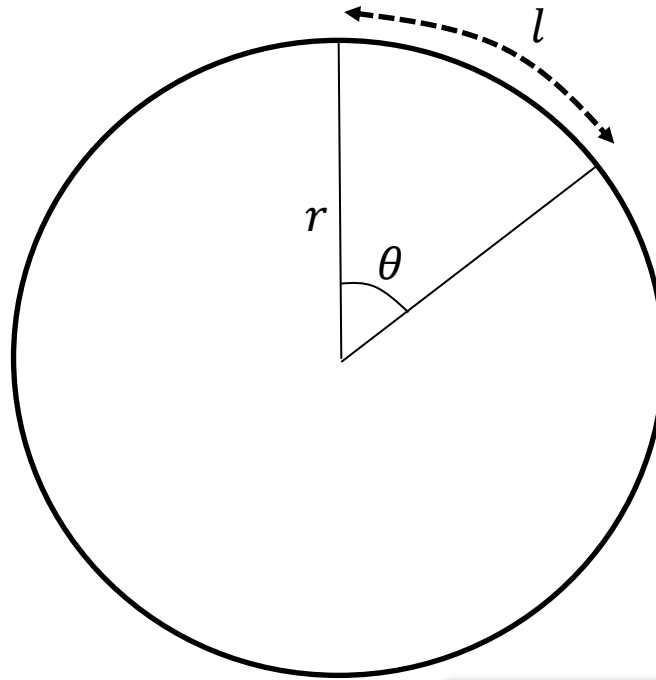


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# P2 Chapter 5: Radians

## Arc Length

# Arc length



Arc length in degrees:

$$l = \frac{\theta}{360} \times 2\pi r$$

Arc length in radians

From before, we know that 1 radian gives an arc of 1 radius in length, so  $\theta$  radians must give a length of...

$$l = r\theta$$

# Examples

[Textbook] Find the length of the arc of a circle of radius 5.2 cm, given that the arc subtends an angle of 0.8 radians at the centre of the circle.

?

[Textbook] An arc  $AB$  of a circle with radius 7 cm and centre  $O$  has a length of 2.45 cm. Find the angle  $\angle AOB$  subtended by the arc at the centre of the circle

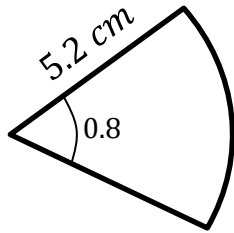
?

**Fro Note:** Whether your calculator is in degrees mode or radians mode is only relevant when using sin/cos/tan – it won't affect simple multiplication!

**Terminology:** 'Subtend' means **opposite** or extending beneath.

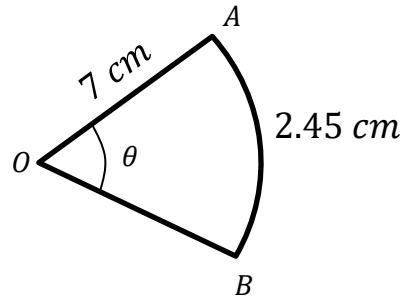
# Examples

[Textbook] Find the length of the arc of a circle of radius 5.2 cm, given that the arc subtends an angle of 0.8 radians at the centre of the circle.



$$0.8 \times 5.2 = 4.16 \text{ cm}$$

[Textbook] An arc  $AB$  of a circle with radius 7 cm and centre  $O$  has a length of 2.45 cm. Find the angle  $\angle AOB$  subtended by the arc at the centre of the circle



$$\theta \times 7 = 2.45$$

$$\theta = \frac{2.45}{7} = 0.35 \text{ rad}$$

**Fro Note:** Whether your calculator is in degrees mode or radians mode is only relevant when using sin/cos/tan – it won't affect simple multiplication!

