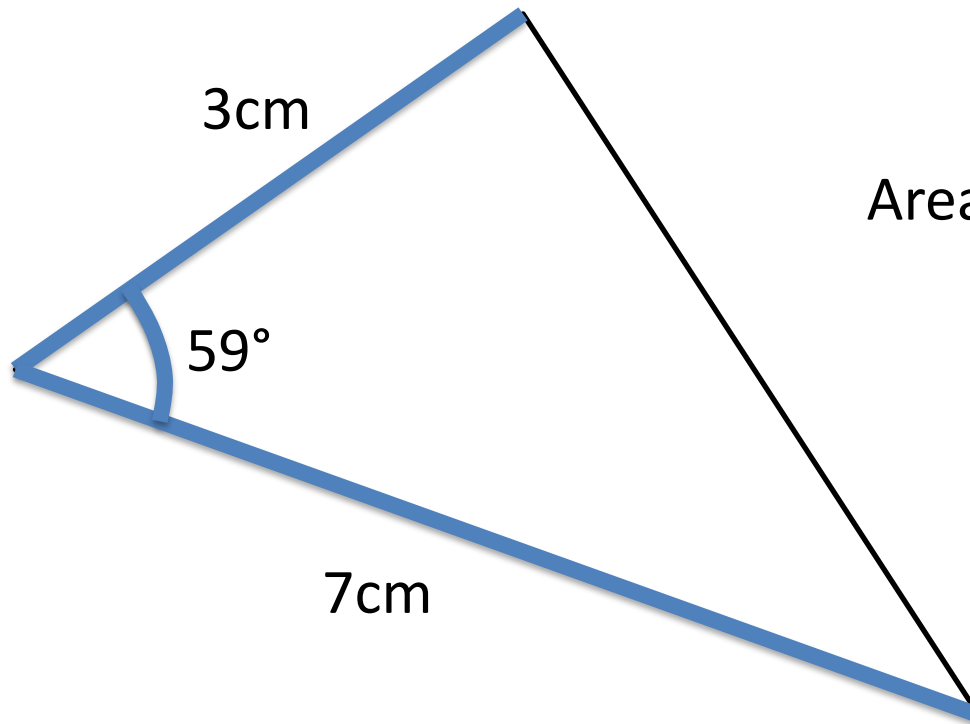

P1 Chapter 9: Trigonometric Ratios

Areas of Any Triangle

Area of Non Right-Angled Triangles



$$\begin{aligned}\text{Area} &= 0.5 \times 3 \times 7 \times \sin(59) \\ &= 9.00\text{cm}^2\end{aligned}$$

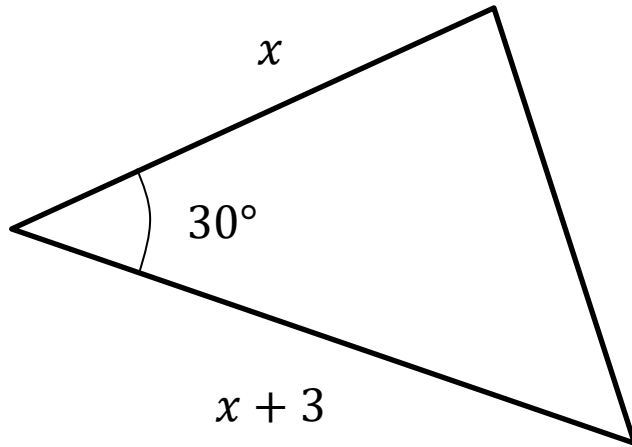


$$\text{Area} = \frac{1}{2} a b \sin(C)$$

where C is the angle between two sides a and b .

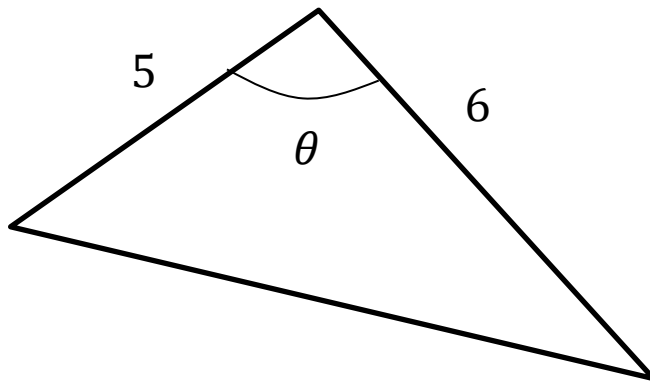
Fro Tip: You shouldn't have to label sides/angles before using the formula. Just remember that the angle is between the two sides.

