Stats Yr2 Chapter 2: Probability Theory

Probability Trees

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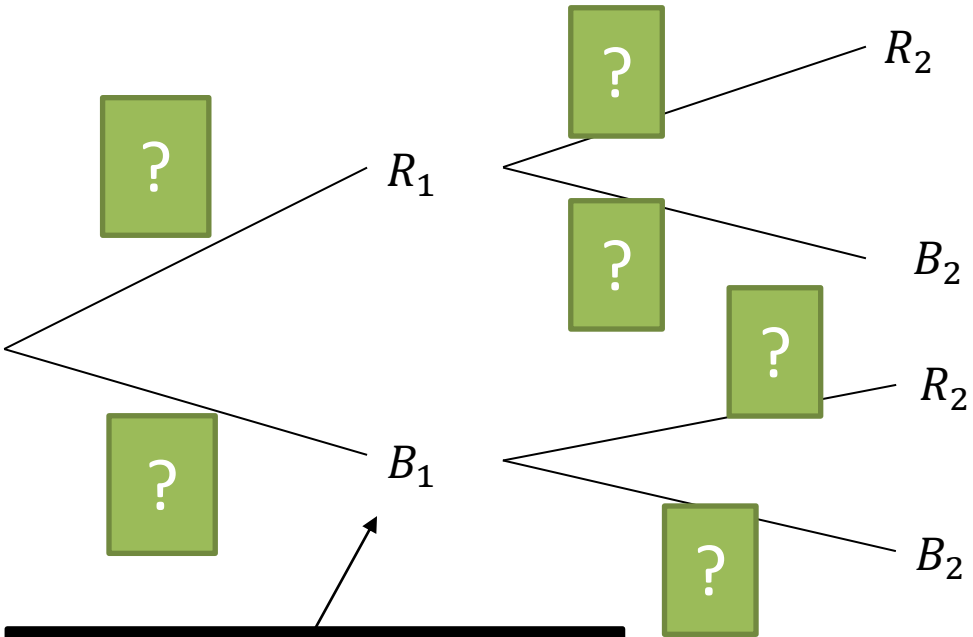
We saw probability trees in Year 1. The only difference here is **determining a conditional probability** using your tree.

Example: You have two bags, the first with 5 red balls and 5 blue balls, and the second with 3 red balls and 6 blue balls. You first pick a ball from the first bag, and place it in the second. You then pick a ball from the second bag. Complete the tree diagram.



Tip: Use variable subscripting to indicate what pick you're referring to.

Hence find the probability that:

a) You pick a red ball on your second pick.

?

b) Given that your second pick was red, the first pick was also red.

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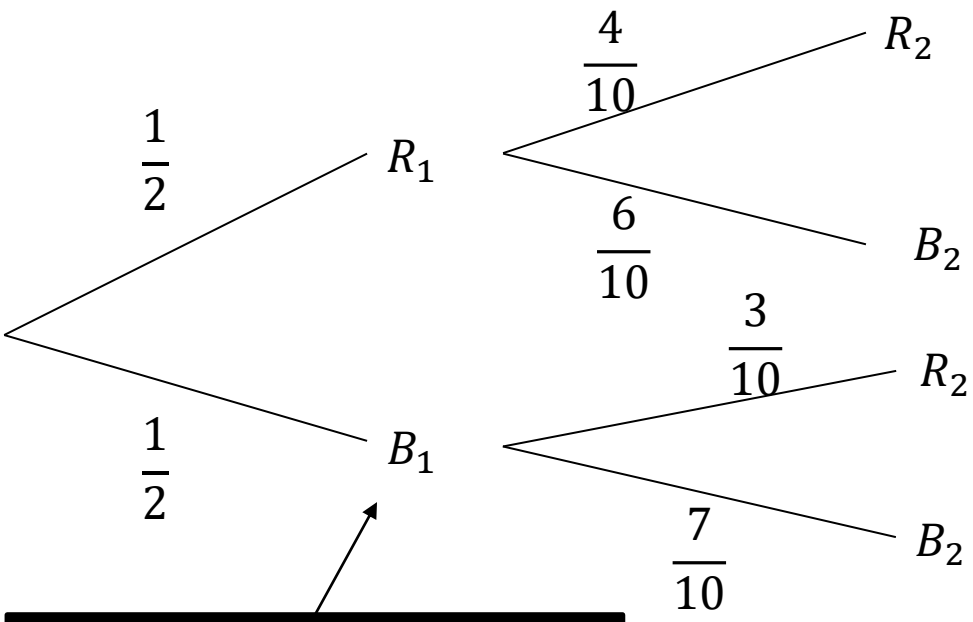


Tip: Use variable subscripting to indicate what pick you're referring to.

