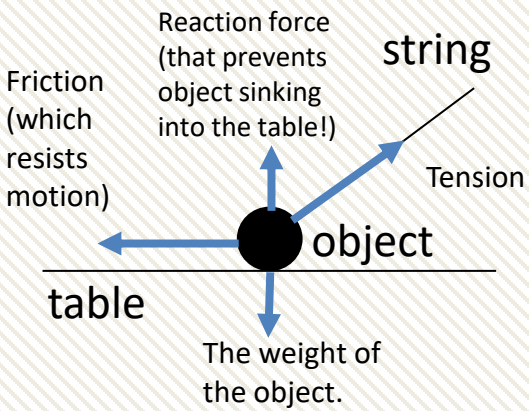
M1 Chapter 8: Modelling

Modelling in Mechanics

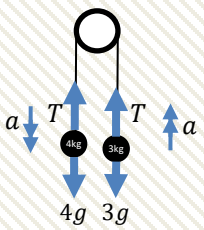
Mechanics, broadly speaking, concerns motion, forces, and how the two interrelate. This chapter just gives you an overview of what you'll be covering in Year 1 and how it all links together.

Forces

You will later encounter force diagrams. This considers the forces acting at a particular point. Some forces you might consider…



- Forces can be considered as vectors.
- The **magnitude** of the force vector gives the 'size' of the force.
- We often **consider forces in a particular direction**. e.g. If the object above is stationary, the forces left must equal the force right, and forces up equal forces down (Newton's 1st Law).
- Often we need to consider the forces at multiple different points if objects are connected, e.g. with pulleys:



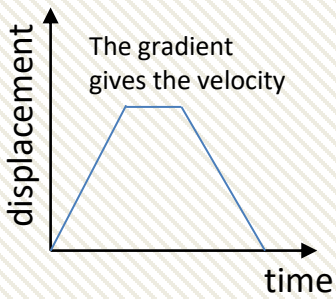
The bridge!

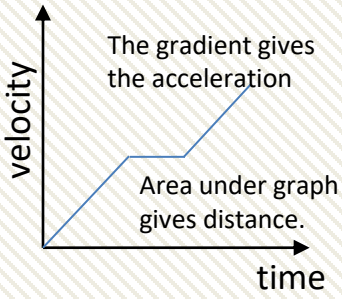
$F = ma$

Newton's 2nd Law allows us to connect the force world (F) with the motion world (acceleration a) if the object is moving.

Motion

At GCSE you may have encountered displacement-time and velocity-time graphs:





Given **constant acceleration** we have 5 quantities of motion ("*suvat*"):

$\boldsymbol{s}$ = displacement
$\boldsymbol{u}$ = initial velocity
$\boldsymbol{v}$ = final velocity
$\boldsymbol{a}$ = acceleration
$\boldsymbol{t}$ = time

which we will see are linked by various equations:

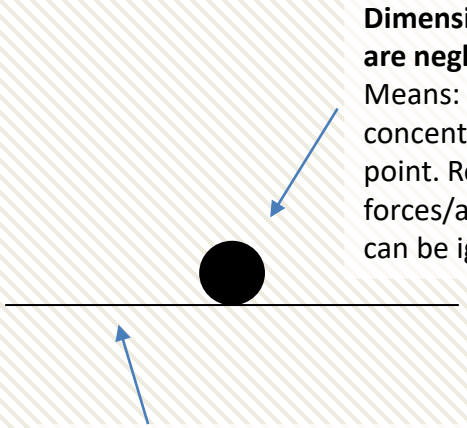
$$s = ut + \frac{1}{2}at^2$$
$$s = \left(\frac{u+v}{2}\right)t$$
$$v^2 = u^2 + 2as$$
$$v = u + at$$

If the **acceleration is not constant**, we can specify displacement/velocity/acceleration as a function of time and differentiate/integrate to change between them.

$s = 2t^3 + 3t \quad \rightarrow \quad v = \frac{ds}{dt} = 6t^2 + 3$

Modelling Assumptions

As with many areas of applied maths, we often have to make various modelling assumptions, to make the maths cleaner or to use well-known mathematical approaches.
Here are common modelling assumptions often made in Mechanics:

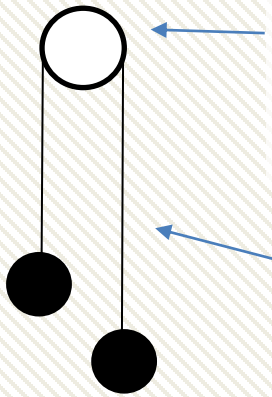


Particle

Dimensions of object are negligible
Means: Mass of object concentrated at single point. Rotational forces/air resistance can be ignored.

Rough/Smooth surface

Means: Objects in contact with surface does/does not experience friction.



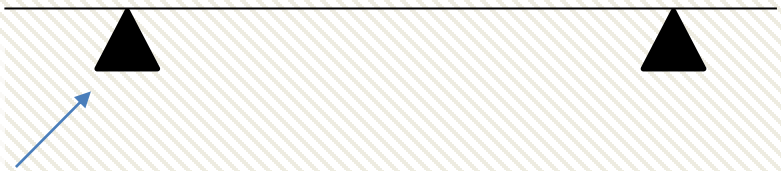
Smooth/light pulley

No friction.
Means: Tension the same in string either side of pulley.
Pulley has no mass.

