M2 Chapter 7: Application of Forces

Friction and Static Particles

Friction

Earlier in the module we saw that the frictional force $F \leq \mu R$, where $F = \mu R$ if the object on the plane is moving. Were the object is not moving, we saw that the **force of friction acts in a direction opposite** to that which it would be moving if the frictional force wasn't there.

[Textbook] A box of mass 10kg rests in limiting equilibrium on a rough plane inclined at $20°$ above the horizontal.
(a) Find the coefficient of friction between the box and the plane.
A horizontal force of magnitude P N is applied to the box. Given that the box remains in equilibrium,
(b) find the maximum possible value of P.

a

? Force Diagram

? Working

b

? Force Diagram

? Working

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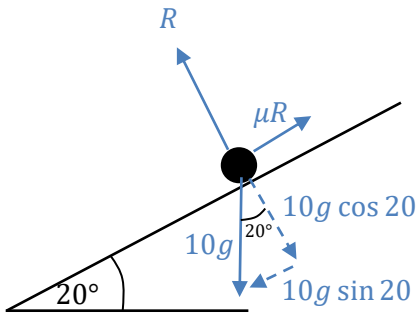
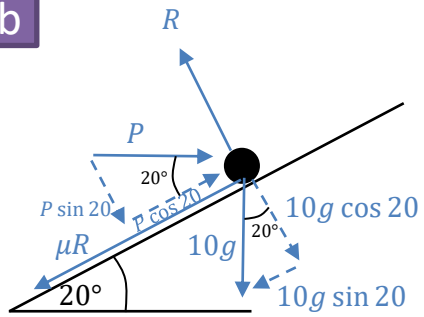


$$R(\nwarrow): \quad R = 10g\cos 20°$$
$$R(\nearrow): \quad \mu R = 10g\sin 20°$$
$$\mu = \frac{10g\sin 20°}{10g\cos 20°} = \tan 20° = 0.36\ (2sf)$$

b

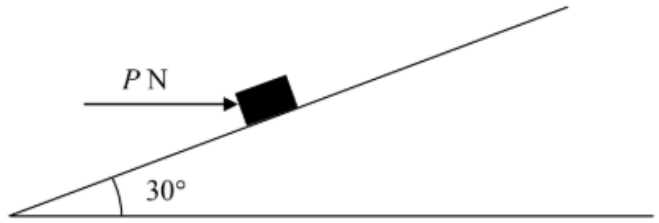


Important: If we increase P to its maximum, the particle is on the verge of moving UP the plane. Thus friction acts downwards to oppose.

$$R(\nwarrow): \quad R = 10g\cos 20° + P\sin 20°$$
$$R(\nearrow): \quad P\cos 20° = 10g\sin 20° + \mu R$$
$$P\cos 20° = 10g\sin 20° + 0.36(10g\cos 20° + P\sin 20°)$$
$$\ldots$$
$$P = 82\ N\ (2\text{sf})$$

Test Your Understanding

Edexcel M1(Old) Jan 2006 Q5



A parcel of weight 10 N lies on a rough plane inclined at an angle of 30° to the horizontal. A horizontal force of magnitude P newtons acts on the parcel, as shown in Figure 2. The parcel is in equilibrium and on the point of slipping up the plane. The normal reaction of the plane on the parcel is 18 N. The coefficient of friction between the parcel and the plane is μ. Find

(a) the value of P, **(4)**

(b) the value of μ. **(5)**

The horizontal force is removed.

(c) Determine whether or not the parcel moves. **(5)**

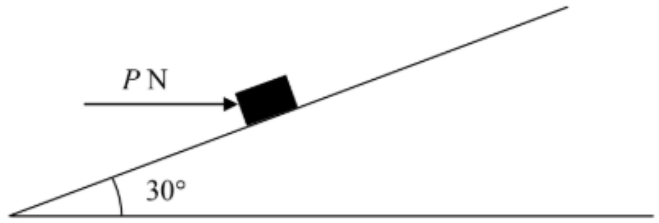
? a

? b

? c

Test Your Understanding

Edexcel M1(Old) Jan 2006 Q5



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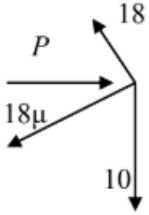


(a) R(perp to plane):

$P \sin 30 + 10 \cos 30 = 18$ M1 A1

↓

Solve: $P \approx \underline{18.7 \text{ N}}$ M1 A1 (4)

