M1 Chapter 11: Variable Acceleration

Velocity Turning Points

Maxima and Minima Problems

Recall from Pure that at minimum/maximum points, the gradient is 0. We could therefore for example find where the velocity is minimum/maximum by finding when $\frac{dv}{dt} = 0$ (i.e. when the acceleration is 0).

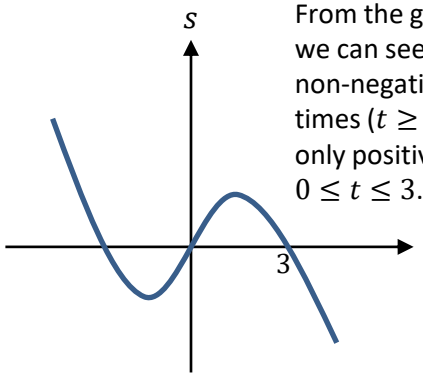
[Textbook] A child is playing with a yo-yo. The yo-yo leaves the child's hand at time $t = 0$ and travels vertically in a straight lien before returning to the child's hand. The distance, s m, of the yo-yo from the child's hand after time t seconds is given by:

$$s = 0.6t + 0.4t^2 - 0.2t^3, \qquad 0 \le t \le 3$$

(a) Justify the restriction $0 \le t \le 3$

(b) Find the maximum distance of the yo-yo from the child's hand, correct to 3sf.

a $s = \frac{1}{5}t(3 + 2t - t^2) = \frac{1}{5}t(3 - t)(1 + t)$



From the graph, we can see that for non-negative times ($t \ge 0$), s is only positive for $0 \le t \le 3$.

b s is maximised when $\frac{ds}{dt} = 0$.

$$\frac{ds}{dt} = 0.6 + 0.8t - 0.6t^2 = 0$$

$$3t^2 - 4t - 3 = 0$$

$$t = \frac{4 \pm \sqrt{52}}{6} = 1.8685 \; or \; -0.5351$$

$s = 0.6(1.8685) + \cdots = 1.21$ m (3sf)

Test Your Understanding

A dolphin escapes from Seaworld and its velocity as it speeds away from the park, is $t^3 - 16t^2 + 64t$ (in ms^{-1}), and maintains this velocity at the point where it would start slowing down.

(a) When does the dolphin reach its maximum velocity?

(b) What is this maximum velocity?

a ?

b ?

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A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geq 0$$

Find

(a) the times when P is instantaneously at rest,

(b) the greatest speed of P in the interval $0 \leq t \leq 4$,

a ?

b ?

Test Your Understanding

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(a) When does the dolphin reach its maximum velocity?

(b) What is this maximum velocity?

a

At the maximum velocity, $\frac{dv}{dt} = 0$

$$\frac{dv}{dt} = 3t^2 - 32t + 64 = 0$$

$$(3t - 8)(t - 8) = 0$$

$$t = \frac{8}{3} \text{ or } t = 8$$

Given the shape of the cubic, $\boldsymbol{t = \frac{8}{3}}$ corresponds to a maximum (and $t = 8$ the minimum)

b

$$v = \left(\frac{8}{3}\right)^3 - 16\left(\frac{8}{3}\right)^2 + 64\left(\frac{8}{3}\right) = 75.9 \text{ ms}^{-1}$$

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(b) the greatest speed of P in the interval $0 \leq t \leq 4$,

a

$v = 0 = 2t^2 - 14t + 20$	M1
$= 2(t-2)(t-5)$	M1
$t = 2$ or $t = 5$	A1 (3)

b

$(t = 0)$, $v = 20$ (m s^{-1})	B1
$a = 4t - 14 = \mathbf{0}$	M1
$t = \frac{7}{2}$, $v = 2 \times \frac{3}{2} \times \frac{-3}{2} = \frac{-9}{2}$	M1A1
Max speed = 20 ms^{-1}	A1 (5)

Exercise 11.3 Maxima and minimum problems

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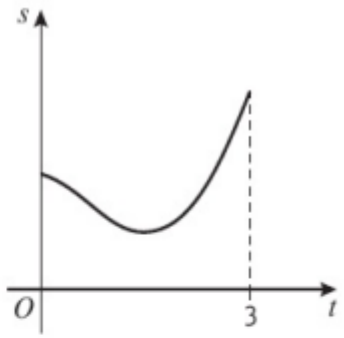
Homework Exercise

1 A particle P moves in a straight line such that its distance, s m, from a fixed point O at time t is given by:

$$s = 0.4t^3 - 0.3t^2 - 1.8t + 5,\ 0 \leqslant t \leqslant 3$$

The diagram shows the displacement–time graph of the motion of P.



a Determine the time at which P is moving with minimum velocity.

b Find the displacement of P from O at this time.

c Find the velocity of P at this time.

