M1 Chapter 11: Variable Acceleration

Chapter Practice

Key Points

Summary of key points

1 If the displacement, s, is expressed as a function of t, then the velocity, v, can be expressed as

$$v = \frac{ds}{dt}$$

2 If the velocity, v, is expressed as a function of t, then the acceleration, a, can be expressed as

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

3

Differentiate ↓

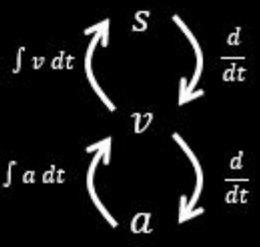
displacement $= s = \int v\,dt$

$\frac{ds}{dt}$ = velocity $= v = \int a\,dt$

$\frac{dv}{dt} = \frac{d^2s}{dt^2}$ = acceleration $= a$

↑ Integrate

Memory Tip: I picture interchanging between s, v, a as differentiating to go downwards and integrating to go upwards:



Chapter Exercises

1 A particle P moves in a horizontal straight line. At time t seconds (where $t \geqslant 0$) the velocity $v\,\text{m s}^{-1}$ of P is given by $v = 15 - 3t$. Find:

a the value of t when P is instantaneously at rest

b the distance travelled by P between the time when $t = 0$ and the time when P is instantaneously at rest.

2 A particle P moves along the x-axis so that, at time t seconds, the displacement of P from O is x metres and the velocity of P is $v\,\text{m s}^{-1}$, where:

$$v = 6t + \tfrac{1}{2}t^3$$

a Find the acceleration of P when $t = 4$.

b Given also that $x = -5$ when $t = 0$, find the distance OP when $t = 4$.

3 A particle P is moving along a straight line. At time $t = 0$, the particle is at a point A and is moving with velocity $8\,\text{m s}^{-1}$ towards a point B on the line, where $AB = 30\,\text{m}$. At time t seconds (where $t \geqslant 0$), the acceleration of P is $(2 - 2t)\,\text{m s}^{-2}$ in the direction $\overrightarrow{AB}$.

a Find an expression, in terms of t, for the displacement of P from A at time t seconds.

b Show that P does not reach B.

c Find the value of t when P returns to A, giving your answer to 3 significant figures.

d Find the total distance travelled by P in the interval between the two instants when it passes through A.

4 A particle starts from rest at a point O and moves along a straight line OP with an acceleration, a, after t seconds given by $a = (8 - 2t^2)\,\text{m s}^{-2}$.

Find:

a the greatest speed of the particle in the direction OP **(5 marks)**

b the distance covered by the particle in the first two seconds of its motion. **(4 marks)**

Chapter Exercises

5 A particle P passes through a point O and moves in a straight line. The displacement, s metres, of P from O, t seconds after passing through O is given by:

$$s = -t^3 + 11t^2 - 24t$$

a Find an expression for the velocity, $v\,\text{m s}^{-1}$, of P at time t seconds. **(2 marks)**

b Calculate the values of t at which P is instantaneously at rest. **(3 marks)**

c Find the value of t at which the acceleration is zero. **(2 marks)**

d Sketch a velocity–time graph to illustrate the motion of P in the interval $0 \leqslant t \leqslant 6$, showing on your sketch the coordinates of the points at which the graph crosses the axes. **(3 marks)**

e Calculate the values of t in the interval $0 \leqslant t \leqslant 6$ between which the speed of P is greater than $16\,\text{m s}^{-1}$. **(6 marks)**

6 A body moves in a straight line. Its velocity, $v\,\text{m s}^{-1}$, at time t seconds is given by $v = 3t^2 - 11t + 10$. Find:

a the values of t when the body is instantaneously at rest **(3 marks)**

b the acceleration of the body when $t = 4$ **(3 marks)**

c the total distance travelled by the body in the interval $0 \leqslant t \leqslant 4$. **(4 marks)**

7 A particle moves along the positive x-axis. At time $t = 0$ the particle passes through the origin with velocity $6\,\text{m s}^{-1}$. The acceleration, $a\,\text{m s}^{-2}$, of the particle at time t seconds is given by $a = 2t^3 - 8t$ for $t \geqslant 0$. Find:

a the velocity of the particle at time t seconds **(3 marks)**

b the displacement of the particle from the origin at time t seconds **(2 marks)**