(b) R(// plane):

$P \cos 30 = 10 \sin 30 + F$ M1 A1

$F = 18\mu$ used M1

↓↓

Sub and solve: $\mu = \underline{0.621 \text{ or } 0.62}$ M1 A1 (5)

(c) Normal reaction now $= 10 \cos 30$ M1 A1

Component of weight down plane $= 10 \sin 30$ $(= 5 \text{ N})$ B1

↓

$F_{max} = \mu R_{new} \approx 5.37 \text{ N}$ (AWRT 5.4) M1

$5.37 > 5 \Rightarrow$ does not slide A1 cso (5)

Exercise 7.3

Pearson Stats/Mechanics Year 2
Pages 58-59

Homework Exercise

1 A book of mass 2 kg rests on a rough horizontal table. When a force of magnitude 8 N acts on the book, at an angle of 20° to the horizontal in an upward direction, the book is on the point of slipping.

Calculate, to three significant figures, the value of the coefficient of friction between the book and the table.

Hint 'On the point of slipping' means that the book is in limiting equilibrium.

2 A block of mass 4 kg rests on a rough horizontal table. When a force of 6 N acts on the block, at an angle of 30° to the horizontal in a downward direction, the block is on the point of slipping. Find the value of the coefficient of friction between the block and the table.

3 A block of weight 10 N is at rest on a rough horizontal surface. A force of magnitude 3 N is applied to the block at an angle of 60° above the horizontal in an upward direction. The coefficient of friction between the block and the surface is 0.3.

a Calculate the force of friction. **b** Determine whether the friction is limiting.

4 A packing crate of mass 10 kg rests on rough horizontal ground. It is filled with books which are evenly distributed through the crate. The coefficient of friction between the crate and the ground is 0.3.

a Find the mass of the books if the crate is in limiting equilibrium under the effect of a horizontal force of magnitude 147 N.

b State what modelling assumptions you have made.

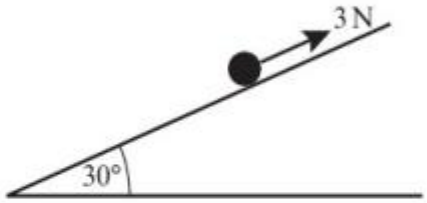
Homework Exercise

5 A block of mass 2 kg rests on a rough horizontal plane. A force P acts on the block at an angle of 45° to the horizontal. The equilibrium is limiting, with $\mu = 0.3$.

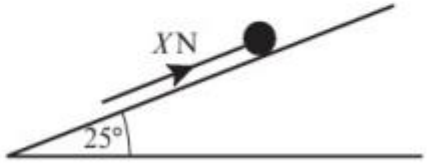
Find the magnitude of P if:

a P acts in a downward direction **b** P acts in an upward direction.

6 A particle of mass 0.3 kg is on a rough plane which is inclined at an angle 30° to the horizontal. The particle is held at rest on the plane by a force of magnitude 3 N acting up the plane, in a direction parallel to a line of greatest slope of the plane. The particle is on the point of slipping up the plane. Find the coefficient of friction between the particle and the plane.



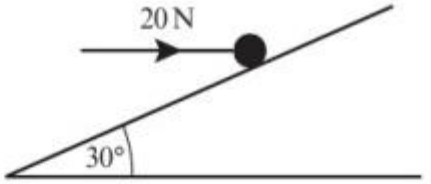
7 A particle of mass 1.5 kg rests in equilibrium on a rough plane under the action of a force of magnitude X N acting up a line of greatest slope of the plane. The plane is inclined at 25° to the horizontal. The particle is in limiting equilibrium and on the point of moving up the plane. The coefficient of friction between the particle and the plane is 0.25. Calculate:



a the normal reaction of the plane on the particle **b** the value of X.

Homework Exercise

8 A horizontal force of magnitude 20 N acts on a block of mass 1.5 kg, which is in equilibrium resting on a rough plane inclined at 30° to the horizontal. The line of action of the force is in the same vertical plane as the line of greatest slope of the inclined plane.

