# P1 Chapter 2: Quadratics

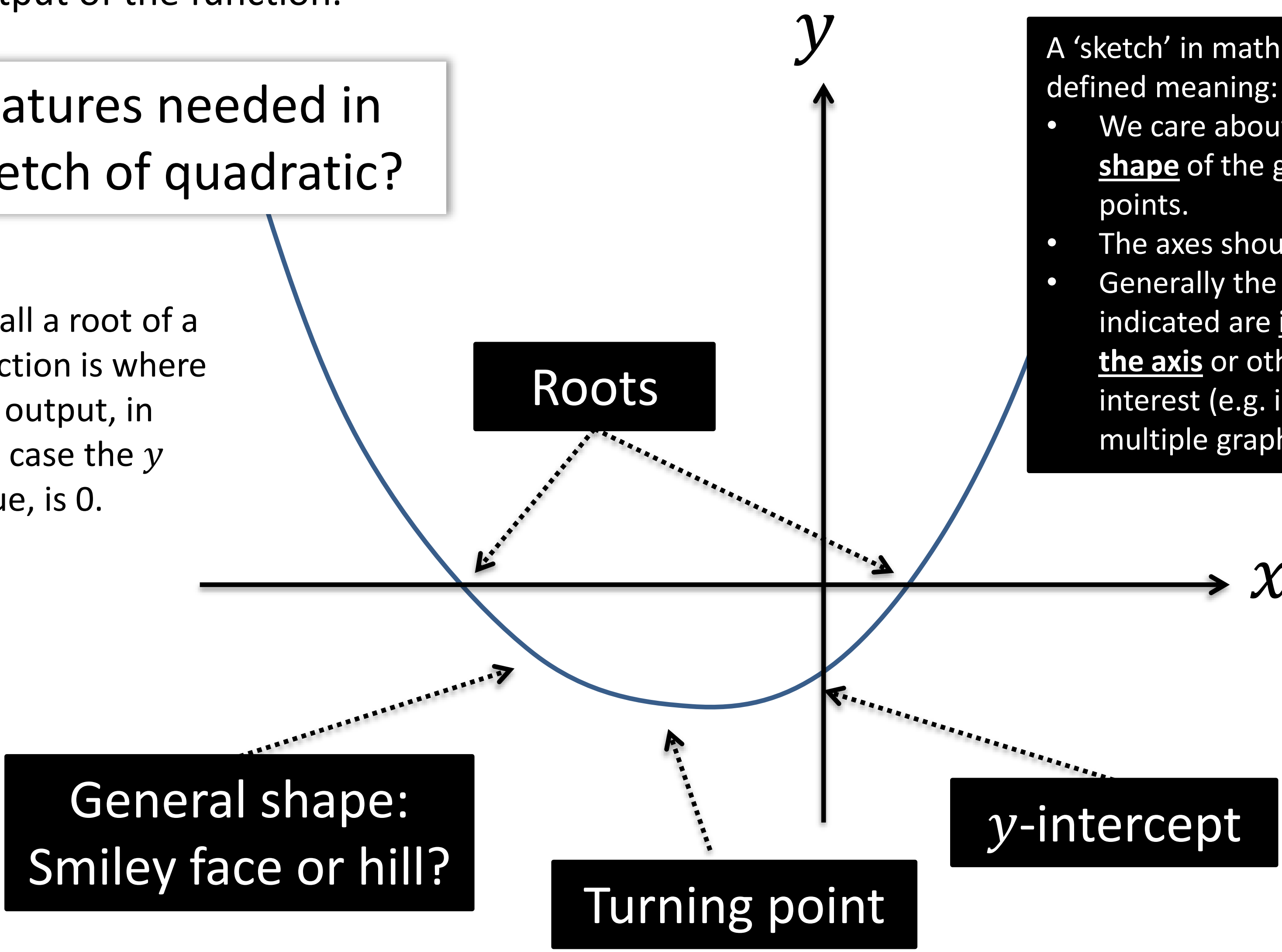
## Quadratic Graphs

# Quadratic Graphs

Recall that $x$ refers to the input of a function, and the expression $f(x)$ refers to the output. For graph sketches, we often write $y = f(x)$, i.e. we set the $y$ values to be the output of the function.

Features needed in sketch of quadratic?

Recall a root of a function is where the output, in this case the $y$ value, is 0.




A 'sketch' in maths has a clearly defined meaning:

- We care about the **general shape** of the graph, not exact points.
- The axes should have **no scale**.
- Generally the only coordinates indicated are **intercepts with the axis** or other points of interest (e.g. intersections of multiple graphs)

# Example

Sketch the graph of $y = x^2 + 3x - 4$ and find the coordinates of the turning point.

Roots:

