Pure1 Chapter 11: Vectors

Chapter Practice

Key Points

1 If $\overrightarrow{PQ} = \overrightarrow{RS}$ then the line segments PQ and RS are equal in length and are parallel.

2 $\overrightarrow{AB} = -\overrightarrow{BA}$ as the line segment AB is equal in length, parallel and in the opposite direction to BA.

3 **Triangle law for vector addition:** $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

If $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$, then $\mathbf{a} + \mathbf{b} = \mathbf{c}$

4 Subtracting a vector is equivalent to 'adding a negative vector': $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$

5 Adding the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QP}$ gives the zero vector $\mathbf{0}$: $\overrightarrow{PQ} + \overrightarrow{QP} = 0$.

6 Any vector parallel to the vector $\mathbf{a}$ may be written as $\lambda\mathbf{a}$, where λ is a non-zero scalar.

7 To multiply a column vector by a scalar, multiply each component by the scalar: $\lambda\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \lambda p \\ \lambda q \end{pmatrix}$

8 To add two column vectors, add the x-components and the y-components $\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p + r \\ q + s \end{pmatrix}$

9 A unit vector is a vector of length 1. The unit vectors along the x- and y-axes are usually denoted by $\mathbf{i}$ and $\mathbf{j}$ respectively. $\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

10 For any two-dimensional vector: $\begin{pmatrix} p \\ q \end{pmatrix} = p\mathbf{i} + q\mathbf{j}$

Key Points

11 For the vector $\mathbf{a} = x\mathbf{i} + y\mathbf{j} = \begin{pmatrix} x \\ y \end{pmatrix}$, the magnitude of the vector is given by: $|\mathbf{a}| = \sqrt{x^2 + y^2}$

12 A unit vector in the direction of $\mathbf{a}$ is $\dfrac{\mathbf{a}}{|\mathbf{a}|}$

13 In general, a point P with coordinates (p, q) has position vector:

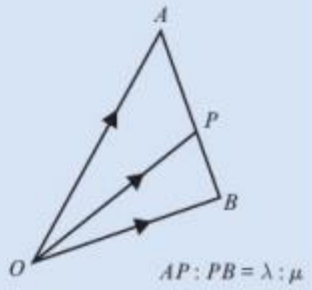
$$\overrightarrow{OP} = p\mathbf{i} + q\mathbf{j} = \begin{pmatrix} p \\ q \end{pmatrix}$$

14 $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$, where $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the position vectors of A and B respectively.

15 If the point P divides the line segment AB in the ratio $\lambda : \mu$, then

$$\overrightarrow{OP} = \overrightarrow{OA} + \frac{\lambda}{\lambda + \mu}\overrightarrow{AB}$$

$$= \overrightarrow{OA} + \frac{\lambda}{\lambda + \mu}(\overrightarrow{OB} - \overrightarrow{OA})$$



16 If $\mathbf{a}$ and $\mathbf{b}$ are two non-parallel vectors and $p\mathbf{a} + q\mathbf{b} = r\mathbf{a} + s\mathbf{b}$ then $p = r$ and $q = s$

Chapter Exercises

1 Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle.

$\mathbf{F}_1 = -3\mathbf{i} + 7\mathbf{j}$ newtons

$\mathbf{F}_2 = \mathbf{i} - \mathbf{j}$ newtons

The resultant force $\mathbf{R}$ acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$.

a Calculate the magnitude of $\mathbf{R}$ in newtons. **(3 marks)**

b Calculate, to the nearest degree, the angle between the line of action of $\mathbf{R}$ and the vector $\mathbf{j}$. **(2 marks)**

2 A small boat S, drifting in the sea, is modelled as a particle moving in a straight line at constant speed. When first sighted at 09:00, S is at a point with position vector $(-2\mathbf{i} - 4\mathbf{j})$ km relative to a fixed origin O, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north respectively. At 09:40, S is at the point with position vector $(4\mathbf{i} - 6\mathbf{j})$ km.

a Calculate the bearing on which S is drifting.

b Find the speed of S.

3 A football player kicks a ball from point A on a flat football field. The motion of the ball is modelled as that of a particle travelling with constant velocity $(4\mathbf{i} + 9\mathbf{j})\,\text{m s}^{-1}$.

a Find the speed of the ball.

b Find the distance of the ball from A after 6 seconds.

c Comment on the validity of this model for large values of t.

4 $ABCD$ is a trapezium with AB parallel to DC and $DC = 4AB$.

M divides DC such that $DM : MC = 3 : 2$, $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.

Find, in terms of $\mathbf{a}$ and $\mathbf{b}$:

a $\overrightarrow{AM}$ **b** $\overrightarrow{BD}$ **c** $\overrightarrow{MB}$ **d** $\overrightarrow{DA}$

Chapter Exercises

5 The vectors $5\mathbf{a} + k\mathbf{b}$ and $8\mathbf{a} + 2\mathbf{b}$ are parallel. Find the value of k. **(3 marks)**

6 Given that $\mathbf{a} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 10 \\ -2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -5 \\ -3 \end{pmatrix}$ find:

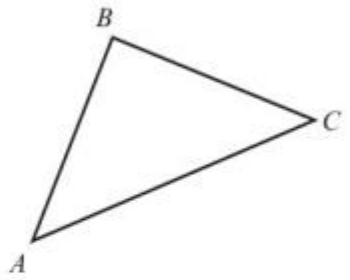
a $\mathbf{a} + \mathbf{b} + \mathbf{c}$ **b** $\mathbf{a} - 2\mathbf{b} + \mathbf{c}$ **c** $2\mathbf{a} + 2\mathbf{b} - 3\mathbf{c}$

