M2 Chapter 7: Application of Forces

Chapter Practice

Key Points

1 A particle or rigid body is in static equilibrium if it is at rest and the resultant force acting on the particle is zero.

2 The maximum value of the frictional force $F_{MAX} = \mu R$ is reached when the body you are considering is on the point of moving. The body is then said to be in limiting equilibrium.

3 In general, the force of friction F is such that $F \leqslant \mu R$, and the direction of the frictional force is opposite to the direction in which the body would move if the frictional force were absent.

4 For a rigid body in static equilibrium:

- the body is stationary
- the resultant force in any direction is zero
- the resultant moment is zero.

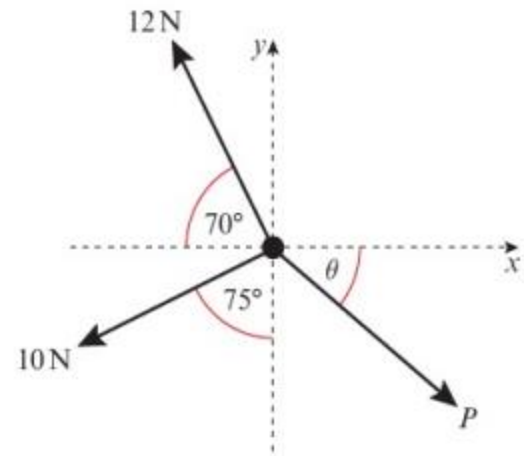
Chapter Exercises

1 A particle is acted upon by three forces as shown in the diagram.

Given that the particle is in equilibrium, work out:

a the size of angle θ

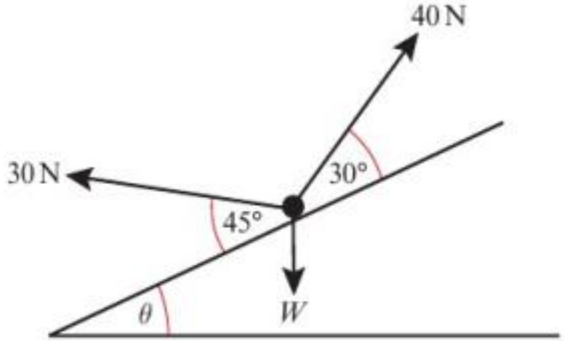
b the magnitude of P.



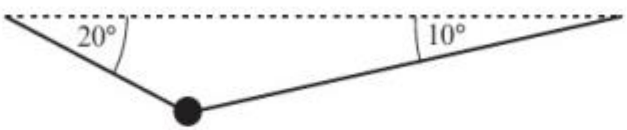
2 A particle is acted upon by three forces as shown in the diagram. Given that it is in equilibrium find:

a the size of angle θ

b the magnitude of W.

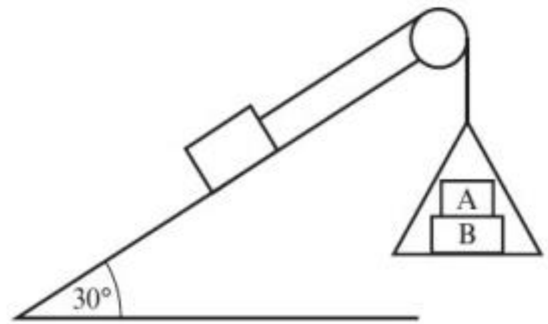


3 An acrobat of mass 55 kg stands on a tightrope. By modelling the acrobat as a particle and the tightrope as two inextensible strings, calculate the tension in the tightrope on each side of the rope.



