M2 Chapter 6: Projectiles

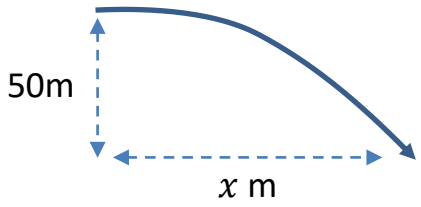
Horizontal Projection

Overview

In Mechanics 1 we already encountered problems of vertical motion of objects when projected upwards. We used "suvat" equations where the acceleration was g ms^{-2}.
In this chapter we allow the object to be **projected sideways**!

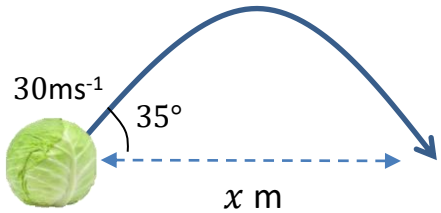
1:: Horizontally projected

"A particle is projected horizontally at 20 ms^{-1}, at a distance 50m above the ground. How far along the ground does it travel?"



2:: Projection at any angle

"A cabbage is projected from ground level at 30 ms^{-1} at an angle of $35°$. How far away is the cabbage when it hits the ground?"



Acceleration in each direction.

The key is separately considering the motion in the vertical and horizontal directions:

In **vertical** direction, acceleration downwards is g ms^{-2}.

Use suvat equations as before.

In **horizontal** direction, acceleration is 0 ms^{-2}.

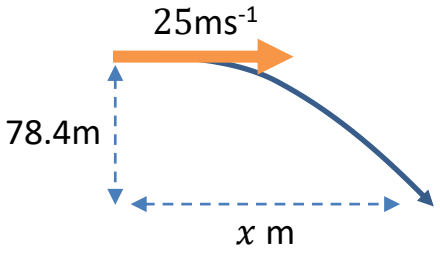
Constant velocity, so can use bog standard $speed = \frac{distance}{time}$

[Textbook] A particle is project horizontally at 25 ms^{-1} from a point 78.4 metres above a horizontal surface. Find:

(a) the time taken by the particle to reach the surface

(b) the horizontal distance travelled in that time.

(c) the distance of the impact point from the original point.



a

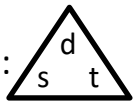
$R(\downarrow)$: $s = 78.4, u = 0, \cancel{v = }, a = 9.8, t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$78.4 = 4.9t^2$$
$$t = 4 \text{ s}$$

b

$R(\rightarrow)$: $x = 25 \times 4 = 100$ m

Just using 'sdt triangle':

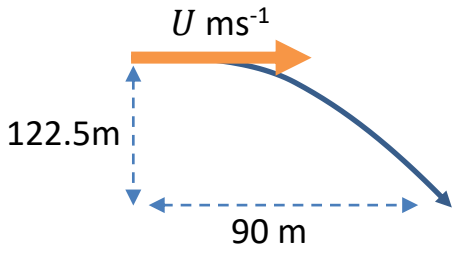


c

$\sqrt{78.4^2 + 100^2} = 130$ m (2sf)

Further Example

[Textbook] A particle is projected horizontally with a speed of U ms^{-1} from a point 122.5m above a horizontal plane. The particle hits the plane at a point which is at a horizontal distance of 90m away from the starting point. Find the initial speed of the particle.



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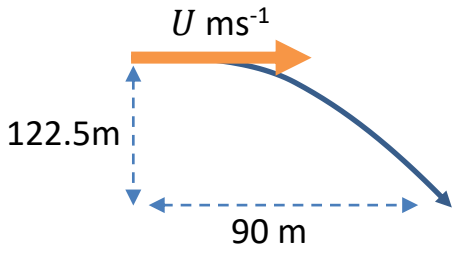


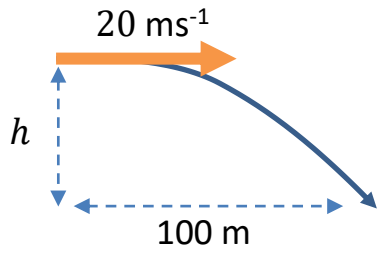
$R(\downarrow)$: $s = 122.5, u = 0, \cancel{v = ,}$
$a = 9.8, t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$122.5 = 4.9t^2$$
$$t = 5 \text{ s}$$

$R(\rightarrow)$: $90 = U \times 5$
$U = 18\, ms^{-1}$

Test Your Understanding

[Textbook] A particle is projected horizontally with a speed of 20 ms^{-1} from a point h m above a horizontal plane. The particle hits the plane at a point which is at a horizontal distance of 100m away from the starting point. Determine the value of h.



