M1 Chapter 9: Constant Acceleration

Chapter Practice

Key Points

1 Velocity is the **rate of change** of displacement.

On a displacement–time graph the **gradient** represents the velocity.

If the displacement–time graph is a straight line, then the velocity is constant.

2 $\text{Average velocity} = \dfrac{\text{displacement from starting point}}{\text{time taken}}$

3 $\text{Average speed} = \dfrac{\text{total distance travelled}}{\text{time taken}}$

4 Acceleration is the **rate of change** of velocity.

In a velocity–time graph the **gradient** represents the acceleration.

If the velocity–time graph is a straight line, then the acceleration is constant.

5 The area between a velocity–time graph and the horizontal axis represents the distance travelled.

For motion in a straight line with positive velocity, the area under the velocity–time graph up to a point t represents the displacement at time t.

6 You need to be able to use and to derive the five formulae for solving problems about particles moving in a straight line with constant acceleration.

- $v = u + at$
- $s = \left(\dfrac{u+v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$

7 The force of **gravity** causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does not depend on the mass of the object.

8 An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of $g = 9.8\ \text{m s}^{-2}$.

Chapter Exercises

1 A car accelerates in a straight line at a constant rate, starting from rest at a point A and reaching a velocity of $45\ \text{km h}^{-1}$ in 20 s. This velocity is then maintained and the car passes a point B 3 minutes after leaving A.

a Sketch a velocity–time graph to illustrate the motion of the car.

b Find the displacement of the car from its starting point after 3 minutes.

2 A particle is moving on an axis Ox. From time $t = 0$ s to time $t = 32$ s, the particle is travelling with constant velocity $15\ \text{m s}^{-1}$. The particle then decelerates from $15\ \text{m s}^{-1}$ to rest in T seconds.

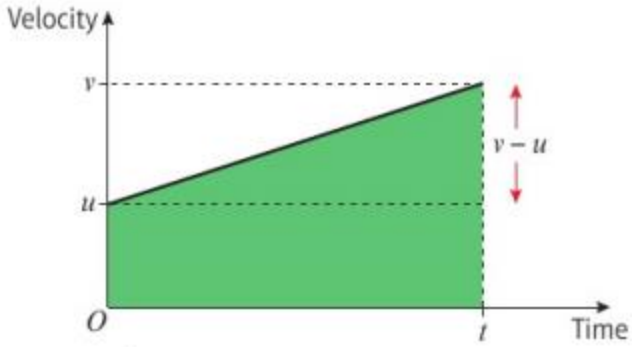
a Sketch a velocity–time graph to illustrate the motion of the particle.

The total distance travelled by the particle is 570 m.

b Find the value of T.

c Sketch a displacement–time graph illustrating the motion of the particle.

3 The velocity–time graph represents the motion of a particle moving in a straight line accelerating from velocity u at time 0 to velocity v at time t.



a Use the graph to show that:

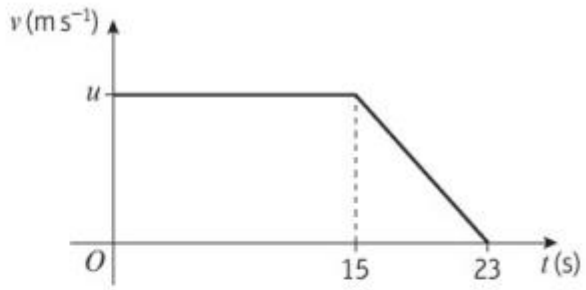
i $v = u + at$ **ii** $s = \left(\frac{u+v}{2}\right)t$

b Hence show that:

i $v^2 = u^2 + 2as$ **ii** $s = ut + \frac{1}{2}at^2$ **iii** $s = vt - \frac{1}{2}at^2$

Chapter Exercises

4 The diagram is a velocity–time graph representing the motion of a cyclist along a straight road. At time $t = 0$ s, the cyclist is moving with velocity u m s^{-1}. The velocity is maintained until time $t = 15$ s, when she slows down with constant deceleration, coming to rest when $t = 23$ s. The total distance she travels in 23 s is 152 m. Find the value of u.



5 A car travelling on a straight road slows down with constant deceleration. The car passes a road sign with velocity 40 km h^{-1} and a post box with velocity of 24 km h^{-1}. The distance between the road sign and the post box is 240 m. Find, in m s^{-2}, the deceleration of the car.

