M2 Chapter 7: Application of Forces

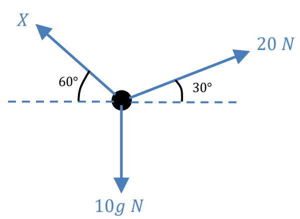
Static Particles

Overview

There is nothing new in this chapter – it just brings together all the individual components we have learnt so far regarding forces: friction, components of forces, $F = ma$, inclined planes and connected particles, for different common types of problems.

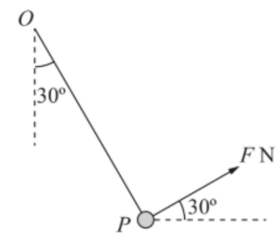
1:: Unknown forces for bodies in equilibrium.

"If the particle is in equilibrium, determine the magnitude of the force X."



2:: Static problem involving weight, tension and pulleys

A particle P of mass 2 kg is attached to one end of a light string, the other end of which is attached to a fixed point O. The particle is held in equilibrium, with OP at 30° to the downward vertical, by a force of magnitude F newtons. The force acts in the same vertical plane as the string and acts at an angle of 30° to the horizontal, as shown in Figure 3.



Find

(i) the value of F,

(ii) the tension in the string.

(8)

Figure 3

3:: Objects in motion on inclined planes

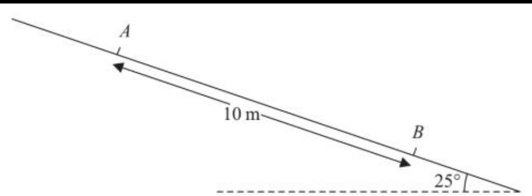


Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B, where $AB = 10$ m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B, **(3)**

(b) the acceleration of P, **(2)**

(c) the coefficient of friction between P and the plane. **(5)**

4:: Connected particles requiring resolution of forces.

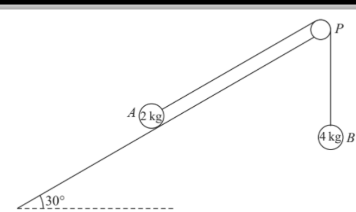


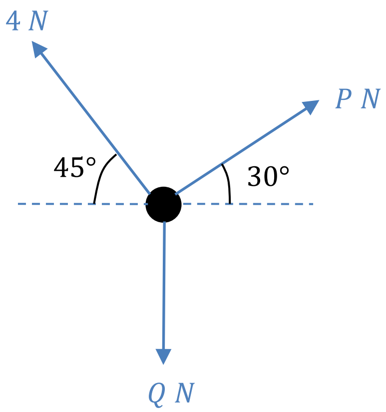
Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

Finding unknown forces by resolving forces



[Textbook] The diagram shows a particle in equilibrium under the forces shown. By resolving horizontally and vertically find the magnitudes of the forces P and Q.

Just resolve separately in the horizontal and vertical directions, as before.

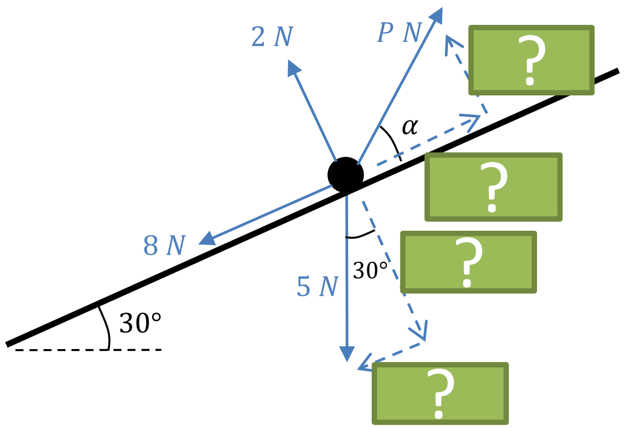
Note: There are two ways of thinking about this. Either "overall force in horizontal direction is 0", thus $P\cos 30° - 4\cos 45° = 0$, or "forces right = force left", as used here

$$R(\rightarrow):\quad P\cos 30° = 4\cos 45°$$
$$R(\uparrow):\quad P\sin 30° + 4\sin 45° = Q$$

$$P = \frac{4\cos 45}{\cos 30} = 3.27\ (3\text{sf})$$
$$Q = \left(\frac{4\cos 45}{\cos 30}\right)\sin 30 + 4\sin 45 = 4.46\ (3\text{sf})$$

Unknown forces on inclined planes

[Textbook] The diagram shows a particle in equilibrium on an inclined plane under the forces shown. Find the magnitude of the force P and the size of the angle α.



