P1 Chapter 8: Binomial Expansion

Estimation

Estimating Powers

Edexcel C2 Jan 2012 Q3

(*a*) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8,$$

giving each term in its simplest form.

(4)

(*b*) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

Tip: Use your calculator to compare against the exact value of 1.025^8.

a

?

b

?

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a

$$\left(1+\frac{x}{4}\right)^8 = \binom{8}{0}(1^8) + \binom{8}{1}(1^7)\left(\frac{x}{4}\right) + \binom{8}{2}(1^6)\left(\frac{x}{4}\right)^2 + \binom{8}{3}(1^5)\left(\frac{x}{4}\right)^3$$

$$= 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3 + \cdots$$

b

Comparing 1.025^8 to $\left(1+\frac{x}{4}\right)^8$, then:

$$1.025 = 1 + \frac{x}{4}$$

$$0.025 = \frac{x}{4}$$

$$x = 0.1$$

Using our expansion with $x = 0.1$:

$$1 + 2(0.1) + \frac{7}{4}(0.1)^2 + \frac{7}{8}(0.1)^3$$

$$= 1.2184 \; to \; 4dp$$

Why should this be a reasonably good approximation of 1.025^8 despite the missing terms in the expansion?

x^r becomes increasingly small when $x < 1$ as the power increases. Thus the 0.1^4 terms and beyond will be negligibly small.

Test Your Understanding

Edexcel C2 Jan 2008 Q3

(*a*) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(*b*) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

(a) ?

(b) ?

Test Your Understanding

Edexcel C2 Jan 2008 Q3

(*a*) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(*b*) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

(a)	$\left(1+\frac{1}{2}x\right)^{10} = 1 + \binom{10}{1}\left(\frac{1}{2}x\right)+\binom{10}{2}\left(\frac{1}{2}x\right)^2+\binom{10}{3}\left(\frac{1}{2}x\right)^3$	M1 A1
	$= 1 + 5x ; + \frac{45}{4}$ (or 11.25)x^2 + $15x^3$ (coeffs need to be these, i.e, simplified)	A1; A1 (4)
	[Allow A1A0, if totally correct with unsimplified, single fraction coefficients)	
(b)	$(1 + \frac{1}{2}\times 0.01)^{10} = 1 + 5(0.01) + (\frac{45}{4}\ or 11.25)(0.01)^2 + 15(0.01)^3$	M1 A1√
	$= 1 + 0.05 + 0.001125 + 0.000015$	
	$= 1.05114$ cao	A1 (3) **[7]**

Exercise 8.5

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Homework Exercise

1 **a** Find the first four terms of the binomial expansion, in ascending powers of x, of $\left(1-\frac{x}{10}\right)^6$.

b By substituting an appropriate value for x, find an approximate value for 0.99^6.

2 **a** Write down the first four terms of the binomial expansion of $\left(2+\frac{x}{5}\right)^{10}$.

b By substituting an appropriate value for x, find an approximate value for 2.1^{10}.

3 If x is so small that terms of x^3 and higher can be ignored, show that:

$$(2+x)(1-3x)^5 \approx 2-29x+165x^2$$

Hint Start by using the binomial expansion to expand $(1-3x)^5$. You can ignore terms of x^3 and higher so you only need to expand up to and including the x^2 term.

4 If x is so small that terms of x^3 and higher can be ignored, and

$$(2-x)(3+x)^4 \approx a+bx+cx^2$$

find the values of the constants a, b and c.

Problem-solving

Find the first 3 terms in the expansion of $(2-x)(3+x)^4$, compare with $a+bx+cx^2$ and write down the values of a, b and c.

5 **a** Write down the first four terms in the expansion of $(1+2x)^8$.

b By substituting an appropriate value of x (which should be stated), find an approximate value of 1.02^8.

6 $f(x)=(1-5x)^{30}$

a Find the first four terms, in ascending powers of x, in the binomial expansion of $f(x)$.

b Use your answer to part **a** to estimate the value of $(0.995)^{30}$, giving your answer to 6 decimal places.

c Use your calculator to evaluate 0.995^{30} and calculate the percentage error in your answer to part **b**.

Homework Exercise

7 a Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(3-\frac{x}{5}\right)^{10}$, giving each term in its simplest form. **(4 marks)**

b Explain how you would use your expansion to give an estimate for the value of 2.98^{10}. **(1 mark)**

8 a Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1-3x)^5$. Give each term in its simplest form. **(4 marks)**

b If x is small, so that x^2 and higher powers can be ignored, show that $(1+x)(1-3x)^5 \approx 1-14x$. **(2 marks)**

9 A microchip company models the probability of having no faulty chips on a single production run as:

$$\text{P(no fault)} = (1-p)^n,\ p < 0.001$$

where p is the probability of a single chip being faulty, and n being the total number of chips produced.

a State why the model is restricted to small values of p. **(1 mark)**

b Given that $n = 200$, find an approximate expression for P(no fault) in the form $a + bp + cp^2$. **(2 marks)**

c The company wants to achieve a 92% likelihood of having no faulty chips on a production run of 200 chips. Use your answer to part **b** to suggest a maximum value of p for this to be the case. **(4 marks)**

Homework Answers

1 **a** $1 - 0.6x + 0.15x^2 - 0.02x^3$

b 0.941 48

2 **a** $1024 + 1024x + 460.8x^2 + 122.88x^3$

b 1666.56

3 $(1 - 3x)^5 = 1^5 + \binom{5}{1}1^4(-3x)^1 + \binom{5}{2}1^3(-3x)^2 = 1 - 15x + 90x^2$

$(2 + x)(1 - 3x)^5 = (2 + x)(1 - 15x + 90x^2)$

$= 2 - 30x + 180x^2 + x - 15x^2 + 90x^3 \approx 2 - 29x + 165x^2$

4 $a = 162$, $b = 135$, $c = 0$

5 **a** $1 + 16x + 112x^2 + 448x^3$

b $x = 0.01$, $1.02^8 \approx 1.171\,648$

6 **a** $1 - 150x + 10875x^2 - 507500x^3$

b 0.860 368

c 0.860 384, 0.0019%

7 **a** $59\,049 - 39\,366x + 11\,809.8x^2$

b Substitute $x = 0.1$ into the expansion.

8 **a** $1 - 15x + 90x^2 - 270x^3$

b $(1 + x)(1 - 3x)^5 \approx (1 + x)(1 - 15x) \approx 1 - 14x$

9 **a** So that higher powers of p can be ignored as they tend to 0

b $1 + 200p - 19\,900p^2$

c $p = 0.000417$ (3 s.f.)