Hence find the probability that:

a) You pick a red ball on your second pick.

$$P(R_2) = P(R_1 \cap R_2) + P(B_1 \cap R_2)$$
$$= \frac{1}{5} + \frac{3}{20} = \frac{7}{20}$$

b) Given that your second pick was red, the first pick was also red.

$$P(R_1|R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)}$$
$$= \frac{\frac{1}{5}}{\frac{7}{20}} = \frac{4}{7}$$

It's vitally important that you use good notation, making use of the | symbol.

Further Example

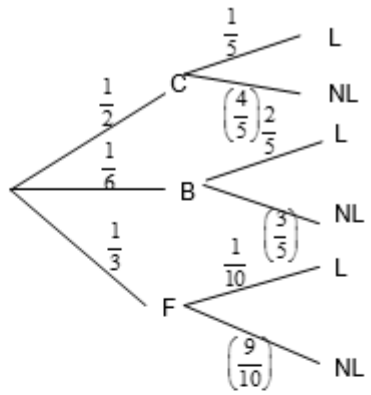
Edexcel S1 May 2009 Q2

On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}, \frac{2}{5}$ and $\frac{1}{10}$ respectively.

(*c*) Given that Bill is late, find the probability that he did not travel on foot. **(4)**

(Part (a) asks for a tree diagram, which may help with this question)

(a)



Correct tree All labels Probabilities on correct branches	B1 B1 B1

?

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Edexcel S1 May 2009 Q2

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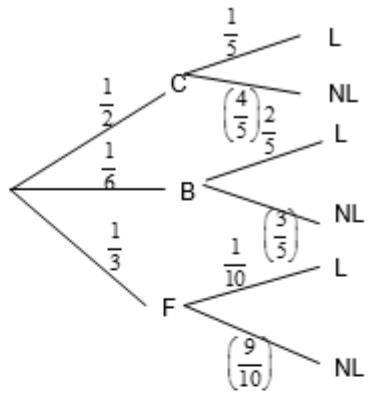

Correct tree
All labels
Probabilities on correct branches

B1
B1
B1

$$P(F'|L) = \frac{P(F' \cap L)}{P(L)}$$

$$= \frac{\left(\frac{1}{6} \times \frac{2}{5}\right) + \left(\frac{1}{2} \times \frac{1}{5}\right)}{\left(\frac{1}{2} \times \frac{1}{5}\right) + \left(\frac{1}{6} \times \frac{2}{5}\right) + \left(\frac{1}{3} \times \frac{1}{10}\right)}$$

$$= \frac{5}{6}$$

Testing Your Understanding

Edexcel S1

6. [Jan 2006 Q4] A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded.

(*a*) Draw a tree diagram to represent the information. **(3)**

Find the probability that

(*a*) the second ball selected is red, **(2)**

(*b*) both balls selected are red, given that the second ball selected is red. **(2)**

(a) ?

(b) ?

(c) ?

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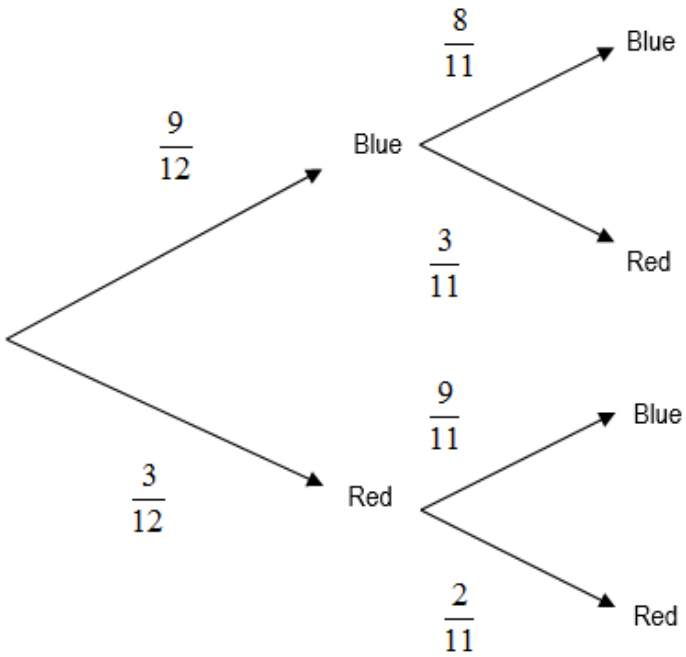
(a)



