P1 Chapter 4: Transforming Graphs

Transforming Functions

Effect of transformation on specific points

Sometimes you will not be given the original function, but will be given a sketch with specific points and features you need to transform.
Where would each of these points end up?

$y = f(x)$	$(4, 3)$	$(1, 0)$	$(6, -4)$
$y = f(x + 1)$	?	?	?
$y = f(2x)$	?	?	?
$y = 3f(x)$	?	?	?
$y = f(x) - 1$	?	?	?
$y = f\left(\dfrac{x}{4}\right)$	?	?	?
$y = f(-x)$	?	?	?
$y = -f(x)$	?	?	?

Effect of transformation on specific points

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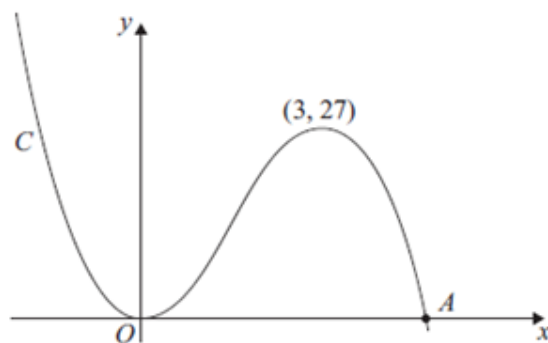| $y = f(x)$	$(4, 3)$	$(1, 0)$	$(6, -4)$
$y = f(x + 1)$	$(3,3)$	$(0,0)$	$(5, -4)$
$y = f(2x)$	$(2,3)$	$(0.5, 0)$	$(3, -4)$
$y = 3f(x)$	$(4,9)$	$(1,0)$	$(6, -12)$
$y = f(x) - 1$	$(4,2)$	$(1, -1)$	$(6, -5)$
$y = f\left(\dfrac{x}{4}\right)$	$(16,3)$	$(4,0)$	$(24, -4)$
$y = f(-x)$	$(-4,3)$	$(-1,0)$	$(-6, -4)$
$y = -f(x)$	$(4, -3)$	$(1,0)$	$(6,4)$

Test Your Understanding

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = x^2(9 - 2x).$$

There is a minimum at the origin, a maximum at the point $(3, 27)$ and C cuts the x-axis at the point A.

(a) Write down the coordinates of the point A.

(1)

(b) On separate diagrams sketch the curve with equation

(i) $y = f(x + 3)$,

(ii) $y = f(3x)$.

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

(6)

The curve with equation $y = f(x) + k$, where k is a constant, has a maximum point at $(3, 10)$.

(c) Write down the value of k.

(1)

(a) ?

(b)(i) ?

(ii) ?

(c) ?

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Figure 1

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The curve with equation $y = f(x) + k$, where k is a constant, has a maximum point at $(3, 10)$.

(c) Write down the value of k.

(1)

(a) {Coordinates of A are} $(4.5, 0)$

(b)(i)

(ii)
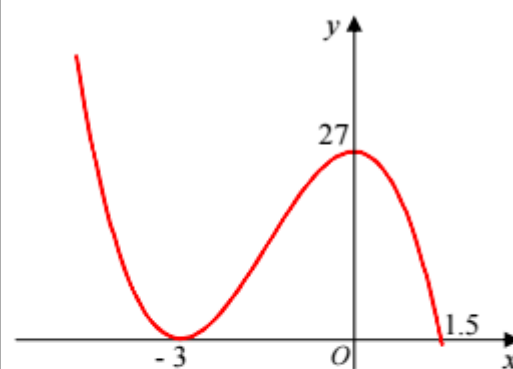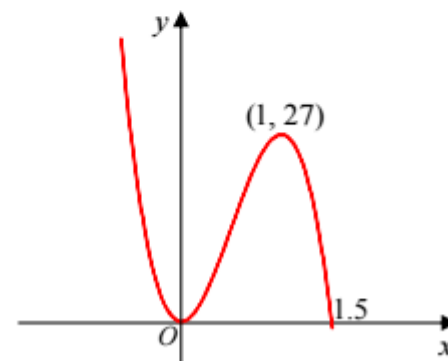
(c) {$k =$} -17

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1 The following diagram shows a sketch of the curve
with equation $y = f(x)$. The points $A(0, 2)$, $B(1, 0)$,
$C(4, 4)$ and $D(6, 0)$ lie on the curve.

