M2 Chapter 5: Inclined Planes

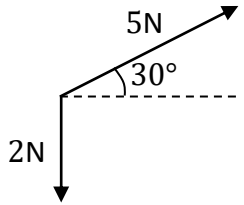
Force Components

Overview

In Year 1 any frictional forces were stated. In this chapter, we will be able to **calculate the frictional force** using the normal reaction force acting on the object.

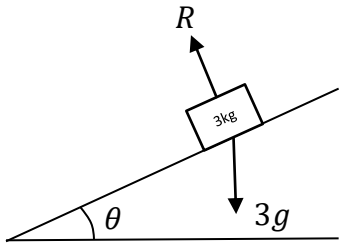
1:: Resolving components

"Determine the magnitude and direction of the resultant force."



2:: Inclined Planes

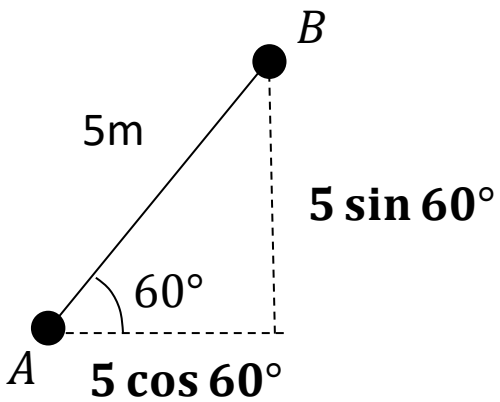
"A block of mass 3kg is placed on a smooth slope with angle of inclination θ where $\tan\theta = \frac{3}{4}$. Determine the acceleration of the block down the slope."



3:: $F \leq \mu R$

Understand that the maximum friction is μR, where μ is the coefficient of friction of the surface, and R is the normal reaction force of the surface on the object. Use to solve inclined plane problems when the surface is rough.

Resolving Forces



In the last chapter/Year 1 we have already taken the '**components**' of a distance in particular directions, for example the horizontal and vertical components.

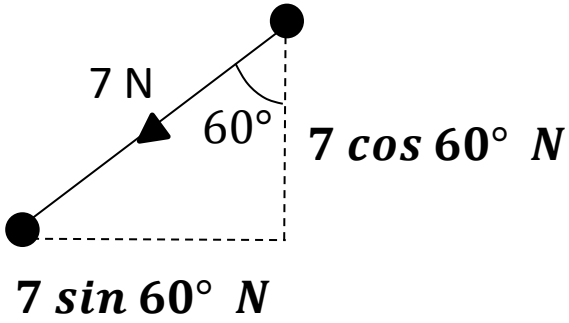
This allowed us for example to convert a displacement (from A to B) from scalar form to vector form:

5m $\Rightarrow$ $\begin{pmatrix} 5\cos 60° \\ 5\sin 60° \end{pmatrix} = \begin{pmatrix} 2.5 \\ 4.33 \end{pmatrix}$ m

And we could convert back to scalar form by finding the magnitude of the displacement vector:

$$\sqrt{(5\cos 60°)^2 + (5\sin 60°)^2} = 5$$

Speed Tip: If x is the magnitude/hypotenuse, use $x\cos\theta$ for the side adjacent to the angle and $x\sin\theta$ for the side opposite it.

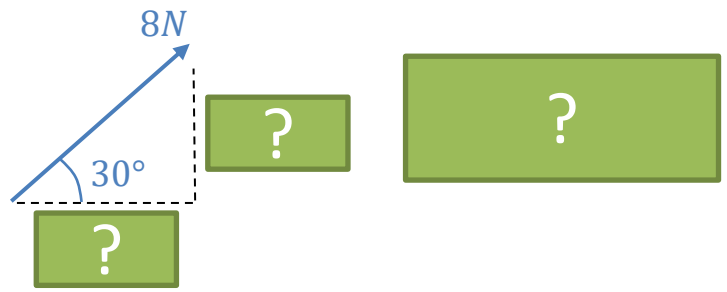


We can use exactly the same principle to find the components of a force, and convert between vector and scalar form.

