M2 Chapter 7: Application of Forces

Further Connected Particles

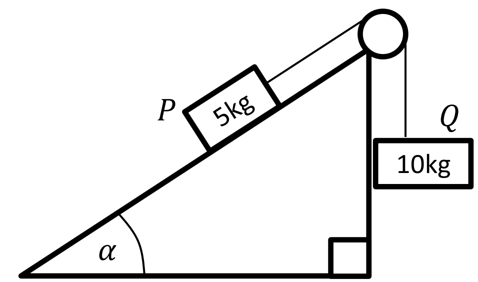
Connected particles involving friction

We have already encountered problems involving connected particles in Mechanics Year 1. We just now throw friction into the mix.

[Textbook] Two particles P and Q of masses 5kg and 10kg respectively are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. P rests on the inclined plane and Q hangs on the edge of the plane with the string vertical and taut. The plane is inclined to the horizontal at an angle α where $\tan\alpha = 0.75$, as shown in the diagram. The coefficient of friction between P and the plane is 0.2. The system is released from rest.

(a) Find the acceleration of the system.

(b) Find the tension in the string.



? Diagram

a

?

b

?

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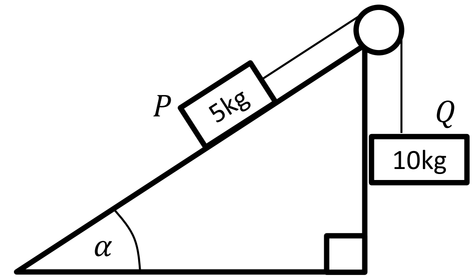
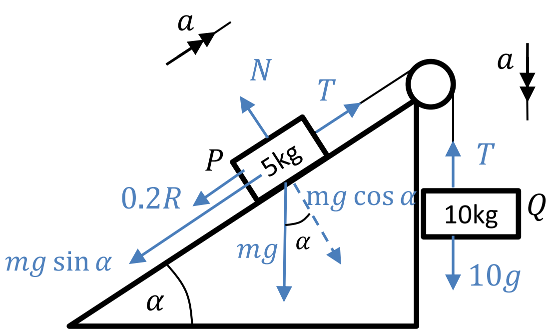
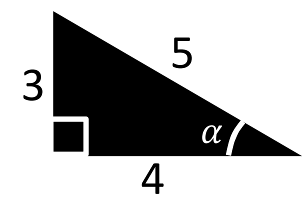




a $\sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$



For P:

$$R(\nwarrow):\ N = 5g\cos\alpha$$

$$N = 5g \times \frac{4}{5} = 4g$$

$$R(\nearrow): T - 5g\sin\alpha - 0.2N = 5a$$

$$T - 3.8g = 5a \quad (1)$$

At Q: $R(\downarrow)$: $10g - T = 10a \quad (2)$

Adding (1) and (2): $6.2g = 15a \ \rightarrow\ a = \frac{31g}{75} = 4.1 \text{ ms}^{-2}$

b Rearranging (1):

$$T = 5a + 3.8g = 57 \text{ N (2sf)}$$

Further Example

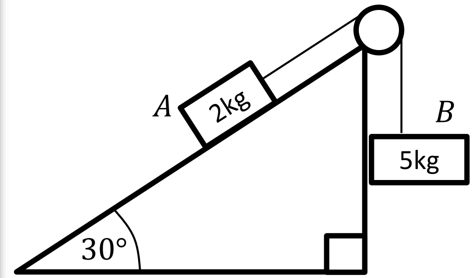
[Textbook] One end of a light inextensible string is attached to a block A of mass 2kg. The block A is held at rest on a **smooth** fixed plane which is inclined to the horizontal at an angle of $30°$. The string lies along the line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a block B of mass 5kg. The system is released from rest. By modelling the blocks as particles and ignoring air resistance,

(a)(i) show that the acceleration of block B is $\frac{4}{7}g$

(ii) find the tension in the string.

(b) State how you have used the fact that the string is inextensible in your calculations.