Tree M1

$\frac{9}{12}, \frac{3}{12}$ A1

Complete & labels A1

(b) $P(\text{Second ball is red}) = \frac{9}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{2}{11} = \frac{1}{4}$ M1A1

(c) $P(\text{Both are red} \mid \text{Second ball is red}) = \frac{\frac{3}{12} \times \frac{2}{11}}{\frac{1}{4}} = \frac{2}{11}$ exact or awrt 0.182 M1A1

Exercise 2E

Pearson Stats/Mechanics Year 2
Pages 17-18

Homework Exercise

1 A bag contains five red and four blue tokens. A token is chosen at random, the colour recorded and the token is not replaced. A second token is chosen and the colour recorded.

a Draw a tree diagram to illustrate this situation.

Find the probability that:

b the second token is red given that the first token is blue

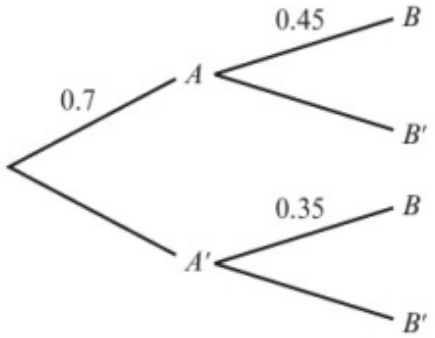
c the first token is red given that the second token is blue

d the first token is blue given that the tokens are different colours

e the tokens are the same colour given that the second token is red.

2 A and B are two events such that $P(B|A) = 0.45$, $P(B|A') = 0.35$ and $P(A) = 0.7$.

a Copy and complete the tree diagram representing this information.



b Find:

i $P(A \cap B)$

ii $P(A' \cap B')$

iii $P(A|B)$

3 A box of 24 chocolates contains 10 dark and 14 milk chocolates. Linda chooses a chocolate at random and eats it, followed by another one.

a Draw a tree diagram to represent this information.

Find the probability that Linda eats:

b two dark chocolates

c one dark and one milk chocolate

d two dark chocolates given that she eats at least one dark chocolate.

Homework Exercise

4 Jean always goes to work by bus or takes a taxi. If one day she goes to work by bus, the probability she goes to work by taxi the next day is 0.4. If one day she goes to work by taxi, the probability she goes to work by bus the next day is 0.7.

Given that Jean takes the bus to work on Monday, find the probability that she takes a taxi to work on Wednesday.

5 Sue has two coins. One is fair, with a head on one side and a tail on the other. The second is a trick coin and has a tail on both sides. Sue picks up one of the coins at random and flips it.

a Find the probability that it lands heads up.

b Given that it lands tails up, find the probability that she picked up the fair coin.

6 A bag contains 4 blue balls and 7 green balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded.

a Draw a tree diagram to represent the information. **(3 marks)**

Find the probability that:

b the second ball selected is green **(2 marks)**

c both balls selected are green, given that the second ball selected is green. **(2 marks)**

Homework Exercise

7 In an engineering company, factories A, B and C are all producing tin sheets of the same type. Factory A produces 25% of the sheets, factory B produces 45% and the rest are produced by factory C. Factories A, B and C produce flawed sheets with probabilities 0.02, 0.07 and 0.04 respectively.

a Draw a tree diagram to represent this information. **(3 marks)**

b Find the probability that a randomly selected sheet is:

i produced by factory B and flawed **(2 marks)**

ii flawed. **(3 marks)**

c Given that a randomly selected sheet is flawed, find the probability that it was produced by factory A. **(3 marks)**

8 A genetic condition is known to be present in 4% of a population. A test is developed to help determine whether or not someone has the genetic condition.
If a person has the condition, the test is positive with probability 0.9.
If a person does not have the condition, the test is positive with probability 0.02.

a Draw a tree diagram to represent this information. **(3 marks)**

A person is selected at random from the population and tested for this condition.

b Find the probability that the test is negative. **(3 marks)**

A doctor randomly selects a person from the population and tests them for the condition.

