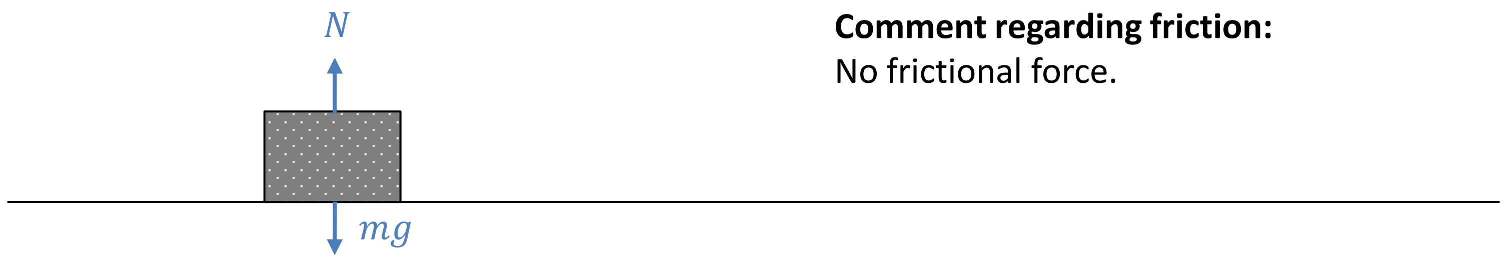
M2 Chapter 5: Inclined Planes

Friction

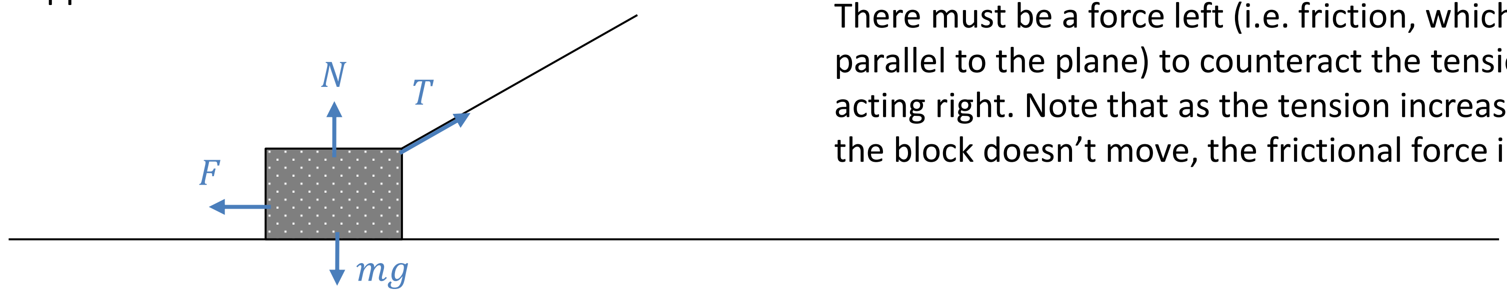
Friction

Scenario 1: A block is on a horizontal rough surface with no forces (other than gravity) acting on it.



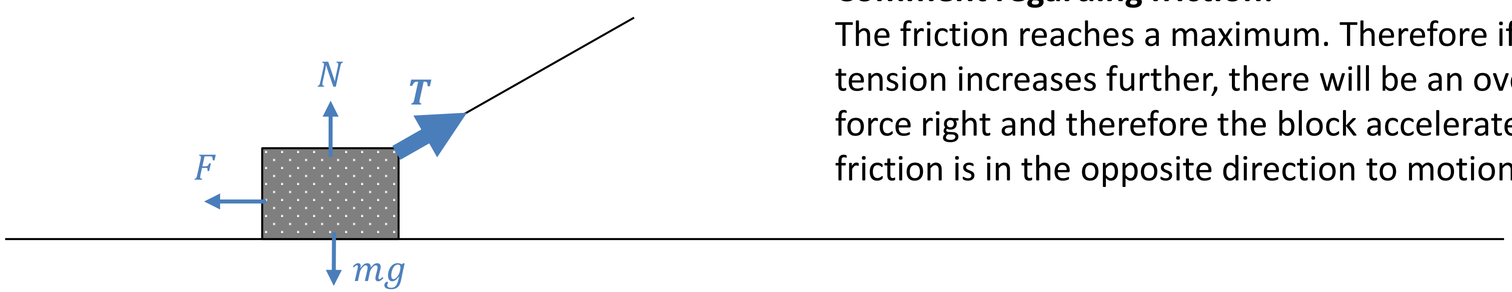
Comment regarding friction:
No frictional force.