Quickfire Questions

Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.

1



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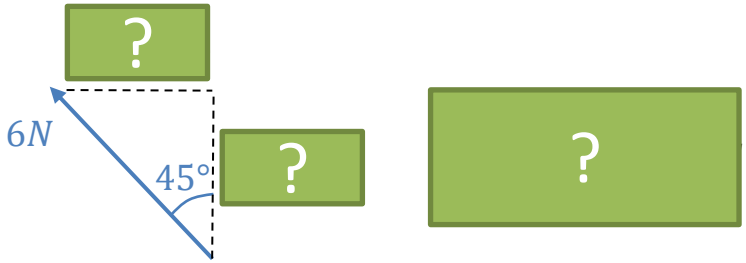

3

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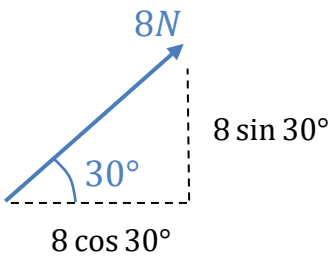
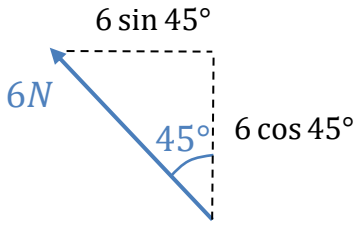


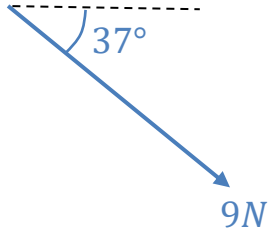
$4\sqrt{3}\,\boldsymbol{i} + 4\boldsymbol{j}$

2



$-3\sqrt{2}\,\boldsymbol{i} + 3\sqrt{2}\boldsymbol{j}$

3



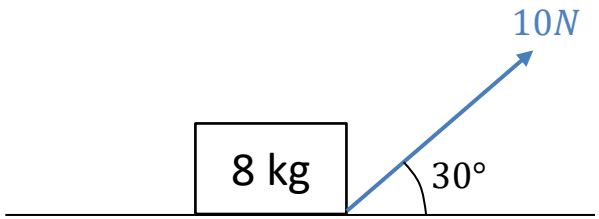
$9\cos 37°\ \boldsymbol{i} - 9\sin 37°\,\boldsymbol{j}$
$= 7.19\boldsymbol{i} - 5.42\boldsymbol{j}$

Applied Example

[Textbook] A box of mass 8kg lies on a smooth horizontal floor. A force of 10N is applied at an angle of $30°$ causing the box to accelerate horizontally along the floor.

(a) Work out the acceleration of the box.

(b) Calculate the normal reaction between the box and the floor.



Recall that $F = ma$, but the force F must be in the **same direction as the acceleration**.

Draw a clear force diagram, obviously!

? Diagram

a ?

b ?

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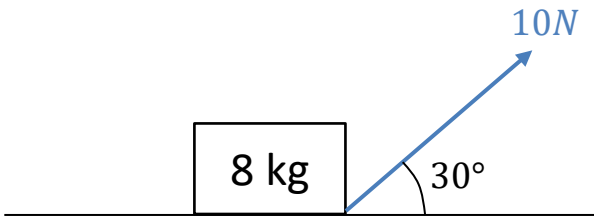

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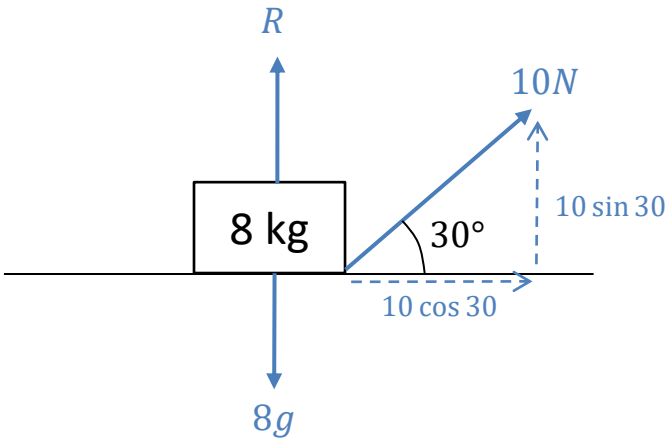