Sketch the following graphs and give the coordinates
of the points, A, B, C and D after each transformation:

a $f(x + 1)$ b $f(x) - 4$ c $f(x + 4)$

d $f(2x)$ e $3f(x)$ f $f(\frac{1}{2}x)$

g $\frac{1}{2}f(x)$ h $f(-x)$
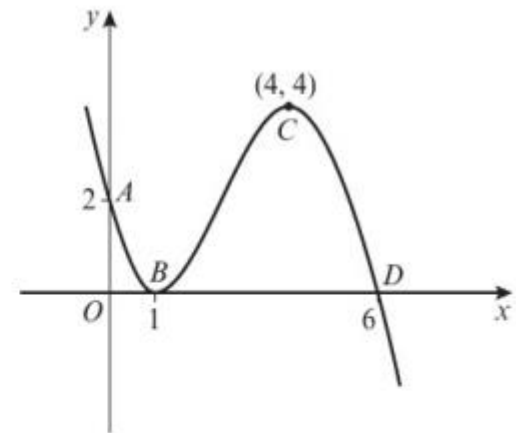
2 The curve $y = f(x)$ passes through the origin and
has horizontal asymptote $y = 2$ and vertical
asymptote $x = 1$, as shown in the diagram.

Sketch the following graphs. Give the equations of
any asymptotes and give the coordinates of
intersections with the axes after each
transformation.

a $f(x) + 2$ b $f(x + 1)$ c $2f(x)$

d $f(x) - 2$ e $f(2x)$ f $f(\frac{1}{2}x)$

g $\frac{1}{2}f(x)$ h $-f(x)$
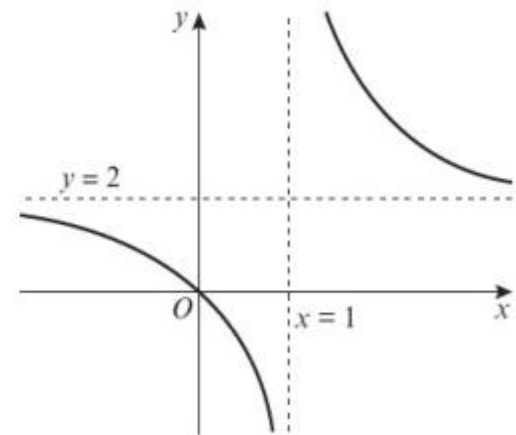
3 The curve with equation $y = f(x)$ passes through the points $A(-4, -6)$, $B(-2, 0)$, $C(0, -3)$ and $D(4, 0)$ as shown in the diagram.

Sketch the following and give the coordinates of the points A, B, C and D after each transformation.

a $f(x - 2)$ **b** $f(x) + 6$ **c** $f(2x)$

d $f(x + 4)$ **e** $f(x) + 3$ **f** $3f(x)$

g $\frac{1}{3}f(x)$ **h** $f(\frac{1}{4}x)$ **i** $-f(x)$

j $f(-x)$
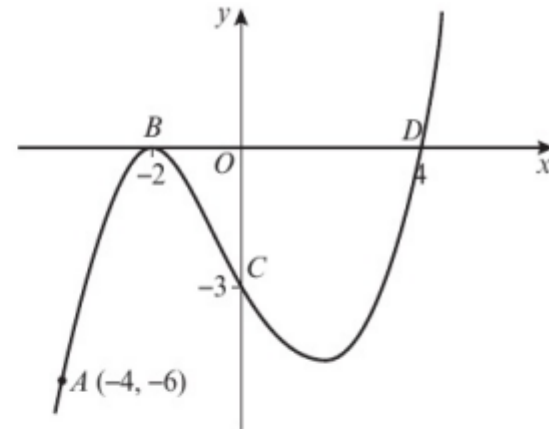
4 A sketch of the curve $y = f(x)$ is shown in the diagram. The curve has a vertical asymptote with equation $x = -2$ and a horizontal asymptote with equation $y = 0$. The curve crosses the y-axis at $(0, 1)$.

