P1 Chapter 12: Differentiation

Quadratic Gradient Function

Differentiating Multiple Terms

Differentiate $y = x^2 + 4x + 3$

First thing to note:
If $y = f(x) + g(x)$ then $\frac{dy}{dx} = f'(x) + g'(x)$
i.e. **differentiate each term individually in a sum/subtraction**.

$$\frac{dy}{dx} = 2x + 4$$

$$y = 4x = 4x^1$$

Therefore applying the usual rule:

$$\frac{dy}{dx} = 4x^0 = 4$$

Alternatively, if you compare $y = 4x$ to $y = mx + c$, it's clear that the gradient is fixed and $m = 4$.

$$y = 3 = 3x^0$$

Therefore applying the usual rule:

$$\frac{dy}{dx} = 0x^{-1} = 0$$

Alternatively, if you sketch $y = 4$, the line is horizontal, so the gradient is 0.

Quickfire Questions

1 $y = 2x^2 - 3x \quad \rightarrow \quad \frac{dy}{dx} =$?

2 $y = 4 - 9x^3 \quad \rightarrow \quad \frac{dy}{dx} =$?

3 $y = 5x + 1 \quad \rightarrow \quad \frac{dy}{dx} =$?

4 $y = ax$ (where a is a constant) $\quad \rightarrow \quad \frac{dy}{dx} =$?

5 $y = 6x - 3 + px^2$ (where p is a constant) $\quad \rightarrow \quad \frac{dy}{dx} =$?

Quickfire Questions

1 $y = 2x^2 - 3x \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = 4x - 3}$

2 $y = 4 - 9x^3 \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = -27x^2}$

3 $y = 5x + 1 \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = 5}$

4 $y = ax \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = a}$
(where a is a constant)

5 $y = 6x - 3 + px^2 \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = 6 + 2px}$
(where p is a constant)

Harder Example

Let $f(x) = 4x^2 - 8x + 3$

a) Find the gradient of $y = f(x)$ at the point $\left(\frac{1}{2}, 0\right)$

b) Find the coordinates of the point on the graph of $y = f(x)$ where the gradient is 8.

c) Find the gradient of $y = f(x)$ at the points where the curve meets the line $y = 4x - 5$.

a

?

Remember that the 'gradient function' allows you to find the gradient for a particular value of x.

b

?

This example is important!
Previously you used a value of x to get the gradient $f'(x)$. This time we're **doing the opposite**: using a known gradient $f'(x)$ to get the value of x. We therefore substitute $f'(x)$ for 8.

Once you have your x, you need to work out y.
Ensure you use the correct equation!

c

?

Harder Example

Let $f(x) = 4x^2 - 8x + 3$

a) Find the gradient of $y = f(x)$ at the point $\left(\frac{1}{2}, 0\right)$

b) Find the coordinates of the point on the graph of $y = f(x)$ where the gradient is 8.

c) Find the gradient of $y = f(x)$ at the points where the curve meets the line $y = 4x - 5$.

a

$f'(x) = 8x - 8$

When $f'\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right) - 8 = -4$

Remember that the 'gradient function' allows you to find the gradient for a particular value of x.

b

$8 = 8x - 8$

$x = 2$

$y = 4(2)^2 - 8(2) + 3 = 3$

Point is $(2,3)$

This example is important!
Previously you used a value of x to get the gradient $f'(x)$.
This time we're **doing the opposite**: using a known gradient $f'(x)$ to get the value of x. We therefore substitute $f'(x)$ for 8.

Once you have your x, you need to work out y.
Ensure you use the correct equation!

c

First find point of intersection:

$4x^2 - 8x + 3 = 4x - 5$

Solving, we obtain: $x = 1$ or $x = 2$

When $x = 1, f'(1) = 0$

When $x = 2, f'(2) = 8$

Test Your Understanding

Let $f(x) = x^2 - 4x + 2$

a) Find the gradient of $y = f(x)$ at the point $(1, -1)$

b) Find the coordinates of the point on the graph of $y = f(x)$ where the gradient is 5.

c) Find the gradient of $y = f(x)$ at the points where the curve meets the line $y = 2 - x$.

a

?

b

?

c

?