6 A particle P is moving along the x-axis with constant deceleration 2.5 m s^{-2}. At time $t = 0$ s, P passes through the origin with velocity 20 m s^{-1} in the direction of x increasing. At time $t = 12$ s, P is at the point A. Find:

a the distance OA

b the total distance P travels in 12 s.

7 A ball is thrown vertically downward from the top of a tower with speed 6 m s^{-1}. The ball strikes the ground with speed 25 m s^{-1}. Find the time the ball takes to move from the top of the tower to the ground.

8 A child drops a ball from a point at the top of a cliff which is 82 m above the sea. The ball is initially at rest. Find:

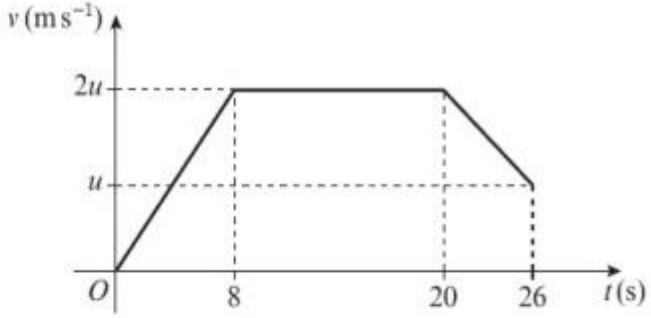
a the time taken for the ball to reach the sea

b the speed with which the ball hits the sea.

c State one physical factor which has been ignored in making your calculation.

Chapter Exercises

9 A particle moves 451 m in a straight line. The diagram shows a speed–time graph illustrating the motion of the particle. The particle starts at rest and accelerates at a constant rate for 8 s reaching a speed of $2u\,\text{m s}^{-1}$. The particle then travels at a constant speed for 12 seconds before decelerating uniformly, reaching a speed of $u\,\text{m s}^{-1}$ at time $t = 26$ s. Find:



a the value of u

b the distance moved by the particle while its speed is less than $u\,\text{m s}^{-1}$.

10 A train is travelling with constant acceleration along a straight track. At time $t = 0$ s, the train passes a point O travelling with velocity $18\,\text{m s}^{-1}$. At time $t = 12$ s, the train passes a point P travelling with velocity $24\,\text{m s}^{-1}$. At time $t = 20$ s, the train passes a point Q. Find:

a the speed of the train at Q **(5 marks)**

b the distance from P to Q. **(2 marks)**

11 A particle moves along a straight line, from a point X to a point Y, with constant acceleration. The distance from X to Y is 104 m. The particle takes 8 s to move from X to Y and the speed of the particle at Y is $18\,\text{m s}^{-1}$. Find:

a the speed of the particle at X **(3 marks)**

b the acceleration of the particle. **(2 marks)**

The particle continues to move with the same acceleration until it reaches a point Z. At Z the speed of the particle is three times the speed of the particle at X.

c Find the distance XZ. **(4 marks)**

Chapter Exercises

12 A pebble is projected vertically upwards with speed 21 m s^{-1} from a point 32 m above the ground. Find:

a the speed with which the pebble strikes the ground **(3 marks)**

b the total time for which the pebble is more than 40 m above the ground. **(4 marks)**

c Sketch a velocity–time graph for the motion of the pebble from the instant it is projected to the instant it hits the ground, showing the values of t at any points where the graph intercepts the horizontal axis. **(4 marks)**

13 A car is moving along a straight road with uniform acceleration. The car passes a checkpoint A with speed 12 m s^{-1} and another checkpoint C with speed 32 m s^{-1}. The distance between A and C is 1100 m.

a Find the time taken by the car to move from A to C. **(2 marks)**

b Given that B is the midpoint of AC, find the speed with which the car passes B. **(2 marks)**

14 A particle is projected vertically upwards with a speed of 30 m s^{-1} from a point A. The point B is h metres above A. The particle moves freely under gravity and is above B for a time 2.4 s. Calculate the value of h. **(5 marks)**

15 Two cars A and B are moving in the same direction along a straight horizontal road. At time $t = 0$, they are side by side, passing a point O on the road. Car A travels at a constant speed of 30 m s^{-1}. Car B passes O with a speed of 20 m s^{-1}, and has constant acceleration of 4 m s^{-2}. Find:

a the speed of B when it has travelled 78 m from O **(2 marks)**

b the distance from O of A when B is 78 m from O **(3 marks)**

c the time when B overtakes A. **(4 marks)**

Chapter Exercises

16 A car is being driven on a straight stretch of motorway at a constant velocity of $34\,\text{m s}^{-1}$, when it passes a velocity restriction sign S warning of road works ahead and requiring speeds to be reduced to $22\,\text{m s}^{-1}$. The driver continues at her velocity for 2 s after passing S. She then reduces her velocity to $22\,\text{m s}^{-1}$ with constant deceleration of $3\,\text{m s}^{-2}$, and continues at the lower velocity.

