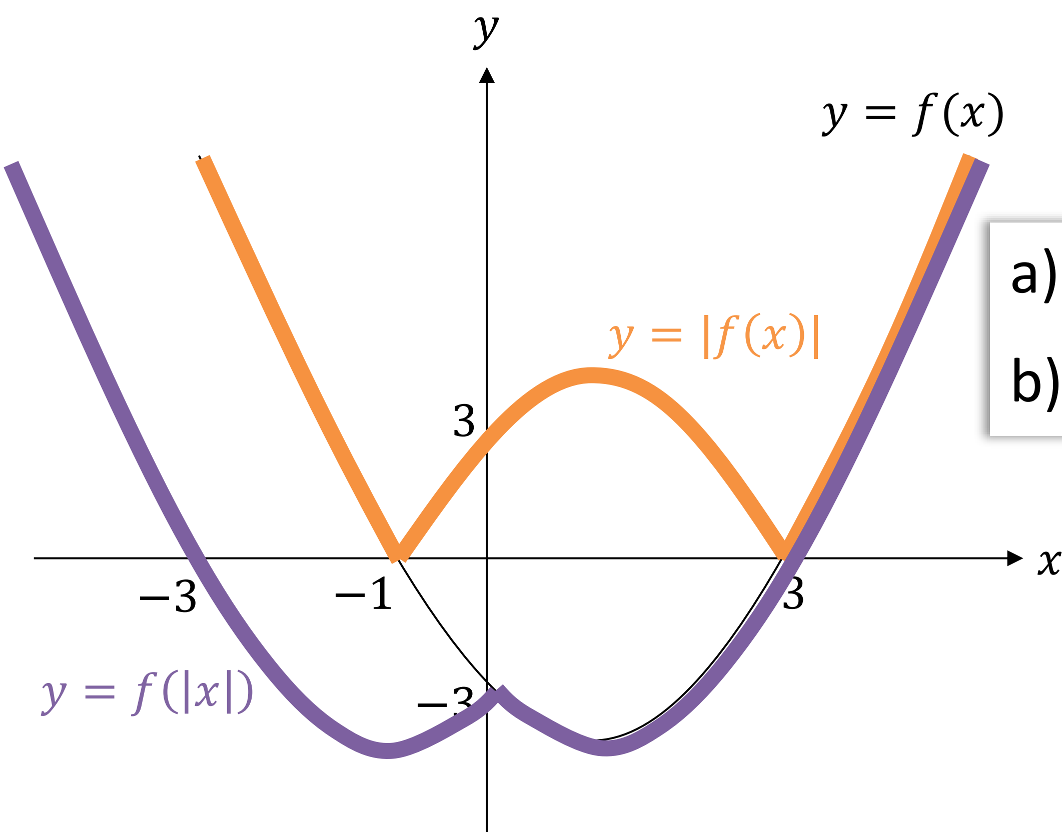
P2 Chapter 2: Graphing Functions

Sketching Modulus Functions

Sketching $y = |f(x)|$ and $y = f(|x|)$



This is a sketch of $y = f(x)$ where $f(x) = (x - 3)(x + 1)$

a) Sketch $y = \lvert f(x) \rvert$	Sketch >
b) Sketch $y = f(\lvert x \rvert)$	Sketch >

The | ... | is outside the function so affects the y value. Any negative y values will be made positive, so any parts of the graph below the x-axis are flipped upwards.
Ensure the y-intercept is indicated.

When $x = -3$ for example, this becomes +3 before being fed into the function, therefore we actually use the y value when x would have been 3 instead of the original -3.
The result is that the graph left of the y-axis is discarded and the graph right of it copied over by reflection in the y-axis.