Scenario 2: A cable is attached to the block and a force applied. The block doesn't move.



Comment regarding friction:
There must be a force left (i.e. friction, which acts parallel to the plane) to counteract the tension force acting right. Note that as the tension increases but the block doesn't move, the frictional force increases.

Scenario 3: The tension is increased until the block starts to move.



Comment regarding friction:
The friction reaches a maximum. Therefore if the tension increases further, there will be an overall force right and therefore the block accelerates. The friction is in the opposite direction to motion.

Friction

This 'maximum friction' depends on two things:

- How **rough** the surface is (i.e. the rougher the surface, the more force required before the block starts moving).
- How much the block is **pressing into** the surface.

✎ The maximum (limiting) friction acting on the two surfaces:

$$F_{max} = \mu N$$

where μ is the coefficient of friction and N is the normal support force acting on the supported object.

Example μ: (source physlink.com)

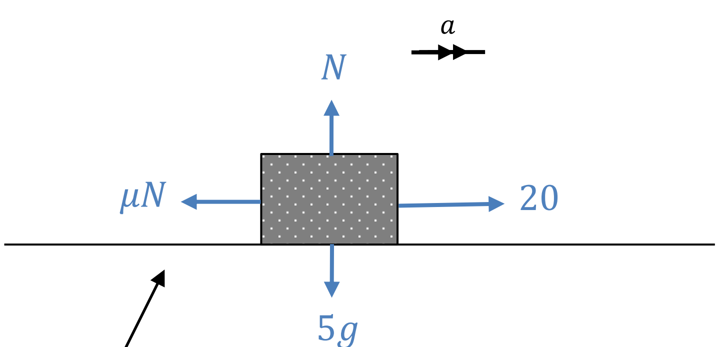
Materials	Coeff. of Static Friction μ_s
Steel on Steel	0.74
Aluminum on Steel	0.61
Copper on Steel	0.53
Rubber on Concrete	1.0
Wood on Wood	0.25-0.5
Glass on Glass	0.94
Waxed wood on Wet snow	0.14
Ice on Ice	0.1
Teflon on Teflon	0.04
Synovial joints in humans	0.01

Example

[Textbook] A particle of mass 5kg is pulled along a rough horizontal surface by a horizontal force of magnitude 20N. The coefficient of friction between the particle and the floor is 0.2. Calculate:

(a) the magnitude of frictional force
(b) the acceleration of the particle.

Note: The particle is moving, so friction will be at its maximum limit, i.e. μN.



Tip: Resolve forces perpendicular to the plane first, as we get an expression for N which can then be used in μN when we resolve parallel to the plane.

Tip: Avoid using *unsubscripted* F in the force diagram (to avoid confusion with the F in $\Sigma F = ma$) and use μN directly.

$R(\uparrow)$: $\quad N = 5g$

$R(\rightarrow)$:
$$20 - \mu N = 5a$$
$$20 - 0.2(5g) = 5a$$
$$a = 2.0\ ms^{-2}$$

Inclined Plane with Friction Example

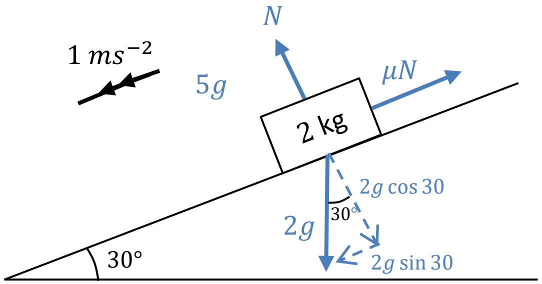
[Textbook] A particle of mass 2kg is sliding down a rough slope that is inclined at $30°$ to the horizontal. Given that the acceleration of the particle is $1ms^{-2}$, find the coefficient of friction μ between the particle and the slope.

? Diagram	? Working

Inclined Plane with Friction Example

[Textbook] A particle of mass 2kg is sliding down a rough slope that is inclined at 30° to the horizontal. Given that the acceleration of the particle is $1 ms^{-2}$, find the coefficient of friction μ between the particle and the slope.