(c) Calculate the magnitude of the force exerted on the pulley by the string.



? Diagram

a

?

b

?

c

?

Further Example

[Textbook] One end of a light inextensible string is attached to a block A of mass 2kg. The block A is held at rest on a **smooth** fixed plane which is inclined to the horizontal at an angle of 30°. The string lies along the line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a block B of mass 5kg. The system is released from rest. By modelling the blocks as particles and ignoring air resistance,

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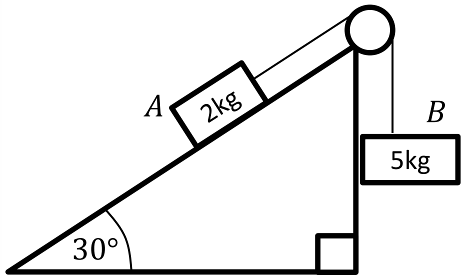
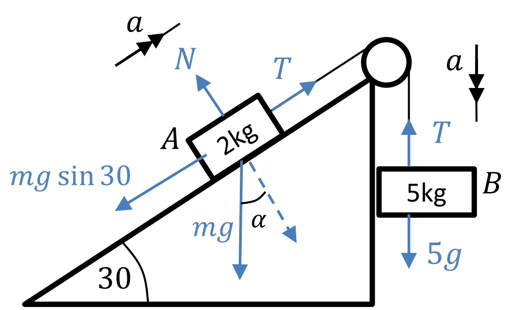




a

For A: $R(\nearrow)$: $T - 2g\sin 30° = 2a$

$$T = 2a + g \quad (1)$$

For B: $R(\downarrow)$: $5g - T = 5a \quad (2)$

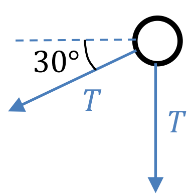
Adding (1) and (2): $5g = 7a + g \rightarrow a = \frac{4}{7}g$

$$T = 2a + g = \frac{8}{7}g + g = \frac{15}{7}g$$

b

String is inextensible so acceleration at A and B the same.

c



Recall that tension acts away from object in direction of string, so two tensions acting on pulley. We want the resultant force.

$$Force = \begin{pmatrix} T\cos 30° \\ T + T\sin 30° \end{pmatrix} = \frac{T}{2}\begin{pmatrix} \sqrt{3} \\ 3 \end{pmatrix}$$

$$Magnitude = \frac{T}{2}\sqrt{\sqrt{3}^2 + 3^2} = \frac{15}{14}g \times \sqrt{12} = \frac{15\sqrt{3}}{7}g \text{ N}$$

We can consider left and down directions the positive ones to avoid negatives. T is constant so can factor out.

Test Your Understanding

Edexcel M1(Old) May 2013(R) Q3

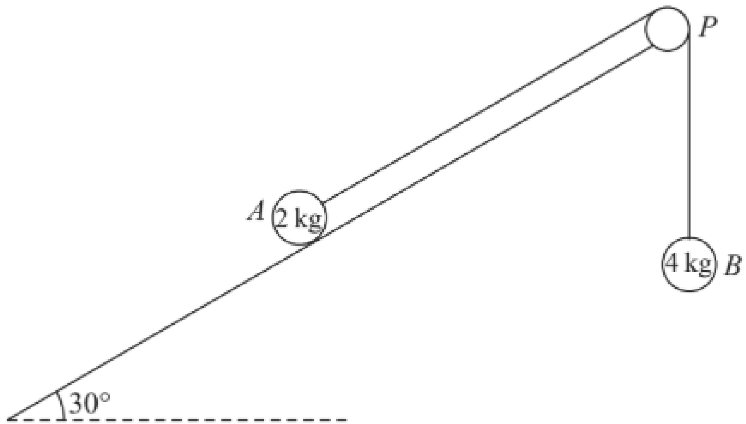


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

?