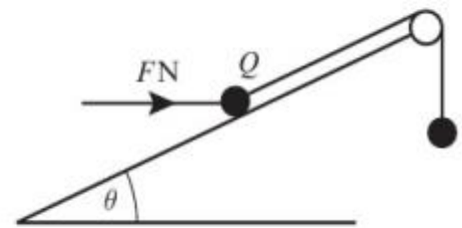
Chapter Exercises

4 A box of mass 5 kg sits on a smooth slope that is angled at 30° to the horizontal. It is attached to a light scale-pan by a light inextensible string which passes over a smooth pulley, as shown in the diagram. The scale-pan carries two masses A and B. The mass of A is 2 kg and the mass of B is 5 kg. Work out the force exerted by A on B. **(8 marks)**



5 A particle Q of mass 5 kg rests in equilibrium on a smooth inclined plane. The plane makes an angle θ with the horizontal, where $\tan\theta = \frac{3}{4}$.

Q is attached to one end of a light inextensible string which passes over a smooth pulley as shown. The other end of the string is attached to a particle of mass 2 kg.



The particle Q is also acted upon by a force of magnitude F N acting horizontally, as shown in the diagram.

Find the magnitude of:

a the force F **(5 marks)**

b the normal reaction between particle Q and the plane. **(3 marks)**

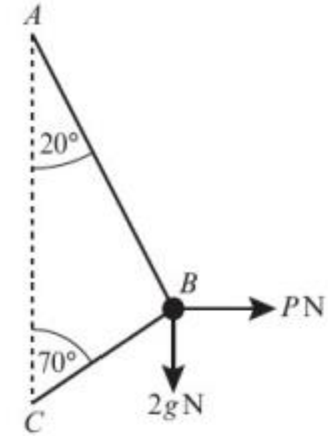
The plane is now assumed to be rough.

c State, with a reason, which of the following statements is true:

1. F will be larger 2. F will be smaller 3. F could be either larger or smaller. **(2 marks)**

Chapter Exercises

6 A smooth bead B of mass 2 kg is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C where A is vertically above C. The bead is held in equilibrium by a horizontal force of magnitude P N. The sections AB and BC make angles of 20° and 70° with the vertical as shown.



a Show that the tension in the string is 33 N (2 s.f.). **(3 marks)**

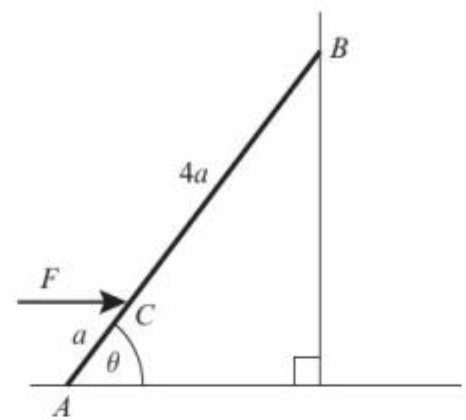
b Calculate the value of P. **(3 marks)**

7 A sledge of mass 50 kg sits on a snowy hill that is angled at 40° to the horizontal. The sledge is held in place by a rope that is angled at 30° above the line of greatest slope of the hill.

a By modelling the sledge as a particle, the hill as a smooth slope and the rope as a light inextensible string, work out the tension in the rope. **(4 marks)**

b Give one criticism of this model. **(1 mark)**

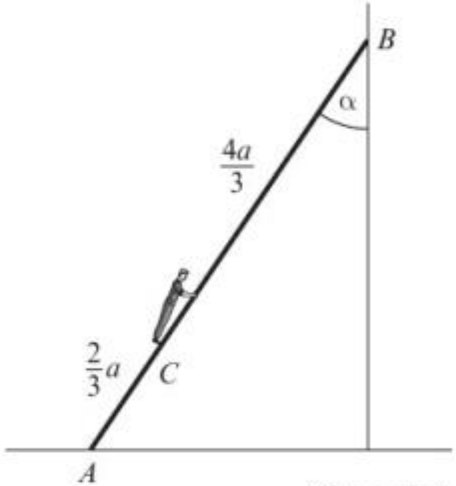
8 A uniform ladder AB has one end A on smooth horizontal ground. The other end B rests against a smooth vertical wall. The ladder is modelled as a uniform rod of mass m and length $5a$. The ladder is kept in equilibrium by a horizontal force F acting at a point C of the ladder where $AC = a$. The force F and the ladder lie in a vertical plane perpendicular to the wall. The ladder is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{9}{5}$, as shown in the diagram.



