M2 Chapter 8: Further Kinematics

Vectors in Kinematics

Overview

This chapter concerns how can use **vectors to represent motion**. In the case of constant acceleration, can we still use our 'suvat' equations? And what if we have variable acceleration with expressions in terms of t?

1:: Vector equations for motion.

The velocity, $\mathbf{v}$ m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$ **(3)**

(b) Find the bearing on which P is moving when $t = 2$ **(2)**

(c) Find the value of t when P is moving

(i) parallel to $\mathbf{j}$,

(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$. **(6)**

2:: Variable acceleration with vectors.

"A particle P of mass 0.8kg is acted on by a single force $\boldsymbol{F}$ N. Relative to a fixed origin O, the position vector of P at time t seconds is $\boldsymbol{r}$ metres, where

$$\boldsymbol{r} = 2t^3\boldsymbol{i} + 50t^{-\frac{1}{2}}\boldsymbol{j}, \qquad t \geq 0$$

Find (a) the speed of P when $t = 4$

(b) The acceleration of P as a vector when $t = 2$

(c) $\boldsymbol{F}$ when $t = 2$."

3:: Integration with vectors to find velocity/displacement

"A particle P is moving in a plane. At time t seconds, its velocity $\boldsymbol{v}$ ms^{-1} is given by $\boldsymbol{v} = 3ti + \frac{1}{2}t^2\boldsymbol{j}, \;\; t \geq 0$ When $t = 0$, the position vector of P with respect to a fixed origin O is $(2\boldsymbol{i} - 3\boldsymbol{j})$ m. Find the position vector of P at time t seconds."

Vector motion

Initially, Kat is at the position vector $\begin{pmatrix}3\\1\end{pmatrix}$m. Each second, she moves with **velocity** $= \begin{pmatrix}4\\2\end{pmatrix}$m/s.

(a) Where will Kat be after 1 second?

(b) Where is Kat after 2 seconds ?

(b) $t = 2 \Rightarrow \vec{s} = \begin{pmatrix}11\\5\end{pmatrix} \equiv 11\vec{\imath} + 5\vec{\jmath}$

$t = 0$

$\begin{pmatrix}3\\1\end{pmatrix}$  $\boldsymbol{v} = \begin{pmatrix}4\\2\end{pmatrix}$

(a) $t = 1 \Rightarrow \vec{s} = \begin{pmatrix}7\\3\end{pmatrix} \equiv 7\vec{\imath} + 3\vec{\jmath}$

In general where would Kat be after t seconds in terms of t?

It'll be $\begin{pmatrix}\mathbf{3}\\\mathbf{1}\end{pmatrix}$ with t lots of $\begin{pmatrix}\mathbf{4}\\\mathbf{2}\end{pmatrix}$ added on:

$$\vec{s} = \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix}t = (3\vec{\imath} + \vec{\jmath}) + (4t\vec{\imath} + 2t\vec{\jmath}) \Rightarrow \vec{s}(t) = \begin{pmatrix}3+4t\\1+2t\end{pmatrix} = (3+4t)\vec{\imath} + (1+2t)\vec{\jmath}$$

✎ Position vector $\boldsymbol{r}$ of particle:

$$\boldsymbol{r} = \boldsymbol{r_0} + \boldsymbol{v}t$$

where r_0 is initial position and $\boldsymbol{v}$ is velocity.

Note: The formula comes from 'common sense' using the reasoning above.

Note II: Further Mathematicians who have finished Vectors in Core Pure Yr1 may see the similarities with vector equations of straight lines.

Example

[Textbook] A particle starts from the position vector $(3\vec{\imath} + 7\vec{\jmath})$ m and moves with constant velocity $(2\vec{\imath} - \vec{\jmath})$ ms^{-1}.

(a) Find the position vector of the particle 4 seconds later.

(b) Find the time at which the particle is due east of the origin.

a

?

b

?

Note: Some people prefer to avoid the $\boldsymbol{i}$ and $\boldsymbol{j}$ notation and write instead as column vectors. This is especially useful when considering directions and parallel vectors.

