Stats1 Chapter 5: Probability

Mutual Exclusion and Independence

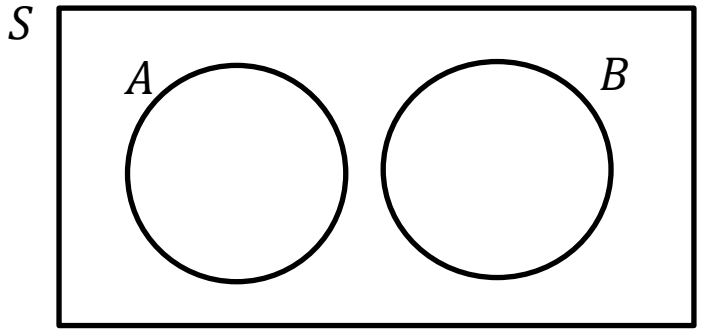
Mutually Exclusive Events

- If two events are mutually exclusive ?
- If A and B are mutually exclusive then:
  - $\boldsymbol{P(A \text{ and } B) =}$?
  - $\boldsymbol{P(A \text{ or } B) =}$?
- The Venn Diagram would look like:

?

Mutually Exclusive Events

- If two events are mutually exclusive **they can't happen at the same time**.
- If A and B are mutually exclusive then:
  - $\boldsymbol{P(A \text{ and } B) = 0}$
  - $\boldsymbol{P(A \text{ or } B) = P(A) + P(B)}$
- The Venn Diagram would look like:



Since $P(A \text{ and } B) = 0$, there can't be any outcomes in the overlap, so we don't have an overlap!

Independent Events

- If two events are independent
  ?
- If A and B are independent then:
  - $\boldsymbol{P(A \text{ and } B) =}$?

Note: Independence does not affect how the circles interact in a Venn Diagram.

Example

1 2 3 4

1 I pick one of the four numbers 1, 2, 3, 4 at random. What's the probability that:

a) I pick a multiple of 2: ?

b) I pick a multiple of 4: ?

2 Explain (conceptually) why these two events are not independent.

?

3 Show that the events are not independent.

?

Independent Events

- If two events are independent
  then whether one event happens does not affect the probability of the other happening.
- If A and B are independent then:
  - $\boldsymbol{P(A \text{ and } B) = P(A) \times P(B)}$

Note: Independence does not affect how the circles interact in a Venn Diagram.

Example

1 2 3 4

1 I pick one of the four numbers 1, 2, 3, 4 at random. What's the probability that:

a) I pick a multiple of 2: $\frac{1}{2}$

b) I pick a multiple of 4: $\frac{1}{4}$

2 Explain (conceptually) why these two events are not independent.
If it is a multiple of 4, then it must also be a multiple of 2. But if it wasn't a multiple of 4 then it may or may not be a multiple of 2. So the events are linked and whether one happened influences the probability of the other happening.

3 Show that the events are not independent.

$$P(multiple\ of\ 2) \times P(multiple\ of\ 4) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

$$P(multiple\ of\ 2\ \boldsymbol{and}\ multiple\ of\ 4) = \frac{1}{4}$$

But $\frac{1}{8} \neq \frac{1}{4}$ therefore not independent.

This is a common exam question. Either show that $P(A \text{ and } B) = P(A) \times P(B)$ or that $P(A \text{ and } B) \neq P(A) \times P(B)$

Further Examples

[Textbook] Events A and B are mutually exclusive and $P(A) = 0.2$ and $P(B) = 0.4$.

a) Find $P(A \text{ or } B)$

b) Find $P(A \text{ but not } B)$

c) Find $P(\text{neither } A \text{ nor } B)$

?

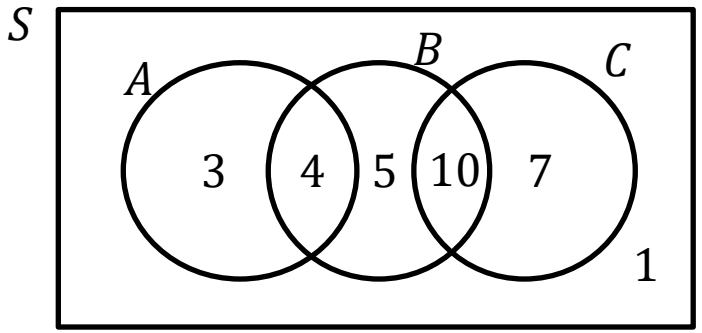
[Textbook] Events A and B are independent and $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$. Find $P(A \text{ and } B)$.