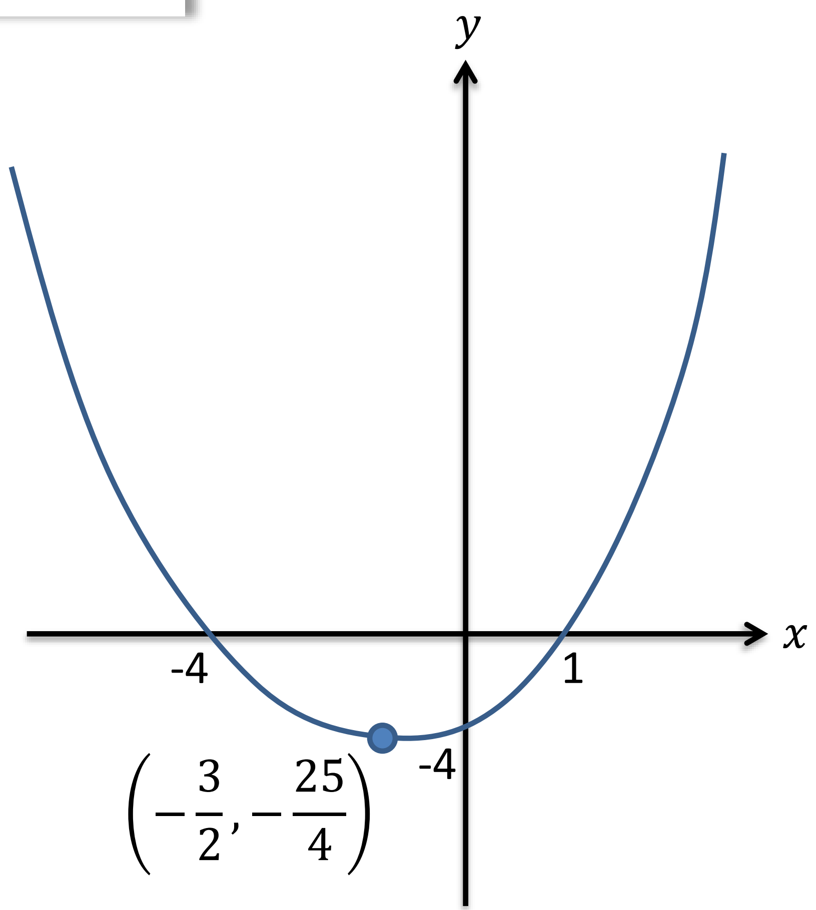
?

$y$-intercept:

?

Turning point:

?




Recall that if $f(x) = (x+a)^2 + b$, the minimum output is $b$ and $-a$ is the $x$ value which minimises it. i.e. Turning point is $(-a, b)$

# Example

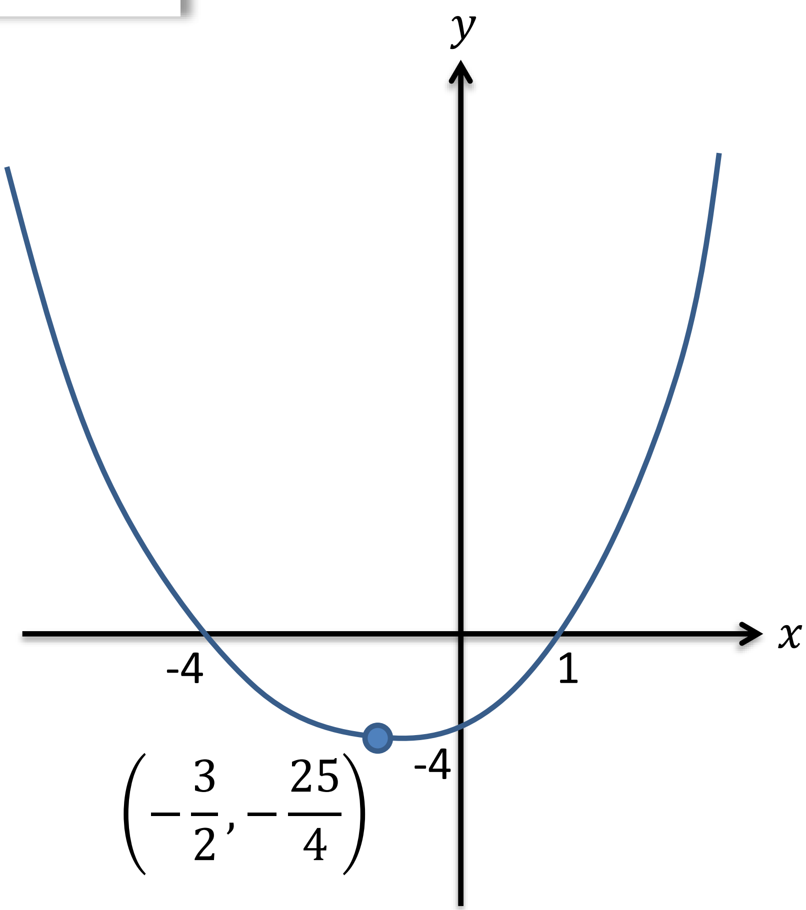
Sketch the graph of $y = x^2 + 3x - 4$ and find the coordinates of the turning point.

Roots: $\boldsymbol{x^2 + 3x - 4 = 0}$
$\boldsymbol{(x+4)(x-1) = 0}$
$\boldsymbol{x = -4 \text{ or } x = 1}$

$y$-intercept: **When $\boldsymbol{x = 0}$,**
$\boldsymbol{y = 0^2 + 3(0) - 4 = -4}$

Turning point: $\boldsymbol{y = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 4}$
$\boldsymbol{y = \left(x + \frac{3}{2}\right)^2 - \frac{25}{4}}$

**Min point:** $\boldsymbol{\left(-\frac{3}{2}, -\frac{25}{4}\right)}$




Recall that if $f(x) = (x+a)^2 + b$, the minimum output is $b$ and $-a$ is the $x$ value which minimises it. i.e. Turning point is $(-a, b)$

