M2 Chapter 5: Inclined Planes

Ramps

Inclined Planes

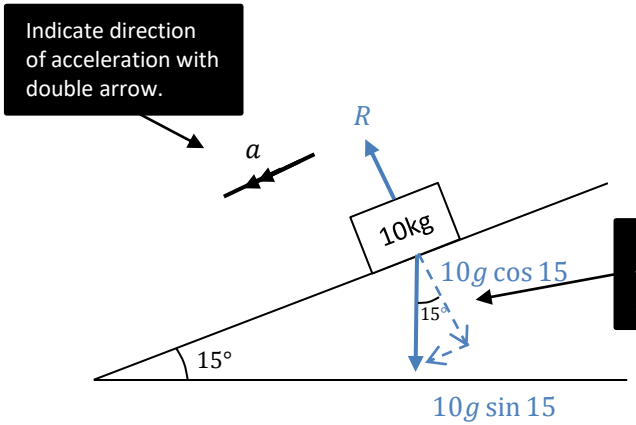
For problems involving inclined planes, resolve forces parallel and perpendicular to the plane.

[Textbook] A block of mass 10kg slides down a smooth slope angled at 15° to the horizontal.

(a) Draw a force diagram to show all the forces acting on the block.

(b) Calculate the magnitude of the normal reaction of the slope on the block.

(c) Find the acceleration of the block.



Mark-schemes: This means "resolving forces in the indicated direction".

$R(\nwarrow)$: $R = 10g\cos 15$

$R(\swarrow)$: $Force = ma$

$10g\sin 15 = 10a$

$a = 2.5\ \text{ms}^{-2}$

Recall that because we used $g = 9.8$, which is only accurate to 2sf, we should only give the final answer to 2sf. In past mark schemes 3sf was condoned but 4 or more penalised.

Inclined Plane with an additional force

[Textbook] A particle of mass m is pushed up a smooth slope, inclined at $30°$ by a force of magnitude $5g$ N acting at angle of $60°$ to the slope, causing the particle to accelerate up the slope at 0.5 ms^{-2}. Show that the mass of the particle is $\left(\frac{5g}{1+g}\right)$ kg

? Diagram

? Working

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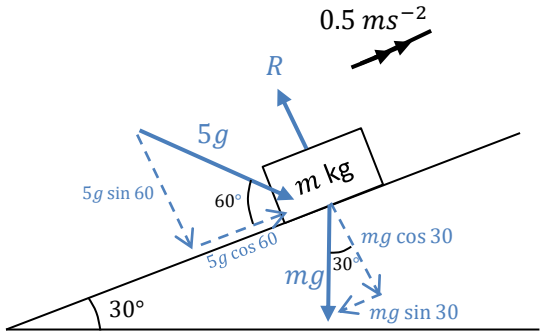
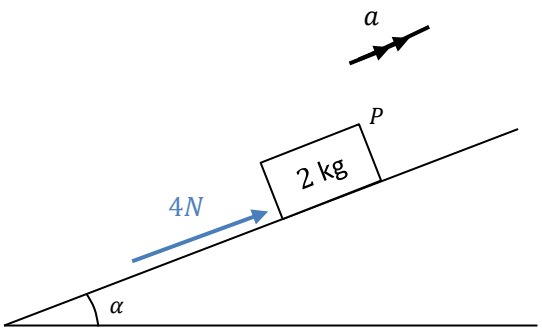


$$R(\nearrow): \quad 5g\cos 60° - mg\sin 30° = 0.5m$$
$$2.5g - 0.5mg = 0.5m$$
$$2.5g = 0.5m + 0.5mg$$
$$5g = m + mg$$
$$5g = m(1+g)$$
$$m = \left(\frac{5g}{1+g}\right) \text{ kg}$$

Test Your Understanding

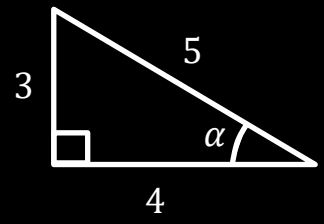
[Textbook] A particle P of mass 2kg is moving on a smooth slope and is being acted on by a force of 4N that acts parallel to the slow, as shown. The slop is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. Work out the acceleration of the particle.



? Diagram

? Working

Help: Don't find α explicitly. We can find $\cos\alpha$ and $\sin\alpha$ by forming a 3-4-5 Pythagorean triangle such that $\tan\alpha = \frac{3}{4}$:



$\cos\alpha =$?

