P2 Chapter 7: Trigonometric Equations

Proving Identities

Proving Trigonometric Identities

Just like Chapter 6 had 'provey' and 'solvey' questions, we also get the 'provey' questions in Chapter 7. Just use the appropriate double angle or addition formula.

Prove that $\tan 2\theta \equiv \frac{2}{\cot\theta - \tan\theta}$

?

Prove that $\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan\theta$

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Prove that $\tan 2\theta \equiv \frac{2}{\cot\theta - \tan\theta}$

$$\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta} \equiv \frac{2}{\frac{1}{\tan\theta} - \tan\theta}$$

$$\equiv \frac{2}{\cot\theta - \tan\theta}$$

Prove that $\frac{1-\cos 2\theta}{\sin 2\theta} \equiv \tan\theta$

$$\frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta} \equiv \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$

$$\equiv \tan\theta$$

Test Your Understanding

[OCR] Prove that $\cot 2x + cosec\ 2x \equiv \cot x$

?

[OCR] By writing $\cos x = \cos\left(2 \times \frac{x}{2}\right)$ or otherwise, prove the identity $\frac{1-\cos x}{1+\cos x} \equiv \tan^2\left(\frac{x}{2}\right)$

?

Test Your Understanding

[OCR] Prove that $\cot 2x + cosec\ 2x \equiv \cot x$

$$\frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x}$$
$$\equiv \frac{\cos 2x + 1}{\sin 2x} \equiv \frac{2\cos^2 x - 1 + 1}{2\sin x \cos x}$$
$$\equiv \frac{2\cos^2 x}{2\sin x \cos x} \equiv \frac{\cos x}{\sin x} \equiv \cot x$$

[OCR] By writing $\cos x = \cos\left(2 \times \frac{x}{2}\right)$ or otherwise, prove the identity $\frac{1-\cos x}{1+\cos x} \equiv \tan^2\left(\frac{x}{2}\right)$

$$\cos x \equiv cos\left(2 \times \frac{x}{2}\right) \equiv 2\cos^2\left(\frac{x}{2}\right) - 1$$
$$\therefore \frac{1 - cos\, x}{1 + cos\, x} \equiv \frac{1 - \left(2\cos^2\left(\frac{x}{2}\right) - 1\right)}{1 + \left(2\, cos^2\left(\frac{x}{2}\right) - 1\right)}$$
$$\equiv \frac{2\left(1 - \cos^2\left(\frac{x}{2}\right)\right)}{2\cos^2\left(\frac{x}{2}\right)} \equiv \frac{\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} \equiv \tan^2\left(\frac{x}{2}\right)$$

Very Challenging Exam Example

Edexcel C3 June 2015 Q8

(a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \; n \in \mathbb{Z}$$

(5)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places.

(4)

? b

? a

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Give your answers to 3 decimal places.

(4)

$$\sec 2\theta + \tan 2\theta = \frac{1}{2} \Rightarrow \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1}{2}$$

$$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$$

$$\Rightarrow \tan\theta = -\frac{1}{3}$$ M1 A1

$$\Rightarrow \theta = awrt\ 2.820, 5.961$$ dM1A1

$$\sec 2A + \tan 2A = \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A}$$

$$= \frac{1 + \sin 2A}{\cos 2A}$$

$$= \frac{1 + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$$ M1

$$= \frac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$$

$$= \frac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}$$ M1

$$= \frac{\cos A + \sin A}{\cos A - \sin A}$$ A1*

We eventually want $\cos A - \sin A$ so $\cos^2 A - \sin^2 A$ is best choice of double-angle formula because this can be factorise to give $\cos A - \sin A$ as a factor.

This is even less obvious. Knowing that we'll have $(\cos A - \sin A)(\cos A + \sin A)$ in the denominator and that the $\cos A + \sin A$ will cancel, we might work backwards from the final result and multiply by $\cos A + \sin A$! Working backwards from the thing we're trying to prove is occasionally a good strategy (provided the steps are reversible!)