a Sketch, on separate diagrams, the graphs of:

i $2f(x)$ **ii** $f(2x)$ **iii** $f(x - 2)$

iv $f(x) - 1$ **v** $f(-x)$ **vi** $-f(x)$

In each case state the equations of any asymptotes and, if possible, points where the curve cuts the axes.

b Suggest a possible equation for $f(x)$.
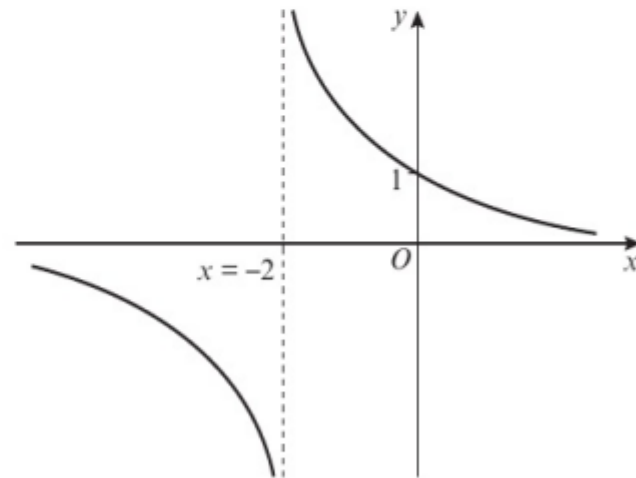
5 The point $P(2, 1)$ lies on the graph with equation $y = f(x)$.

a On the graph of $y = f(ax)$, the point P is mapped to the point $Q(4, 1)$.
Determine the value of a. **(1 mark)**

b Write down the coordinates of the point to which P maps under each transformation

i $f(x - 4)$ **ii** $3f(x)$ **iii** $\frac{1}{2}f(x) - 4$ **(3 marks)**

6 The diagram shows a sketch of a curve with equation $y = f(x)$.
The points $A(-1, 0)$, $B(0, 2)$, $C(1, 2)$ and $D(2, 0)$ lie on the curve.

Sketch the following graphs and give the coordinates of the points
A, B, C and D after each transformation:

a $y + 2 = f(x)$ **b** $\frac{1}{2}y = f(x)$

c $y - 3 = f(x)$ **d** $3y = f(x)$

e $2y - 1 = f(x)$

Problem-solving

Rearrange each equation into the form $y = \ldots$
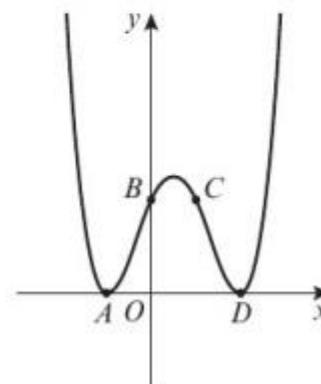
1

a
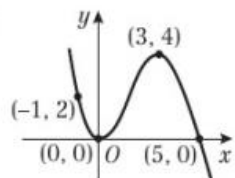
b
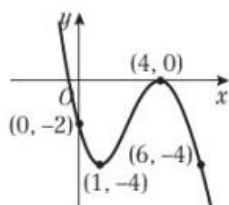
c
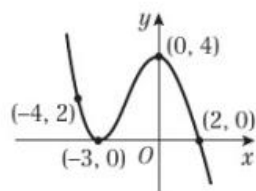
d
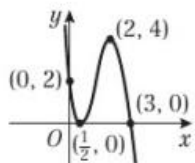
e
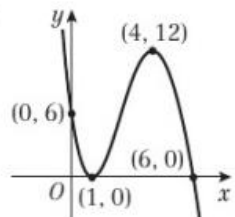
f
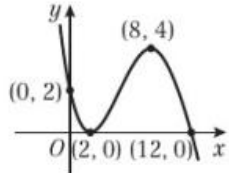
g
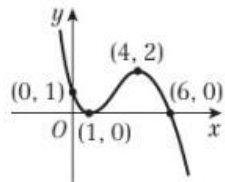
h
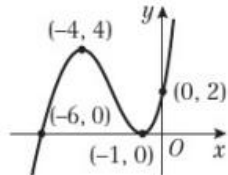
2