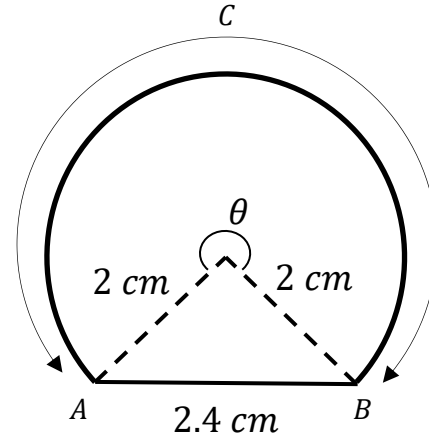
**Terminology:** 'Subtend' means **opposite** or extending beneath.

# Further Examples

[Textbook] An arc  $AB$  of a circle, with centre  $O$  and radius  $r$  cm, subtends an angle of  $\theta$  radians at  $O$ . The perimeter of the sector  $AOB$  is  $P$  cm. Express  $r$  in terms of  $P$  and  $\theta$ .

?

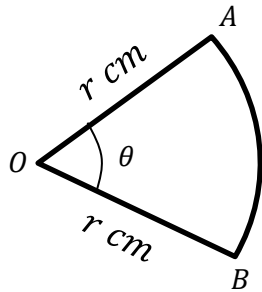
[Textbook] The border of a garden pond consists of a straight edge  $AB$  of length 2.4m, and a curved part  $C$ , as shown in the diagram. The curve part is an arc of a circle, centre  $O$  and radius 2m. Find the length of  $C$ .



?

# Further Examples

[Textbook] An arc  $AB$  of a circle, with centre  $O$  and radius  $r$  cm, subtends an angle of  $\theta$  radians at  $O$ . The perimeter of the sector  $AOB$  is  $P$  cm. Express  $r$  in terms of  $P$  and  $\theta$ .



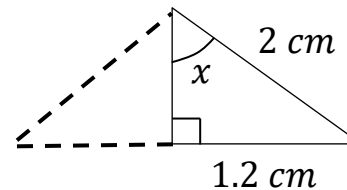
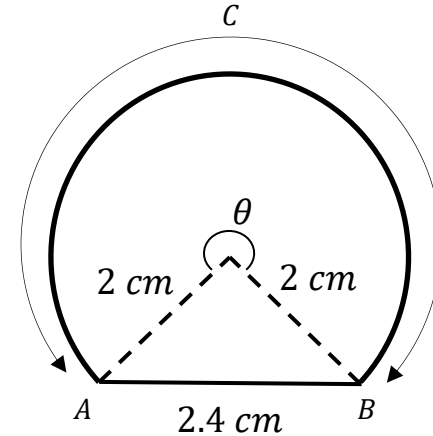
$$P = 2r + \theta r$$

$$P = r(2 + \theta)$$

$$r = \frac{P}{2 + \theta}$$

[Textbook] The border of a garden pond consists of a straight edge  $AB$  of length 2.4m, and a curved part  $C$ , as shown in the diagram. The curve part is an arc of a circle, centre  $O$  and radius 2m.

Find the length of  $C$ .



**Fro Tip:** Trigonometry on right-angled triangles is always simpler than using sine/cosine rule.

$$x = \sin^{-1}\left(\frac{1.2}{2}\right) = 0.6435 \dots \text{rad}$$

Angles round a point add to  $2\pi$ .

