Stats Yr2 Chapter 2: Probability Theory

Probability Formulae

Full Laws of Probability

If events A and B are <u>independent</u>.

$$\mathrm{P}(A \cap B) = P(A) \times P(B)$$
$$P(A|B) = P(A)$$

If events A and B are <u>mutually exclusive</u>:

$$P(A \cap B) = 0$$
$$P(A \cup B) = P(A) + P(B)$$

In general:

We first encountered this in the previous section.

$$\mathrm{P}(A|B) = \frac{P(A \cap B)}{P(B)}$$

This is known as the **<u>Addition Law</u>.** **Informal Proof**: If we added the probabilities in the A and B sets in the Venn Diagram, we'd be double counting the intersection, so subtract so that it's only counted once.

$$\mathrm{P}(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example

Edexcel S1

6. Explain what you understand by

(a) a sample space, **(1)**

(b) an event. **(1)**

Two events A and B are independent,

such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$

Find

(c) $P(A \cap B)$, **(1)**

(d) $P(A \mid B)$, **(2)**

(e) $P(A \cup B)$. **(2)**

a)	?
b)	?
c)	?
d)	?
e)	?

Example

Edexcel S1

6. Explain what you understand by
(a) a sample space, **(1)**
(b) an event. **(1)**

Two events A and B are independent,

such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$

Find
(c) $P(A \cap B)$, **(1)**
(d) $P(A \mid B)$, **(2)**
(e) $P(A \cup B)$. **(2)**

a) The set of **<u>all</u>** outcomes.
b) A set of one or more outcomes (that is a subset of the sample space).
c) $P(A \cap B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$
d) $P(A|B) = P(A) = \frac{1}{3}$
e) $P(A \cup B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{1}{2}$

Further Examples

[Textbook] C and D are two events such that $P(C) = 0.2$, $P(D) = 0.6$ and $P(C|D) = 0.3$. Find:
a. $P(C \cap D)$ b. $P(D|C)$ c. $P(C \cup D)$

? a

? b

? c

Hints for (b): You saw the words "are independent". So write out $P(A \cap B) = P(A)P(B)$.
Also, you're given $P(A \cup B)$ which suggests you might be able to use the Addition Rule.

10. [Jan 2012 Q2] (a) State in words the relationship between two events R and S when $P(R \cap S) = 0$. **(1)**

The events A and B are independent with $P(A) = \frac{1}{4}$ and $P(A \cup B) = \frac{2}{3}$. Find

(*b*) $P(B)$, **(4)**
(*c*) $P(A' \cap B)$, **(2)**
(*d*) $P(B' | A)$. **(2)**

(a) ?

(b) ?

(c) ?

(d) ?

Further Examples

[Textbook] C and D are two events such that $P(C) = 0.2$, $P(D) = 0.6$ and $P(C|D) = 0.3$. Find:
a. $P(C \cap D)$ b. $P(D|C)$ c. $P(C \cup D)$

$$P(C|D) = \frac{P(C \cap D)}{P(D)} \rightarrow 0.3 = \frac{P(C \cap D)}{0.6}$$
$$P(C \cap D) = 0.18$$

$$P(D|C) = \frac{P(C \cap D)}{P(C)} = \frac{0.18}{0.2} = 0.9$$

$$P(C \cup D) = P(C) + P(D) - P(C \cap D)$$
$$= 0.2 + 0.6 - 0.18 = 0.62$$

10. [Jan 2012 Q2] (a) State in words the relationship between two events R and S when $P(R \cap S) = 0$. **(1)**

The events A and B are independent with $P(A) = \frac{1}{4}$ and $P(A \cup B) = \frac{2}{3}$. Find

(*b*) $P(B)$, **(4)**
(*c*) $P(A' \cap B)$, **(2)**
(*d*) $P(B'|A)$. **(2)**

Hints for (b): You saw the words "are independent". So write out $P(A \cap B) = P(A)P(B)$.
Also, you're given $P(A \cup B)$ which suggests you might be able to use the Addition Rule.

(a)	(*R* and *S* are mutually) exclusive.	B1
(b)	$\frac{2}{3} = \frac{1}{4} + P(B) - P(A \cap B)$ use of Addition Rule	M1
	$\frac{2}{3} = \frac{1}{4} + P(B) - \frac{1}{4} \times P(B)$ use of independence	M1 A1
	$\frac{5}{12} = \frac{3}{4} P(B)$	
	$P(B) = \frac{5}{9}$	A1
(c)	$P(A' \cap B) = \frac{3}{4} \times \frac{5}{9} = \frac{15}{36} = \frac{5}{12}$	M1A1ft
(d)	$P(B'|A) = \frac{(1-(b)) \times 0.25}{0.25}$ or $P(B')$ or $\frac{\frac{1}{9}}{\frac{1}{4}}$	M1
	$= \frac{4}{9}$	A1

Test Your Understanding

Edexcel S1

9. Three events A, B and C are defined in the sample space S. The events A and B are mutually exclusive and A and C are independent.

