M2 Chapter 7: Application of Forces

Inclined Plane Dynamics

Particles moving on a rough plane

We've previously considered particles moving on smooth planes.
And particles in equilibrium (and not moving) on rough planes.
So let's consider particles moving on rough planes!

[Textbook] A particle is held at rest on a rough plane which is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is 0.5. The particle is released and slides down the plane. Find:
(a) the acceleration of the particle.
(b) the distance it slides in the first 2 seconds.

? Diagram

? a

? b

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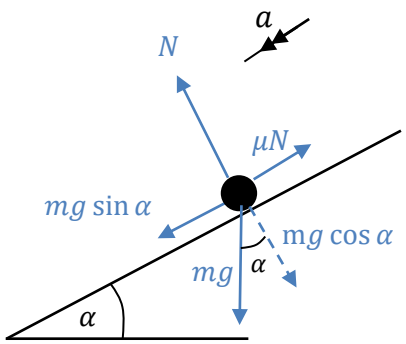
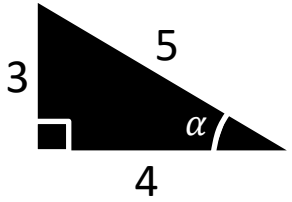


If $\tan\alpha = \frac{3}{4}$ then $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$



$$R(\nwarrow): \quad N = mg\cos\alpha$$
$$N = \frac{4}{5}mg$$
$$R(\swarrow): \quad mg\sin\alpha - \mu N = ma$$
$$\frac{3}{4}mg - \frac{2}{5}mg = ma$$
$$a = 0.2g = 2.0\ \text{ms}^{-2}$$

$$s = ?, u = 0, \cancel{v = ,} a = 0.2g, t = 2$$
$$s = ut + \frac{1}{2}at^2 = 3.9\ (3\text{sf})$$

Test Your Understanding

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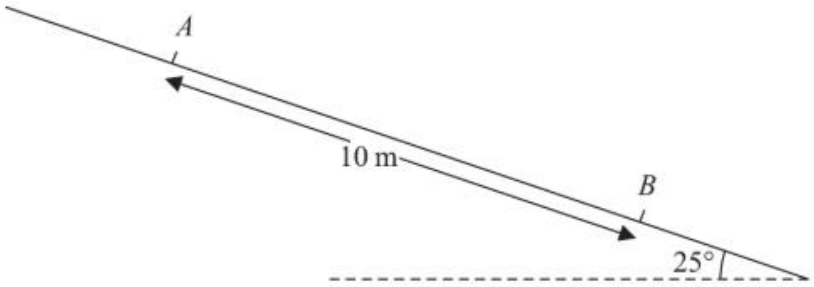

Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B, where $AB = 10$ m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B,

(b) the acceleration of P,

(c) the coefficient of friction between P and the plane.

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Test Your Understanding

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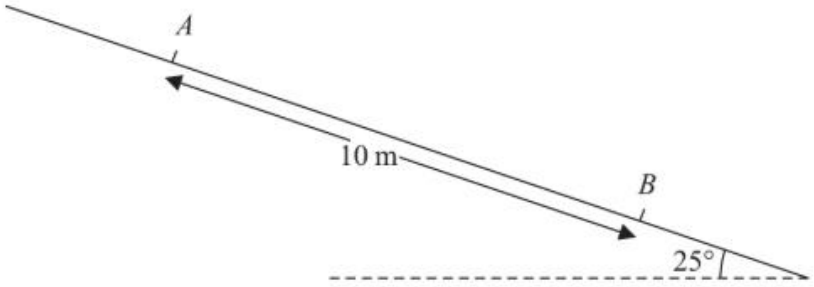

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(a) the speed of P at B, **(3)**

(b) the acceleration of P, **(2)**

(c) the coefficient of friction between P and the plane. **(5)**

$= \frac{2+v}{2} \times 3.5$ M1A1

$v = \frac{20}{3.5} - 2 = \frac{26}{7} = 3.71 \ (\text{m s}^{-1})$ A1

(b)

$a = \frac{v-u}{t} = \frac{\frac{26}{7} - 2}{3.5} = \frac{24}{49} = 0.490 \ (\text{m s}^{-2})$ M1A1

(c)

Normal support: $\mathsf{N} = 0.6g\cos 25°$ B1

Resolve parallel to the slope : $0.6g\sin 25° - \mu \times \mathsf{N} = 0.6 \times a$ M1A2

$\mu = 0.41$ or 0.411 A1

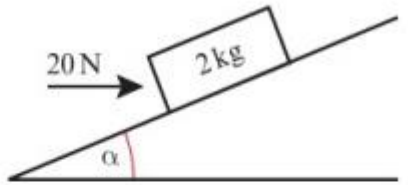
Exercise 7.5

Pearson Stats/Mechanics Year 2
Pages 62-63

Homework Exercise

1 A particle of mass 0.5 kg is placed on a smooth inclined plane. Given that the plane makes an angle of 20° with the horizontal, find the acceleration of the particle.