?

[Textbook] The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes.

a) Find the probability that a student chosen at random watches B or C or both.

b) Determine whether watching A and watching B are statistically independent.



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Further Examples

[Textbook] Events A and B are mutually exclusive and $P(A) = 0.2$ and $P(B) = 0.4$.

a) Find $P(A \text{ or } B)$

b) Find $P(A \text{ but not } B)$

c) Find $P(\text{neither } A \text{ nor } B)$

$P(A \text{ or } B) = 0.2 + 0.4 = 0.6$

$P(A \text{ but not } B) = P(A) = 0.2$

$P(\text{neither } A \text{ nor } B) = 1 - 0.2 - 0.4 = 0.4$

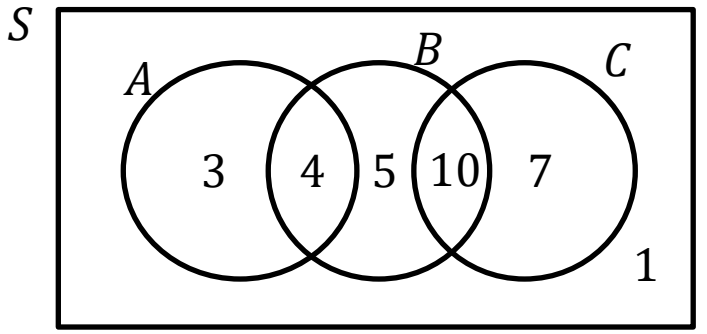
[Textbook] Events A and B are independent and $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$. Find $P(A \text{ and } B)$.

$$P(A \text{ and } B) = \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$$

[Textbook] The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes.

a) Find the probability that a student chosen at random watches B or C or both.

b) Determine whether watching A and watching B are statistically independent.



$$P(\text{watches } B \text{ or } C) = \frac{26}{30}$$

$$P(A) = \frac{7}{30} \quad P(B) = \frac{19}{30}$$

$$P(A \text{ and } B) = \frac{4}{30}$$

$$P(A) \times P(B) = \frac{7}{30} \times \frac{19}{30} = \frac{133}{900} \neq \frac{4}{30}$$

$\therefore$ not independent.

Test Your Understanding

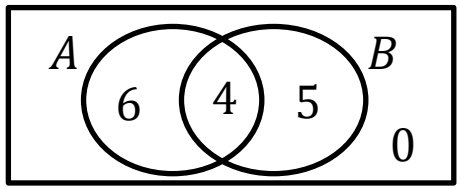
There are three events A, B, C. The events A and B are mutually exclusive.

a) Draw a Venn diagram which represents this information.

b) If $P(A) = 0.1$ and $P(B) = 0.6$, determine $P(neither\ A\ nor\ B)$

?

The Venn diagram shows the number of people who like each of two different colours. Determine if A and B are independent.



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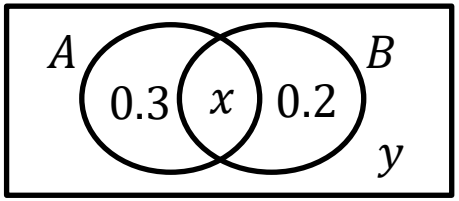
The Venn diagram shows the probability of each event. Given that A and B are independent, determine the possible values of x.



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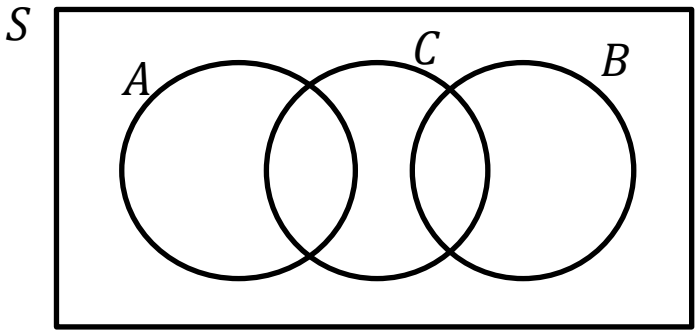
Test Your Understanding

There are three events A, B, C. The events A and B are mutually exclusive.

a) Draw a Venn diagram which represents this information.