Recall the recommendation to use dotted labelled arrow for the components of forces in diagrams.

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Unknown forces on inclined planes

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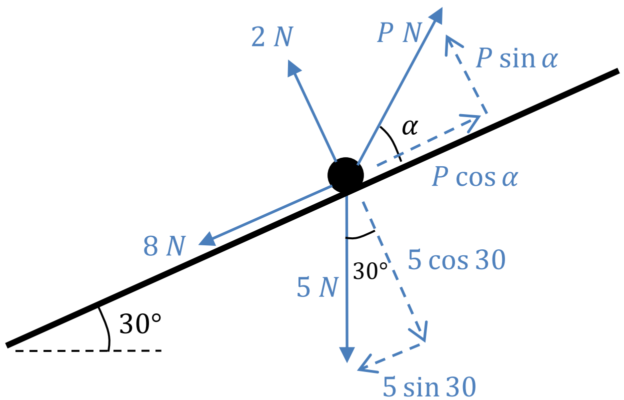


$$R(\nearrow):\quad P\cos\alpha = 8 + 5\sin 30°$$
$$R(\nwarrow):\quad P\sin\alpha + 2 = 5\cos 30°$$

When we have $P \sin \alpha$ and $P \cos \alpha$, a good strategy is to divide them so we get just $\tan \alpha$. You may have seen this in Year 2 trigonometry when expressing trig sums in the form $R \sin(x + \alpha)$.

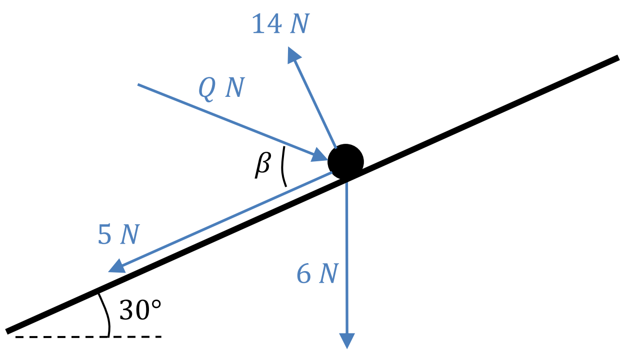
Recall the recommendation to use dotted labelled arrow for the components of forces in diagrams.

$$P\sin\alpha = 5\cos 30° - 2$$
$$P\cos\alpha = 8 + 5\sin 30°$$
$$\therefore \tan\alpha = \frac{5\cos 30° - 2}{8 + 5\sin 30°} = 0.222\ldots$$
$$\alpha = 12.5°\ (3sf)$$

$$P\sin 12.5\ldots = 5\cos 30° - 2$$
$$P = 10.8\ (3sf)$$

Test Your Understanding

The diagram shows a particle in equilibrium on an inclined plane under the forces shown. Find the magnitude of the force Q and the size of the angle β.



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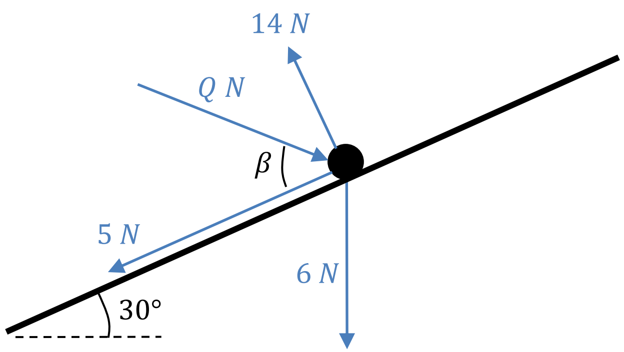
Test Your Understanding

The diagram shows a particle in equilibrium on an inclined plane under the forces shown. Find the magnitude of the force Q and the size of the angle β.



$$R(\nearrow):\quad Q\cos\beta = 5 + 6\sin 30°$$
$$R(\nwarrow):\quad Q\sin\beta + 6\cos 30° = 14$$

$$Q\sin\beta = 14 - 6\cos 30°$$
$$Q\cos\beta = 5 + 6\sin 30°$$
$$\therefore \tan\beta = \frac{14 - 6\cos 30°}{5 + 6\sin 30°}$$
$$\beta = 47.7°\ (3\text{sf})$$