Test Your Understanding

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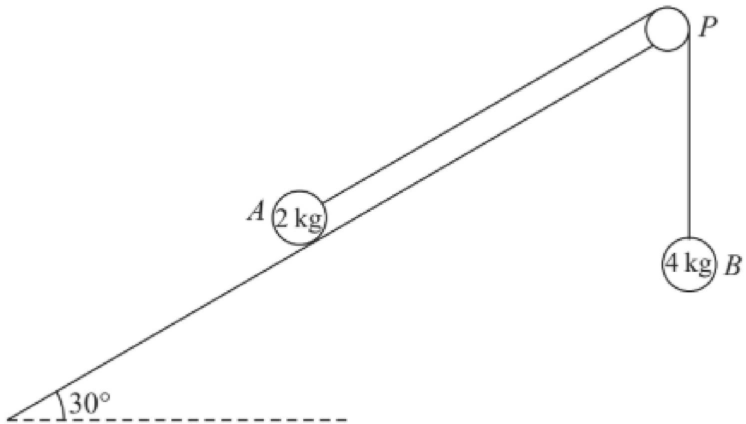

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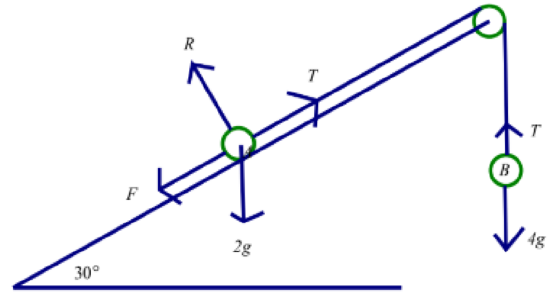


Equation of motion of B: $4g - T = 4a$	M1A1
Equation of motion of A: $T - F - 2g\sin 30 = 2a$	M1A2
OR: $4g - F - 2g\sin 30 = 6a$	
Resolve perpendicular to the plane at A: $R = 2g\cos 30$	B1
Use of $F = \mu R$: $F = \frac{1}{\sqrt{3}} \times 2g\cos 30 (= g)$	M1
$T - g - g = T - 2g = 2a$	
$2T - 4g = 4g - T$, $3T = 8g$, $T = \frac{8g}{3} (\approx 26)$ 26.1(N)	**DM1**A1

Exercise 7.6

Pearson Stats/Mechanics Year 2

Pages 63-65

Homework Exercise

1 Two particles P and Q of equal mass are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a smooth inclined plane. The plane is inclined to the horizontal at an angle α where $\tan \alpha = 0.75$. Particle P is held at rest on the inclined plane at a distance of 2 m from the pulley and Q hangs freely on the edge of the plane at a distance of 3 m above the ground with the string vertical and taut. Particle P is released. Find the speed with which it hits the pulley.

2 A van of mass 900 kg is towing a trailer of mass 500 kg up a straight road which is inclined to the horizontal at an angle α where $\tan \alpha = 0.75$. The van and the trailer are connected by a light inextensible tow-bar. The engine of the van exerts a driving force of magnitude 12 kN and the van and the trailer experience constant resistances to motion of magnitudes 1600 N and 600 N respectively.

a Find the acceleration of the van.

b Find the tension in the tow-bar.

c Comment on the modelling assumption that the resistances to motion of the van and trailer are constant.

3 Two particles P and Q of mass 2 kg and 3 kg respectively are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle of 30°. Particle P is held at rest on the inclined plane and Q hangs freely with the string vertical and taut. Particle P is released and it accelerates up the plane at $2.5\,\mathrm{m\,s^{-2}}$. Find:

