# P2 Chapter 5: Radians

## Solving Trigonometric Equations

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- $\sin(x) = \sin(\pi - x)$
- $\cos(x) = \cos(2\pi - x)$
- $sin, cos$ repeat every $2\pi$ but $tan$ every $\pi$

Solving trigonometric equations is virtually the same as you did in Year 1, except:

(a) Your calculator needs to be in radians mode.

(b) We use $\pi -$ instead of $180° -$, and so on.

[Textbook] Solve the equation $\sin 3\theta = \frac{\sqrt{3}}{2}$ in the interval $0 \leq \theta \leq 2\pi$.

$$0 \leq 3\theta \leq 6\pi$$

Adjust interval.

$$3\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$$

$$\therefore \theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}$$

Use $\pi -$ to get second value in each 'pair'. Then go to next cycle by adding $2\pi$ to each value in pair.

Only $\div$ 3 once all values obtained in range.

**[Jan 07 Q6]** Find all the solutions, in the interval $0 \leq x < 2\pi$, of the equation

$$2\cos^2 x + 1 = 5\sin x,$$

giving each solution in terms of $\pi$. **(6)**

$$2(1 - \sin^2 x) + 1 = 5\sin x$$
$$2 - 2\sin^2 x + 1 = 5\sin x$$
$$2\sin^2 x + 5\sin x - 3 = 0$$
$$(2\sin x - 1)(\sin x + 3) = 0$$
$$\sin x = \frac{1}{2} \quad or \sin x = -3$$
$$x = \frac{\pi}{6}$$
$$or\ x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

# Exercise 5.4

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**Extension**

[MAT 2010 1C] In the range $0 \leq x \leq 2\pi$, the equation $\sin^2 x + 3\sin x\cos x + 2\cos^2 x = 0$ has how many solutions?

?

# Exercise 5.4

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**Extension**

[MAT 2010 1C] In the range $0 \leq x \leq 2\pi$, the equation $\sin^2 x + 3\sin x \cos x + 2\cos^2 x = 0$ has how many solutions?

$\boldsymbol{(\sin x + 2\cos x)(\sin x + \cos x) = 0}$
$\boldsymbol{\sin x = -2\cos x}$ **or** $\boldsymbol{\sin x = -\cos x}$
$\boldsymbol{\tan x = -2}$ **or** $\boldsymbol{\tan x = -1}$
**The tan graph always has 1 solution per each cycle of $\boldsymbol{\pi}$ radians, so 4 solutions.**

# Homework Exercise

**1** Solve the following equations for $\theta$, in the interval $0 \leqslant \theta \leqslant 2\pi$, giving your answers to 3 significant figures where they are not exact.

**a** $\cos\theta = 0.7$ **b** $\sin\theta = -0.2$ **c** $\tan\theta = 5$ **d** $\cos\theta = -1$

**2** Solve the following equations for $\theta$, in the interval $0 \leqslant \theta \leqslant 2\pi$, giving your answers to 3 significant figures where they are not exact.

**a** $4\sin\theta = 3$ **b** $7\tan\theta = 1$ **c** $8\tan\theta = 15$ **d** $\sqrt{5}\cos\theta = \sqrt{2}$

**3** Solve the following equations for $\theta$, in the interval $0 \leqslant \theta \leqslant 2\pi$, giving your answers to 3 significant figures where they are not exact.

**a** $5\cos\theta + 1 = 3$ **b** $\sqrt{5}\sin\theta + 2 = 1$ **c** $8\tan\theta - 5 = 5$ **d** $\sqrt{7}\cos\theta - 1 = \sqrt{2}$

**4** Solve the following equations for $\theta$, giving your answers to 3 significant figures where appropriate, in the intervals indicated:

**a** $\sqrt{3}\tan\theta - 1 = 0,\ -\pi \leqslant \theta \leqslant \pi$ **b** $5\sin\theta = 1,\ -\pi \leqslant \theta \leqslant 2\pi$

**c** $8\cos\theta = 5,\ -2\pi \leqslant \theta \leqslant 2\pi$ **d** $3\cos\theta - 1 = 0.02,\ -\pi \leqslant \theta \leqslant 3\pi$

**e** $0.4\tan\theta - 5 = -7,\ 0 \leqslant \theta \leqslant 4\pi$ **f** $\cos\theta - 1 = -0.82,\ \frac{\pi}{2} \leqslant \theta \leqslant \frac{7\pi}{3}$

**5** Solve the following equations for $\theta$, in the interval $0 \leqslant \theta \leqslant 2\pi$, giving your answers to 3 significant figures where they are not exact.

**a** $5\cos 2\theta = 4$ **b** $5\sin 3\theta + 3 = 1$

**c** $\sqrt{3}\tan 4\theta - 5 = -4$ **d** $\sqrt{10}\cos 2\theta + \sqrt{2} = 3\sqrt{2}$

# Homework Exercise

**6** Solve the following equations for $\theta$, giving your answers to 3 significant figures where appropriate, in the intervals indicated.

**a** $\sqrt{2}\sin 3\theta - 1 = 0, \quad -\pi \leqslant \theta \leqslant \pi$

**b** $2\cos 4\theta = -1, \quad -\pi \leqslant \theta \leqslant 2\pi$

**c** $8\tan 2\theta = 7, \quad -2\pi \leqslant \theta \leqslant 2\pi$

**d** $6\cos 2\theta - 1 = 0.2, \quad -\pi \leqslant \theta \leqslant 3\pi$

**7** Solve the following equations for $\theta$, in the interval $0 \leqslant \theta \leqslant 2\pi$, giving your answers to 3 significant figures where they are not exact.

