M2 Chapter 7: Application of Forces

Static Rigid Bodies

Static rigid bodies

Recall from the chapter on moments that for a stationary rigid body:

- The **resultant force is 0**.
- The **resultant moment is 0**.

The problems are the same as in the moments chapter, except now we may need to consider frictional forces.

[Textbook] A uniform rod AB of mass 40kg and length 10m rests with the end A on rough horizontal ground. The rod rests against a smooth peg C where $AC = 8$ m. The rod is in limiting equilibrium at an angle of $15°$ to the horizontal. Find:

(a) the magnitude of the reaction of C

(b) the coefficient of friction between the rod and the ground.

? Diagram

a

Taking moments about A:

?

b

?

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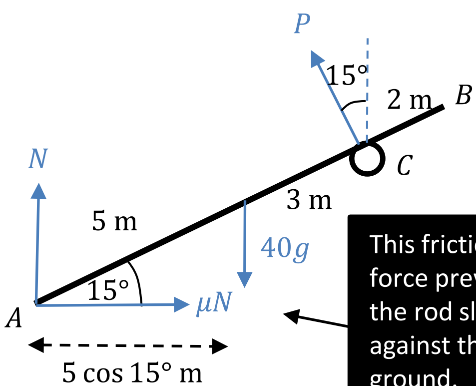

This frictional force prevents the rod slides against the ground.

a

Taking moments about A:

$$40g \times 5\cos 15° = P \times 8$$
$$P = 236.65$$

The normal support at C has magnitude 240N (2sf).

b

$$R(\rightarrow):\quad \mu N = P\sin 15° = 61.25$$
$$R(\uparrow):\quad N + P\cos 15° = 40g$$
$$\therefore N = 40g - P\cos 15° = 163.41$$
$$\therefore \mu = \frac{61.25}{163.41} = 0.37\ (2\text{sf})$$

Further Example

[Textbook] A ladder AB, of mass m and length $3a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $2m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium at an angle of 60° at an angle of 60° with the ground.

(a) Find the coefficient of friction between the ladder and the ground.

(b) State how you have used the assumption that the ladder is uniform in your calculations.

? Diagram

a

$R(\rightarrow)$: ?

$R(\uparrow)$: ?

Taking moments about B:

?

b

?

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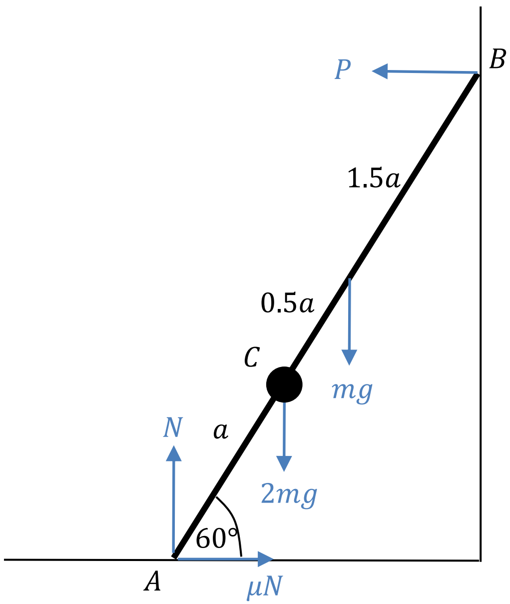


As before, two different reaction forces so need different variables.

a

$R(\rightarrow): \mu N = P$

$R(\uparrow): \quad N = 2mg + mg = 3mg$

Taking moments about B:

$(2mg \times 2a\cos 60°) + (mg \times 1.5a\cos 60°) + (\mu N \times 3a\sin 60°)$
$= N \times 3a\cos 60°$

$$2.75mg + \frac{3\sqrt{3}}{2}\mu N = 1.5$$

$$2.75mg + \frac{3\sqrt{3}}{2}\mu(3mg) = 1.5(3mg)$$

$$2.75 + \frac{9\sqrt{3}}{2}\mu = 4.5$$

$$\mu = 0.225 \text{ (3sf)}$$

b You can assume its weight acts at its midpoint.