Exercise 7.6

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Extension:

[STEP I 2005 Q4]

(a) Given that $\cos\theta = \frac{3}{5}$ and that $\frac{3\pi}{2} \leqslant \theta \leqslant 2\pi$, show that $\sin 2\theta = -\frac{24}{25}$, and evaluate $\cos 3\theta$.

(b) Prove the identity $\tan 3\theta \equiv \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.

Hence evaluate $\tan\theta$, given that $\tan 3\theta = \frac{11}{2}$ and that $\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{2}$.

(a) ? (b) ?

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Hence evaluate $\tan\theta$, given that $\tan 3\theta = \frac{11}{2}$ and that $\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{2}$.

(a) $-\frac{117}{125}$ (b) $tan\ \theta = 8 + \sqrt{75}$

Homework Exercise

1 Prove the following identities.

a $\dfrac{\cos 2A}{\cos A + \sin A} \equiv \cos A - \sin A$

b $\dfrac{\sin B}{\sin A} - \dfrac{\cos B}{\cos A} \equiv 2 \operatorname{cosec} 2A \sin (B - A)$

c $\dfrac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta$

d $\dfrac{\sec^2 \theta}{1 - \tan^2 \theta} \equiv \sec 2\theta$

e $2(\sin^3 \theta \cos \theta + \cos^3 \theta \sin \theta) \equiv \sin 2\theta$

f $\dfrac{\sin 3\theta}{\sin \theta} - \dfrac{\cos 3\theta}{\cos \theta} \equiv 2$

g $\operatorname{cosec} \theta - 2 \cot 2\theta \cos \theta \equiv 2 \sin \theta$

h $\dfrac{\sec \theta - 1}{\sec \theta + 1} \equiv \tan^2 \dfrac{\theta}{2}$

i $\tan \left(\dfrac{\pi}{4} - x\right) \equiv \dfrac{1 - \sin 2x}{\cos 2x}$

2 Prove the identities:

a $\sin (A + 60°) + \sin (A - 60°) \equiv \sin A$

b $\dfrac{\cos A}{\sin B} - \dfrac{\sin A}{\cos B} \equiv \dfrac{\cos (A + B)}{\sin B \cos B}$

c $\dfrac{\sin (x + y)}{\cos x \cos y} \equiv \tan x + \tan y$

d $\dfrac{\cos (x + y)}{\sin x \sin y} + 1 \equiv \cot x \cot y$

e $\cos \left(\theta + \dfrac{\pi}{3}\right) + \sqrt{3} \sin \theta \equiv \sin \left(\theta + \dfrac{\pi}{6}\right)$

f $\cot (A + B) \equiv \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$

g $\sin^2 (45° + \theta) + \sin^2 (45° - \theta) \equiv 1$

h $\cos (A + B) \cos (A - B) \equiv \cos^2 A - \sin^2 B$

Homework Exercise

3 **a** Show that $\tan\theta + \cot\theta \equiv 2\operatorname{cosec} 2\theta$. **(3 marks)**

b Hence find the value of $\tan 75° + \cot 75°$. **(2 marks)**

4 **a** Show that $\sin 3\theta \equiv 3\sin\theta\cos^2\theta - \sin^3\theta$. **(3 marks)**

b Show that $\cos 3\theta \equiv \cos^3\theta - 3\sin^2\theta\cos\theta$. **(3 marks)**

c Hence, or otherwise, show that $\tan 3\theta \equiv \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$ **(4 marks)**

d Given that θ is acute and that $\cos\theta = \frac{1}{3}$, show that $\tan 3\theta = \dfrac{10\sqrt{2}}{23}$ **(3 marks)**

5 **a** Using $\cos 2A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$, show that:

i $\cos^2\dfrac{x}{2} \equiv \dfrac{1 + \cos x}{2}$ **ii** $\sin^2\dfrac{x}{2} \equiv \dfrac{1 - \cos x}{2}$

b Given that $\cos\theta = 0.6$, and that θ is acute, write down the values of:

i $\cos\dfrac{\theta}{2}$ **ii** $\sin\dfrac{\theta}{2}$ **iii** $\tan\dfrac{\theta}{2}$

c Show that $\cos^4\dfrac{A}{2} \equiv \dfrac{1}{8}(3 + 4\cos A + \cos 2A)$.