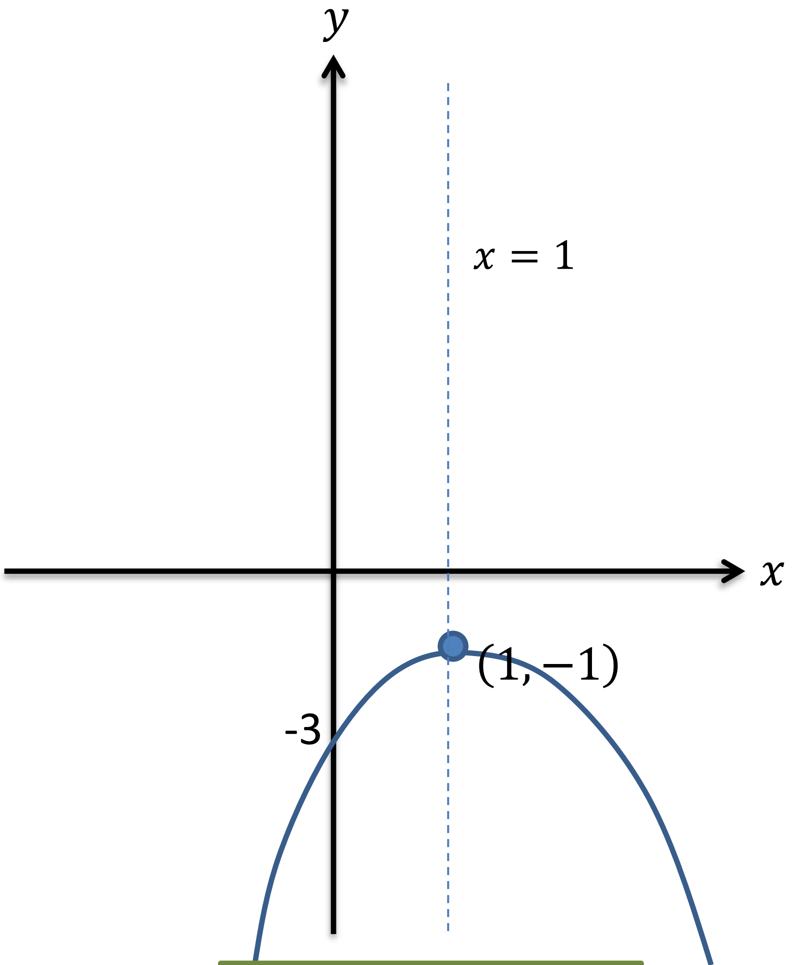
# Example

Sketch the graph of $y = 4x - 2x^2 - 3$ and find the coordinates of the turning point. Write down the equation of the line of symmetry.

Roots: ?

$y$-intercept: ?

Turning point: ?




Line of reflection: ?

# Example

Sketch the graph of $y = 4x - 2x^2 - 3$ and find the coordinates of the turning point. Write down the equation of the line of symmetry.

Roots: $-2x^2 + 4x - 3 = 0$

$a = -2, b = 4, c = -3$

$$x = \frac{-4 \pm \sqrt{4^2 - (4 \times -2 \times -3)}}{-4}$$

$$= \frac{-4 \pm \sqrt{-8}}{-4}$$

**This has no solutions, so the $y$ value can never be 0, i.e. the parabola does not touch the $x$-axis.**

$y$-intercept: **-3 (by inspection)**

Turning point: $y = -2x^2 + 4x - 3$

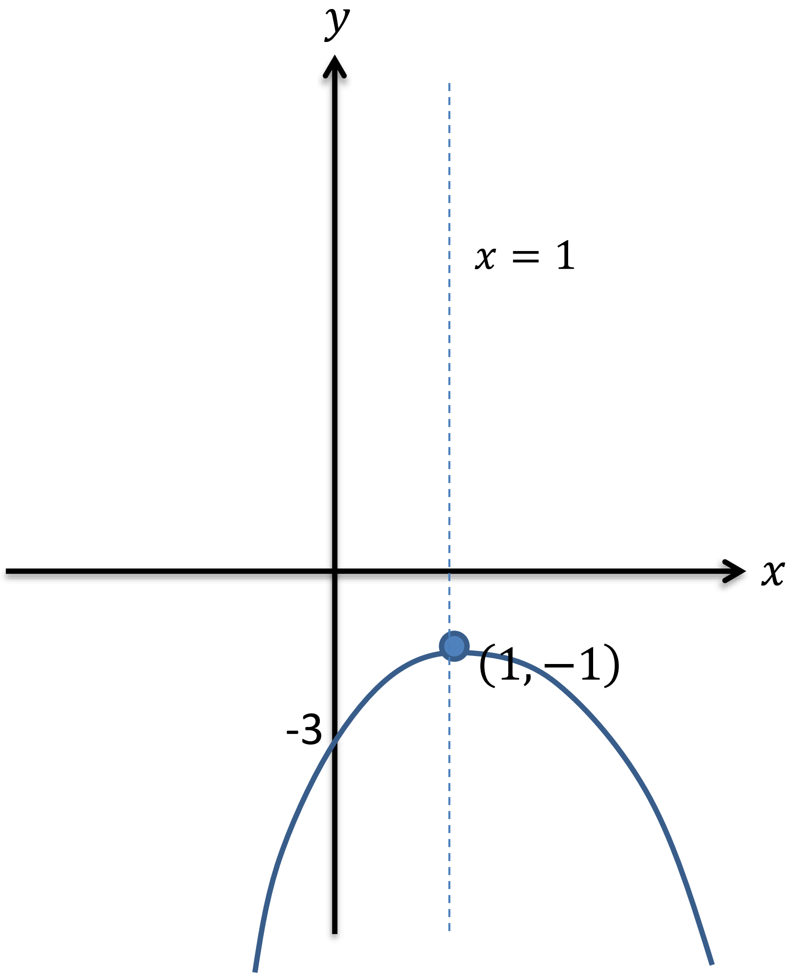
$= -2(x^2 - 2x) - 3$

$= -2\left((x-1)^2 - 1\right) - 3$

$= -2(x-1)^2 + 2 - 3$

$= -2(x-1)^2 - 1$

**Max point is $(1, -1)$**




Line of reflection: $x = 1$

# Test Your Understanding

Sketch the following, indicating any intercepts with the axis, the turning point and the equation of the line of symmetry.