Test Your Understanding

Edexcel C4 June 2012 Q4

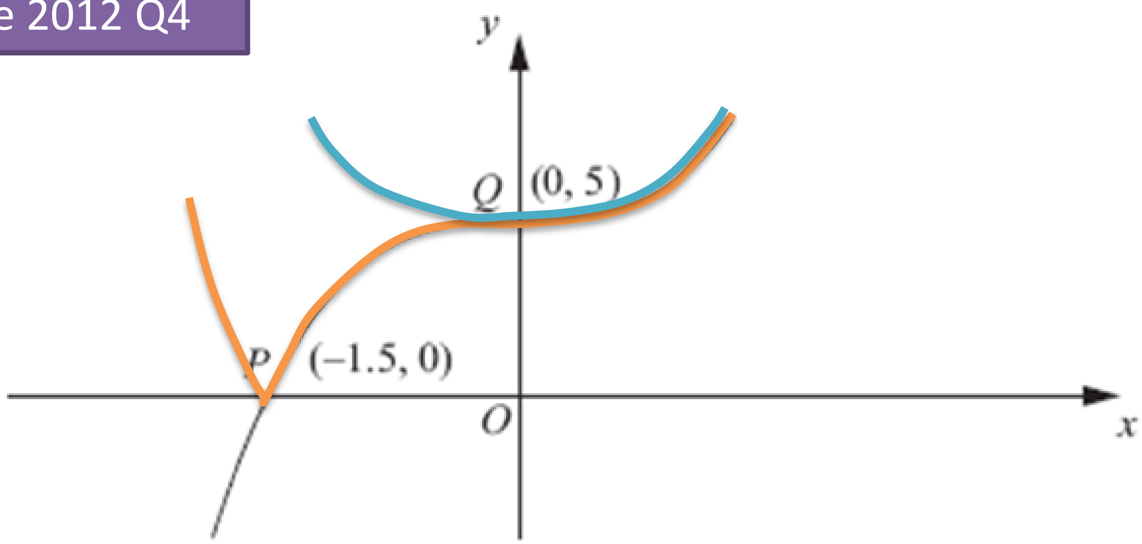


Figure 2 shows part of the curve with equation $y = f(x)$.
The curve passes through the points $P(-1.5, 0)$ and $Q(0, 5)$ as shown.

On separate diagrams, sketch the curve with equation

(*a*) $y = |f(x)|$ Sketch >

(2)

(*b*) $y = f(|x|)$ Sketch >

(2)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Further Test Your Understanding

[Textbook] Sketch for $-2\pi \leq x \leq 2\pi$:

a) $y = |\sin(x)|$

b) $y = \sin(|x|)$

a

?

b

?

