Stats1 Chapter 7: Hypothesis Testing

Two Tailed Tests

Two-Tailed Tests

We have already seen that if we're interest in bias 'either way', we have two tails, and therefore have to split the critical region by **halving the significance level at each end**.

Over a long period of time it has been found that in Enrico's restaurant the ratio of non-veg to veg meals is 2 to 1. In Manuel's restaurant in a random sample of 10 people ordering meals, 1 ordered a vegetarian meal. Using a 5% level of significance, test whether or not the proportion of people eating veg meals in Manuel's restaurant is different to that in Enrico's restaurant.

?

Two-Tailed Tests

We have already seen that if we're interest in bias 'either way', we have two tails, and therefore have to split the critical region by **halving the significance level at each end**.

Over a long period of time it has been found that in Enrico's restaurant the ratio of non-veg to veg meals is 2 to 1. In Manuel's restaurant in a random sample of 10 people ordering meals, 1 ordered a vegetarian meal. Using a 5% level of significance, test whether or not the proportion of people eating veg meals in Manuel's restaurant is different to that in Enrico's restaurant.

Proportion eating veg meals at Enrico's is $\frac{1}{3}$

Let p be the proportion of people at Manuel's that order veg.

Let X be number of people eating veg meals.

$$H_0: p = \frac{1}{3} \qquad H_1: p \neq \frac{1}{3}$$

If H_0 true then $X \sim B\left(10, \frac{1}{3}\right)$

$$P(X \leq 1) = P(X = 0) + P(X = 1)$$
$$= 0.104 \ (3sf)$$

$0.104 > 0.025$ therefore insufficient evidence to reject H_0.

There is no evidence that proportion of veg meals at Manuel's restaurant is different to Enrico's.

Half significance as 2 tailed.

Conclusion and what it means in context.

Test Your Understanding

Edexcel S2 Jan 2006 Q7a

A teacher thinks that 20% of the pupils in a school read the Deano comic regularly.

He chooses 20 pupils at random and finds 9 of them read the Deano.

(*a*) (i) Test, at the 5% level of significance, whether or not there is evidence that the percentage of pupils that read the Deano is different from 20%. State your hypotheses clearly.

(ii) State all the possible numbers of pupils that read the Deano from a sample of size 20 that will make the test in part (*a*)(i) significant at the 5% level. **(9)**

(a)(i)

?

(ii)

?

Test Your Understanding

Edexcel S2 Jan 2006 Q7a

A teacher thinks that 20% of the pupils in a school read the Deano comic regularly.

He chooses 20 pupils at random and finds 9 of them read the Deano.

(*a*) (i) Test, at the 5% level of significance, whether or not there is evidence that the percentage of pupils that read the Deano is different from 20%. State your hypotheses clearly.

(ii) State all the possible numbers of pupils that read the Deano from a sample of size 20 that will make the test in part (*a*)(i) significant at the 5% level. **(9)**

(a)(i)	$H_0 : p = 0.2, H_1 : p \neq 0.2$ $\quad p =$	B1B1
	$P(X \geq 9) = 1 - P(X \leq 8)$ or attempt critical value/region	M1
	$= 1 - 0.9900 = 0.01$ CR $X \geq 9$	
	$0.01 < 0.025$ or $9 \geq 9$ or $0.99 > 0.975$ or $0.02 < 0.05$ or lies in interval with correct interval stated.	A1
	Evidence that the percentage of pupils that read Deano is not 20%	A1
(ii)	X ~ Bin (20, 0.2) $\quad$ may be implied or seen in (i) or (ii)	B1
	So 0 or [9,20] make test significant. $\quad$ 0,9,between "their 9" and 20	B1B1B1
		(9)

Exercise 7.4

Pearson Applied Year 1/AS

Pages 48-49

Homework Exercise

1 A single observation, x, is taken from a binomial distribution $B(30, p)$ and a value of 10 is obtained. Use this observation to test H_0: $p = 0.5$ against H_1: $p \neq 0.5$ using a 5% significance level.

2 A random variable has distribution $X \sim B(25, p)$. A single observation of $x = 10$ is taken from this distribution. Test, at the 10% significance level, H_0: $p = 0.3$ against H_1: $p \neq 0.3$.

3 A single observation, x, is taken from a binomial distribution $B(10, p)$ and a value of 9 is obtained. Use this observation to test H_0: $p = 0.75$ against H_1: $p \neq 0.75$ using a 5% significance level.

