
P1 Chapter 1: Inequalities

Chapter Practice

Key Points

- 1** Linear simultaneous equations can be solved using elimination or substitution.
- 2** Simultaneous equations with one linear and one quadratic equation can have up to two pairs of solutions. You need to make sure the solutions are paired correctly.
- 3** The solutions of a pair of simultaneous equations represent the points of intersection of their graphs.
- 4** For a pair of simultaneous equations that produce a quadratic equation of the form $ax^2 + bx + c = 0$:
 - $b^2 - 4ac > 0$ two real solutions
 - $b^2 - 4ac = 0$ one real solution
 - $b^2 - 4ac < 0$ no real solutions
- 5** The solution of an inequality is the set of all real numbers x that make the inequality true.
- 6** To solve a quadratic inequality:
 - Rearrange so that the right-hand side of the inequality is 0
 - Solve the corresponding quadratic equation to find the critical values
 - Sketch the graph of the quadratic function
 - Use your sketch to find the required set of values.

Key Points

- 7 The values of x for which the curve $y = f(x)$ is **below** the curve $y = g(x)$ satisfy the inequality $f(x) < g(x)$.
The values of x for which the curve $y = f(x)$ is **above** the curve $y = g(x)$ satisfy the inequality $f(x) > g(x)$.
- 8 $y < f(x)$ represents the points on the coordinate grid below the curve $y = f(x)$.
 $y > f(x)$ represents the points on the coordinate grid above the curve $y = f(x)$.
- 9 If $y > f(x)$ or $y < f(x)$ then the curve $y = f(x)$ is not included in the region and is represented by a dotted line.
If $y \geq f(x)$ or $y \leq f(x)$ then the curve $y = f(x)$ is included in the region and is represented by a solid line.

Chapter Exercises

1 $2kx - y = 4$

$$4kx + 3y = -2$$

are two simultaneous equations, where k is a constant.

a Show that $y = -2$.

(3 marks)

b Find an expression for x in terms of the constant k .

(1 mark)

2 Solve the simultaneous equations:

$$x + 2y = 3$$

$$x^2 - 4y^2 = -33$$

(7 marks)

3 Given the simultaneous equations

$$x - 2y = 1$$

$$3xy - y^2 = 8$$

a Show that $5y^2 + 3y - 8 = 0$.

(2 marks)

b Hence find the pairs (x, y) for which the simultaneous equations are satisfied.

(5 marks)

4 a By eliminating y from the equations

$$x + y = 2$$

$$x^2 + xy - y^2 = -1$$

show that $x^2 - 6x + 3 = 0$.

(2 marks)

b Hence, or otherwise solve the simultaneous equations

$$x + y = 2$$

$$x^2 + xy - y^2 = -1$$

giving x and y in the form $a \pm b\sqrt{6}$, where a and b are integers.

(5 marks)

Chapter Exercises

5 a Given that $3^x = 9^{y-1}$, show that $x = 2y - 2$.

(1 mark)

b Solve the simultaneous equations:

$$x = 2y - 2$$

$$x^2 = y^2 + 7$$

(6 marks)

6 Solve the simultaneous equations:

$$x + 2y = 3$$

$$x^2 - 2y + 4y^2 = 18$$

(7 marks)

7 The curve and the line given by the equations

$$kx^2 - xy + (k + 1)x = 1$$

$$-\frac{k}{2}x + y = 1$$

where k is a non-zero constant, intersect at a single point.

a Find the value of k .

(5 marks)

b Give the coordinates of the point of intersection of the line and the curve.