$$\theta = 2\pi - 2x = 4.9961 \dots \text{rad}$$

$$\therefore C = 2 \times 4.9961 = 9.99 \text{ m (3sf)}$$

# Test Your Understanding

Edexcel C2 Jan 2005 Q7

Figure 1

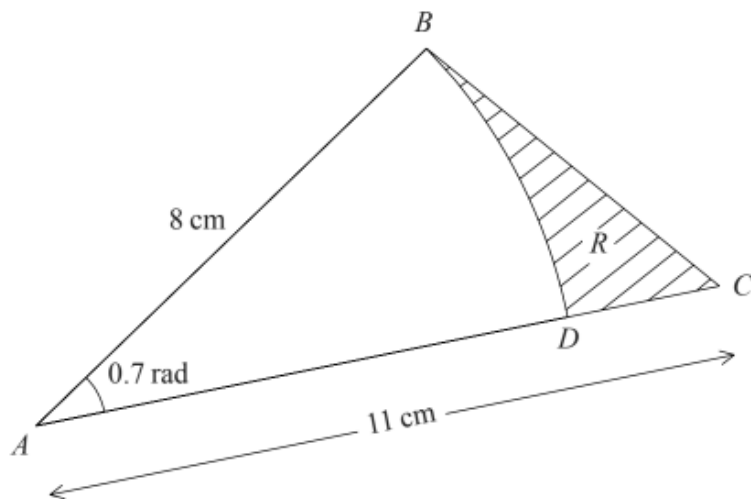


Figure 1 shows the triangle  $ABC$ , with  $AB = 8\text{ cm}$ ,  $AC = 11\text{ cm}$  and  $\angle BAC = 0.7$  radians. The arc  $BD$ , where  $D$  lies on  $AC$ , is an arc of a circle with centre  $A$  and radius  $8\text{ cm}$ . The region  $R$ , shown shaded in Figure 1, is bounded by the straight lines  $BC$  and  $CD$  and the arc  $BD$ .

Find

- (a) The length of the arc  $BD$ .
- (b) The perimeter of  $R$ , giving your answer to 3 significant figures.

a

?

b

?

# Test Your Understanding

Edexcel C2 Jan 2005 Q7

Figure 1

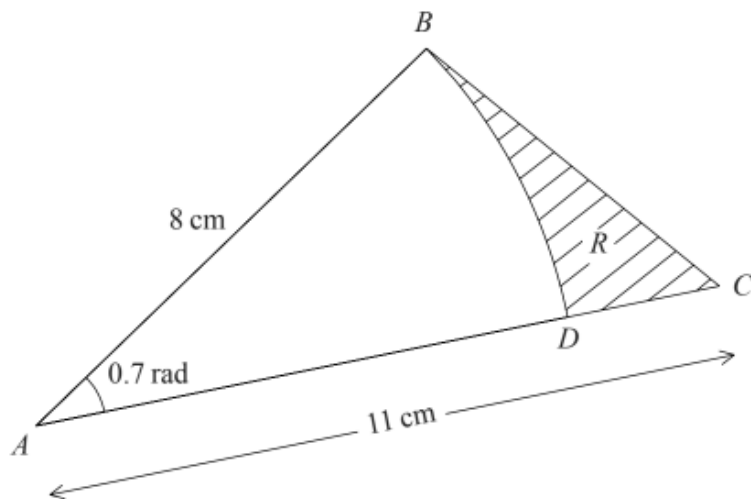


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Find

- (a) The length of the arc  $BD$ .
- (b) The perimeter of  $R$ , giving your answer to 3 significant figures.

a Length of arc  $BD = 0.7 \times 8 = 5.6 \text{ cm}$

b Perimeter  $= BD + CD + BC$

$$CD = 11 - 8 = 3$$

$$BC = \sqrt{8^2 + 11^2 - 2 \times 8 \times 11 \times \cos(0.7)} = 7.09 \text{ cm}$$

$$\therefore P = 5.6 + 3 + 7.09 = 15.7 \text{ cm (to 3sf)}$$

# Exercise 5.2

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# Homework Exercise

- 1 An arc  $AB$  of a circle, centre  $O$  and radius  $r$  cm, subtends an angle  $\theta$  radians at  $O$ .

The length of  $AB$  is  $l$  cm.

- a Find  $l$  when:    i  $r = 6, \theta = 0.45$     ii  $r = 4.5, \theta = 0.45$     iii  $r = 20, \theta = \frac{3}{8}\pi$   
 b Find  $r$  when:    i  $l = 10, \theta = 0.6$     ii  $l = 1.26, \theta = 0.7$     iii  $l = 1.5\pi, \theta = \frac{5}{12}\pi$   
 c Find  $\theta$  when:    i  $l = 10, r = 7.5$     ii  $l = 4.5, r = 5.625$     iii  $l = \sqrt{12}, r = \sqrt{3}$

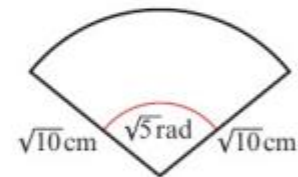
- 2 A minor arc  $AB$  of a circle, centre  $O$  and radius 10 cm, subtends an angle  $x$  at  $O$ . The major arc  $AB$  subtends an angle  $5x$  at  $O$ . Find, in terms of  $\pi$ , the length of the minor arc  $AB$ .

