
P2 Chapter 1: Algebra Techniques

Algebraic Fractions

Multiplying/Dividing Algebraic Fractions

As you saw at GCSE level, multiplying algebraic fractions is no different to multiplying numeric fractions.

You may however need to **cancel common factors, by factorising where possible.**

$$\frac{a}{b} \times \frac{c}{a} = \frac{ac}{ab} = \frac{c}{b}$$

$$\frac{x+1}{2} \times \frac{3}{x^2 - 1} = \frac{3(x+1)}{2(x+1)(x-1)} = \frac{3}{2(x-1)}$$

To divide by a fraction, **multiply by the reciprocal of the second fraction.**

$$\frac{p}{q} \div \frac{r}{q} = \frac{p}{q} \times \frac{q}{r} = \frac{p}{r}$$

$$\frac{x+2}{x+4} \div \frac{3x+6}{x^2 - 16} = \frac{x+2}{x+4} \times \frac{(x+4)(x-4)}{3(x+2)} = \frac{x-4}{3}$$

Test Your Understanding

$$\frac{x+3}{5} \times \frac{10}{x^2 - 9}$$

$$= ?$$

$$\frac{x^2 + x}{y} \div \frac{x^2 - x - 2}{y^2}$$

$$= ?$$

“Crimes against Mathematics”:

$$\frac{x^2 + y}{2y} = \frac{x^2}{2}$$

$$? = ?$$

Test Your Understanding

$$\frac{x+3}{5} \times \frac{10}{x^2 - 9} = \frac{x+3}{5} \times \frac{10}{(x+3)(x-3)} = \frac{2}{x-3}$$

$$\frac{x^2 + x}{y} \div \frac{x^2 - x - 2}{y^2} = \frac{x(x+1)}{y} \times \frac{y^2}{(x+1)(x-2)} = \frac{xy}{x-2}$$

“Crimes against Mathematics”:

$$\frac{x^2 + y}{2y} = \frac{x^2}{2}$$

Cancelling in fractions only occurs by dividing. But the student has subtracted y in the numerator.

Exercise 1.2

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Homework Exercise

1 Simplify:

a $\frac{a}{d} \times \frac{a}{c}$

b $\frac{a^2}{c} \times \frac{c}{a}$

c $\frac{2}{x} \times \frac{x}{4}$

d $\frac{3}{x} \div \frac{6}{x}$

e $\frac{4}{xy} \div \frac{x}{y}$

f $\frac{2r^2}{5} \div \frac{4}{r^3}$

2 Simplify:

a $(x+2) \times \frac{1}{x^2 - 4}$

b $\frac{1}{a^2 + 6a + 9} \times \frac{a^2 - 9}{2}$

c $\frac{x^2 - 3x}{y^2 + y} \times \frac{y + 1}{x}$

d $\frac{y}{y+3} \div \frac{y^2}{y^2 + 4y + 3}$

e $\frac{x^2}{3} \div \frac{2x^3 - 6x^2}{x^2 - 3x}$

f $\frac{4x^2 - 25}{4x - 10} \div \frac{2x + 5}{8}$

g $\frac{x+3}{x^2 + 10x + 25} \times \frac{x^2 + 5x}{x^2 + 3x}$

h $\frac{3y^2 + 4y - 4}{10} \div \frac{3y + 6}{15}$

i $\frac{x^2 + 2xy + y^2}{2} \times \frac{4}{(x-y)^2}$

3 Show that $\frac{x^2 - 64}{x^2 - 36} \div \frac{64 - x^2}{x^2 - 36} = -1$ (4 marks)

4 Show that $\frac{2x^2 - 11x - 40}{x^2 - 4x - 32} \times \frac{x^2 + 8x + 16}{6x^2 - 3x - 45} \div \frac{8x^2 + 20x - 48}{10x^2 - 45x + 45} = \frac{a}{b}$ and find the values of the constants a and b , where a and b are integers. (4 marks)

Homework Exercise

5 a Simplify fully $\frac{x^2 + 2x - 24}{2x^2 + 10x} \times \frac{x^2 - 3x}{x^2 + 3x - 18}$ (3 marks)

Hint Simplify and then solve the logarithmic equation.

← Year 1, Section 14.6

b Given that

$$\ln((x^2 + 2x - 24)(x^2 - 3x)) = 2 + \ln((2x^2 + 10x)(x^2 + 3x - 18)) \text{ find } x \text{ in terms of e. (4 marks)}$$

6 $f(x) = \frac{2x^2 - 3x - 2}{6x - 8} \div \frac{x - 2}{3x^2 + 14x - 24}$

a Show that $f(x) = \frac{2x^2 + 13x + 6}{2}$ (4 marks)

Hint Differentiate each term separately.