(a) Draw a Venn diagram to illustrate the relationships between the 3 events and the sample space. **(3)**

Given that $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup C) = 0.7$, find

(b) $P(A | C)$, **(2)**

(c) $P(A \cup B)$, **(2)**

(d) $P(C)$. **(4)**

a) ?

b) ?

c) ?

d) ?

Test Your Understanding

Edexcel S1

9. Three events A, B and C are defined in the sample space S. The events A and B are mutually exclusive and A and C are independent.

(a) Draw a Venn diagram to illustrate the relationships between the 3 events and the sample space. **(3)**

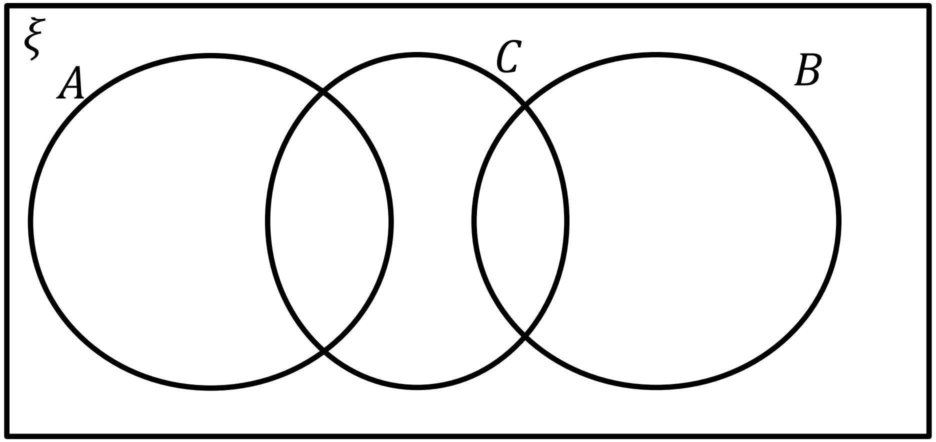
Given that $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup C) = 0.7$, find

(b) $P(A \mid C)$, **(2)**

(c) $P(A \cup B)$, **(2)**

(d) $P(C)$. **(4)**

a)



b) $P(A|C) = P(A) = 0.2$

c) $P(A \cup B) = P(A) + P(B) = 0.6$

d) $P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$$0.7 = 0.2 + P(C) - 0.2P(C)$$

$$0.5 = 0.8P(C)$$

$$P(C) = \frac{5}{8}$$

SUPER IMPORTANT TIPS

If I were to identify two tips that will possible help you the most in probability questions:

If you see the words '**given that**', <u>Immediately</u> write out the law for conditional probability.

Example: "Given Bob walks to school, find the probability that he's not late…"

First thing you should write: $P(L'|W) = \frac{P(L' \cap W)}{P(W)} = \cdots$

If you see the words '**are independent**', <u>Immediately</u> write out the laws for independence.
(Even before you've finished reading the question!)

Example: "A is independent from B…"

First thing you should write: $P(A \cap B) = P(A)P(B)$
$P(A|B) = P(A)$

If you're stuck on a question where you have to find a probability given others, it's probably because you've failed to take into account that two events are independent or mutually exclusive, or you need to use the conditional probability or additional law.

Exercise 2.4

Pearson Stats/Mechanics Year 2
Pages 15-17

Homework Exercise

1 A and B are two events where $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B) = 0.6$.
Find:

a $P(A \cap B)$ **b** $P(A')$ **c** $P(A \cup B')$ **d** $P(A' \cup B)$

2 C and D are two events where $P(C) = 0.55$, $P(D) = 0.65$ and $P(C \cap D) = 0.4$.

a Find $P(C \cup D)$.

b Draw a Venn diagram and use it to find:
i $P(C' \cap D')$ **ii** $P(C|D)$ **iii** $P(C|D')$

c Explain why events C and D are not statistically independent.

3 E and F are two events where $P(E) = 0.7$, $P(F) = 0.8$ and $P(E \cap F) = 0.6$.

a Find $P(E \cup F)$.

b Draw a Venn diagram and use it to find:
i $P(E \cup F')$ **ii** $P(E' \cap F)$ **iii** $P(E|F')$

4 There are two events T and Q where $P(T) = P(Q) = 3P(T \cap Q)$ and $P(T \cup Q) = 0.75$.
Find:

a $P(T \cap Q)$ **b** $P(T)$ **c** $P(Q')$ **d** $P(T' \cap Q')$ **e** $P(T \cap Q')$

5 A survey of all the households in the town of Bury was carried out. The survey showed that 70% have a freezer and 20% have a dishwasher and 80% have either a dishwasher or a freezer or both appliances. Find the probability that a randomly chosen household in Bury has both appliances.

Homework Exercise

6 A and B are two events such that $P(A) = 0.4$, $P(B) = 0.5$ and $P(A|B) = 0.4$. Find:

a $P(B|A)$ **b** $P(A' \cap B')$ **c** $P(A' \cap B)$.