Test Your Understanding



The area of this triangle is 10.
Determine x .

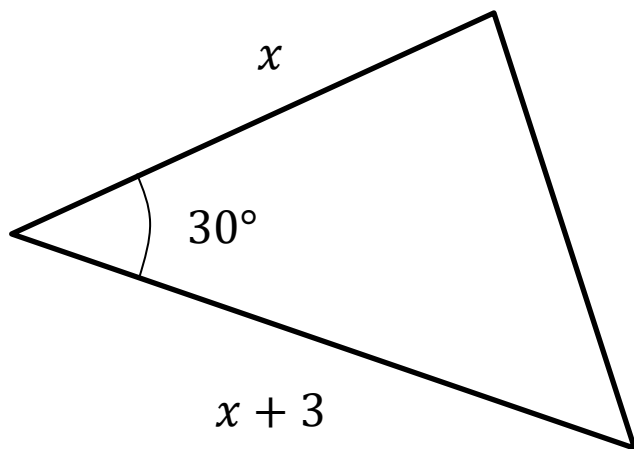
?



The area of this triangle is also 10.
If θ is obtuse, determine θ .

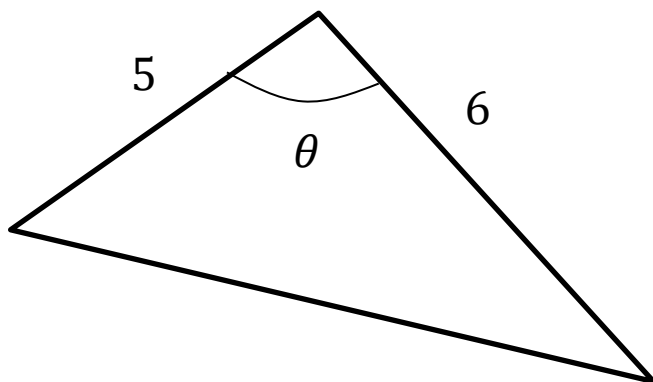
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Test Your Understanding



The area of this triangle is 10.
Determine x .

$$\begin{aligned}\frac{1}{2}x(x+3)\sin(30^\circ) &= 10 \\ \frac{1}{4}x(x+3) &= 10 \\ x(x+3) &= 40 \\ x^2 + 3x - 40 &= 0 \\ (x+8)(x-5) &= 0 \\ \text{As } x > 0, \quad x &= 5\end{aligned}$$



The area of this triangle is also 10.
If θ is obtuse, determine θ .

$$\begin{aligned}\frac{1}{2} \times 5 \times 6 \times \sin(\theta) &= 10 \\ \sin(\theta) &= \frac{10}{15} \\ \theta &= 180^\circ - 41.8^\circ = 138.2^\circ\end{aligned}$$

Exercise 9.3

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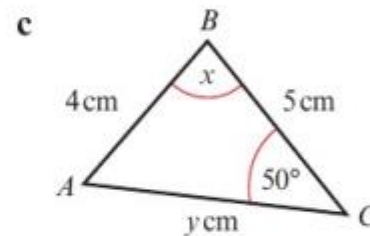
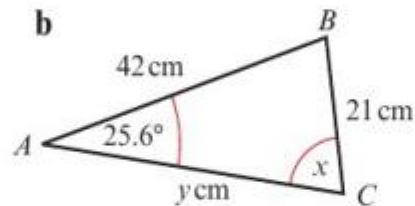
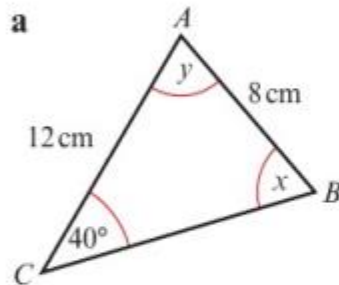
Homework Exercise

Give answers to 3 significant figures, where appropriate.

1 In $\triangle ABC$, $BC = 6$ cm, $AC = 4.5$ cm and $\angle ABC = 45^\circ$.

- Calculate the two possible values of $\angle BAC$.
- Draw a diagram to illustrate your answers.