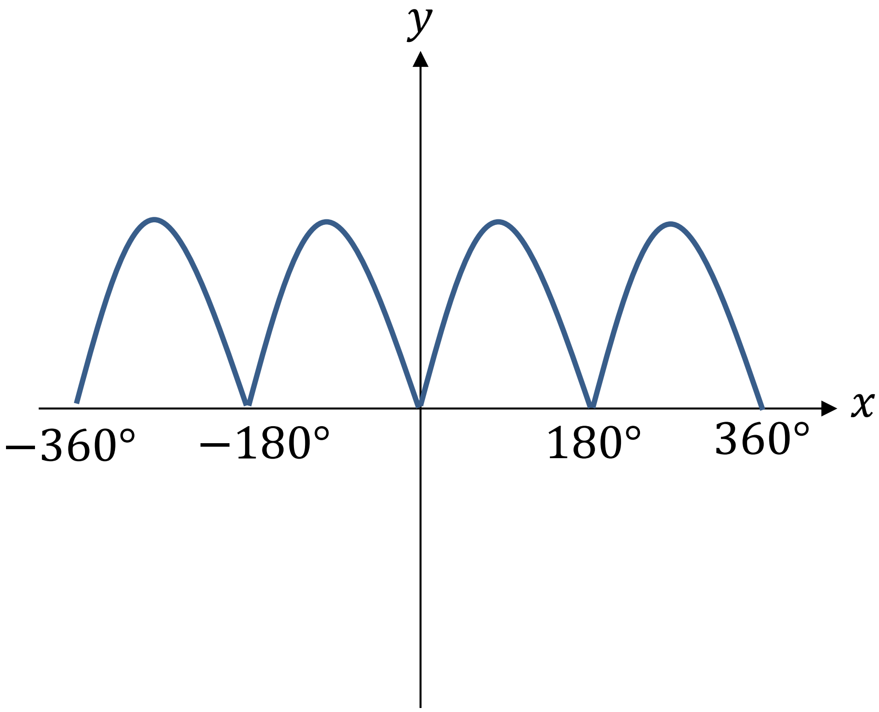
Further Test Your Understanding

[Textbook] Sketch for $-2\pi \leq x \leq 2\pi$:

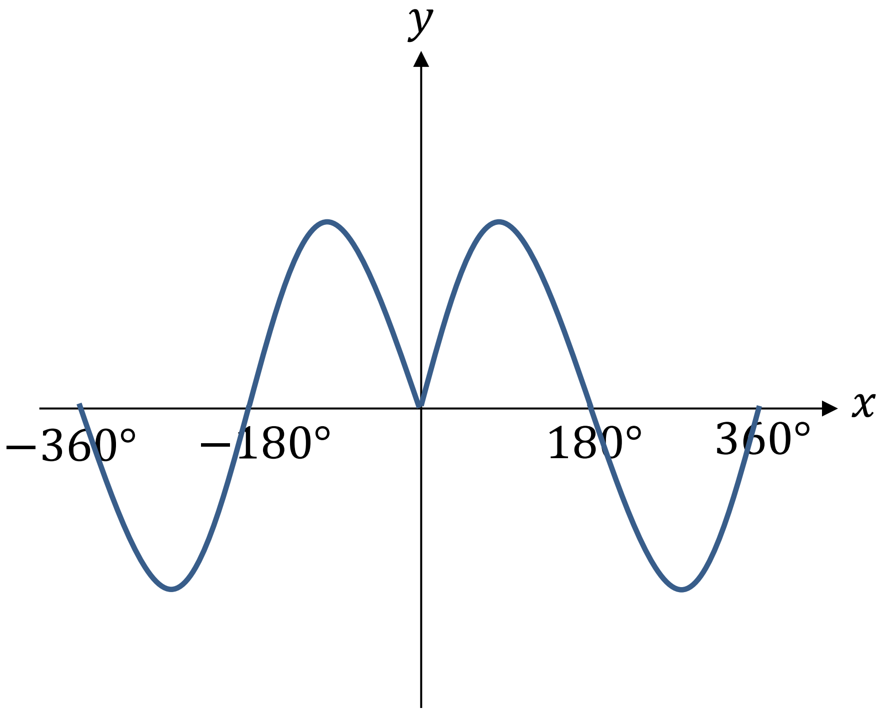
a) $y = |\sin(x)|$

b) $y = \sin(|x|)$

a



b



Exercise 2E

Pearson Pure Mathematics Year 2/AS
Pages 42-44

Extension

1 [SMC 2008 Q25] What is the area of the polygon forms by all the points (x, y) in the plane satisfying the inequality $\big||x|-2\big|+\big||y|-2\big| \leq 4$?

A 24 B 32 C 64 D 96 E 112

?

Exercise 2E

Pearson Pure Mathematics Year 2/AS
Pages 42-44

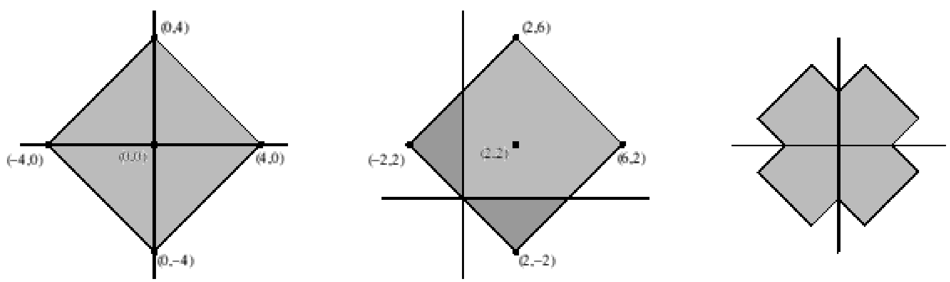
Extension

1 [SMC 2008 Q25] What is the area of the polygon forms by all the points (x, y) in the plane satisfying the inequality $\big||x|-2\big| + \big||y|-2\big| \leq 4$?