6 Show that $\cos^4\theta \equiv \frac{3}{8} + \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$. You must show each stage of your working. **(6 marks)**

7 Prove that $\sin^2(x + y) - \sin^2(x - y) \equiv \sin 2x \sin 2y$. **(5 marks)**

Homework Exercise

8 Prove that $\cos 2\theta - \sqrt{3}\sin 2\theta \equiv 2\cos\left(2\theta + \frac{\pi}{3}\right)$. **(4 marks)**

9 Prove that $4\cos\left(2\theta - \frac{\pi}{6}\right) \equiv 2\sqrt{3} - 4\sqrt{3}\sin^2\theta + 4\sin\theta\cos\theta$. **(4 marks)**

10 Show that:

a $\cos\theta + \sin\theta \equiv \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

b $\sqrt{3}\sin 2\theta - \cos 2\theta \equiv 2\sin\left(2\theta - \frac{\pi}{6}\right)$

Challenge

1 a Show that $\cos(A + B) - \cos(A - B) \equiv -2\sin A\sin B$.

b Hence show that $\cos P - \cos Q \equiv -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$.

c Express $3\sin x\sin 7x$ as the difference of cosines.

2 a Prove that $\sin P + \sin Q \equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$.

b Hence, or otherwise, show that $2\sin\frac{11\pi}{24}\cos\frac{5\pi}{24} = \frac{\sqrt{3}+\sqrt{2}}{2}$

Homework Answers

1 **a** L.H.S. $= \dfrac{\cos^2 A - \sin^2 A}{\cos A + \sin A} = \dfrac{(\cancel{\cos A} + \cancel{\sin A})(\cos A - \sin A)}{\cancel{\cos A} + \cancel{\sin A}}$

$= \cos A - \sin A =$ R.H.S.

b R.H.S. $= \dfrac{\cancel{2}}{\cancel{2}\sin A \cos A}\{\sin B \cos A - \cos B \sin A\}$

$= \dfrac{\sin B}{\sin A} - \dfrac{\cos B}{\cos A} =$ L.H.S.

c L.H.S. $= \dfrac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta} = \dfrac{2\sin^2\theta}{2\sin\theta\cos\theta} = \tan\theta =$ R.H.S.

d L.H.S. $= \dfrac{1 + \tan^2\theta}{1 - \tan^2\theta} = \dfrac{1 + \dfrac{\sin^2\theta}{\cos^2\theta}}{1 - \dfrac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta - \sin^2\theta} = \dfrac{1}{\cos 2\theta} = \sec 2\theta =$ R.H.S.

e L.H.S. $= 2\sin\theta\cos\theta(\sin^2\theta + \cos^2\theta)$

$= 2\sin\theta\cos\theta = \sin 2\theta =$ R.H.S.

f L.H.S. $= \dfrac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\sin(3\theta - \theta)}{\sin\theta\cos\theta}$

$= \dfrac{\sin 2\theta}{\sin\theta\cos\theta} = \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 =$ R.H.S.

g L.H.S. $= \dfrac{1}{\sin\theta} - \dfrac{2\cos 2\theta\cos\theta}{\sin 2\theta} = \dfrac{1}{\sin\theta} - \dfrac{\cancel{2}\cos 2\theta\,\cancel{\cos\theta}}{\cancel{2}\sin\theta\,\cancel{\cos\theta}}$

$= \dfrac{1 - \cos 2\theta}{\sin\theta} = \dfrac{1 - (1 - 2\sin^2\theta)}{\sin\theta} = 2\sin\theta =$ R.H.S.