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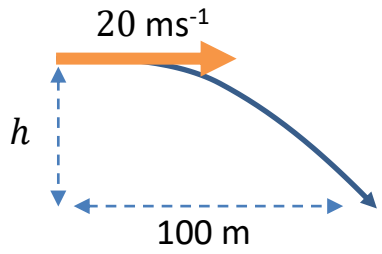

$$R(\rightarrow):\ t = \frac{100}{20} = 5\ \text{s}$$

$$R(\downarrow):\ s = h, u = 0, \cancel{v = ,}$$
$$a = 9.8, t = 5$$

$$s = ut + \frac{1}{2}at^2$$
$$h = \frac{1}{2} \times 9.8 \times 5^2$$
$$= 122.5\ \text{m}$$

Exercise 6A

Pearson Stats/Mechanics Year 2
Pages 49-50

Homework Exercise

1 A particle is projected horizontally at $20\,\text{m s}^{-1}$ from a point h metres above horizontal ground. It lands on the ground 5 seconds later. Find:

a the value of h

b the horizontal distance travelled between the time the particle is projected and the time it hits the ground.

2 A particle is projected horizontally with a velocity of $18\,\text{m s}^{-1}$. Find:

a the horizontal and vertical components of the displacement of the particle from the point of projection after 2 seconds

b the distance of the particle from the point of projection after 2 seconds.

3 A particle is projected horizontally with a speed of $U\,\text{m s}^{-1}$ from a point 160 m above a horizontal plane. The particle hits the plane at a point which is at a horizontal distance of 95 m away from the point of projection. Find the initial speed of the particle.

4 A particle is projected horizontally from a point A which is 16 m above horizontal ground. The projectile strikes the ground at a point B which is at a horizontal distance of 140 m from A. Find the speed of projection of the particle.

5 A particle is projected horizontally with velocity $20\,\text{m s}^{-1}$ along a flat smooth table-top from a point 2 m from the table edge. The particle then leaves the table-top which is at a height of 1.2 m from the floor. Work out the total time taken for the particle to travel from the point of projection until it lands on the floor.

Homework Exercise

6 A darts player throws darts at a dart board which hangs vertically. The motion of a dart is modelled as that of a particle moving freely under gravity. The darts move in a vertical plane which is perpendicular to the plane of the dart board. A dart is thrown horizontally with an initial velocity of $14\,\text{m s}^{-1}$. It hits the board at a point which is 9 cm below the level from which it was thrown.

Find the horizontal distance from the point where the dart was thrown to the dart board.

(4 marks)

7 A particle of mass 2.5 kg is projected along a horizontal rough surface with a velocity of $5\,\text{m s}^{-1}$. After travelling a distance of 2 m the ball leaves the rough surface as a projectile and lands on the ground which is 1.2 m vertically below. Given that the total time taken for the ball to travel from the initial point of projection to the point when it lands is 1.0 seconds, find:

a the time for which the particle is in contact with the surface **(4 marks)**

b the coefficient of friction between the particle and the surface **(6 marks)**

c the horizontal distance travelled from the point of projection to the point where the particle hits the ground. **(3 marks)**

Homework Answers

1 **a** 122.5 m **b** 100 m

2 **a** $x = 36\,\text{m}, y = 19.6\,\text{m}$ **b** 41 m

3 $u = 16.6\,\text{m s}^{-1}$

4 $77.5\,\text{m s}^{-1}$

5 0.59 s

6 1.9 m

7 **a** 0.5 s **b** 0.42 **c** 3.5 m