P2 Chapter 2: Graphing Functions

The Modulus Function

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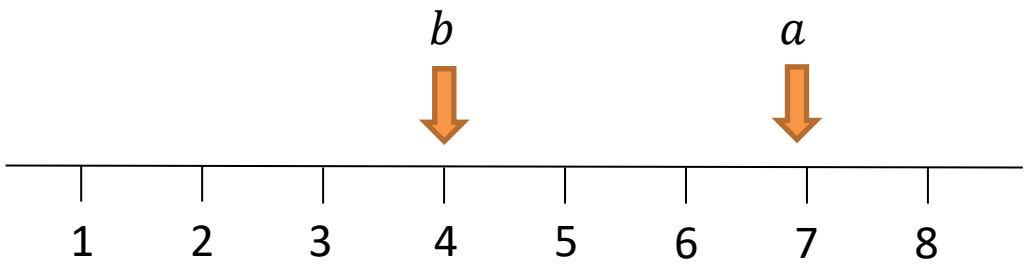
The modulus of a number a, written $|a|$, is its **non-negative** numerical value.
e.g. $|6| = 6$ and $|-7.1| = 7.1$



The modulus function is particularly useful in expressing a **difference**. We generally like to quote differences as positive values, but $b - a$ may be negative if b is smaller than a. By using $|b - a|$, we get round this problem!

More fundamentally, the modulus of a value gives us its '**magnitude**', i.e. size; from Mechanics, you should also be used to the notion the distances and speeds are quoted as positive values.
And in Pure Year 1 we saw the same notation used for vectors: $|\boldsymbol{a}|$ gives us the magnitude/length of the vector $\boldsymbol{a}$. It's the same function!

Examples

If $f(x) = |2x - 3| + 1$, find

a) $f(5)$

b) $f(-2)$

c) $f(1)$

?

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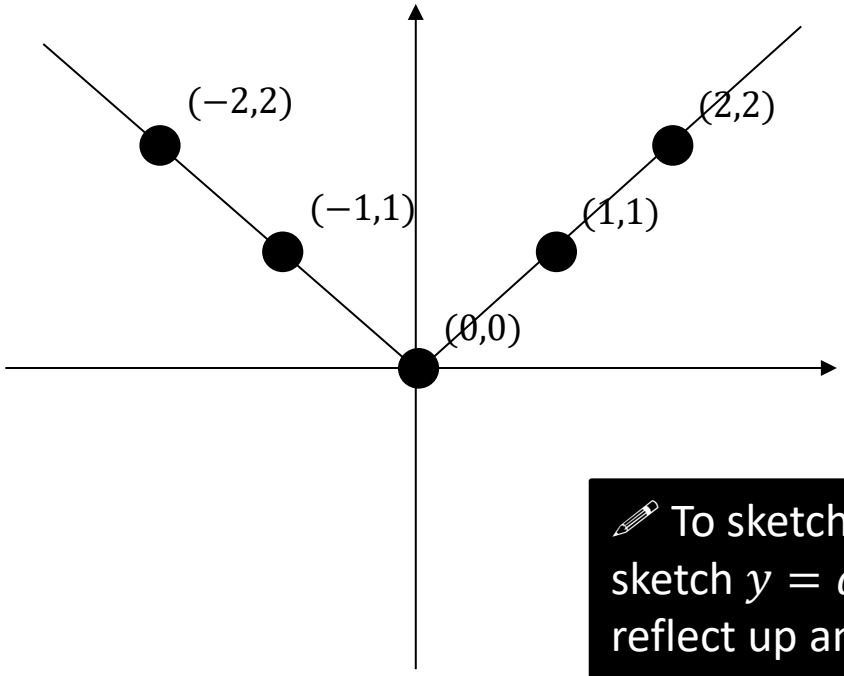
$$\boldsymbol{f(5) = |2(5) - 3| + 1 = |7| + 1 = 8}$$
$$\boldsymbol{f(-2) = |2(-2) - 3| + 1 = |-7| + 1 = 8}$$
$$\boldsymbol{f(1) = |2(1) - 3| + 1 = |-1| + 1 = 2}$$

Modulus Graphs

$y = |x|$

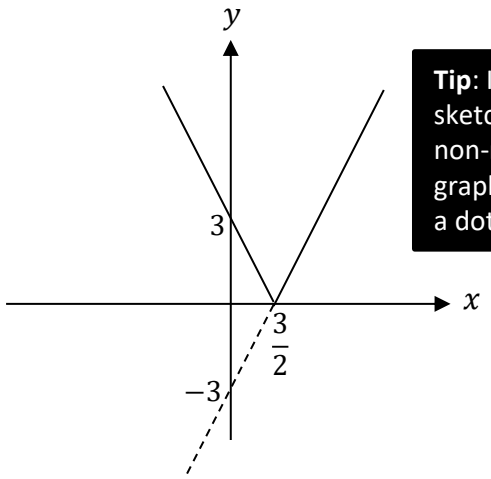
x	-2	-1	0	1	2
y	2	1	0	1	2



To sketch $y = |ax + b|$, sketch $y = ax + b$ then reflect up any section below the x-axis.

