M2 Chapter 8: Further Kinematics

Vector Projectiles

Vector methods for projectiles

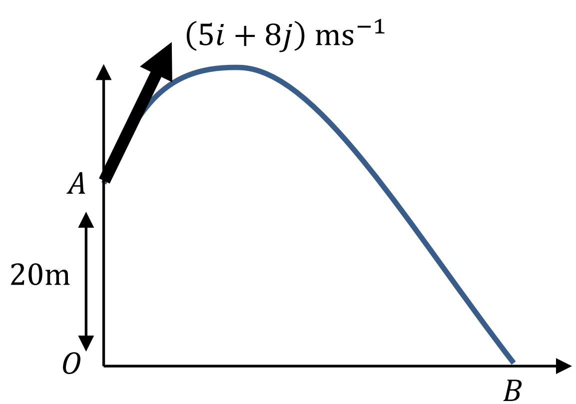
Previously we considered the initial speed of the projectile and the angle of projection. But we could also **use a velocity vector to represent the initial projection** (vectors have both direction and magnitude) and subsequent motion.

[Textbook] A ball is struck by a racket from a point A which has position vector $20j$ m relative to a fixed origin O. Immediately after being struck, the ball has velocity $(5i + 8j)$ ms^{-1}, where i and j are unit vectors horizontally and vertically respectively. After being struck, the ball travels freely under gravity until it strikes the ground at point B.

(a) Find the speed of the ball 1.5 seconds after being struck.

(b) Find an expression for the position vector, r, of the ball relative to O at time t seconds.

(c) Hence determine the distance OB.



a

?

b

?

c

?

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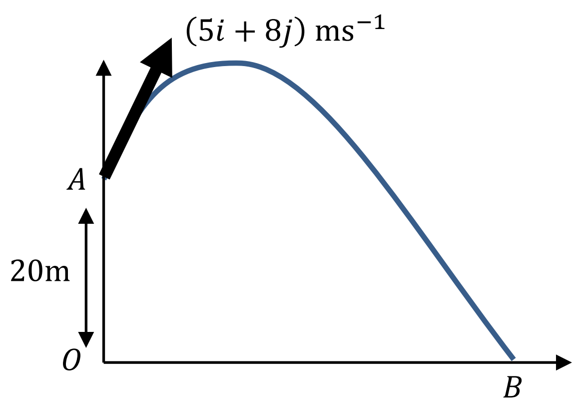

a

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$= \begin{pmatrix}5\\8\end{pmatrix} + \begin{pmatrix}0\\-9.8\end{pmatrix}t = \begin{pmatrix}5\\8-9.8t\end{pmatrix}$$

When $t = 1.5, \boldsymbol{v} = \begin{pmatrix}5\\-6.7\end{pmatrix}$

Speed $= |\boldsymbol{v}| = \sqrt{5^2 + 6.7^2} = 8.4$ms^{-1}

b

Displacement relative to A:

$$\boldsymbol{r}_A = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$
$$= \begin{pmatrix}5\\8\end{pmatrix}t + \frac{1}{2}\begin{pmatrix}0\\-9.8\end{pmatrix}t^2 = \begin{pmatrix}5t\\8t-4.9t^2\end{pmatrix}$$

So position relative to O:

$$r_O = \begin{pmatrix}0\\20\end{pmatrix} + \begin{pmatrix}5t\\8t-4.9t^2\end{pmatrix}$$
$$= \begin{pmatrix}5t\\8t-4.9t^2+20\end{pmatrix}$$

c

When $\boldsymbol{j}$-component is 0:

$8t - 4.9t^2 + 20 = 0 \quad \rightarrow \quad t = 2.995\ldots$

i-component: $5t = 15$ m (2sf)

Test Your Understanding

Edexcel M2(Old) Jan 2012 Q7

[In this question, the unit vectors **i** *and* **j** *are horizontal and vertical respectively.]*