(3 marks)

8 A person throws a ball in a sports hall. The height of the ball, h m, can be modelled in relation to the horizontal distance from the point it was thrown from by the quadratic equation:

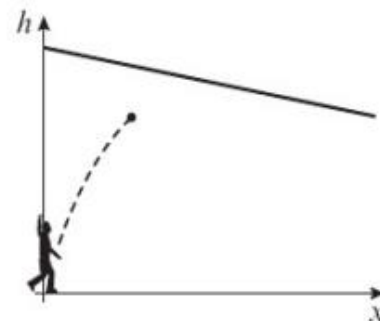
$$h = -\frac{3}{10}x^2 + \frac{5}{2}x + \frac{3}{2}$$

The hall has a sloping ceiling which can be modelled with equation

$$h = \frac{15}{2} - \frac{1}{5}x.$$

Determine whether the model predicts that the ball will hit the ceiling.

(5 marks)



Chapter Exercises

- 9 Give your answers in set notation.
- a Solve the inequality $3x - 8 > x + 13$. (2 marks)
 - b Solve the inequality $x^2 - 5x - 14 > 0$. (4 marks)
- 10 Find the set of values of x for which $(x - 1)(x - 4) < 2(x - 4)$. (6 marks)
- 11 a Use algebra to solve $(x - 1)(x + 2) = 18$. (2 marks)
- b Hence, or otherwise, find the set of values of x for which $(x - 1)(x + 2) > 18$.
Give your answer in set notation. (2 marks)
- 12 Find the set of values of x for which:
- a $6x - 7 < 2x + 3$ (2 marks)
 - b $2x^2 - 11x + 5 < 0$ (4 marks)
 - c $5 < \frac{20}{x}$ (4 marks)
 - d both $6x - 7 < 2x + 3$ and $2x^2 - 11x + 5 < 0$. (2 marks)
- 13 Find the set of values of x that satisfy $\frac{8}{x^2} + 1 \leq \frac{9}{x}$, $x \neq 0$ (5 marks)
- 14 Find the values of k for which $kx^2 + 8x + 5 = 0$ has real roots. (3 marks)
- 15 The equation $2x^2 + 4kx - 5k = 0$, where k is a constant, has no real roots.
Prove that k satisfies the inequality $-\frac{5}{2} < k < 0$. (3 marks)

Chapter Exercises

- 16 a** Sketch the graphs of $y = f(x) = x^2 + 2x - 15$ and $g(x) = 6 - 2x$ on the same axes. (4 marks)
- b** Find the coordinates of any points of intersection. (3 marks)
- c** Write down the set of values of x for which $f(x) > g(x)$. (1 mark)
- 17** Find the set of values of x for which the curve with equation $y = 2x^2 + 3x - 15$ is below the line with equation $y = 8 + 2x$. (5 marks)
- 18** On a coordinate grid, shade the region that satisfies the inequalities:
 $y > x^2 + 4x - 12$ and $y < 4 - x^2$. (5 marks)
- 19 a** On a coordinate grid, shade the region that satisfies the inequalities
 $y + x < 6$, $y < 2x + 9$, $y > 3$ and $x > 0$. (6 marks)
- b** Work out the area of the shaded region. (2 marks)

Challenge

- 1** Find the possible values of k for the quadratic equation $2kx^2 + 5kx + 5k - 3 = 0$ to have real roots.
- 2** A straight line has equation $y = 2x - k$ and a parabola has equation $y = 3x^2 + 2kx + 5$ where k is a constant. Find the range of values of k for which the line and the parabola do not intersect.

