M2 Chapter 5: Inclined Planes

Chapter Practice

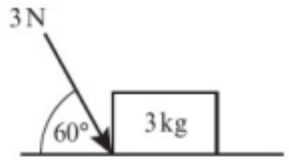
Key Points

Summary of key points

1. If a force is applied at an angle to the direction of motion, you can resolve it to find the component of the force that acts in the direction of motion.
2. The component of a force of magnitude F in a certain direction is $F\cos\theta$, where θ is the size of the angle between the force and the direction.
3. To solve problems involving inclined planes, it is usually easier to resolve parallel to and at right angles to the plane.
4. The maximum or limiting value of the friction between two surfaces, F_{MAX}, is given by $\mathbf{F_{MAX}} = \mu\mathbf{N}$ where μ is the coefficient of friction and **N** is the normal reaction between the two surfaces.

Chapter Exercises

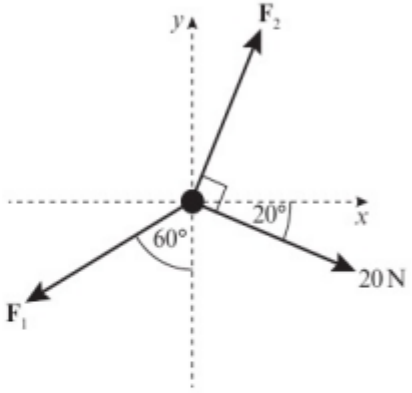
1 A box of mass 3 kg lies on a smooth horizontal floor. A force of 3 N is applied at an angle of 60° to the horizontal, causing the box to accelerate horizontally along the floor.



a Find the magnitude of the normal reaction of the floor on the box.

b Find the acceleration of the box.

2 A system of forces acts upon a particle as shown in the diagram. The resultant force on the particle is $(3\mathbf{i} + 2\mathbf{j})$ N. Calculate the magnitudes $\mathbf{F}_1$ and $\mathbf{F}_2$.



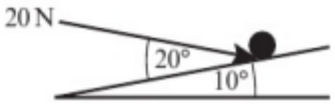
3 A force of 20 N is pulling a particle of mass 2 kg up a rough slope that is inclined at 45° to the horizontal. The force acts parallel to the slope, and the resistance due to friction is constant and has magnitude 4 N.

a Draw a force diagram to represent all the forces acting on the particle.

b Work out the normal reaction between the particle and the plane.

c Show that the acceleration of the particle is 1.1 m s^{-2} (2 s.f.).

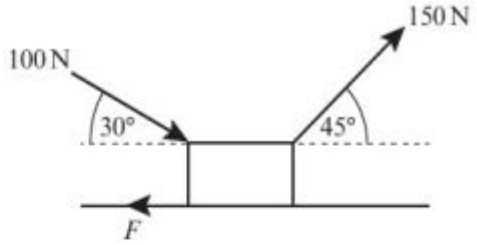
4 A particle of mass 5 kg sits on a smooth slope that is inclined at 10° to the horizontal. A force of 20 N acts on the particle at an angle of 20° to the plane, as shown in the diagram. Find the acceleration of the particle.



(5 marks)

Chapter Exercises

5 A box is being pushed and pulled across a rough surface by constant forces as shown in the diagram.
The box is moving at a constant speed. By modelling the box as a particle, show that the magnitude of the resistance due to friction F is $25(3\sqrt{2} + 2\sqrt{3})$ N.
(4 marks)



6 A trailer of mass 20 kg sits at rest on a rough horizontal plane. A force of 20 N acts on the trailer at an angle of 30° above the horizontal. Given that the trailer is in limiting equilibrium, work out the value of the coefficient of friction. **(6 marks)**

7 A particle of mass 2 kg is moving down a rough plane that is inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$. A force of P N acts horizontally upon the particle towards the plane.
Given that the coefficient of friction is 0.3 and that the particle is moving at a constant velocity, calculate the value of P. **(7 marks)**

