P1 Chapter 2: Quadratics

Modelling With Quadratics

Modelling

The A Level has a particular emphasis of the application of theory to real-life situations. A mathematical model is the maths used to model such a situation, possibly with some simplifying assumptions.

Example (from textbook): A spear is thrown over level ground from the top of a tower. The height, in metres, of the spear above the ground after t seconds is modelled by the function: $h(t) = 12.25 + 14.7t - 4.9t^2, \quad t \geq 0$

a) Interpret the meaning of the constant term 12.25 in the model.
b) After how many seconds does the spear hit the ground?
c) Write $h(t)$ in the form $A - B(t - C)^2$, where A, B and C are constants to be found.
d) Using your answer to part c or otherwise, find the maximum height of the spear above the ground, and the time at which this maximum height is reached?

a ?

b ?

c ?

d ?

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a When the time is 0, clearly $h(0) = 12.25$. So the 12.25m is the height of the tower. (In general, the constant term of an expression is often the 'initial value'.

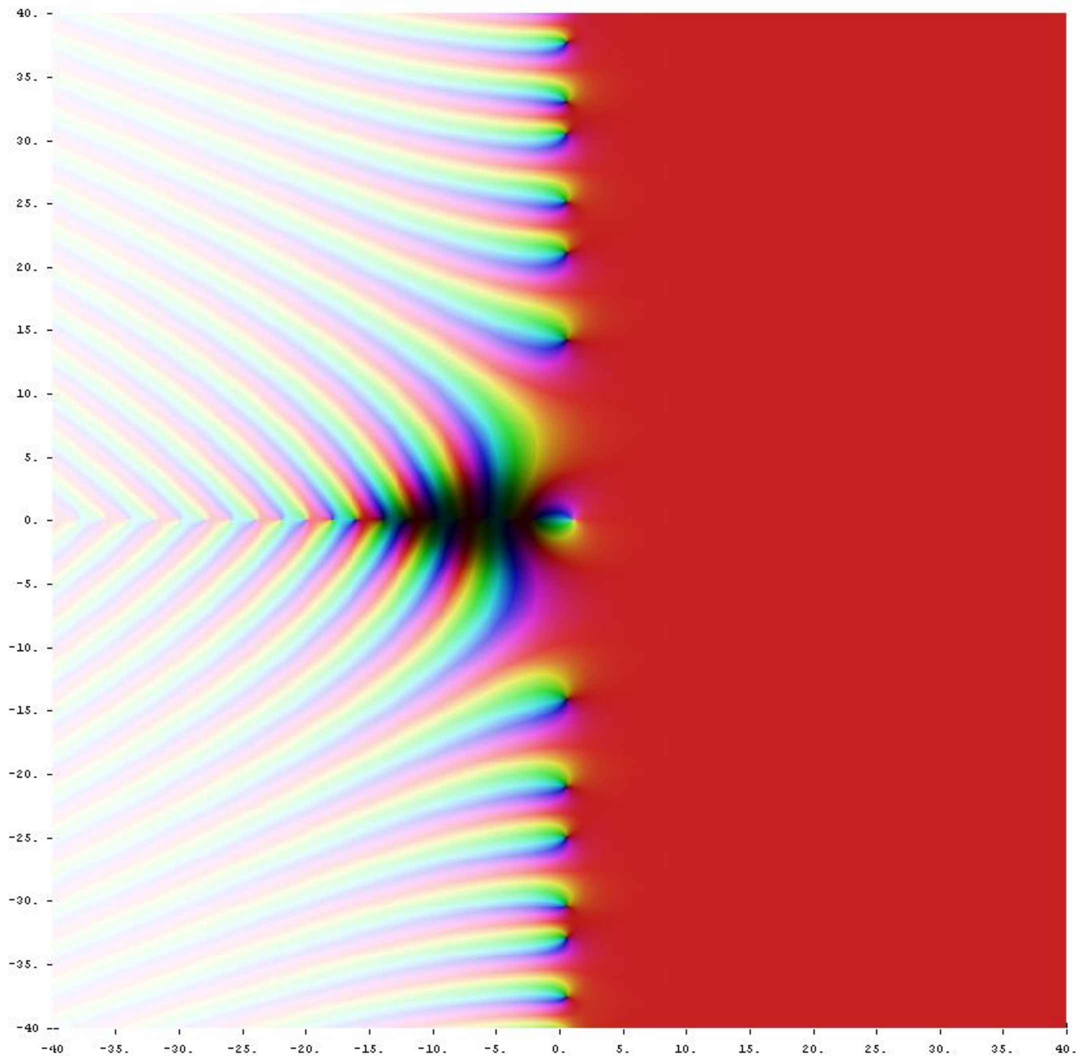
b When it hits the ground, $h(t) = 0$, thus: $12.25 + 14.7t - 4.9t^2$
Solving gives $t = -0.679 \text{ } or \text{ } t = 3.68$ (to 3sf) but as $t \geq 0$, $t = 3.68$ (to 3sf)

c By completing the square: $23.275 - 4.9(t - 1.5)^2$

d The maximum height of the spear is 23.275m, 1.5 seconds after spear is thrown.