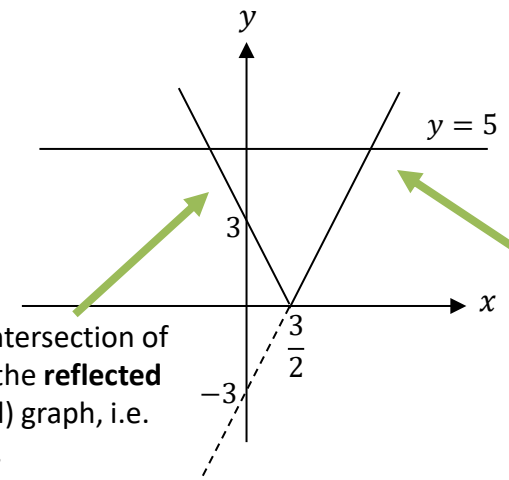
Modulus Graphs

Sketch $y = |2x - 3|$



Tip: I like to sketch the non-modulus graph first with a dotted line.

Solve $|2x - 3| = 5$



As you would have done in Pure Year 1, sketch a line for each side of the equation, so that we can use the points of intersection.

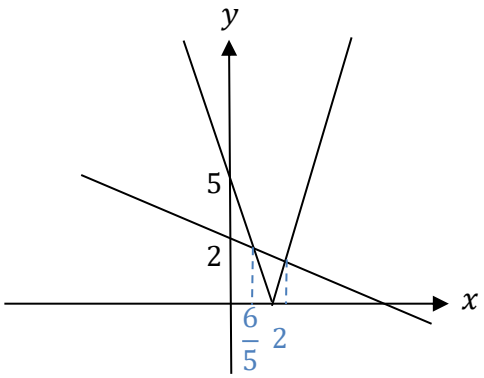
This is the intersection of $y = 5$ with the **reflected** (i.e. negated) graph, i.e. $y = 3 - 2x$.

$$\mathbf{3 - 2x = 5}$$
$$\boldsymbol{x = -1}$$

This is the intersection of $y = 5$ with the original **unreflected** graph, i.e. $y = 2x - 3$.

$$\mathbf{2x - 3 = 5}$$
$$\boldsymbol{x = 4}$$

Solve $|3x - 5| = 2 - \frac{1}{2}x$



$$\mathbf{3x - 5 = 2 - \frac{1}{2}x}$$
$$\boldsymbol{x = 2}$$

$$\mathbf{5 - 3x = 2 - \frac{1}{2}x}$$
$$\boldsymbol{x = \frac{6}{5}}$$

Solve $|3x - 5| > 2 - \frac{1}{2}x$

The graph of $y = |3x - 5|$ needs to be above $y = 2 - \frac{1}{2}x$.

By observation (and using our points of intersection), this occurs when

$$x < \frac{6}{5} \text{ or } x > 2$$

Test Your Understanding

Solve $|x + 1| = 2x + 5$
(be careful – there's only one solution!)

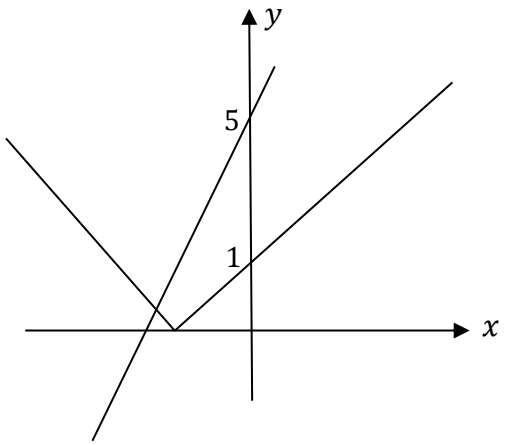
?

Solve $|4x - 1| < 2x$

?

Test Your Understanding

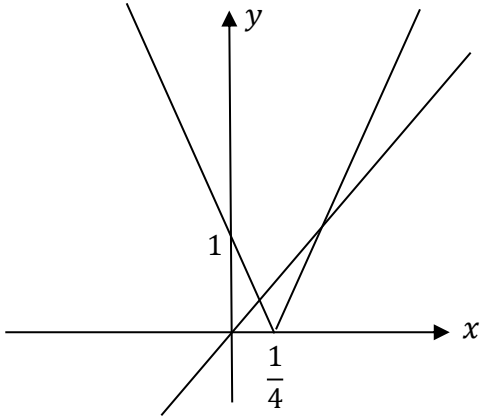
Solve $|x + 1| = 2x + 5$
(be careful – there's only one solution!)



