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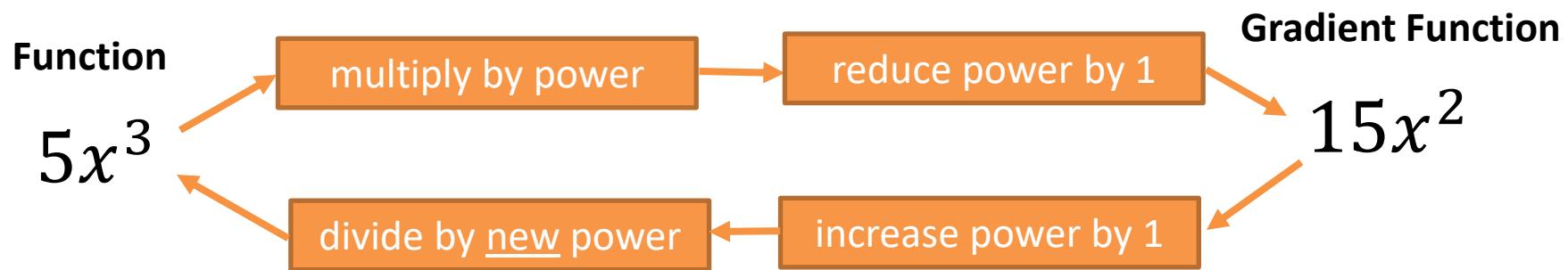
# P1 Chapter 13: Integration

## The Anti-Derivative

# Integrating $x^n$ terms

Integration is the **opposite of differentiation**.

Integrals are also called the '*anti-derivative*'.



However, there's one added complication...

$$\text{Find } y \text{ when } \frac{dy}{dx} = 3x^2$$

Adding 1 to the power and dividing by this power give us:

$$y = x^3$$

However, other functions would also have differentiated to  $3x^2$ :

$$y = x^3 + 1, \quad y = x^3 - 4, \quad \dots$$

Clearly we could have had any constant, as it disappears upon differentiation.

$$\therefore y = x^3 + c$$

$c$  is known as a **constant of integration**

# Examples

Find  $y$  when:

$$\frac{dy}{dx} = 4x^3 \quad y = x^4 + c$$

**Exam Note:** Historically ‘C1’ penalised the lack of  $+c$ , but modules thereafter didn’t. You should always include it.

$$\frac{dy}{dx} = x^5 \quad y = \frac{1}{6}x^6 + c$$

You could also write as  $\frac{x^6}{6}$ . It’s a matter of personal preference.

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} \quad y = 3\left(\frac{2}{3}\right)x^{\frac{3}{2}} + c = 2x^{\frac{3}{2}} + c$$

**Textbook Error** on Pg289: “When integrating polynomials”, but Example 3 is not a polynomial because it has negative and fractional powers.

**Fro Tip:** Many students are taught to write  $\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$  (as does textbook!). This is ugly and students then often struggle to simplify it. Instead remember back to Year 7: **When you divide by a fraction, you multiply by the reciprocal.**

# More Fractional Examples

Find  $y$  when:

$$\frac{dy}{dx} = \frac{4}{\sqrt{x}} = 4x^{-\frac{1}{2}}$$

$$y = 8x^{\frac{1}{2}} + c$$

When we divide by  $\frac{1}{2}$  we multiply by the reciprocal, i.e. 2.

**Fro Tip:** I recommend doing in your head when the simplification would be simple.

$$\frac{dy}{dx} = 5x^{-2}$$

$$y = -5x^{-1} + c$$

$$\frac{dy}{dx} = 4x^{\frac{2}{3}}$$

$$y = 4 \left( \frac{3}{5} \right) x^{\frac{5}{3}} + c = \frac{12}{5} x^{\frac{5}{3}} + c$$

$$\frac{dy}{dx} = 10x^{-\frac{2}{7}}$$

$$y = 10 \left( \frac{7}{5} \right) x^{\frac{5}{7}} + c = 14x^{\frac{5}{7}} + c$$

# Test Your Understanding

Find  $f(x)$  when:

$$f'(x) = 2x + 7$$

$$f(x) = ?$$

$$f'(x) = x^2 - 1$$

$$f(x) = ?$$

$$f'(x) = \frac{2}{x^7} = ?$$

$$f(x) = ?$$

$$f'(x) = \sqrt[3]{x} = ?$$

$$f(x) = ?$$

$$f'(x) = 33x^{\frac{5}{6}}$$

$$f(x) = ?$$

**Note:** In case you're wondering what happens if  $\frac{dy}{dx} = \frac{1}{x} = x^{-1}$ , the problem is that after adding 1 to the power, we'd be dividing by 0. You will learn how to integrate  $\frac{1}{x}$  in Year 2.

# Test Your Understanding

Find  $f(x)$  when:

$$f'(x) = 2x + 7$$

$$f(x) = x^2 + 7x + c$$

$$f'(x) = x^2 - 1$$

$$f(x) = \frac{1}{3}x^3 - x + c$$

$$f'(x) = \frac{2}{x^7} = 2x^{-7}$$

$$f(x) = -\frac{1}{3}x^{-6} + c$$

$$f'(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$f(x) = \frac{3}{4}x^{\frac{4}{3}} + c$$

$$f'(x) = 33x^{\frac{5}{6}}$$

$$f(x) = 18x^{\frac{11}{6}} + c$$

**Note:** In case you're wondering what happens if  $\frac{dy}{dx} = \frac{1}{x} = x^{-1}$ , the problem is that after adding 1 to the power, we'd be dividing by 0. You will learn how to integrate  $\frac{1}{x}$  in Year 2.

