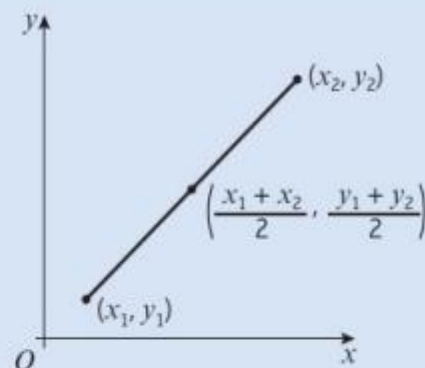

P1 Chapter 6: Circles

Chapter Practice

Key Points

- 1** The midpoint of a line segment with endpoints

(x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



- 2** The perpendicular bisector of a line segment AB is the straight line that is perpendicular to AB and passes through the midpoint of AB .

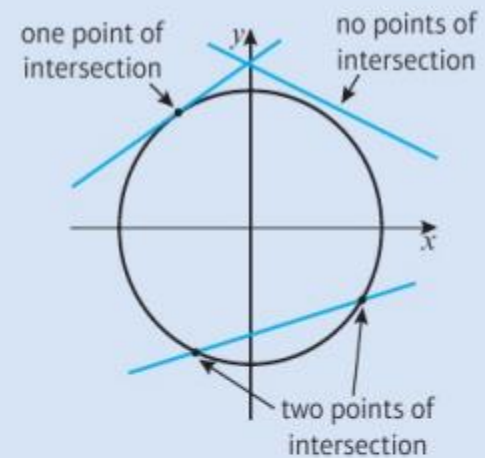


If the gradient of AB is m then the gradient of its perpendicular bisector, l , will be $-\frac{1}{m}$.

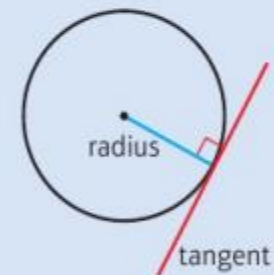
- 3** The equation of a circle with centre $(0, 0)$ and radius r is $x^2 + y^2 = r^2$.
- 4** The equation of the circle with centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$.
- 5** The equation of a circle can be given in the form: $x^2 + y^2 + 2fx + 2gy + c = 0$
This circle has centre $(-f, -g)$ and radius $\sqrt{f^2 + g^2 - c}$

Key Points

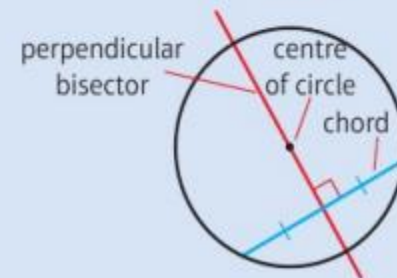
- 6** A straight line can intersect a circle once, by just touching the circle, or twice. Not all straight lines will intersect a given circle.



- 7** A tangent to a circle is perpendicular to the radius of the circle at the point of intersection.

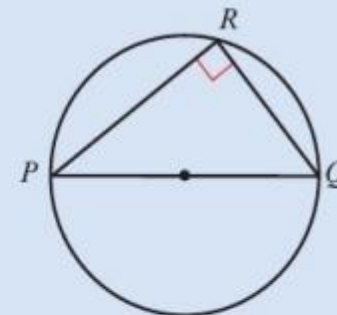


- 8** The perpendicular bisector of a chord will go through the centre of a circle.



Key Points

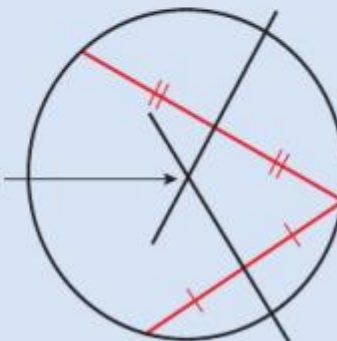
- 9** • If $\angle PRQ = 90^\circ$ then R lies on the circle with diameter PQ .
• The angle in a semicircle is always a right angle.



- 10** To find the centre of a circle given any three points:

- Find the equations of the perpendicular bisectors of two different chords.
- Find the coordinates of intersection of the perpendicular bisectors.

