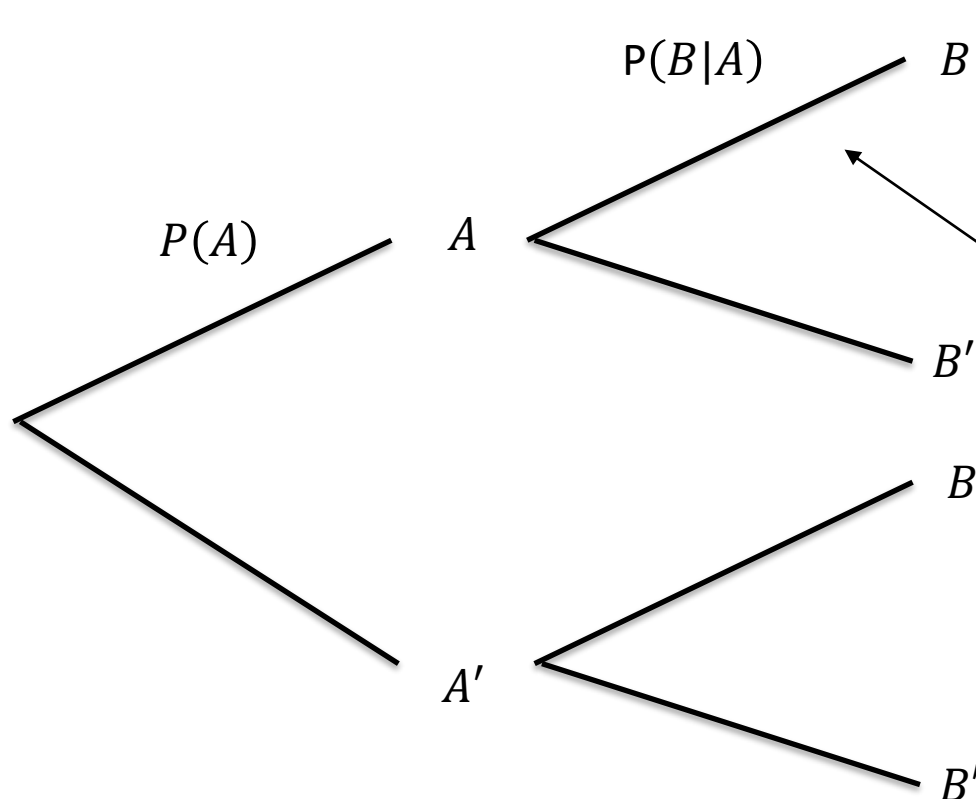

Stats Yr2 Chapter 2: Probability Theory

Conditional Probability

Conditional Probability

Think about how we formed a probability tree at GCSE:



$$P(A \cap B) = P(A) \times P(B|A)$$

Read the ' $|$ ' symbol as "**given that**". i.e. "B occurred **given that** A occurred".

Alternatively (and more commonly):

$$\pencil P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Memory Tip: You're dividing by the event you're conditioning on.

Examples

- 1 The following two-way table shows what foreign language students in Year 9 study.

B is the event that the student is a boy. F is the event they chose French as their language.

	B	B'	Total
F	14	38	52
F'	26	22	48
Total	40	60	100

- a Determine the probability of:
 $P(F|B')$

Method 1: Using the formula:

$$P(F|B') = \frac{P(F \cap B')}{P(B')} = \frac{\frac{38}{100}}{\frac{60}{100}} = \frac{38}{60}$$

Method 2: Restricted sample space.

We're finding the "probability they study French **given** they're not a boy", i.e. we're choosing only from the non-boys:

$$P(F|B') = \frac{38}{60}$$

- b $P(B|F') = \frac{26}{48}$

Check your understanding

The events E and F are such that

$$P(E) = 0.28 \quad P(E \cup F) = 0.76 \quad P(E \cap F') = 0.11$$

Find

a) $P(E \cap F) =$

b) $P(F) =$

c) $P(E' | F') =$

Click to reveal Venn Diagram

Check your understanding

The events E and F are such that

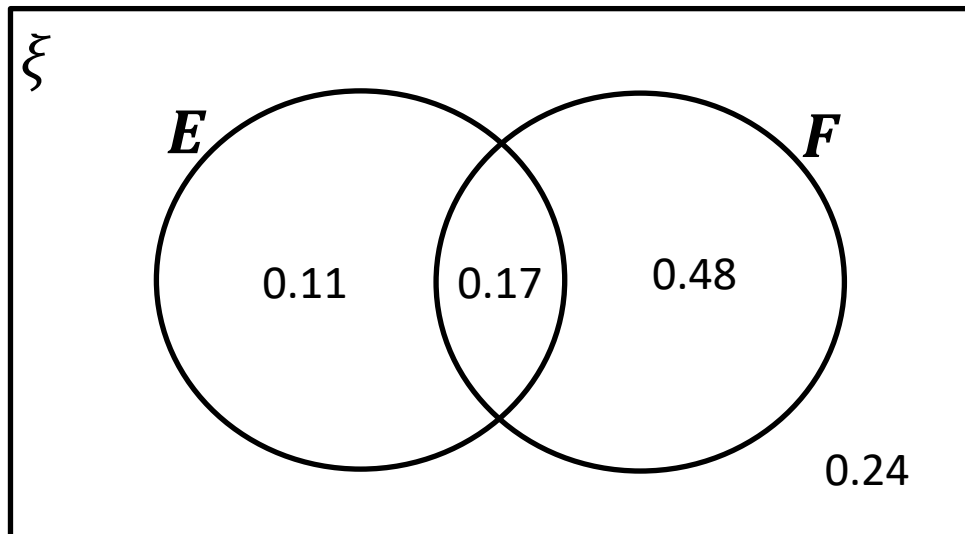
$$P(E) = 0.28 \quad P(E \cup F) = 0.76 \quad P(E \cap F') = 0.11$$

Find

a) $P(E \cap F) = 0.17$

b) $P(F) = 0.65$

c) $P(E'|F') = \frac{P(E' \cap F')}{P(F')} = \frac{0.24}{0.35} = \frac{24}{35}$



Further Practice

1

$$P(A \cap B') = 0.4, P(A \cup B) = 0.75$$

Then:

$$P(B) = \boxed{?}$$

$$P(A' \cap B') = \boxed{?}$$

2

$$P(A) = 0.47 \text{ and } P(A \cap B) = 0.12 \text{ and } P(A' \cap B') = 0.03$$

Then:

$$P(A|B') = \boxed{?}$$

3

$$P(A') = 0.7, P(B') = 0.2, P(A \cap B') = 0.1$$

Then:

$$P(A \cup B') = \boxed{?}$$

$$P(B|A') = \boxed{?}$$

Further Practice

1

$$P(A \cap B') = 0.4, P(A \cup B) = 0.75$$

Then:

$$P(B) = \mathbf{0.35}$$

$$P(A' \cap B') = \mathbf{0.25}$$

2

$$P(A) = 0.47 \text{ and } P(A \cap B) = 0.12 \text{ and } P(A' \cap B') = 0.03$$

Then:

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{\mathbf{0.35}}{\mathbf{0.38}} = \frac{\mathbf{35}}{\mathbf{38}}$$

3

$$P(A') = 0.7, P(B') = 0.2, P(A \cap B') = 0.1$$

Then:

$$\begin{aligned} P(A \cup B') &= P(A) + P(B') - P(A \cap B') \\ &= \mathbf{0.3} + \mathbf{0.2} - \mathbf{0.1} \\ &= \mathbf{0.4} \end{aligned}$$

$$P(B|A') = \frac{P(A' \cap B)}{P(A')} = \frac{\mathbf{0.6}}{\mathbf{0.7}} = \frac{\mathbf{6}}{\mathbf{7}}$$

Exercise 2.2

Pearson Stats/Mechanics Year 2

Pages 11-13

Homework Exercise

- 1 The two-way table shows the fast-food preferences of 60 students in a sixth-form.

	Pizza	Curry	Total
Male	11	18	29
Female	14	17	31
Total	25	35	60

Find:

- a $P(\text{Male})$ b $P(\text{Curry}|\text{Male})$ c $P(\text{Male}|\text{Curry})$ d $P(\text{Pizza}|\text{Female})$

- 2 In a sports club, there are 75 members of whom 32 are female. Of the female members, 15 play badminton and 17 play squash. There are 22 men who play squash and the rest play badminton.

a Draw a two-way table to illustrate this situation.

b Find:

- i $P(\text{Male}|\text{Squash})$ ii $P(\text{Female}|\text{Badminton})$ iii $P(\text{Squash}|\text{Female})$

- 3 A group of 80 children are asked about their favourite ice-cream flavour. Of the 45 girls, 13 like vanilla, 12 like chocolate and the rest like strawberry. Of the boys, 2 like vanilla and 23 like strawberry. The rest like chocolate.

a Draw a two-way table to show this situation.

b Find:

- i $P(\text{Boy}|\text{Strawberry})$ ii $P(\text{Girl}|\text{Vanilla})$ iii $P(\text{Chocolate}|\text{Boy})$

- 4 A red and a blue spinner each have four equally likely outcomes, numbered 1 to 4. The two spinners are spun at the same time, and the sum of the numbers shown, X , is recorded.

a Draw a sample space diagram for X .

b Find:

- i $P(X = 5)$ ii $P(X = 3|\text{Red spinner is 2})$ iii $P(\text{Blue spinner is 3}|X = 5)$