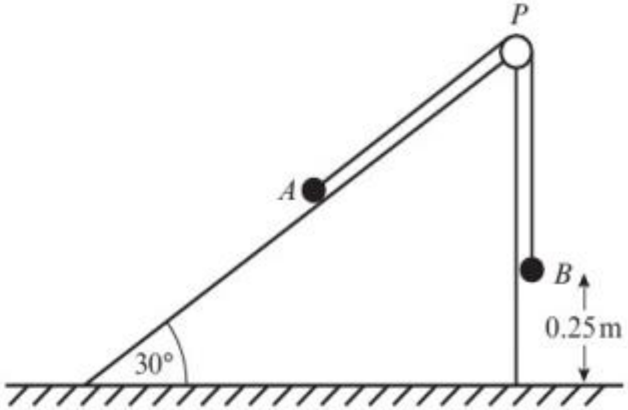
a the tension in the string **(2 marks)**

b the coefficient of friction between P and the plane **(4 marks)**

c the force exerted by the string on the pulley. **(3 marks)**

Homework Exercise

4 Two particles A and B, of mass m kg and 3 kg respectively, are connected by a light inextensible string. The particle A is held resting on a smooth fixed plane inclined at 30° to the horizontal. The string passes over a smooth pulley P fixed at the top of the plane. The portion AP of the string lies along a line of greatest slope of the plane and B hangs freely from the pulley, as shown in the figure. The system is released from rest with B at a height of 0.25 m above horizontal ground.



Immediately after release, B descends with an acceleration of $\frac{2}{5}g$. Given that A does not reach P, calculate:

a the tension in the string while B is descending **(2 marks)**

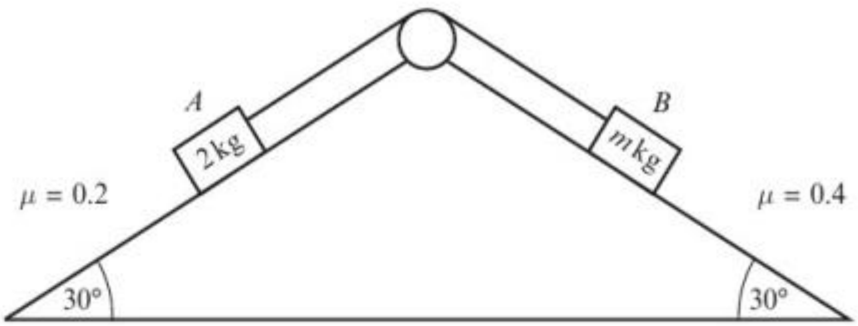
b the value of m. **(4 marks)**

The particle B strikes the ground and does not rebound. Find:

c the time between the instant when B strikes the ground and the instant when A reaches its highest point on the plane. **(6 marks)**

Homework Exercise

5 Two particles A and B on back-to-back rough slopes are connected by a light inextensible string that passes over a smooth pulley as shown in the diagram. A has mass 2 kg and B has mass m kg.



The coefficient of friction between A and the slope is 0.2 and the coefficient of friction between B and the slope is 0.4.

a Show that the maximum value that m can take before the particles begin to move is

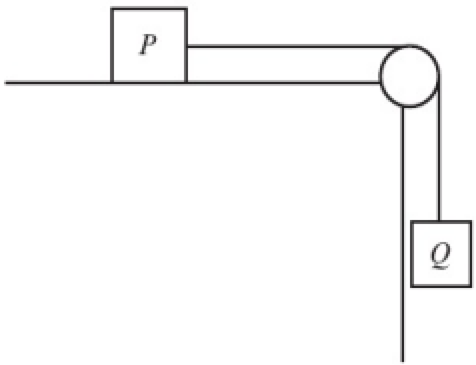
$$\frac{10 + 2\sqrt{3}}{5 - 2\sqrt{3}}$$

(6 marks)

b Given that $m = 10$, find the acceleration of the particles. **(6 marks)**

Homework Exercise

6 A block of metal P of mass 1.5 kg rests on a rough horizontal work bench and is attached to one end of a light inextensible string. The string passes over a small smooth pulley fixed at the edge of the bench. The other end of the string is attached to a box Q of mass 1.6 kg which hangs freely below the pulley, as shown in the diagram. The coefficient of friction between P and the table is μ. The system is released from rest with the string taut. Two seconds after release, Q has velocity $6\,\mathrm{m\,s^{-1}}$. Modelling P and Q as particles,



a calculate the acceleration of Q **(3 marks)**

b find the tension in the string **(4 marks)**

c show that μ is 0.434 (3 s.f.). **(5 marks)**

d State how in your calculations you have used the information that the string is inextensible. **(1 mark)**