## Notation

The **minor arc**  $AB$  is the shorter arc between points  $A$  and  $B$  on a circle.

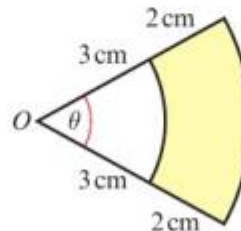
- 3 An arc  $AB$  of a circle, centre  $O$  and radius 6 cm, has length  $l$  cm. Given that the chord  $AB$  has length 6 cm, find the value of  $l$ , giving your answer in terms of  $\pi$ .

- 4 The sector of a circle of radius  $\sqrt{10}$  cm contains an angle of  $\sqrt{5}$  radians, as shown in the diagram. Find the length of the arc, giving your answer in the form  $p\sqrt{q}$  cm, where  $p$  and  $q$  are integers.



- 5 Referring to the diagram, find:

- a the perimeter of the shaded region when  $\theta = 0.8$  radians.  
 b the value of  $\theta$  when the perimeter of the shaded region is 14 cm.



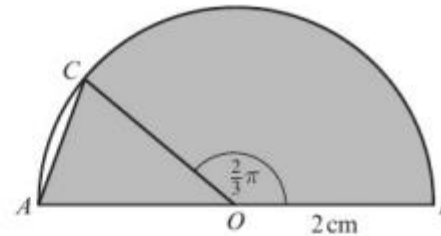
## Problem-solving

The radius of the larger arc is  $3 + 2 = 5$  cm.

- 6 A sector of a circle of radius  $r$  cm contains an angle of 1.2 radians. Given that the sector has the same perimeter as a square of area  $36 \text{ cm}^2$ , find the value of  $r$ .  
 7 A sector of a circle of radius 15 cm contains an angle of  $\theta$  radians. Given that the perimeter of the sector is 42 cm, find the value of  $\theta$ .

# Homework Exercise

- 8 In the diagram  $AB$  is the diameter of a circle, centre  $O$  and radius 2 cm.  
The point  $C$  is on the circumference such that  $\angle COB = \frac{2}{3}\pi$  radians.



- a State the value, in radians, of  $\angle COA$ .

(1 mark)

The shaded region enclosed by the chord  $AC$ , arc  $CB$  and  $AB$  is the template for a brooch.

- b Find the exact value of the perimeter of the brooch.

(5 marks)

- 9 The points  $A$  and  $B$  lie on the circumference of a circle with centre  $O$  and radius 8.5 cm.  
The point  $C$  lies on the major arc  $AB$ . Given that  $\angle ACB = 0.4$  radians, calculate the length of the minor arc  $AB$ .

- 10 In the diagram  $OAB$  is a sector of a circle, centre  $O$  and radius  $R$  cm, and  $\angle AOB = 2\theta$  radians. A circle, centre  $C$  and radius  $r$  cm, touches the arc  $AB$  at  $T$ , and touches  $OA$  and  $OB$  at  $D$  and  $E$  respectively, as shown.

- a Write down, in terms of  $R$  and  $r$ , the length of  $OC$ .

(1 mark)

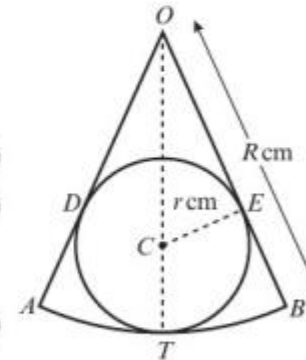
- b Using  $\triangle OCE$ , show that  $R \sin \theta = r(1 + \sin \theta)$ .

(3 marks)

- c Given that  $\sin \theta = \frac{3}{4}$  and that the perimeter of the sector  $OAB$  is

21 cm, find  $r$ , giving your answer to 3 significant figures.