$\sin\alpha =$?

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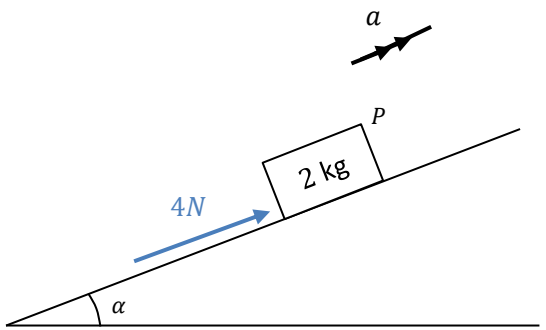
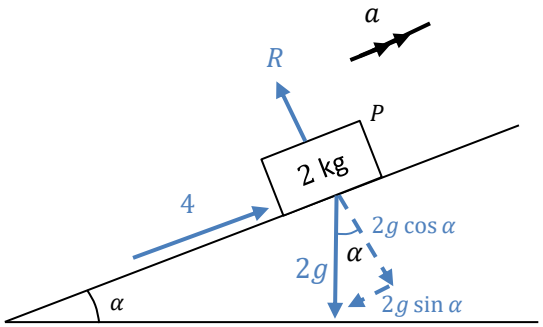




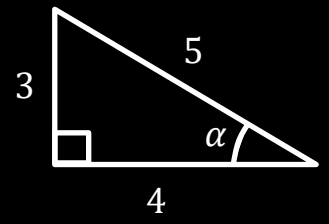
$$R(\nearrow): \quad 4 - 2g\sin\alpha = 2a$$

$$4 - 2 \times 9.8 \times \frac{3}{5} = 2a$$

$$a = -3.9\ ms^{-2}$$

The particles accelerates **down** the slop at 3.9 ms^{-2}.

Help: Don't find α explicitly. We can find $\cos\alpha$ and $\sin\alpha$ by forming a 3-4-5 Pythagorean triangle such that $\tan\alpha = \frac{3}{4}$:



$\cos\alpha =$?

$\sin\alpha =$?

Exercise 5.2

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Homework Exercise

1 A particle of mass 3 kg slides down a smooth slope that is inclined at 20° to the horizontal.

a Draw a force diagram to represent all the forces acting on the particle.

b Work out the normal reaction between the particle and the plane.

c Find the acceleration of the particle.

2 A force of 50 N is pulling a particle of mass 5 kg up a smooth plane that is inclined at 30° to the horizontal. Given that the force acts parallel to the plane,

a draw a force diagram to represent all the forces acting on the particle

b work out the normal reaction between the particle and the plane

c find the acceleration of the particle.

3 A particle of mass 0.5 kg is held at rest on a smooth slope that is inclined at an angle α to the horizontal. The particle is released. Given that $\tan\alpha = \frac{3}{4}$, calculate:

a the normal reaction between the particle and the plane

b the acceleration of the particle.

4 A force of 30 N is pulling a particle of mass 6 kg up a rough slope that is inclined at 15° to the horizontal. The force acts in the direction of motion of the particle and the particle experiences a constant resistance due to friction.

a Draw a force diagram to represent all the forces acting on the particle. **(4 marks)**

Given that the particle is moving with constant speed,

b calculate the magnitude of the resistance due to friction. **(5 marks)**

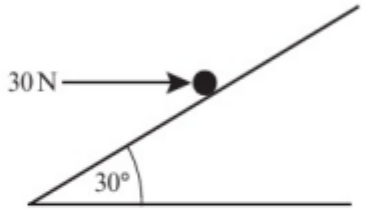
Homework Exercise

5 A particle of mass m kg is sliding down a smooth slope that is angled at 30° to the horizontal. The normal reaction between the plane and the particle is 5 N.

a Calculate the mass m of the particle. **(3 marks)**

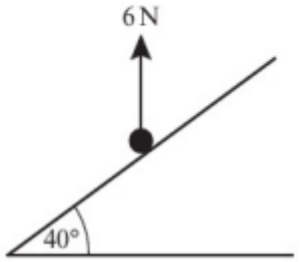
b Calculate the acceleration of the particle. **(3 marks)**

6 A force of 30 N acts horizontally on a particle of mass 5 kg that rests on a smooth slope that is inclined at 30° to the horizontal as shown in the diagram.
Find the acceleration of the particle.



(4 marks)

7 A particle of mass 3 kg is moving on a rough slope that is inclined at 40° to the horizontal. A force of 6 N acts vertically upon the particle. Given that the particle is moving at a constant velocity, calculate the value of F, the constant resistance due to friction.