8 A particle of mass 0.5 kg is being pulled up a rough slope that is angled at 30° to the horizontal by a force of 5 N. The force acts at an angle of 30° above the slope. Given that the coefficient of friction is 0.1, calculate the acceleration of the particle. **(7 marks)**

Chapter Exercises

9 A car of mass 2150 kg is travelling down a rough road that is inclined at 10° to the horizontal. The engine of the car applies a constant driving force of magnitude 700 N, which acts in the direction of travel of the car. Any friction between the road and the tyres is initially ignored, and air resistance is modelled as a single constant force of magnitude F N that acts to oppose the motion of the car.

a Given that the car is travelling in a straight line at a constant speed of 22 m s^{-1}, find the magnitude of F. **(3 marks)**

The driver brakes suddenly. In the subsequent motion the car continues to travel in a straight line, and the tyres skid along the road, bringing the car to a standstill after 40 m. The driving force is removed, and the force due to air resistance is modelled as remaining constant.

b Find the coefficient of friction between the tyres and the road. **(7 marks)**

c Criticise this model with relation to

i the frictional forces acting on the car

ii the motion of the car. **(2 marks)**

Challenge

A boat of mass 400 kg is being pulled up a rough slipway at a constant speed of 5 m s^{-1} by a winch. The slipway is modelled as a plane inclined at an angle of 15° to the horizontal, and the boat is modelled as a particle. The coefficient of friction between the boat and the slipway is 0.2.

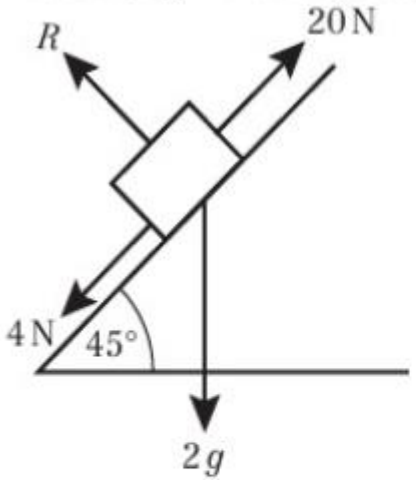
At the point when the boat is 8 m from the water-line, as measure along the line of greatest slope of the slipway, the winch cable snaps. Show that the boat will slide back down into the water, and calculate the total time from the winch cable breaking to the boat reaching the water-line.

Chapter Answers

1 **a** 32.0 N (3 s.f.) **b** 0.5 m s^{-2}

2 $F_1 = 27.8\,\text{N}$, $F_2 = 24.2\,\text{N}$ (3 s.f.)

3 **a**



b 13.9 N (3 s.f.)

c Res (↗): $16 - 2g\sin 45 = 2a$

$$a = \frac{16 - 2g\sin 45}{2} = 1.1\,\text{m s}^{-2}\ (2\ \text{s.f})$$

4 2.06 m s^{-2} (3 s.f.)

5 R(→): $F = 150\cos 45 + 100\cos 30$

$$= \frac{150\sqrt{2}}{2} + \frac{100\sqrt{3}}{2}$$

$$= 25(3\sqrt{2} + 2\sqrt{3})\,\text{N}$$

6 $\mu = \frac{5\sqrt{3}}{93}$

7 11.4 N

8 3.41 m s^{-2} (3 s.f.)

9 **a** 4400 N (2 s.f.)

b 0.59 (2 s.f.)

c **i** e.g. The force due to air resistance will not remain constant in the subsequent motion of the car.

ii e.g. Whilst skidding the car is unlikely to travel in a straight line.

Challenge

$F_{MAX} = 0.2 \times 400g\cos 15^\circ = 760\,\text{N}$ (2 s.f.)

Component of weight that acts down the slipway:

$400g\sin 15^\circ = 1000\,\text{N}$ (2 s.f.)

1000 N > 760 N so boat will come to momentary rest then accelerate back down the slope.

The boat will take 6.9 seconds to reach the water.