We can see the line $y = 2x + 5$ only intersects with the reflected part of the modulus graph.

$$-x - 1 = 2x + 5$$
$$3x = -6$$
$$x = -2$$

Solve $|4x - 1| < 2x$



Determine 'critical values':

$$4x - 1 = 2x$$
$$x = \frac{1}{2}$$

$$1 - 4x = 2x$$
$$x = \frac{1}{6}$$

$$\therefore \frac{1}{6} < x < \frac{1}{2}$$

Exercise 2.1

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Homework Exercise

1 Write down the values of

a $\left|\frac{3}{4}\right|$ **b** $|-0.28|$ **c** $|3-11|$ **d** $\left|\frac{5}{7}-\frac{3}{8}\right|$ **e** $|20-6\times4|$ **f** $|4^2\times2-3\times7|$

2 $f(x)=|7-5x|+3$. Write down the values of:

a $f(1)$ **b** $f(10)$ **c** $f(-6)$

3 $g(x)=|x^2-8x|$. Write down the values of:

a $g(4)$ **b** $g(-5)$ **c** $g(8)$

4 Sketch the graph of each of the following. In each case, write down the coordinates of any points at which the graph meets the coordinate axes.

a $y=|x-1|$ **b** $y=|2x+3|$ **c** $y=|4x-7|$ **d** $y=\left|\frac{1}{2}x-5\right|$

e $y=|7-x|$ **f** $y=|6-4x|$

g $y=-|x|$ **h** $y=-|3x-1|$

Hint $y=-|x|$ is a reflection of $y=|x|$ in the x-axis. ← Year 1, Chapter 4

5 $g(x)=\left|4-\frac{3}{2}x\right|$ and $h(x)=5$

a On the same axes, sketch the graphs of $y=g(x)$ and $y=h(x)$.

b Hence solve the equation $\left|4-\frac{3}{2}x\right|=5$.

Homework Exercise

6 Solve:

a $|3x - 1| = 5$ **b** $\left|\frac{x-5}{2}\right| = 1$ **c** $|4x + 3| = -2$

d $|7x - 3| = 4$ **e** $\left|\frac{4-5x}{3}\right| = 2$ **f** $\left|\frac{x}{6} - 1\right| = 3$

7 **a** On the same diagram, sketch the graphs $y = -2x$ and $y = \left|\frac{1}{2}x - 2\right|$.

b Solve the equation $-2x = \left|\frac{1}{2}x - 2\right|$.

8 Solve $|3x - 5| = 11 - x$. **(4 marks)**

9 **a** On the same set of axes, sketch $y = |6 - x|$ and $y = \frac{1}{2}x - 5$.

b State with a reason whether there are any solutions to the equation $|6 - x| = \frac{1}{2}x - 5$.

10 A student attempts to solve the equation $|3x + 4| = x$. The student writes the following working:

$3x + 4 = x$		$-(3x + 4) = x$
$4 = -2x$	or	$-3x - 4 = x$
$x = -2$		$-4 = 4x$
		$x = -1$

Solutions are $x = -2$ and $x = -1$.

Explain the error made by the student.

Homework Exercise

11 **a** On the same diagram, sketch the graphs of $y = -|3x + 4|$ and $y = 2x - 9$.

b Solve the inequality $-|3x + 4| < 2x - 9$.

12 Solve the inequality $|2x + 9| < 14 - x$. **(4 marks)**

13 The equation $|6 - x| = \frac{1}{2}x + k$ has exactly one solution.

a Find the value of k. **(2 marks)**

b State the solution to the equation. **(2 marks)**

Problem-solving

The solution must be at the vertex of the graph of the modulus function.

Challenge

$f(x) = |x^2 + 9x + 8|$ and $g(x) = 1 - x$

a On the same axes, sketch graphs of $y = f(x)$ and $y = g(x)$.

b Use your sketch to find all the solutions to $|x^2 + 9x + 8| = 1 - x$.

Homework Answers

1 **a** $\frac{3}{4}$ **b** 0.28 **c** 8 **d** $\frac{19}{56}$ **e** 4 **f** 11

2 **a** 5 **b** 46 **c** 40

3 **a** 16 **b** 65 **c** 0

4 **a** Positive $|x|$ graph with vertex at (1, 0), y-intercept at (0, 1)

b Positive $|x|$ graph with vertex at $(-1\frac{1}{2}, 0)$, y-intercept at (0, 3)

c Positive $|x|$ graph with vertex at $(\frac{7}{4}, 0)$, y-intercept at (0, 7)

d Positive $|x|$ graph with vertex at (10, 0), y-intercept at (0, 5)