a $R(\rightarrow): 10\cos 30° = 8a$

$$a = \frac{5\sqrt{3}}{8}\ \text{ms}^{-2}$$

b $R(\uparrow): R + 10\sin 30° = 8g$

$$R = 73\ N\ (2\text{sf})$$

HIGHLY RECOMMENDED Tips:

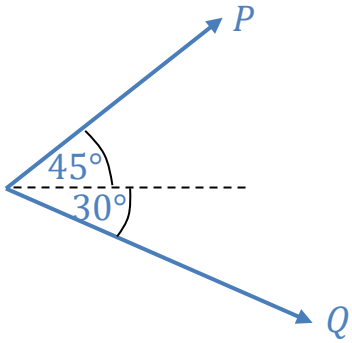
(a) Use dotted lines for components of a force, to distinguish from solid lines for full forces.

(b) Make sure the direction of your components is clear in the diagram, to ensure you get the sign right in calculations.

(c) Write the values of your components in the diagram (as above).

Combining Forces

[Textbook] Two forces P and Q act on a particle as shown. P has a magnitude of 10N and Q has a magnitude of 8N. Work out the magnitude and direction of the resultant force.



Method 1: Finding total x and y components of force.

?

Method 2: Using Triangle Law for vector addition.

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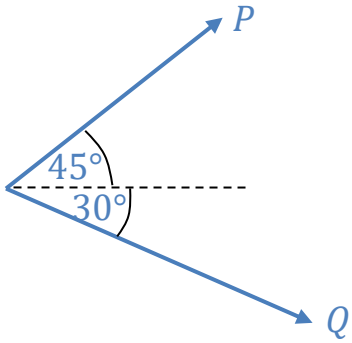
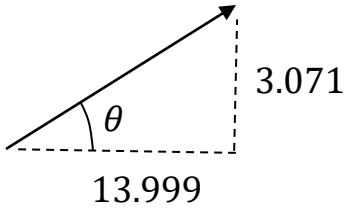


Method 1: Finding total x and y components of force.

Resultant force:

$$\begin{pmatrix} 10\cos 45° + 8\,cos\,30° \\ 10\sin 45° - 8\sin 30° \end{pmatrix} = \begin{pmatrix} 13.999 \\ 3.071 \end{pmatrix}$$

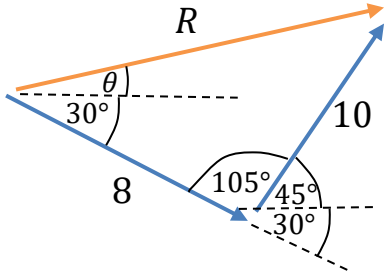
Magnitude: $\sqrt{13.999^2 + 3.071^2} = 14.3\ N$



Do a quick sketch of the force vector to get the direction:

$$\theta = \tan^{-1}\left(\frac{3.071}{13.999}\right) = 12.4°$$

Method 2: Using Triangle Law for vector addition.



We can avoid resolving components by drawing the force vectors in a chain, then finding the vector from the start to end point. The resultant vector (orange) geometrically represents the same of the vectors.

Use cosine rule to get magnitude of R:

$R^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos(105°)$

$R = 14.3\ N$

Use sine rule to get θ:

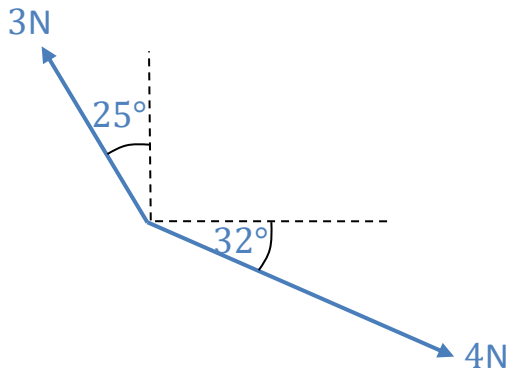
$$\frac{\sin(\theta + 30°)}{10} = \frac{\sin(105°)}{14.332}$$

$$\sin(\theta + 30°) = \frac{10\sin(105°)}{14.332} \quad \ldots$$

$\theta = 12.4°$

Test Your Understanding

A particle has forces acting on it as indicated in the diagram. Determine the magnitude and direction (anticlockwise from the positive x direction) of the resultant force.



