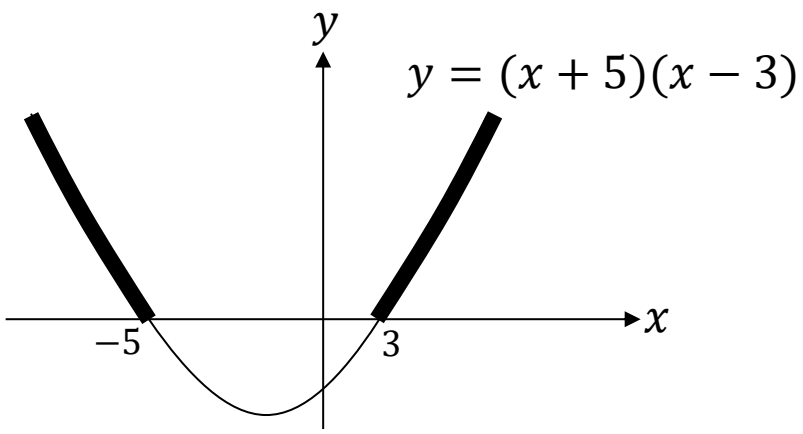
P1 Chapter 3: Inequalities

Graph Inequality

Inequalities on Graphs



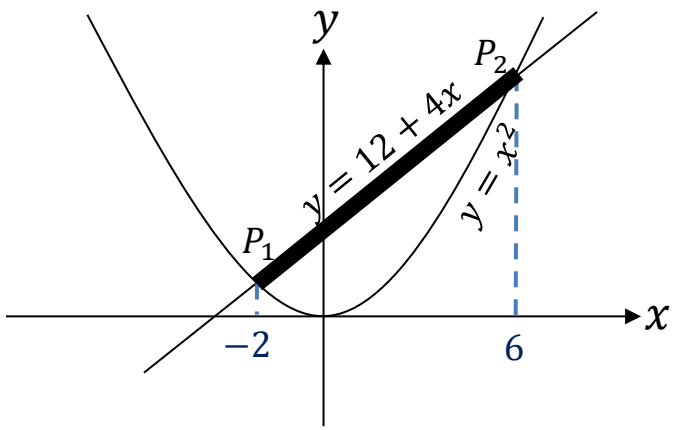
When we solved quadratic inequalities, e.g. $(x+5)(x-3) > 0$
We plotted $y = (x+5)(x-3)$ and observed the values of x for which $y > 0$.

Can we use a similar method when we don't have 0 on one side?

Example: L_1 has equation $y = 12 + 4x$. L_2 has equation $y = x^2$.
The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of P_1 and P_2, the points of intersection.

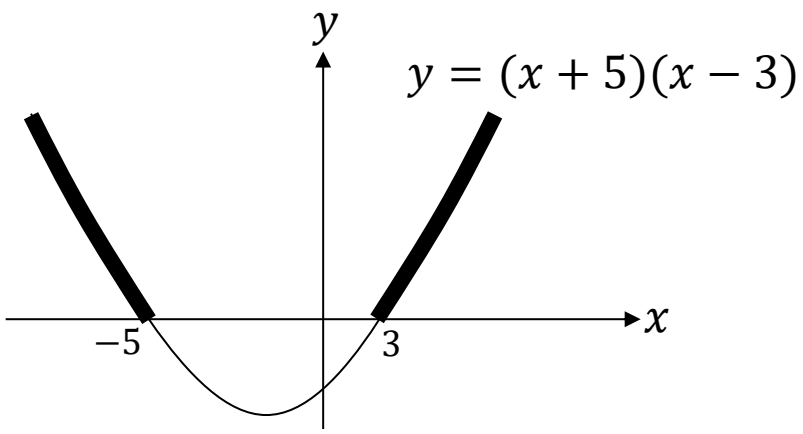
b) Hence write down the solution to the inequality $12 + 4x > x^2$.



a ?

b ?

Inequalities on Graphs



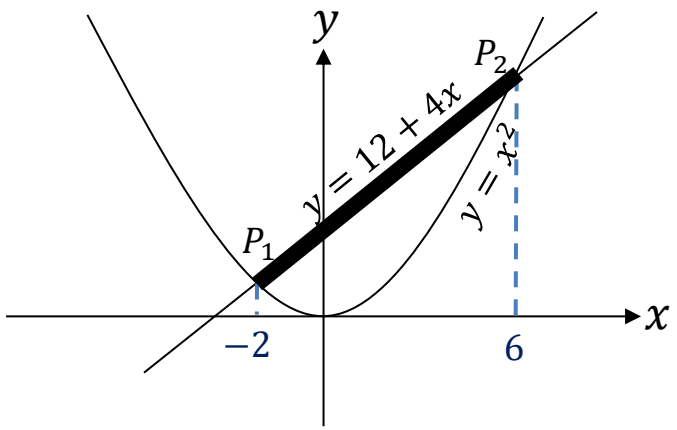
When we solved quadratic inequalities, e.g. $(x+5)(x-3) > 0$ We plotted $y = (x+5)(x-3)$ and observed the values of x for which $y > 0$.

Can we use a similar method when we don't have 0 on one side?

Example: L_1 has equation $y = 12 + 4x$. L_2 has equation $y = x^2$.
The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of P_1 and P_2, the points of intersection.

b) Hence write down the solution to the inequality $12 + 4x > x^2$.



a Solve simultaneously to find points of intersection:

$$x^2 = 12 + 4x$$
$$x^2 - 4x - 12 = 0$$
$$x = 6, x = -2 \quad \rightarrow \quad y = 36, y = 4$$
$$P_1(-2,4), \qquad P_2(6,36)$$

b When $12 + 4x > x^2$ the L_1 graph is above the L_2 graph. This happens when $-2 < x < 6$.