**a** $y = x^2 + 4$

?

**b** $y = x^2 - 7x + 10$

?

**c** $y = 5x + 3 - 2x^2$

?

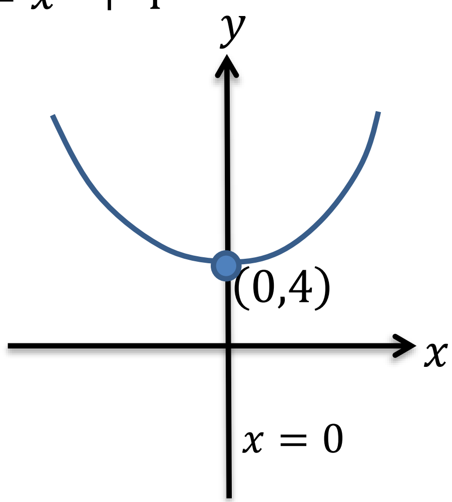
**d** $y = x^2 + 4x + 11$

?

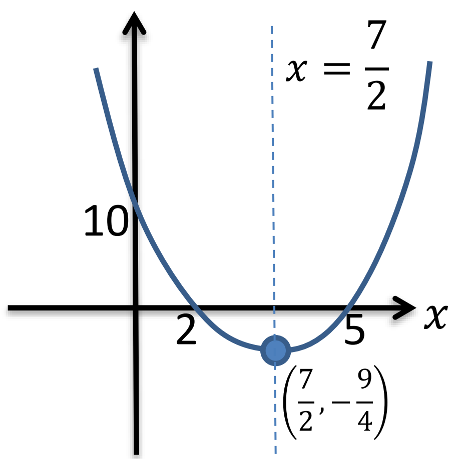
# Test Your Understanding

Sketch the following, indicating any intercepts with the axis, the turning point and the equation of the line of symmetry.