a $y = 4$, $x = 1$, $(0, 2)$
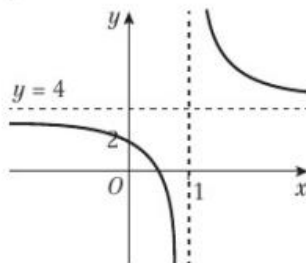
b $y = 2$, $x = 0$, $(-1, 0)$
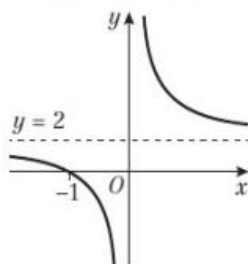
c $y = 4$, $x = 1$, $(0, 0)$
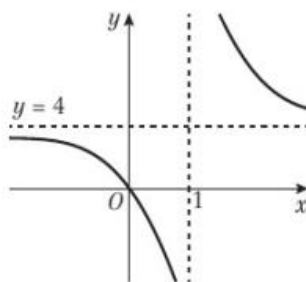
d $y = 0$, $x = 1$, $(0, -2)$
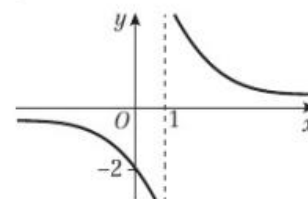
e $y = 2$, $x = \frac{1}{2}$, $(0, 0)$
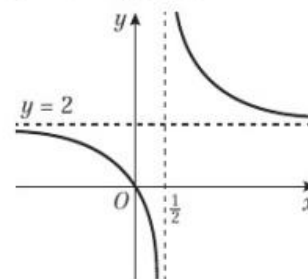
f $y = 2$, $x = 2$, $(0, 0)$
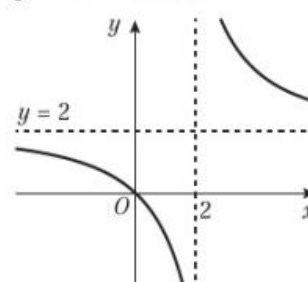
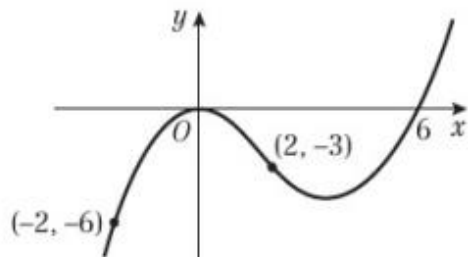**g** $y = 1$, $x = 1$, $(0, 0)$
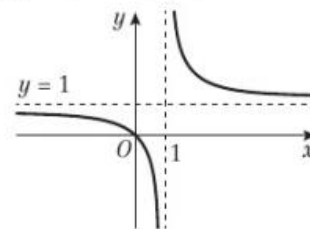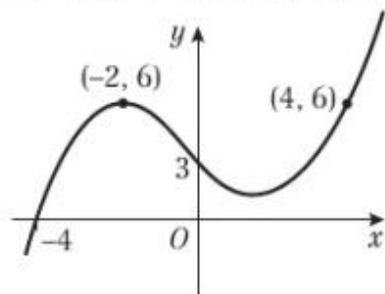

h $y = -2$, $x = 1$, $(0, 0)$
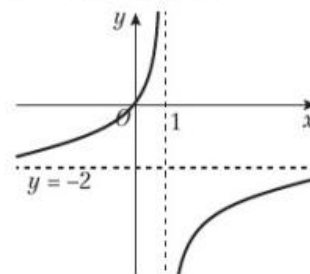
Homework Answers