7 Let A and B be events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$.
Find:

a $P(A|B)$ **b** $P(A' \cap B)$ **c** $P(A' \cap B')$

8 C and D are two events where $P(C|D) = \frac{1}{3}$, $P(C|D') = \frac{1}{5}$ and $P(D) = \frac{1}{4}$. Find:

a $P(C \cap D)$ **b** $P(C \cap D')$ **c** $P(C)$

d $P(D|C)$ **e** $P(D'|C)$ **f** $P(D'|C')$

9 Given that $P(A) = 0.42$, $P(B) = 0.37$ and $P(A \cap B) = 0.12$. Find:

a $P(A \cup B)$ **(2 marks)**

b $P(A \mid B')$ **(2 marks)**

The event C has $P(C) = 0.3$.

The events B and C are mutually exclusive and the events A and C are independent.

c Find $P(A \cap C)$. **(2 marks)**

d Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region. **(4 marks)**

e Find $P((A' \cup C)')$. **(2 marks)**

Homework Exercise

10 Three events A, B and C are such that $P(A) = 0.4$, $P(B) = 0.7$, $P(C) = 0.4$ and $P(A \cap B) = 0.3$. Given that A and C are mutually exclusive and that B and C are independent, find:

a $P(B \cap C)$ **(1 mark)**

b $P(B|C)$ **(1 mark)**

c $P(A|B')$ **(1 mark)**

d $P((B \cap C)|A')$ **(1 mark)**

11 Anna and Bella are sometimes late for school. The events A and B are defined as follows:
A is the event that Anna is late for school
B is the event that Bella is late for school
$P(A) = 0.3$, $P(B) = 0.7$ and $P(A' \cap B') = 0.1$. On a randomly selected day, find the probability that:

a both Anna and Bella are late to school **(1 mark)**

b Anna is late to school given that Bella is late to school. **(2 marks)**

Their teacher suspects that Anna and Bella being late for school is linked in some way.

c Comment on his suspicion, showing your working. **(2 marks)**

12 John and Kayleigh play darts in the same team. The events J and K are defined as follows:
J is the event that John wins his match
K is the event that Kayleigh wins her match
$P(J) = 0.6$, $P(K) = 0.7$ and $P(J \cup K) = 0.8$.
Find the probability that:

a both John and Kayleigh win their matches **(1 mark)**

b John wins his match given that Kayleigh loses hers **(2 marks)**

c Kayleigh wins her match given that John wins his. **(2 marks)**

d Determine whether the events J and K are statistically independent.
You must show all your working. **(2 marks)**

Homework Exercise

Challenge

The discrete random variable X has probability function:

$P(X = x) = kx$, $x = 1, 2, 3, 4, 5$

Find:

a the value of k

b $P(X = 5 | X > 2)$

c $P(X \text{ is odd} | X \text{ is prime})$

Homework Answers

1 **a** 0.3 **b** 0.6 **c** 0.8 **d** 0.9

2 **a** 0.8
b **i** 0.2 **ii** 0.615 (3 s.f.) **iii** 0.429 (3 s.f.)
c $P(C \cap D) \neq P(C) \times P(D)$

3 **a** 0.9
b **i** 0.8 **ii** 0.2 **iii** 0.5

4 **a** 0.15 **b** 0.45 **c** 0.55 **d** 0.25 **e** 0.3

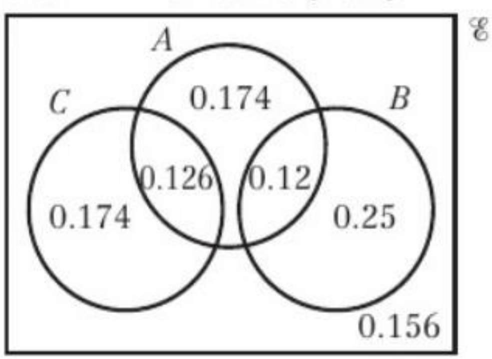
5 0.1

6 **a** 0.5 **b** 0.3 **c** 0.3

7 **a** 0.3 **b** 0.35 **c** 0.4

8 **a** 0.0833 (3 s.f.) **b** 0.15
c 0.233 (3 s.f.) **d** 0.357 (3 s.f.)
e 0.643 (3 s.f.) **f** 0.783 (3 s.f.)

9 **a** 0.67 **b** 0.476 (3 s.f.) **c** 0.126
d



e 0.294

10 **a** 0.28 **b** 0.7
c 0.333 (3 s.f.) **d** 0.467 (3 s.f.)

11 **a** 0.1 **b** 0.143 (3 s.f.)
c $P(A) \times P(B) = 0.3 \times 0.7 = 0.21$, $P(A \cap B) = 0.15$
This suggests that the events are not independent. If Anna is late, Bella is *less* likely to be late and vice versa.

12 **a** 0.5 **b** 0.333 (3 s.f.) **c** 0.833 (3 s.f.)
d $P(K|J) = 0.833\ldots \neq P(K) = 0.7$. So J and K are not independent

Challenge

a $\frac{1}{15}$ **b** $\frac{5}{12}$ **c** $\frac{2}{3}$