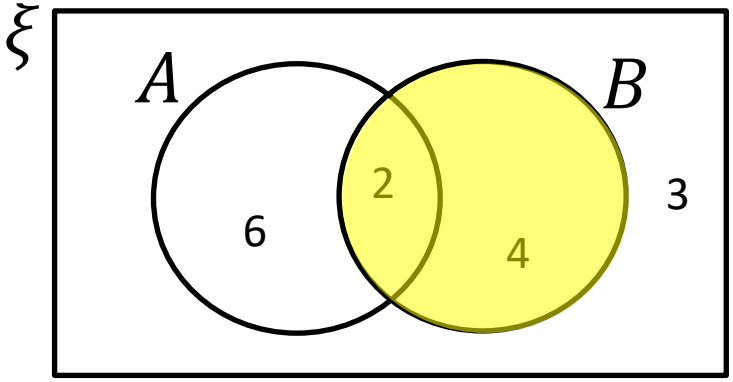
Stats Yr2 Chapter 2: Probability Theory

Probability with Venn Diagrams

Examples

1 Using the Venn Diagram, determine:



a $P(A|B)$

Method 1: Using the formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/15}{6/15} = \frac{1}{3}$$

Method 2: Restricted sample space

$$P(A|B) = \frac{2}{6} = \frac{1}{3}$$

Out of 6 things in B, 2 are in $A \cap B$.

b $P(A'|B') = \frac{3}{9} = \frac{1}{3}$

c $P(B|A \cup B) = \frac{6}{12} = \frac{1}{2}$

Further Examples

a Given that $P(A) = 0.5$ and $P(A \cap B) = 0.3$, what is $P(B|A)$?

?

Fro Tip: The 'restricted sample space' method also works for Venn Diagrams with probabilities.

b Given that $P(Y) = 0.6$ and $P(X \cap Y) = 0.4$, what is $P(X'|Y)$?
(Hint: Drawing a Venn Diagram will help!)

?

c Given that $P(A) = 0.5$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, what is $P(B|A')$?

?

Further Examples

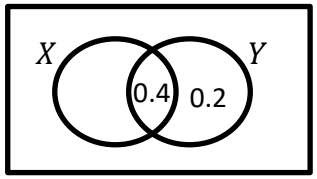
a Given that $P(A) = 0.5$ and $P(A \cap B) = 0.3$, what is $P(B|A)$?

$$\boldsymbol{P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.3}{0.5} = 0.6}$$

Fro Tip: The 'restricted sample space' method also works for Venn Diagrams with probabilities.

b Given that $P(Y) = 0.6$ and $P(X \cap Y) = 0.4$, what is $P(X'|Y)$?
(Hint: Drawing a Venn Diagram will help!)

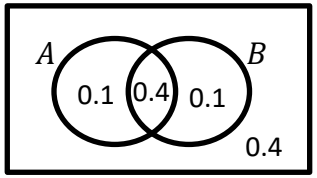
$$\boldsymbol{P(X'|Y) = \frac{P(X' \cap Y)}{P(Y)}}$$



$$\boldsymbol{\therefore P(X'|Y) = \frac{0.2}{0.6} = \frac{1}{3}}$$

c Given that $P(A) = 0.5$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, what is $P(B|A')$?

