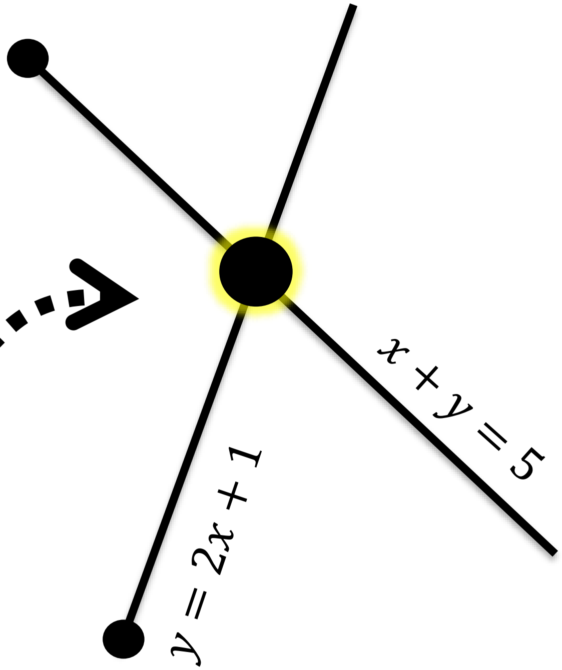
P1 Chapter 3: Inequalities

Solving Graphically

Simultaneous Equations and Graphs



Recall that a line with a given equation is **the set of all points that satisfy the equation**.

i.e. It is a graphical representation of the solution set where each (x, y) point represents each of the solutions x and y to the equation. e.g. $x = 3, y = 7$

Now suppose we introduced a second simultaneous equation:

$$y = 2x + 1$$
$$x + y = 5$$

The second line again consists of all points (x, y) which satisfy the equation. So what point must satisfy both equations simultaneously?
The point of intersection!

Example

a) On the same axes, draw the graphs of:

$$2x + y = 3$$
$$y = x^2 - 3x + 1$$

? a

b) Use your graph to write down the solutions to the simultaneous equations.

? b

c) What algebraic method (perhaps thinking about the previous chapter), could we have used to show the graphs would have intersected twice?

? c

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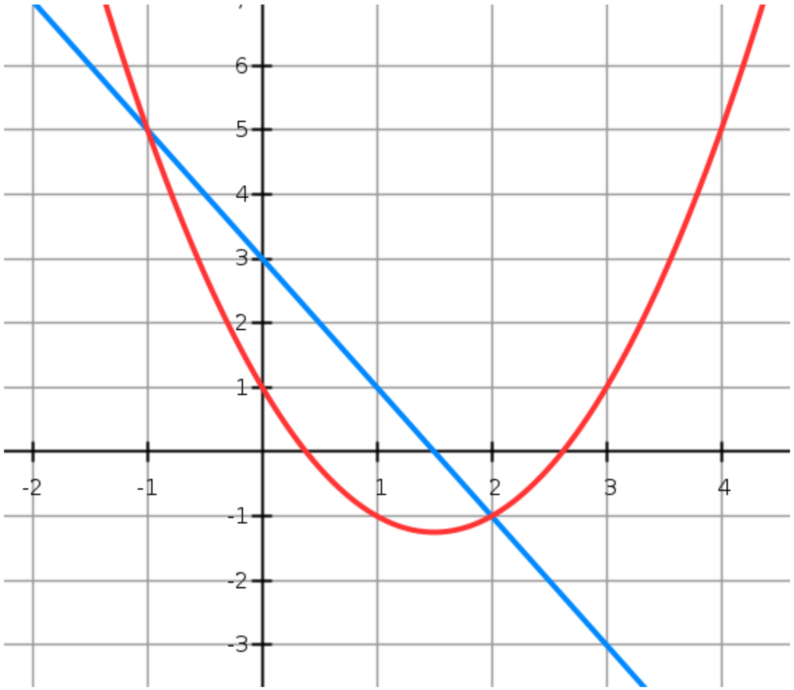

b) Use your graph to write down the solutions to the simultaneous equations.

$$\boldsymbol{x = -1, y = 5 \text{ } or}$$
$$\boldsymbol{x = 2, y = -1}$$

(We could always substitute into the original equations to check they work)

c) What algebraic method (perhaps thinking about the previous chapter), could we have used to show the graphs would have intersected twice?

Substituting linear equation into quadratic:

$$\boldsymbol{y = 3 - 2x}$$
$$\boldsymbol{\therefore 3 - 2x = x^2 - 3x + 1}$$
$$\boldsymbol{x^2 - x - 2 = 0}$$

Since there were two points of intersection, the equation must have two distinct solutions. Thus $\boldsymbol{b^2 - 4ac > 0}$

$$\boldsymbol{a = 1, b = -1, c = -2}$$
$$\boldsymbol{1 + 8 = 9 > 0}$$

Thus the quadratic has two distinct solutions, i.e. we have two points of intersection.