4 A random variable has distribution $X \sim B(20, p)$. A single observation of $x = 1$ is taken from this distribution. Test, at the 1% significance level, H_0: $p = 0.6$ against H_1: $p \neq 0.6$.

5 A random variable has distribution $X \sim B(50, p)$. A single observation of $x = 4$ is taken from this distribution. Test, at the 2% significance level, H_0: $p = 0.02$ against H_1: $p \neq 0.02$.

Watch out Although the observed value of 4 appears to be small, the expected value of X is actually $50 \times 0.02 = 1$. You need to consider the upper tail of the distribution: $P(X \geqslant 4)$.

6 A coin is tossed 20 times, and lands on heads 6 times. Use a two-tailed test with a 5% significance level to determine whether there is sufficient evidence to conclude that the coin is biased.

Homework Exercise

7 The national proportion of people experiencing complications after having a particular operation in hospitals is 20%. A hospital decides to take a sample of size 20 from their records.

a Find critical regions, at the 5% level of significance, to test whether or not their proportion of complications differs from the national proportion. The probability in each tail should be as close to 2.5% as possible. **(5 marks)**

b State the actual significance level of the test. **(1 mark)**

The hospital finds that 8 of their 20 patients experienced complications.

c Comment on this finding in light of your critical regions. **(2 marks)**

8 A machine makes glass bowls and it is observed that one in ten of the bowls have hairline cracks in them. The production process is modified and a sample of 20 bowls is taken. 1 of the bowls is cracked. Test, at the 10% level of significance, the hypothesis that the proportion of cracked bowls has changed as a result of the change in the production process. State your hypotheses clearly. **(7 marks)**

9 Over a period of time, Agnetha has discovered that the carrots that she grows have a 25% chance of being longer than 7 cm. She tries a new type of fertiliser. In a random sample of 30 carrots, 13 are longer than 7 cm. Agnetha claims that the new fertiliser has changed the probability of a carrot being longer than 7 cm. Test Agnetha's claim at the 5% significance level. State your hypotheses clearly. **(7 marks)**

10 A standard blood test is able to diagnose a particular disease with probability 0.96. A manufacturer suggests that a cheaper test will have the same probability of success. It conducts a clinical trial on 75 patients. The new test correctly diagnoses 63 of these patients. Test the manufacturer's claim at the 10% level, stating your hypotheses clearly. **(7 marks)**

Homework Answers

1 $P(X \leqslant 10) = (0.0494 > 0.025$ (two-tailed)
There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.5$

2 $P(X \geqslant 10) = 0.189 > 0.05$ (two tailed)
There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.3$

3 $(X \geqslant 9) = 0.244 > 0.025$ (two-tailed)
There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.75$

4 $P(X \leqslant 1) = 0.00000034 < 0.005$ (two-tailed)
$X = 1$ lies within the critical region, so we can reject the null hypothesis.

5 $P(X \geqslant 4) = 0.0178 > 0.01$ (two-tailed)
There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.02$

6 $P(X \leqslant 6) = 0.0577 > 0.025$ (two-tailed)
$X = 6$ does not lie in the critical region, so there is no reason to think that the coin is biased.

7 **a** Critical region $X = 0$ and $X \geqslant 8$
b 4.36%
c H_0: $p = 0.2$, H_1: $p \neq 0.2$
$X = 8$ is in the critical region. There is enough evidence to reject H_0. The hospital's proportion of complications differs from the national figure.

8 Test statistic: the number of cracked bowls.
H_0: $p = 0.1$, H_1: $p \neq 0.1$
$P(X \leqslant 1) = 0.3917 = 39.17\%$
$39.17\% > 5\%$ (two-tailed) so there is not enough evidence to reject H_0. The proportion of cracked bowls has not changed.

9 Test statistic: the number of carrots longer than 7 cm
H_0: $p = 0.25$, H_1: $p \neq 0.25$
$P(X \geqslant 13) = 1 - P(X \leqslant 12) = 0.0216 = 2.16\%$
$2.16\% < 2.5\%$ (two-tailed) so there is enough evidence to reject the null hypothesis. The probability of a carrot being longer than 7 cm has increased.

10 Test statistic: the number of patients correctly diagnosed.
H_0: $p = 0.96$, H_1: $p \neq 0.96$
$P(X \leqslant 63) = 0.0000417 < 0.05$ (two-tailed) so there is enough evidence to reject the null hypothesis. The new test does not have the same probability of success as the old test.