P1 Chapter 12: Differentiation

Polynomial Gradients

Differentiating Harder Expressions

If your expression isn't a sum of x^n terms, simply manipulate it until it is!

1. Turn roots into powers:

$$y = \sqrt{x} = \boxed{?} \rightarrow \frac{dy}{dx} = \boxed{?}$$

$$y = \frac{1}{\sqrt[3]{x}} = \boxed{?} \rightarrow \frac{dy}{dx} = \boxed{?}$$

2. Split up fractions.

$$y = \frac{x^2 + 3}{\sqrt{x}} = \boxed{?}$$

$$\rightarrow \frac{dy}{dx} = \boxed{?}$$

3. Expand out brackets.

$$y = x^2(x - 3) = \boxed{?}$$

$$\rightarrow \frac{dy}{dx} = \boxed{?}$$

4. Beware of numbers in denominators!

NOT $3x^{-1}$!!!

$$y = \frac{1}{3x} = \boxed{?} \rightarrow \frac{dy}{dx} = \boxed{?}$$

Differentiating Harder Expressions

If your expression isn't a sum of x^n terms, simply manipulate it until it is!

1. Turn roots into powers:

$$y = \sqrt{x} = x^{\frac{1}{2}} \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}}$$

$$y = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}} \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = -\frac{1}{3}x^{-\frac{4}{3}}}$$

2. Split up fractions.

$$y = \frac{x^2+3}{\sqrt{x}} = \frac{x^2+3}{x^{\frac{1}{2}}} = x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$$

$$\rightarrow \quad \boldsymbol{\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}}$$

3. Expand out brackets.

$$y = x^2(x-3) = x^3 - 3x^2$$

$$\rightarrow \quad \boldsymbol{\frac{dy}{dx} = 3x^2 - 6x}$$

NOT $3x^{-1}$!!!

4. Beware of numbers in denominators!

$$y = \frac{1}{3x} = \frac{1}{3}x^{-1} \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = -\frac{1}{3}x^{-2}}$$

Test Your Understanding

Differentiate the following.

$$y = \frac{1}{\sqrt{x}} = \text{?}$$

$$y = \frac{2 + x^3}{x^2} = \text{?}$$

$$y = \frac{1 + 2x}{3x\sqrt{x}} = \text{?}$$

Test Your Understanding

Differentiate the following.

$$y = \frac{1}{\sqrt{x}} = \boldsymbol{x^{-\frac{1}{2}}} \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}}$$

$$y = \frac{2 + x^3}{x^2} = \boldsymbol{2x^{-2} + x} \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = -4x^{-3} + 1}$$

$$y = \frac{1 + 2x}{3x\sqrt{x}} = \boldsymbol{\frac{1 + 2x}{3x^{\frac{3}{2}}}} = \boldsymbol{\frac{1}{3}x^{-\frac{3}{2}}} + \boldsymbol{\frac{2}{3}x^{-\frac{1}{2}}} \quad \rightarrow \quad \boldsymbol{\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{5}{2}} - \frac{1}{3}x^{-\frac{3}{2}}}$$

Exercise 12.5

Pearson Pure Mathematics Year 1/AS
Page 95

Extension

This just means "differentiate twice". We'll be looking at the 'second derivative' later in this chapter.

[MAT 2013 1E]

The expression $\frac{d^2}{dx^2}[(2x-1)^4(1-x)^5]+\frac{d}{dx}[(2x+1)^4(3x^2-2)^2]$ is a polynomial of degree:

A) 9
B) 8
C) 7
D) less than 7

?

Exercise 12.5

Pearson Pure Mathematics Year 1/AS
Page 95

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[MAT 2013 1E]

The expression $\frac{d^2}{dx^2}[(2x-1)^4(1-x)^5]+\frac{d}{dx}[(2x+1)^4(3x^2-2)^2]$ is a polynomial of degree:

A) 9
B) 8
C) 7
D) less than 7

Full expansion is not needed. The highest power term in the first polynomial is $16x^4\times(-x)^5=-16x^9$, differentiating twice to give $(-16\times 9\times 8)^9x^7$. The highest power term in the second polynomial is $16x^4\times 9x^2=(16\times 9)x^8$, differentiating once to give $(16\times 9\times 8)x^7$. These terms cancel leaving a polynomial of order (at most) 6. The answer is (D).