7 In triangle ABC, $\overrightarrow{AB} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{AC} = 6\mathbf{i} + 3\mathbf{j}$, find:

a $\overrightarrow{BC}$ **(2 marks)**

b $\angle BAC$ **(4 marks)**

c the area of the triangle. **(2 marks)**



8 The resultant of the vectors $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j}$ and $\mathbf{b} = 2p\mathbf{i} - p\mathbf{j}$ is parallel to the vector $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$. Find:

a the value of p **(3 marks)**

b the resultant of vectors **a** and **b**. **(1 mark)**

9 For each of the following vectors, find

i a unit vector in the same direction **ii** the angle the vector makes with **i**

a $\mathbf{a} = 8\mathbf{i} + 15\mathbf{j}$ **b** $\mathbf{b} = 24\mathbf{i} - 7\mathbf{j}$ **c** $\mathbf{c} = -9\mathbf{i} + 40\mathbf{j}$ **d** $\mathbf{d} = 3\mathbf{i} - 2\mathbf{j}$

10 The vector $\mathbf{a} = p\mathbf{i} + q\mathbf{j}$, where p and q are positive constants, is such that $|\mathbf{a}| = 15$. Given that a makes an angle of 55° with **i**, find the values of p and q.

Chapter Exercises

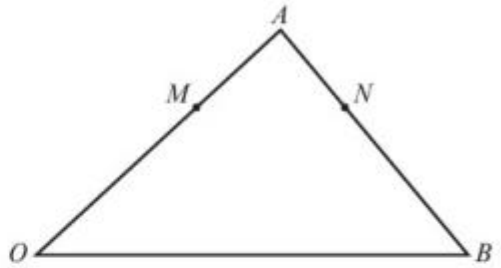
11 Given that $|3\mathbf{i} - k\mathbf{j}| = 3\sqrt{5}$, find the value of k. **(3 marks)**

12 OAB is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point M divides OA in the ratio 3 : 2. MN is parallel to OB.

a Express the vector $\overrightarrow{ON}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. **(4 marks)**

b Find vector $\overrightarrow{MN}$. **(2 marks)**

c Show that $AN : NB = 2 : 3$. **(2 marks)**



13 Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, are given by the vectors $\mathbf{F}_1 = (4\mathbf{i} - 5\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N. The resultant force, $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$ acts in a direction which is parallel to the vector $(3\mathbf{i} - \mathbf{j})$

a Find the angle between $\mathbf{R}$ and the vector $\mathbf{i}$. **(3 marks)**

b Show that $p + 3q = 11$. **(4 marks)**

c Given that $p = 2$, find the magnitude of $\mathbf{R}$. **(2 marks)**

14 A particle P is accelerating at a constant speed. When $t = 0$, P has velocity $\mathbf{u} = (3\mathbf{i} + 4\mathbf{j})$ m s^{-1} and at time $t = 2$ s, P has velocity $\mathbf{v} = (15\mathbf{i} - 3\mathbf{j})$ m s^{-1}.

The acceleration vector of the particle is given by the formula: $\mathbf{a} = \dfrac{\mathbf{v} - \mathbf{u}}{t}$

Find the magnitude of the acceleration of P. **(3 marks)**

Challenge

The point B lies on the line with equation $3y = 15 - 5x$.

Given that $|\overrightarrow{OB}| = \dfrac{\sqrt{34}}{2}$, find two possible expressions for $\overrightarrow{OB}$ In the form $p\mathbf{i} + q\mathbf{j}$.

Chapter Answers

1 **a** $2\sqrt{10}$ newtons **b** 18°

2 **a** 108° **b** 9.48 km h^{-1}

3 **a** 9.85 m s^{-1} **b** 59.1 m

c The model ignores friction and air resistance.
The model will become less accurate as t increases.

4 **a** $\mathbf{b} - \frac{3}{5}\mathbf{a}$ **b** $\mathbf{b} - 4\mathbf{a}$ **c** $\frac{8}{5}\mathbf{a} - \mathbf{b}$ **d** $3\mathbf{a} - \mathbf{b}$

5 1.25

6 **a** $\begin{pmatrix}12\\-1\end{pmatrix}$ **b** $\begin{pmatrix}-18\\5\end{pmatrix}$ **c** $\begin{pmatrix}49\\13\end{pmatrix}$

7 **a** $3\mathbf{i} - 2\mathbf{j}$ **b** 32.5° **c** 10.5

8 **a** $p = -1.5$ **b** $\mathbf{i} - 1.5\mathbf{j}$

9 **a** **i** $\frac{1}{17}(8\mathbf{i} + 15\mathbf{j})$ **ii** 61.9° above

b **i** $\frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$ **ii** 16.3° below

c **i** $\frac{1}{41}(-9\mathbf{i} + 40\mathbf{j})$ **ii** 102.7° above

d **i** $\frac{1}{\sqrt{13}}(3\mathbf{i} - 2\mathbf{j})$ **ii** 33.7° below

10 $p = 8.6$, $q = 12.3$

11 ±6

12 **a** $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ **b** $\frac{2}{5}\mathbf{b}$

c $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{AN} = \frac{2}{5}(\mathbf{b} - \mathbf{a})$ so $AN : NB = 2 : 3$

13 **a** 18.4° below

b $\mathbf{R} = (4 + p)\mathbf{i} + (5 - q)\mathbf{j}$, $4 + p = 3\lambda$ and $5 - q = -\lambda$
$4 + p = 3(q - 5)$ so $p + 3q = 11$

c $2\sqrt{10}$ newtons

14 $\frac{\sqrt{193}}{2}$

Challenge

$\overrightarrow{OB} = \frac{3}{2}\mathbf{i} + \frac{5}{2}\mathbf{j}$ or $\frac{99}{34}\mathbf{i} + \frac{5}{34}\mathbf{j}$