2 In each of the diagrams shown below, calculate the possible values of x and the corresponding values of y .



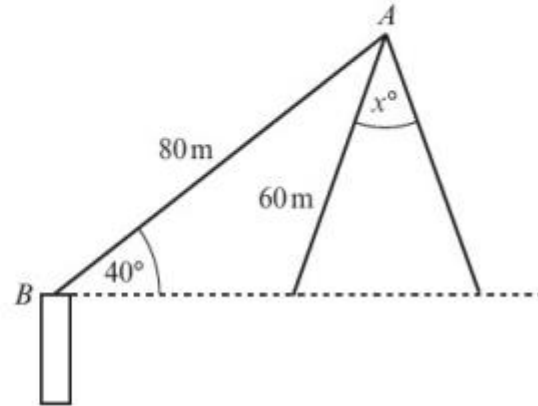
3 In each of the following cases $\triangle ABC$ has $\angle ABC = 30^\circ$ and $AB = 10$ cm.

- Calculate the least possible length that AC could be.
- Given that $AC = 12$ cm, calculate $\angle ACB$.
- Given instead that $AC = 7$ cm, calculate the two possible values of $\angle ACB$.

4 Triangle ABC is such that $AB = 4$ cm, $BC = 6$ cm and $\angle ACB = 36^\circ$. Show that one of the possible values of $\angle ABC$ is 25.8° (to 3 s.f.). Using this value, calculate the length of AC .

Homework Exercise

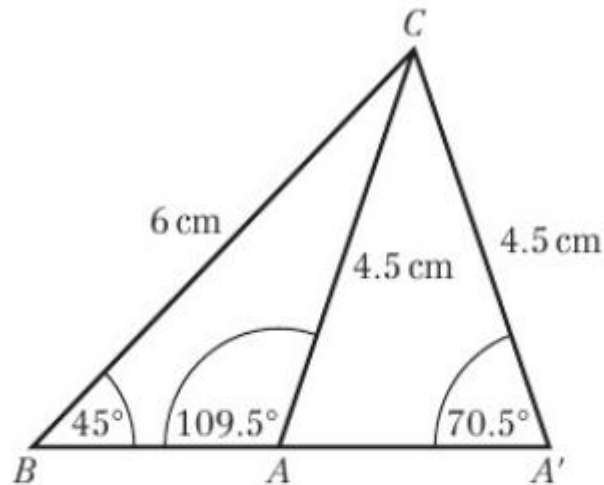
- 5 Two triangles ABC are such that $AB = 4.5$ cm, $BC = 6.8$ cm and $\angle ACB = 30^\circ$. Work out the value of the largest angle in each of the triangles.
- 6 a A crane arm AB of length 80 m is anchored at point B at an angle of 40° to the horizontal. A wrecking ball is suspended on a cable of length 60 m from A . Find the angle x through which the wrecking ball rotates as it passes the two points level with the base of the crane arm at B . (6 marks)
- b Write down one modelling assumption you have made. (1 mark)



Homework Answers

1 a $70.5^\circ, 109^\circ$ (109.5°)

b



2 a $x = 74.6^\circ, y = 65.4^\circ$

$x = 105^\circ, y = 34.6^\circ$

b $x = 59.8^\circ, y = 48.4 \text{ cm}$

$x = 120^\circ, y = 27.3 \text{ cm}$

c $x = 56.8^\circ, y = 4.37 \text{ cm}$

$x = 23.2^\circ, y = 2.06 \text{ cm}$

3 a 5 cm ($ACB = 90^\circ$)

b 24.6°

c $45.6^\circ, 134(.4)^\circ$

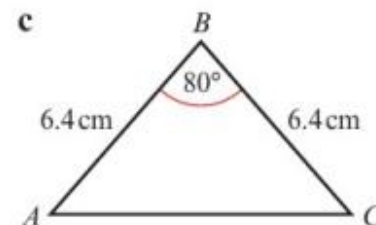
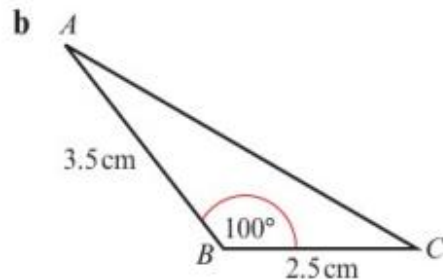
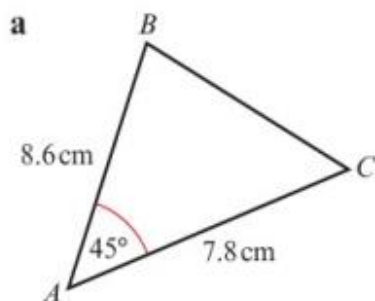
4 2.96 cm