Recall that friction ***always*** acts in ***opposite direction*** to motion.



$$R(\nwarrow): \quad N = 2g\cos 30°$$

$$R(\swarrow): \quad 2g\sin 30° - \mu N = 2a$$

$$g - 2\mu g\cos 30° = 2a$$

…

$$\mu = \frac{7.8}{16.974\ldots} = 0.46$$

Test Your Understanding

Edexcel M1(Old) May 2013 Q3

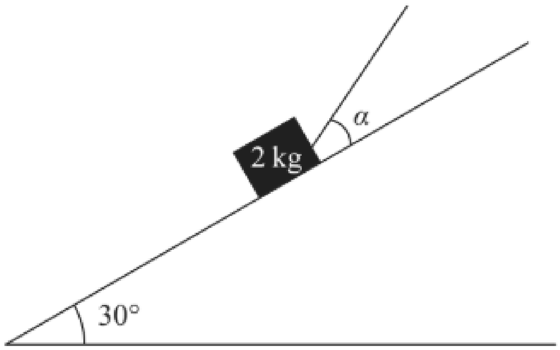


Figure 1

A box of mass 2 kg is held in equilibrium on a fixed rough inclined plane by a rope. The rope lies in a vertical plane containing a line of greatest slope of the inclined plane. The rope is inclined to the plane at an angle α, where $\tan \alpha = \frac{3}{4}$, and the plane is at an angle of 30° to the horizontal, as shown in Figure 1. The coefficient of friction between the box and the inclined plane is $\frac{1}{3}$ and the box is on the point of slipping up the plane. By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

(8)

?

Test Your Understanding

Edexcel M1(Old) May 2013 Q3

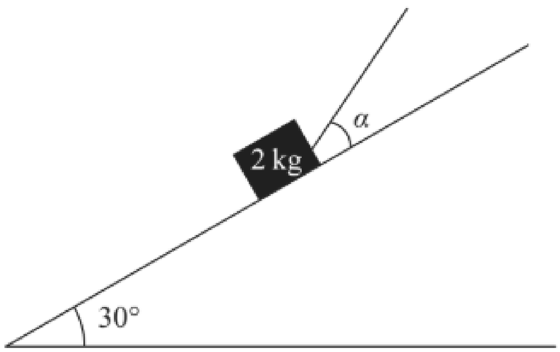


Figure 1

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(8)

$T\cos\alpha - F = 2g\cos 60^o$	M1 A1
$T\sin\alpha + R = 2g\cos 30^o$	M1 A1
$F = \frac{1}{3}R$	B1
eliminating F and R	**DM**1
$T = g(1+\frac{1}{\sqrt{3}})$, 1.6g (or better), 15.5, 15 (N)	**DM**1 A1
	(8)
	[8]

Exercise 5.3

Pearson Stats/Mechanics Year 2
Pages 46-47

Additional question:

1 A block lies on a rough plane at an incline of θ. The coefficient of friction between the block and plane is μ. If the block is on the verge of sliding down the plane, prove that $\mu = \tan\theta$.

?

Side Note: Since $\tan 45 = 1$, the implication is that it's very much possible to have values of μ greater than 1, i.e. if we have to raise the angle of the plane beyond $45°$ before the block starts sliding.

Exercise 5.3

Pearson Stats/Mechanics Year 2
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Additional question:

1 A block lies on a rough plane at an incline of θ. The coefficient of friction between the block and plane is μ. If the block is on the verge of sliding down the plane, prove that $\mu = \tan\theta$.

Let mass of block be m.

$$R(\nwarrow): \quad N = mg\cos\theta$$
$$R(\swarrow): \quad mg\sin\theta = \mu N$$
$$mg\sin\theta = \mu mg\cos\theta$$
$$\mu = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

Side Note: Since $\tan 45 = 1$, the implication is that it's very much possible to have values of μ greater than 1, i.e. if we have to raise the angle of the plane beyond $45°$ before the block starts sliding.

Homework Exercise

1 Each of the following diagrams shows a body of mass 5 kg lying initially at rest on rough horizontal ground. The coefficient of friction between the body and the ground is $\frac{1}{7}$. In each diagram R is the normal reaction of the ground on the body and F is the frictional force exerted on the body. Any other forces applied to the body are as shown on the diagrams.

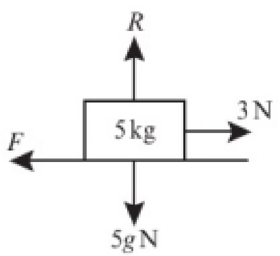
In each case

i find the magnitude of F,

ii state whether the body will remain at rest or accelerate from rest along the ground,

iii find, when appropriate, the magnitude of this acceleration.

