6}\right) =$?

To find $\cos\left(\frac{4\pi}{3}\right)$ directly using your calculator, you need to switch to radians mode. Press $SHIFT \rightarrow SETUP$, then $ANGLE\ UNIT$, then $Radians$. An R will appear at the top of your screen, instead of D.

sin, cos, tan of angles in radians

Reminder of laws from Year 1:

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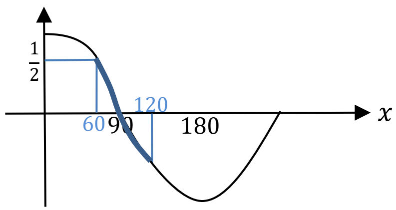
- sin, cos repeat every 2π but tan every π

To find sin/cos/tan of a '**common**' angle in radians without using a calculator, it is easiest to just **convert to degrees first**.

$$\cos\left(\frac{4\pi}{3}\right) = \mathbf{\cos(240°) = \cos(120°) = -\cos(60°) = -\frac{1}{2}}$$



$$\sin\left(-\frac{7\pi}{6}\right) = \mathbf{\sin(-210°) = \sin(150°) = \sin(30°) = \frac{1}{2}}$$

To find $\cos\left(\frac{4\pi}{3}\right)$ directly using your calculator, you need to switch to radians mode. Press $SHIFT \rightarrow SETUP$, then $ANGLE\ UNIT$, then $Radians$. An R will appear at the top of your screen, instead of D.

Exercise 5.1

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Homework Exercise

1 Convert the following angles in radians to degrees.

a $\frac{\pi}{20}$ **b** $\frac{\pi}{15}$ **c** $\frac{5\pi}{12}$ **d** $\frac{5\pi}{4}$ **e** $\frac{3\pi}{2}$ **f** 3π

2 Convert the following angles to degrees, giving your answer to 1 d.p.

a 0.46 rad **b** 1 rad **c** 1.135 rad **d** $\sqrt{3}$ rad

3 Evaluate the following, giving your answers to 3 significant figures.

a $\sin(0.5\,\text{rad})$ **b** $\cos(\sqrt{2}\,\text{rad})$ **c** $\tan(1.05\,\text{rad})$ **d** $\sin(2\,\text{rad})$ **e** $\sin(3.6\,\text{rad})$

4 Convert the following angles to radians, giving your answers as multiples of π.

a 8° **b** 10° **c** 22.5° **d** 30° **e** 112.5°
f 240° **g** 270° **h** 315° **i** 330°

5 Convert the following angles to radians, giving your answers to 3 significant figures.

a 50° **b** 75° **c** 100° **d** 160° **e** 230° **f** 320°

6 Sketch the graphs of:

a $y = \tan x$ for $0 \leqslant x \leqslant 2\pi$ **b** $y = \cos x$ for $-\pi \leqslant x \leqslant \pi$

Mark any points where the graphs cut the coordinate axes.

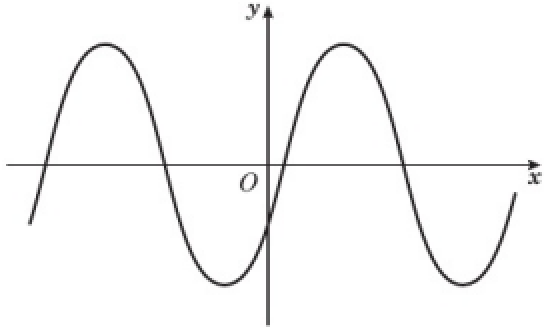
7 Sketch the following graphs for the given ranges, marking any points where the graphs cut the coordinate axes.

a $y = \sin(x - \pi)$ for $-\pi \leqslant x \leqslant \pi$ **b** $y = \cos 2x$ for $0 \leqslant x \leqslant 2\pi$

c $y = \tan\left(x + \frac{\pi}{2}\right)$ for $-\pi \leqslant x \leqslant \pi$ **d** $y = \sin\frac{1}{3}x + 1$ for $0 \leqslant x \leqslant 6\pi$

Homework Exercise

8 The diagram shows the curve with equation $y = \cos\left(x - \frac{2\pi}{3}\right)$, $-2\pi \leqslant x \leqslant 2\pi$.