Perpendicular
bisectors intersect
at the centre of
the circle



Chapter Exercises

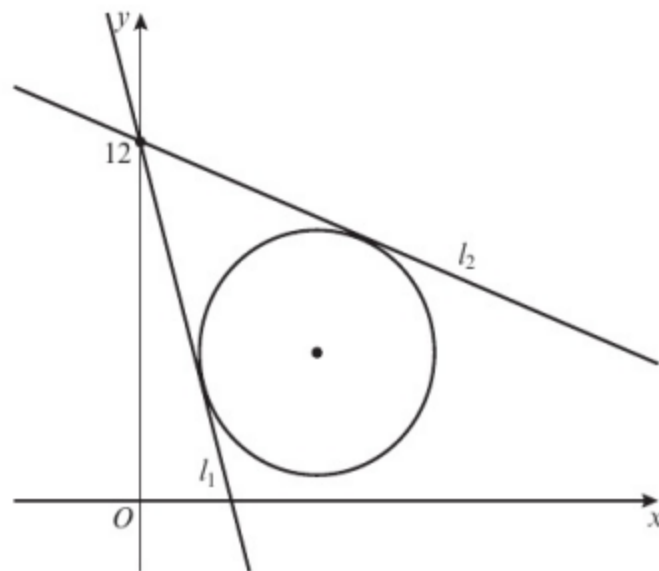
- 1 The line segment QR is a diameter of the circle centre C , where Q and R have coordinates $(11, 12)$ and $(-5, 0)$ respectively. The point P has coordinates $(13, 6)$.
 - a Find the coordinates of C .
 - b Find the radius of the circle.
 - c Write down the equation of the circle.
 - d Show that P lies on the circle.
- 2 Show that $(0, 0)$ lies inside the circle $(x - 5)^2 + (y + 2)^2 = 30$.
- 3 The circle C has equation $x^2 + 3x + y^2 + 6y = 3x - 2y - 7$.
 - a Find the centre and radius of the circle. **(4 marks)**
 - b Find the points of intersection of the circle and the y -axis. **(3 marks)**
 - c Show that the circle does not intersect the x -axis. **(2 marks)**
- 4 The centres of the circles $(x - 8)^2 + (y - 8)^2 = 117$ and $(x + 1)^2 + (y - 3)^2 = 106$ are P and Q respectively.
 - a Show that P lies on $(x + 1)^2 + (y - 3)^2 = 106$.
 - b Find the length of PQ .
- 5 The points $A(-1, 0)$, $B\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $C\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ are the vertices of a triangle.
 - a Show that the circle $x^2 + y^2 = 1$ passes through the vertices of the triangle.
 - b Show that $\triangle ABC$ is equilateral.

Chapter Exercises

- 6 A circle with equation $(x - k)^2 + (y - 3k)^2 = 13$ passes through the point $(3, 0)$.
- a Find two possible values of k . (6 marks)
 - b Given that $k > 0$, write down the equation of the circle. (1 mark)
- 7 The line with $3x - y - 9 = 0$ does not intersect the circle with equation $x^2 + px + y^2 + 4y = 20$.
Show that $42 - \sqrt{10} < p < 42 + 10\sqrt{10}$. (6 marks)
- 8 The line $y = 2x - 8$ meets the coordinate axes at A and B . The line segment AB is a diameter of the circle. Find the equation of the circle.
- 9 The circle centre $(8, 10)$ meets the x -axis at $(4, 0)$ and $(a, 0)$.
- a Find the radius of the circle.
 - b Find the value of a .
- 10 The circle $(x - 5)^2 + y^2 = 36$ meets the x -axis at P and Q . Find the coordinates of P and Q .
- 11 The circle $(x + 4)^2 + (y - 7)^2 = 121$ meets the y -axis at $(0, m)$ and $(0, n)$.
Find the values of m and n .
- 12 The circle C with equation $(x + 5)^2 + (y + 2)^2 = 125$ meets the positive coordinate axes at $A(a, 0)$ and $B(0, b)$.
- a Find the values of a and b . (2 marks)
 - b Find the equation of the line AB . (2 marks)
 - c Find the area of the triangle OAB , where O is the origin. (2 marks)