Another Example

a) On the same axes, draw the graphs of:

$$y = 2x - 2$$
$$y = x^2 + 4x + 1$$

? a

b) Prove algebraically that the lines never meet.

? b

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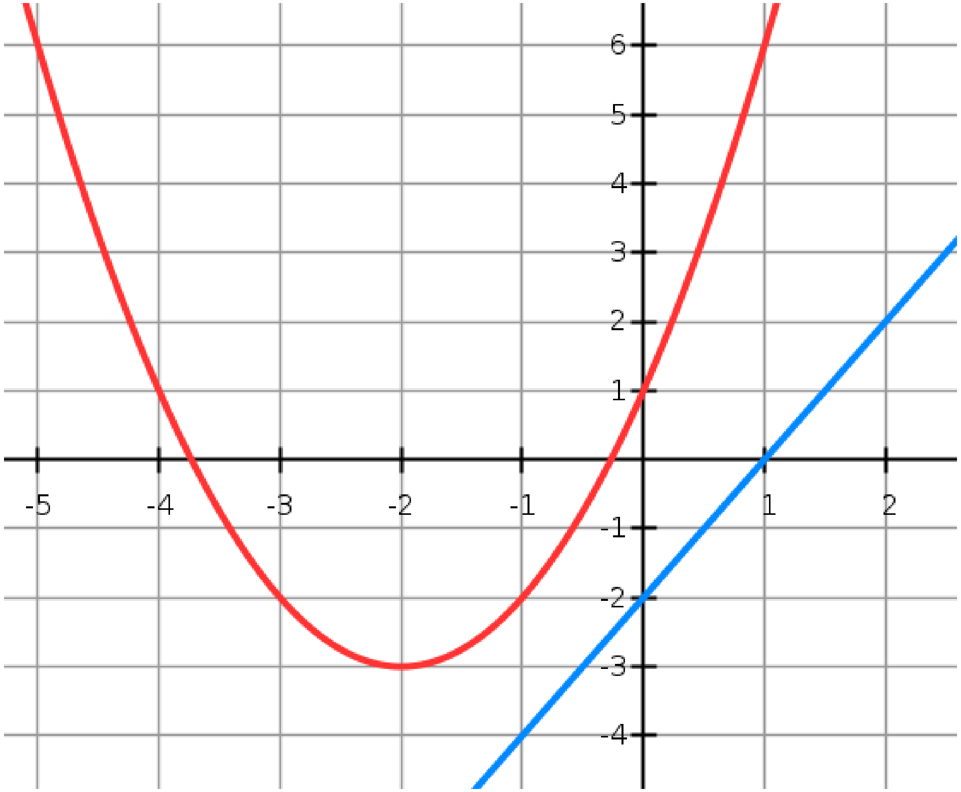

b) Prove algebraically that the lines never meet.

When we try to solve simultaneously by substitution, the equation must have no solutions.

$$\boldsymbol{x^2 + 4x + 1 = 2x - 2}$$
$$\boldsymbol{x^2 + 2x + 3 = 0}$$
$$\boldsymbol{a = 1, b = 2, c = 3}$$
$$\boldsymbol{b^2 - 4ac = 4 - 12 = -8}$$

$\boldsymbol{-8 < 0}$ **therefore no solutions, and therefore no points of intersection.**

Final Example

The line with equation $y = 2x + 1$ meets the curve with equation $kx^2 + 2y + (k-2) = 0$ at exactly one point. Given that k is a positive constant:

a) Find the value of k.

b) For this value of k, find the coordinates of this point of intersection.

a

? a

b

? a

Final Example

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a) Find the value of k.
b) For this value of k, find the coordinates of this point of intersection.

a Substituting:

$$kx^2 + 2(2x + 1) + k - 2 = 0$$
$$kx^2 + 4x + 2 + k - 2 = 0$$
$$kx^2 + 4x + k = 0$$

Since one point of intersection, equation has one solution, so $b^2 - 4ac = 0$.

$$a = k, b = 4, c = k$$
$$16 - 4k^2 = 0$$
$$k = \pm 2$$

But k is positive so $k = 2$.

b When $k = 2$, $\quad 2x^2 + 4x + 2 = 0$

$$x^2 + 2x + 1 = 0$$
$$(x + 1)^2 = 0 \quad \rightarrow \quad x = -1$$
$$y = 2(-1) + 1 = -1 \quad \rightarrow \quad (-1, -1)$$

We can breathe a sigh of relief as we were expecting one solution only.

Exercise 3.3

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Homework Exercise

1 In each case:

i draw the graphs for each pair of equations on the same axes

ii find the coordinates of the point of intersection.

a $y = 3x - 5$
$y = 3 - x$

b $y = 2x - 7$
$y = 8 - 3x$

c $y = 3x + 2$
$3x + y + 1 = 0$

2 **a** Use graph paper to draw accurately the graphs of $2y = 2x + 11$ and $y = 2x^2 - 3x - 5$ on the same axes.

b Use your graph to find the coordinates of the points of intersection.

c Verify your solutions by substitution.