Problem-solving

Make sure you write down the coordinates of all four points of intersection with the x-axis and the coordinates of the y-intercept.

Write down the coordinates of the points at which the curve meets the coordinate axes. **(3 marks)**

Challenge

Describe all the angles, θ, in radians, that satisfy:

a $\cos\theta = 1$

b $\sin\theta = -1$

c $\tan\theta$ is undefined.

Hint You can use $n\pi$, where n is an integer, to describe any integer multiple of π.

Homework Exercise

1 Express the following as trigonometric ratios of either $\frac{\pi}{6}$, $\frac{\pi}{4}$ or $\frac{\pi}{3}$, and hence find their exact values.

a $\sin\frac{3\pi}{4}$ **b** $\sin\left(-\frac{\pi}{3}\right)$ **c** $\sin\frac{11\pi}{6}$

d $\cos\frac{2\pi}{3}$ **e** $\cos\frac{5\pi}{3}$ **f** $\cos\frac{5\pi}{4}$

g $\tan\frac{3\pi}{4}$ **h** $\tan\left(-\frac{5\pi}{4}\right)$ **i** $\tan\frac{7\pi}{6}$

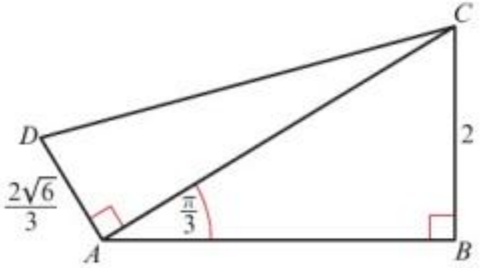
2 Without using a calculator, find the exact values of the following trigonometric ratios.

a $\sin\frac{7\pi}{3}$ **b** $\sin\left(-\frac{5\pi}{3}\right)$ **c** $\cos\left(-\frac{7\pi}{6}\right)$

d $\cos\frac{11\pi}{4}$ **e** $\tan\frac{5\pi}{3}$ **f** $\tan\left(-\frac{2\pi}{3}\right)$

3 The diagram shows a right-angled triangle ACD on another right-angled triangle ABC with $AD = \frac{2\sqrt{6}}{3}$ and $BC = 2$.

Show that $DC = k\sqrt{2}$, where k is a constant to be determined.



Homework Answers

1 **a** 9° **b** 12° **c** 75° **d** 225°
e 270° **f** 540°

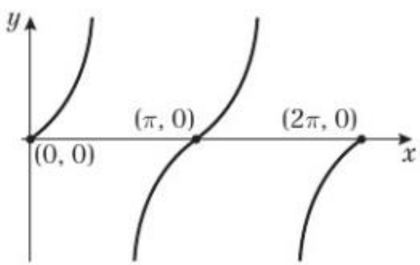
2 **a** 26.4° **b** 57.3° **c** 65.0° **d** 99.2°

3 **a** 0.479 **b** 0.156 **c** 1.74 **d** 0.909
e −0.443

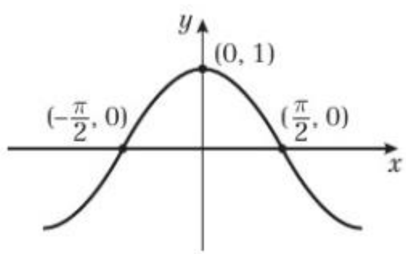
4 **a** $\frac{2\pi}{45}$ **b** $\frac{\pi}{18}$ **c** $\frac{\pi}{8}$ **d** $\frac{\pi}{6}$
e $\frac{5\pi}{8}$ **f** $\frac{4\pi}{3}$ **g** $\frac{3\pi}{2}$ **h** $\frac{7\pi}{4}$
i $\frac{11\pi}{6}$