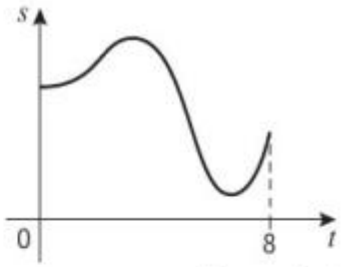
c the values of t at which the particle is instantaneously at rest. **(3 marks)**

Chapter Exercises

8 A remote control drone hovers such that its vertical height, s m, above ground level at time t seconds is given by the equation:

$$x = \frac{t^4 - 12t^3 + 28t^2 + 400}{50}, \quad 0 \leqslant t \leqslant 8$$

The diagram shows a sketch of a displacement–time graph of the drone's motion.



Determine the maximum and minimum height of the drone. **(7 marks)**

9 A rocket sled is used to test a parachute braking mechanism for a space capsule.

At the moment the parachute is deployed, the sled is 1.5 km from its launch site and is travelling away from it at a speed of 800 m s^{-1}. The sled comes to rest 25 seconds after the parachute is deployed.

The rocket sled is modelled as a particle moving in a straight horizontal line with constant acceleration. At a time t seconds after the parachute is deployed, its distance, s m, from the launch site is given by:

$$s = a + bt + ct^2, \ 0 \leqslant t \leqslant 25$$

Find the values of a, b and c in this model. **(6 marks)**

10 A particle P moves along the x-axis. It passes through the origin O at time $t = 0$ with speed 7 m s^{-1} in the direction of x increasing.

At time t seconds the acceleration of P in the direction of x increasing is $(20 - 6t)$ m s^{-2}.

a Show that the velocity v m s^{-1} of P at time t seconds is given by:

$$v = 7 + 20t - 3t^2$$ **(3 marks)**

b Show that $v = 0$ when $t = 7$ and find the greatest speed of P in the interval $0 \leqslant t \leqslant 7$. **(4 marks)**

c Find the distance travelled by P in the interval $0 \leqslant t \leqslant 7$. **(4 marks)**

Chapter Exercises

11 A particle P moves along a straight line. Initially, P is at rest at a point O on the line. At time t seconds (where $t \geqslant 0$) the acceleration of P is proportional to $(7 - t^2)$ and the displacement of P from O is s metres. When $t = 3$, the velocity of P is $6\,\text{m s}^{-1}$.

Show that $s = \frac{1}{24}t^2(42 - t^2)$. **(7 marks)**

12 A mouse leaves its hole and makes a short journey along a straight wall before returning to its hole. The mouse is modelled as a particle moving in a straight line. The distance of the mouse, s m, from its hole at time t minutes is given by:

$$s = t^4 - 10t^3 + 25t^2,\ 0 \leqslant t \leqslant 5$$

a Explain the restriction $0 \leqslant t \leqslant 5$. **(3 marks)**

b Find the greatest distance of the mouse from its hole. **(6 marks)**

13 At a time t seconds after launch, the space shuttle can be modelled as a particle moving in a straight line with acceleration, $a\,\text{m s}^{-2}$, given by the equation:

$$a = (6.77 \times 10^{-7})t^3 - (3.98 \times 10^{-4})t^2 + 0.105t + 0.859,\quad 124 \leqslant t \leqslant 446$$

a Suggest two reasons why the space shuttle might experience variable acceleration during its launch phase.

Given that the velocity of the space shuttle at time $t = 124$ is $974\,\text{m s}^{-1}$:

b find an expression for the velocity $v\,\text{m s}^{-1}$ of the space shuttle at time t. Give your coefficients to 3 significant figures.

c Hence find the velocity of the space shuttle at time $t = 446$, correct to 3 s.f.

From $t = 446$, the space shuttle maintains a constant acceleration of $28.6\,\text{m s}^{-2}$ until it reaches its escape velocity of $7.85\,\text{km s}^{-1}$. It then cuts its main engines.

d Calculate the time at which the space shuttle cuts its main engines.

Chapter Exercises

Challenge

1 A particle starts at rest and moves in a straight line. At time t seconds after the beginning of its motion, the acceleration of the particle, a m s^{-2}, is given by:

$$a = 3t^2 - 18t + 20, \ t \geqslant 0$$

Find the distance travelled by the particle in the first 5 seconds of its motion.

2 A particle travels in a straight line with an acceleration, a m s^{-2}, given by $a = 6t + 2$.

The particle travels 50 metres in the fourth second. Find the velocity of the particle when $t = 5$ seconds.