a Draw a velocity–time graph to illustrate the motion of the car after it passes S. **(2 marks)**

b Find the shortest distance before the road works that S should be placed on the road to ensure that a car driven in this way has had its velocity reduced to $22\,\text{m s}^{-1}$ by the time it reaches the start of the road works. **(4 marks)**

17 A train starts from rest at station A and accelerates uniformly at $3x\,\text{m s}^{-2}$ until it reaches a velocity of $30\,\text{m s}^{-1}$. For the next T seconds the train maintains this constant velocity. The train then decelerates uniformly at $x\,\text{m s}^{-2}$ until it comes to rest at a station B. The distance between the stations is 6 km and the time taken from A to B is 5 minutes.

a Sketch a velocity–time graph to illustrate this journey. **(2 marks)**

b Show that $\frac{40}{x} + T = 300$. **(4 marks)**

c Find the value of T and the value of x. **(2 marks)**

d Calculate the distance the train travels at constant velocity. **(2 marks)**

e Calculate the time taken from leaving A until reaching the point halfway between the stations. **(3 marks)**

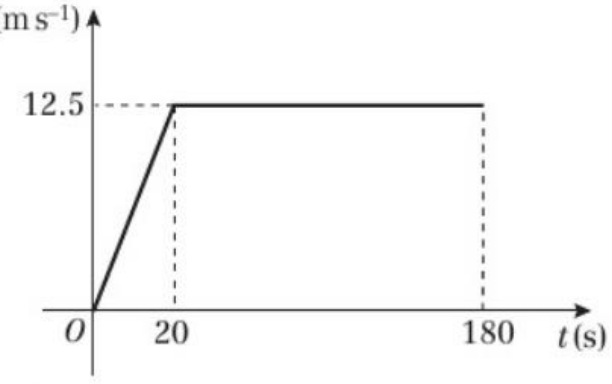
Chapter Exercises

Challenge

A ball is projected vertically upwards with speed $10\,\text{m s}^{-1}$ from a point X, which is 50 m above the ground. T seconds after the first ball is projected upwards, a second ball is dropped from X. Initially the second ball is at rest. The balls collide 25 m above the ground. Find the value of T.

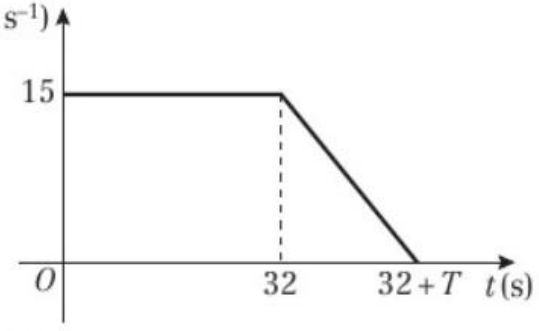
Chapter Answers

1 a



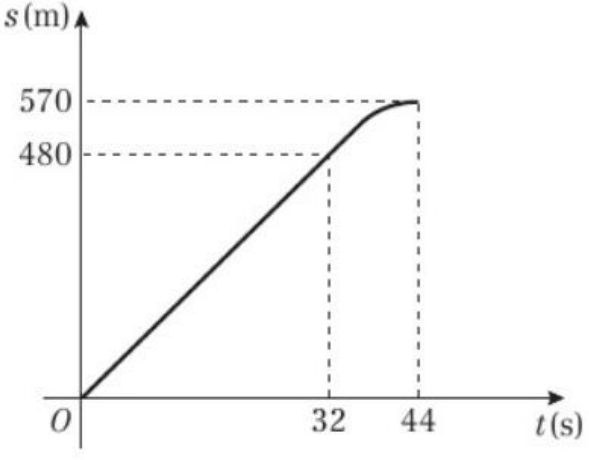
b 2125 m

2 a



b $T = 12$

c



3 a i a = gradient of line. Using the formula for the gradient of a line, $a = \frac{v-u}{t}$, which can be rearranged to give $v = u + at$

ii s = area under the graph. Using the formula for the area of a trapezium, $s = \left(\frac{u+v}{2}\right)t$

b i Substitute $t = \frac{v-u}{a}$ into $s = \left(\frac{u+v}{2}\right)t$

ii Substitute $v = u + at$ into $s = \left(\frac{u+v}{2}\right)t$

iii Substitute $u = v - at$ into $s = \left(\frac{u+v}{2}\right)t$

4 $u = 8$

5 0.165 m s^{-2} (3 d.p.)

