Mech1 Chapter 7: Modelling

Chapter Practice

Key Points

1. Mathematical models can be constructed to simulate real-life situations.
2. Modelling assumptions can be used to simplify your calculations.
3. The base SI units most commonly used in mechanics are:

Quantity	Unit	Symbol
Mass	kilogram	kg
Length/displacement	metre	m
Time	second	s

4. A vector is a quantity which has both magnitude and direction.
5. A scalar quantity has magnitude only.
6. Distance is the magnitude of the displacement vector.
7. Speed is the magnitude of the velocity vector.

Chapter Exercises

1 The motion of a cricket ball after it is hit until it lands on the cricket pitch can be modelled using the equation $h = \frac{1}{10}(24x - 3x^2)$, where h m is the vertical height of the ball above the cricket pitch and x m is the horizontal distance from where it was hit. Find:

Hint The path of the cricket ball is modelled as a quadratic curve. Draw a sketch for the model and use the symmetry of the curve.

a the vertical height of the ball when it is at a horizontal distance of 2 m from where it was hit

b the two horizontal distances for which the height of the ball was 2.1 m.

Given that the model is valid from when the ball is hit to when it lands on the cricket pitch:

c find the values of x for which the model is valid

d work out the maximum height of the cricket ball.

2 A diver dives from a diving board into a swimming pool with a depth of 4.5 m. The height of the diver above the water, h m, can be modelled using $h = 10 - 0.58x^2$ for $0 \leqslant x \leqslant 5$, where x m is the horizontal distance from the end of the diving board.

a Find the height of the diver when $x = 2$ m.

b Find the horizontal distance from the end of the diving board to the point where the diver enters the water.

In this model the diver is modelled as particle.

c Describe the effects of this modelling assumption.

d Comment on the validity of this modelling assumption for the motion of the diver after she enters the water.

Chapter Exercises

3 Make a list of the assumptions you might make to create simple models of the following:

a The motion of a man skiing down a snow-covered slope.

b The motion of a yo-yo on a string.

In each case, describe the effects of the modelling assumptions.

4 Convert to SI units:

a 2.5 km per minute **b** $0.6\,\text{kg cm}^{-2}$ **c** $1.2 \times 10^3\,\text{g cm}^{-3}$

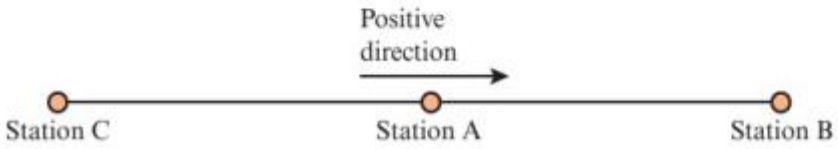
5 A man throws a bowling ball in a bowling alley.

a Make a list of the assumptions you might make to create a simple model of the motion of the bowling ball.

b Taking the direction that the ball travels in as the positive direction, state with a reason whether each of the following are likely to be positive or negative:

i the velocity **ii** the acceleration.

6 A train engine pulling a truck starts at station A then travels in a straight line to station B. It then moves back from station B to station A and on to station C as shown in the diagram.



Hint The **sign** of something means whether it is positive or negative.

What is the sign of the velocity and displacement on the journey from:

a station A to station B **b** station B to station A **c** station A to station C?

Chapter Exercises

7 The acceleration of a boat is given by $\mathbf{a} = -0.05\mathbf{i} + 0.15\mathbf{j}\,\text{m s}^{-2}$. Find:

a the magnitude of the acceleration

b the angle the direction of the acceleration vector makes with the unit vector $\mathbf{i}$.

8 The velocity of a toy car is given by $\mathbf{v} = 3.5\mathbf{i} - 2.5\mathbf{j}\,\text{m s}^{-1}$. Find:

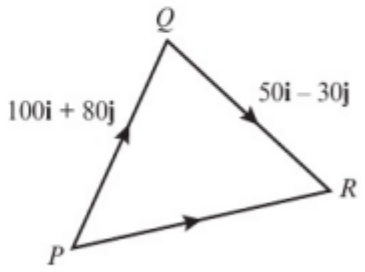
a the speed of the toy car

b the angle the direction of motion of the toy car makes with the unit vector $\mathbf{j}$.

9 A plane flies from P to Q and then from Q to R.

The displacement from P to Q is $100\mathbf{i} + 80\mathbf{j}$ m.

The displacement from Q to R is $50\mathbf{i} - 30\mathbf{j}$ m.



a Find the magnitude of the displacement from P to R.

b Find the total distance the plane has travelled in getting from P to R.

c Find the angle the vector $\overrightarrow{PQ}$ makes with the unit vector $\mathbf{j}$.

Chapter Answers

1 **a** 3.6 m **b** 1 m and 7 m
c $0 \leqslant x \leqslant 8$ **d** 4.8 m

2 **a** 7.68 m **b** 4.15 m
c Ignore the effects of air resistance on the diver and rotational effects of external forces.
d Assumption not valid, diver experiences drag and buoyancy in the water.

3 **a** Model the man on skis as a particle – ignore the rotational effect of any forces that are acting on the man as well as any effects due to air resistance.
Consider the snow-covered slope as smooth – assume there is no friction between the skis and the snow-covered slope.
b Model the yo-yo as a particle – ignore the rotational effect of any forces that are acting on the yo-yo as well as any effects due to air resistance.
Consider the string as light and inextensible – ignore the weight of the string and assume it does not stretch.
Model the yo-yo as smooth – assume there is no friction between the yo-yo and the string.

4 **a** $41.7\ \text{m s}^{-1}$
b $6000\ \text{kg m}^{-2}$
c $1.2 \times 10^{6}\ \text{kg m}^{-3}$

5 **a** Model ball as a particle. Assume the floor is smooth.
b **i** Positive – the positive direction is defined as the direction in which the ball is travelling.
ii Negative – the ball will be slowing down.

6 **a** Velocity is positive, displacement is positive
b Velocity is negative, displacement is positive
c Velocity is negative, displacement is negative

7 **a** $0.158\ \text{ms}^{-2}$ **b** 108.4°

8 **a** $4.3\ \text{ms}^{-1}$ **b** 125.5°

9 **a** 158.1 m **b** 186.4 m **c** 51.3°