Chapter Answers

1 **a** $t = 5$ **b** 37.5 m

2 **a** $30\,\text{m s}^{-2}$ **b** 75 m

3 **a** Displacement $= 8t + t^2 - \frac{t^3}{3}$

b Max displacement when $t = 4$, $s = 26\frac{2}{3}$ m, which is less than 30 m so P does not reach B.

c $t = 6.62$ s

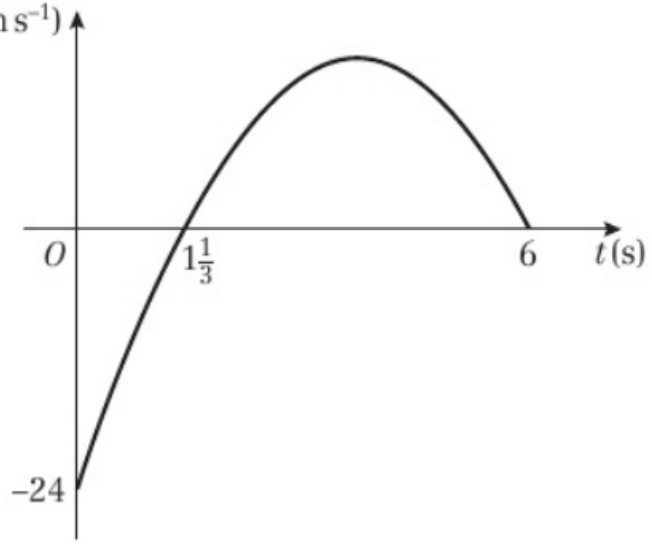
d $53\frac{1}{3}$ m

4 **a** $\frac{32}{3}\,\text{m s}^{-1}$ **b** $\frac{40}{3}$ m

5 **a** $(-3t^2 + 22t - 24)\,\text{m s}^{-1}$ **b** $t = \frac{4}{3}$ and $t = 6$

c $t = \frac{11}{3}$

d



e $0 \leqslant t < 0.38$, $\frac{10}{3} < t < 4$

6 **a** $t = \frac{5}{3}$ and $t = 2$ **b** $13\,\text{m s}^{-2}$ **c** $\frac{433}{27}$ m

7 **a** $v = \frac{t^4}{2} - 4t^2 + 6$ **b** $s = \frac{t^5}{10} - \frac{4t^3}{3} + 6t$

c $t = \sqrt{2}$ and $t = \sqrt{6}$

8 max = 8.64 m, min = 1.14 m

9 $a = 1500$, $b = 800$, $c = -16$

10 **a** $v = \int 20 - 6t\,\text{d}t = 20t - 3t^2 + c$

At $t = 0$, $v = 7$ so $c = 7$ and $v = 7 + 20t - 3t^2$

b The greatest speed is $40\frac{1}{3}\,\text{m s}^{-1}$

c 196 m

11 $v = \int k(7 - t^2)\text{d}t \Rightarrow v = k\left(7t - \frac{t^3}{3}\right) + c$

$t = 0$, $v = 0$ $\therefore c = 0$; $t = 3$, $v = 6$ $\therefore k = \frac{1}{2}$

$v = \frac{7}{2}t - \frac{t^3}{6}$

$s = \int v\text{d}t = \int \left(\frac{7}{2}t - \frac{t^3}{6}\right)\text{d}t = \frac{7t^2}{4} - \frac{t^4}{24} + c$

$t = 0$, $s = 0$ $\therefore c = 0$

$s = \frac{7t^2}{4} - \frac{t^4}{24} = \frac{1}{24}t^2(42 - t^2)$

12 **a** Time cannot be negative so $t \geqslant 0$

at $t = 5$ $s = 0$ so mouse has returned to its hole.

b 39.1 m

13 **a** Mass is not constant as fuel is used.
Gravity is not constant so weight not constant.
Thrust may not be constant.

b $v = (1.69 \times 10^{-7})\,t^4 - (1.33 \times 10^{-4})\,t^3 + 0.0525\,t^2 + 0.859\,t + 274\,\text{m s}^{-1}$

c $v = 5990\,\text{m s}^{-1}$

d 510 seconds (2 s.f.) after launch

Challenge

1 32.75 m

2 $91\,\text{m s}^{-1}$