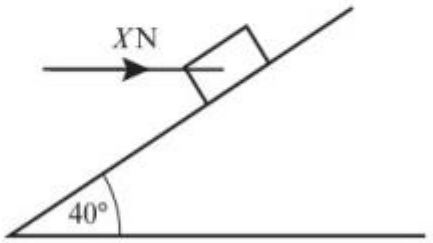


a Find the normal reaction between the block and the plane. **(4 marks)**

b Find the magnitude and direction of the frictional force acting on the block. **(3 marks)**

c Hence find the minimum value of the coefficient of friction between the block and the plane. **(2 marks)**

9 A box of mass 3 kg lies on a rough plane inclined at 40° to the horizontal. The box is held in equilibrium by means of a horizontal force of magnitude X N. The line of action of the force is in the same vertical plane as the line of greatest slope of the inclined plane. The coefficient of friction between the box and the plane is 0.3 and the box is in limiting equilibrium and is about to move up the plane.

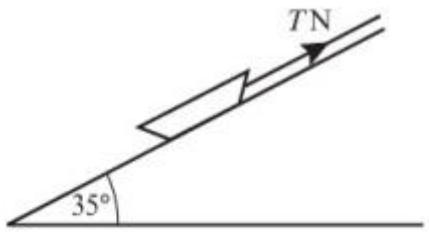


a Find X. **(6 marks)**

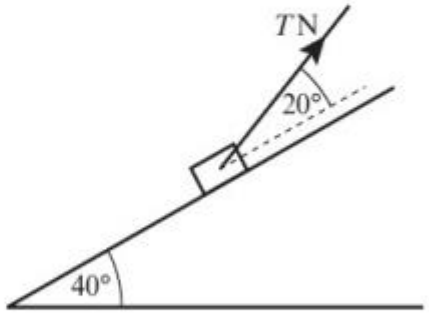
b Find the normal reaction between the box and the plane. **(2 marks)**

Homework Exercise

10 A small child, sitting on a sledge, rests in equilibrium on an inclined slope. The sledge is held by a rope which lies along the slope and is under tension. The sledge is on the point of slipping down the plane. Modelling the child and sledge as a particle and the rope as a light inextensible string, calculate the tension in the rope, given that the mass of the child and sledge is 22 kg, the coefficient of friction is 0.125 and that the slope is a plane inclined at 35° to the horizontal.



11 A box of mass 0.5 kg is placed on a plane which is inclined at an angle of 40° to the horizontal. The coefficient of friction between the box and the plane is $\frac{1}{5}$. The box is kept in equilibrium by a light string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20° with the plane, as shown in the diagram. The box is in limiting equilibrium and may be modelled as a particle. The tension in the string is T N.



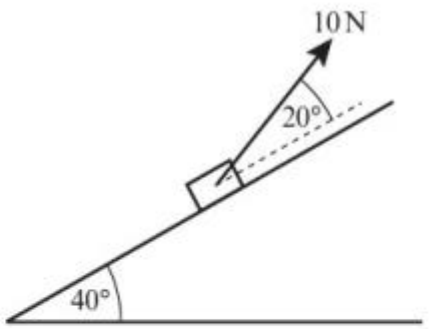
Find the range of possible values of T. **(8 marks)**

Problem-solving

The box might be about to move up or down the slope.

Homework Exercise

12 A box of mass 1 kg is placed on a plane, which is inclined at an angle of 40° to the horizontal. The box is kept in equilibrium on the point of moving up the plane by a light string, which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20° with the plane, as shown in the diagram. The box is in limiting equilibrium and may be modelled as a particle. The tension in the string is 10 N and the coefficient of friction between the box and the plane is μ. Find μ.



(7 marks)

13 A particle of mass 2 kg rests in limiting equilibrium on a rough plane angled at θ above the horizontal where $\tan\theta = \frac{3}{4}$. A horizontal force of magnitude P N acting into the plane is applied to the box. Given that the box remains in equilibrium, find the maximum possible value of P. **(8 marks)**

Problem-solving

First find the coefficient of friction between the box and the plane.

Homework Answers

1 0.446

2 0.123

3 **a** 1.5 N **b** Not limiting

4 **a** 40 kg

b The assumption is that the crate and books may be modelled as a particle.

5 **a** 11.9 N **b** 6.40 N

6 0.601 (accept 0.6)

7 **a** 13.3 N **b** $X = 10.7$ N

8 **a** 22.7 N

b 9.97 N down the plane

c $\mu \geqslant 0.439$

9 **a** $X = 44.8$ (accept 44.7) **b** $R = 51.3$ N

10 $T = 102$ (3 s.f.)

11 $2.75 \leqslant T \leqslant 3.87$ N

12 0.758

13 **a** 0.75 **b** 67.2 N