Show that $F = \dfrac{25mg}{72}$. **(8 marks)**

9 A uniform ladder AB, of mass m and length $2a$, has one end A on rough horizontal ground. The other end B rests against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The ladder makes an angle α with the vertical, where $\tan \alpha = \frac{3}{4}$. A child of mass $2m$ stands on the ladder at C where $AC = \frac{2}{3}a$, as shown in the diagram. The ladder and the child are in equilibrium.

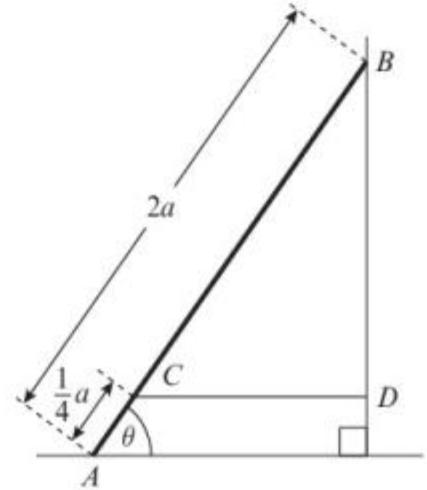
By modelling the ladder as a rod and the child as a particle, calculate the least possible value of the coefficient of friction between the ladder and the ground.



(8 marks)

10 A uniform ladder, of weight W and length $2a$, rests in equilibrium with one end A on a smooth horizontal floor and the other end B against a rough vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the ladder is μ. The ladder makes an angle θ with the floor, where $\tan \theta = \frac{4}{3}$. A horizontal light inextensible string CD is attached to the ladder at the point C, where $AC = \frac{1}{4}a$. The string is attached to the wall at the point D, with BD vertical, as shown in the diagram.

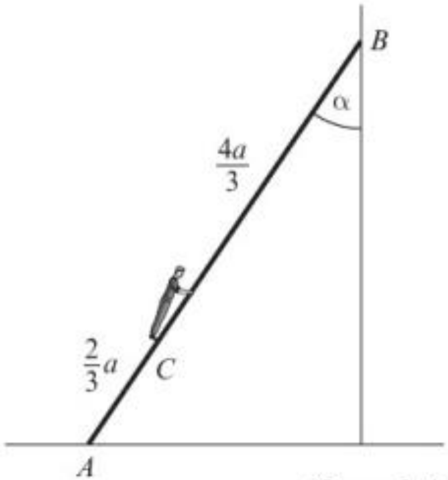
The tension in the string is $\frac{1}{3}W$. By modelling the ladder as a rod,



a find the magnitude of the force of the floor on the ladder **(5 marks)**

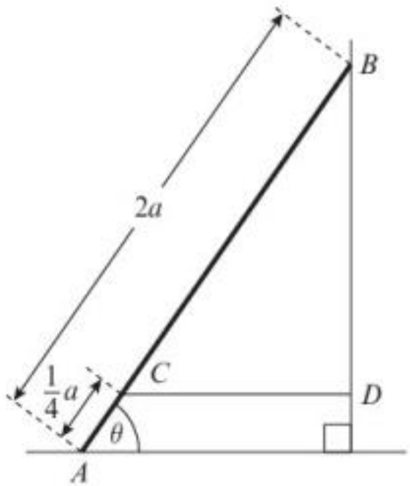
Chapter Exercises

9 A uniform ladder AB, of mass m and length $2a$, has one end A on rough horizontal ground. The other end B rests against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The ladder makes an angle α with the vertical, where $\tan\alpha = \frac{3}{4}$. A child of mass $2m$ stands on the ladder at C where $AC = \frac{2}{3}a$, as shown in the diagram. The ladder and the child are in equilibrium.



