
P2 Chapter 3: Sequences and Series

Modelling with Series

Modelling

Anything involving compound changes (e.g. bank interest) will form a geometric sequence, as there is a constant ratio between terms.
We can therefore use formulae such as S_n to solve problems.

[Textbook] Bruce starts a new company. In year 1 his profits will be £20 000. He predicts his profits to increase by £5000 each year, so that his profits in year 2 are modelled to be £25 000, in year 3, £30 000 and so on. He predicts this will continue until he reaches annual profits of £100 000. He then models his annual profits to remain at £100 000.

- Calculate the profits for Bruce's business in the first 20 years.
- State one reason why this may not be a suitable model.
- Bruce's financial advisor says the yearly profits are likely to increase by 5% per annum. Using this model, calculate the profits for Bruce's business in the first 20 years.

a

b

? b

? a

c

? c

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a

The sequence is arithmetic up until £100 000, but stops increasing thereafter. We need to know the position of this term.

$$\begin{aligned}a &= 20\ 000, & d &= 5000 \\u_n &= a + (n - 1)d \rightarrow 100\ 000 \\&= 20\ 000 + (n - 1)d \\&\therefore n = 17\end{aligned}$$

First find sum of first 17 terms (where sequence is arithmetic):

$$\begin{aligned}S_{17} &= \frac{17}{2}(2(20\ 000) + 16(5000) + 1\ 020\ 000 \\&\therefore S_{20} = 1\ 020\ 000 + 3(100\ 000) = 1\ 320\ 000\end{aligned}$$

b

It is unlikely that Bruce's profits will increase by exactly the same amount each year.

c

$$\begin{aligned}a &= 20\ 000 & r &= 1.05 \\S_{20} &= \frac{20\ 000(1.05^{20} - 1)}{1.05 - 1} \\&= £\ 661\ 319.08\end{aligned}$$

Geometric Modelling Example

[Textbook] A piece of A4 paper is folded in half repeatedly. The thickness of the A4 paper is 0.5 mm.

- (a) Work out the thickness of the paper after four folds.
- (b) Work out the thickness of the paper after 20 folds.
- (c) State one reason why this might be an unrealistic model.

a

?

b

?

c

?

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a $a = 0.5 \text{ mm}$, $r = 2$

After 4 folds:

$$u_5 = 0.5 \times 2^4 = 8 \text{ mm}$$

b After 20 folds

$$u_{21} = 0.5 \times 2^{20} = 524\,288 \text{ mm}$$

c It is impossible to fold the paper that many times so the model is unrealistic.

Test Your Understanding

Edexcel C2 Jan 2013 Q3

A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05.

- (a) Show that the predicted profit in the year 2016 is £138 915. (1)
- (b) Find the first year in which the yearly predicted profit exceeds £200 000. (5)
- (c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound. (3)

(a)

? a

(b)

? b

A1

(c)

? c

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(a)	$120000 \times (1.05)^3 = 138915^*$	Or $120000 \times 1.05 \times 1.05 \times 1.05 = 138915$ Or 120000, 126000, 132000, 138915 Or $a = 120000$ and $a \times (1.05)^3 = 138915$	B1
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(b)	$120000 \times (1.05)^{n-1} > 200000$	Allow n or $n - 1$ and " $>$ ", " $<$ ", or " $=$ " etc.	M1
	$\log 1.05^{n-1} > \log \left(\frac{5}{3}\right)$	Takes logs correctly Allow n or $n - 1$ and " $>$ ", " $<$ ", or " $=$ " etc.	M1

	$\frac{\log \left(\frac{5}{3}\right)}{(n-1)} > \frac{\log 1.05}{\log 1.05}$ or equivalent $\log \left(\frac{5}{4}\right)$ e.g. $(n-1) > \frac{\log \left(\frac{5}{4}\right)}{\log 1.05}$	Allow n or $n - 1$ and " $>$ ", " $<$ ", or " $=$ " etc. Allow 1.6 or awrt 1.67 for 5/3.	A1
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	2024	M1: Identifies a calendar year using their value of n or $n - 1$ A1: 2024 only cso	M1A1
2024 with no working = no marks			
See appendix for alternative taking logs base 1.05 and mis-read as total profit			

(c)	$\frac{a(1-r^n)}{1-r} = \frac{120000(1-1.05^{11})}{1-1.05}$	M1: Correct sum formula with $n = 10, 11$ or 12 A1: Correct numerical expression with $n = 11$	M1 A1
	1704814		Cao (Allow 1704814.00)