Exercise 7.4

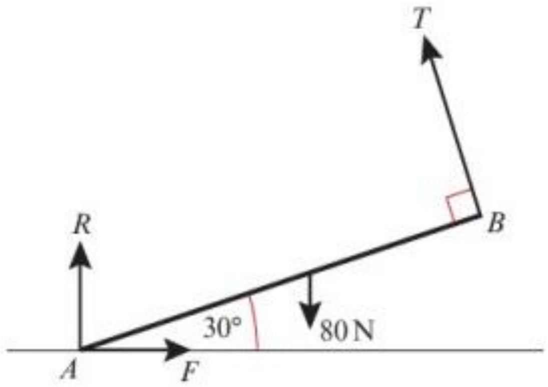
Pearson Stats/Mechanics Year 2

Pages 60-61

Homework Exercise

Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$.

1 A uniform rod AB of weight 80 N rests with its lower end A on a rough horizontal floor. A string attached to end B keeps the rod in equilibrium. The string is held at 90° to the rod. The tension in the string is T. The coefficient of friction between the rod and the ground is μ. R is the normal reaction at A and F is the frictional force at A. Find the magnitudes of T, R and F, and the least possible value of μ.



2 A uniform ladder of mass 10 kg and length 5 m rests against a smooth vertical wall with its lower end on rough horizontal ground. The ladder rests in equilibrium at an angle of 65° to the horizontal. Find:

a the magnitude of the normal reaction S at the wall

b the magnitude of the normal reaction R at the ground and the frictional force at the ground

c the least possible value of the coefficient of friction between the ladder and the ground.

d State how you have used the assumption that the ladder is uniform in your calculations.

3 A uniform ladder AB of mass 20 kg rests with its top A against a smooth vertical wall and its base B on rough horizontal ground. The coefficient of friction between the ladder and the ground is $\frac{3}{4}$. A mass of 10 kg is attached to the ladder. Given that the ladder is about to slip, find the inclination of the ladder to the horizontal,

a if the 10 kg mass is attached at A

b if the 10 kg mass is attached at B.

c State how you have used the assumption that the wall is smooth in your calculations.

Homework Exercise

Whenever a numerical value of g is required, take $g = 9.8\ \text{m s}^{-2}$.

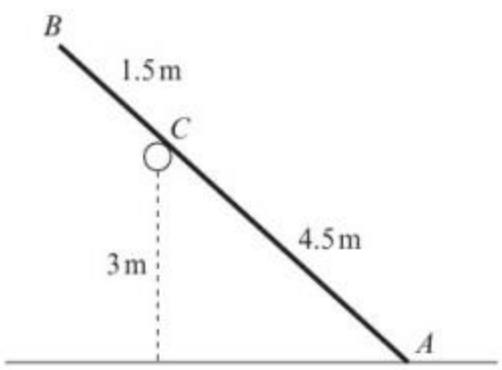
4 A uniform ladder of mass 20 kg and length 8 m rests against a smooth vertical wall with its lower end on rough horizontal ground. The coefficient of friction between the ground and the ladder is 0.3. The ladder is inclined at an angle θ to the horizontal, where $\tan\theta = 2$. A boy of mass 30 kg climbs up the ladder. By modelling the ladder as a uniform rod, the boy as a particle and the wall as smooth and vertical,

a find how far up the ladder the boy can climb before the ladder slips. **(8 marks)**

b Criticise this model with respect to:

i the ladder **ii** the wall. **(2 marks)**

5 A smooth horizontal rail is fixed at a height of 3 m above a rough horizontal surface. A uniform pole AB of weight 4 N and length 6 m is resting with end A on the rough ground and touching the rail at point C. The vertical plane containing the pole is perpendicular to the rail. The distance AC is 4.5 m and the pole is in limiting equilibrium. Calculate:



a the magnitude of the force exerted by the rail on the pole

b the coefficient of friction between the pole and the ground.