**a** $4\cos^2\theta = 2$

**b** $3\tan^2\theta + \tan\theta = 0$

**c** $\cos^2\theta - 2\cos\theta = 3$

**d** $2\sin^2 2\theta - 5\cos 2\theta = -2$

**8** Solve the following equations for $\theta$, in the interval $0 \leqslant \theta \leqslant 2\pi$, giving your answers to 3 significant figures where they are not exact.

**a** $\cos\theta + 2\sin^2\theta + 1 = 0$

**b** $10\sin^2\theta = 3\cos^2\theta$

**c** $4\cos^2\theta + 8\sin^2\theta = 2\sin^2\theta - 2\cos^2\theta$

**d** $2\sin^2\theta - 7 + 12\cos\theta = 0$

**9** Solve, for $0 \leqslant x < 2\pi$,

**a** $\cos\left(x - \frac{\pi}{12}\right) = \frac{1}{\sqrt{2}}$

**b** $\sin 3x = -\frac{1}{2}$

**c** $\cos(2\theta + 0.2) = -0.2, -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$

**d** $\tan\left(2\theta + \frac{\pi}{4}\right) = 1, 0 \leqslant \theta \leqslant 2\pi$

**10** **a** Solve, for $-\pi \leqslant \theta < \pi$, $(1 + \tan\theta)(5\sin\theta - 2) = 0$. **(4 marks)**

**b** Solve, for $0 \leqslant x < 2\pi$, $4\tan x = 5\sin x$. **(6 marks)**

**11** Find all the solutions, in the interval $0 \leqslant x \leqslant 2\pi$, to the equation $8\cos^2 x + 6\sin x - 6 = 3$ giving each solution to one decimal place. **(6 marks)**

# Homework Exercise

**12** Find, for $0 \leqslant x \leqslant 2\pi$, all the solutions of $\cos^2 x - 1 = \frac{7}{2}\sin^2 x - 2$ giving each solution to one decimal place. **(6 marks)**

**13** Show that the equation $8\sin^2 x + 4\sin x - 20 = 4$ has no solutions. **(3 marks)**

**14** **a** Show that the equation $\tan^2 x - 2\tan x - 6 = 0$ can be written as $\tan x = p \pm \sqrt{q}$ where $p$ and $q$ are numbers to be found. **(3 marks)**

**b** Hence solve, for $0 \leqslant x \leqslant 3\pi$, the equation $\tan^2 x - 2\tan x - 6 = 0$ giving your answers to 1 decimal place where appropriate. **(5 marks)**

**15** In the triangle $ABC$, $AB = 5$ cm, $AC = 4$ cm, $\angle ABC = 0.5$ radians and $\angle ACB = x$ radians.

**a** Use the sine rule to find the value of $\sin x$, giving your answer to 3 decimal places. **(3 marks)**

Given that there are two possible values of $x$,

**b** find these values of $x$, giving your answers to 2 decimal places. **(3 marks)**

# Homework Answers

1 **a** $\frac{2}{3}$ **b** 1 **c** 1

2 **a** $\dfrac{\sin 3\theta}{\theta \sin 4\theta} \approx \dfrac{3\theta}{\theta \times 4\theta} = \dfrac{3\theta}{4\theta^2} = \dfrac{3}{4\theta}$

**b** $\dfrac{\cos\theta - 1}{\tan 2\theta} \approx \dfrac{1 - \dfrac{\theta^2}{2} - 1}{2\theta} = \dfrac{-\dfrac{\theta^2}{2}}{2\theta} = -\dfrac{\theta}{4}$

**c** $\dfrac{\tan 4\theta + \theta^2}{3\theta - \sin 2\theta} \approx \dfrac{4\theta + \theta^2}{3\theta - 2\theta} = \dfrac{4\theta + \theta^2}{\theta} = 4 + \theta$

3 **a** 0.970379 **b** 0.970232

**c** –0.015% **d** –1.77%

**e** The larger the value of $\theta$ the less accurate the approximation is.

4 $\dfrac{\theta - \sin\theta}{\sin\theta} \times 100 = 1 \Rightarrow (\theta - \sin\theta) \times 100 = \sin\theta$

$\Rightarrow 100\theta - 100\sin\theta = \sin\theta \Rightarrow 100\theta = 101\sin\theta.$

5 **a** $\dfrac{4\cos 3\theta - 2 + 5\sin\theta}{1 - \sin 2\theta} \approx \dfrac{4\left(1 - \dfrac{(3\theta)^2}{2}\right) - 2 + 5\theta}{1 - 2\theta}$

$= \dfrac{4\left(1 - \dfrac{9\theta^2}{2}\right) - 2 + 5\theta}{1 - 2\theta} = \dfrac{4 - 18\theta^2 - 2 + 5\theta}{1 - 2\theta}$

$= \dfrac{(1 - 2\theta)(9\theta + 2)}{1 - 2\theta} = 9\theta + 2$

**b** 2

**Challenge**

1 **a** $CD = AC\theta$

**b** $\sin\theta \approx \dfrac{CD}{AD} = \dfrac{r\theta}{r} = \theta$

$\tan\theta \approx \dfrac{CD}{AC} = \dfrac{r\theta}{r} = \theta$

2 **a** $1 - \dfrac{x^2}{2}$

**b** $\cos\theta = \sqrt{1 - \sin^2\theta} = 1 - \dfrac{\sin^2\theta}{2}$, if $\sin\theta \approx \theta$ then this becomes $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$