M1 Chapter 10: Forces and Motion

Pulleys

Pulleys

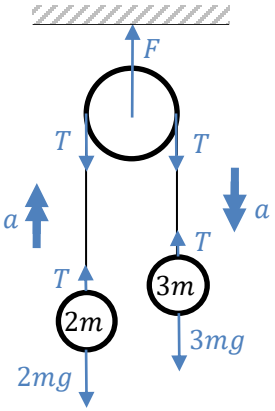
A pulley is a wheel on which a rope/string/cable passes.
For now the two particles hanging either side will either be horizontal or vertical.
Later they could be on a slope which works the same way, but needs more geometry.

Why can't we just model both particles as a single particle as before?
The particles are moving in different directions (it is possible, but now separate just works better)**.**

Under what conditions is the tension in each part of the string the same?
Single piece of string and a <u>smooth pulley</u>.

[Textbook] Particles P and Q, of masses $2m$ and $3m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

(a) Write down an equation of motion for P and for Q.
(b) Find the acceleration of each mass.
(c) Find the tension in the string.
(d) Find the force exerted on the pulley by the string.
(e) Find the speed of Q after the first 4 s, assuming that P does not reach the pulley.



Remember that tension acts away from each object in the direction of the string. This includes the pulley!

Ensure that you add the direction of acceleration at each moving particle, and resolve forces in this direction.

a For P, $R(\uparrow)$: $T - 2mg = 2ma$ (1)
For Q, $R(\downarrow)$: $3mg - T = 3ma$ (2)

b Adding equations:
$mg = 5ma \quad \rightarrow \quad a = \frac{1}{5}g$ ms^{-2}

c Substituting back into (1):
$T = 2ma + 2mg = \frac{2}{5}mg + 2mg = \frac{12}{5}mg$ N

d Force exerted on pulley by string:
$= 2T = \frac{24}{5}mg$ N

e
$v = at$
$= 7.84$ m/s (3sf)

Horizontal and vertical string

?

Diagram

[Textbook] Two particles A and B of masses 0.4kg and 0.8kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 4.5m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 0.5m above horizontal ground. A frictional force of magnitude 0.08g opposes the motion of particle A. The system is released from rest. Find:

(a) The acceleration of the system

(b) The speed for B after 0.4s

(c) The tension in the string.

(d) Find the force exerted on the pulley by the string.

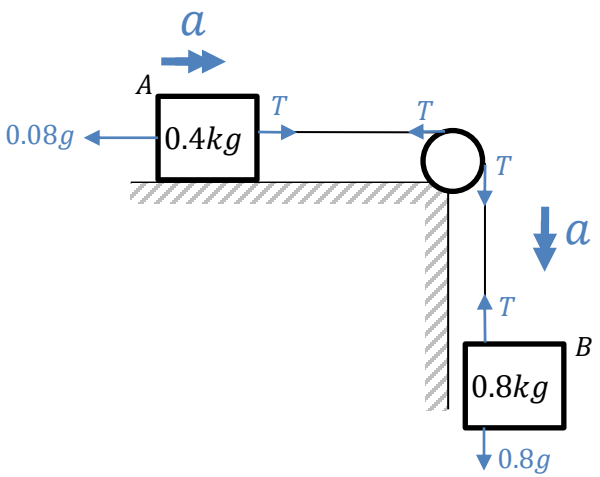
a ?

b ?

c ?

d ?

Horizontal and vertical string



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(a) The acceleration of the system
(b) The speed for B after 0.4s
(c) The tension in the string.
(d) Find the force exerted on the pulley by the string.

a For A, $R(\rightarrow)$, $T - 0.08g = 0.4a$ (1)
For B, $R(\downarrow)$, $0.8g - T = 0.8a$ (2)
Adding:

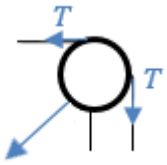
$$0.72g = 1.2a \quad \rightarrow \quad a = 0.6g$$

b

$$v = at, \qquad a = 5.88$$
$$v = 2.4\ m/s\ (2sf)$$

c From A (eqn-1), $T = 0.4a + 0.08g \rightarrow \quad T = 1.0N$

d The resultant force of the two tensions against the pulley:



$$R^2 = T^2 + T^2$$
$$R = \sqrt{2}\,T$$
$$\rightarrow \quad R = 1.4N$$

Test Your Understanding

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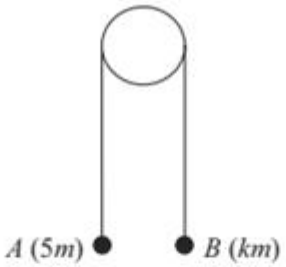


Figure 4

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in Figure 4. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

(a) Show that the tension in the string as A descends is $\frac{15}{4}mg$.