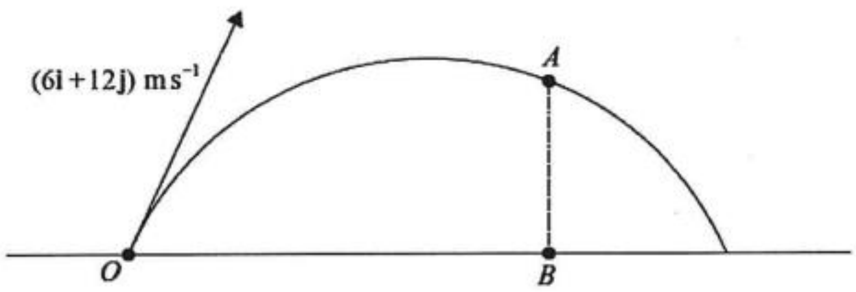

Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(*a*) the value of t,

(7)

(*b*) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(*c*) Find the time taken for the ball to travel from O to C.

(3)

(a) ?

(b) ?

(c) ?

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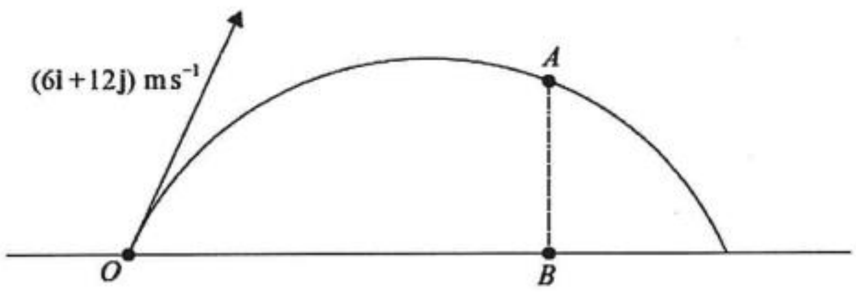

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(a)	$\mathbf{i} \rightarrow$ distance $= 6t$	B1
	$\mathbf{j} \uparrow$ distance $= 12t - \frac{1}{2}gt^2$	M1 A1
	At B, $2\left(12t - \frac{1}{2}gt^2\right) = 6t$	M1 A1
	$(24-6)t = gt^2$	DM1
	$18 = gt$, $t = \frac{18}{g} (= 1.84\text{s})$	A1
(b)	$\mathbf{i} \rightarrow$ speed $= 6$	B1
	$\mathbf{j} \uparrow$ velocity $= 12 - gt = -6$	M1 A1
	$\therefore$ speed at A	
	$= \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2} (= 8.49)(\text{ms}^{-1})$	M1 A1
(c)	$\uparrow$ speed $= 12 - gt = +6$	M1 A1 ft
	$t = \frac{6}{g} (= 0.61\text{s})$	A1

Exercise 8.2

Pearson Stats/Mechanics Year 2

Pages 69-70

Homework Exercise

For all questions in this exercise **i** and **j** are unit vectors horizontally and vertically respectively.

Unless stated otherwise, take $g = 9.8\ \text{m s}^{-2}$.

1 A particle P is projected from the origin with velocity $(12\mathbf{i} + 24\mathbf{j})\ \text{m s}^{-1}$. The particle moves freely under gravity. Find:

a the position vector of P after 3 s

b the speed of P after 3 s.

2 In this question use $g = 10\ \text{m s}^{-2}$

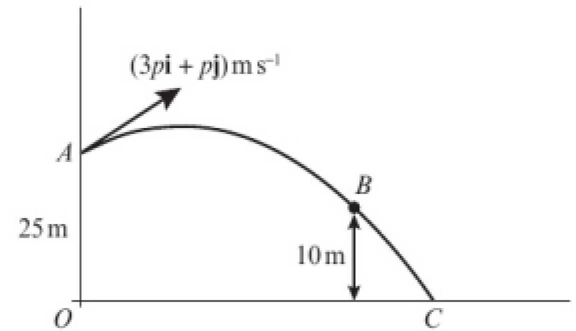
A particle P is projected from the origin with velocity $(4\mathbf{i} + 5\mathbf{j})\ \text{m s}^{-1}$. The particle moves freely under gravity. Find:

a the position vector of P after t s

b the greatest height of the particle.

Hint When the particle is at its greatest height, the **j**-component of the velocity will be 0.