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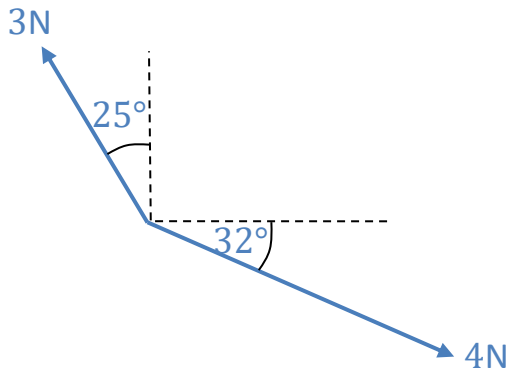


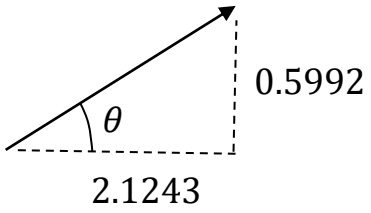
Resultant force:

$$\begin{pmatrix} 4\cos 32° - 3\sin 25° \\ 3\cos 25° - 4\sin 32° \end{pmatrix} = \begin{pmatrix} 2.1243 \\ 0.5992 \end{pmatrix}$$

Magnitude: $\sqrt{2.1243^2 + 0.5992^2}$

$= 2.21\ N$

Direction:



$$\theta = \tan^{-1}\left(\frac{0.5992}{2.1243}\right)$$

$$= 15.8°$$

Exercise 5.1

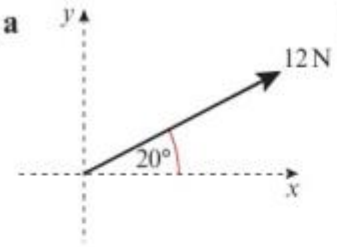
Pearson Stats/Mechanics Year 2
Pages 43-44

Homework Exercise

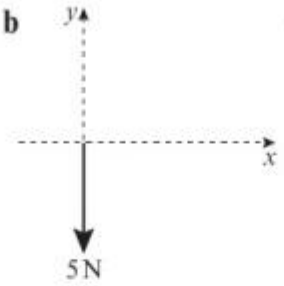
1 Find the component of each force in **i** the x-direction **ii** the y-direction

iii Hence write each force in the form $p\mathbf{i} + q\mathbf{j}$ where **i** and **j** are the unit vectors in the x and y directions respectively.

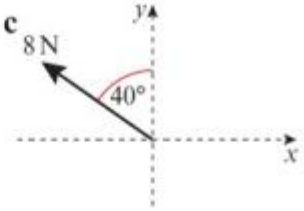
a



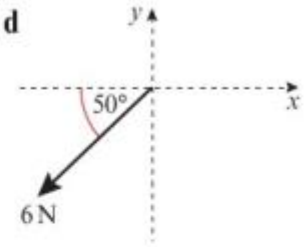
b



c

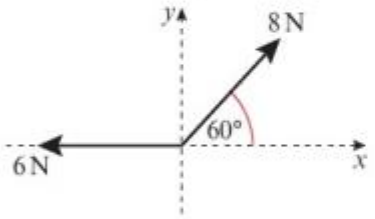


d

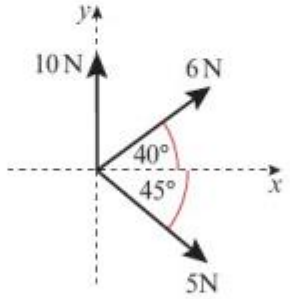


2 For each of the following systems of forces, find the sum of the components in **i** the x-direction, **ii** the y-direction.