Hint The forces acting on the body can affect the magnitude of the normal reaction. In part **d** the normal reaction is $(5g + 14)$ N, so $F_{MAX} = \mu(5g + 14)$ N.

a


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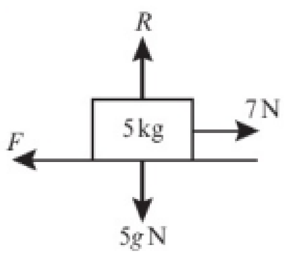

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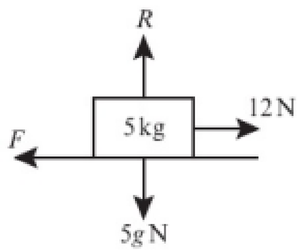

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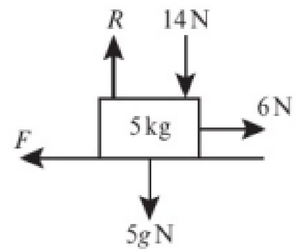

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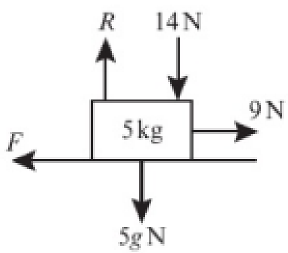
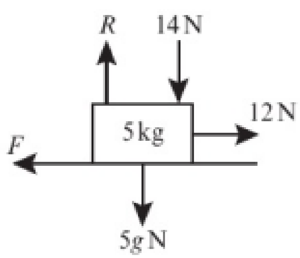


f


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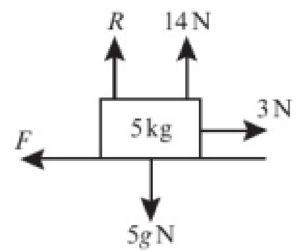

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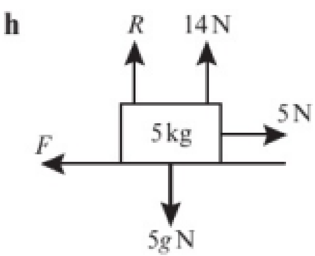
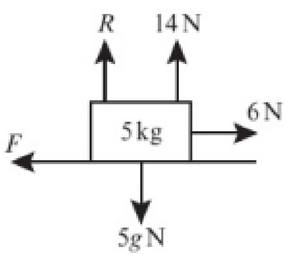


i


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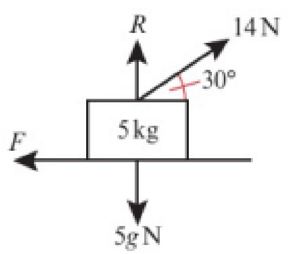

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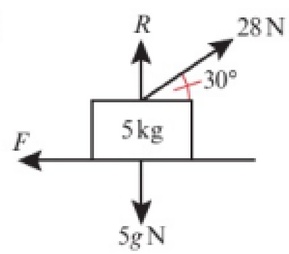

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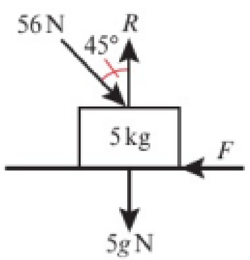
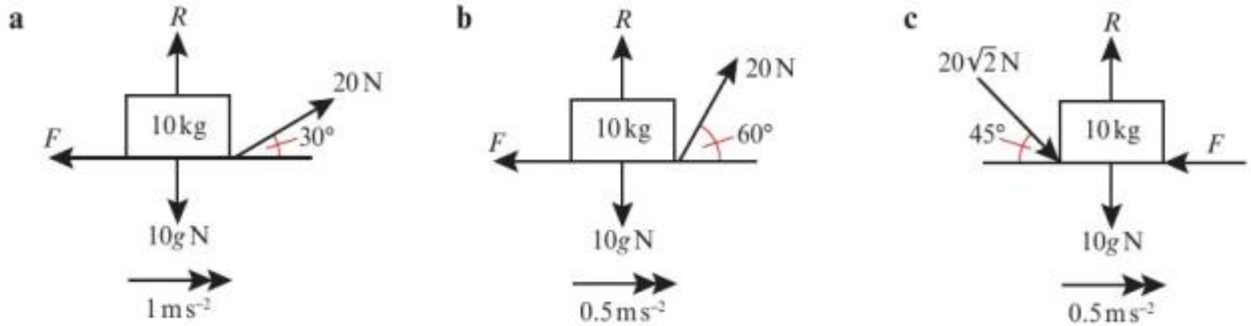


Homework Exercise

2 In each of the following diagrams, the forces shown cause the body of mass 10 kg to accelerate as shown along the rough horizontal plane. R is the normal reaction and F is the frictional force. Find the normal reaction and the coefficient of friction in each case.



