P1 Chapter 2: Quadratics

Chapter Practice

Key Points

1 To solve a quadratic equation by factorising:

- Write the equation in the form $ax^2 + bx + c = 0$
- Factorise the left-hand side
- Set each factor equal to zero and solve to find the value(s) of x

2 The solutions of the equation $ax^2 + bx + c = 0$ where $a \neq 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3 $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

4 $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

5 The set of possible inputs for a function is called the **domain**.

The set of possible outputs of a function is called the **range**.

6 The **roots** of a function are the values of x for which $f(x) = 0$.

7 You can find the coordinates of a **turning point** of a quadratic graph by completing the square. If $f(x) = a(x + p)^2 + q$, the graph of $y = f(x)$ has a turning point at $(-p, q)$.

8 For the quadratic function $f(x) = ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is called the **discriminant**. The value of the discriminant shows how many roots $f(x)$ has:

- If $b^2 - 4ac > 0$ then a quadratic function has two distinct real roots.
- If $b^2 - 4ac = 0$ then a quadratic function has one repeated real root.
- If $b^2 - 4ac < 0$ then a quadratic function has no real roots

9 Quadratics can be used to model real-life situations.

Chapter Exercises

1 Solve the following equations without a calculator. Leave your answers in surd form where necessary.

a $y^2 + 3y + 2 = 0$ **b** $3x^2 + 13x - 10 = 0$ **c** $5x^2 - 10x = 4x + 3$ **d** $(2x - 5)^2 = 7$

2 Sketch graphs of the following equations:

a $y = x^2 + 5x + 4$ **b** $y = 2x^2 + x - 3$ **c** $y = 6 - 10x - 4x^2$ **d** $y = 15x - 2x^2$

3 $\mathrm{f}(x) = x^2 + 3x - 5$ and $\mathrm{g}(x) = 4x + k$, where k is a constant.

a Given that $\mathrm{f}(3) = \mathrm{g}(3)$, find the value of k. **(3 marks)**

b Find the values of x for which $\mathrm{f}(x) = \mathrm{g}(x)$. **(3 marks)**

4 Solve the following equations, giving your answers correct to 3 significant figures:

a $k^2 + 11k - 1 = 0$ **b** $2t^2 - 5t + 1 = 0$ **c** $10 - x - x^2 = 7$ **d** $(3x - 1)^2 = 3 - x^2$

5 Write each of these expressions in the form $p(x + q)^2 + r$, where p, q and r are constants to be found:

a $x^2 + 12x - 9$ **b** $5x^2 - 40x + 13$ **c** $8x - 2x^2$ **d** $3x^2 - (x + 1)^2$

6 Find the value k for which the equation $5x^2 - 2x + k = 0$ has exactly one solution. **(2 marks)**

7 Given that for all values of x:

$$3x^2 + 12x + 5 = p(x + q)^2 + r$$

a find the values of p, q and r. **(3 marks)**

b Hence solve the equation $3x^2 + 12x + 5 = 0$. **(2 marks)**

8 The function f is defined as $\mathrm{f}(x) = 2^{2x} - 20(2^x) + 64$, $x \in \mathbb{R}$.

a Write $\mathrm{f}(x)$ in the form $(2^x - a)(2^x - b)$, where a and b are real constants. **(2 marks)**

b Hence find the two roots of $\mathrm{f}(x)$. **(2 marks)**

9 Find, as surds, the roots of the equation:

$$2(x+1)(x-4)-(x-2)^2=0.$$

10 Use algebra to solve $(x-1)(x+2)=18$.

11 A diver launches herself off a springboard. The height of the diver, in metres, above the pool t seconds after launch can be modelled by the following function:

$$\mathrm{h}(t)=5t-10t^2+10,\ t\geqslant 0$$

a How high is the springboard above the water? **(1 mark)**

b Use the model to find the time at which the diver hits the water. **(3 marks)**

c Rearrange $\mathrm{h}(t)$ into the form $A-B(t-C)^2$ and give the values of the constants A, B and C. **(3 marks)**

d Using your answer to part **c** or otherwise, find the maximum height of the diver, and the time at which this maximum height is reached. **(2 marks)**

12 For this question, $\mathrm{f}(x)=4kx^2+(4k+2)x+1$, where k is a real constant.

a Find the discriminant of $\mathrm{f}(x)$ in terms of k. **(3 marks)**

b By simplifying your answer to part **a** or otherwise, prove that $\mathrm{f}(x)$ has two distinct real roots for all non-zero values of k. **(2 marks)**

c Explain why $\mathrm{f}(x)$ cannot have two distinct real roots when $k=0$. **(1 mark)**

13 Find all of the roots of the function $\mathrm{r}(x)=x^8-17x^4+16$. **(5 marks)**

Chapter Exercises

13 Find all of the roots of the function $\mathrm{r}(x) = x^8 - 17x^4 + 16$. **(5 marks)**

14 Lynn is selling cushions as part of an enterprise project. On her first attempt, she sold 80 cushions at the cost of £15 each. She hopes to sell more cushions next time. Her adviser suggests that she can expect to sell 10 more cushions for every £1 that she lowers the price.

