P2 Chapter 1: Algebra Techniques

Partial Fractions

Partial Fractions

If the **denominator is a product of a linear terms**, it can be split into the sum of 'partial fractions', where **each denominator is a single linear term**.

$$\frac{6x-2}{(x-3)(x+1)} \equiv \frac{A}{x-3} + \frac{B}{x+1}$$

Notation reminder: $\equiv$ means 'equivalent/identical to', and indicates that both sides are equal for **all** values of x.

Method 1: Substitution

$$6x-2 \equiv A(x+1) + B(x-3)$$

We don't like fractions, so multiply through by denominator of LHS. *See note below.*

Let $x=-1$:

$$-8 = -4B \quad \rightarrow \quad B = 2$$

Let $x=3$:

$$16 = 4A \quad \rightarrow \quad A = 4$$

Choose values of x that make one of brackets disappear.

$$\therefore \frac{6x-2}{(x-3)(x+1)} \equiv \frac{4}{x-3} + \frac{2}{x+1}$$

When we multiply $\frac{A}{x-3}$ by $(x-3)(x+1)$, multiplying initially by $(x-3)$ cancels out the "over $x-3$", but we still need to multiply by the $(x+1)$, giving $A(x+1)$. The textbook instead has an intermediate step of adding the fractions on the RHS first, but I feel this is unnecessary.

Method 2: Comparing Coefficients

$$6x-2 \equiv A(x+1) + B(x-3)$$

Comparing coefficients of x terms:

$$6 = A + B$$

Comparing constant terms:

$$-2 = A - 3B$$

If two sides are identical, x terms must match, constant terms must match and so on, e.g. if $3x+4 \equiv ax+b$ Then $a=3$ and $b=4$, whereas this is not necessarily true for a normal equality.

Solving simultaneously:

$$A = 4, \qquad B = 2$$

$$\therefore \frac{6x-2}{(x-3)(x+1)} \equiv \frac{4}{x-3} + \frac{2}{x+1}$$

Further Example

Given that $\frac{6x^2+5x-2}{x(x-1)(2x+1)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1}$, find the values of the constants A, B, C.

$$6x^2 + 5x - 2 \equiv A(x-1)(2x+1) + Bx(2x+1) + Cx(x-1)$$

Let $x = 1$:

$$9 = 3B \quad \rightarrow \quad B = 3$$

Let $x = -\frac{1}{2}$:

$$-3 = \frac{3}{4}C \quad \rightarrow \quad C = -4$$

Let $x = 0$:

$$-2 = -A \quad \rightarrow \quad A = 2$$

$$\therefore \frac{6x^2 + 5x - 2}{x(x-1)(2x+1)} \equiv \frac{2}{x} + \frac{3}{x-1} - \frac{4}{2x+1}$$

Test Your Understanding

C4 June 2005 Q3a

Express $\dfrac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

(3)

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Test Your Understanding

C4 June 2005 Q3a

Express $\dfrac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

(3)

(a)	$$\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$$	
	$$5x+3 = A(x+2) + B(2x-3)$$	
	Substituting $x=-2$ or $x=\frac{3}{2}$ and obtaining A or B; or equating coefficients and solving a pair of simultaneous equations to obtain A or B.	M1
	$$A=3, B=1$$	A1, A1 **(3)**
	If the cover-up rule is used, give M1 A1 for the first of A or B found, A1 for the second.	

Exercise 1.3

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Homework Exercise

1 Express the following as partial fractions:

a $\dfrac{6x-2}{(x-2)(x+3)}$ **b** $\dfrac{2x+11}{(x+1)(x+4)}$ **c** $\dfrac{-7x-12}{2x(x-4)}$

d $\dfrac{2x-13}{(2x+1)(x-3)}$ **e** $\dfrac{6x+6}{x^2-9}$

Hint First factorise the denominator.

f $\dfrac{7-3x}{x^2-3x-4}$ **g** $\dfrac{8-x}{x^2+4x}$ **h** $\dfrac{2x-14}{x^2+2x-15}$

2 Show that $\dfrac{-2x-5}{(4+x)(2-x)}$ can be written in the form $\dfrac{A}{4+x}+\dfrac{B}{2-x}$ where A and B are constants to be found. **(3 marks)**

3 The expression $\dfrac{A}{(x-4)(x+8)}$ can be written in partial fractions as $\dfrac{2}{x-4}+\dfrac{B}{x+8}$

Find the values of the constants A and B.

4 $\mathrm{h}(x)=\dfrac{2x^2-12x-26}{(x+1)(x-2)(x+5)}, x>2$

Given that $\mathrm{h}(x)$ can be expressed in the form $\dfrac{A}{x+1}+\dfrac{B}{x-2}+\dfrac{C}{x+5}$, find the values of A, B and C. **(4 marks)**

Homework Exercise

E **5** Given that, for $x < -1$, $\dfrac{-10x^2 - 8x + 2}{x(2x+1)(3x-2)} \equiv \dfrac{D}{x} + \dfrac{E}{2x+1} + \dfrac{F}{3x-2}$, where D, E and F are constants. Find the values of D, E and F. **(4 marks)**

6 Express the following as partial fractions:

a $\dfrac{2x^2 - 12x - 26}{(x+1)(x-2)(x+5)}$ **b** $\dfrac{-10x^2 - 8x + 2}{x(2x+1)(3x-2)}$ **c** $\dfrac{-5x^2 - 19x - 32}{(x+1)(x+2)(x-5)}$

P **7** Express the following as partial fractions:

a $\dfrac{6x^2 + 7x - 3}{x^3 - x}$ **b** $\dfrac{8x + 9}{10x^2 + 3x - 4}$

Hint First factorise the denominator.

Challenge

Express $\dfrac{5x^2 - 15x - 8}{x^3 - 4x^2 + x + 6}$ as a sum of fractions with linear denominators.

Homework Answers

1 **a** $\dfrac{4}{x+3}+\dfrac{2}{x-2}$ **b** $\dfrac{3}{x+1}-\dfrac{1}{x+4}$

c $\dfrac{3}{2x}-\dfrac{5}{x-4}$ **d** $\dfrac{4}{2x+1}-\dfrac{1}{x-3}$

e $\dfrac{2}{x+3}+\dfrac{4}{x-3}$ **f** $-\dfrac{2}{x+1}-\dfrac{1}{x-4}$

g $\dfrac{2}{x}-\dfrac{3}{x+4}$ **h** $\dfrac{3}{x+5}-\dfrac{1}{x-3}$

2 $A=\frac{1}{2}, B=-\frac{3}{2}$

3 $A=24, B=-2$

4 $A=1, B=-2, C=3$

5 $D=-1, E=2, F=-5$

6 $\dfrac{3}{x+1}-\dfrac{2}{x+2}-\dfrac{6}{x-5}$

7 **a** $\dfrac{3}{x}-\dfrac{2}{x+1}+\dfrac{5}{x-1}$

b $\dfrac{-1}{5x+4}+\dfrac{2}{2x-1}$

Challenge

$\dfrac{6}{x-2}+\dfrac{1}{x+1}-\dfrac{2}{x-3}$