By modelling the ladder as a rod and the child as a particle, calculate the least possible value of the coefficient of friction between the ladder and the ground. **(8 marks)**

10 A uniform ladder, of weight W and length $2a$, rests in equilibrium with one end A on a smooth horizontal floor and the other end B against a rough vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the ladder is μ. The ladder makes an angle θ with the floor, where $\tan\theta = \frac{4}{3}$. A horizontal light inextensible string CD is attached to the ladder at the point C, where $AC = \frac{1}{4}a$. The string is attached to the wall at the point D, with BD vertical, as shown in the diagram.



The tension in the string is $\frac{1}{3}W$. By modelling the ladder as a rod,

a find the magnitude of the force of the floor on the ladder **(5 marks)**

b show that $\mu \geqslant \frac{1}{3}$. **(3 marks)**

c State how you have used the modelling assumption that the ladder is a rod. **(1 mark)**

11 A uniform ladder, of weight W and length 5 m, has one end on rough horizontal ground and the other touching a smooth vertical wall. The coefficient of friction between the ladder and the ground is 0.3.

The top of the ladder touches the wall at a point 4 m vertically above the level of the ground.

a Show that the ladder can not rest in equilibrium in this position. **(6 marks)**

In order to enable the ladder to rest in equilibrium in the position described above, a brick is attached to the bottom of the ladder.

Assuming that this brick is at the lowest point of the ladder, but not touching the ground,

b show that the horizontal frictional force exerted by the ladder on the ground is independent of the mass of the brick **(4 marks)**

c find, in terms of W and g, the smallest mass of the brick for which the ladder will rest in equilibrium. **(3 marks)**

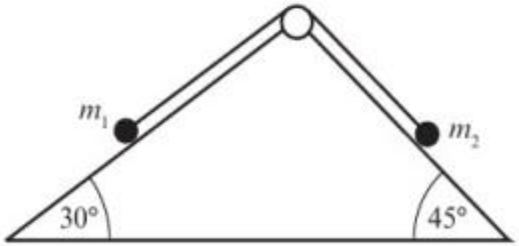
12 A non-uniform ladder PQ of mass 20 kg and length 4 metres, rests with P on smooth horizontal ground and Q against a rough vertical wall. The coefficient of friction between the ladder and the wall is 0.2. The centre of mass of the ladder is 1 m from P. The ladder is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{2}$. A horizontal force F applied to the base of the ladder can just prevent it from slipping. By modelling the ladder as a rod determine the value of F. **(10 marks)**

13 A particle of mass 3 kg is released from rest on a rough slope that is angled at α to the horizontal where $\tan\alpha = \frac{3}{4}$. After 1.5 seconds the particle has travelled 6 m. Work out the coefficient of friction μ. **(6 marks)**

14 A particle of mass 5 kg is pushed up a rough slope, inclined at 30° to the horizontal, by a force of 80 N applied at an angle of 10° slope. Given that the coefficient of friction of the slope is 0.4, find the acceleration of the particle. **(6 marks)**

15 Two particles, A of mass m_1 kg and B of mass m_2 kg are connected by a light inextensible string. The string passes over a smooth pulley, P. A sits on a rough horizontal table, where the coefficient of friction between A and the table is μ, and B lies directly below P. Given that $m_2 > \mu m_1$, show that the acceleration of the system is $\dfrac{g(m_2 - \mu m_1)}{m_1 + m_2}$. **(5 marks)**

16 Two particles of masses m_1 and m_2 are connected by a light inextensible string that passes over a smooth pulley. The particles are released from rest on smooth slopes angled at 30° and 45° to the horizontal as shown in the diagram. Given that m_2 is accelerating down the 45° slope at $\frac{1}{2}$ m s^{-2}, show that



$$\frac{m_1}{m_2} = \frac{g\sqrt{2} - 1}{1 + g}.$$

(6 marks)