Example

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a

$$\boldsymbol{r} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} + 4\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 11 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 11 \\ 3 \end{pmatrix} \text{ m}$$

b

If due East, then the $\boldsymbol{j}$ component is 0:

$$r = \begin{pmatrix} 3 \\ 7 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 + 2t \\ 7 - t \end{pmatrix}$$

$7 - t = 0 \quad \rightarrow \quad t = 7$ seconds

Note: Some people prefer to avoid the $\boldsymbol{i}$ and $\boldsymbol{j}$ notation and write instead as column vectors. This is especially useful when considering directions and parallel vectors.

$suvat$... but with vectors!

Some $suvat$ equations work with vectors. By convention, we use $\boldsymbol{r}$ instead of $\boldsymbol{s}$ for displacement in 2D/3D (as we did in the previous exercise). In 2D, which of the quantities are vectors and which are scalars?

$$\boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$
$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$r =$ **vector**
$u =$?
$v =$?
$a =$?
$t =$?

Note that as $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors, we can't for example use $v^2 = u^2 + 2as$, as you can't square a vector.

[Textbook] A particle P has velocity $(-3\boldsymbol{i} + \boldsymbol{j})$ ms^{-1}. The particle moves with constant acceleration $\boldsymbol{a} = (2\boldsymbol{i} + 3\boldsymbol{j})$ ms^{-2}. Find (a) the speed of the particle and (b) the bearing on which it is travelling at time $t = 3$ seconds.

a

?

b

?

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r = **vector**
u = **vector**
v = **vector**
a = **vector**
t = scalar

$$\boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$
$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

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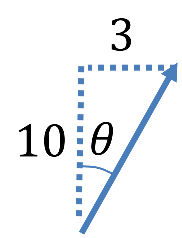
a

$$v = \boldsymbol{u} + \boldsymbol{a}t$$
$$= \begin{pmatrix} -3 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \end{pmatrix} \text{ ms}^{-1}$$

Speed $= \sqrt{3^2 + 10^2} = 10.4$ ms^{-1} (3sf)

Remember that speed is the scalar for of velocity, so find magnitude of the vector.

b



$$\tan\theta = \frac{3}{10} \Rightarrow \theta = 16.7°$$

Bearing is $017°$

The velocity vector gives the direction of motion. Just draw it out to establish angles.

Further Example

[Textbook] An ice skater is skating on a large flat ice rink. At time $t = 0$ the skater is at a fixed point O and is travelling with velocity $(2.4\boldsymbol{i} - 0.6\boldsymbol{j})$ ms^{-1}.
At time $t = 20$ s the skater is travelling with velocity $(-5.6\boldsymbol{i} + 3.4\boldsymbol{j})$ ms^{-1}.
Relative to O, the skater has position vector $\boldsymbol{s}$ at time t seconds.
Modelling the ice skater as a particle with constant acceleration, find:
(a) The acceleration of the ice skater
(b) An expression for $\boldsymbol{s}$ in terms of t
(c) The time at which the skater is directly north-east of O.
A second skater travels so that she has position vector $\boldsymbol{r} = (1.1t - 6)\boldsymbol{j}$ m relative to O at time t.
(d) Show that the two skaters will meet.

a

?

b

?

c

?

d

?

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a

Using $v = u + at$,

$$\begin{pmatrix} -5.6 \\ 3.4 \end{pmatrix} = \begin{pmatrix} 2.4 \\ -0.6 \end{pmatrix} + 20a$$

$$20a = \begin{pmatrix} -8 \\ 4 \end{pmatrix}$$

$$a = \begin{pmatrix} -0.4 \\ 0.2 \end{pmatrix} \text{ ms}^{-2}$$

b

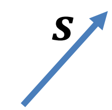
Using $s = ut + \frac{1}{2}at^2$,

$$s = \begin{pmatrix} 2.4 \\ -0.6 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} -0.4 \\ 0.2 \end{pmatrix} t^2$$

$$= \begin{pmatrix} 2.4t - 0.2t^2 \\ -0.6t + 0.1t^2 \end{pmatrix} \text{ m}$$

c

When north-east of O, the $\boldsymbol{i}$ component will be the same as the $\boldsymbol{j}$ component.



$$2.4t - 0.2t^2 = -0.6t + 0.1t^2$$

$$3t(1 - 0.1t) = 0$$

$$\cancel{t = 0} \text{ or } t = 10$$

d

When they meet, two position vectors will be the same:

$$\begin{pmatrix} 2.4t - 0.2t^2 \\ -0.6t + 0.1t^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1.1t - 6 \end{pmatrix}$$

$$2.4t - 0.2t^2 = 0 \quad \rightarrow \quad 2t(12 - t) = 0$$

$$t = 0 \text{ or } t = 12$$

When $t = 0$, $-0.6(0) + 0.1(0^2) = 0$
But $1.1(0) - 6 = -6$ so do not meet when $t = 0$
When $t = 12$, $-0.6(12) + 0.1(12^2) = 7.2$ and $1.1(12) - 6 = 7.2$ so skaters meet when $t = 12$ seconds.