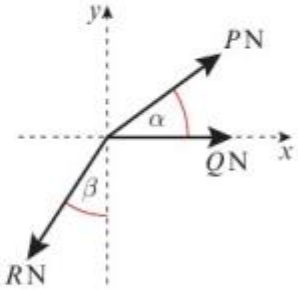
a



b

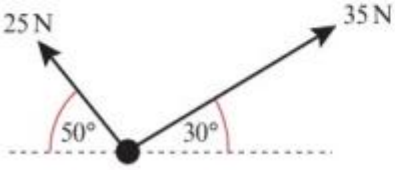


c

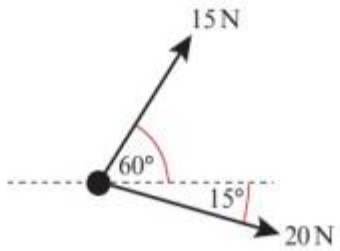


3 Find the magnitude and direction of the resultant force acting on each of the particles shown below.

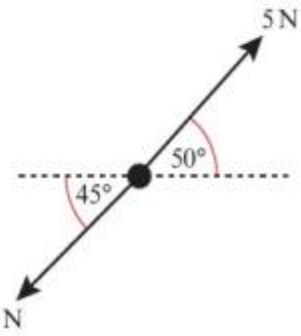
a



b



c

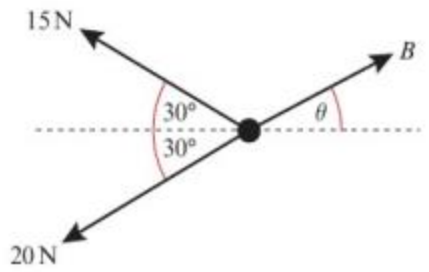


Homework Exercise

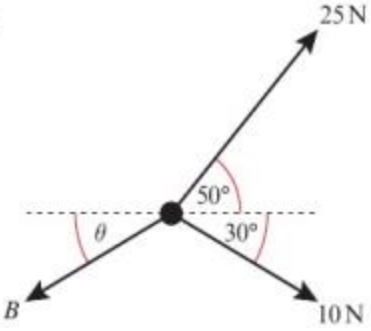
4 Three forces act upon a particle as shown in the diagrams below.

Given that the particle is in equilibrium, calculate the magnitude of B and the value of θ.

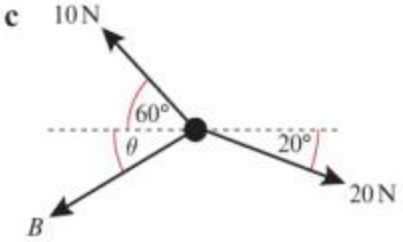
a



b



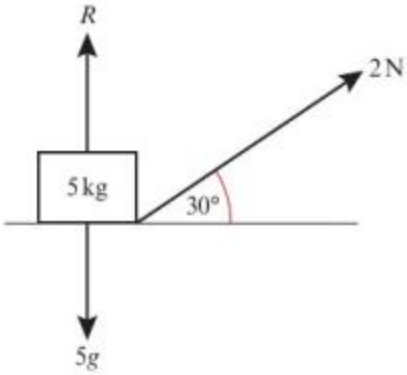
c



5 A box of mass 5 kg lies on a smooth horizontal floor. The box is pulled by a force of 2 N applied at an angle of 30° to the horizontal, causing the box to accelerate horizontally along the floor.

a Work out the acceleration of the box.

b Work out the normal reaction of the box with the floor.