$$\boldsymbol{P(B|A') = \frac{P(A' \cap B)}{P(A')}}$$

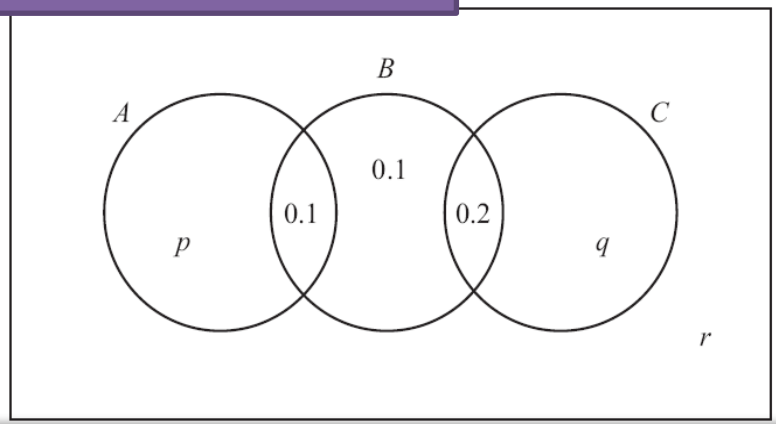


$$\boldsymbol{= \frac{0.1}{0.5}}$$

$$\boldsymbol{= 0.2}$$

Further Test Your Understanding

May 2013 (R) Q6



The Venn diagram in Figure 1 shows three events A, B and C and the probabilities associated with each region of B. The constants p, q and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent.

(a) Find the value of p. **(3)**

Given that $P(B|C) = \frac{5}{11}$,

(b) find the value of q and the value of r (4)

(c) Find $P(A \cup C|B)$ (2)

(a) **(From earlier)**

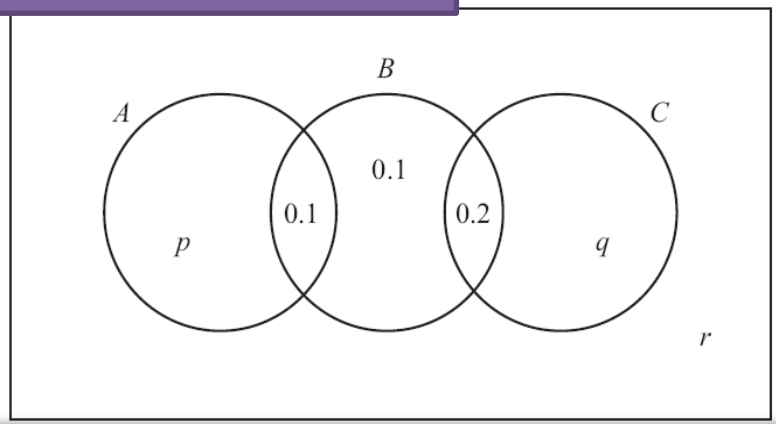
$$0.1 = (p + 0.1) \times 0.4$$
$$p + 0.1 = 0.25$$
$$p = 0.15$$

(b) ?

(c) ?

Further Test Your Understanding

May 2013 (R) Q6



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(a) **(From earlier)**

$$0.1 = (p + 0.1) \times 0.4$$
$$p + 0.1 = 0.25$$
$$p = 0.15$$

(b) $P(B|C) = \frac{P(B \cap C)}{P(C)}$

$$\frac{5}{11} = \frac{0.2}{0.2 + q}$$
$$q = 0.24$$
$$r = 1 - 0.15 - 0.1 - 0.1 - 0.2 - 0.24$$
$$= 0.21$$

(c) $P(A \cup C|B) = \frac{P((A \cup C) \cap B)}{P(B)}$

$$= \frac{0.1 + 0.2}{0.4} = 0.75$$

Exercise 2.3

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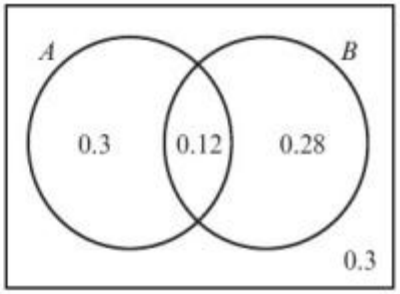
Homework Exercise

1 The Venn diagram shows the probabilities for two events, A and B.



Find:

a $P(A \cup B)$ **b** $P(A|B)$

c $P(B|A')$ **d** $P(B|A \cup B)$

2 C and D are two events such that $P(C) = 0.8$, $P(D) = 0.4$ and $P(C \cap D) = 0.25$.

a Draw a Venn diagram showing the probabilities for events C and D.