Chapter Exercises

- 13** The circle, centre (p, q) radius 25, meets the x -axis at $(-7, 0)$ and $(7, 0)$, where $q > 0$.
- a** Find the values of p and q .
 - b** Find the coordinates of the points where the circle meets the y -axis.
- 14** The point $A(-3, -7)$ lies on the circle centre $(5, 1)$. Find the equation of the tangent to the circle at A .
- 15** The line segment AB is a chord of a circle centre $(2, -1)$, where A and B are $(3, 7)$ and $(-5, 3)$ respectively. AC is a diameter of the circle. Find the area of $\triangle ABC$.
- 16** The circle C has equation $(x - 6)^2 + (y - 5)^2 = 17$.
The lines l_1 and l_2 are each a tangent to the circle and intersect at the point $(0, 12)$.
Find the equations of l_1 and l_2 , giving your answers in the form $y = mx + c$. **(8 marks)**



Chapter Exercises

- 17 The points A and B lie on a circle with centre C , as shown in the diagram.

The point A has coordinates $(3, 7)$ and the point B has coordinates $(5, 1)$.

M is the midpoint of the line segment AB .

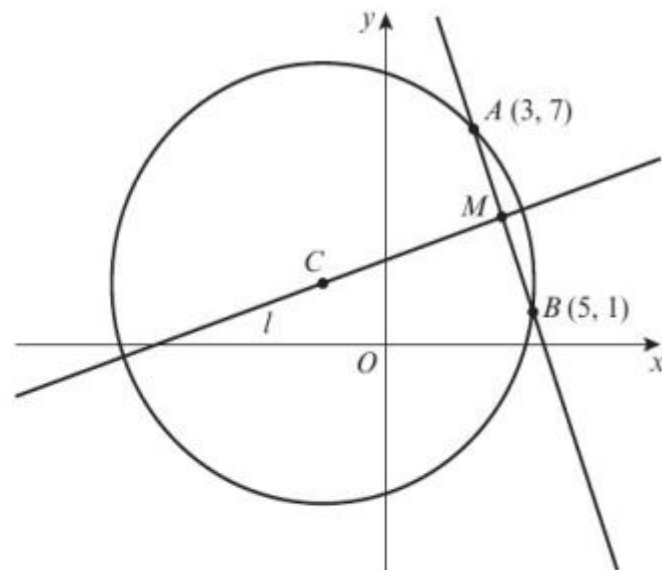
The line l passes through the points M and C .

- a Find an equation for l . (4 marks)

Given that the x -coordinate of C is -2 :

- b find an equation of the circle (4 marks)

- c find the area of the triangle ABC . (3 marks)

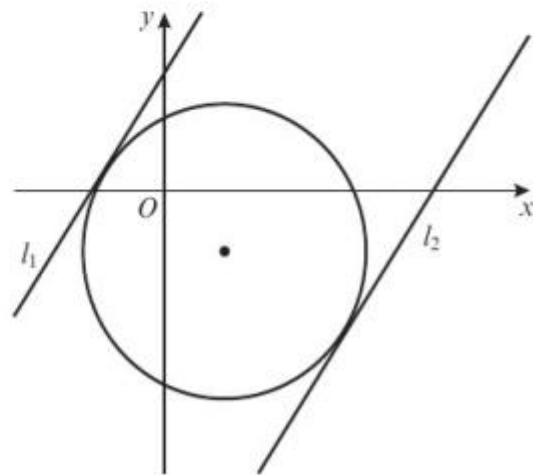


- 18 The circle C has equation $(x - 3)^2 + (y + 3)^2 = 52$.

The baselines l_1 and l_2 are tangents to the circle and have gradient $\frac{3}{2}$.

- a Find the points of intersection, P and Q , of the tangents and the circle. (6 marks)

- b Find the equations of lines l_1 and l_2 , giving your answers in the form $ax + by + c = 0$. (2 marks)



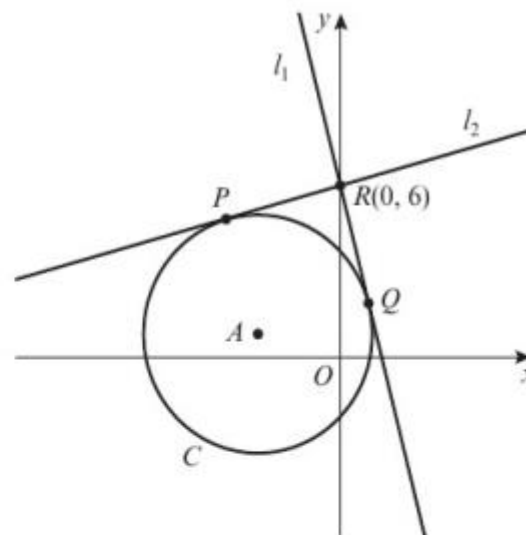
Chapter Exercises

- 19 The circle C has equation $x^2 + 6x + y^2 - 2y = 7$.