A 24 B 32 C 64 D 96 E 112

25. **D** To work out the area of $\big||x|-2\big| + \big||y|-2\big| \leqslant 4$, we first consider the region $|x| + |y| \leqslant 4$ which is shown in (a). This region is then translated to give $|x-2| + |y-2| \leqslant 4$ as shown in (b).

By properties of the modulus, if the point (x, y) lies in the polygon, then so do $(x, -y)$, $(-x, y)$ and $(-x, -y)$. Thus $\big||x|-2\big| + \big||y|-2\big| \leqslant 4$ can be obtained from (b) by reflecting in the axes and the origin, as shown in (c).



(a) (b) (c)

Hence the required area is 4 times the area in the first quadrant. From (b), the required area in the first quadrant is the area of a square of side $4\sqrt{2}$ minus two triangles (cut off by the axes) which, combined, make up a square of side $2\sqrt{2}$. So the area in the first quadrant is $(4\sqrt{2})^2 - (2\sqrt{2})^2 = 32 - 8 = 24$.

Hence the area of the polygon is $4 \times 24 = 96$ square units.

Exercise 2.5

Pearson Pure Mathematics Year 2/AS
Page 10

Extension

1 [SMC 2008 Q25] What is the area of the polygon forms by all the points (x, y) in the plane satisfying the inequality $\big||x| - 2\big| + \big||y| - 2\big| \leq 4$?

A 24 B 32 C 64 D 96 E 112

?

Exercise 2.5

Pearson Pure Mathematics Year 2/AS
Page 10

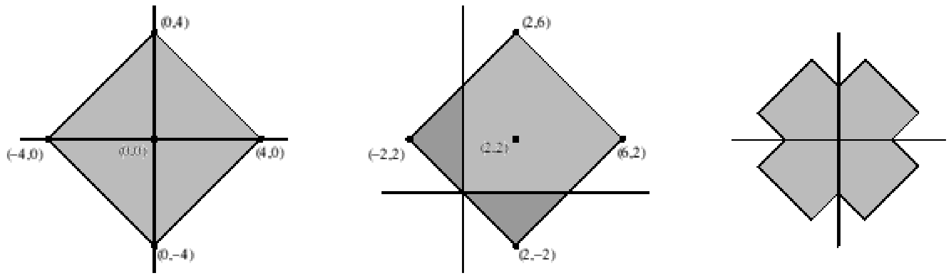
Extension

1 [SMC 2008 Q25] What is the area of the polygon forms by all the points (x, y) in the plane satisfying the inequality $\big||x|-2\big| + \big||y|-2\big| \leq 4$?

A 24 B 32 C 64 D 96 E 112

25. **D** To work out the area of $\big||x|-2\big| + \big||y|-2\big| \leqslant 4$, we first consider the region $|x| + |y| \leqslant 4$ which is shown in (a). This region is then translated to give $|x-2| + |y-2| \leqslant 4$ as shown in (b).

By properties of the modulus, if the point (x, y) lies in the polygon, then so do $(x, -y)$, $(-x, y)$ and $(-x, -y)$. Thus $\big||x|-2\big| + \big||y|-2\big| \leqslant 4$ can be obtained from (b) by reflecting in the axes and the origin, as shown in (c).