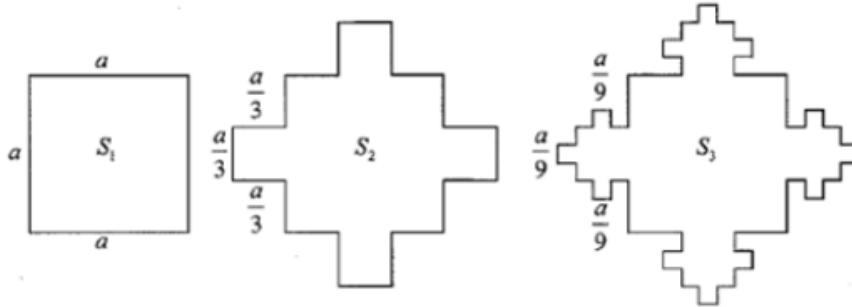
Exercise 3.8

Pearson Pure Mathematics Year 2/AS

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Extension

1 [AEA 2007 Q5]



The figure shows part of a sequence S_1, S_2, S_3, \dots , of model snowflakes. The first term S_1 consist of a single square of side a . To obtain S_2 , the middle third of each edge is replaced with a new square, of side $\frac{a}{3}$, as shown. Subsequent terms are added by replacing the middle third of each external edge of a new square formed in the previous snowflake, by a square $\frac{1}{3}$ of the size, as illustrated by S_3 .

- Deduce that to form S_4 , 36 new squares of side $\frac{a}{27}$ must be added to S_3 .
- Show that the perimeters of S_2 and S_3 are $\frac{20a}{3}$ and $\frac{28a}{3}$ respectively.
- Find the perimeter of S_n .

?

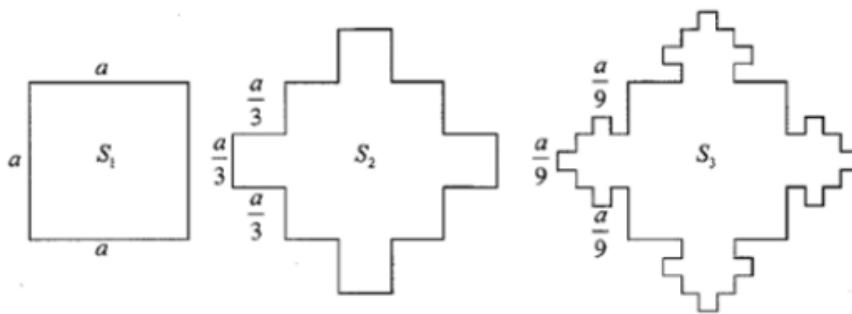
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(a) Each $(\frac{a}{9})$ square has 3 sides, therefore $4 \times 3(\frac{a}{9})$ squares.
 $\therefore 3 \times 4 \times 3 = 36 (\frac{a}{27})$ squares.

Convincing argument
or calculation.

3x12 or 9x4 OK
6x6 or 18x2 NOT

B1

(1)

(b) Let P_i = perimeter of S_i

$$P_1 = 4a, \quad P_2 = 4a + 2 \times \frac{a}{3} \times 4 = 4a + \frac{8a}{3} = \frac{20a}{3} \text{ (A)}$$

$$P_3 = P_2 + 2 \times \frac{a}{9} \times 3 \times 4 = P_2 + \frac{8a}{3} = \frac{28a}{3} \text{ (B)}$$

Clear counting
method

No incorrect work
seen

M1

A1 c/o (2)

(c) $P_1 = 4a$, Common difference = $\frac{8a}{3}$

$$\therefore P_n = 4a + (n-1) \frac{8a}{3}$$

$$= \underline{\underline{\frac{4a}{3}}} + \underline{\underline{\frac{8a}{3}(n-1)}} \quad \text{or} \quad \underline{\underline{\frac{4a}{3}(2n+1)}} \text{ or } \underline{\underline{\frac{4a}{3}(n-1)\frac{8a}{3}}} \text{ oe.}$$

Identify Arithmetic
use of n^{th} term
formula

M1

A1 (4)

Homework Exercise

- 1 An investor puts £4000 in an account. Every month thereafter she deposits another £200. How much money in total will she have invested at the start of **a** the 10th month and **b** the m th month?

Hint At the start of the 6th month she will have only made 5 deposits of £200.

- 2 Carol starts a new job on a salary of £20 000. She is given an annual wage rise of £500 at the end of every year until she reaches her maximum salary of £25 000. Find the total amount she earns (assuming no other rises), **a** in the first 10 years, **b** over 15 years and **c** state one reason why this may be an unsuitable model.

Problem-solving

This is an arithmetic series with $a = 20\,000$ and $d = 500$. First find how many years it will take her to reach her maximum salary.

- 3 James decides to save some money during the six-week holiday. He saves 1p on the first day, 2p on the second, 3p on the third and so on.

- a** How much will he have at the end of the holiday (42 days)?
b If he carried on, how long would it be before he has saved £100?

- 4 A population of ants is growing at a rate of 10% a year. If there were 200 ants in the initial population, write down the number of ants after:

- a** 1 year **b** 2 years **c** 3 years **d** 10 years.

Problem-solving

This is a geometric sequence.
 $a = 200$ and $r = 1.1$

- 5 A motorcycle has four gears. The maximum speed in bottom gear is 40 km h^{-1} and the maximum speed in top gear is 120 km h^{-1} . Given that the maximum speeds in each successive gear form a geometric progression, calculate, in km h^{-1} to one decimal place, the maximum speeds in the two intermediate gears.