(3)

(b) Find the value of k.

(3)

(c) State how you have used the information that the pulley is smooth.

(1)

("N2L" = Newton's 2nd Law)

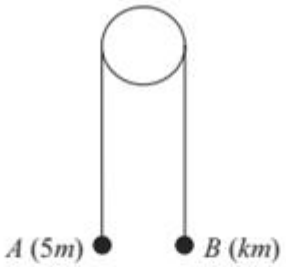
?

?

?

Test Your Understanding

Edexcel M1 Jan 2010 Q6



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(1)

("N2L" = Newton's 2nd Law)

(a) N2L A: $5mg - T = 5m \times \frac{1}{4}g$ **M1 A1**

$T = \frac{15}{4}mg$ * cso **A1**

(b) N2L B: $T - kmg = km \times \frac{1}{4}g$ **M1 A1**

$k = 3$ **A1**

(c) The tensions in the two parts of the string are the same **B1**

Classwork Exercise 10.6

Pearson Stats/Mechanics Year 1 Exercise Book
Pages 74-75
Questions 1 to 4

Homework Exercise

1 Two particles A and B of masses 4 kg and 3 kg respectively are connected by a light inextensible string which passes over a small smooth fixed pulley. The particles are released from rest with the string taut.

a Find the tension in the string.

When A has travelled a distance of 2 m it strikes the ground and immediately comes to rest.

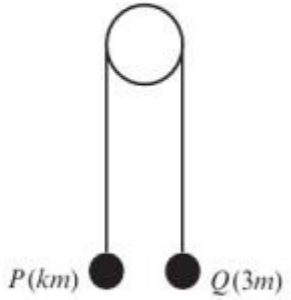
b Find the speed of A when it hits the ground.

c Assuming that B does not hit the pulley, find the greatest height that B reaches above its initial position.

Problem-solving

After A hits the ground B behaves like a particle moving freely under gravity.

2 Two particles P and Q have masses km and $3m$ respectively, where $k < 3$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with P and Q at the same height above a horizontal plane, as shown in the diagram. The system is released from rest. After release, Q descends with acceleration $\frac{1}{3}g$.



a Calculate the tension in the string as Q descends. **(3 marks)**

b Show that $k = 1.5$ **(3 marks)**

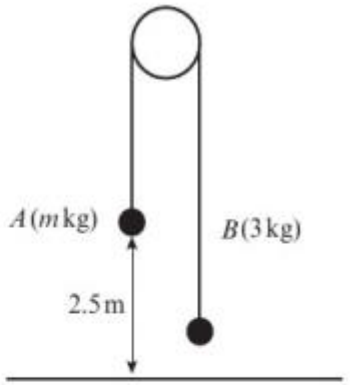
c State how you have used the information that the pulley is smooth. **(1 mark)**

After descending for 1.8 s, the particle Q reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between P and the pulley is such that, in the subsequent motion, P does not reach the pulley.

d Show that the greatest height, in metres, reached by P above the plane is $1.26g$. **(7 marks)**

Homework Exercise

3 Two particles A and B have masses m kg and 3 kg respectively, where $m > 3$. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially A is 2.5 m above horizontal ground. The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in the figure. After A has been descending for 1.25 s, it strikes the ground. Particle A reaches the ground before B has reached the pulley.



a Show that the acceleration of B as it ascends is $3.2\,\text{m s}^{-2}$. **(3 marks)**

b Find the tension in the string as A descends. **(3 marks)**

c Show that $m = \frac{65}{11}$. **(4 marks)**

d State how you have used the information that the string is inextensible. **(1 mark)**