3 A ball is projected from a point A at the top of a cliff, with position vector $25\mathbf{j}$ m relative to the base of the cliff O. The base of the cliff is at sea level. The velocity of projection is $(3p\mathbf{i} + p\mathbf{j})\ \text{m s}^{-1}$, where p is a constant. After 2 seconds, the ball passes a point B with position vector $(q\mathbf{i} + 10\mathbf{j})$ m, where q is a constant, before hitting the sea at point C. The ball is modelled as a particle moving freely under gravity and the sea is modelled as a horizontal plane.



a Suggest, with reasons, which of these two modelling assumptions is most realistic. **(2 marks)**

b Find the velocity vector of the ball at point B. **(6 marks)**

A remote-control boat leaves O at the same time the ball is projected, and travels in a straight line towards C with constant acceleration. Given that the ball lands on the boat,

c find the acceleration of the boat. **(6 marks)**

Homework Exercise

5 A particle is projected with velocity $(8\mathbf{i} + 10\mathbf{j})\,\text{m s}^{-1}$ from a point O at the top of a cliff and moves freely under gravity. Six seconds after projection, the particle strikes the sea at the point S. Calculate:

a the horizontal distance between O and S **(2 marks)**

b the vertical distance between O and S. **(3 marks)**

At time T seconds after projection, the particle is moving with velocity $(8\mathbf{i} - 14.5\mathbf{j})\,\text{m s}^{-1}$.

c Find the value of T and the position vector, relative to O, of the particle at this instant. **(6 marks)**

6 In this question use $g = 10\,\text{m s}^{-2}$

A body B is projected from a fixed point O on horizontal ground with velocity $a\mathbf{i} + b\mathbf{j}\,\text{m s}^{-1}$, where a and b are positive constants. The body moves freely under gravity until it hits the ground at the point P, where it immediately comes to rest.

The position vector of a point on the path of B relative to O is $(x\mathbf{i} + y\mathbf{j})\,\text{m}$.

a Show that $y = \dfrac{bx}{a} - \dfrac{5x^2}{a^2}$ **(5 marks)**

Given that $a = 8$, $OP = X\,\text{m}$ and the maximum vertical height of B above the ground is $Y\,\text{m}$,

b find, in terms of b,

i X **ii** Y **(6 marks)**

Homework Answers

1 **a** $(36\mathbf{i} + 27.9\mathbf{j})$ m **b** $13\,\text{m s}^{-1}$ (2 s.f.)

2 **a** $\mathbf{r} = (4t)\mathbf{i} + (5t - 5t^2)\mathbf{j}$ **b** 1.25 m

3 **a** Either answer with justification
e.g. The sea is likely to be horizontal and relatively flat, whereas the ball is subject to air resistance, so the assumption that sea is a horizontal plane is most reasonable.
Or e.g. Although the sea is horizontal it is unlikely to be flat because of waves, so the assumption that the ball is a particle is most reasonable.

b $\mathbf{v} = (6.9\mathbf{i} - 17\mathbf{j})\,\text{m s}^{-1}$ (both values to 2 s.f.)

c $5.5\,\text{m s}^{-2}$ (2 s.f.)

4 **a** $R(\uparrow)$: $0 = 4ut - \frac{g}{2}t^2 \Rightarrow t = \frac{8u}{g}$

$R(\rightarrow)$: $750 = 3ut = \frac{24u^2}{g} \Rightarrow u^2 = \frac{750g}{24} \Rightarrow u = 17.5$

b 250m **c** 22° (nearest degree)

5 **a** 48 m **b** 120 m (2 s.f.)

c $T = 2.5$ s, $\mathbf{r} = (20\mathbf{i} - \frac{45}{8}\mathbf{j})$ m

6 **a** $x = at \Rightarrow t = \frac{x}{a}$

$y = bt - 5t^2 \Rightarrow y = b\left(\frac{x}{a}\right) - 5\left(\frac{x}{a}\right)^2 \Rightarrow y = \frac{bx}{a} - \frac{5x^2}{a^2}$

b **i** $X = 1.6b$ **ii** $Y = 0.05b^2$