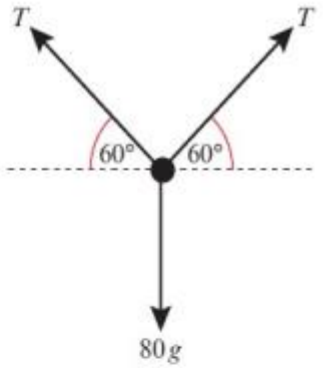


6 A force P is applied to a box of mass 10 kg causing the box to accelerate at $2\,\text{m s}^{-2}$ along a smooth, horizontal plane. Given that the force causing the acceleration is applied at 45° to the plane, work out the value of P. **(3 marks)**

7 A force of 20 N is applied to a box of mass m kg causing the box to accelerate at $0.5\,\text{m s}^{-2}$ along a smooth, horizontal plane. Given that the force causing the acceleration is applied at 25° to the plane, work out the value of m. **(3 marks)**

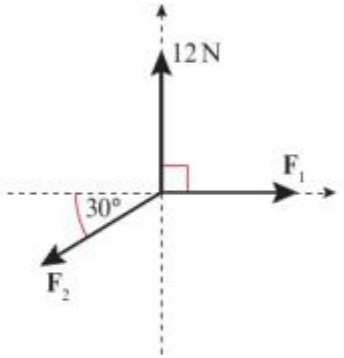
Homework Exercise

8 A parachutist of mass 80 kg is attached to a canopy by two lines, each with tension T. The parachutist is falling with constant velocity, and experiences a resistance to motion due to air resistance equal to one quarter of her weight. Show that the tension in each line, T, is $20\sqrt{3}\,g$ N.



(3 marks)

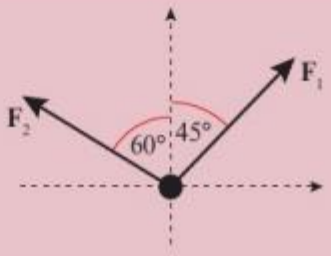
9 A system of forces act upon a particle as shown in the diagram. The resultant force on the particle is $(2\sqrt{3}\mathbf{i} + 2\mathbf{j})$ N. Calculate the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.



(3 marks)

Challenge

Two forces act upon a particle as shown in the diagram. The resultant force on the particle is $(3\mathbf{i} + 5\mathbf{j})$ N. Calculate the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.



Homework Answers

1 **a** **i** 11.3 N (3 s.f.) **ii** 4.10 N (3 s.f.)
iii $(11.3\mathbf{i} + 4.10\mathbf{j})$ N
b **i** 0 N **ii** −5 N **iii** $-5\mathbf{j}$ N
c **i** −5.14 N (3 s.f.) **ii** 6.13 N (3 s.f.)
iii $(-5.14\mathbf{i} + 6.13\mathbf{j})$ N
d **i** −3.86 N (3 s.f.) **ii** −4.60 N (3 s.f.)
iii $(-3.86\mathbf{i} - 4.60\mathbf{j})$ N

2 **a** **i** −2 N **ii** 6.93 N (3 s.f.)
b **i** 8.13 N (3 s.f.) **ii** 10.3 N (3 s.f.)
c **i** $(P\cos\alpha + Q - R\sin\beta)$ N
ii $(P\sin\alpha - R\cos\beta)$ N

3 **a** 39.3 N (3 s.f.) at an angle of 68.8° above the horizontal
b 27.9 N (3 s.f.) at an angle of 16.2° above the horizontal
c 3.01 N (3 s.f.) at an angle of 53.3° above the horizontal

4 **a** $B = 30.4$ N, $\theta = 4.72°$
b $B = 28.5$ N, $\theta = 29.8°$
c $B = 13.9$ N, $\theta = 7.52°$

5 **a** $\frac{\sqrt{3}}{5}$ m s^{-2} **b** 48 N

6 $20\sqrt{2}$ N

7 36.3 kg (3 s.f.)

8 $2T\sin 60 + 20g = 80g$

$$T = \frac{60g}{2\sin 60} = 20\sqrt{3}\,g$$

9 $\mathbf{F}_1 = 12\sqrt{3}$ N, $\mathbf{F}_2 = 20$ N

Challenge
$\mathbf{F}_1 = 6\sqrt{2} - \sqrt{6}$ N, $\mathbf{F}_2 = 2\sqrt{3} - 2$