Given that the test is negative,

c find the probability that they do have the condition. **(2 marks)**

d Comment on the effectiveness of this test. **(1 mark)**

Homework Exercise

9 On a randomly chosen day the probabilities that Bill travels to work by car, by bus or by train are 0.1, 0.6 and 0.3 respectively. The probabilities of being late when using these methods of travel are 0.55, 0.3 and 0.05 respectively.

a Draw a tree diagram to represent this information. **(3 marks)**

b Find the probability that on a randomly chosen day,

i Bill travels by train and is late **(2 marks)**

ii Bill is late. **(2 marks)**

c Given that Bill is late, find the probability that he did not travel by car. **(4 marks)**

10 A box A contains 7 counters of which 4 are green and 3 are blue.
A box B contains 5 counters of which 2 are green and 3 are blue.
A counter is drawn at random from box A and placed in box B. A second counter is drawn at random from box A and placed in box B.
A third counter is then drawn at random from the counters in box B.

a Draw a tree diagram to show this situation. **(4 marks)**

The event C occurs when the 2 counters drawn from box A are of the same colour.
The event D occurs when the counter drawn from box B is blue.

b Find $P(C)$. **(3 marks)**

c Show that $P(D) = \frac{27}{49}$ **(3 marks)**

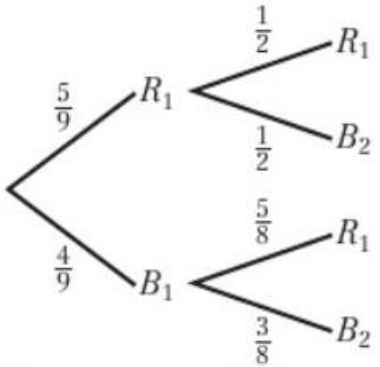
d Show that $P(C \cap D) = \frac{11}{49}$ **(2 marks)**

e Hence find $P(C \cup D)$. **(2 marks)**

f Given that all three counters drawn are the same colour, find the probability that they are all green. **(3 marks)**

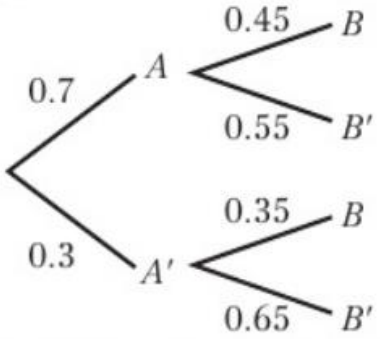
Homework Answers

1 **a**



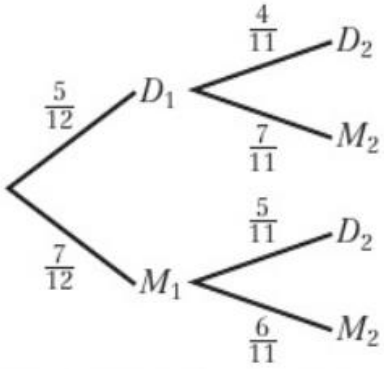
b $\frac{5}{8}$ **c** $\frac{5}{8}$ **d** $\frac{1}{2}$ **e** $\frac{1}{2}$

2 **a**



b **i** 0.315 **ii** 0.195 **iii** 0.75

3 **a**

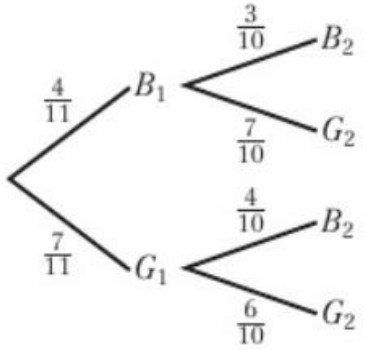


b 0.152 (3 s.f.) **c** 0.530 (3 s.f.) **d** 0.222 (3 s.f.)