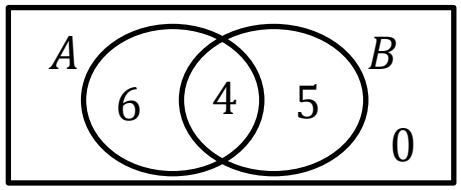
b) If $P(A) = 0.1$ and $P(B) = 0.6$, determine $P(neither\ A\ nor\ B)$



Note that C may occur with A or with B, as we're not told otherwise.

$$\boldsymbol{P(neither\ A\ nor\ B)} = \mathbf{1 - 0.1 - 0.6 = 0.3}$$

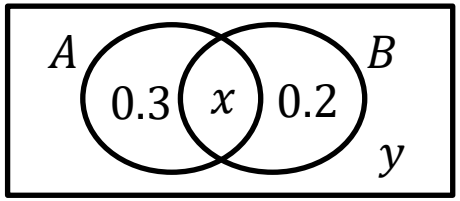
The Venn diagram shows the number of people who like each of two different colours. Determine if A and B are independent.



$$\boldsymbol{P(A) = \frac{10}{15} = \frac{2}{3}} \quad \boldsymbol{P(B) = \frac{9}{15} = \frac{3}{5}, P(A\ and\ B) = \frac{4}{15}}$$

$$\boldsymbol{P(A) \times P(B) = \frac{2}{3} \times \frac{3}{5} = \frac{6}{15} \neq \frac{4}{15}} \quad \boldsymbol{\therefore}\ \textbf{not independent.}$$

The Venn diagram shows the probability of each event. Given that A and B are independent, determine the possible values of x.



$$\boldsymbol{P(A\ and\ B) = P(A) \times P(B)}$$

$$\boldsymbol{x = (0.3 + x)(0.2 + x)}$$

Solving the quadratic, $x = 0.2$ or $x = 0.3$

Exercise 5.3

Pearson Applied Year 1/AS
Pages 34-35

Homework Exercise

1 Events A and B are mutually exclusive. $P(A) = 0.2$ and $P(B) = 0.5$.

a Draw a Venn diagram to represent these two events.

b Find $P(A \text{ or } B)$.

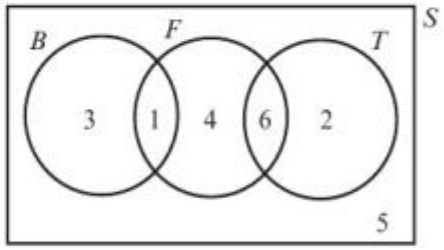
c Find P(neither A nor B).

2 Two fair dice are rolled and the result on each die is recorded. Show that the events 'the sum of the scores on the dice is 4' and 'both dice land on the same number' are not mutually exclusive.

3 $P(A) = 0.5$ and $P(B) = 0.3$. Given that events A and B are independent, find $P(A \text{ and } B)$.

4 $P(A) = 0.15$ and $P(A \text{ and } B) = 0.045$. Given that events A and B are independent, find $P(B)$.

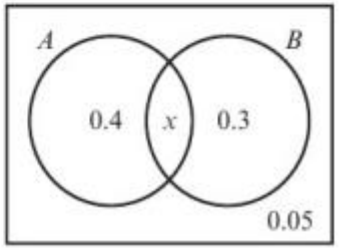
5 The Venn diagram shows the number of children in a play group that like playing with bricks (B), action figures (F) or trains (T).



a State, with a reason, which two types of toy are mutually exclusive.

b Determine whether the events 'plays with bricks' and 'plays with action figures' are independent.

6 The Venn diagram shows the probabilities that a group of students like pasta (A) or pizza (B).



a Write down the value of x. **(1 mark)**

b Determine whether the events 'like pasta' and 'like pizza' are independent. **(3 marks)**

Homework Exercise

7 S and T are two events such that $P(S) = 0.3$, $P(T) = 0.4$ and $P(S \text{ but not } T) = 0.18$.

a Show that S and T are independent.