(a) (b) (c)

Hence the required area is 4 times the area in the first quadrant. From (b), the required area in the first quadrant is the area of a square of side $4\sqrt{2}$ minus two triangles (cut off by the axes) which, combined, make up a square of side $2\sqrt{2}$. So the area in the first quadrant is $(4\sqrt{2})^2 - (2\sqrt{2})^2 = 32 - 8 = 24$.

Hence the area of the polygon is $4 \times 24 = 96$ square units.

Homework Exercise

1 $f(x) = x^2 - 7x - 8$

a Sketch the graph of $y = f(x)$.

b Sketch the graph of $y = |f(x)|$.

c Sketch the graph of $y = f(|x|)$.

2 $g: x \mapsto \cos x, -360° \leqslant x \leqslant 360°$

a Sketch the graph of $y = g(x)$.

b Sketch the graph of $y = |g(x)|$.

c Sketch the graph of $y = g(|x|)$.

3 $h: x \mapsto (x - 1)(x - 2)(x + 3)$

a Sketch the graph of $y = h(x)$.

b Sketch the graph of $y = |h(x)|$.

c Sketch the graph of $y = h(|x|)$.

4 The function k is defined by $k(x) = \frac{a}{x^2}, a > 0, x \in \mathbb{R}, x \neq 0$.

a Sketch the graph of $y = k(x)$.

b Explain why it is not necessary to sketch $y = |k(x)|$ and $y = k(|x|)$.

The function m is defined by $m(x) = \frac{a}{x^2}, a < 0, x \in \mathbb{R}, x \neq 0$.

c Sketch the graph of $y = m(x)$.

d State with a reason whether the following statements are true or false.

i $|k(x)| = |m(x)|$ **ii** $k(|x|) = m(|x|)$ **iii** $m(x) = m(|x|)$

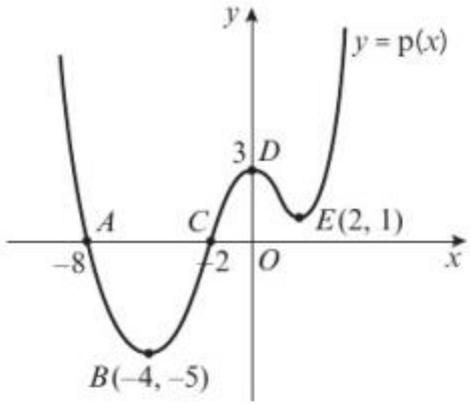
Homework Exercise

5 The diagram shows the graph of $y = \mathrm{p}(x)$ with 5 points labelled.

Sketch each of the following graphs, labelling the points corresponding to A, B, C, D and E, and any points of intersection with the coordinate axes.

a $y = |\mathrm{p}(x)|$ **(3 marks)**

b $y = \mathrm{p}(|x|)$ **(3 marks)**

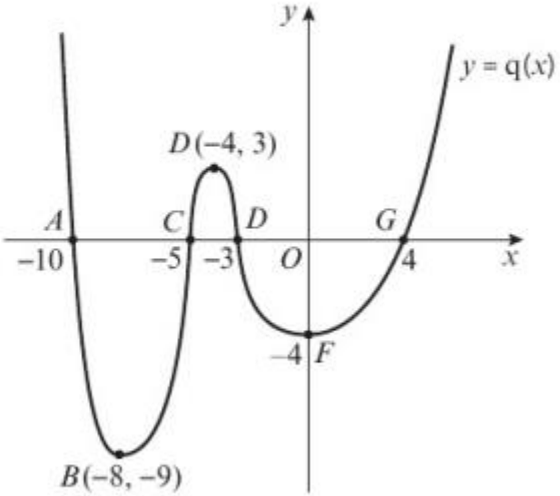


6 The diagram shows the graph of $y = \mathrm{q}(x)$ with 7 points labelled.

Sketch each of the following graphs, labelling the points corresponding to A, B, C, D and E, and any points of intersection with the coordinate axes.

a $y = |\mathrm{q}(x)|$ **(4 marks)**

b $y = \mathrm{q}(|x|)$ **(3 marks)**



7 $\mathrm{k}(x) = \frac{a}{x}$, $a > 0$, $x \neq 0$

a Sketch the graph of $y = \mathrm{k}(x)$.

b Sketch the graph of $y = |\mathrm{k}(x)|$.

c Sketch the graph of $y = \mathrm{k}(|x|)$.