h L.H.S. $= \dfrac{\dfrac{1}{\cos\theta} - 1}{\dfrac{1}{\cos\theta} + 1} = \dfrac{1 - \cos\theta}{1 + \cos\theta} = \dfrac{1 - \left(1 - 2\sin^2\dfrac{\theta}{2}\right)}{1 + \left(2\cos^2\dfrac{\theta}{2} - 1\right)}$

$= \dfrac{2\sin^2\dfrac{\theta}{2}}{2\cos^2\dfrac{\theta}{2}} = \tan^2\dfrac{\theta}{2} =$ R.H.S.

i L.H.S. $= \dfrac{1 - \tan x}{1 + \tan x} = \dfrac{\cos x - \sin x}{\cos x + \sin x}$

$= \dfrac{(\cos x - \sin x)(\cos x - \sin x)}{\cos^2 x - \sin^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x - 2\sin x\cos x}{\cos^2 x - \sin^2 x} = \dfrac{1 - \sin 2x}{\cos 2x} =$ R.H.S.

Homework Answers

2 **a** L.H.S. $= \sin(A + 60°) + \sin(A - 60°) = \sin A \cos 60°$
$+ \cos A \sin 60° + \sin A \cos 60° - \cos A \sin 60°$
$= 2 \sin A \cos 60° \equiv \sin A =$ R.H.S.

b L.H.S. $= \dfrac{\cos A}{\sin B} - \dfrac{\sin A}{\cos B} = \dfrac{\cos A \cos B - \sin A \sin B}{\sin B \cos B}$

$\equiv \dfrac{\cos(A + B)}{\sin B \cos B} =$ R.H.S.

c L.H.S. $= \dfrac{\sin(x + y)}{\cos x \cos y} = \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}$

$= \dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y} \equiv \tan x + \tan y =$ R.H.S.

d L.H.S. $= \dfrac{\cos(x + y)}{\sin x \sin y} + 1 = \dfrac{\cos x \cos y - \sin x \sin y}{\sin x \sin y} + 1$

$= \dfrac{\cos x \cos y}{\sin x \sin y} - \dfrac{\sin x \sin y}{\sin x \sin y} + 1 = \dfrac{\cos x \cos y}{\sin x \sin y}$

$\equiv \cot x \cot y =$ R.H.S.

e L.H.S. $= \cos\left(\theta + \frac{\pi}{3}\right) + \sqrt{3} \sin\theta = \cos\theta \cos\frac{\pi}{3}$
$- \sin\theta \sin\frac{\pi}{3} + \sqrt{3} \sin\theta = \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta + \sqrt{3}\sin\theta$
$= \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta \equiv \sin\left(\theta + \frac{\pi}{6}\right) =$ R.H.S.

f L.H.S. $= \cot(A + B) = \dfrac{\cos(A + B)}{\sin(A + B)}$

$= \dfrac{\cos A \cos B - \sin A \sin B}{\sin A \cos B + \cos A \sin B}$

$= \dfrac{\dfrac{\cos A \cos B}{\sin A \sin B} - \dfrac{\sin A \sin B}{\sin A \sin B}}{\dfrac{\sin A \cos B}{\sin A \sin B} + \dfrac{\cos A \sin B}{\sin A \sin B}} \equiv \dfrac{\cot A \cot B - 1}{\cot A + \cot B} =$ R.H.S.

g L.H.S. $= \sin^2(45° + \theta) + \sin^2(45° - \theta) = (\sin(45° + \theta))^2$
$+ (\sin(45° - \theta))^2 = (\sin 45° \cos\theta + \cos 45° \sin\theta)^2$
$+ (\sin 45° \cos\theta - \cos 45° \sin\theta)^2$