5 In one triangle $ABC = 101^\circ$ (100.9°); in the other
 $BAC = 131^\circ$ (130.9°)

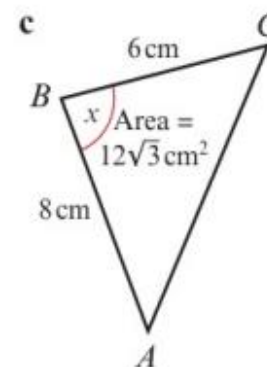
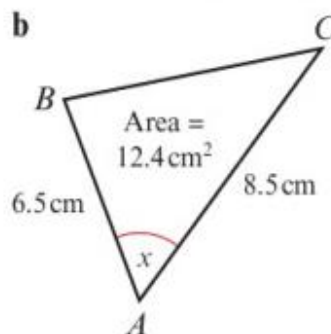
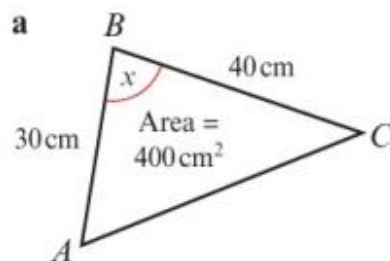
6 a 62.0° b The swing is symmetrical

Homework Exercise

1 Calculate the area of each triangle.

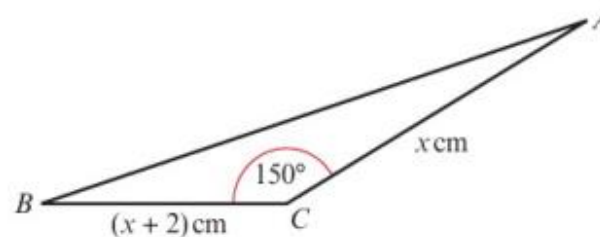


2 Work out the possible sizes of x in the following triangles.



3 A fenced triangular plot of ground has area 1200 m^2 . The fences along the two smaller sides are 60 m and 80 m respectively and the angle between them is θ . Show that $\theta = 150^\circ$, and work out the total length of fencing.

4 In triangle ABC , $BC = (x + 2)\text{ cm}$, $AC = x\text{ cm}$ and $\angle BCA = 150^\circ$. Given that the area of the triangle is 5 cm^2 , work out the value of x , giving your answer to 3 significant figures.



Homework Exercise

- 5 In $\triangle PQR$, $PQ = (x + 2)$ cm, $PR = (5 - x)$ cm and $\angle QPR = 30^\circ$.

The area of the triangle is A cm².

- a Show that $A = \frac{1}{4}(10 + 3x - x^2)$. (3 marks)

- b Use the method of completing the square, or otherwise, to find the maximum value of A , and give the corresponding value of x . (4 marks)

- 6 In $\triangle ABC$, $AB = x$ cm, $AC = (5 + x)$ cm and $\angle BAC = 150^\circ$. Given that the area of the triangle is $3\frac{3}{4}$ cm²

Problem-solving

x represents a length so it must be positive.

- a Show that x satisfies the equation $x^2 + 5x - 15 = 0$. (3 marks)

- b Calculate the value of x , giving your answer to 3 significant figures. (3 marks)

Homework Answers

1 **a** 23.7 cm^2 **b** 4.31 cm^2 **c** 20.2 cm^2

2 **a** $x = 41.8^\circ$ or $138(.2)^\circ$

b $x = 26.7^\circ$ or $153(.3)^\circ$

c $x = 60^\circ$ or 120°

3 $275(.3) \text{ m}$ (third side = 135.3 m)

4 3.58

5 **a** Area $= \frac{1}{2}(x + 2)(5 - x) \sin 30^\circ$

$$= \frac{1}{2}(10 + 3x - x^2) \times \frac{1}{2}$$

$$= \frac{1}{4}(10 + 3x - x^2)$$

b Maximum $A = 3\frac{1}{16}$, when $x = 1\frac{1}{2}$

6 **a** $\frac{1}{2}x(5 + x) \sin 150^\circ = \frac{15}{4}$

$$\frac{1}{2}(5x + x^2) \times \frac{1}{2} = \frac{15}{4}$$

$$5x + x^2 = 15$$

$$x^2 + 5x - 15 = 0$$

b 2.11