Test Your Understanding

Edexcel M1(Old) May 2013(R) Q6

[In this question **i** *and* **j** *are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O.]*

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j})$ km.

(a) Find the position vector of S at time t hours. **(2)**

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j})$ km. The two ships meet at the point P.

(b) Find the value of n. **(5)**

(c) Find the distance OP. **(4)**

(a) ?

(b) ?

(c) ?

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(a)	Use of $\mathrm{r} = \mathrm{r}_0 + \mathrm{v}t$	M1	
	$(-4\mathrm{i} + 2\mathrm{j}) + (3\mathrm{i} + 3\mathrm{j})t = (-4 + 3t)\mathrm{i} + (2 + 3t)\mathrm{j}$	A1	
			(2)
(b)	$(6\mathrm{i} + \mathrm{j}) + (-2\mathrm{i} + n\mathrm{j})t = (6 - 2t)\mathrm{i} + (1 + nt)\mathrm{j}$	B1	
	Position vectors identical $\Rightarrow -4 + 3t = 6 - 2t$ **AND** $5t = 10$,	M1	
	Either equation	A1	
	$2 + 3 \times 2 = 1 + 2n$,	**DM1**	
	$n = 3.5$	A1	
			(5)
(c)	Position vector of P is $(-4 + 6)\mathrm{i} + (2 + 6)\mathrm{j} = 2\mathrm{i} + 8\mathrm{j}$	M1A1	
	Distance OP $= \sqrt{2^2 + 8^2} = \sqrt{68} = 8.25$ (km)	M1A1	
			(4)

Exercise 8.1

Pearson Stats/Mechanics Year 2

Pages 68-69

Homework Exercise

For all questions in this exercise, take **i** and **j** to be the unit vectors due east and north respectively.

1 A particle P starts at the point with position vector $\mathbf{r}_0$. P moves with constant velocity $\mathbf{v}$ m s^{-1}. After t seconds, P is at the point with position vector $\mathbf{r}$.

a Find $\mathbf{r}$ if $\mathbf{r}_0 = 2\mathbf{i}$, $\mathbf{v} = \mathbf{i} + 3\mathbf{j}$, and $t = 4$.

b Find $\mathbf{r}$ if $\mathbf{r}_0 = 3\mathbf{i} - \mathbf{j}$, $\mathbf{v} = -2\mathbf{i} + \mathbf{j}$, and $t = 5$.

c Find $\mathbf{r}_0$ if $\mathbf{r} = 4\mathbf{i} + 3\mathbf{j}$, $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$, and $t = 3$.

d Find $\mathbf{r}_0$ if $\mathbf{r} = -2\mathbf{i} + 5\mathbf{j}$, $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$, and $t = 6$.

e Find $\mathbf{v}$ if $\mathbf{r}_0 = 2\mathbf{i} + 2\mathbf{j}$, $\mathbf{r} = 8\mathbf{i} - 7\mathbf{j}$, and $t = 3$.

f Find t if $\mathbf{r}_0 = 4\mathbf{i} + \mathbf{j}$, $\mathbf{r} = 12\mathbf{i} - 11\mathbf{j}$, and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$.

2 A radio-controlled boat starts from position vector $(10\mathbf{i} - 5\mathbf{j})$ m relative to a fixed origin and travels with constant velocity, passing a point with position vector $(-2\mathbf{i} + 9\mathbf{j})$ m after 4 seconds. Find the speed and bearing of the boat.

3 A clockwork mouse starts from a point with position vector $(-2\mathbf{i} + 3\mathbf{j})$ m relative to a fixed origin and moves in a straight line with a constant speed of 4 m s^{-1}. Find the time taken for the mouse to travel to the point with position vector $(6\mathbf{i} - 3\mathbf{j})$ m.