When A strikes the ground it does not rebound and the string becomes slack. Particle B then moves freely under gravity, without reaching the pulley, until the string becomes taut again.

e Find the time between the instant when A strikes the ground and the instant when the string becomes taut again. **(6 marks)**

4 Two particles A and B of masses 5 kg and 3 kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table and the string passes over a small smooth pulley which is fixed at the edge of the table. Particle B hangs freely. The friction between A and the table is 24.5 N. The system is released from rest. Find:

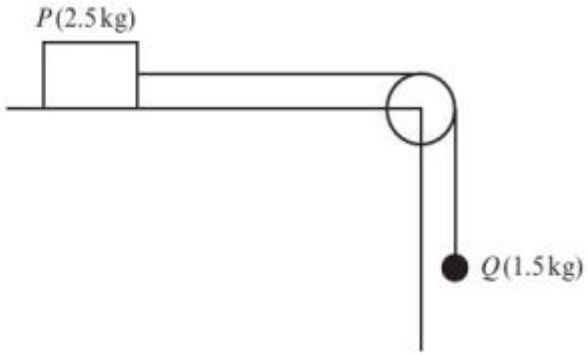
a the acceleration of the system

b the tension in the string

c the magnitude of the force exerted on the pulley by the string.

Homework Exercise

5 A box P of mass 2.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley fixed at the edge of the table. The other end of the string is attached to a sphere Q of mass 1.5 kg which hangs freely below the pulley. The magnitude of the frictional force between P and the table is k N. The system is released from rest with the string taut. After release, Q descends a distance of 0.8 m in 0.75 s.



a Modelling P and Q as particles:

i calculate the acceleration of Q **(3 marks)**

ii show that the tension in the string is 10.4 N (to 3 s.f.) **(4 marks)**

iii find the value of k. **(3 marks)**

b State how in your calculations you have used the information that the string is inextensible. **(1 mark)**

Homework Answers

1 a 33.6 N (3 s.f.)

b 2.37 m s^{-1} (3 s.f.)

c 2.29 m (3 s.f.)

2 a $2mg$ N

b For P: $2mg - kmg = \frac{1}{3}kmg$

So $2 - k = \frac{1}{3}k$ and $k = 1.5$

c Smooth $\Rightarrow$ no friction so magnitude of acceleration is the same in objects connected by a taut inextensible string.

d While Q is descending, distance travelled by $P = s_1$

$s = ut + \frac{1}{2}at^2 \Rightarrow s_1 = \frac{1}{6}g \times 1.8^2 = 0.54g$

Speed of P at this time $= v_1$

$v^2 = u^2 + 2as \Rightarrow v_1^2 = 0^2 + \left(2 \times \frac{g}{3} \times 0.54g\right) = 0.36g^2$

After Q hits the ground, P travels freely under gravity and travels a further distance s_2.

$v^2 = u^2 + 2as \Rightarrow 0^2 = 0.36g^2 - 2gs_2 \Rightarrow s_2 = 0.18g$

Total distance travelled $= s_1 + s_2 = 0.54g + 0.18g$
$= 0.72g$ m

As particles started at same height P must be s_1 metres above the plane at the start.

Maximum height reached by P above the plane
$= 0.72g + s_1 = 0.72g + 0.54g = 1.26g$ m

3 a $s = ut + \frac{1}{2}at^2$ so $2.5 = 0 + \frac{1}{2} \times a \times 1.25^2$, $a = 3.2\ \text{m s}^{-2}$

b 39 N

c For A, R(↓): $mg - T = ma$

$T = m(9.8 - 3.2)$, $T = 6.6m$

Substituting for T: $39 = 6.6m$

$m = \frac{65}{11}$

d Same tension in string either side of the pulley.

e $\frac{40}{49}$ s

4 a 0.613 m s^{-2} (3 s.f.)

b 27.6 N (3 s.f.)

c 39.0 N (3 s.f.)

5 a **i** 2.84 m s^{-2} (3 s.f.)

ii $2.84(1.5) = 1.5g - T$

$T = 1.5g - 4.26 = 10.4$ N (3 s.f.)

iii 3.3 N

b Same tension in string either side of the pulley.