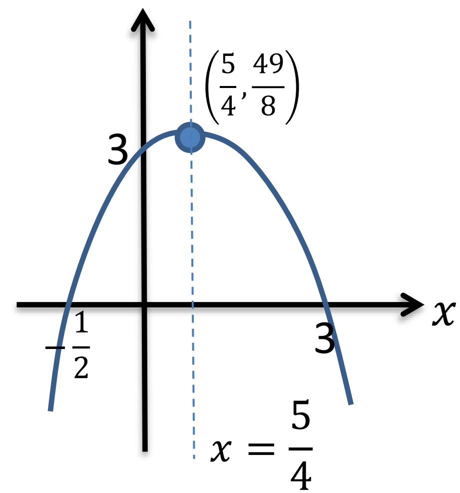
a $y = x^2 + 4$



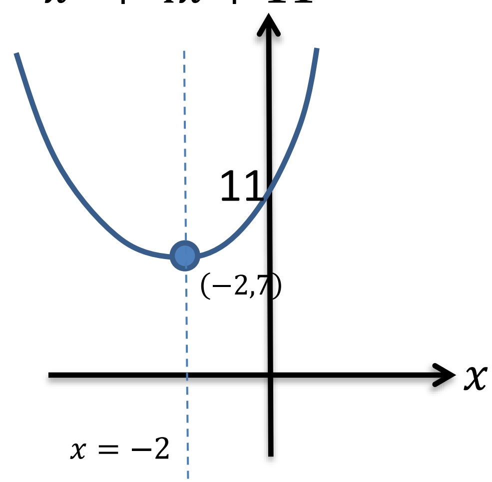

b $y = x^2 - 7x + 10$



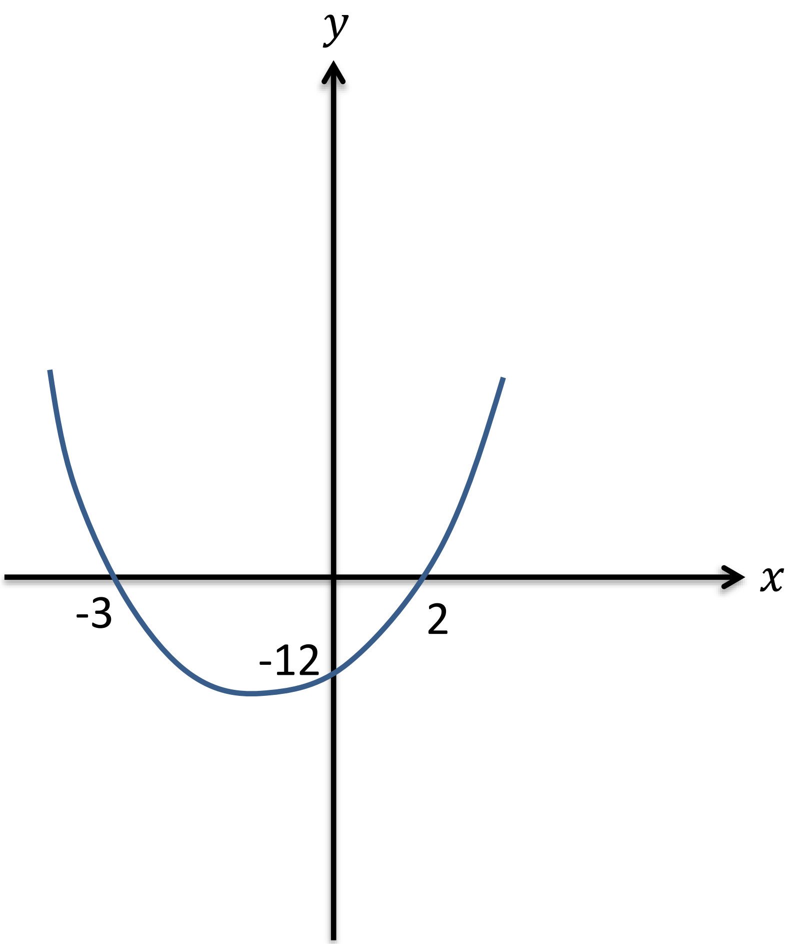

c $y = 5x + 3 - 2x^2$




d $y = x^2 + 4x + 11$



# Determining the Equation using a Graph




Determine the equation of this quadratic graph, in the form $y = ax^2 + bx + c$.

Since the roots are $-3$ and 2, a good start would be:

$$y = (x+3)(x-2)$$

noting that this gives us the correct 'positive quadratic' shape.

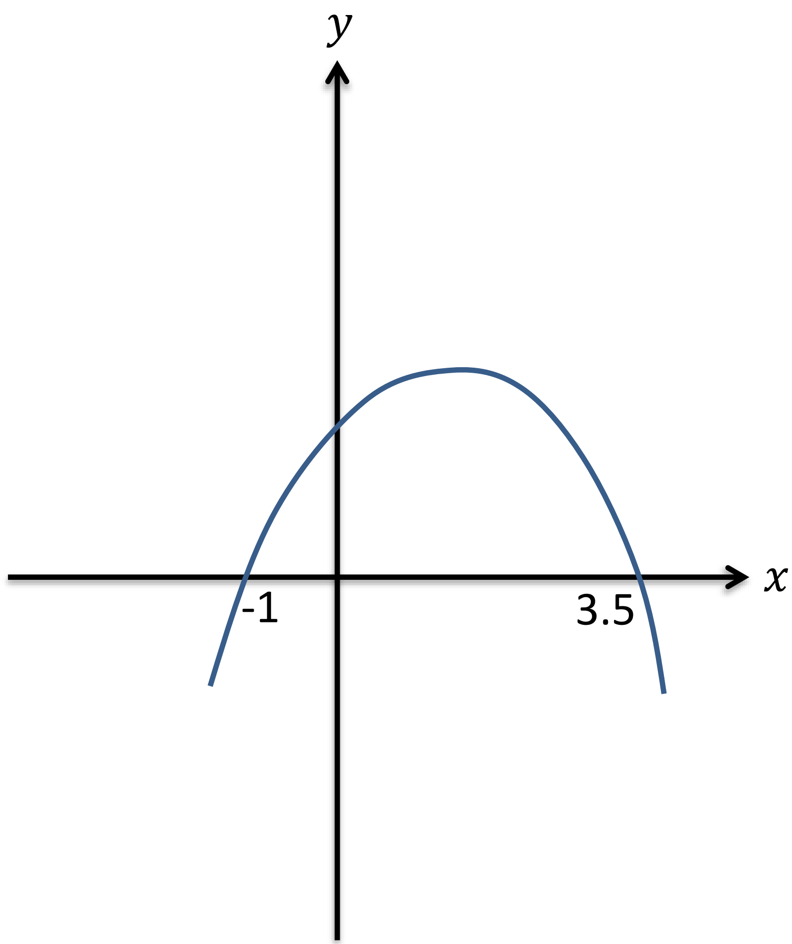
However, expanding, we get the wrong $y$-intercept of -6:

$$y = x^2 + x - 6$$

We can simply scale the $y$-value appropriately without affecting the roots:

$$y = 2(x+3)(x-2)$$
$$= 2x^2 + 2x - 12$$

# Further Example




Determine the equation of this quadratic graph, in the form $y = ax^2 + bx + c$, where $a, b, c$ are integers.

A good start would be:

$$y = -(x+1)(x-3.5)$$

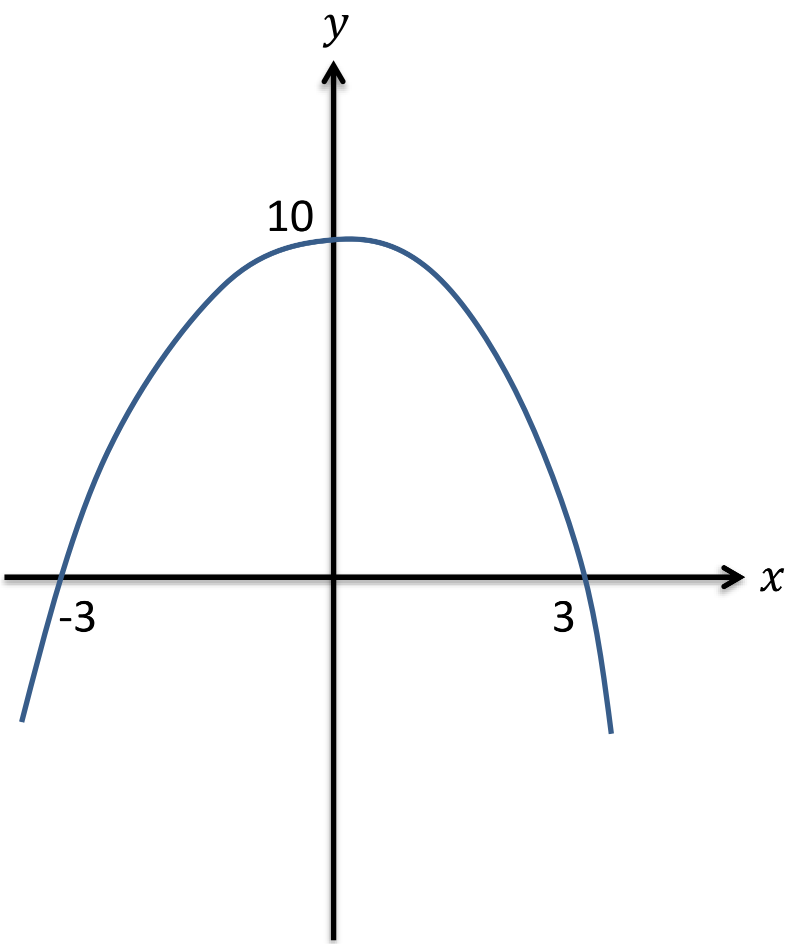
The minus on the front ensures we get a negative $x^2$ term for the correct shape.
Expanding:

$$y = -x^2 + 2.5x + 3.5$$

Note that no $y$-intercept has been specified, so we can scale the whole expression without affecting the roots (i.e. we only scale in the $y$ direction). Doubling:

$$\boldsymbol{y = -2x^2 + 5x + 7}$$

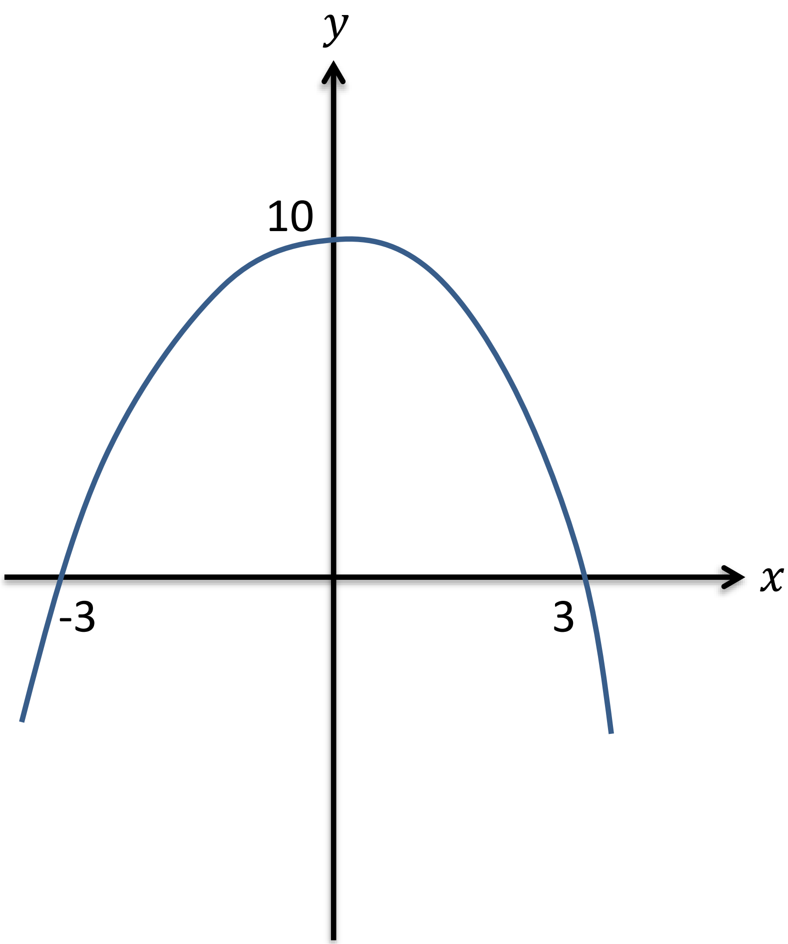
# Test Your Understanding




Determine an equation of this quadratic graph.

?

# Test Your Understanding




Determine an equation of this quadratic graph.

A good start to get the roots correct is:

$$y = -(x+3)(x-3)$$

The minus on the front ensures we get a negative $x^2$ term for the correct shape.

Expanding this gives us the incorrect $y$-intercept of 9:

$$y = -x^2 + 9$$

To scale 9 to 10, without affecting the root, we need to scale by $\frac{10}{9}$:

$$y = -\frac{10}{9}(x+3)(x-3)$$

Or more cleanly:

$$\boldsymbol{y = \frac{10}{9}(x+3)(3-x)}$$

# Exercise 2.4

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**Extension Question:**

[MAT 2003 1H] Into how many regions is the plane divided when the following three parabolas are drawn?

$$y = x^2$$
$$y = x^2 - 2x$$
$$y = x^2 + 2x + 2$$

?