Homework Exercise

1 Differentiate:

a $x^4 + x^{-1}$ **b** $2x^5 + 3x^{-2}$ **c** $6x^{\frac{3}{2}} + 2x^{-\frac{1}{2}} + 4$

2 Find the gradient of the curve with equation $y = \mathrm{f}(x)$ at the point A where:

a $\mathrm{f}(x) = x^3 - 3x + 2$ and A is at $(-1, 4)$ **b** $\mathrm{f}(x) = 3x^2 + 2x^{-1}$ and A is at $(2, 13)$

3 Find the point or points on the curve with equation $y = \mathrm{f}(x)$, where the gradient is zero:

a $\mathrm{f}(x) = x^2 - 5x$ **b** $\mathrm{f}(x) = x^3 - 9x^2 + 24x - 20$

c $\mathrm{f}(x) = x^{\frac{3}{2}} - 6x + 1$ **d** $\mathrm{f}(x) = x^{-1} + 4x$

4 Differentiate:

a $2\sqrt{x}$ **b** $\dfrac{3}{x^2}$ **c** $\dfrac{1}{3x^3}$ **d** $\frac{1}{3}x^3(x - 2)$

e $\dfrac{2}{x^3} + \sqrt{x}$ **f** $\sqrt[3]{x} + \dfrac{1}{2x}$ **g** $\dfrac{2x + 3}{x}$ **h** $\dfrac{3x^2 - 6}{x}$

i $\dfrac{2x^3 + 3x}{\sqrt{x}}$ **j** $x(x^2 - x + 2)$ **k** $3x^2(x^2 + 2x)$ **l** $(3x - 2)\left(4x + \dfrac{1}{x}\right)$

5 Find the gradient of the curve with equation $y = \mathrm{f}(x)$ at the point A where:

a $\mathrm{f}(x) = x(x + 1)$ and A is at $(0, 0)$ **b** $\mathrm{f}(x) = \dfrac{2x - 6}{x^2}$ and A is at $(3, 0)$

c $\mathrm{f}(x) = \dfrac{1}{\sqrt{x}}$ and A is at $\left(\frac{1}{4}, 2\right)$ **d** $\mathrm{f}(x) = 3x - \dfrac{4}{x^2}$ and A is at $(2, 5)$

Homework Exercise

6 $f(x) = \dfrac{12}{p\sqrt{x}} + x$, where p is a real constant and $x > 0$.

Given that $f'(2) = 3$, find p, giving your answer in the form $a\sqrt{2}$ where a is a rational number. **(4 marks)**

7 $f(x) = (2 - x)^9$

a Find the first 3 terms, in ascending powers of x, of the binomial expansion of $f(x)$, giving each term in its simplest form.

b If x is small, so that x^2 and higher powers can be ignored, show that $f'(x) \approx 9216x - 2304$.

Hint Use the binomial expansion with $a = 2$, $b = -x$ and $n = 9$. ← Section 8.3

Homework Answers

1 **a** $4x^3 - x^{-2}$ **b** $10x^4 - 6x^{-3}$ **c** $9x^{\frac{1}{2}} - x^{-\frac{3}{2}}$

2 **a** 0 **b** $11\frac{1}{2}$

3 **a** $(2\frac{1}{2}, -6\frac{1}{4})$ **b** $(4, -4)$ and $(2, 0)$

c $(16, -31)$ **d** $(\frac{1}{2}, 4)$ $(-\frac{1}{2}, -4)$

4 **a** $x^{-\frac{1}{2}}$ **b** $-6x^{-3}$ **c** $-x^{-4}$

d $\frac{4}{3}x^3 - 2x^2$ **e** $\frac{1}{2}x^{-\frac{1}{2}} - 6x^{-4}$ **f** $\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-2}$

g $-3x^{-2}$ **h** $3 + 6x^{-2}$ **i** $5x^{\frac{3}{2}} + \frac{3}{2}x^{-\frac{1}{2}}$

j $3x^2 - 2x + 2$ **k** $12x^3 + 18x^2$ **l** $24x - 8 + 2x^{-2}$

5 **a** 1 **b** $\frac{2}{9}$ **c** -4 **d** 4

6 $-\frac{3}{4}\sqrt{2}$

7 **a** $512 - 2304x + 4608x^2$

b
$$\begin{aligned} f'(x) &\approx \frac{d}{dx}(512 - 2304x + 4608x^2) \\ &= -2304 + 2 \times 4608x \\ &= 9216x - 2304 \end{aligned}$$