Stats1 Chapter 6: Distributions

Chapter Practice

Key Points

1 A **probability distribution** fully describes the probability of any outcome in the sample space.

2 The sum of the probabilities of all outcomes of an event add up to 1. For a random variable X, you can write $\sum P(X = x) = 1$ for all x.

3 You can model X with a **binomial distribution**, **B(n, p)**, if:

- there are a fixed number of trials, n
- there are two possible outcomes (success or failure)
- there is a fixed probabillity of success, p
- the trials are independent of each other.

4 If a random variable X has the binomial distribution B(n, p) then its probability mass function is given by

$$P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$$

Chapter Exercises

1 The random variable X has probability function

$$P(X = x) = \frac{x}{21} \qquad x = 1, 2, 3, 4, 5, 6.$$

a Construct a table giving the probability distribution of X.

b Find $P(2 < X \leqslant 5)$.

2 The discrete random variable X has the probability distribution shown.

x	-2	-1	0	1	2	3
$\mathbf{P(X = x)}$	0.1	0.2	0.3	r	0.1	0.1

a Find r. **(1 mark)**

b Calculate $P(-1 \leqslant x < 2)$. **(2 marks)**

3 The random variable X has probability function

$$P(X = x) = \frac{(3x - 1)}{26} \qquad x = 1, 2, 3, 4.$$

a Construct a table giving the probability distribution of X. **(2 marks)**

b Find $P(2 < X \leqslant 4)$. **(2 marks)**

4 Sixteen counters are numbered 1 to 16 and placed in a bag. One counter is chosen at random and the number, X, recorded.

a Write down one condition on selecting a counter if X is to be modelled as a discrete uniform distribution. **(1 mark)**

b Find:

i $P(X = 5)$ **(1 mark)**

ii $P(X \text{ is prime})$ **(2 marks)**

iii $P(3 \leqslant X < 11)$ **(2 marks)**

Chapter Exercises

5 The random variable Y has probability function

$$P(Y = y) = \frac{y}{k} \qquad y = 1, 2, 3, 4, 5$$

a Find the value of k. **(2 marks)**

b Construct a table giving the probability distribution of Y. **(2 marks)**

c Find $P(Y > 3)$. **(1 mark)**

6 Stuart rolls a biased dice four times. P(six) = $\frac{1}{4}$. The random variable T represents the number of times he rolls a six.

a Construct a table giving the probability distribution of T. **(3 marks)**

b Find $P(T < 3)$. **(2 marks)**

He rolls the dice again, this time recording the number of rolls required to roll a six. He rolls the dice a maximum of five times. Let the random variable S stand for the number of times he rolls the dice.

c Construct a table giving the probability distribution of S. **(3 marks)**

d Find $P(S > 2)$. **(2 marks)**

7 The discrete random variable $X \sim B(30, 0.73)$. Find:

a $P(X = 20)$ **(1 mark)**

b $P(X \leqslant 13)$ **(1 mark)**

c $P(11 < X \leqslant 25)$ **(2 marks)**

8 A coin is biased so that the probability of a head is $\frac{2}{3}$.The coin is tossed repeatedly. Find the probability that:

a the first tail will occur on the sixth toss

b in the first 8 tosses there will be exactly 2 tails.

Chapter Exercises

9 Records kept in a hospital show that 3 out of every 10 patients who visit the accident and emergency department have to wait more than half an hour. Find, to 3 decimal places, the probability that of the first 12 patients who come to the accident and emergency department:

a none

b more than 2

will have to wait more than half an hour.

10 **a** State clearly the conditions under which it is appropriate to assume that a random variable has a binomial distribution. **(2 marks)**

A door-to-door canvasser tries to persuade people to have a certain type of double glazing installed. The probability that his canvassing at a house is successful is 0.05.

b Find the probability that he will have at least 2 successes out of the first 10 houses he canvasses. **(2 marks)**

c Calculate the smallest number of houses he must canvass so that the probability of his getting at least one success exceeds 0.99. **(4 marks)**

11 A completely unprepared student is given a true/false-type test with 10 questions. Assuming that the student answers all the questions at random:

a find the probability that the student gets all the answers correct.

It is decided that a pass will be awarded for 8 or more correct answers.

b Find the probability that the student passes the test.

12 A six-sided die is biased. When the die is thrown the number 5 is twice as likely to appear as any other number. All the other faces are equally likely to appear. The die is thrown repeatedly. Find the probability that:

a the first 5 will occur on the sixth throw

b in the first eight throws there will be exactly three 5s.

13 A manufacturer produces large quantities of plastic chairs. It is known from previous records that 15% of these chairs are green. A random sample of 10 chairs is taken.

a Define a suitable distribution to model the number of green chairs in this sample. **(1 mark)**

b Find the probability of at least 5 green chairs in this sample. **(3 marks)**

c Find the probability of exactly 2 green chairs in this sample. **(3 marks)**

14 A bag contains a large number of beads of which 45% are yellow. A random sample of 20 beads is taken from the bag. Use the binomial distribution to find the probability that the sample contains:

a fewer than 12 yellow beads **(2 marks)**

b exactly 12 yellow beads. **(3 marks)**

15 An archer hits the bullseye with probability 0.6. She shoots 20 arrows at a time.

a Find the probability that she hits the bullseye with at least 50% of her arrows. **(3 marks)**

She shoots 12 sets of 20 arrows.

b Find the probability that she hits the bullseye with at least 50% of her arrows in 7 of the 12 sets of arrows. **(2 marks)**

c Find the probability that she hits the bullseye with at least 50% of her arrows in fewer than 6 sets of arrows. **(2 marks)**

Challenge

A driving theory test has 50 questions. Each question has four answers, of which only one is correct.

Annabelle is certain she got 32 answers correct, but she guessed the remaining answers. She needs to get 43 correct answers to pass the test.

Find the probability that Annabelle passed the test.

Chapter Answers

1 a

x	$P(X = x)$
1	$\frac{1}{21}$
2	$\frac{2}{21}$
3	$\frac{3}{21}$
4	$\frac{4}{21}$
5	$\frac{5}{21}$
6	$\frac{6}{21}$

b $\frac{12}{21}$

2 a 0.2 **b** 0.7

3 a

x	1	2	3	4
$P(X = x)$	0.0769	0.1923	0.3077	0.4231

b $\frac{19}{26}$

4 a The probabilities must be the same.
b i 0.0625 **ii** 0.375 **iii** 0.5

5 a 15
b

y	1	2	3	4	5
$P(X = y)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

c $\frac{9}{15}$ or $\frac{3}{5}$

6 a

t	0	1	2	3	4
$P(T = t)$	0.316	0.422	0.211	0.0469	0.00391

b 0.949
c

s	1	2	3	4	5
$P(S = s)$	0.25	0.188	0.141	0.105	0.316

d 0.562

7 a 0.114 **b** 0.0005799
c 0.9373

8 a 0.0439 or $\frac{32}{729}$ **b** 0.273

9 a 0.014 (3 d.p.) **b** 0.747 (3 d.p.)

10 a 1 There are n independent trials.
2 n is a fixed number.
3 The outcome of each trial is success or failure.
4 The probability of success at each trial is constant.
5 The outcome of any trial is independant of any other trial.

11 a 0.000977 **b** 0.0547

12 a 0.0531 **b** 0.243

13 a $X \sim B(10, 0.15)$
b 0.0099 **c** 0.2759

14 a 0.8692 **b** 0.0727

15 a 0.8725 **b** 0.01027 **c** 0.0002407

Challenge
0.001244