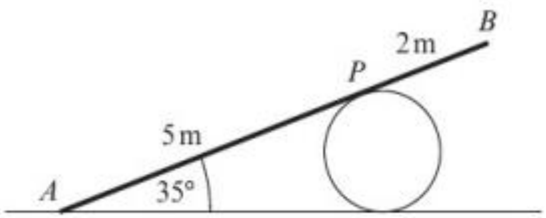
c State how you have used the assumption that the rail is smooth in your calculations

6 A uniform ladder rests in limiting equilibrium with its top against a smooth vertical wall and its base on a rough horizontal floor. The coefficient of friction between the ladder and the floor is μ. Given that the ladder makes an angle θ with the floor, show that $2\mu\tan\theta = 1$.

Homework Exercise

Whenever a numerical value of g is required, take $g = 9.8\ \mathrm{m\,s^{-2}}$.

7 A uniform ladder AB has length 7 m and mass 20 kg. The ladder is resting against a smooth cylindrical drum at P, where AP is 5 m, with end A in contact with rough horizontal ground. The ladder is inclined at 35° to the horizontal.



Find the normal and frictional components of the contact force at A, and hence find the least possible value of the coefficient of friction between the ladder and the ground.

8 A uniform ladder rests in limiting equilibrium with one end on rough horizontal ground and the other end against a rough vertical wall. The coefficient of friction between the ladder and the ground is μ_1 and the coefficient of friction between the ladder and the wall is μ_2. Given that the ladder makes an angle θ with the horizontal, show that $\tan\theta = \dfrac{1 - \mu_1\mu_2}{2\mu_1}$. **(8 marks)**

9 A uniform ladder of weight W rests in equilibrium with one end on rough horizontal ground and the other resting against a smooth vertical wall. The vertical plane containing the ladder is at right angles to the wall and the ladder is inclined at 60° to the horizontal. The coefficient of friction between the ladder and the ground is μ.

a Find, in terms of W, the magnitude of the force exerted by the wall on the ladder. **(6 marks)**

b Show that $\mu \geqslant \dfrac{\sqrt{3}}{6}$. **(3 marks)**

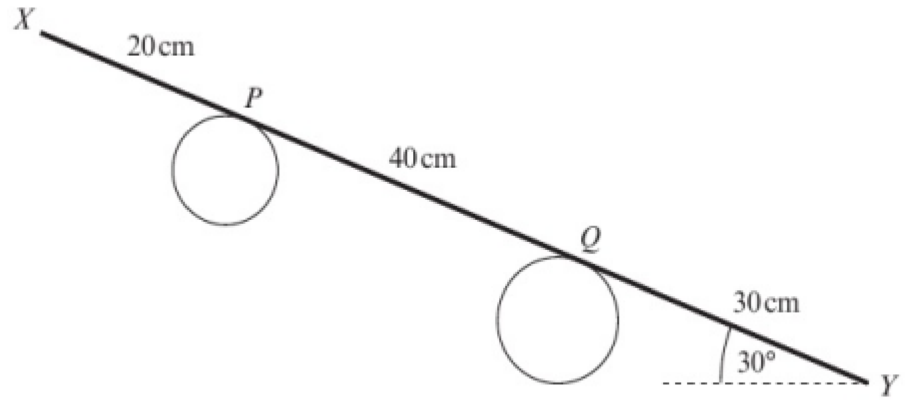
A load of weight w is attached to the ladder at its upper end (resting against the wall).

c Given that $\mu = \dfrac{\sqrt{3}}{5}$ and that the equilibrium is limiting, find w in terms of W. **(8 marks)**

Homework Exercise

Whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$.