# Exercise 13.1

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# Homework Exercise

1 Find an expression for  $y$  when  $\frac{dy}{dx}$  is the following:

- |                        |                      |                       |                      |                     |                      |
|------------------------|----------------------|-----------------------|----------------------|---------------------|----------------------|
| a $x^5$                | b $10x^4$            | c $-x^{-2}$           | d $-4x^{-3}$         | e $x^{\frac{2}{3}}$ | f $4x^{\frac{1}{2}}$ |
| g $-2x^6$              | h $x^{-\frac{1}{2}}$ | i $5x^{-\frac{3}{2}}$ | j $6x^{\frac{1}{3}}$ | k $36x^{11}$        | l $-14x^{-8}$        |
| m $-3x^{-\frac{2}{3}}$ | n $-5$               | o $6x$                | p $2x^{-0.4}$        |                     |                      |

2 Find  $y$  when  $\frac{dy}{dx}$  is given by the following expressions. In each case simplify your answer.

- |   |   |  |
|---|---|--|
| a $x^3 - \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-2}$ | b $4x^3 + x^{-\frac{2}{3}} - x^{-2}$      | c $4 - 12x^{-4} + 2x^{-\frac{1}{2}}$   |
| d $5x^{\frac{2}{3}} - 10x^4 + x^{-3}$           | e $-\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x$ | f $5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$ |

3 Find  $f(x)$  when  $f'(x)$  is given by the following expressions. In each case simplify your answer.

- |   |  |   |
|---|--|---|
| a $12x + \frac{3}{2}x^{-\frac{3}{2}} + 5$ | b $6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$ | c $\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$ |
| d $10x^4 + 8x^{-3}$                       | e $2x^{-\frac{1}{3}} + 4x^{-\frac{5}{3}}$        | f $9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$              |

4 Find  $y$  given that  $\frac{dy}{dx} = (2x + 3)^2$ . (4 marks)

## Problem-solving

Start by expanding the brackets.

5 Find  $f(x)$  given that  $f'(x) = 3x^{-2} + 6x^{\frac{1}{2}} + x - 4$ . (4 marks)

## Challenge

$$\text{Find } y \text{ when } \frac{dy}{dx} = (2\sqrt{x} - x^2) \left( \frac{3+x}{x^5} \right)$$

# Homework Answers

- |   |   |   |   |   |
|---|---|---|---|---|
| 1 | a | $y = \frac{1}{6}x^6 + c$                                      | b | $y = 2x^5 + c$  |
|   | c | $y = x^{-1} + c$  | d | $y = 2x^{-2} + c$                                     |
|   | e | $y = \frac{3}{5}x^{\frac{5}{3}} + c$                          | f | $y = \frac{8}{3}x^{\frac{3}{2}} + c$                  |
|   | g | $y = -\frac{2}{7}x^7 + c$                                     | h | $y = 2x^{\frac{1}{2}} + c$                            |
|   | i | $y = -10x^{-\frac{1}{2}} + c$                                 | j | $y = \frac{9}{2}x^{\frac{4}{3}} + c$                  |
|   | k | $y = 3x^{12} + c$   | l | $y = 2x^{-7} + c$                                     |
|   | m | $y = -9x^{\frac{1}{3}} + c$                                   | n | $y = -5x + c$   |
|   | o | $y = 3x^2 + c$  | p | $y = \frac{10}{3}x^{0.6} + c$                         |
| 2 | a | $y = \frac{1}{4}x^4 - 3x^{\frac{1}{2}} + 6x^{-1} + c$         | b | $y = x^4 + 3x^{\frac{1}{3}} + x^{-1} + c$             |
|   | c | $y = 4x + 4x^{-3} + 4x^{\frac{1}{2}} + c$                     | d | $y = 3x^{\frac{5}{3}} - 2x^5 - \frac{1}{2}x^{-2} + c$ |
|   | e | $y = 4x^{-\frac{1}{3}} - 3x + 4x^2 + c$                       | f | $y = x^5 + 2x^{-\frac{1}{2}} + 3x^{-4} + c$           |
| 3 | a | $f(x) = 6x^2 - 3x^{-\frac{1}{2}} + 5x + c$                    | b | $f(x) = x^6 - x^{-6} + x^{-\frac{1}{6}} + c$          |
|   | c | $f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} + c$               | d | $f(x) = 2x^5 - 4x^{-2} + c$                           |
|   | e | $f(x) = 3x^{\frac{5}{3}} - 6x^{-\frac{2}{3}} + c$             |   |   |
|   | f | $f(x) = 3x^3 - 2x^{-2} + \frac{1}{2}x^{\frac{1}{2}} + c$      |   |   |
| 4 |   | $y = \frac{4x^3}{3} + 6x^2 + 9x + c$                          |   |   |
| 5 |   | $f(x) = -3x^{-1} + 4x^{\frac{3}{2}} + \frac{x^2}{2} - 4x + c$ |   |   |

## Challenge

$$y = -\frac{12}{7}x^{\frac{7}{2}} - \frac{4}{5}x^{\frac{5}{2}} + \frac{3}{2}x^2 + \frac{1}{x} + c$$