Exercise 3.6

Pearson Pure Mathematics Year 1/AS

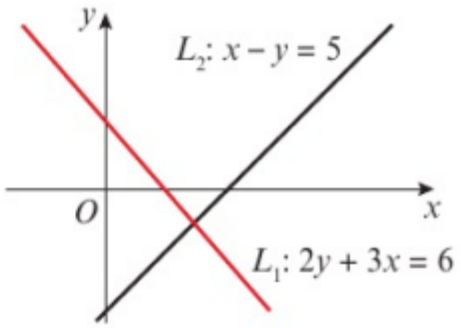
Page 21

Homework Exercise

1 L_1 has equation $2y + 3x = 6$.
L_2 has the equation $x - y = 5$.
The diagram shows a sketch of L_1 and L_2.

a Find the coordinates of P, the point of intersection.

b Hence write down the solution to the inequality $2y + 3x > x - y$.



2 For each pair of functions:

i Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same axes.

ii Find the coordinates of any points of intersection.

iii Write down the solutions to the inequality $f(x) \leqslant g(x)$.

a $f(x) = 3x - 7$
$g(x) = 13 - 2x$

b $f(x) = 8 - 5x$
$g(x) = 14 - 3x$

c $f(x) = x^2 + 5$
$g(x) = 5 - 2x$

d $f(x) = 3 - x^2$
$g(x) = 2x - 12$

e $f(x) = x^2 - 5$
$g(x) = 7x + 13$

f $f(x) = 7 - x^2$
$g(x) = 2x - 8$

Homework Exercise

3 Find the set of values of x for which the curve with equation $y = f(x)$ is below the line with equation $y = g(x)$.

a $f(x) = 3x^2 - 2x - 1$
$g(x) = x + 5$

b $f(x) = 2x^2 - 4x + 1$
$g(x) = 3x - 2$

c $f(x) = 5x - 2x^2 - 4$
$g(x) = -2x - 1$

d $f(x) = \frac{2}{x}, x \neq 0$
$g(x) = 1$

e $f(x) = \frac{3}{x^2} - \frac{4}{x}, x \neq 0$
$g(x) = -1$

f $f(x) = \frac{2}{x+1}, x \neq -1$
$g(x) = 8$

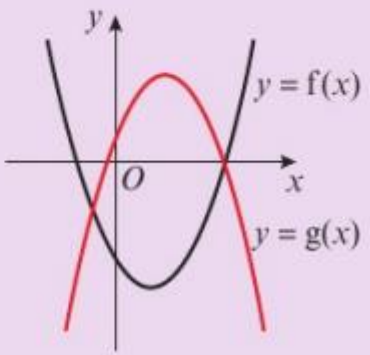
Challenge

The sketch shows the graphs of

$f(x) = x^2 - 4x - 12$
$g(x) = 6 + 5x - x^2$

a Find the coordinates of the points of intersection.

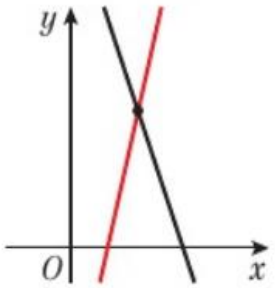
b Find the set of values of x for which $f(x) < g(x)$.
Give your answer in set notation.



Homework Answers

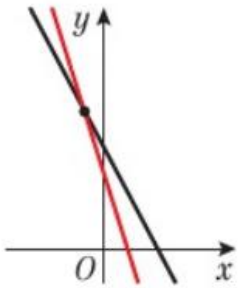
1 **a** P(3.2, −1.8) **b** $x < 3.2$

2 **a** **i**



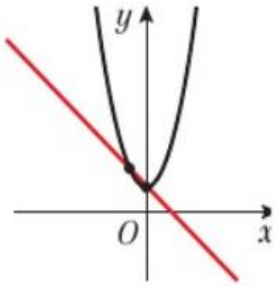
ii (4, 5) **iii** $x \leqslant 4$

b **i**



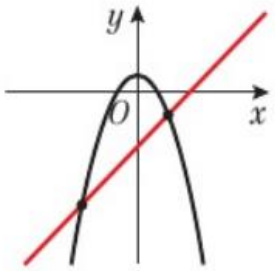
ii (−3, 23) **iii** $x \geqslant -3$

c **i**



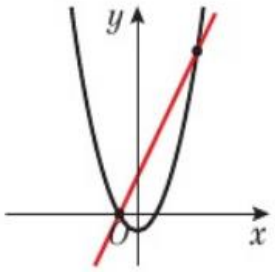
ii (−2, 9), (0, 5) **iii** $-2 \leqslant x \leqslant 0$

d **i**



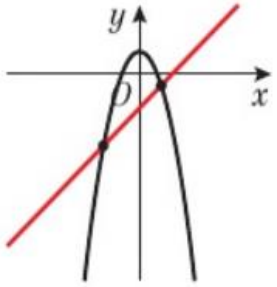
ii (−5, −22), (3, −6) **iii** $x \leqslant -5$ or $x \geqslant 3$

e **i**



ii (−2, −1), (9, 76) **iii** $-2 \leqslant x \leqslant 9$

f **i**



ii (−5, −18), (3, −2) **iii** $x \leqslant -5$ or $x \geqslant 3$

3 **a** $-1 < x < 2$
b $0.5 < x < 3$
c $x < 0.5$ or $x > 3$
d $x < 0$ or $x > 2$
e $1 < x < 3$
f $x < -1$ or $x > -0.75$

Challenge
a (−1.5, −3.75), (6, 0)
b $\{x: -1.5 < x < 6\}$