3 A particle of mass 0.5 kg is sliding down a rough slope that is angled at 15° to the horizontal. The acceleration of the particle is $0.25\,\text{m}\,\text{s}^{-2}$. Calculate the coefficient of friction between the particle and the slope. **(3 marks)**

4 A particle of mass 2 kg is sliding down a rough slope that is angled at 20° to the horizontal. A force of magnitude P acts parallel to the slope and is attempting to pull the particle up the slope. The acceleration of the particle is $0.2\,\text{m}\,\text{s}^{-2}$ down the slope and the coefficient of friction between the particle and the slope is 0.3. Find the value of P. **(4 marks)**

5 A particle of mass 5 kg is being pushed up a rough slope that is angled at 30° to the horizontal by a horizontal force P. Given that the coefficient of friction is 0.2 and the acceleration of the particle is $2\,\text{m}\,\text{s}^{-2}$ calculate the value of P.

Homework Exercise

6 A sled of mass 10 kg is being pulled along a rough horizontal plane by a force P that acts at an angle of 45° to the horizontal. The coefficient of friction between the sled and the plane is 0.1. Given that the sled accelerates at 0.3 m s^{-2}, find the value of P.

(7 marks)

7 A train of mass m kg is travelling at 20 m s^{-1} when it applies its brakes, causing the wheels to lock up. The train decelerates at a constant rate, coming to a complete stop in 30 seconds.

a By modelling the train as a particle, show that the coefficient of friction between the railway track and the wheels of the train is $\mu = \frac{2}{3g}$. **(6 marks)**

Problem-solving

Use the formulae for constant acceleration.

← Year 1, Chapter 9

The train is no longer modelled as a particle, so that the effects of air resistance can be taken into account.

b State, with a reason, whether the coefficient of friction between the track and the wheels will increase or decrease in this revised model. **(2 marks)**

Challenge

A particle of mass m kg is sliding down a rough slope that is angled at α to the horizontal. The coefficient of friction between the particle and the slope is μ. Show that the acceleration of the particle is independent of its mass.

Homework Answers

1 **a** **i** 3 N **ii** $F = 3$ N and body remains at rest
b **i** 7 N **ii** $F = 7$ N and body remains at rest
c **i** 7 N **ii** $F = 7$ N and body accelerates
iii $1\,\mathrm{m\,s^{-2}}$
d **i** 6 N **ii** $F = 6$ N and body remains at rest
e **i** 9 N
ii $F = 9$ N and body remains at rest in limiting equilibrium
f **i** 9 N
ii $F = 9$ N and body accelerates
iii $0.6\,\mathrm{m\,s^{-2}}$
g **i** 3 N **ii** $F = 3$ N and body remains at rest
h **i** 5 N
ii $F = 5$ N and body remains at rest in limiting equilibrium
i **i** 5 N **ii** $F = 5$ N and body accelerates
iii $0.2\,\mathrm{m\,s^{-2}}$
j **i** 6 N **ii** $F = 6$ N and body accelerates
iii $1.22\,\mathrm{m\,s^{-2}}$ (3 s.f.)
k **i** 5 N **ii** $F = 5$ N and body accelerates
iii $3.85\,\mathrm{m\,s^{-2}}$ (3 s.f.)
l **i** 12.7 N (3 s.f.)
ii The body accelerates.
iii $5.39\,\mathrm{m\,s^{-2}}$ (3 s.f.)

2 **a** $R = 88$ N, $\mu = 0.083$ (2 s.f.)
b $R = 80.679$ N, $\mu = 0.062$ (2 s.f.)
c $R = 118$ N, $\mu = 0.13$ (2 s.f.)

3 0.242 (3 s.f.)

4 0.778 N (3 s.f.)

5 56.1 N (3 s.f.)

6 16.5 N (3 s.f.)

7 **a** Use $v = u + at$ to find $a = -\frac{2}{3}\mathrm{ms^{-2}}$
R(→): $-\mu mg = -\frac{2}{3}m$
$\mu = \frac{2}{3g}$
b The coefficient of friction remains unchanged. The air resistance has no effect on the coefficient of friction, which is dependent on the properties of the wheels and the rails.

Challenge
R(↗): $mg\sin\alpha - \mu mg\cos\alpha = ma$
$g\sin\alpha - \mu g\cos\alpha = a$