6 a 60 m **b** 100 m

7 1.9 s

8 a 4.1 s (2 s.f.) **b** 40 m s^{-1} (2 s.f.)

c air resistance

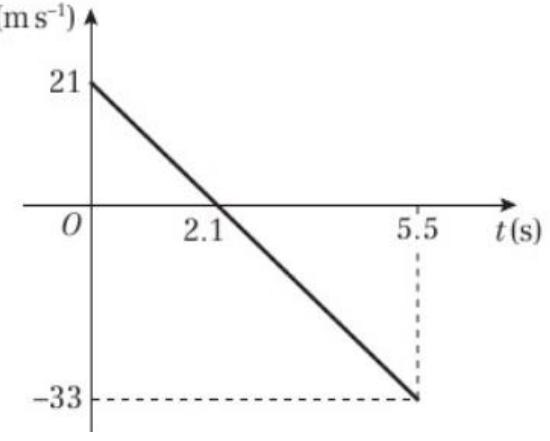
9 a $u = 11$ **b** 22 m

10 a 28 m s^{-1} **b** 208 m

11 a 8 m s^{-1} **b** 1.25 m s^{-2} **c** 204.8 m

12 a 33 m s^{-1} (2 s.f.) **b** 3.4 s (2 s.f.)

c



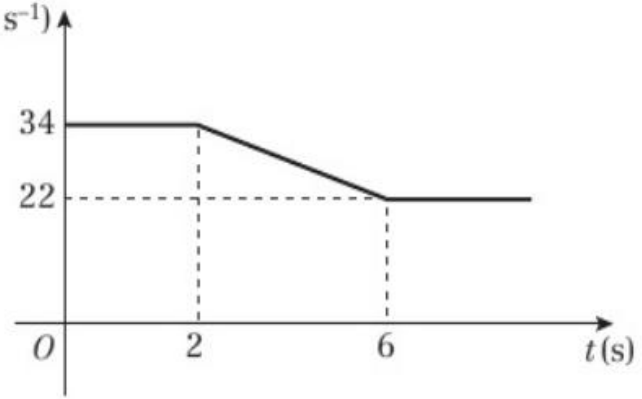
Chapter Answers

13 **a** 50 s **b** 24.2 m s^{-1} (3 s.f.)

14 $h = 39$ (2 s.f.)

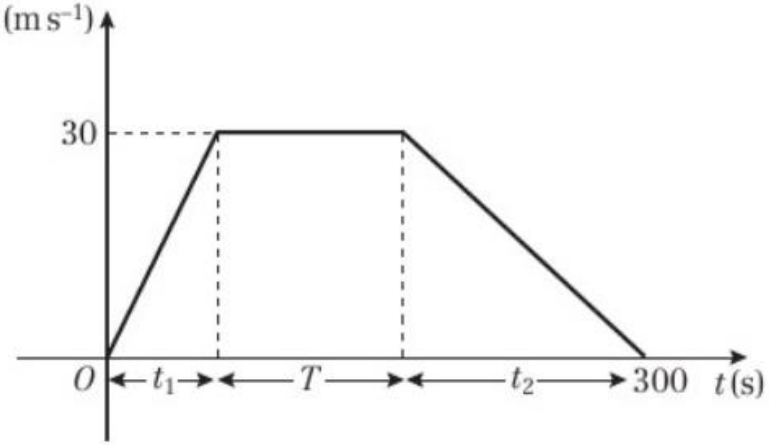
15 **a** 32 m s^{-1} **b** 90 m **c** 5 s

16 **a**



b 180 m

17 **a**



b $\frac{30}{t_1} = 3x \Rightarrow t_1 = \frac{1}{x}, \frac{-30}{t_2} = -x \Rightarrow t_2 = \frac{30}{x}$

So $\frac{10}{x} + T + \frac{30}{x} = 300 \Rightarrow \frac{40}{x} + T = 300$

c $T = 100, x = 0.2$ **d** 3 km **e** 125 s

Challenge

1.2 s