Test Your Understanding

Let $f(x) = x^2 - 4x + 2$

a) Find the gradient of $y = f(x)$ at the point $(1, -1)$

b) Find the coordinates of the point on the graph of $y = f(x)$ where the gradient is 5.

c) Find the gradient of $y = f(x)$ at the points where the curve meets the line $y = 2 - x$.

a

$f'(x) = 2x - 4$

When $f'(1) = 2(1) - 4 = -2$

b

$5 = 2x - 4$

$$x = \frac{9}{2}$$

$$y = \left(\frac{9}{2}\right)^2 - 4\left(\frac{9}{2}\right) + 2 = \frac{17}{4}$$

Point is $\left(\frac{9}{2}, \frac{17}{4}\right)$

c

$x^2 - 4x + 2 = 2 - x$

Solving: $x = 0$ or $x = 3$

When $x = 0, f'(0) = -4$

When $x = 3, f'(3) = 2$

Exercise 12.4

Pearson Pure Mathematics Year 1/AS
Page 94

Homework Exercise

1 Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when y equals:

a $2x^2 - 6x + 3$ **b** $\frac{1}{2}x^2 + 12x$ **c** $4x^2 - 6$

d $8x^2 + 7x + 12$ **e** $5 + 4x - 5x^2$

2 Find the gradient of the curve with equation:

a $y = 3x^2$ at the point (2, 12) **b** $y = x^2 + 4x$ at the point (1, 5)

c $y = 2x^2 - x - 1$ at the point (2, 5) **d** $y = \frac{1}{2}x^2 + \frac{3}{2}x$ at the point (1, 2)

e $y = 3 - x^2$ at the point (1, 2) **f** $y = 4 - 2x^2$ at the point (−1, 2)

3 Find the y-coordinate and the value of the gradient at the point P with x-coordinate 1 on the curve with equation $y = 3 + 2x - x^2$.

4 Find the coordinates of the point on the curve with equation $y = x^2 + 5x - 4$ where the gradient is 3.

5 Find the gradients of the curve $y = x^2 - 5x + 10$ at the points A and B where the curve meets the line $y = 4$.

6 Find the gradients of the curve $y = 2x^2$ at the points C and D where the curve meets the line $y = x + 3$.

7 $\mathrm{f}(x) = x^2 - 2x - 8$

a Sketch the graph of $y = \mathrm{f}(x)$.

b On the same set of axes, sketch the graph of $y = \mathrm{f}'(x)$.

c Explain why the x-coordinate of the turning point of $y = \mathrm{f}(x)$ is the same as the x-coordinate of the point where the graph of $y = \mathrm{f}'(x)$ crosses the x-axis.

Homework Answers

1 **a** $4x - 6$ **b** $x + 12$ **c** $8x$ **d** $16x + 7$
e $4 - 10x$

2 **a** 12 **b** 6 **c** 7 **d** $2\frac{1}{2}$
e -2 **f** 4

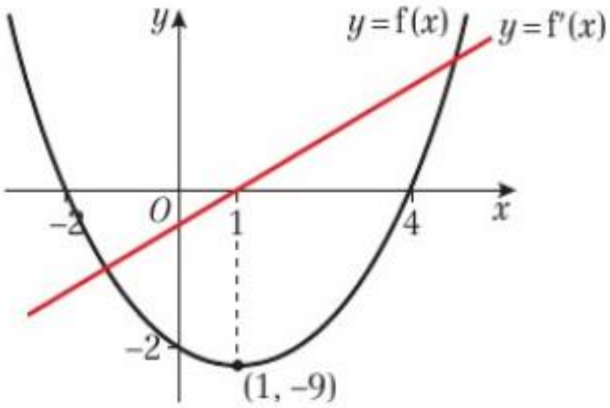
3 4, 0

4 $(-1, -8)$

5 1, -1

6 6, -4

7 **a, b**



c At the turning point, the gradient of $y = f(x)$ is zero, i.e. $f'(x) = 0$.