The lines l_1 and l_2 are tangents to the circle.

They intersect at the point $R(0, 6)$.

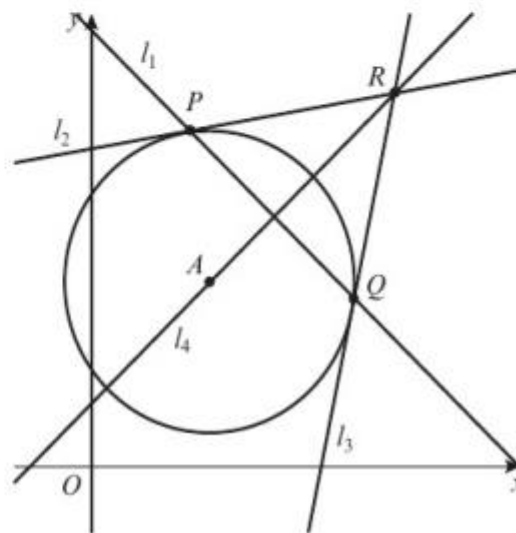
- a Find the equations of lines l_1 and l_2 , giving your answers in the form $y = mx + b$. (6 marks)
- b Find the points of intersection, P and Q , of the tangents and the circle. (4 marks)
- c Find the area of quadrilateral $APRQ$. (2 marks)



- 20 The circle C has a centre at $(6, 9)$ and a radius of $\sqrt{50}$.

The line l_1 with equation $x + y - 21 = 0$ intersects the circle at the points P and Q .

- a Find the coordinates of the point P and the point Q . (5 marks)
- b Find the equations of l_2 and l_3 , the tangents at the points P and Q respectively. (4 marks)
- c Find the equation of l_4 , the perpendicular bisector of the chord PQ . (4 marks)
- d Show that the two tangents and the perpendicular bisector intersect and find the coordinates of R , the point of intersection. (2 marks)
- e Calculate the area of the kite $APRQ$. (3 marks)



Chapter Exercises

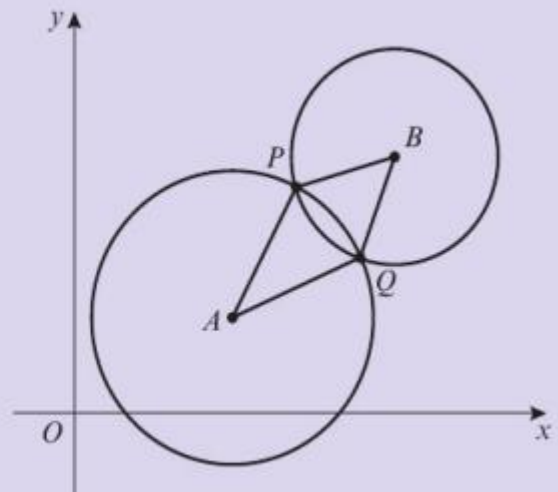
- 21 The line $y = -3x + 12$ meets the coordinate axes at A and B .
- a Find the coordinates of A and B .
 - b Find the coordinates of the midpoint of AB .
 - c Find the equation of the circle that passes through A , B and O , where O is the origin.
- 22 The points $A(-3, -2)$, $B(-6, 0)$ and $C(1, q)$ lie on the circumference of a circle such that $\angle BAC = 90^\circ$.
- a Find the value of q . (4 marks)
 - b Find the equation of the circle. (4 marks)
- 23 The points $R(-4, 3)$, $S(7, 4)$ and $T(8, -7)$ lie on the circumference of a circle.
- a Show that RT is the diameter of the circle. (4 marks)
 - b Find the equation of the circle. (4 marks)
- 24 The points $A(-4, 0)$, $B(4, 8)$ and $C(6, 0)$ lie on the circumference of circle C .
Find the equation of the circle.
- 25 The points $A(-7, 7)$, $B(1, 9)$, $C(3, 1)$ and $D(-7, 1)$ lie on a circle.
- a Find the equation of the perpendicular bisector of:
 - i AB
 - ii CD
 - b Find the equation of the circle.