a The number of cushions sold c can be modelled by the equation $c = 230 - Hp$, where £p is the price of each cushion and H is a constant. Determine the value of H. **(1 mark)**

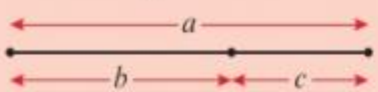
To model her total revenue, £r, Lynn multiplies the number of cushions sold by the price of each cushion. She writes this as $r = p(230 - Hp)$.

b Rearrange r into the form $A - B(p - C)^2$, where A, B and C are constants to be found. **(3 marks)**

c Using your answer to part **b** or otherwise, show that Lynn can increase her revenue by £122.50 through lowering her prices, and state the optimum selling price of a cushion. **(2 marks)**

Challenge

a The ratio of the lengths $a:b$ in this line is the same as the ratio of the lengths $b:c$.



Show that this ratio is $\frac{1 + \sqrt{5}}{2} : 1$.

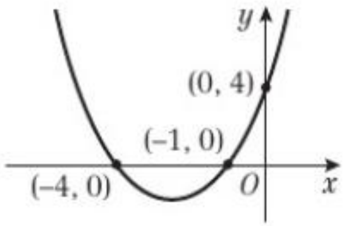
b Show also that the infinite square root

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}} = \frac{1 + \sqrt{5}}{2}$$

1 **a** $y = -1$ or -2 **b** $x = \frac{2}{3}$ or $x = -5$

c $x = -\frac{1}{5}$ or 3 **d** $\dfrac{5 \pm \sqrt{7}}{2}$

2 **a**



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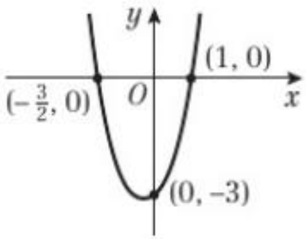

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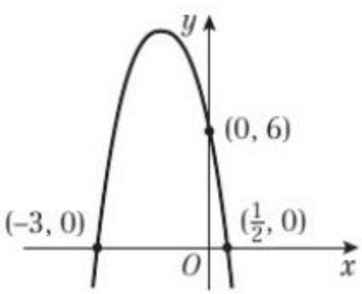

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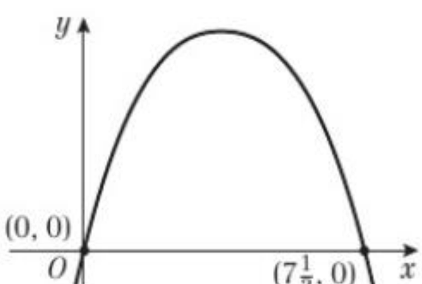

3 **a** $k = 1$ **b** $x = 3$ and $x = -2$

4 **a** $k = 0.0902$ or $k = -11.1$
b $t = 2.28$ or $t = 0.219$
c $x = -2.30$ or $x = 1.30$
d $x = 0.839$ or $x = -0.239$

5 **a** $(x + 6)^2 - 45$; $p = 1$, $q = 6$, $r = -45$
b $5(x - 4)^2 - 67$; $p = 5$, $q = -4$, $r = -67$
c $-2(x - 2)^2 + 8$; $p = -2$, $q = -2$, $r = 8$
d $2\left(x - \frac{1}{2}\right)^2 - \frac{3}{2}$; $p = 2$, $q = -\frac{1}{2}$, $r = -\frac{3}{2}$

6 $k = \frac{1}{5}$

7 **a** $p = 3$, $q = 2$, $r = -7$ **b** $-2 \pm \sqrt{\frac{7}{3}}$

8 **a** $f(x) = (2^x - 16)(2^x - 4)$ **b** 4 and 2

9 $1 \pm \sqrt{13}$

10 $x = -5$ or $x = 4$

11 **a** 10 m **b** 1.28 s
c $h(t) = 10.625 - 10(t - 0.25)^2$
$A = 10.625$, $B = 10$, $C = 0.25$
d 10.625 m at 0.25 s

12 **a** $16k^2 + 4$
b $k^2 \geqslant 0$ for all k, so $16k^2 + 4 > 0$
c When $k = 0$, $f(x) = 2x + 1$; this is a linear function with only one root

13 1, −1, 2 and −2

14 **a** $H = 10$
b $r = 1322.5 - 10(p - 11.5)^2$
$A = 1322.5$, $B = 10$, $C = 11.5$
c Old revenue is 80 × £15 = £1200; new revenue is £1322.50; difference is £122.50. The best selling price of a cushion is £11.50.

Challenge

a $\dfrac{a+b}{a} = \dfrac{a}{b}$

$a^2 - ba - b^2 = 0$

Using quadratic formula: $a = \dfrac{b + \sqrt{5b^2}}{2}$

So $a : b$ is $\dfrac{b + \sqrt{5b^2}}{2} : b$

Dividing by b: $\dfrac{1 + \sqrt{5}}{2} : 1$

b Let $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$

So $x = \sqrt{1 + x} \Rightarrow x^2 - x - 1 = 0$

Using quadratic formula: $x = \dfrac{1 + \sqrt{5}}{2}$