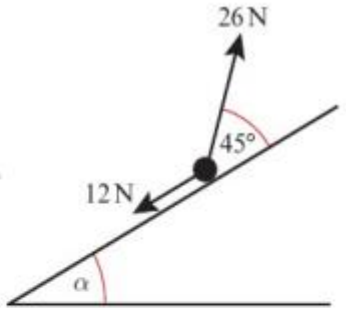


(4 marks)

Homework Exercise

8 A particle of mass m kg is pulled up a rough slope by a force of 26 N that acts at an angle of 45° to the slope. The particle experiences a constant frictional force of magnitude 12 N.

Given that $\tan\alpha = \frac{1}{\sqrt{3}}$ and that the acceleration of the particle is $1\,\text{m s}^{-2}$, show that $m = 1.08$ kg (3 s.f.).



(5 marks)

Challenge

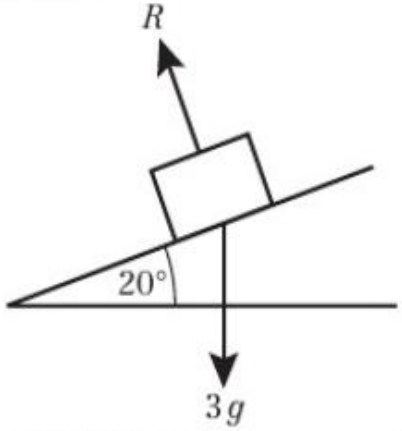
A particle is sliding down a smooth slope inclined at an angle θ to the horizontal, where $0 < \theta < 30°$. The angle of inclination of the slope is increased by 60°, and the magnitude of the acceleration of the particle increases from a to $4a$.

a Show that $\tan\theta = \frac{\sqrt{3}}{7}$

b Hence find θ, giving your answer to 3 significant figures.

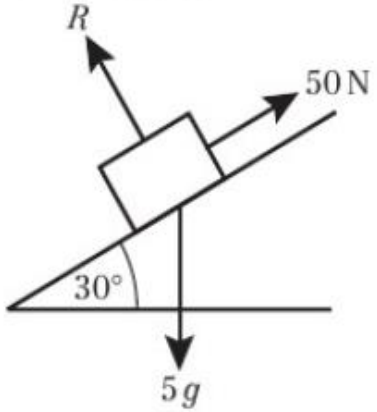
Homework Answers

1 **a**



b 27.6 N (3 s.f.) **c** 3.35 m s^{-2}

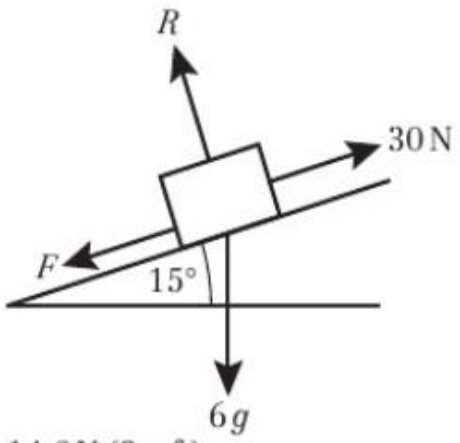
2 **a**



b 42.4 N (3 s.f.) **c** 5.1 m s^{-2}

3 **a** 3.92 N (3 s.f.) **b** 5.88 m s^{-2} (3 s.f.)

4 **a**



b 14.8 N (3 s.f.)

5 **a** 0.589 kg (3 s.f.) **b** 4.9 m s^{-2}

6 0.296 m s^{-2} (3 s.f.)

7 15.0 N (3 s.f.)

8 R(↗): $26\cos 45 - mg\sin\alpha - 12 = m \times 1$

$13\sqrt{2} - 12 = m + \frac{1}{2}mg$

$m = \frac{13\sqrt{2} - 12}{1 + \frac{g}{2}}$

$m = 1.08$ kg (3 s.f.)

Challenge

a $mg\sin\theta = ma$ and $mg\sin(\theta + 60) = 4ma$

$4\sin\theta = \sin(\theta + 60)$

$4\sin\theta = \sin\theta\cos 60 + \cos\theta\sin 60$

$4\sin\theta = \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$

$\frac{7}{2}\sin\theta = \frac{\sqrt{3}}{2}\cos\theta$

$\tan\theta = \frac{\sqrt{3}}{7}$

b 13.9°