$$Q\cos 47.7\ldots = 5 + 6\sin 30°$$
$$Q = 11.9\ N\ (3\text{sf})$$

Exercise 7.1

Pearson Stats/Mechanics Year 2

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Homework Exercise

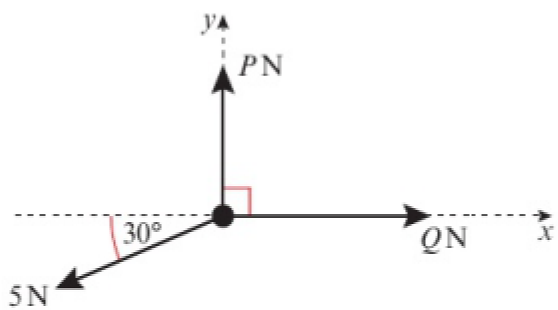
1 Each of the following diagrams shows a particle in static equilibrium. For each particle:

i resolve the components in the x-direction

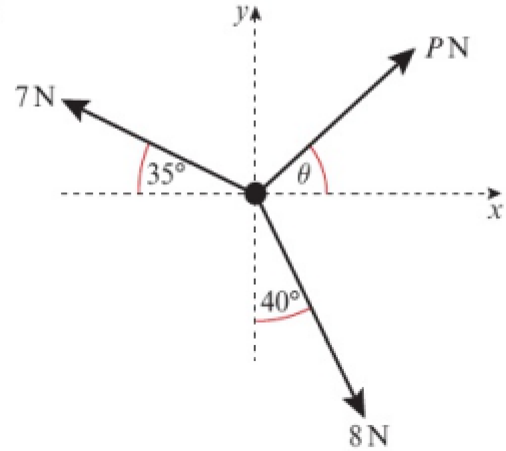
ii resolve the components in the y-direction

iii find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).

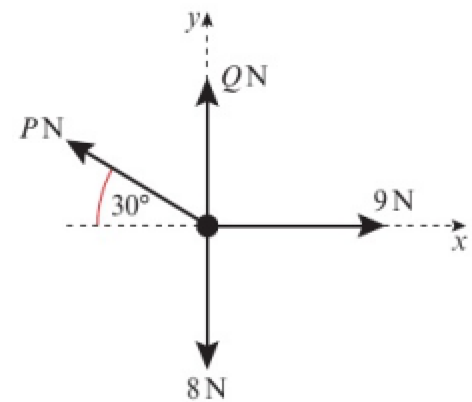
a



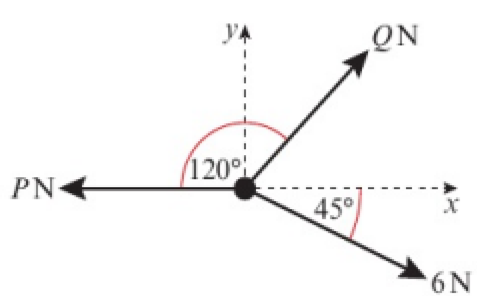
b



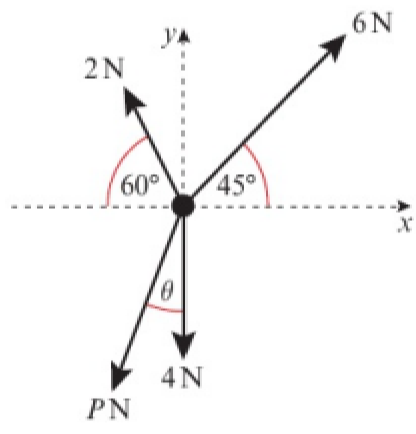
c



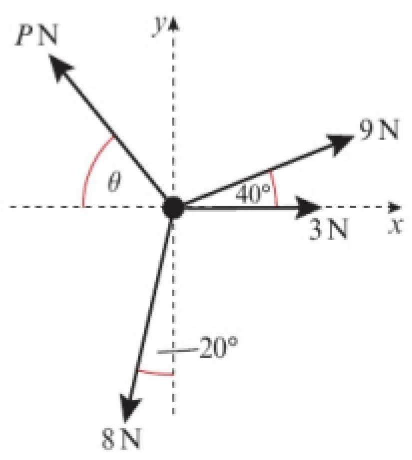
d



e



f



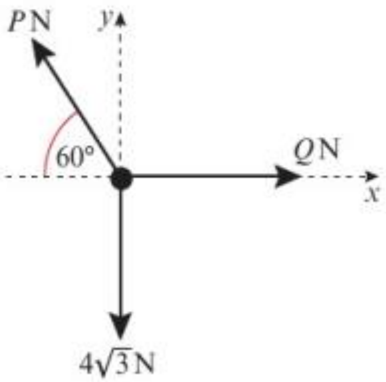
Homework Exercise

2 For each of the following particles in static equilibrium:

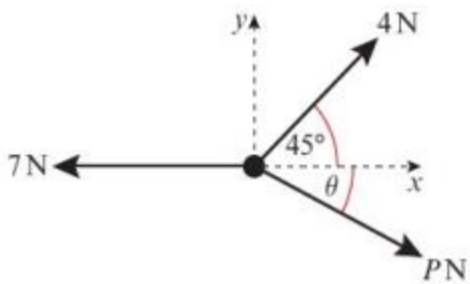
i draw a triangle of forces diagram.

ii Use trigonometry to find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).