2 The diagram shows a box of mass 2 kg being pushed up a smooth plane by a horizontal force of magnitude 20 N. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.



Find:

a the normal reaction between the box and the plane

b the acceleration of the box up the plane.

3 A boy of mass 40 kg slides from rest down a straight slide of length 5 m. The slide is inclined to the horizontal at an angle of 20°. The coefficient of friction between the boy and the slide is 0.1. By modelling the boy as a particle, find:

a the acceleration of the boy

b the speed of the boy at the bottom of the slide.

4 A block of mass 20 kg is released from rest at the top of a rough slope. The slope is inclined to the horizontal at an angle of 30°. After 6 s the speed of the block is 21 m s^{-1}. Find the coefficient of friction between the block and the slope.

Homework Exercise

5 A book of mass 2 kg slides down a rough plane inclined at 20° to the horizontal. The acceleration of the book is $1.5\,\text{m s}^{-2}$. Find the coefficient of friction between the book and the plane.

6 A block of mass 4 kg is pulled up a rough slope, inclined at 25° to the horizontal, by means of a rope. The rope lies along the line of the slope. The tension in the rope is 30 N. Given that the acceleration of the block is $2\,\text{m s}^{-2}$ find the coefficient of friction between the block and the plane.

7 A parcel of mass 10 kg is released from rest on a rough plane which is inclined at 25° to the horizontal.

a Find the normal reaction between the parcel and the plane. **(2 marks)**

Two seconds after being released the parcel has moved 4 m down the plane.

b Find the coefficient of friction between the parcel and the plane. **(2 marks)**

8 A particle P is projected up a rough plane which is inclined at an angle a to the horizontal, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{3}$. The particle is projected from the point A with speed $20\,\text{m s}^{-1}$ and comes to instantaneous rest at the point B.

a Show that while P is moving up the plane its deceleration is $\frac{13g}{15}$. **(5 marks)**

b Find, to two significant figures, the distance AB. **(2 marks)**

c Find, to two significant figures, the time taken for P to move from A to B. **(2 marks)**

d Find the speed of P when it returns to A. **(7 marks)**

Homework Exercise

9 A particle of mass 2 kg is released from rest on a rough slope that is angled at α to the horizontal where $\tan\alpha = \frac{2}{5}$. After 3 seconds the speed of the particle is $6\,\text{m s}^{-1}$. Work out the coefficient of friction μ. **(8 marks)**

10 A particle of mass m kg is released from rest on a rough slope that is angled at α to the horizontal. The particle begins to accelerate down the slope. Show that the acceleration of the particle is independent of its mass.

11 A particle of mass 5 kg is projected up a rough slope at $16\,\text{m s}^{-1}$ and comes to rest at a point P after 5 s. Given that the slope is inclined at 10° to the horizontal,

a work out the coefficient of friction μ. **(7 marks)**

b State, with supporting calculations, whether the particle will remain at rest at P or will begin to slide back down the slope. **(2 marks)**

Homework Answers

1 $3.35\,\text{m s}^{-2}$ (3 s.f.)

2 **a** 27.7 N (3 s.f.) **b** $2.12\,\text{m s}^{-2}$

3 **a** $2.43\,\text{m s}^{-2}$ (3 s.f.) **b** $4.93\,\text{m s}^{-1}$ (3 s.f.)

4 0.165 (3 s.f.)

5 0.20 (2 s.f.)

6 0.15 (2 s.f.)

7 **a** 88.8 N (3 s.f.) **b** 0.24 (2 s.f.)

8 **a** $\frac{13g}{15}$ **b** 23.5 m (3 s.f.)

c 2.35 s (3 s.f.) **d** $12.4\,\text{m s}^{-1}$ (3 s.f.)

9 0.180 (3 s.f.)

10 R(↖): $R = mg\cos\alpha$, R(↗): $\mu R - mg\sin\alpha = ma$

$\mu mg\cos\alpha - mg\sin\alpha = ma$

$\mu g\cos\alpha - g\sin\alpha = a$

11 **a** 0.155

b The particle will slide down the hill. The component of weight down the slope $5g\sin 10 = 8.51$ N is greater than the friction $0.155 \times 5g\cos 10 = 7.48$ N.