$= \left(\dfrac{\sqrt{2}}{2}\cos\theta + \dfrac{\sqrt{2}}{2}\sin\theta\right)^2 + \left(\dfrac{\sqrt{2}}{2}\cos\theta - \dfrac{\sqrt{2}}{2}\sin\theta\right)^2$

$= \dfrac{1}{2}\cos^2\theta + \cos\theta\sin\theta + \dfrac{1}{2}\sin^2\theta + \dfrac{1}{2}\cos^2\theta$

$- \cos\theta\sin\theta + \dfrac{1}{2}\sin^2\theta = \cos^2\theta + \sin^2\theta \equiv 1 =$ R.H.S.

h L.H.S. $= \cos(A + B)\cos(A - B)$
$= (\cos A \cos B - \sin A \sin B) \times (\cos A \cos B + \sin A \sin B)$
$= (\cos^2 A \cos^2 B) - (\sin^2 A \sin^2 B) = (\cos^2 A(1 - \sin^2 B))$
$- ((1 - \cos^2 A)\sin^2 B) = \cos^2 A - \cos^2 A \sin^2 B$
$- \sin^2 B + \cos^2 A \sin^2 B \equiv \cos^2 A - \sin^2 B =$ R.H.S.

Homework Answers

3 **a** L.H.S. $= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$$= \frac{1}{\left(\frac{1}{2}\right)\sin 2\theta} = 2\operatorname{cosec} 2\theta = \text{R.H.S.}$$

b 4

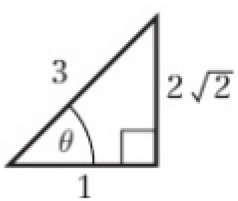
4 **a** Use $\sin 3\theta \equiv \sin(2\theta + \theta)$ and substitute $\cos 2\theta \equiv \cos^2\theta - \sin^2\theta$.

b Use $\cos 3\theta \equiv \cos(2\theta + \theta)$ and substitute $\cos 2\theta \equiv \cos^2\theta - \sin^2\theta$.

c $\tan 3\theta \equiv \dfrac{\sin 3\theta}{\cos 3\theta} = \dfrac{3\sin\theta\cos^2\theta - \sin^3\theta}{\cos^3\theta - 3\sin^2\theta\cos\theta}$

$$= \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

d



$\tan\theta = 2\sqrt{2}$

so $\tan 3\theta = \dfrac{6\sqrt{2} - 16\sqrt{2}}{1 - 24} = \dfrac{-10\sqrt{2}}{-23} = \dfrac{10\sqrt{2}}{23}$

5 **a** **i** $\cos x \equiv 2\cos^2\dfrac{x}{2} - 1$

$$\Rightarrow 2\cos^2\frac{x}{2} \equiv 1 + \cos x \Rightarrow \cos^2\frac{x}{2} \equiv \frac{1 + \cos x}{2}$$

ii $\cos x \equiv 1 - 2\sin^2\dfrac{x}{2}$

$$\Rightarrow 2\sin^2\frac{x}{2} \equiv 1 - \cos x \Rightarrow \sin^2\frac{x}{2} \equiv \frac{1 - \cos x}{2}$$

b **i** $\dfrac{2\sqrt{5}}{5}$ **ii** $\dfrac{\sqrt{5}}{5}$ **iii** $\frac{1}{2}$

c $\cos^4\dfrac{A}{2} \equiv \left(\dfrac{1 + \cos A}{2}\right)^2 \equiv \dfrac{1 + 2\cos A + \cos^2 A}{4}$

$$\equiv \frac{1 + 2\cos A + \left(\frac{1 + \cos 2A}{2}\right)}{4}$$

$$\equiv \frac{2 + 4\cos A + 1 + \cos 2A}{8} \equiv \frac{3 + 4\cos A + \cos 2A}{8}$$