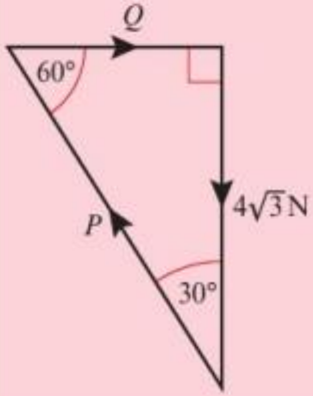
a



b

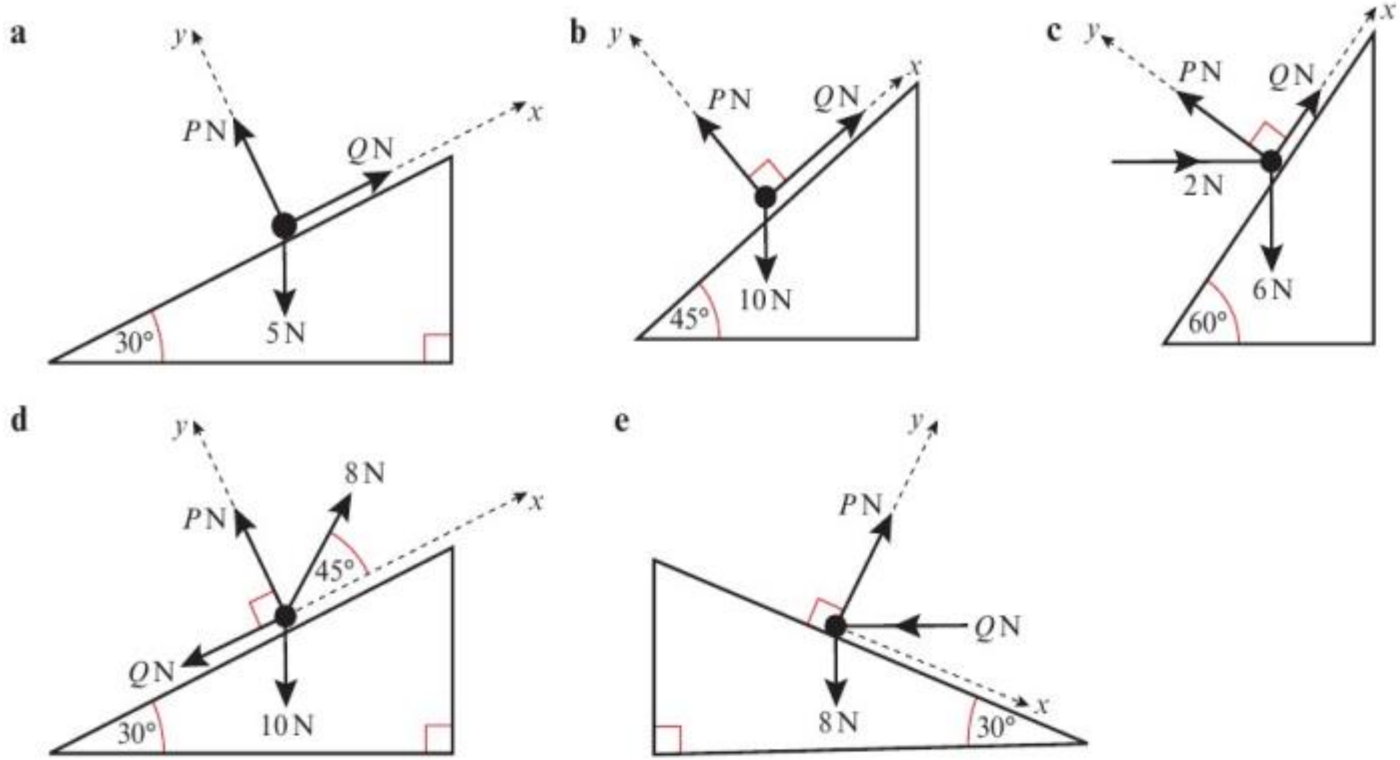


Hint The triangle of forces diagram for part **a** is:



Homework Exercise

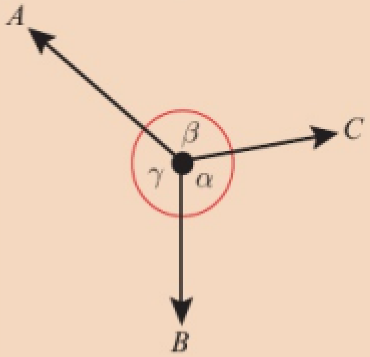
3 Each of these particles rests in equilibrium on a sloping plane under the forces shown. In each case, find the magnitude of forces P and Q.