Homework Exercise

- 6 A car depreciates in value by 15% a year.

After 3 years it is worth £11 054.25.

- a What was the car's initial price?
- b When will the car's value first be less than £5000?

Problem-solving

Use your answer to part a to write an inequality, then solve it using logarithms.

- 7 A salesman is paid commission of £10 per week for each life insurance policy that he has sold.

Each week he sells one new policy so that he is paid £10 commission in the first week, £20 commission in the second week, £30 commission in the third week and so on.

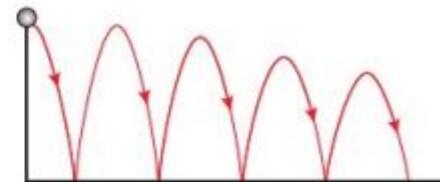
- a Find his total commission in the first year of 52 weeks. (2 marks)
 - b In the second year the commission increases to £11 per week on new policies sold, although it remains at £10 per week for policies sold in the first year. He continues to sell one policy per week. Show that he is paid £542 in the second week of his second year. (3 marks)
 - c Find the total commission paid to him in the second year. (2 marks)
- 8 Prospectors are drilling for oil. The cost of drilling to a depth of 50 m is £500. To drill a further 50 m costs £640 and, hence, the total cost of drilling to a depth of 100 m is £1140. Each subsequent extra depth of 50 m costs £140 more to drill than the previous 50 m.
- a Show that the cost of drilling to a depth of 500 m is £11 300. (3 marks)
 - b The total sum of money available for drilling is £76 000. Find, to the nearest 50 m, the greatest depth that can be drilled. (3 marks)

Homework Exercise

- 9 Each year, for 40 years, Anne will pay money into a savings scheme. In the first year she pays in £500. Her payments then increase by £50 each year, so that she pays in £550 in the second year, £600 in the third year, and so on.
- Find the amount that Anne will pay in the 40th year. **(2 marks)**
 - Find the total amount that Anne will pay in over the 40 years. **(3 marks)**
 - Over the same 40 years, Brian will also pay money into the savings scheme. In the first year he pays in £890 and his payments then increase by £ d each year. Given that Brian and Anne will pay in exactly the same amount over the 40 years, find the value of d . **(4 marks)**
- 10 A virus is spreading such that the number of people infected increases by 4% a day. Initially 100 people were diagnosed with the virus. How many days will it be before 1000 are infected?
- 11 I invest £ A in the bank at a rate of interest of 3.5% per annum. How long will it be before I double my money?
- 12 The fish in a particular area of the North Sea are being reduced by 6% each year due to overfishing. How long will it be before the fish stocks are halved?
- 13 The man who invented the game of chess was asked to name his reward. He asked for 1 grain of corn to be placed on the first square of his chessboard, 2 on the second, 4 on the third and so on until all 64 squares were covered. He then said he would like as many grains of corn as the chessboard carried. How many grains of corn did he claim as his prize?

Homework Exercise

- 14 A ball is dropped from a height of 10 m. It bounces to a height of 7 m and continues to bounce. Subsequent heights to which it bounces follow a geometric sequence. Find out:
- how high it will bounce after the fourth bounce
 - the total vertical distance travelled up to the point when the ball hits the ground for the sixth time.
- 15 Richard is doing a sponsored cycle. He plans to cycle 1000 miles over a number of days. He plans to cycle 10 miles on day 1 and increase the distance by 10% a day.
- How long will it take Richard to complete the challenge?
 - What will be his greatest number of miles completed in a day?
- 16 A savings scheme is offering a rate of interest of 3.5% per annum for the lifetime of the plan. Alan wants to save up £20 000. He works out that he can afford to save £500 every year, which he will deposit on 1 January. If interest is paid on 31 December, how many years will it be before he has saved up his £20 000?



Homework Answers

1 a £5800 b $\text{£}(3800 + 200m)$

2 a £222 500 b £347 500

c It is unlikely her salary will rise by the same amount each year.

3 a £9.03 b 141 days

4 a 220 b 242 c 266 d 519

5 57.7, 83.2

6 a £18 000 b after 7.88 years

7 a £13 780

b Let a denote term of first year and u denote term of second year

$$a_{52} = 10 + 51(10) = 520$$

$$u_1 = 520 + 11$$

$$u_2 = 531 + 11 = 542$$

c £42 198

8 a 500 m is 10 terms,

$$S_{10} = \frac{10}{2}(1000 + 9(140)) = 11\ 300$$

b 1500 m

9 a £2450 b £59 000 c $d = 30$

10 59 days

11 20.15 years

12 11.2 years

$$13 2^{64} - 1 = 1.84 \times 10^{19}$$

14 a 2.401 m

b 48.8234 m

15 a 26 days

b 98.5 miles on 25th day

16 25 years