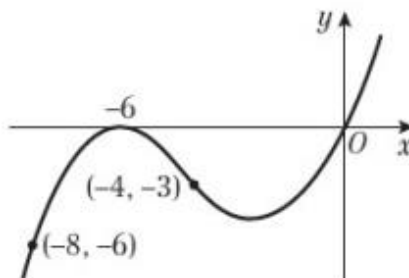**3** **a** $A(-2, -6), B(0, 0), C(2, -3), D(6, 0)$

b $A(-4, 0), B(-2, 6), C(0, 3), D(4, 6)$

c $A(-2, -6), B(-1, 0), C(0, -3), D(2, 0)$
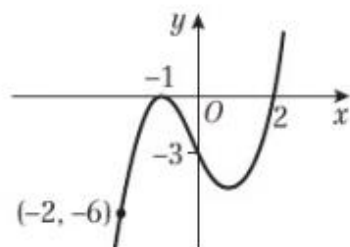
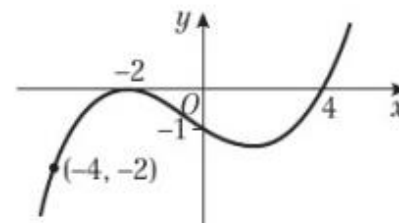**d** $A(-8, -6), B(-6, 0), C(-4, -3), D(0, 0)$

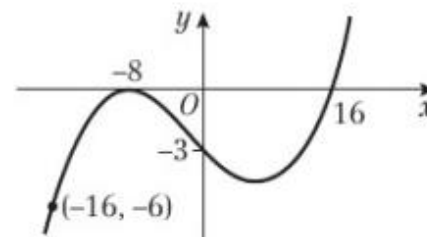**e** $A(-4, -3), B(-2, 3), C(0, 0), D(4, 3)$
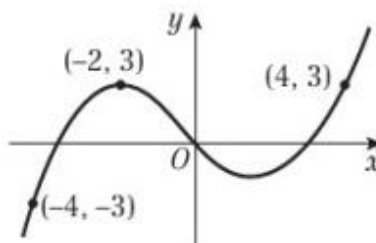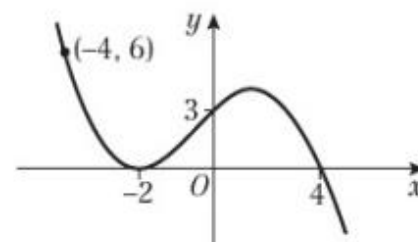

f $A(-4, -18), B(-2, 0), C(0, -9), D(4, 0)$
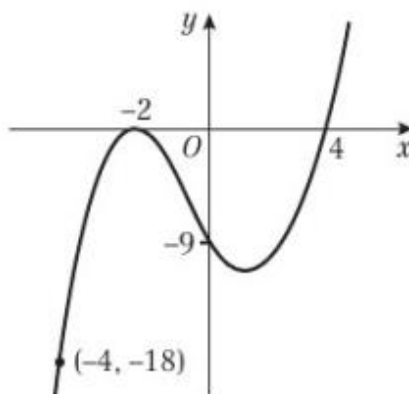
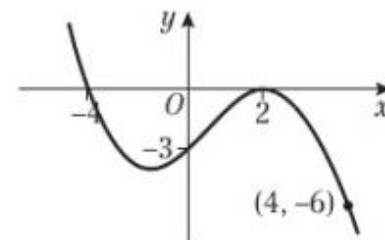**g** $A(-4, -2), B(-2, 0), C(0, -1), D(4, 0)$