b Find:

i $P(C \cup D)$ **ii** $P(C|D)$ **iii** $P(D|C)$ **iv** $P(D'|C')$

3 S and T are two events such that $P(S) = 0.5$ and $P(T) = 0.7$. Given that S and T are independent,

a draw a Venn diagram showing the probabilities for events S and T.

b Find:

i $P(S \cap T)$ **ii** $P(S|T)$ **iii** $P(T|S')$ **iv** $P(S|S' \cup T')$

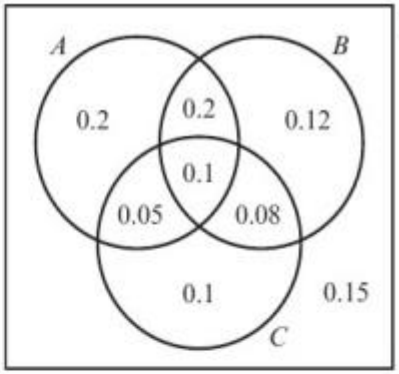
4 120 members of a youth club play either snooker (A) or pool (B) or neither. Given that 65 play snooker, 50 play pool and 20 play both, find:

Hint Draw a Venn diagram.

a $P(A \cap B')$ **b** $P(A|B)$ **c** $P(B|A')$ **d** $P(A|A \cup B)$

Homework Exercise

6 The Venn diagram shows the probabilities of three events, A, B and C.

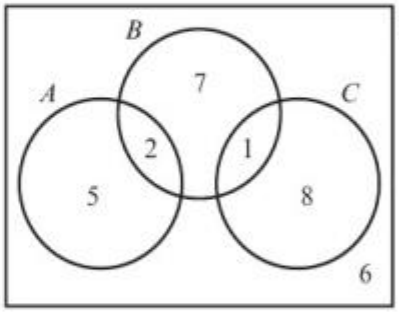


Find:

a $P(A|B)$ **b** $P(C|A')$

c $P((A \cap B)|C')$ **d** $P(C|(A' \cup B'))$

7 The Venn diagram shows the number of students in a class who watch any of 3 popular TV programmes A, B and C.



One of these students is selected at random. Given that the student watches at least one of the TV programmes, find the probability that the student watches:

a programme C **(2 marks)**

b exactly two of the programmes. **(2 marks)**

c Determine whether or not watching programme B and watching programme C are statistically independent. **(3 marks)**

Problem-solving

If $P(A|B) = P(A)$ then events A and B are independent.

Homework Exercise

8 Three events, A, B and C are such that A and B are mutually exclusive and B and C are independent. $P(A) = 0.2$, $P(B) = 0.6$ and $P(C) = 0.5$. Given that $P(A' \cap B' \cap C') = 0.1$,

a draw a Venn diagram to show the probabilities for events A, B and C. **(4 marks)**

b Find:

i $P(A|C)$ **(1 mark)**

ii $P(B|C')$ **(1 mark)**

iii $P(C|(A \cup B))$ **(1 mark)**

9 A doctor completes a medical study of 100 people, 5 of whom are known to have an illness and 95 of whom are known not to. A diagnostic test is applied. All 5 of the people with the illness test positive, and 10 people without the illness also test positive. Given that event A = person has the disease and event B = person tests positive,

a draw a Venn diagram to represent this situation. **(3 marks)**

b Calculate $P(A|B)$. **(2 marks)**

c With reference to your answer to part **b**, comment on the usefulness of the diagnostic test. **(2 marks)**

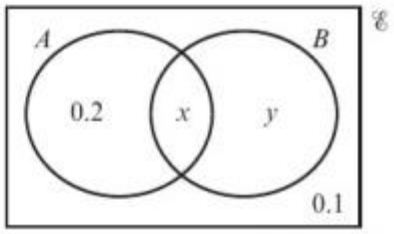
10 Events A and B are such that $P(A) = 0.6$ and $P(B) = 0.7$. Given that $P(A' \cap B') = 0.12$, find:

a $P(B|A')$ **b** $P(B|A)$

c Explain what your answers to parts **a** and **b** tell you about events A and B.