Homework Exercise

Challenge

The diagram shows three coplanar forces of A, B and C acting on a particle in equilibrium.



Show that $\dfrac{A}{\sin \alpha} = \dfrac{B}{\sin \beta} = \dfrac{C}{\sin \gamma}$

Notation This result is known as **Lami's Theorem**.

Homework Answers

1 **a** **i** $Q - 5\cos 30° = 0$ **ii** $P - 5\sin 30° = 0$

iii $Q = 4.33\,\text{N}$ $P = 2.5\,\text{N}$

b **i** $P\cos\theta + 8\sin 40° - 7\cos 35° = 0$

ii $P\sin\theta + 7\sin 35° - 8\cos 40° = 0$

iii $\theta = 74.4°$ (allow $74.3°$) $P = 2.20\,\text{N}$ (allow 2.19)

c **i** $9 - P\cos 30° = 0$

ii $Q + P\sin 30° - 8 = 0$

iii $Q = 2.80\,\text{N}$ $P = 10.4\,\text{N}$

d **i** $Q\cos 60° + 6\cos 45° - P = 0$

ii $Q\sin 60° - 6\sin 45° = 0$

iii $Q = 4.90\,\text{N}$ $P = 6.69\,\text{N}$

e **i** $6\cos 45° - 2\cos 60° - P\sin\theta = 0$

ii $6\sin 45° + 2\sin 60° - P\cos\theta - 4 = 0$

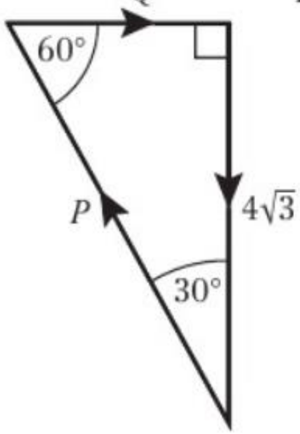
iii $\theta = 58.7°$ $P = 3.80\,\text{N}$

f **i** $9\cos 40° + 3 - P\cos\theta - 8\sin 20° = 0$

ii $P\sin\theta + 9\sin 40° - 8\cos 20° = 0$

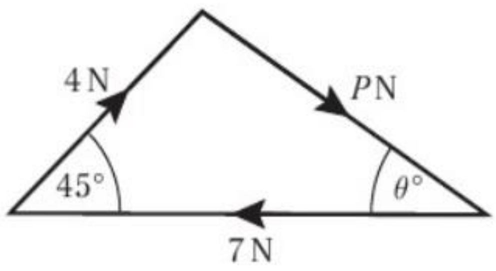
iii $\theta = 13.6°$ $P = 7.36\,\text{N}$

2 **a** **i**



ii $Q = 4\,\text{N}, P = 8\,\text{N}$

b **i**



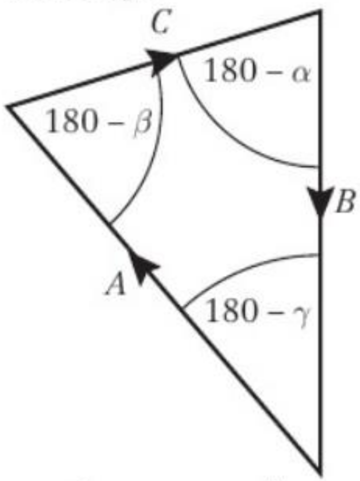
ii $\theta = 34.1°, P = 5.04\,\text{N}$

3 **a** $P = 4.33\,\text{N}, Q = 2.5\,\text{N}$ **b** $P = 7.07\,\text{N}, Q = 7.07\,\text{N}$

c $P = 4.73\,\text{N}, Q = 4.20\,\text{N}$ **d** $P = 3.00\,\text{N}, Q = 0.657\,\text{N}$

e $P = 9.24\,\text{N}, Q = 4.62\,\text{N}$

Challenge



$$\frac{A}{\sin(180 - \alpha)} = \frac{B}{\sin(180 - \beta)} = \frac{C}{\sin(180 - \gamma)}$$

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$