# Exercise 2.4

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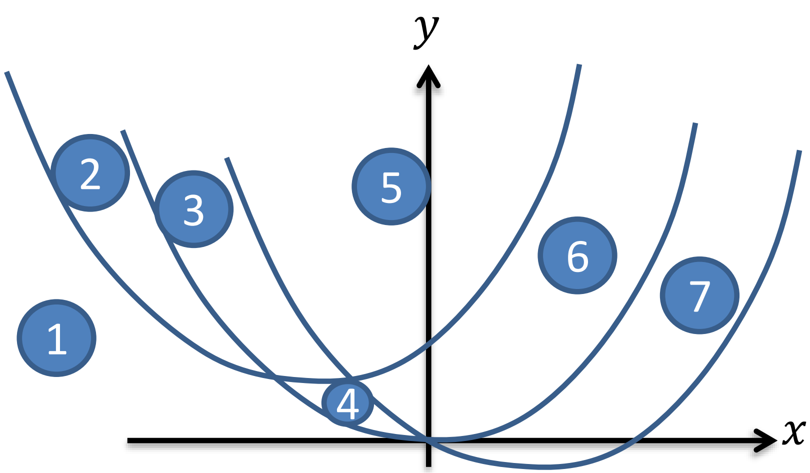
**Extension Question:**

[MAT 2003 1H] Into how many regions is the plane divided when the following three parabolas are drawn?

$$y = x^2$$
$$y = x^2 - 2x$$
$$y = x^2 + 2x + 2$$

**Completing the square:**

$$\boldsymbol{y = x^2}$$
$$\boldsymbol{y = (x-1)^2 - 1}$$
$$\boldsymbol{y = (x+1)^2 + 1}$$

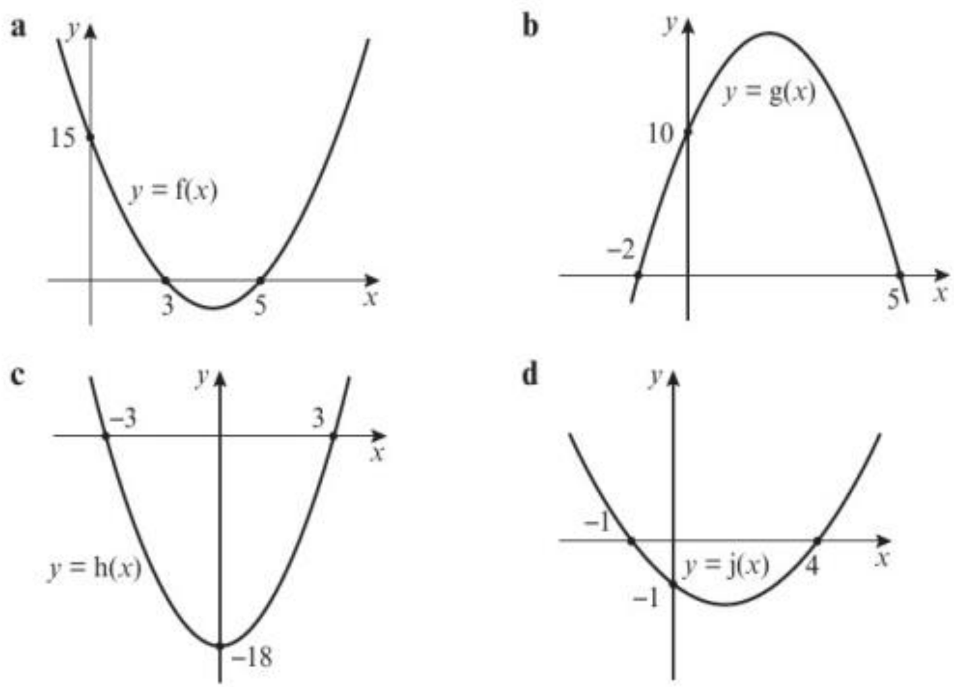


# Homework Exercise

1 Sketch the graphs of the following equations. For each graph, show the coordinates of the point(s) where the graph crosses the coordinate axes, and write down the coordinate of the turning point and the equation of the line of symmetry.

**a** $y = x^2 - 6x + 8$ **b** $y = x^2 + 2x - 15$ **c** $y = 25 - x^2$ **d** $y = x^2 + 3x + 2$
**e** $y = -x^2 + 6x + 7$ **f** $y = 2x^2 + 4x + 10$ **g** $y = 2x^2 + 7x - 15$ **h** $y = 6x^2 - 19x + 10$
**i** $y = 4 - 7x - 2x^2$ **j** $y = 0.5x^2 + 0.2x + 0.02$

2 These sketches are graphs of quadratic functions of the form $ax^2 + bx + c$.
Find the values of $a$, $b$ and $c$ for each function.



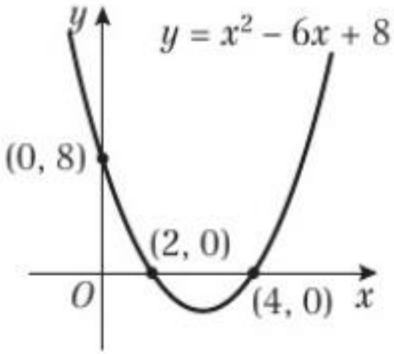

**Problem-solving**

Check your answers by substituting values into the function. In part **c** the graph passes through (0, −18), so h(0) should be −18.