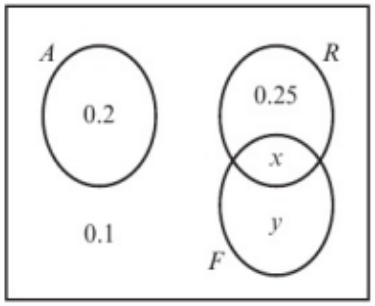
b Find:

i $P(S \text{ and } T)$ **ii** $P(\text{neither } S \text{ nor } T)$.

8 W and X are two events such that $P(W) = 0.5$, $P(W \text{ and not } X) = 0.25$ and $P(\text{neither } W \text{ nor } X) = 0.3$. State, with a reason, whether W and X are independent events.

(3 marks)

9 The Venn diagram shows the probabilities of members of a social club taking part in charitable activities.



A represents taking part in an archery competition.

R represents taking part in a raffle.

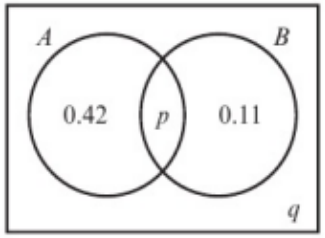
F represents taking part in a fun run.

The probability that a member takes part in the archery competition or the raffle is 0.6.

a Find the value of x and the value of y. **(2 marks)**

b Show that events R and F are not independent. **(3 marks)**

10 Given that A and B are independent, find the two possible values for p and q.



Challenge

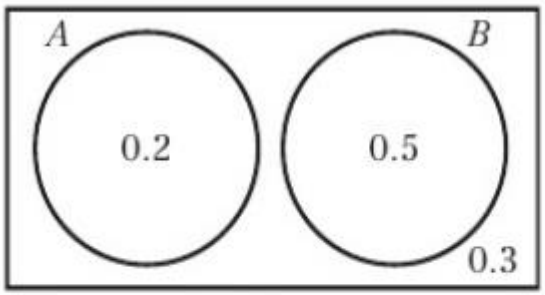
A and B are independent events in a sample space S. Given that A and B are independent, prove that:

a A and 'not B' are independent

b 'not A' and 'not B' are independent.

Homework Answers

1 a



b 0.7 **c** 0.3

2 P(sum of 4) + P(same number) ≠ P(sum of 4 or same number), so the events are not mutually exclusive.

3 0.15

4 0.3

5 a Bricks and trains; their curves do not overlap.

b Not independent.

6 a 0.25 **b** Not independent

7 a $P(S \text{ and } T) = 0.3 - 0.18 = 0.12$

$P(S) \times P(T) = 0.3 \times 0.4 = 0.12 = P(S \text{ and } T)$

So S and T are independent.

b i 0.12 **ii** 0.42

8 $P(W) \times P(X) = 0.5 \times 0.45 = 0.225$

$P(W \text{ and } X) = 0.25$, so W and X are **not** independent.

9 a $x = 0.15$, $y = 0.3$

b $P(F \text{ and } R) = 0.15 \neq P(F) \times P(R) = 0.45 \times 0.4 = 0.18$

10 $p = 0.14$ and $q = 0.33$ or $p = 0.33$ and $q = 0.14$

Homework Answers

Challenge

a Set $P(A) = p$ and $P(B) = q$, then $P(A \text{ and } B) = pq$

$P(A \text{ and not } B) = P(A) - P(A \text{ and } B) = p - pq$

$P(\text{not } B) = 1 - q$

$\Rightarrow P(A) \times P(\text{not } B) = p(1 - q) = p - pq = P(A \text{ and not } B)$

b $P(\text{not } A \text{ and not } B) = 1 - P(A \text{ or } B)$

$= 1 - P(A) - P(B) + P(A \text{ and } B)$

$= 1 - p - q + pq = (1 - p)(1 - q)$

But $P(\text{not } A) = 1 - p$ and $P(\text{not } B) = 1 - q$, so

$P(\text{not } A \text{ and not } B) = P(\text{not } A) \times P(\text{not } B)$