Homework Exercise

8 $\mathrm{m}(x) = \frac{a}{x}$, $a < 0$, $x \neq 0$

a Sketch the graph of $y = \mathrm{m}(x)$.

b Describe the relationship between $y = |\mathrm{m}(x)|$ and $y = \mathrm{m}(|x|)$.

9 $\mathrm{f}(x) = \mathrm{e}^x$ and $\mathrm{g}(x) = \mathrm{e}^{-x}$

a Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ on the same axes.

b Explain why it is not necessary to sketch $y = |\mathrm{f}(x)|$ and $y = |\mathrm{g}(x)|$.

c Sketch the graphs of $y = \mathrm{f}(|x|)$ and $y = \mathrm{g}(|x|)$ on the same axes.

10 The function $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = \begin{cases} -2x - 6, & -5 \leqslant x < -1 \\ (x+1)^2, & -1 \leqslant x \leqslant 2 \end{cases}$$

a Sketch $\mathrm{f}(x)$ stating its range. **(5 marks)**

b Sketch the graph of $y = |\mathrm{f}(x)|$. **(3 marks)**

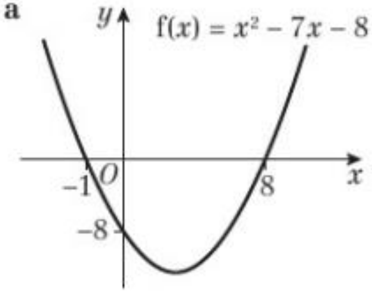
c Sketch the graph of $y = \mathrm{f}(|x|)$. **(3 marks)**

Problem-solving

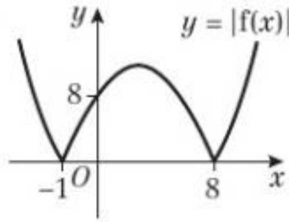
A piecewise function like this does not have to be continuous. Work out the value of both expressions when $x = -1$ to help you with your sketch.