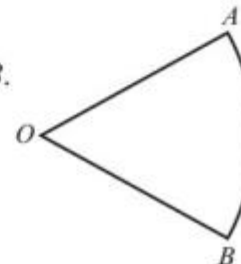
(7 marks)



- 11 The diagram shows a sector  $AOB$ .

The perimeter of the sector is twice the length of the arc  $AB$ .

Find the size of angle  $AOB$ .



# Homework Exercise

- 12 A circular Ferris wheel has 24 pods equally spaced on its circumference.

Given the arc length between each pod is  $\frac{3\pi}{2}$  m, and modelling each pod as a particle,

**a** calculate the diameter of the Ferris wheel.

Given that it takes approximately 30 seconds for a pod to complete one revolution,

**b** estimate the speed of the pod in km/h.

- 13 The diagram above shows a triangular garden,  $PQR$ , with  $PQ = 12$  m,  $PR = 7$  m and  $\angle QPR = 0.5$  radians. The curve  $SR$  is a small path separating the shaded patio area and the lawn, and is an arc of a circle with centre at  $P$  and radius 7 m.

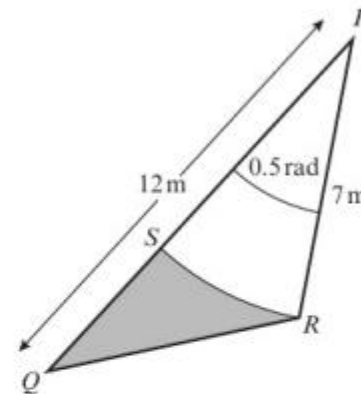
Find:

**a** the length of the path  $SR$

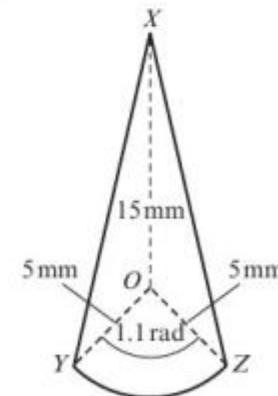
(2 marks)

**b** the perimeter of the shaded patio, giving your answer to 3 significant figures.

(4 marks)



- 14 The shape  $XYZ$  shown is a design for an earring.



The straight lines  $XY$  and  $XZ$  are equal in length. The curve  $YZ$  is an arc of a circle with centre  $O$  and radius 5 mm. The size of  $\angle YOZ$  is 1.1 radians and  $XO = 15$  mm.

**a** Find the size of  $\angle XOZ$ , in radians, to 3 significant figures.

(2 marks)

**b** Find the total perimeter of the earring, to the nearest mm.

(6 marks)

# Homework Answers

- 1   **a**   **i**   2.7                      **ii**   2.025                      **iii**    $7.5\pi$   
     **b**   **i**    $\frac{50}{3}$                       **ii**   1.8                      **iii**   3.6  
     **c**   **i**    $\frac{4}{3}$                       **ii**   0.8                      **iii**   2
- 2    $\frac{10}{3}\pi$  cm                      3    $2\pi$                       4    $5\sqrt{2}$  cm
- 5   **a**   10.4 cm                      **b**   1.25 rad
- 6   7.5                      7   0.8
- 8   **a**    $\frac{1}{3}\pi$                       **b**    $6 + \frac{4}{3}\pi$  cm
- 9   6.8 cm
- 10 **a**    $R - r$   
     **b**    $\sin \theta = \frac{r}{R - r} \Rightarrow (R - r) \sin \theta = r \Rightarrow (R \sin \theta - r \sin \theta) = r$   
          $\Rightarrow R \sin \theta = r + r \sin \theta \Rightarrow R \sin \theta = r(1 + \sin \theta).$   
     **c**   2.43 cm
- 11 2 rad
- 12 **a**   36 m                      **b**   13.6 km/h
- 13 **a**   3.5 m                      **b**   15.3 m
- 14 **a**   2.59 rad                      **b**   44 mm