P1 Chapter 1: Algebra

Factorising

Factorising

Informally, factorising is the opposite of expanding brackets.
More formally, a factorised expression is one which is expressed as **a product of expressions**.

$x(x+1)(x+2)$ ➡ Factorised as it is the product of 3 linear factors, $x, x+1$ and $x+2$.

Note: A linear expression is of the form $ax+b$. It is called linear because plotting $y=ax+b$ would form a straight line.

$x(x+1)+(x-1)(x+1)$ ➡ Not factorised because the outer-most operation is a sum, not a product.

Basic Examples:

$$x^3+x^2 = \boldsymbol{x^2(x+1)}$$
$$4x-8xy = \boldsymbol{4x(1-2y)}$$

Factorising Quadratics

Recap:

We find two numbers which multiply to give the coefficient of x and multiply to give the constant term.

$$x^2 - 5x - 14 = (x - 7)(x + 2)$$

Fro Note: The *coefficient* of a term is the constant on front of it, e.g. the coefficient of $4x^2$ is 4.

But what if the coefficient of x^2 is not 1?

$$2x^2 + 5x - 12 = (2x - 3)(x + 4)$$

The easiest way is to use your common sense to guess the brackets. What multiplies to give the $2x^2$? What multiplies to give the constant term of -12?

Or you can 'split the middle term' (don't be embarrassed if you've forgotten how to!)

$$2x^2 + 5x - 12 \quad \begin{matrix} \oplus 5 \\ \otimes -24 \end{matrix}$$

$$= 2x^2 + 8x - 3x - 12$$

$$= 2x(x + 4) - 3(x + 4)$$

$$= (x + 4)(2x - 3)$$

STEP 1: Find two numbers which add to give the middle number and multiply to give the first times last.

STEP 2: Split the middle term.

STEP 3: Factorise first half and second half ensuring bracket is duplicated..

STEP 4: Factorise out bracket.

Other Factorisations

Difference of two squares:

$$4x^2 - 9 = \boxed{?}$$

Using multiple factorisations:

$$x^3 - x$$
$$= \boxed{?}$$
$$=$$

Tip: Always look for a common factor first before using other factorisation techniques.

$$x^3 + 3x^2 + 2x$$
$$= \boxed{?}$$
$$=$$

Other Factorisations

Difference of two squares:

$$4x^2 - 9 = (\mathbf{2x + 3})(\mathbf{2x - 3})$$

Using multiple factorisations:

$$x^3 - x$$
$$= \mathbf{x}(\mathbf{x^2 - 1})$$
$$= \mathbf{x}(\mathbf{x + 1})(\mathbf{x - 1})$$

$$x^3 + 3x^2 + 2x$$
$$= \mathbf{x}(\mathbf{x^2 + 3x + 2})$$
$$= \mathbf{x}(\mathbf{x + 2})(\mathbf{x + 1})$$

Tip: Always look for a common factor first before using other factorisation techniques.

Test Your Understanding

1 Factorise completely:

$$6x^2 + x - 2$$

?

2 Factorise completely:

$$x^3 - 7x^2 + 12x$$

?

☠ Factorise completely:

$$x^4 - 1$$

?

☠ Factorise completely:

$$x^3 - 1$$

?

Test Your Understanding

1 Factorise completely:

$$6x^2 + x - 2$$

$6x^2 + x - 2 \quad \oplus 1 \otimes -12$
$= 6x^2 + 4x - 3x - 2$
$= 2x(3x + 2) - 1(3x + 2)$
$= (3x + 2)(2x - 1)$

2 Factorise completely:

$$x^3 - 7x^2 + 12x$$

$= x(x^2 - 7x + 12)$
$= x(x - 3)(x - 4)$

☠ Factorise completely:

$$x^4 - 1$$

$= (x^2 + 1)(x^2 - 1)$
$= (x^2 + 1)(x + 1)(x - 1)$

☠ Factorise completely:

$$x^3 - 1$$

$= (x - 1)(x^2 + x + 1)$

Note: You would not be expected to factorise this at A Level (but you would in STEP!).
In general, the *difference of two cubes*:

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Exercise 1.3

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Homework Exercise

1 Factorise these expressions completely:

a $4x + 8$ **b** $6x - 24$ **c** $20x + 15$

d $2x^2 + 4$ **e** $4x^2 + 20$ **f** $6x^2 - 18x$

g $x^2 - 7x$ **h** $2x^2 + 4x$ **i** $3x^2 - x$

j $6x^2 - 2x$ **k** $10y^2 - 5y$ **l** $35x^2 - 28x$

m $x^2 + 2x$ **n** $3y^2 + 2y$ **o** $4x^2 + 12x$

p $5y^2 - 20y$ **q** $9xy^2 + 12x^2y$ **r** $6ab - 2ab^2$

s $5x^2 - 25xy$ **t** $12x^2y + 8xy^2$ **u** $15y - 20yz^2$

v $12x^2 - 30$ **w** $xy^2 - x^2y$ **x** $12y^2 - 4yx$

2 Factorise:

a $x^2 + 4x$ **b** $2x^2 + 6x$ **c** $x^2 + 11x + 24$

d $x^2 + 8x + 12$ **e** $x^2 + 3x - 40$ **f** $x^2 - 8x + 12$

g $x^2 + 5x + 6$ **h** $x^2 - 2x - 24$ **i** $x^2 - 3x - 10$

j $x^2 + x - 20$ **k** $2x^2 + 5x + 2$ **l** $3x^2 + 10x - 8$

m $5x^2 - 16x + 3$ **n** $6x^2 - 8x - 8$

o $2x^2 + 7x - 15$ **p** $2x^4 + 14x^2 + 24$

q $x^2 - 4$ **r** $x^2 - 49$

s $4x^2 - 25$ **t** $9x^2 - 25y^2$ **u** $36x^2 - 4$

v $2x^2 - 50$ **w** $6x^2 - 10x + 4$ **x** $15x^2 + 42x - 9$

Hint For part **n**, take 2 out as a common factor first. For part **p**, let $y = x^2$.

Homework Exercise

3 Factorise completely:

a $x^3 + 2x$	**b** $x^3 - x^2 + x$	**c** $x^3 - 5x$
d $x^3 - 9x$	**e** $x^3 - x^2 - 12x$	**f** $x^3 + 11x^2 + 30x$
g $x^3 - 7x^2 + 6x$	**h** $x^3 - 64x$	**i** $2x^3 - 5x^2 - 3x$
j $2x^3 + 13x^2 + 15x$	**k** $x^3 - 4x$	**l** $3x^3 + 27x^2 + 60x$

4 Factorise completely $x^4 - y^4$. **(2 marks)**

Problem-solving

Watch out for terms that can be written as a function of a function: $x^4 = (x^2)^2$

5 Factorise completely $6x^3 + 7x^2 - 5x$. **(2 marks)**

Challenge

Write $4x^4 - 13x^2 + 9$ as the product of four linear factors.

Homework Answers

1

a	$4(x + 2)$	**b**	$6(x - 4)$
c	$5(4x + 3)$	**d**	$2(x^2 + 2)$
e	$4(x^2 + 5)$	**f**	$6x(x - 3)$
g	$x(x - 7)$	**h**	$2x(x + 2)$
i	$x(3x - 1)$	**j**	$2x(3x - 1)$
k	$5y(2y - 1)$	**l**	$7x(5x - 4)$
m	$x(x + 2)$	**n**	$y(3y + 2)$
o	$4x(x + 3)$	**p**	$5y(y - 4)$
q	$3xy(3y + 4x)$	**r**	$2ab(3 - b)$
s	$5x(x - 5y)$	**t**	$4xy(3x + 2y)$
u	$5y(3 - 4z^2)$	**v**	$6(2x^2 - 5)$
w	$xy(y - x)$	**x**	$4y(3y - x)$

2

a	$x(x + 4)$	**b**	$2x(x + 3)$
c	$(x + 8)(x + 3)$	**d**	$(x + 6)(x + 2)$
e	$(x + 8)(x - 5)$	**f**	$(x - 6)(x - 2)$
g	$(x + 2)(x + 3)$	**h**	$(x - 6)(x + 4)$
i	$(x - 5)(x + 2)$	**j**	$(x + 5)(x - 4)$
k	$(2x + 1)(x + 2)$	**l**	$(3x - 2)(x + 4)$
m	$(5x - 1)(x - 3)$	**n**	$2(3x + 2)(x - 2)$
o	$(2x - 3)(x + 5)$	**p**	$2(x^2 + 3)(x^2 + 4)$
q	$(x + 2)(x - 2)$	**r**	$(x + 7)(x - 7)$
s	$(2x + 5)(2x - 5)$	**t**	$(3x + 5y)(3x - 5y)$
u	$4(3x + 1)(3x - 1)$	**v**	$2(x + 5)(x - 5)$
w	$2(3x - 2)(x - 1)$	**x**	$3(5x - 1)(x + 3)$

3

a	$x(x^2 + 2)$	**b**	$x(x^2 - x + 1)$
c	$x(x^2 - 5)$	**d**	$x(x + 3)(x - 3)$
e	$x(x - 4)(x + 3)$	**f**	$x(x + 5)(x + 6)$
g	$x(x - 1)(x - 6)$	**h**	$x(x + 8)(x - 8)$
i	$x(2x + 1)(x - 3)$	**j**	$x(2x + 3)(x + 5)$
k	$x(x + 2)(x - 2)$	**l**	$3x(x + 4)(x + 5)$

4 $(x^2 + y^2)(x + y)(x - y)$

5 $x(3x + 5)(2x - 1)$

Challenge

$(x - 1)(x + 1)(2x + 3)(2x - 3)$