P2 Chapter 4: Binomial Terms

Expanding Partial Fractions

Using Partial Fractions

Partial fractions allows us to split up a fraction into ones we can then find the binomial expansion of.

[Textbook]

a) Express $\frac{4-5x}{(1+x)(2-x)}$ as partial fractions.

b) Hence show that the cubic approximation of $\frac{4-5x}{(1+x)(2-x)}$ is $2-\frac{7}{2}x+\frac{11}{4}x^2-\frac{25}{8}x^3$

c) State the range of values of x for which the expansion is valid.

a

?

b

?

c

?

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a

$$\frac{4-5x}{(1+x)(2-x)}=\frac{A}{1+x}+\frac{B}{2-x}$$

$$4-5x=A(2-x)+B(1+x)$$

Let $x=2$: $\quad -6=3B \quad \rightarrow \quad B=-2$

Let $x=-1$: $\quad 9=3A \quad \rightarrow \quad A=3$

Therefore $\frac{4-5x}{(1+x)(2-x)}=\frac{3}{1+x}-\frac{2}{2-x}$

b

$$=3(1+x)^{-1}-2(2-x)^{-1}$$

Expansion of $3(1+x)^{-1}$:

$$3\left[1+(-1)x+\frac{(-1)(-2)}{2!}x^2+\frac{(-1)(-2)(-3)}{3!}x^3+\cdots\right]$$

$$=3-3x+3x^2-3x^3+\cdots$$

Similarly $2(2-x)^{-1}=2\times 2^{-1}\left(1-\frac{1}{2}x\right)^{-1}=\cdots=1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{8}x^3$

Hence $\frac{4-5x}{(1+x)(2-x)}\approx 3-3x+3x^2-3x^3-\left(1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{8}x^3\right)$

$$=2-\frac{7}{2}x+\frac{11}{4}x^2-\frac{25}{8}x^3$$

c

First find the valid range for each of the expansions:

$$|x|<1$$

$$\left|\frac{x}{2}\right|<1 \quad \rightarrow \quad |x|<2$$

$$\therefore |x|<1$$

$-1<x<1$ is more restrictive than $-2<x<2$. If <u>both</u> have to be true, **the more restrictive one applies** (this can be seen by drawing the inequalities a number line).

Test Your Understanding

[C4 June 2010 Q5]

10. $$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

(*a*) Find the values of the constants A, B and C. **(4)**

(*b*) Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction. **(7)**

? a

? b

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(a)	$A = 2$	B1	
	$2x^2+5x-10 = A(x-1)(x+2)+B(x+2)+C(x-1)$		
$x \to 1$	$-3 = 3B \Rightarrow B = -1$	M1 A1	
$x \to -2$	$-12 = -3C \Rightarrow C = 4$	A1	**(4)**
(b)	$\frac{2x^2+5x-10}{(x-1)(x+2)} = 2+(1-x)^{-1}+2\left(1+\frac{x}{2}\right)^{-1}$	M1	
	$(1-x)^{-1} = 1+x+x^2+\ldots$	B1	
	$\left(1+\frac{x}{2}\right)^{-1} = 1-\frac{x}{2}+\frac{x^2}{4}+\ldots$	B1	
	$\frac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2)+(1-1)x+\left(1+\frac{1}{2}\right)x^2+\ldots$	M1	
	$= 5+\ldots$ ft their $A-B+\frac{1}{2}C$	A1 ft	
	$= \ldots+\frac{3}{2}x^2+\ldots$ $0x$ stated or implied	A1 A1	**(7)**
			[11]

Exercise 4.3

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Homework Exercise

1 a Express $\frac{8x+4}{(1-x)(2+x)}$ as partial fractions.

b Hence or otherwise expand $\frac{8x+4}{(1-x)(2+x)}$ in ascending powers of x as far as the term in x^2.

c State the set of values of x for which the expansion is valid.

2 a Express $-\frac{2x}{(2+x)^2}$ as partial fractions.

b Hence prove that $-\frac{2x}{(2+x)^2}$ can be expressed in the form $-\frac{1}{2}x + Bx^2 + Cx^3$ where constants B and C are to be determined.

c State the set of values of x for which the expansion is valid.

3 a Express $\frac{6+7x+5x^2}{(1+x)(1-x)(2+x)}$ as partial fractions.

b Hence or otherwise expand $\frac{6+7x+5x^2}{(1+x)(1-x)(2+x)}$ in ascending powers of x as far as the term in x^3.

c State the set of values of x for which the expansion is valid.

4 $g(x) = \frac{12x-1}{(1+2x)(1-3x)}$, $|x| < \frac{1}{3}$

Given that $g(x)$ can be expressed in the form $g(x) = \frac{A}{1+2x} + \frac{B}{1-3x}$

a Find the values of A and B. **(3 marks)**

b Hence, or otherwise, find the series expansion of $g(x)$, in ascending powers of x, up to and including the x^2 term. Simplify each term. **(6 marks)**

Homework Exercise

5 a Express $\frac{2x^2 + 7x - 6}{(x + 5)(x - 4)}$ in partial fractions.

Hint First divide the numerator by the denominator.

b Hence, or otherwise, expand $\frac{2x^2 + 7x - 6}{(x + 5)(x - 4)}$ in ascending powers of x as far as the term in x^2.

c State the set of values of x for which the expansion is valid.

6 $\frac{3x^2 + 4x - 5}{(x + 3)(x - 2)} = A + \frac{B}{x + 3} + \frac{C}{x - 2}$

a Find the values of the constants A, B and C. **(4 marks)**

b Hence, or otherwise, expand $\frac{3x^2 + 4x - 5}{(x + 3)(x - 2)}$ in ascending powers of x, as far as the term in x^2.

Give each coefficient as a simplified fraction. **(7 marks)**

7 $f(x) = \frac{2x^2 + 5x + 11}{(2x - 1)^2(x + 1)}$, $|x| < \frac{1}{2}$

$f(x)$ can be expressed in the form $f(x) = \frac{A}{2x - 1} + \frac{B}{(2x - 1)^2} + \frac{C}{x + 1}$

a Find the values of A, B and C. **(4 marks)**

b Hence or otherwise, find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2. Simplify each term. **(6 marks)**

c Find the percentage error made in using the series expansion in part **b** to estimate the value of $f(0.05)$. Give your answer to 2 significant figures. **(4 marks)**

Homework Answers

1 **a** $\frac{4}{1-x} - \frac{4}{2+x}$ **b** $2 + 5x + \frac{7}{2}x^2$

c valid $|x| < 1$

2 **a** $-\frac{2}{2+x} + \frac{4}{(2+x)^2}$ **b** $B = \frac{1}{2}$, $C = -\frac{3}{8}$

c $|x| < 2$

3 **a** $\frac{2}{1+x} + \frac{3}{1-x} - \frac{4}{2+x}$ **b** $3 + 2x + \frac{9}{2}x^2 + \frac{5}{4}x^3$

c $|x| < 1$

4 **a** $A = -\frac{14}{5}$ and $B = \frac{9}{5}$ **b** $-1 + 11x + 5x^2$

5 **a** $2 - \frac{1}{x+5} + \frac{6}{x-4}$ **b** $\frac{3}{10} - \frac{67}{200}x - \frac{407}{4000}x^2$

c $|x| < 4$

6 **a** $A = 3$, $B = -2$ and $C = 3$ **b** $\frac{5}{6} - \frac{19}{36}x - \frac{97}{216}x^2$

7 **a** $A = -\frac{7}{9}$, $B = \frac{28}{3}$ and $C = \frac{8}{9}$ **b** $11 + 38x + 116x^2$

c 0.33%