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# Stats Yr2 Chapter 3: Distribution-N

## Gaussian Probability

# Getting normal values from your calculator

## Note:

If for some reason you haven't got an fx-CG50, buy one now!

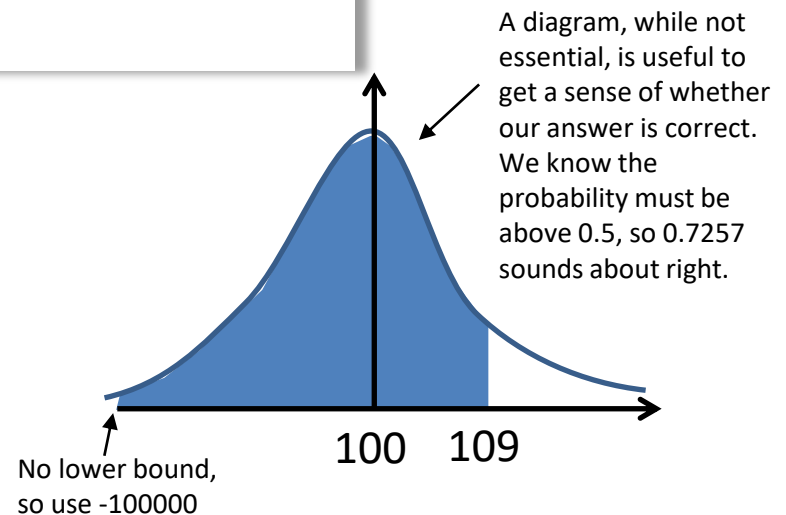
These instructions below assume you have a Classwiz. TODO: check and update for fx-CG50.

Just like a cumulative frequency graph gives the running total of the frequency up to a given value, a **cumulative distribution** gives the **running total of the probability** up to a given value.

IQ is distributed using  $X \sim N(100, 15^2)$ . Find

- (a)  $P(X < 109)$
- (b)  $P(X \geq 93)$
- (c)  $P(110 < X < 120)$
- (d)  $P(X < 80 \text{ or } X > 106)$

- a
1. Press MODE.
  2. Choose DISTRIBUTION (option 7)
  3. Choose Normal CD (i.e. "Cumulative Distribution")
  4. Since the lower value is effectively  $-\infty$ , use any value at least  $5\sigma$  below the mean ( $-100000$  will do!). Press = after each value.
  5. Put the upper value as 109.
  6. Set  $\sigma = 15$  and  $\mu = 100$
  7. You should obtain  $P(X < 109) = 0.7257$  (4dp)



# Getting normal values from your calculator

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b

?

**For Note:**  $\geq$  vs  $>$  makes no difference as the distribution is continuous.

c

?

d

?

# Getting normal values from your calculator

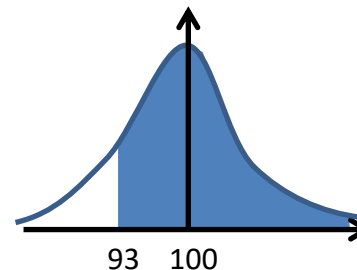
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b

Use 93 for the lower limit and something arbitrarily high (e.g. 100000) for the upper limit.

$$P(X \geq 93) = 0.6796 \text{ (4dp)}$$



**Fro Note:**  $\geq$  vs  $>$  makes no difference as the distribution is continuous.

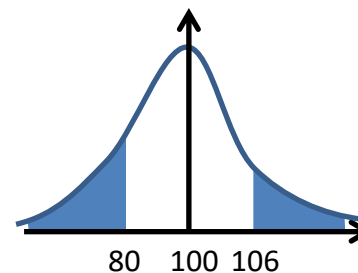
c

$$P(110 < X < 120) = 0.1613 \text{ (4dp)}$$

d

From the graph, we can see that we can find the probability between 80 and 106, and take away from 1:

$$\begin{aligned} P(X < 80 \text{ or } X > 106) &= 1 - P(80 < X < 106) \\ &= 1 - 0.5642 \\ &= 0.4358 \end{aligned}$$



**Froculator Tip:** You will need to switch back to normal calculation mode to do the  $1 - \dots$ . Note however that the ANS key will store the last result from the Distribution Mode!

# Test Your Understanding

The criteria for joining Mensa is an IQ of at least 131.

Assuming that IQ has the distribution  $X \sim N(100, 15^2)$  for a population, determine:

- a) What percentage of people are eligible to join Mensa.
- b) If 30 adults are randomly chosen, the probability that at least 3 of them will be eligible to join. (Hint: Binomial distribution?)

a

?

b

?

# Test Your Understanding

The criteria for joining Mensa is an IQ of at least 131.


Assuming that IQ has the distribution  $X \sim N(100, 15^2)$  for a population, determine:

- a) What percentage of people are eligible to join Mensa.
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a  $P(X \geq 131) = 0.01938 \text{ (4sf)}$

b Let  $Y$  be the number of adults among 30 who are eligible to join.  
 $Y \sim B(30, 0.0193827)$

$$\begin{aligned} P(Y \geq 3) &= 1 - P(Y \leq 2) \\ &= 1 - 0.979986 \\ &= 0.02001 \text{ (4sf)} \end{aligned}$$



**Froculator Note:** You absolutely can't use tables in a formula booklet here. You'll need to use a calculator.

# Exercise 3.2

Pearson Stats/Mechanics Year 2

Pages 22-23

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# Homework Exercise

- 1 The random variable  $X \sim N(30, 2^2)$ .

Find:   **a**  $P(X < 33)$                       **b**  $P(X > 26)$                       **c**  $P(X \geq 31.6)$

- 2 The random variable  $X \sim N(40, 9)$ .