2 A body starts at rest and moves in a straight line. At time t seconds the displacement of the body from its starting point, s m, is given by:

$$s = 4t^3 - t^4,\ 0 \leqslant t \leqslant 4.$$

a Show that the body returns to its starting position at $t = 4$.

b Explain why s is always non-negative.

c Find the maximum displacement of the body from its starting point.

Hint Write $s = t^3(4 - t)$ and consider the sign of each factor in the range $0 \leqslant t \leqslant 4$.

3 At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is v m s^{-1}, where:

$$v = t^2(6 - t)^2,\ t \geqslant 0$$

a Sketch a velocity–time graph for the motion of P.

b Find the maximum value of v and the time at which it occurs.

Homework Exercise

4 A particle P moves along the x-axis. Its velocity, $v\,\text{m s}^{-1}$ in the positive x-direction, at time t seconds is given by:

$$v = 2t^2 - 3t + 5,\ t \geqslant 0$$

a Show that P never comes to rest.

b Find the minimum velocity of P.

5 A particle P starts at the origin O at time $t = 0$ and moves along the x-axis. At time t seconds the distance of the particle, $s\,\text{m}$, from the origin is given by:

$$s = \frac{9t^2}{2} - t^3,\ 0 \leqslant t \leqslant 4.5$$

a Sketch a displacement–time graph for the motion of P. **(2 marks)**

b Hence justify the restriction $0 \leqslant t \leqslant 4.5$. **(2 marks)**

c Find the maximum distance of the particle from O. **(5 marks)**

d Find the magnitude of the acceleration of the particle at this point. **(3 marks)**

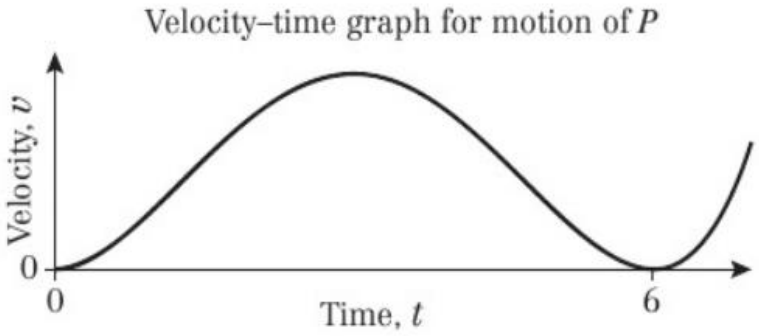
6 A train moves in a straight line along a 4 km test track. The motion of the train is modelled as a particle travelling in a straight line, and the distance, $s\,\text{m}$, of the train from the start of the track after time t seconds is given by $s = 3.6t + 1.76t^2 - 0.02t^3$, $0 \leqslant t \leqslant 90$. Show that the train never reaches the end of the track. **(7 marks)**

Homework Answers

1 a 0.25 s
b 4.54 m
c $v = -1.88\,\text{m s}^{-1}$

2 a The body returns to its starting position 4 s after leaving it.
b Since $t \geqslant 0$, t^3 is always positive.
Since $t \leqslant 4$, $4 - t$ is always non-negative.
c 27 m

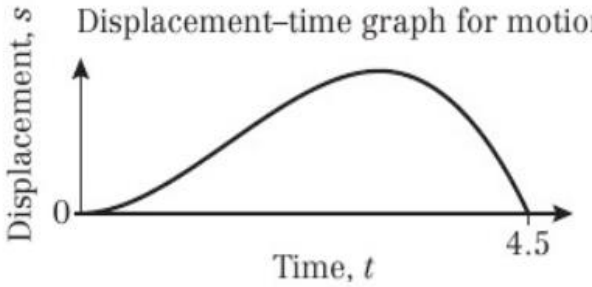
3 a



b $v = 81\,\text{m s}^{-1}$ when $t = 3$ s

4 a Discriminant of $2t^2 - 3t + 5$ is <0, so no solutions for $v = 0$
b $3.88\,\text{m s}^{-1}$ (3 s.f.)

5 a



b s is a distance so cannot be negative.
c 13.5 m **d** $9\,\text{m s}^{-2}$

6 Max distance is when $\frac{ds}{dt} = 3.6 + 3.52t - 0.06t^2 = 0$, so $t = 59.7$ (3 s.f.)
$\therefore$ Max distance = 2.23 km (3 s.f.), so the train never reaches the end of the track.