4 A helicopter starts from the point with position vector $\begin{pmatrix}120\\-10\end{pmatrix}$ m relative to a fixed origin, and moves with constant velocity $\begin{pmatrix}-30\\40\end{pmatrix}$ m s^{-1}. Find:

a the position vector of the helicopter t seconds later

b the time at which the helicopter is due north of the origin.

Hint When the helicopter is due north of the origin, the **i**-component of its position vector will be 0.

5 At time $t = 0$, the particle P is at the point with position vector $4\mathbf{i}$, and moving with constant velocity $\mathbf{i} + \mathbf{j}$ m s^{-1}. A second particle Q is at the point with position vector $-3\mathbf{j}$ and moving with velocity $\mathbf{v}$ m s^{-1}. After 8 seconds, the paths of P and Q meet. Find the speed of Q.

Homework Exercise

6 At noon, a ferry F is 400 m due north of an observation point O and is moving with a constant velocity of $(7\mathbf{i} + 7\mathbf{j})\,\text{m s}^{-1}$, and a speedboat S is 500 m due east of O, moving with a constant velocity of $(-3\mathbf{i} + 15\mathbf{j})\,\text{m s}^{-1}$.

a Write down the position vectors of F and S at time t seconds after noon.

b Show that F and S will collide, and find the position vector of the point of collision.

7 A particle starts at rest and moves with constant acceleration. After 5 seconds its velocity is $\begin{pmatrix} 3 \\ 4 \end{pmatrix}\,\text{m s}^{-1}$.

a Find the acceleration of the particle.

b The displacement vector of the particle from its starting position after 5 seconds.

8 An object moves with constant acceleration so that its velocity changes from $(15\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ to $(5\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$ in 4 seconds. Find:

a the acceleration of the particle

Given that the initial position vector of the particle relative to a fixed origin O is $10\mathbf{i} - 8\mathbf{j}$ m,

b find the position vector of the particle after t seconds.

9 A plane moves with constant acceleration $\begin{pmatrix} -1 \\ 1.5 \end{pmatrix}\,\text{m s}^{-2}$.

When $t = 0$, the velocity of the plane is $\begin{pmatrix} 70 \\ -30 \end{pmatrix}\,\text{m s}^{-1}$. Find:

a the velocity of the plane after 10 seconds

b the distance of the plane from its starting point after 10 seconds.

10 A model boat moves with constant acceleration $(0.2\mathbf{i} + 0.6\mathbf{j})\,\text{m s}^{-2}$. After 20 seconds its velocity is $(4\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$. Find the displacement vector of the boat from its starting position after 20 seconds.

Homework Exercise

11 A particle A starts at the point with position vector $12\mathbf{i} + 12\mathbf{j}$. The initial velocity of A is $(-\mathbf{i} + \mathbf{j})\,\mathrm{m\,s^{-1}}$, and it has constant acceleration $(2\mathbf{i} - 4\mathbf{j})\,\mathrm{m\,s^{-2}}$. Another particle, B, has initial velocity $\mathbf{i}\,\mathrm{m\,s^{-1}}$ and constant acceleration $2\mathbf{j}\,\mathrm{m\,s^{-2}}$. After 3 seconds the two particles collide. Find:

a the speeds of the two particles when they collide

b the position vector of the point where the two particles collide

c the position vector of B's starting point.

12 A ship is moving such that at time 12:00 its position is O and its velocity is $(-4\mathbf{i} + 8\mathbf{j})\,\mathrm{km\,h^{-1}}$. At 14:00, the ship is travelling with velocity $(-2\mathbf{i} - 6\mathbf{j})\,\mathrm{km\,h^{-1}}$.

Relative to O, the ship has displacement $\mathbf{s}$ at time t hours after 12:00 where $t \geqslant 0$.