h $A(-16, -6), B(-8, 0), C(0, -3), D(16, 0)$
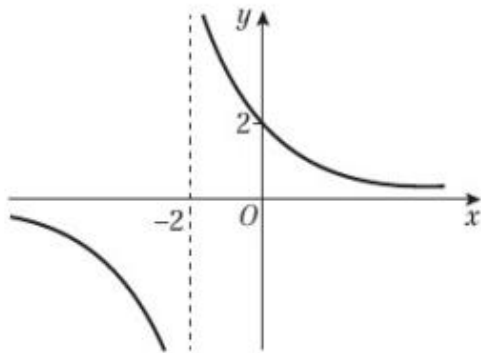

i $A(-4, 6), B(-2, 0), C(0, 3), D(4, 0)$
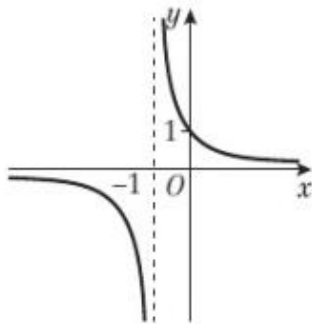

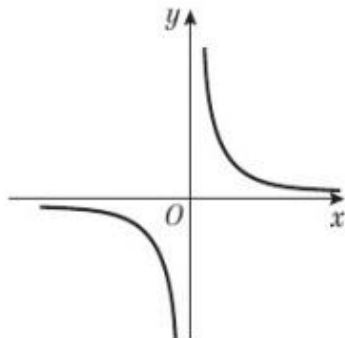**j** $A(4, -6), B(2, 0), C(0, -3), D(-4, 0)$

Homework Answers

4 **a** **i** $x = -2, y = 0, (0, 2)$

iv $x = -2\ y = -1\ (0, 0)$
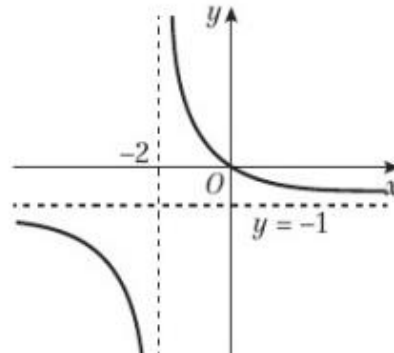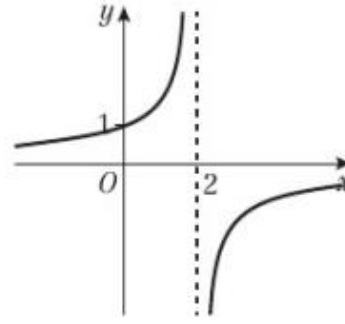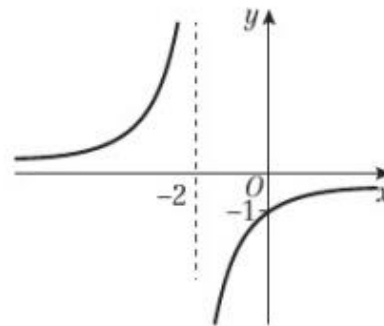
4 **b** $f(x) = \dfrac{2}{x + 1}$

5 **a** $\frac{1}{2}$
    b **i** $(6, 1)$
       ii $(2, 3)$
       iii $(2, -3.5)$

ii $x = -1, y = 0, (0, 1)$

v $x = 2\ y = 0\ (0, 1)$

iii $x = 0\ y = 0$

vi $x = -2\ y = 0\ (0, -1)$

Homework Answers

6　a　$A(-1, -2)\ B(0, 0)\ C(1, 0)\ D(2, -2)$

$y + 2 = f(x)$

b　$A(-1, 0)\ B(0, 4)\ C(1, 4)\ D(2, 0)$

$\frac{1}{2}y = f(x)$

c　$A(-1, 3)\ B(0, 5)\ C(1, 5)\ D(2, 3)$

$y - 3 = f(x)$

d　$A(-1, 0)\ B(0, \frac{2}{3})\ C(1, \frac{2}{3})\ D(2, 0)$

$3y = f(x)$

e　$A(-1, 0.5)\ B(0, 1.5)\ C(1, 1.5)\ D(2, 0.5)$

$2y - 1 = f(x)$