Homework Exercise

- 5 Two fair six-sided dice are thrown and the product is recorded.
- a Draw a sample-space diagram to illustrate the possible outcomes.
 - b Given that the first dice shows a 5, find the probability that the product is 20.
 - c Given that the product is 12, find the probability that the second dice shows a 6.
 - d Explain the importance of the word 'fair' in this context.
- 6 A card is drawn at random from a pack of 52 playing cards. Given that the card is a diamond, find the probability that the card is an ace.
- 7 Two coins are flipped and the results are recorded. Given that one coin lands on a head, find the probability of:
- a two heads
 - b a head and a tail.
 - c State one modelling assumption used in your calculations.
- 8 120 students are asked about their viewing habits. 56 say they watch sports (S) and 77 say they watch dramas (D). Of those who watch dramas, 18 also watch sports.
- a Draw a two-way table to show this information. **(2 marks)**
 - b One student is chosen at random. Find:
 - i $P(D')$ **(1 mark)**
 - ii $P(S' \cap D')$ **(1 mark)**
 - iii $P(S|D)$ **(1 mark)**
 - iv $P(D'|S)$ **(1 mark)**

Homework Exercise

- 9 A rambling group is made up of 63 women and 47 men. 26 of the women and 18 of the men use a walking stick.

a Draw a two-way table to show this information. (2 marks)

b One Rambler is chosen at random. Find:

i $P(\text{Uses a stick})$ (1 mark)

ii $P(\text{Uses a stick}|\text{Female})$ (1 mark)

iii $P(\text{Male}|\text{Uses a stick})$ (1 mark)

- 10 A veterinary surgery has 750 registered pet owners. Of these 450 are female. 320 of the pet owners own a cat and 250 own a dog. Of the remaining pet owners, 25 are males who own another type of pet. No one owns more than one type of pet. 175 female owners have a cat.

One owner is chosen at random. Given that:

F is the event that an owner is female

D is the event that an owner has a dog

C is the event that an owner has a cat.

Find:

a $P(D' \cap C')$ b $P(D|F')$ c $P(F'|C)$ d $P((D' \cap C')|F)$

Homework Answers

1 a $\frac{29}{60}$ b $\frac{18}{29}$ c $\frac{18}{35}$ d $\frac{14}{31}$

2 a

	Badminton	Squash	Total
Male	21	22	43
Female	15	17	32
Total	36	39	75

b i $\frac{22}{39}$ ii $\frac{15}{36}$ or $\frac{5}{12}$ iii $\frac{17}{32}$

3 a

	Girls	Boys	Total
Vanilla	13	2	15
Chocolate	12	10	22
Strawberry	20	23	43
Total	45	35	80

b i $\frac{23}{43}$ ii $\frac{13}{15}$ iii $\frac{10}{35}$ or $\frac{2}{7}$

4 a Blue spinner

Red spinner

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

b i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{1}{4}$

Homework Answers

1 a $\frac{29}{60}$ b $\frac{18}{29}$ c $\frac{18}{35}$ d $\frac{14}{31}$

2 a

	Badminton	Squash	Total
Male	21	22	43
Female	15	17	32
Total	36	39	75

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Total	45	35	80

b i $\frac{23}{43}$ ii $\frac{13}{15}$ iii $\frac{10}{35}$ or $\frac{2}{7}$

4 a

Blue spinner

Red spinner	1	2	3	4
	1	2	3	4
	2	3	4	5
	3	4	5	6
	4	5	6	7

b i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{1}{4}$

5 a

Dice 1

Dice 2	1	2	3	4	5	6
	1	1	2	3	4	5
	2	2	4	6	8	10
	3	3	6	9	12	15
	4	4	8	12	16	20
	5	5	10	15	20	25
	6	6	12	18	24	30

b $\frac{1}{6}$ c $\frac{1}{4}$
 d All outcomes are equally likely.

Homework Answers

6 0.0769 (3 s.f.) or $\frac{1}{13}$

7 **a** 0.333 **b** 0.667

c Assume that the coins are not biased.

8 **a**

	D	D'	Total
S	18	38	56
S'	59	5	64
Total	77	43	120

b **i** $\frac{43}{120}$ **ii** $\frac{5}{120}$ **iii** $\frac{18}{77}$ **iv** $\frac{38}{56}$

9 **a**

	Women	Men	Total
Stick	26	18	44
No stick	37	29	66
Total	63	47	110

b **i** $\frac{44}{110}$ or $\frac{2}{5}$ **ii** $\frac{26}{63}$ **iii** $\frac{18}{44}$ or $\frac{9}{22}$

10 **a** $\frac{6}{25}$ **b** $\frac{13}{30}$ **c** $\frac{29}{64}$ **d** $\frac{31}{90}$