P1 Chapter 12: Differentiation

Gradient Functions

Differentiating x^n

Thankfully, there's a quick way to differentiate terms of the form x^n (where n is a constant) with having to use first principles every time:

If $y = ax^n$ then $\frac{dy}{dx} = nax^{n-1}$ (where a, n are constants)
i.e. multiply by the power and reduce the power by 1

Examples:

$y = x^5 \rightarrow \frac{dy}{dx} =$?

← Power is 5, so multiply by 5 then reduce power by 5.

$f(x) = x^{\frac{1}{2}} \rightarrow$?

← The power need not be an integer!
Remember to use $f'(x)$ not $\frac{dy}{dx}$

$y = 2x^6 \rightarrow$?

$f(x) = \frac{x}{x^4} =$? $\rightarrow$?

$y = \sqrt{x^6} =$? $\rightarrow$?

Why would it be incorrect to say that $y = 2^x$ differentiates to $\frac{dy}{dx} = x\,2^{x-1}$?

?

Differentiating x^n

Thankfully, there's a quick way to differentiate terms of the form x^n (where n is a constant) with having to use first principles every time:

> If $y = ax^n$ then $\frac{dy}{dx} = nax^{n-1}$ (where a, n are constants)
> i.e. multiply by the power and reduce the power by 1

Examples:

$$y = x^5 \quad \rightarrow \quad \frac{dy}{dx} = 5x^4$$

Power is 5, so multiply by 5 then reduce power by 5.

$$f(x) = x^{\frac{1}{2}} \quad \rightarrow \quad f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

The power need not be an integer!
Remember to use $f'(x)$ not $\frac{dy}{dx}$

$$y = 2x^6 \quad \rightarrow \quad \frac{dy}{dx} = 12x^5$$

$$f(x) = \frac{x}{x^4} = x^{-3} \quad \rightarrow \quad f'(x) = -3x^{-4}$$

$$y = \sqrt{x^6} = x^3 \quad \rightarrow \quad \frac{dy}{dx} = 3x^2$$

Why would it be incorrect to say that $y = 2^x$ differentiates to $\frac{dy}{dx} = x\,2^{x-1}$?

The rule only works when the base is x and the power is a constant. Neither is true here! Note that x^n is "a power of x" whereas 2^x is an exponential term (which you will encounter more in Chp14), and therefore differentiate differently. You will learn how to differentiate exponential terms in Year 2.

Test Your Understanding

1. $y = x^7 \rightarrow \boldsymbol{\frac{dy}{dx}} =$?

2. $y = 3x^{10} \rightarrow$?

3. $f(x) = \frac{x^{\frac{1}{2}}}{x^2} =$? $\rightarrow$?

4. $y = ax^a \rightarrow$?

5. $f(x) = \sqrt{49x^7} =$? $\rightarrow \boldsymbol{f'(x)} =$?

Test Your Understanding

1. $y = x^7 \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = 7x^6}$

2. $y = 3x^{10} \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = 30x^9}$

3. $f(x) = \frac{x^{\frac{1}{2}}}{x^2} = x^{-\frac{3}{2}} \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = -\frac{3}{2}x^{-\frac{5}{2}}}$

4. $y = ax^a \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = a^2x^{a-1}}$

5. $f(x) = \sqrt{49x^7} = 7x^{\frac{7}{2}} \quad \rightarrow \quad \boldsymbol{f'(x) = \frac{49}{2}x^{\frac{5}{2}}}$

Exercise 12.3

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Homework Exercise

1 Find $f'(x)$ given that $f(x)$ equals:

a x^7 **b** x^8 **c** x^4 **d** $x^{\frac{1}{3}}$

g x^{-3} **h** x^{-4} **e** $x^{\frac{1}{4}}$ **f** $\sqrt[3]{x}$

i $\frac{1}{x^2}$ **j** $\frac{1}{x^5}$ **k** $\frac{1}{\sqrt{x}}$ **l** $\frac{1}{\sqrt[3]{x}}$

m $x^3 \times x^6$ **n** $x^2 \times x^3$ **o** $x \times x^2$ **p** $\frac{x^2}{x^4}$ **q** $\frac{x^3}{x^2}$ **r** $\frac{x^6}{x^3}$

Hint Make sure that the functions are in the form x^n before you differentiate.

2 Find $\frac{dy}{dx}$ given that y equals:

a $3x^2$ **b** $6x^9$ **c** $\frac{1}{2}x^4$ **d** $20x^{\frac{1}{4}}$ **e** $6x^{\frac{5}{4}}$

f $10x^{-1}$ **g** $\frac{4x^6}{2x^3}$ **h** $\frac{x}{8x^5}$ **i** $-\frac{2}{\sqrt{x}}$ **j** $\sqrt{\frac{5x^4 \times 10x}{2x^2}}$

3 Find the gradient of the curve with equation $y = 3\sqrt{x}$ at the point where:

a $x = 4$ **b** $x = 9$

c $x = \frac{1}{4}$ **d** $x = \frac{9}{16}$

4 Given that $2y^2 - x^3 = 0$ and $y > 0$, find $\frac{dy}{dx}$ **(2 marks)**

Problem-solving

Try rearranging unfamiliar equations into a form you recognise.

Homework Answers

1	**a**	$7x^6$	**b**	$8x^7$	**c**	$4x^3$	**d**	$\frac{1}{3}x^{-\frac{2}{3}}$
	e	$\frac{1}{4}x^{-\frac{3}{4}}$	**f**	$\frac{1}{3}x^{-\frac{2}{3}}$	**g**	$-3x^{-4}$	**h**	$-4x^{-5}$
	i	$-2x^{-3}$	**j**	$-5x^{-6}$	**k**	$-\frac{1}{2}x^{-\frac{3}{2}}$	**l**	$-\frac{1}{3}x^{-\frac{4}{3}}$
	m	$9x^8$	**n**	$5x^4$	**o**	$3x^2$	**p**	$-2x^{-3}$
	q	1	**r**	$3x^2$				
2	**a**	$6x$	**b**	$54x^8$	**c**	$2x^3$	**d**	$5x^{-\frac{3}{4}}$
	e	$\frac{15}{2}x^{\frac{1}{2}}$	**f**	$-10x^{-2}$	**g**	$6x^2$	**h**	$-\frac{1}{2x^5}$
	i	$x^{-\frac{3}{2}}$	**j**	$\frac{15}{2}\sqrt{x}$				
3	**a**	$\frac{3}{4}$	**b**	$\frac{1}{2}$	**c**	3	**d**	2

4 $\frac{dy}{dx} = \frac{3}{2}\sqrt{\frac{x}{2}}$