6 L.H.S. $\equiv \cos^4\theta \equiv (\cos^2\theta)^2 \equiv \left(\dfrac{1 + \cos 2\theta}{2}\right)^2$

$$\equiv \frac{1}{4}(1 + 2\cos 2\theta + \cos^2 2\theta) \equiv \frac{1}{4} + \frac{1}{2}\cos 2\theta$$

$$+ \frac{1}{4}\left(\frac{1 + \cos 4\theta}{2}\right) \equiv \frac{1}{4} + \frac{1}{2}\cos 2\theta + \frac{1}{8} + \frac{1}{8}\cos 4\theta$$

$$\equiv \frac{3}{8} + \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta \equiv \text{R.H.S.}$$

Homework Answers

7 $[\sin(x+y)+\sin(x-y)][\sin(x+y)-\sin(x-y)]$
$\equiv [2\sin x\cos y][2\cos x\sin y]$
$\equiv [2\sin x\cos x][2\cos y\sin y]$
$\equiv \sin 2x\sin 2y$

8 $2\cos\left(2\theta+\frac{\pi}{3}\right)\equiv 2\left(\cos 2\theta\cos\frac{\pi}{3}-\sin 2\theta\sin\frac{\pi}{3}\right)$
$\equiv 2\left(\cos 2\theta\frac{1}{2}-\sin 2\theta\frac{\sqrt{3}}{2}\right)\equiv \cos 2\theta-\sqrt{3}\sin 2\theta$

9 $4\cos\left(2\theta-\frac{\pi}{6}\right)\equiv 4\cos 2\theta\cos\frac{\pi}{6}+4\sin 2\theta\sin\frac{\pi}{6}$
$\equiv 2\sqrt{3}\cos 2\theta+2\sin 2\theta\equiv 2\sqrt{3}(1-2\sin^2\theta)+4\sin\theta\cos\theta$
$\equiv 2\sqrt{3}-4\sqrt{3}\sin^2\theta+4\sin\theta\cos\theta$

10 **a** R.H.S. $=\sqrt{2}\left\{\sin\theta\cos\frac{\pi}{4}+\cos\theta\sin\frac{\pi}{4}\right\}$
$=\sqrt{2}\left\{\sin\theta\frac{1}{\sqrt{2}}+\cos\theta\frac{1}{\sqrt{2}}\right\}=\sin\theta+\cos\theta=$ L.H.S.

b R.H.S. $=2\left\{\sin 2\theta\cos\frac{\pi}{6}-\cos 2\theta\sin\frac{\pi}{6}\right\}$
$=2\left\{\sin 2\theta\frac{\sqrt{3}}{2}-\cos 2\theta\frac{1}{2}\right\}=\sqrt{3}\sin 2\theta-\cos 2\theta=$ L.H.S.

Challenge

1 **a** $\cos(A+B)-\cos(A-B)$
$\equiv \cos A\cos B-\sin A\sin B-(\cos A\cos B+\sin A\sin B)$
$\equiv -2\sin A\sin B$

b Let $A+B=P$ and $A-B=Q$. Solve to get $A=\frac{P+Q}{2}$ and $B=\frac{P-Q}{2}$. Then use result from part **a** to get
$\cos P-\cos Q=-2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$

c $-\frac{3}{2}(\cos 8x-\cos 6x)$

2 **a** $\sin(A+B)+\sin(A-B)$
$=\sin A\cos B+\cos A\sin B+\sin A\cos B-\cos A\sin B$
$=2\sin A\cos B$
Let $A+B=P$ and $A-B=Q$
$\therefore A=\frac{P+Q}{2}$ and $B=\frac{P-Q}{2}$
$\therefore \sin P+\sin Q=2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$

b $\frac{11\pi}{24}=\frac{P+Q}{2},\frac{5\pi}{24}=\frac{P-Q}{2}$
$\frac{22\pi}{24}=P+Q,\frac{10\pi}{24}=P-Q$
$\frac{32\pi}{24}=2P\Rightarrow P=\frac{2\pi}{3},Q=\frac{\pi}{4},$
$\sin\left(\frac{2\pi}{3}\right)+\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{3}+\sqrt{2}}{2}$