Chapter Answers

1 a $4kx - 2y = 8$

$$4kx + 3y = -2$$

$$-5y = 10$$

$$y = -2$$

b $x = \frac{1}{k}$

2 $x = -4, y = 3\frac{1}{2}$

3 a Substitute $x = 1 + 2y$ into $3xy - y^2 = 8$

b $(3, 1)$ and $(-\frac{11}{5}, -\frac{8}{5})$

4 a Substitute $y = 2 - x$ into $x^2 + xy - y^2 = 0$

b $x = 3 \pm \sqrt{6}, y = -1 \pm \sqrt{6}$

5 a $3^x = (3^2)^{y-1} = 3^{2y-2} \Rightarrow x = 2y - 2$

b $x = 4, y = 3$ and $x = -2\frac{2}{3}, y = -\frac{1}{3}$

6 $x = -1\frac{1}{2}$, $y = 2\frac{1}{4}$ and $x = 4$, $y = -\frac{1}{2}$

7 **a** $k = -2$ **b** $(-1, 2)$

8 Yes, the ball will hit the ceiling

9 a $\{x: x > 10\frac{1}{2}\}$

b $\{x: x < -2\} \cup \{x: x > 7\}$

10 $3 < x < 4$

11 a $x = -5, x = 4$

b $\{x: x < -5\} \cup \{x: x > 4\}$

12 a $x < 2\frac{1}{2}$

b $\frac{1}{2} < x < 5$

c $0 < x < 4$

d $\frac{1}{2} < x < 2\frac{1}{2}$

13 $1 \leq x \leq 8$

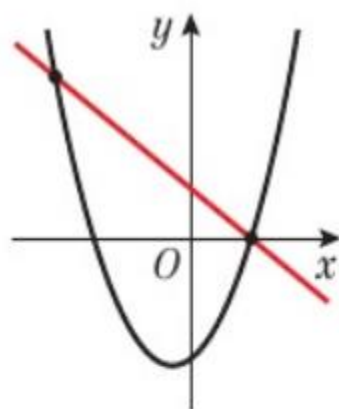
14 $k \leq 3\frac{1}{5}$

15 $b^2 < 4ac$ so $16k^2 < -40k$

$$8k(2k + 5) < 0 \text{ so } -\frac{5}{2} < k < 0$$

Chapter Answers

16 a

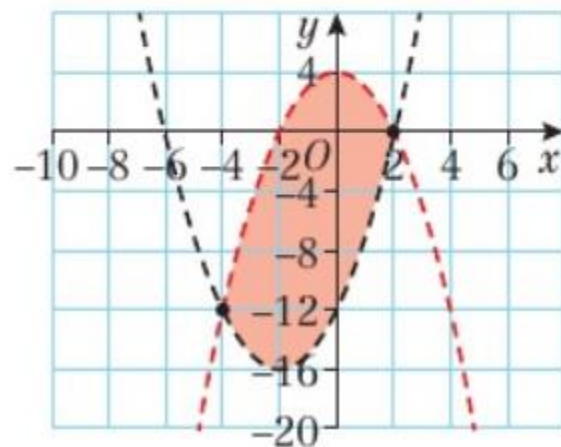


b $(-7, 20), (3, 0)$

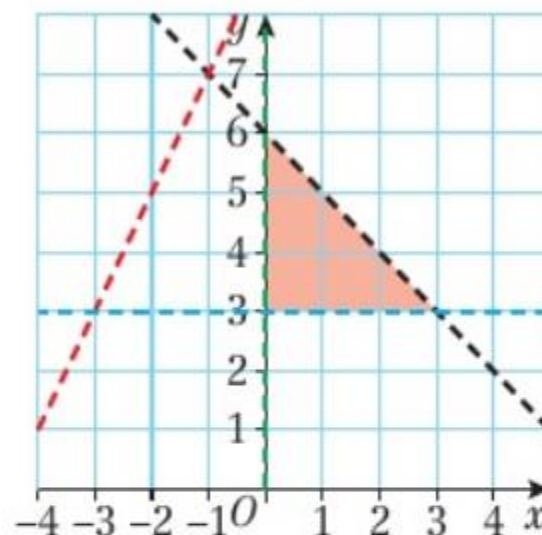
c $x < -7, x > 3$

17 $\frac{1}{4}(-1 - \sqrt{185}) < x < \frac{1}{4}(-1 + \sqrt{185})$

18



19 a



b $\frac{9}{2}$

Challenge

1 $0 < x < 1.6$

2 $-2 < k < 7$