Would you like $1,000,000 for finding roots?

We saw earlier that the roots of a function f are the values x such that $f(x) = 0$.



The **Riemann Zeta Function** is a function that allows you to do the infinite sum of powers of reciprocals, e.g.

$$\zeta(2) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$
$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots$$

One of the 8 '**Clay Millennium Problems**' (for which solving any attracts a $1,000,000 prize) is to **showing all roots of this function have some particular form**, i.e. the form of x such that $\zeta(x) = 0$.

Exercise 2.6

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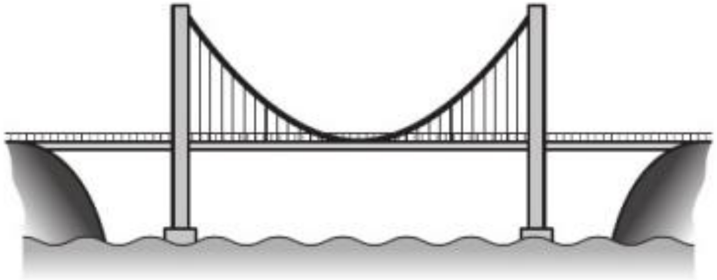
Homework Exercise

1 The diagram shows a section of a suspension bridge carrying a road over water.



Problem-solving

For part **a**, make sure your answer is in the context of the model.

The height of the cables above water level in metres can be modelled by the function $h(x) = 0.000\,12x^2 + 200$, where x is the displacement in metres from the centre of the bridge.

a Interpret the meaning of the constant term 200 in the model. **(1 mark)**

b Use the model to find the two values of x at which the height is 346 m. **(3 marks)**

c Given that the towers at each end are 346 m tall, use your answer to part **b** to calculate the length of the bridge to the nearest metre. **(1 mark)**

2 A car manufacturer uses a model to predict the fuel consumption, y miles per gallon (mpg), for a specific model of car travelling at a speed of x mph.

$$y = -0.01x^2 + 0.975x + 16, \; x > 0$$

a Use the model to find two speeds at which the car has a fuel consumption of 32.5 mpg. **(3 marks)**

b Rewrite y in the form $A - B(x - C)^2$, where A, B and C are constants to be found. **(3 marks)**

c Using your answer to part **b**, find the speed at which the car has the greatest fuel efficiency. **(1 mark)**

d Use the model to calculate the fuel consumption of a car travelling at 120 mph. Comment on the validity of using this model for very high speeds. **(2 marks)**

Homework Exercise

3 A fertiliser company uses a model to determine how the amount of fertiliser used, f kilograms per hectare, affects the grain yield g, measured in tonnes per hectare.

$$g = 6 + 0.03f - 0.000\,06f^2$$

a According to the model, how much grain would each hectare yield without any fertiliser? **(1 mark)**

b One farmer currently uses 20 kilograms of fertiliser per hectare. How much more fertiliser would he need to use to increase his grain yield by 1 tonne per hectare? **(4 marks)**

4 A football stadium has 25 000 seats. The football club know from past experience that they will sell only 10 000 tickets if each ticket costs £30. They also expect to sell 1000 more tickets every time the price goes down by £1.

a The number of tickets sold t can be modelled by the linear equation $t = M - 1000p$, where £p is the price of each ticket and M is a constant. Find the value of M. **(1 mark)**

The total revenue, £r, can be calculated by multiplying the number of tickets sold by the price of each ticket. This can be written as $r = p(M - 1000p)$.

b Rearrange r into the form $A - B(p - C)^2$, where A, B and C are constants to be found. **(3 marks)**

c Using your answer to part **b** or otherwise, work out how much the football club should charge for each ticket if they want to make the maximum amount of money. **(2 marks)**

Homework Exercise

Challenge

Accident investigators are studying the stopping distance of a particular car.
When the car is travelling at 20 mph, its stopping distance is 6 feet.
When the car is travelling at 30 mph, its stopping distance is 14 feet.
When the car is travelling at 40 mph, its stopping distance is 24 feet.

The investigators suggest that the stopping distance in feet, d, is a quadratic function of the speed in miles per hour, s.

a Given that $d(s) = as^2 + bs + c$, find the values of the constants a, b and c.

b At an accident scene a car has left behind a skid that is 20 feet long.
Use your model to calculate the speed that this car was going at before the accident.

Hint Start by setting up three simultaneous equations. Combine two different pairs of equations to eliminate c. Use the results to find the values of a and b first.

Homework Answers

1 **a** The height of the bridge above ground level
  b $x = 1103$ and $x = -1103$
  c 2206 m

2 **a** 21.8 mph and 75.7 mph
  b $A = 39.77$, $B = 0.01$, $C = 48.75$
  c 48.75mph
  d –11 mpg; a negative answer is impossible so this model is not valid for very high speeds

3 **a** 6 tonnes
  b 39.6 kilograms per hectare.

4 **a** $M = 40\,000$
  b $r = 400\,000 - 1000(p - 20)^2$
  $A = 400\,000$, $B = 1000$, $C = 20$
  c £20

Challenge

a $a = 0.01$, $b = 0.3$, $c = -4$
b 36.2 mph