Inextensible string

String does not stretch under load.
Means: Acceleration the same in any connected objects.

Fro Tip:
Particularly make note of underlined text!



Rod

One dimension is negligible, like a pole or beam.
Means: Mass is concentrated along line. Rigid.

Peg/Support

A support from which a body can be suspended or rested.
Means: Dimensionless and fixed. Can be rough or smooth depending on question.

Exercise 8A, 8B

Pearson Stats/Mechanics Year 1

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1) Exercise 8.1 pages 52-53
2) Exercise 8.2 pages 53-54

Homework Exercise

1 The motion of a golf ball after it is struck by a golfer can be modelled using the equation $h = 0.36x - 0.003x^2$, where h m is the height of the golf ball above the ground and x m is the horizontal distance travelled.

a Find the height of the golf ball when it is:

i struck **ii** at a horizontal distance of 100 m.

b Use the model to predict the height of the golf ball when it is 200 m from the golfer.

c Comment on the validity of this prediction.

2 A stone is thrown into the sea from the top of a cliff. The height of the stone above sea level, h m at time t s after it is thrown can be modelled by the equation $h = -5t^2 + 15t + 90$.

a Write down the height of the cliff above sea level.

b Find the height of the stone:

i when $t = 3$ **ii** when $t = 5$.

c Use the model to predict the height of the stone after 20 seconds.

d Comment on the validity of this prediction.

3 The motion of a basketball as it leaves a player's hand and passes through the net is modelled using the equation $h = 2 + 1.1x - 0.1x^2$, where h m is the height of the basketball above the ground and x m is the horizontal distance travelled.

a Find the two values of x for which the basketball is exactly 4 m above the ground.

This model is valid for $0 \leqslant x \leqslant k$, where k m is the horizontal distance of the net from the player. Given that the height of the net is 3 m:

b Find the value of k.

c Explain why the model is not valid for $x > k$.

Homework Exercise

4 A car accelerates from rest to 60 mph in 10 seconds. A quadratic equation of the form $d = kt^2$ can be used to model the distance travelled, d metres in time t seconds.

Problem-solving

Use the information given to work out the value of k.

a Given that when $t = 1$ second the distance travelled by the car is 13.2 metres, use the model to find the distance travelled when the car reaches 60 mph.

b Write down the range of values of t for which the model is valid.

5 The model for the motion of a golf ball given in question 1 is only valid when h is positive. Find the range of values of x for which the model is valid.

6 The model for the height of the stone above sea level given in question 2 is only valid from the time the stone is thrown until the time it enters the sea. Find the range of values of t for which the model is valid.

7 A football is kicked by the goalkeeper from one end of the football pitch.

State the effect of the following assumptions on any calculations made using this model:

a The football is modelled as a particle. **b** Air resistance is negligible.

8 An ice puck is hit and slides across the ice.

State the effect of the following assumptions on any calculations made using this model:

a The ice puck is modelled as a particle. **b** The ice is smooth.

Homework Exercise

9 A parachute jumper wants to model her descent from an aeroplane to the ground. She models herself and her parachute as particles connected by a light inextensible string. Explain why this may not be a suitable modelling assumption for this situation.

10 A fishing rod manufacturer constructs a mathematical model to predict the behaviour of a particular fishing rod. The fishing rod is modelled as a light rod.

a Describe the effects of this modelling assumption.

b Comment on its validity in this situation.

11 Make a list of the assumptions you might make to create simple models of the following:

a The motion of a golf ball after it is hit

b The motion of a child on a sledge going down a snow-covered hill

c The motion of two objects of different masses connected by a string that passes over a pulley

d The motion of a suitcase on wheels being pulled along a path by its handle.

Homework Answers

1 **a** **i** $h = 0$ **ii** $h = 6\,\text{m}$

b $h = -48\,\text{m}$.

c Model is not valid when $x = 200$ as height would be 48 m below ground level.

2 **a** 90 m **b** **i** $h = 90\,\text{m}$ **ii** $h = 40\,\text{m}$

c $h = -1610\,\text{m}$

d Model is not valid when $t = 20$ as height would be 1610 m below sea level.

3 **a** $x = 2.30\,\text{m}$ or $8.70\,\text{m}$ **b** $k = 10\,\text{m}$

c When $k = 10$ seconds the ball passes through the net so model not valid for $k > 10$

4 **a** 1320 m

b Model is valid for $0 \leqslant t \leqslant 10$

5 $0 \leqslant x \leqslant 120$

6 $0 \leqslant t \leqslant 6$

7 **a** Ignore the rotational effect of any external forces that are acting on it, and the effects of air resistance.

b Ignore the frictional effects on the football due to air resistance.

8 **a** Ignore the rotational effect of any external forces that are acting on it, and the effects of air resistance.

b Ignore any friction between the ice puck and the ice surface.

9 Parachute jumper and parachute should be considered together as one particle as they move together.

10 **a** If modelled as a light rod, the fishing rod is considered to have no thickness and is rigid.

b If the fishing rod had no thickness and was rigid it would be unsuitable for fishing.

11 **a** Model golf ball as a particle, ignore the effects of air resistance.

b Model child on sledge as a particle, consider the hill as smooth.

c Model objects as particles, string as light and inextensible, pulley as smooth.

d Model suitcase and handle as a particle, path as smooth, ignore friction.

Homework Answers