10 A uniform rod XY has weight 20 N and length 90 cm. The rod rests on two parallel pegs, with X above Y, in a vertical plane which is perpendicular to the axes of the pegs, as shown in the diagram. The rod makes an angle of 30° to the horizontal and touches the two pegs at P and Q, where $XP = 20$ cm and $XQ = 60$ cm.



a Calculate the normal components of the forces on the rod at P and at Q. **(8 marks)**

The coefficient of friction between the rod and each peg is μ.

b Given that the rod is about to slip, find μ. **(2 marks)**

Homework Exercise

Whenever a numerical value of g is required, take $g = 9.8\text{ m s}^{-2}$.

11 A ladder XY, of length l and weight W, has its end X on rough horizontal ground. The coefficient of friction between the ladder and the ground is $\frac{1}{5}$. The end Y of the ladder is resting against a smooth vertical wall. A window cleaner of weight $9W$ stands at the top of the ladder. To stop the ladder from slipping, the window cleaner's assistant applies a horizontal force of magnitude P to the ladder at X, towards the wall. The force acts in a direction which is perpendicular to the wall. The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle θ with the horizontal ground, where $\tan\theta = \sqrt{3}$. The window cleaner is modelled as a particle and the ladder is modelled as a uniform rod.

a Find, in terms of W, the reaction of the wall on the ladder at Y. **(5 marks)**

b Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium. **(5 marks)**

c State how you have used the modelling assumption that the ladder is uniform in your calculations. **(1 mark)**

In practice, the ladder is wider and heavier at the bottom. The model is adjusted so the ladder is modelled as a non-uniform rod with its centre of mass closer to the base.

d State, with a reason, the effect this will have on

i the magnitude of the reaction of the wall on the ladder at Y

ii the range of possible values of P for which the ladder remains in equilibrium. **(4 marks)**

Homework Answers

1 34.6 N, 50 N, 17.3 N, 0.35

2 **a** 22.8 N **b** 98 N, 22.8 N **c** 0.233

d The weight of a uniform ladder passes through its midpoint.

3 **a** 41.6° **b** 24.0°

c No friction at the wall.

4 **a** $5\frac{1}{3}$ m

b i The ladder may not be uniform.

ii There would be friction between the ladder and the vertical wall.

5 **a** 1.99 N **b** 0.526

c There is no friction between the rail and the pole.

6 R(↑): $R = mg$, R(→): $N = \mu mg$

Let the length of the ladder be $2l$

Taking moments about A,

$mgl\cos\theta = 2\mu mgl\sin\theta$

Cancelling and rearranging gives

$2\mu\tan\theta = 1$

7 104 N, 64.5 N, 0.620

8 R(↑): $R + \mu_2 N = mg$, R(→): $N = \mu_1 R$

Taking moments about the base of the ladder

$2N\tan\theta + 2\mu_2 N = mg$

$2\tan\theta + 2\mu_2 = \frac{1}{\mu_1} + \mu_2$

$2\tan\theta = \frac{1}{\mu_1} - \mu_2$

$\tan\theta = \frac{1 - \mu_1\mu_2}{2\mu_1}$

9 **a** $\frac{W\sqrt{3}}{6}$

b $F = P = \frac{W\sqrt{3}}{6}$, $R = W$, $F \leqslant \mu R$, $\mu \geqslant \frac{\sqrt{3}}{6}$

c $\frac{W}{4}$

10 **a** 6.50 N, 10.8 N **b** $\frac{1}{\sqrt{3}}$

11 **a** $\frac{19}{2\sqrt{3}}W$ **b** $\left(\frac{19}{2\sqrt{3}} - 2\right)W \leqslant P \leqslant \left(\frac{19}{2\sqrt{3}} + 2\right)W$

c It allows us to assume that the weight of the ladder acts through its midpoint.

d i The magnitude of the reaction at Y will get smaller.

ii The range of values for P will get smaller.