Homework Answers

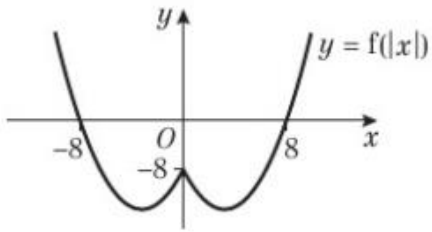
1 **a**



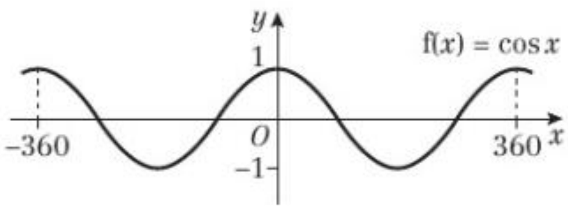
b



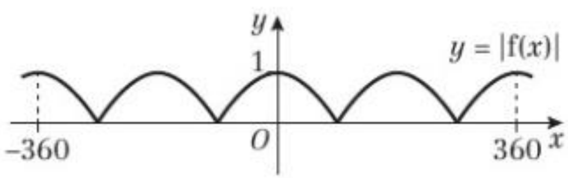
c



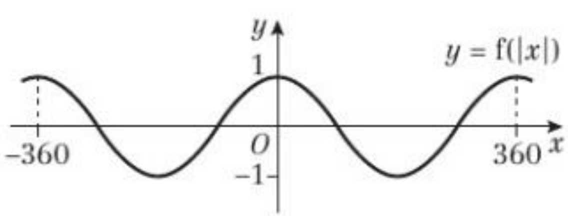
2 **a**



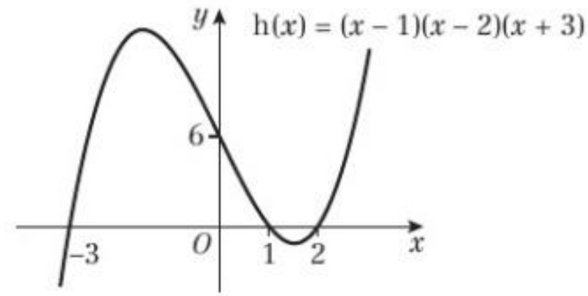
b



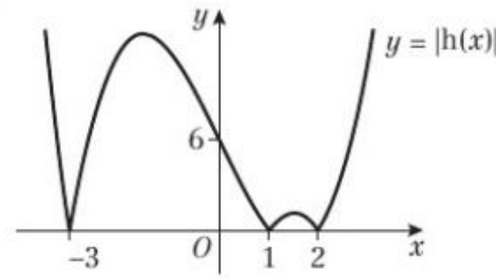
c



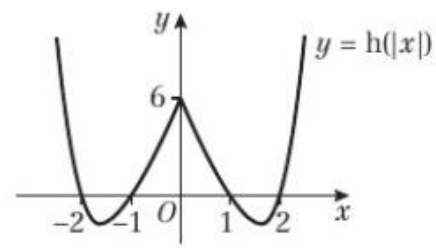
3 **a**



b

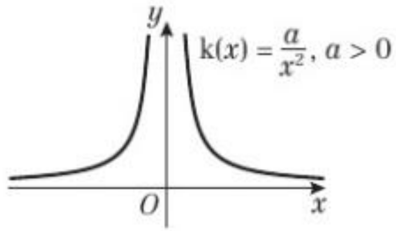


c



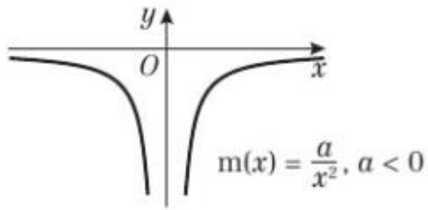
Homework Answers

4 a



b Both these graphs would match the original graph

c

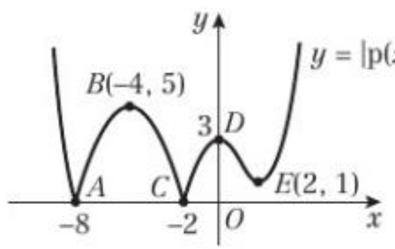


d i True, $|k(x)| = \left|\frac{a}{x^2}\right| = \left|\frac{-a}{x^2}\right| = |m(x)|$

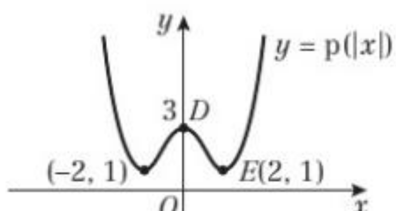
ii False, $k(|x|) = \frac{a}{|x|^2} \neq \frac{-a}{|x|^2} = m(|x|)$