← Year 1, Section 12.5

b Hence differentiate $f(x)$ and find $f'(4)$. (3 marks)

Homework Exercise

1 Write as a single fraction:

a $\frac{1}{3} + \frac{1}{4}$

b $\frac{3}{4} - \frac{2}{5}$

c $\frac{1}{p} + \frac{1}{q}$

d $\frac{3}{4x} + \frac{1}{8x}$

e $\frac{3}{x^2} - \frac{1}{x}$

f $\frac{a}{5b} - \frac{3}{2b}$

2 Write as a single fraction:

a $\frac{3}{x} - \frac{2}{x+1}$

b $\frac{2}{x-1} - \frac{3}{x+2}$

c $\frac{4}{2x+1} + \frac{2}{x-1}$

d $\frac{1}{3}(x+2) - \frac{1}{2}(x+3)$

e $\frac{3x}{(x+4)^2} - \frac{1}{x+4}$

f $\frac{5}{2(x+3)} + \frac{4}{3(x-1)}$

3 Write as a single fraction:

a $\frac{2}{x^2+2x+1} + \frac{1}{x+1}$

b $\frac{7}{x^2-4} + \frac{3}{x+2}$

c $\frac{2}{x^2+6x+9} - \frac{3}{x^2+4x+3}$

d $\frac{2}{y^2-x^2} + \frac{3}{y-x}$

e $\frac{3}{x^2+3x+2} - \frac{1}{x^2+4x+4}$

f $\frac{x+2}{x^2-x-12} - \frac{x+1}{x^2+5x+6}$

4 Express $\frac{6x+1}{x^2+2x-15} - \frac{4}{x-3}$ as a single fraction in its simplest form. (4 marks)

5 Express each of the following as a fraction in its simplest form.

a $\frac{3}{x} + \frac{2}{x+1} + \frac{1}{x+2}$

b $\frac{4}{3x} - \frac{2}{x-2} + \frac{1}{2x+1}$

c $\frac{3}{x-1} + \frac{2}{x+1} + \frac{4}{x-3}$

6 Express $\frac{4(2x-1)}{36x^2-1} + \frac{7}{6x-1}$ as a single fraction in its simplest form. (4 marks)

7 $g(x) = x + \frac{6}{x+2} + \frac{36}{x^2-2x-8}$, $x \in \mathbb{R}, x \neq -2, x \neq 4$

a Show that $g(x) = \frac{x^3 - 2x^2 - 2x + 12}{(x+2)(x-4)}$ (4 marks)

b Using algebraic long division, or otherwise, further show that $g(x) = \frac{x^2 - 4x + 6}{x-4}$ (4 marks)

Homework Answers

1 a $\frac{a^2}{cd}$ b a c $\frac{1}{2}$ d $\frac{1}{2}$ e $\frac{4}{x^2}$ f $\frac{r^5}{10}$

2 a $\frac{1}{x-2}$ b $\frac{a-3}{2(a+3)}$ c $\frac{x-3}{y}$ d $\frac{y+1}{y}$
e $\frac{x}{6}$ f 4 g $\frac{1}{x+5}$ h $\frac{3y-2}{2}$ i $\frac{2(x+y)^2}{(x-y)^2}$

3 All factors cancel exactly except $\frac{x-8}{8-x} = \frac{x-8}{-(x-8)} = -1$

4 $a = 5, b = 12$

5 a $\frac{x-4}{2x+10}$ b $x = \frac{10e^2 + 4}{1 - 2e^2}$

6 a
$$\begin{aligned} & \frac{2x^2 - 3x - 2}{6x - 8} \div \frac{x - 2}{3x^2 + 14x - 24} = \frac{2x^2 - 3x - 2}{6x - 8} \\ & \times \frac{3x^2 + 14x - 24}{x - 2} = \frac{(2x + 1)(x - 2)}{2(3x - 4)} \times \frac{(3x - 4)(x + 6)}{x - 2} \\ & = \frac{(2x + 1)(x + 6)}{2} = \frac{2x^2 + 13x + 6}{2} \end{aligned}$$

b $f(x) = 2x + \frac{13}{2}; f(4) = \frac{29}{2}$

Homework Answers

1 a $\frac{7}{12}$ b $\frac{7}{20}$ c $\frac{p+q}{pq}$ d $\frac{7}{8x}$ e $\frac{3-x}{x^2}$ f $\frac{2a-15}{10b}$

2 a $\frac{x+3}{x(x+1)}$ b $\frac{-x+7}{(x-1)(x+2)}$ c $\frac{8x-2}{(2x+1)(x-1)}$
d $\frac{-x-5}{6}$ e $\frac{2x-4}{(x+4)^2}$ f $\frac{23x+9}{6(x+3)(x-1)}$

3 a $\frac{x+3}{(x+1)^2}$ b $\frac{3x+1}{(x-2)(x+2)}$ c $\frac{-x-7}{(x+1)(x+3)^2}$
d $\frac{3x+3y+2}{(y-x)(y+x)}$ e $\frac{2x+5}{(x+2)^2(x+1)}$ f $\frac{7x+8}{(x+2)(x+3)(x-4)}$

4 $\frac{2x-19}{(x+5)(x-3)}$

5 a $\frac{6x^2+14x+6}{x(x+1)(x+2)}$ b $\frac{-x^2-24x-8}{3x(x-2)(2x+1)}$
c $\frac{9x^2-14x-7}{(x-1)(x+1)(x-3)}$

6 $\frac{50x+3}{(6x+1)(6x-1)}$

7 a $x + \frac{6}{x+2} + \frac{36}{x^2-2x-8}$
 $= \frac{x(x+2)(x-4)}{(x+2)(x-4)} + \frac{6(x-4)}{(x+2)(x-4)} + \frac{36}{(x+2)(x-4)}$
 $= \frac{x^3-2x^2-2x+12}{(x+2)(x-4)}$

b Divide $x^3 - 2x^2 - 2x + 12$ by $(x+2)$ to give $x^2 - 4x + 6$