Homework Exercise

Challenge

Two particles of mass m_1 and m_2 lie in static equilibrium on a triangular wedge as shown in the diagram. The particles are connected by a light inextensible string that passes over a smooth pulley.

a Given that the wedge is smooth, show that $\dfrac{m_1}{m_2} = \sqrt{3}$.

b Given instead that the wedge is rough, and that the coefficient of friction between each particle and the wedge is μ, show that

$$\frac{\sqrt{3} - \mu}{1 + \mu\sqrt{3}} \leqslant \frac{m_1}{m_2} \leqslant \frac{\sqrt{3} + \mu}{1 - \mu\sqrt{3}}$$

Homework Answers

1 $2.8\,\mathrm{m\,s^{-1}}$

2 **a** $1.12\,\mathrm{m\,s^{-2}}$ **b** $4100\,\mathrm{N}$

c The resistances are unlikely to be constant as the resistance will increase as the speed increases.

3 **a** $21.9\,\mathrm{N}$ **b** 0.418 (3 s.f.) **c** $38\,\mathrm{N}$ (2 s.f.)

4 **a** $18\,\mathrm{N}$ (2 s.f.) **b** 2 **c** $\frac{2}{7}\,\mathrm{s}$

5 **a** For particle on LHS

$\mathrm{R}(\nwarrow)$: $R_1 = \sqrt{3}\,g$, $\mathrm{R}(\nearrow)$: $T = \frac{\sqrt{3}}{5}g + g$

For particle on RHS

$\mathrm{R}(\nearrow)$: $R_2 = \frac{\sqrt{3}}{2}mg$, $\mathrm{R}(\searrow)$: $T = \frac{1}{2}mg - \frac{\sqrt{3}}{5}mg$

For maximum value of m, $\mathrm{R}(\searrow)$ is equal to $\mathrm{R}(\nearrow)$:

$\frac{1}{2}mg - \frac{\sqrt{3}}{5}mg = \frac{\sqrt{3}}{5}g + g$

$\Rightarrow \frac{1}{2}m - \frac{\sqrt{3}}{5}m = \frac{\sqrt{3}}{5} + 1$

$\Rightarrow \left(\frac{1}{2} - \frac{\sqrt{3}}{5}\right)m = \frac{\sqrt{3}}{5} + 1$

$\Rightarrow m = \frac{10 + 2\sqrt{3}}{5 - 2\sqrt{3}}$

b $2.70\,\mathrm{m\,s^{-2}}$ (3 s.f.)

6 **a** $3\,\mathrm{ms^{-2}}$ **b** $10.88\,\mathrm{N}$

c $\mathrm{R}(\rightarrow)$: $T - 1.5\mu g = 4.5$

$\mu = \frac{10.88 - 4.5}{1.5g} = \frac{319}{735} = 0.434$ (3 s.f.)

d The string doesn't stretch so the tension in the string is constant.

Challenge

a For particle on LHS: $\mathrm{R}(\nearrow)$: $T = \frac{1}{2}m_1 g$

For particle on RHS: $\mathrm{R}(\nwarrow)$: $T = \frac{\sqrt{3}}{2}m_2 g$

To prove, equate values of T.

b If (attempted) motion is down slope on RHS

Consider particle on LHS

$T = \frac{1}{2}m_1 g + \frac{\sqrt{3}}{2}\mu m_1 g$

Consider particle on RHS

$T = \frac{\sqrt{3}}{2}m_2 g - \frac{1}{2}\mu m_2 g$

Equate values of T to find $\frac{m_1}{m_2}$

If (attempted) motion is down slope on LHS

Consider particle on LHS

$T = \frac{1}{2}m_1 g - \frac{\sqrt{3}}{2}\mu m_1 g$

Consider particle on RHS

$T = \frac{1}{2}\mu m_2 g + \frac{\sqrt{3}}{2}m_2 g$

Equate values of T to find $\frac{m_1}{m_2}$