5 **a** 0.873 rad **b** 1.31 rad **c** 1.75 rad **d** 2.79 rad
e 4.01 rad **f** 5.59 rad

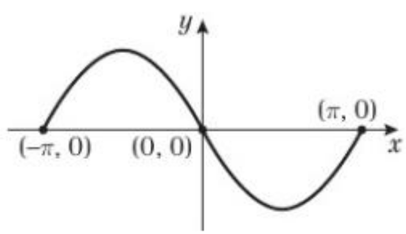
6 **a**



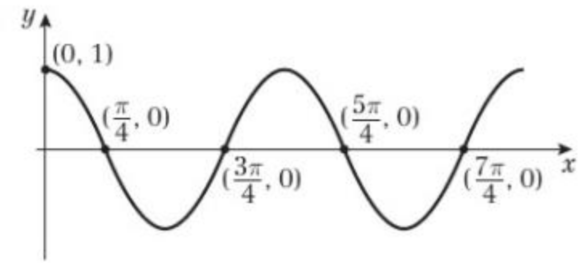
b



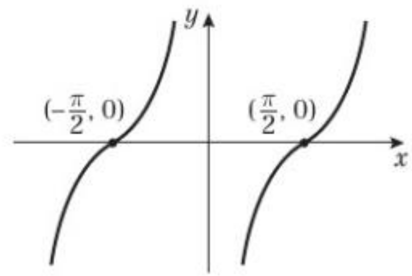
7 **a**



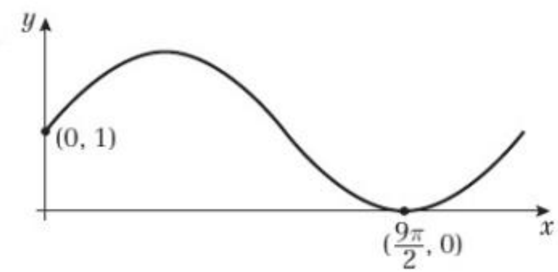
7 **b**



c



d



8 (0, −0.5)
$\left(-\frac{11\pi}{6},0\right), \left(-\frac{5\pi}{6},0\right), \left(\frac{\pi}{6},0\right), \left(\frac{7\pi}{6},0\right)$

Challenge

a $2\pi n,\ n \in \mathbb{Z}$ **b** $\frac{3\pi}{2} + 2\pi n,\ n \in \mathbb{Z}$ **c** $\frac{\pi}{2} + \pi n,\ n \in \mathbb{Z}$

Homework Answers

1 **a** $\sin\left(\pi - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ **b** $-\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

c $\sin\left(2\pi - \frac{\pi}{6}\right) = -\frac{1}{2}$ **d** $\cos\left(\pi - \frac{\pi}{3}\right) = -\frac{1}{2}$

e $\cos\left(2\pi - \frac{\pi}{3}\right) = \frac{1}{2}$ **f** $\cos\left(\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

g $\tan\left(\pi - \frac{\pi}{4}\right) = -1$ **h** $-\tan\left(\pi + \frac{\pi}{4}\right) = -1$

i $\tan\left(\pi + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$

2 **a** $\frac{\sqrt{3}}{2}$ **b** $\frac{\sqrt{3}}{2}$ **c** $-\frac{\sqrt{3}}{2}$

d $-\frac{\sqrt{2}}{2}$ **e** $-\sqrt{3}$ **f** $\sqrt{3}$

3 $AC = \dfrac{2}{\sin\left(\frac{\pi}{3}\right)} = \dfrac{4\sqrt{3}}{3}$

$DC^2 = AD^2 + AC^2 = \left(\frac{2\sqrt{6}}{3}\right)^2 + \left(\frac{4\sqrt{3}}{3}\right)^2 = 8$

$DC = 2\sqrt{2}$