P1 Chapter 10: Trigonometry Equations

Exact Values

Examples

Without a calculator, work out the value of each below.

$\tan(225°) =$?

tan repeats every 180° so can subtract 180°

$\tan(210°) =$?

$\sin(150°) =$?

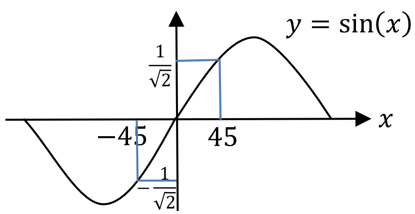
For *sin* we can subtract from 180°.

$\cos(300°) =$?

For *cos* we can subtract from 360°.

$\sin(-45°) =$?

We have to resort to a sketch for this one.

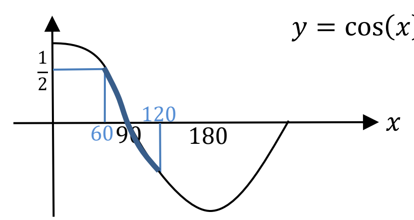


$\cos(750°) =$?

cos repeats every 360°.

$\cos(120°) =$?

Again, let's just use a graph.



The graph is rotationally symmetric about 90°. Since 120° is 30° above 90°, we get the same y value for $90° - 30° = 60°$, except negative.

Use the 'laws' where you can, but otherwise just draw out a quick sketch of the graph.

- $\sin(x) = \sin(180 - x)$
- $\cos(x) = \cos(360 - x)$
- *sin*, *cos* repeat every 360° but *tan* every 180°

Notes: It's not hard to see from the graph that in general, $\sin(-x) = -\sin(x)$. Even more generally, a function f is known as an '**odd function**' if $f(-x) = -f(x)$. *tan* is similarly 'odd' as $\tan(-x) = -\tan(x)$.

A function is **even** if $f(-x) = f(x)$. Examples are $f(x) = \cos(x)$ and $f(x) = x^2$ as $\cos(-x) = \cos(x)$ and $(-x)^2 = (x)^2$. You do not need to know this for the exam.

Examples

Without a calculator, work out the value of each below.

$$\tan(225°) = \mathbf{\tan(45°) = 1}$$

tan repeats every 180° so can subtract 180°

$$\tan(210°) = \mathbf{\tan(30°) = \frac{1}{\sqrt{3}}}$$

$$\sin(150°) = \mathbf{\sin(30°) = \frac{1}{2}}$$

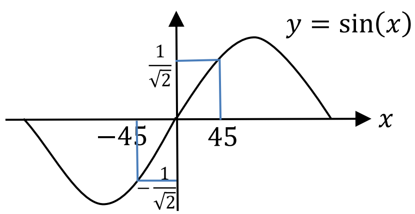
For *sin* we can subtract from 180°.

$$\cos(300°) = \mathbf{\cos(60°) = \frac{1}{2}}$$

For *cos* we can subtract from 360°.

$$\sin(-45°) = \mathbf{-\sin(45°) = -\frac{1}{\sqrt{2}}}$$

We have to resort to a sketch for this one.

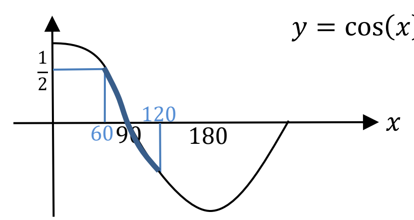


$$\cos(750°) = \mathbf{\cos(30°) = \frac{\sqrt{3}}{2}}$$

cos repeats every 360°.

$$\cos(120°) = \mathbf{-\cos(60°) = -\frac{1}{2}}$$

Again, let's just use a graph.



The graph is rotationally symmetric about 90°. Since 120° is 30° above 90°, we get the same y value for $90° - 30° = 60°$, except negative.

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- $\cos(x) = \cos(360 - x)$
- sin, cos repeat every 360° but tan every 180°

Notes: It's not hard to see from the graph that in general, $\sin(-x) = -\sin(x)$.
Even more generally, a function f is known as an '**odd function**' if $f(-x) = -f(x)$.
tan is similarly 'odd' as $\tan(-x) = -\tan(x)$.

A function is **even** if $f(-x) = f(x)$. Examples are $f(x) = \cos(x)$ and $f(x) = x^2$ as $\cos(-x) = \cos(x)$ and $(-x)^2 = (x)^2$. You do not need to know this for the exam.

Test Your Understanding

Without a calculator, work out the value of each below.

$\cos(315°) =$?

$\sin(420°) =$?

$\tan(-120°) =$?

$\tan(-45°) =$?

$\sin(135°) =$?

- $\sin(x) = \sin(180 - x)$
- $\cos(x) = \cos(360 - x)$
- sin, cos repeat every 360° but tan every 180°

Test Your Understanding

Without a calculator, work out the value of each below.

$$\cos(315°) = \mathbf{\cos(45°)} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$$

$$\sin(420°) = \mathbf{\sin(60°)} = \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$$

$$\tan(-120°) = \mathbf{\tan(60°)} = \sqrt{\mathbf{3}}$$

$$\tan(-45°) = -\boldsymbol{tan}(\mathbf{45°}) = -\mathbf{1}$$

$$\sin(135°) = \mathbf{\sin(45°)} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$$

- $\sin(x) = \sin(180 - x)$
- $\cos(x) = \cos(360 - x)$
- sin, cos repeat every 360° but tan every 180°

Exercise 10.2

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Homework Exercise

1 Express the following as trigonometric ratios of either 30°, 45° or 60°, and hence find their exact values.

a sin 135° **b** sin (–60°) **c** sin 330° **d** sin 420° **e** sin (–300°)

f cos 120° **g** cos 300° **h** cos 225° **i** cos (–210°) **j** cos 495°

k tan 135° **l** tan (–225°) **m** tan 210° **n** tan 300° **o** tan (–120°)

Challenge

The diagram shows an isosceles right-angled triangle ABC.
$AE = DE = 1$ unit. Angle $ACD = 30°$.

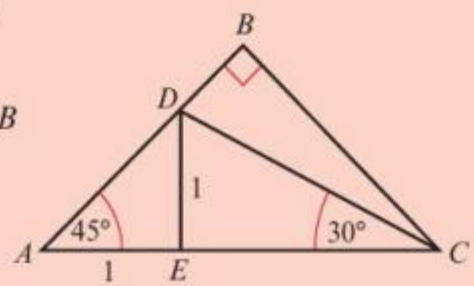
a Calculate the exact lengths of

i CE **ii** DC **iii** BC **iv** DB

b State the size of angle BCD.

c Hence find exact values for

i sin 15° **ii** cos 15°



Homework Answers

1 a $\frac{\sqrt{2}}{2}$ b $-\frac{\sqrt{3}}{2}$ c $-\frac{1}{2}$ d $\frac{\sqrt{3}}{2}$

e $\frac{\sqrt{3}}{2}$ f $-\frac{1}{2}$ g $\frac{1}{2}$ h $-\frac{\sqrt{2}}{2}$

i $-\frac{\sqrt{3}}{2}$ j $-\frac{\sqrt{2}}{2}$ k -1 l -1

m $\frac{\sqrt{3}}{3}$ n $-\sqrt{3}$ o $\sqrt{3}$

Challenge

a i $\sqrt{3}$ ii 2 iii $\sqrt{2+\sqrt{3}}$ iv $\sqrt{2+\sqrt{3}}-\sqrt{2}$

b $15°$

c i $\frac{\sqrt{2+\sqrt{3}}-\sqrt{2}}{2}$ ii $\frac{\sqrt{2+\sqrt{3}}}{2}$