Chapter Exercises

Challenge

The circle with equation $(x - 5)^2 + (y - 3)^2 = 20$ with centre A intersects the circle with equation $(x - 10)^2 + (y - 8)^2 = 10$ with centre B at the points P and Q .

- a** Find the equation of the line containing the points P and Q in the form $ax + by + c = 0$.
- b** Find the coordinates of the points P and Q .
- c** Find the area of the kite $APBQ$.



Chapter Answers

- 1 a $C(3, 6)$
b $r = 10$
c $(x - 3)^2 + (y - 6)^2 = 100$
d P satisfies the equation of the circle.
- 2 $(0 - 5)^2 + (0 + 2)^2 = 5^2 + 2^2 = 29 < 30$ therefore point is inside the circle
- 3 a Centre $(0, -4)$ and radius $= 3$
b $(0, -1)$ and $(0, -7)$
c Students' own work. Equation $x^2 = -7$ has no real solutions.
- 4 a $P(8, 8)$, $(8 + 1)^2 + (8 - 3)^2 = 9^2 + 5^2 = 81 + 25 = 106$
b $\sqrt{106}$
- 5 a All points satisfy $x^2 + y^2 = 1$, therefore all lie on circle.
b $AB = BC = CA$
- 6 a $k = 1$, $k = -\frac{2}{5}$
b $(x - 1)^2 + (y - 3)^2 = 13$
- 7 Substitute $y = 3x - 9$ into the equation
 $x^2 + px + y^2 + 4y = 20$
 $x^2 + px + (3x - 9)^2 + 4(3x - 9) = 20$
 $10x^2 + (p - 42)x + 25 = 0$
Using the discriminant: $(p - 42)^2 - 1000 < 0$
 $42 - 10\sqrt{10} < p < 42 + 10\sqrt{10}$
- 8 $(x - 2)^2 + (y + 4)^2 = 20$
- 9 a $2\sqrt{29}$ b 12
- 10 $(-1, 0)$, $(11, 0)$
- 11 The values of m and n are $7 - \sqrt{105}$ and $7 + \sqrt{105}$.
- 12 a $a = 6$ and $b = 8$ b $y = -\frac{4}{3}x + 8$ c 24
- 13 a $p = 0$, $q = 24$ b $(0, 49)$, $(0, -1)$
- 14 $x + y + 10 = 0$
- 15 60
- 16 $l_1: y = -4x + 12$ and $l_2: y = -\frac{8}{19}x + 12$
- 17 a $y = \frac{1}{3}x + \frac{8}{3}$
b $(x + 2)^2 + (y - 2)^2 = 50$
c 20
- 18 a $P(-3, 1)$ and $Q(9, -7)$
b $y = \frac{3}{2}x + \frac{11}{2}$ and $y = \frac{3}{2}x - \frac{41}{2}$
- 19 a $y = -4x + 6$ and $y = \frac{1}{4}x + 6$
b $P(-4, 5)$ and $Q(1, 2)$
c 17
- 20 a $P(5, 16)$ and $Q(13, 8)$
b $l_2: y = \frac{1}{7}x + \frac{107}{7}$ and $l_3: y = 7x - 83$
c $l_4: y = x + 3$
d All 3 equations have solution $x = 15$, $y = 18$
so $R(15, 18)$
e $\frac{200}{3}$

Chapter Answers

21 a $(4,0), (0,12)$

b $(2,6)$

c $(x-2)^2 + (y-6)^2 = 40$

22 a $q = 4$

b $(x + \frac{5}{2})^2 + (y-2)^2 = -\frac{65}{4}$

23 a $RS^2 + ST^2 = RT^2$

b $(x-2)^2 + (y+2)^2 = 61$

24 $(x-1)^2 + (y-3)^2 = 34$

25 a i $y = -4x - 4$ ii $x = -2$

b $(x+2)^2 + (y-4)^2 = 34$

Challenge

a $x + y - 14 = 0$

b $P(7, 7)$ and $Q(9, 5)$

c 10