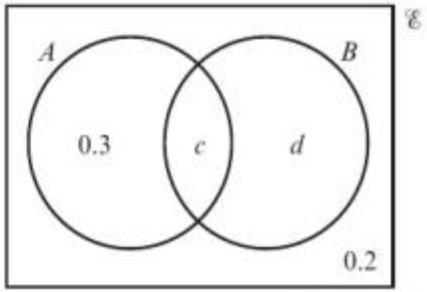
Homework Exercise

11 The Venn diagram shows the probabilities for two events, A and B. Given that $P(A|B) = P(B')$, find the values of x and y.



(3 marks)

12 The Venn diagram shows the probabilities for two events, A and B. Given that $P(A|B) = P(A')$, find the values of c and d.

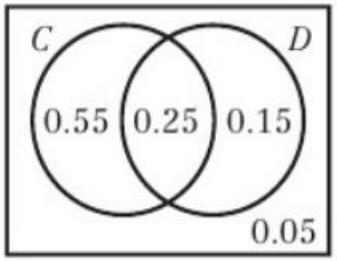


(3 marks)

Homework Answers

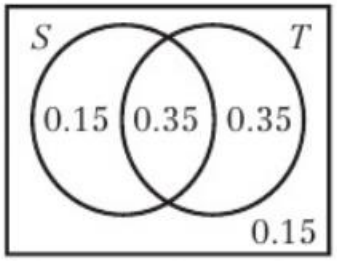
1 **a** 0.7 **b** 0.3
c 0.483 (3 s.f.) **d** 0.571 (3 s.f.)

2 **a**



b **i** 0.95 **ii** 0.625 **iii** 0.313 (3 s.f.) **iv** 0.25

3 **a**



b **i** 0.35 **ii** 0.5 **iii** 0.7 **iv** 0.231 (3 s.f.)

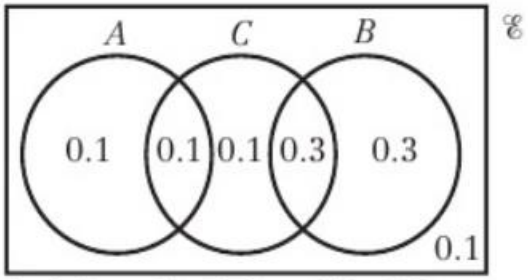
4 **a** $\frac{3}{8}$ **b** $\frac{2}{5}$ **c** $\frac{6}{11}$ **d** $\frac{13}{19}$

5 **a** $\frac{9}{80}$ **b** $\frac{9}{32}$ **c** $\frac{1}{5}$ **d** $\frac{12}{35}$

6 **a** 0.6 **b** 0.4
c 0.299 (3 s.f.) **d** 0.329 (3 s.f.)

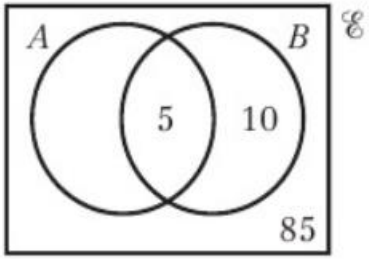
7 **a** $\frac{9}{23}$ **b** $\frac{3}{23}$
c $P(B|C) = 0.111\ldots \neq P(B) = 0.345\ldots$ So B and C are not independent

8 **a**



b **i** 0.2 **ii** 0.6 **iii** 0.5

9 **a**



b $\frac{1}{3}$

c No one who doesn't have the disease would be given a false negative result. However, only $\frac{1}{3}$ of the people who have a positive result would have the disease.

10 **a** 0.7 **b** 0.7 **c** They are independent.

11 $x = 0.21, y = 0.49$

12 $c = \frac{7}{30}, d = \frac{4}{15}$