3 **a** On the same axes sketch the curve with equation $x^2 + y = 9$ and the line with equation $2x + y = 6$.

b Find the coordinates of the points of intersection.

c Verify your solutions by substitution.

4 **a** On the same axes sketch the curve with equation $y = (x - 2)^2$ and the line with equation $y = 3x - 2$.

b Find the coordinates of the point of intersection.

Hint You need to use algebra in part **b** to find the coordinates.

5 Find the coordinates of the points at which the line with equation $y = x - 4$ intersects the curve with equation $y^2 = 2x^2 - 17$.

6 Find the coordinates of the points at which the line with equation $y = 3x - 1$ intersects the curve with equation $y^2 = xy + 15$.

Homework Exercise

7 Determine the number of points of intersection for these pairs of simultaneous equations.

a $y = 6x^2 + 3x - 7$
$y = 2x + 8$

b $y = 4x^2 - 18x + 40$
$y = 10x - 9$

c $y = 3x^2 - 2x + 4$
$7x + y + 3 = 0$

8 Given the simultaneous equations

$$2x - y = 1$$
$$x^2 + 4ky + 5k = 0$$

where k is a non-zero constant

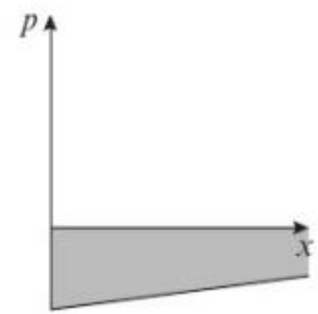
a show that $x^2 + 8kx + k = 0$. **(2 marks)**

Given that $x^2 + 8kx + k = 0$ has equal roots,

b find the value of k **(3 marks)**

c for this value of k, find the solution of the simultaneous equations. **(3 marks)**

9 A swimmer dives into a pool. Her position, p m, underwater can be modelled in relation to her horizontal distance, x m, from the point she entered the water as a quadratic equation $p = \frac{1}{2}x^2 - 3x$.

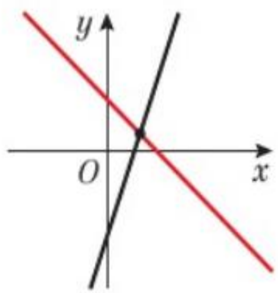


The position of the bottom of the pool can be modelled by the linear equation $p = 0.3x - 6$.

Determine whether this model predicts that the swimmer will touch the bottom of the pool. **(5 marks)**

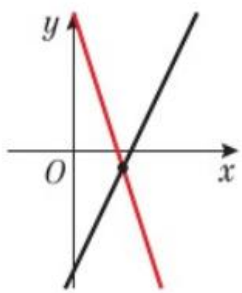
Homework Answers

1 **a** **i**



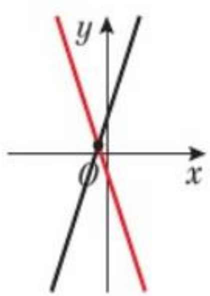
ii (2, 1)

b **i**



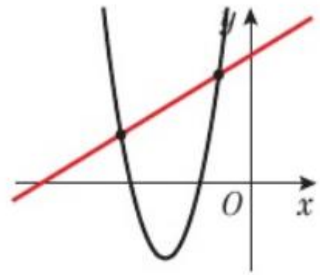
ii (3, –1)

c **i**



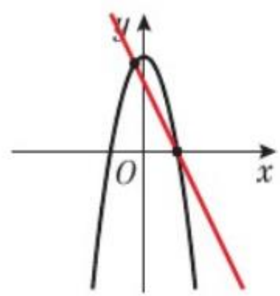
ii (–0.5, 0.5)

2 **a**



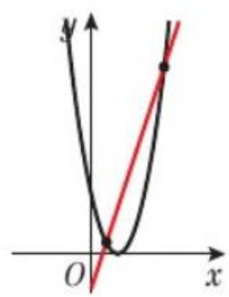
b (3.5, 9) and (–1.5, 4)

3 **a**



b (–1, 8) and (3, 0)

4 **a**



b (6, 16) and (1, 1)

5 (–11, –15) and (3, –1)

6 $(-1\frac{1}{6}, -4\frac{1}{2})$ and (2, 5)

7 **a** 2 points **b** 1 point **c** 0 points

8 **a** $y = 2x - 1$

$x^2 + 4k(2x - 1) + 5k = 0$

$x^2 + 8kx - 4k + 5k = 0$ $\quad x^2 + 8kx + k = 0$

b $k = \frac{1}{16}$ **c** $x = -\frac{1}{4}, y = -\frac{3}{2}$

9 If swimmer reaches the bottom of the pool

$0.5x^2 - 3x = 0.3x - 6$

$0.5x^2 - 3.3x + 6 = 0$

$b^2 - 4ac = (-3.3)^2 - 4 \times 0.5 \times 6 = -1.11$

negative so no points of intersection and diver does not reach the bottom of the pool