3 The graph of $y = ax^2 + bx + c$ has a minimum at (5, −3) and passes through (4, 0).
Find the values of $a$, $b$ and $c$. **(3 marks)**

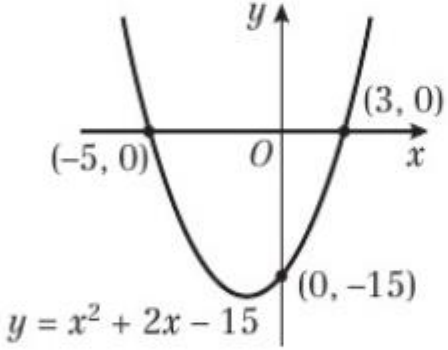
# Homework Answers

**1** **a**



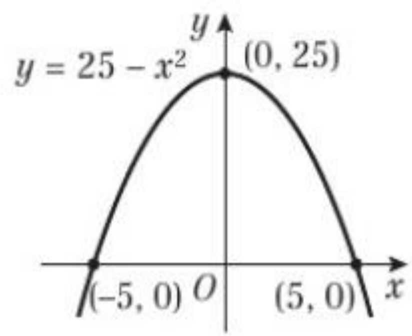

Turning point: (3, –1)
Line of symmetry: $x = 3$

**b**



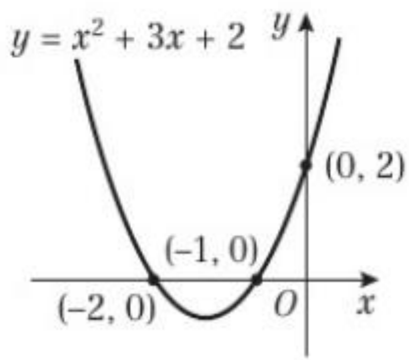

Turning point: (–1, –16)
Line of symmetry: $x = -1$

**c**




Turning point: (0, 25)
Line of symmetry: $x = 0$

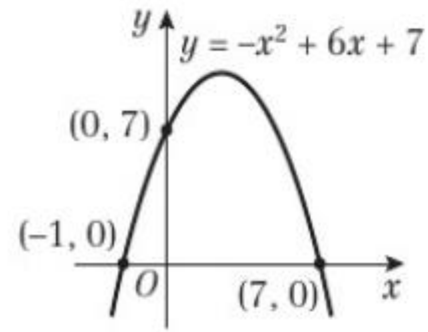
**d**




Turning point: $\left(-\frac{3}{2}, -\frac{1}{4}\right)$

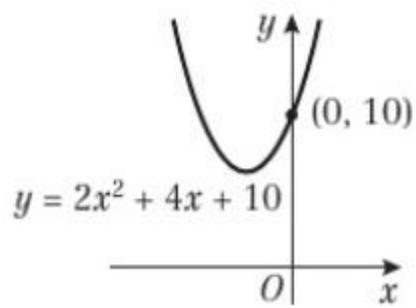
Line of symmetry: $x = -\frac{3}{2}$

**e**




Turning point: (3, 16)
Line of symmetry: $x = 3$

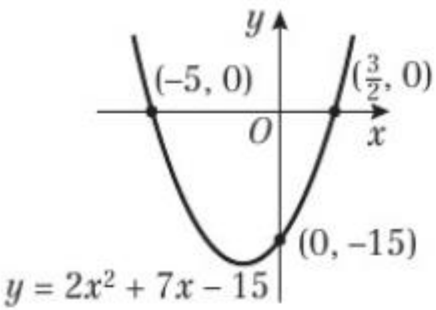
**f**




Turning point: (–1, 8)
Line of symmetry: $x = -1$

# Homework Answers

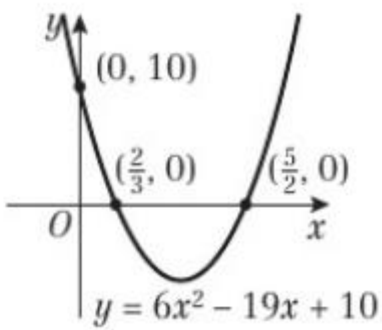
**1 g**




Turning point: $\left(-\frac{7}{4}, -\frac{169}{8}\right)$

Line of symmetry: $x = -\frac{7}{4}$

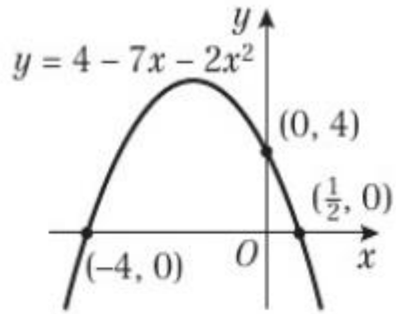
**h**




Turning point: $\left(\frac{19}{12}, -\frac{121}{24}\right)$

Line of symmetry: $x = \frac{19}{12}$

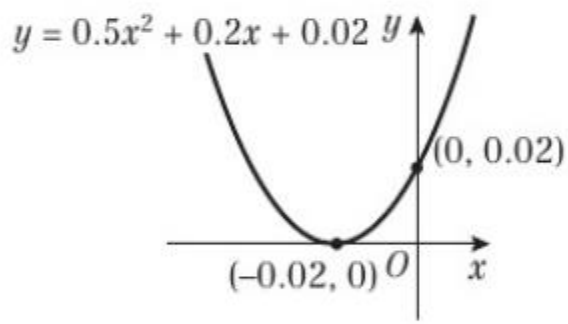
**i**




Turning point: $\left(-\frac{7}{4}, \frac{81}{8}\right)$

Line of symmetry: $x = -\frac{7}{4}$

**j**




Turning point: (–0.2, 0)
Line of symmetry: $x = -0.2$

**2**
- **a** $a = 1, b = -8, c = 15$
- **b** $a = -1, b = 3, c = 10$
- **c** $a = 2, b = 0, c = -18$
- **d** $a = \frac{1}{4}, b = -\frac{3}{4}, c = -1$

**3** $a = 3, b = -30, c = 72$