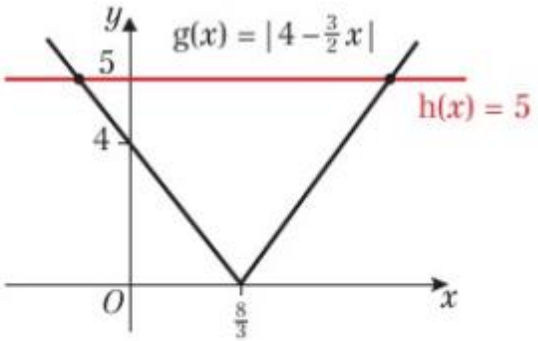
e Positive $|x|$ graph with vertex at (7, 0), y-intercept at (0, 7)

f Positive $|x|$ graph with vertex at $(\frac{3}{2}, 0)$, y-intercept at (0, 6)

g Negative $|x|$ graph with vertex and y-intercept at (0, 0)

h Negative $|x|$ graph with vertex at $(\frac{1}{3}, 0)$, y-intercept at (0, –1)

5 **a**



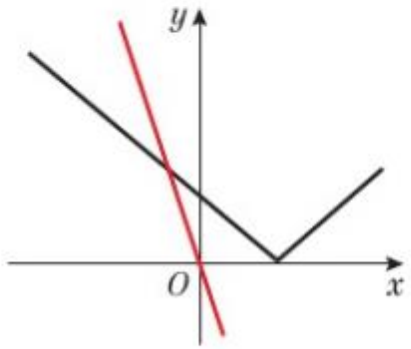
b $x = -\frac{2}{3}$ and $x = 6$

6 **a** $x = 2$ and $x = -\frac{4}{3}$ **b** $x = 7$ or $x = 3$

c No solution **d** $x = 1$ and $x = -\frac{1}{7}$

e $x = -\frac{2}{5}$ or $x = 2$ **f** $x = 24$ or $x = -12$

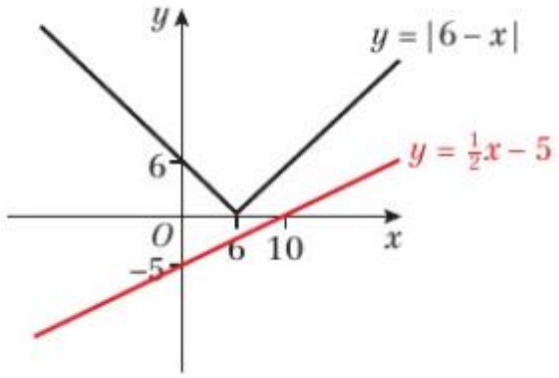
7 **a**



b $x = -\frac{4}{3}$

8 $x = -3$, $x = 4$

9 **a**

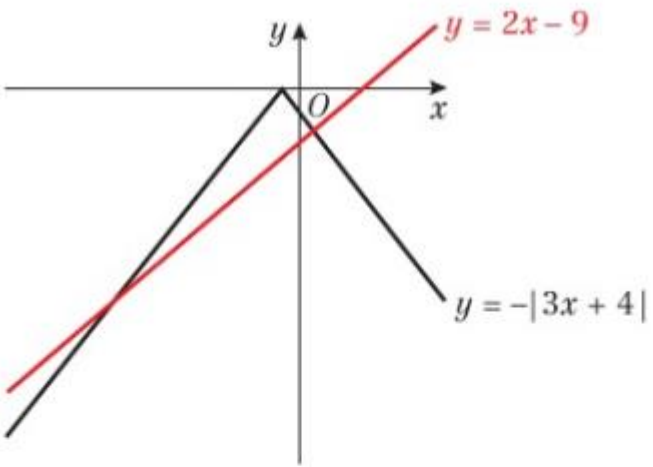


b The two graphs do not intersect, therefore there are no solutions to the equation $|6 - x| = \frac{1}{2}x - 5$.

10 Value for x cannot be negative as it equals a modulus.

Homework Answers

11 a



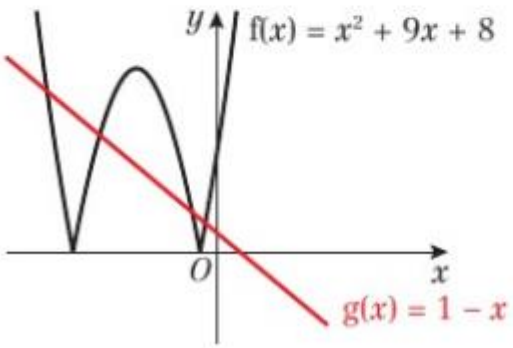
b $x < -13$ and $x > 1$

12 $-23 < x < \frac{5}{3}$

13 a $k = -3$ **b** Solution is $x = 6$.

Challenge

a



b There are 4 solutions: $x = -5 \pm 3\sqrt{2}$ and $x = -4 \pm \sqrt{7}$