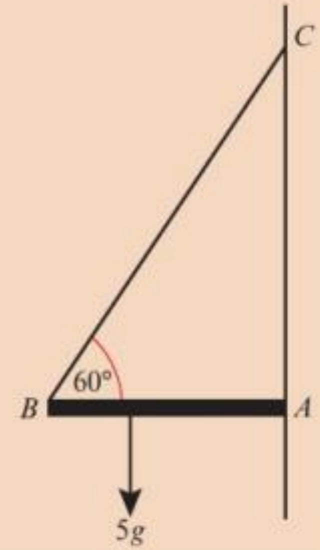
Chapter Exercises

Challenge

The diagram shows a uniform rod AB of length 3 m and of mass 10 kg. The rod is smoothly hinged at A which lies on a vertical wall. A particle of mass 5 kg is suspended 1 m from B. The rod is kept in a horizontal position by a light inextensible string BC, where C lies on the wall directly above A. The plane ABC is perpendicular to the wall and $\angle ABC$ is 60°.

a Calculate the tension in the string.

b Work out the magnitude and direction of the reaction at the hinge.



Watch out The reaction at the hinge does not have to be normal (perpendicular) to the wall.

Chapter Answers

1 **a** 32.3° (3 s.f.) **b** 16.3 N (3 s.f.)

2 **a** 18.0° (3 s.f.) **b** 43.3 N (3 s.f.)

3 $T_1 = 1062$ N, $T_2 = 1013$ N

4 12 N (2 s.f.)

5 **a** 12.25 N **b** 46.6 N (3 s.f.)

c F will be smaller

6 **a** $R(\uparrow)$: $T\cos 20 = 2g + T\cos 70$

$$T = \frac{2g}{\cos 20 - \cos 70}$$

$$= 33\text{ N (2 s.f.)}$$

b 42 N (2 s.f.)

7 **a** 364 N (3 s.f.) **b** Hill unlikely to be smooth.

8 $R(\rightarrow)$: $F = N$

Taking moments about A

$$Fa\sin\theta + \tfrac{5}{2}mga\cos\theta = 5aN\sin\theta$$

$$\tfrac{5}{2}mg\cos\theta = 4F\sin\theta$$

$$\tfrac{5}{8}mg = F\tan\theta$$

9 $\frac{7}{24}$

10 **a** $\frac{8W}{9}$

b $R(\uparrow)$: $R + \mu N \geqslant W$, $R(\rightarrow)$: $N = \frac{W}{3}$

$$\frac{\mu W}{3} \geqslant W - \frac{8W}{9}$$

c The ladder has negligible thickness/the ladder does not bend.

11 **a** Taking moments about point where ladder touches the ground
R(↑): $R = W$, R(→): $N = 0.3R$
$1.5W = 1.2W$. This cannot be true so the ladder cannot rest in this position.

b R(→): $F = N$
Taking moments about point where ladder touches the ground $1.5W = 4N$, $F = N = \frac{3W}{8}$

c $\frac{W}{4g}$

12 18 N (2 s.f.)

13 0.070 (2 s.f.)

14 6.35 ms^{-2} (3 s.f.)

15 R(→): $T - \mu m_1 g = m_1 a$, R(↑): $T = m_2 g - m_2 a$
$m_1 a + \mu m_1 g = m_2 g - m_2 a$
$g(m_2 - \mu m_1) = a(m_1 + m_2)$

16 R(↗): $T - m_1 g \sin 30 = \frac{1}{2} m_1$,
R(↘): $m_2 g \cos 45 - T = \frac{1}{2} m_2$
and $T = \frac{\sqrt{2}}{2} m_2 g - \frac{1}{2} m_2$, $T = \frac{1}{2} m_1 + \frac{1}{2} m_1 g$
$\sqrt{2}\, m_2 g - m_2 = m_1 + m_1 g$
$m_2(\sqrt{2}\, g - 1) = m_1(1 + g)$

Challenge

a 94.3 N (3 s.f.) **b** 80.6 N (3 s.f.), 54.2° (3 s.f.) to the horizontal