Find:   **a**  $P(X > 45)$                       **b**  $P(X \leq 38)$                       **c**  $P(41 \leq X \leq 44)$

**Watch out**

In the normal distribution  $N(40, 9)$  the second parameter is the **variance**. The standard deviation in this normal distribution is  $\sqrt{9} = 3$ .

- 3 The random variable  $X \sim N(25, 25)$ .

Find:   **a**  $P(Y < 20)$                       **b**  $P(18 < Y < 26)$                       **c**  $P(Y > 23.8)$

- 4 The random variable  $X \sim N(18, 10)$ .

Find:   **a**  $P(X \geq 20)$                       **b**  $P(X < 15)$                       **c**  $P(18.4 < X < 18.7)$

- 5 The random variable  $M \sim N(15, 1.5^2)$ .

**a** Find:   **i**  $P(M > 14)$                       **ii**  $P(M < 14)$

**b** Calculate the sum of your answers to **a i** and **ii** and comment on your answer.

- 6 The random variable  $T \sim N(4.5, 0.4)$ .

**a** Find  $P(T < 4.2)$ .  
**b** Without further calculation, write down  $P(T > 4.2)$ .

- 7 The random variable  $Y \sim N(45, 2^2)$ . Find:

**a**  $P(Y < 41 \text{ or } Y > 47)$                       **b**  $P(Y < 44 \text{ or } 46.5 < Y < 47.5)$



# Homework Exercise

- 8 The volume of soap dispensed by a soap-dispenser on each press,  $X$  ml, is modelled as  $X \sim N(6, 0.8^2)$ .
- a Find: i  $P(X > 7)$       ii  $P(X < 5)$  (2 marks)
- The soap dispenser is pressed three times.
- b Find the probability that on all three presses, less than 5 ml of soap is dispensed. (2 marks)
- 9 The amount of mineral water,  $W$  ml, in a bottle produced by a certain manufacturer is modelled as  $W \sim N(500, 14^2)$ .
- a Find: i  $P(W > 505)$       ii  $P(W < 490)$  (2 marks)
- A sample of 4 bottles is taken.
- b Find the probability that all of the bottles contain more than 490 ml. (2 marks)
- 10 The heights of a large group of women are normally distributed with a mean of 165 cm and a standard deviation of 3.5 cm. A woman is selected at random from this group.
- a Find the probability that she is shorter than 160 cm.
- Steven is looking for a woman whose height is between 168 cm and 174 cm for a part in his next film.
- b Find the proportion of women from this group who meet Steven's criteria.
- A sample of 20 women is taken from the group.
- c Find the probability that at least 5 of the women meet Steven's criteria.

## Problem-solving

For part c, formulate a binomial random variable to represent the number of women in the sample who meet Steven's criteria.

# Homework Exercise

- 11 The diameters of bolts,  $D$  mm, made by a particular machine are modelled as  $D \sim N(13, 0.1^2)$ .
- a Find the probability that a bolt, chosen at random, has a diameter less than 12.8 mm. **(1 mark)**

Bolts are considered to be 'perfect' if the diameter lies between 12.9 mm and 13.1 mm.  
A random sample of 40 bolts is taken.

- b Find the probability that more than 25 of the bolts are 'perfect'. **(4 marks)**
- 12 The masses,  $X$  grams, of a large population of squirrels are modelled as a normal distribution with  $X \sim N(480, 40^2)$ .
- a Find the probability that a squirrel chosen at random has a mass greater than 490 g. **(1 mark)**
- A naturalist takes a random sample of 30 squirrels from the population.
- b Find the probability that at least 15 of the squirrels have a mass between 470 g and 490 g. **(4 marks)**

# Homework Answers

*For Chapter 3, student answers may differ slightly from those shown here when calculators are used rather than table values.*

- |           |          |  |           |           |          |        |
|-----------|----------|--|-----------|-----------|----------|--------|
| <b>1</b>  | <b>a</b> | 0.9332   | <b>b</b>  | 0.9772    | <b>c</b> | 0.2119 |
| <b>2</b>  | <b>a</b> | 0.0478   | <b>b</b>  | 0.2525    | <b>c</b> | 0.2782 |
| <b>3</b>  | <b>a</b> | 0.1587   | <b>b</b>  | 0.4985    | <b>c</b> | 0.5948 |
| <b>4</b>  | <b>a</b> | 0.2635   | <b>b</b>  | 0.1714    | <b>c</b> | 0.0373 |
| <b>5</b>  | <b>a</b> | <b>i</b> 0.7475  | <b>ii</b> | 0.2525    |          |        |
|           | <b>b</b> | Sum is 1, combined probabilities include every possible value. |           |           |          |        |
| <b>6</b>  | <b>a</b> | 0.3176   | <b>b</b>  | 0.6824    |          |        |
| <b>7</b>  | <b>a</b> | 0.1814   | <b>b</b>  | 0.4295    |          |        |
| <b>8</b>  | <b>a</b> | <b>i</b> 0.1056  | <b>ii</b> | 0.1056    | <b>b</b> | 0.0012 |
| <b>9</b>  | <b>a</b> | <b>i</b> 0.3605  | <b>ii</b> | 0.2375    | <b>b</b> | 0.3380 |
| <b>10</b> | <b>a</b> | 0.0766   | <b>b</b>  | 0.1906    | <b>c</b> | 0.3296 |
| <b>11</b> | <b>a</b> | 0.0228   | <b>b</b>  | 0.7345    |          |        |
| <b>12</b> | <b>a</b> | 0.4013   | <b>b</b>  | 0.0001986 |          |        |