Modelling the ship as a particle with constant acceleration, find:

a the acceleration of the ship **(2 marks)**

b an expression for $\mathbf{s}$ in terms of t **(2 marks)**

c the time at which the ship is directly south-west of O. **(3 marks)**

At time t hours after 12:00, another ship has displacement $\mathbf{r} = (40 - 25t)\mathbf{j}$ relative to O.

d Find the position vector of the point where the two ships meet. **(4 marks)**

13 A particle moves so that its position vector, in metres, relative to a fixed origin O at time t seconds is $\mathbf{r} = (2t^2 - 3)\mathbf{i} + (7 - 4t)\mathbf{j}$, where $t \geqslant 0$.

a Show that the particle is north-east of O when $t^2 + 2t - 5 = 0$. **(2 marks)**

b Hence determine the distance of the particle from O when it is north-east of O, giving your answer correct to 3 significant figures. **(3 marks)**

A second particle moves with constant acceleration $(3a\mathbf{i} - 2a\mathbf{j})\,\mathrm{m\,s^{-2}}$. When $t = 0$ the velocity of the particle is $(5\mathbf{i} + 6\mathbf{j})\,\mathrm{m\,s^{-1}}$ and its position vector relative to O is $5\mathbf{j}\,\mathrm{m}$. When $t = 2$ seconds the particle is travelling with velocity $(b\mathbf{i} + 2b\mathbf{j})\,\mathrm{m\,s^{-1}}$.

c Find the speed and direction of the particle when $t = 2$. **(6 marks)**

d Find the distance between the two particles at this time. **(4 marks)**

Homework Exercise

Challenge

During an air show, a stunt aeroplane passes over a control tower with velocity $(20\mathbf{i} - 100\mathbf{j})$ m s^{-1}, and flies in a horizontal plane with constant acceleration $6\mathbf{j}$ m s^{-2}. A second aeroplane passes over the same control tower at time t seconds later, where $t > 0$, travelling with velocity $(70\mathbf{i} + 40\mathbf{j})$ m s^{-1}. The second aeroplane is flying in a higher horizontal plane with constant acceleration $-8\mathbf{j}$ m s^{-2}.

Given that the two aeroplanes pass directly over one another in their subsequent motion, find the value of t.

Homework Answers

1 **a** $6\mathbf{i} + 12\mathbf{j}$ **b** $-7\mathbf{i} + 4\mathbf{j}$ **c** $-2\mathbf{i} + 6\mathbf{j}$
d $10\mathbf{i} - 13\mathbf{j}$ **e** $2\mathbf{i} - 3\mathbf{j}$ **f** $4\,\text{s}$

2 $\frac{\sqrt{85}}{2}\,\text{m s}^{-1}$, $319°$

3 $2.5\,\text{s}$

4 **a** $\begin{pmatrix} 120 - 30t \\ -10 + 40t \end{pmatrix}$ **b** $4\,\text{s}$

5 $2.03\,\text{m s}^{-1}$

6 **a** $7t\mathbf{i} + (400 + 7t)\mathbf{j}$, $(500 - 3t)\mathbf{i} + 15t\mathbf{j}$
b $350\mathbf{i} + 750\mathbf{j}$

7 **a** $\begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}\text{m s}^{-2}$ **b** $\begin{pmatrix} \frac{15}{2} \\ 10 \end{pmatrix}\text{m}$

8 **a** $-\frac{5}{2}\mathbf{i} - \frac{7}{4}\mathbf{j}\,\text{m s}^{-2}$
b $\left(10 + 15t - \frac{5}{4}t^2\right)\mathbf{i} + \left(-8 + 4t - \frac{7}{8}t^2\right)\mathbf{j}\,\text{m}$

9 **a** $\begin{pmatrix} 60 \\ -15 \end{pmatrix}\text{m s}^{-1}$ **b** $688\,\text{m}$

10 $\begin{pmatrix} 40 \\ -60 \end{pmatrix}\text{m}$

11 **a** $12.1\,\text{m s}^{-1}$, $6.08\,\text{m s}^{-1}$ **b** $18\mathbf{i} - 3\mathbf{j}$
c $15\mathbf{i} - 12\mathbf{j}$

12 **a** $\mathbf{i} - 7\mathbf{j}\,\text{m s}^{-2}$ **b** $s = (-4t + 0.5t^2)\mathbf{i} + (8t - 3.5t^2)\mathbf{j}$
c 15:00 **d** $-160\mathbf{j}$

13 **a** North-east of O when **i** and **j** components are equal
$2t^2 - 3 = 7 - 4t \Rightarrow 2t^2 + 4t - 10 = 0 \Rightarrow t^2 + 2t - 5 = 0$
b $1.70\,\text{m}$ **c** $7.83\,\text{m s}^{-1}$, $026.6°$ **d** $19.3\,\text{m}$

Challenge
$24\,\text{s}$