iii True, $m(|x|) = \frac{-a}{|x|^2} = \frac{-a}{x^2} = m(x)$

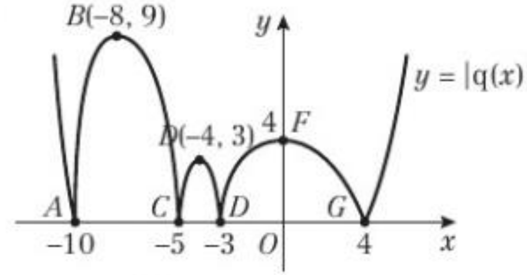
5 a



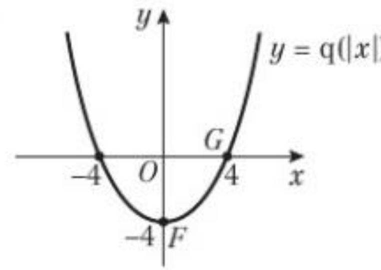
b



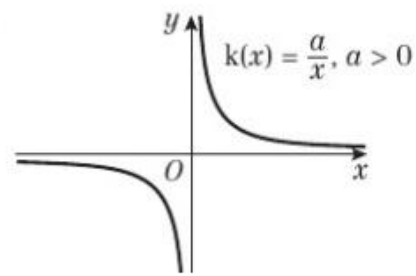
6 a



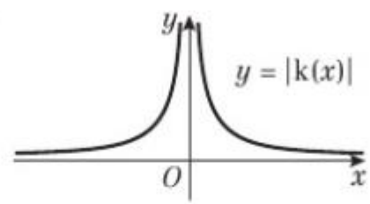
b



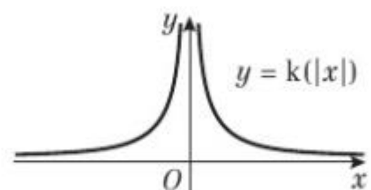
7 a



b

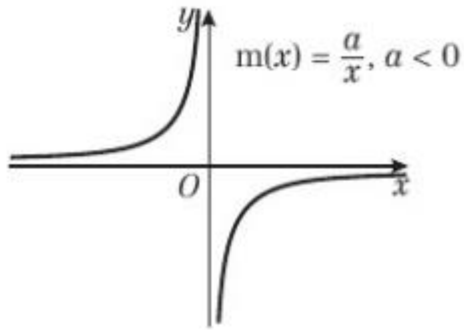


c



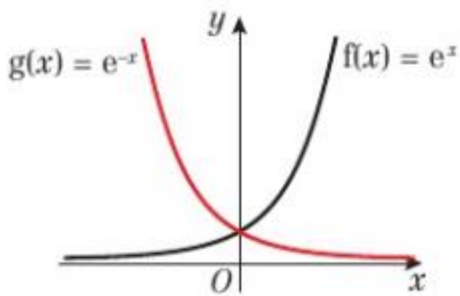
Homework Answers

8 **a**



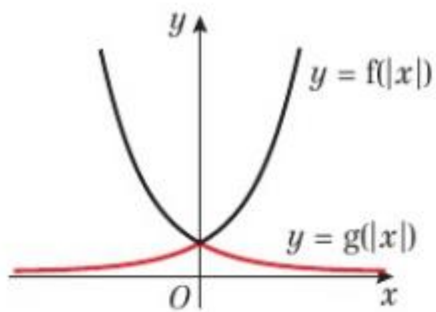
b They are reflections of each other in the x-axis.
$|\mathrm{m}(x)| = -\mathrm{m}(|x|)$

9 **a**

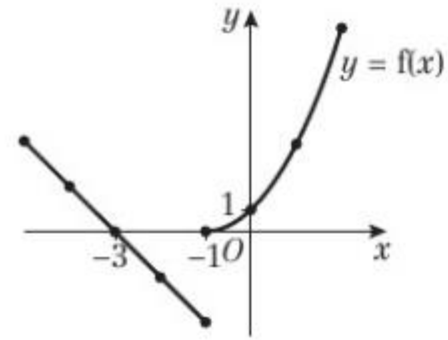


b They would be the same as the original graph.

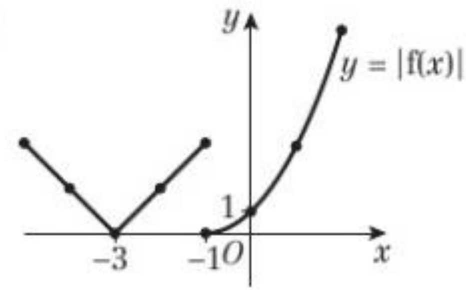
c



10 **a** $-4 < \mathrm{f}(x) \leqslant 9$



b



c