4 0.36

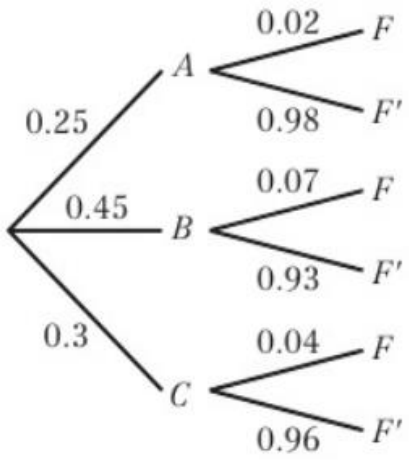
5 **a** 0.25 **b** 0.333

6 **a**



b $\frac{7}{11}$ **c** $\frac{3}{5}$

7 **a**

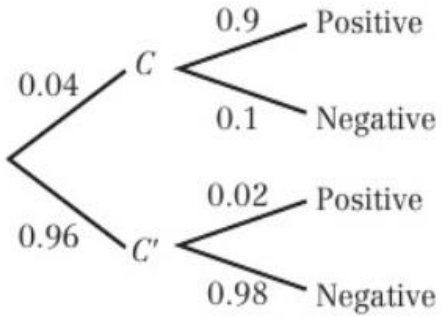


b **i** 0.0315 **ii** 0.0485

c 0.103 (3 s.f.)

Homework Answers

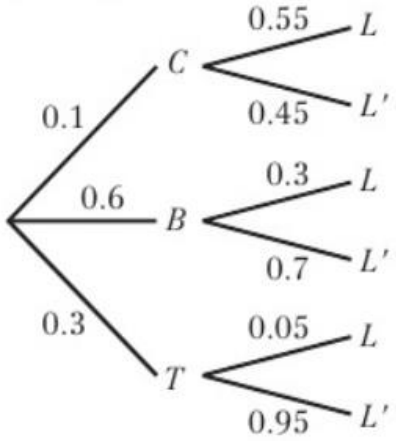
8 a



b 0.945 (3 s.f.) **c** 0.00423

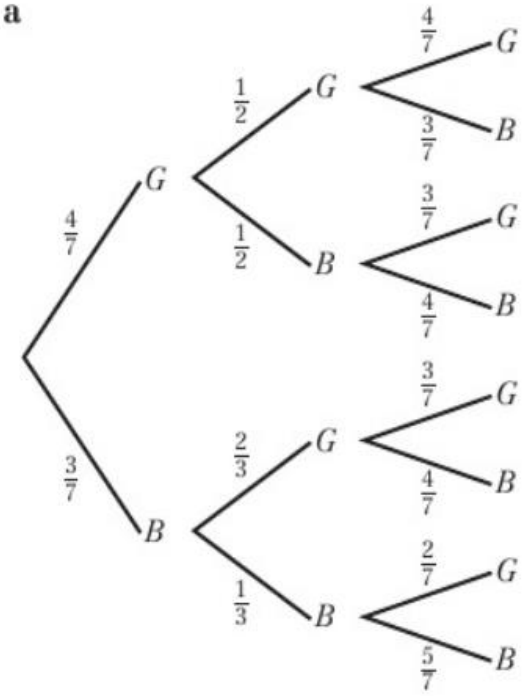
d The probability that a positive result is a false positive (positive result for someone without the condition) = $P(-|+) = 0.348$. Over one third of positive results are false positives and 10% of people with the condition give negative results.

9 a



b i 0.015 **ii** 0.25 **c** 0.78

10 a



b $\frac{3}{7}$

c Adding together the probabilities on the 4 branches of the tree diagram where the counter from box B is blue: $\frac{12}{98} + \frac{16}{98} + \frac{24}{147} + \frac{15}{157} = \frac{27}{49}$

d Adding together the probabilities on the 2 branches of the tree diagram where events C and D both occur. $\frac{12}{98} + \frac{15}{147} = \frac{11}{49}$

e $\frac{37}{49}$ **f** $\frac{8}{13}$