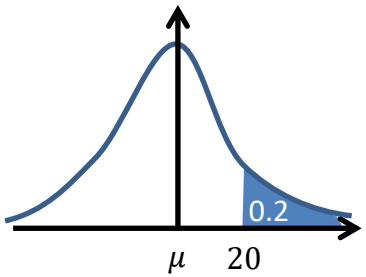
Stats Yr2 Chapter 3: Distribution-N

Mean and Variance

Missing μ and σ

In the last section, you may have thought, "what's the point of standardising to Z when I can just use the DISTRIBUTION mode on my calculator?"
Fair point, but both forward and reverse normal lookups on the calculator **required you to specify μ and σ.**

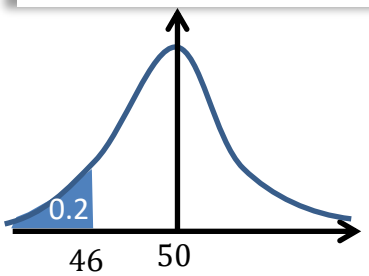
[Textbook] $X \sim N(\mu, 3^2)$. Given that $P(X > 20) = 0.2$, find the value of μ.



$$P(X > 20) = 0.2$$
$$0.8416 = \frac{20 - \mu}{3}$$
$$\mu = 17.5 \ (3sf)$$

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

[Textbook] A machine makes metal sheets with width, X cm, modelled as a normal distribution such that $X \sim N(50, \sigma^2)$.
(a) Given that $P(X < 46) = 0.2119$, find the value of σ.
(b) Find the 90th percentile of the widths.



$$P(X < 46) = 0.2119$$
$$-0.80 = \frac{46 - 50}{\sigma}$$
$$\sigma = 5$$

To use inverse normal distribution on calculator, note that $Z \sim N(0,1^2)$, so use $\mu = 0$ and $\sigma = 1$.

$$X \sim N(50, 5^2)$$
$$P(X < a) = 0.9$$
$$a = 56.4 \ cm \ (1dp)$$

The method here is exactly the same as before:
1. Using a sketch, determine whether you're expecting a positive or negative z value.
2. Look up z value, using tables if you can (otherwise your calculator). Make negative if in bottom half.
3. Use $Z = \frac{X - \mu}{\sigma}$

When both are missing

If both μ and σ are missing, we end up with simultaneous equations which we must solve.

Edexcel S1 Jan 2011

The weight, Y grams, of soup put into a carton by machine B is normally distributed with mean μ grams and standard deviation σ grams.

(c) Given that $P(Y < 160) = 0.99$ and $P(Y > 152) = 0.90$, find the value of μ and the value of σ.

(6)

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(6)

$\frac{160-\mu}{\sigma} = 2.3263$	M1 B1
$\frac{152-\mu}{\sigma} = -1.2816$	B1
$160 - \mu = 2.3263\sigma$	
$152 - \mu = -1.2816\sigma$	
$8 = 3.6079\sigma$	M1
$\sigma = 2.21\ldots$ **awrt 2.22**	A1
$\mu = 154.84\ldots$ **awrt 155**	A1

Test Your Understanding

Edexcel S1 May 2013 (R)

The time taken to fly from London to Berlin has a normal distribution with mean 100 minutes and standard deviation d minutes.

Given that 15% of the flights from London to Berlin take longer than 115 minutes,

(b) find the value of the standard deviation d.

(4)

?

Edexcel S1 Jan 2002

5. The duration of the pregnancy of a certain breed of cow is normally distributed with mean μ days and standard deviation σ days. Only 2.5% of all pregnancies are shorter than 235 days and 15% are longer than 286 days.

(a) Show that $\mu - 235 = 1.96\sigma$.

(2)

(b) Obtain a second equation in μ and σ.

(3)

(c) Find the value of μ and the value of σ.

(4)

(d) Find the values between which the middle 68.3% of pregnancies lie.

(2)

(a) ?

(b) ?

(c) ?

(d) ?

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$$[\mathrm{P}(B > 115) = 0.15 \Rightarrow] \quad \frac{115-100}{d} = 1.0364$$

$$\underline{\boldsymbol{d = 14.5}}$$

Edexcel S1 Jan 2002

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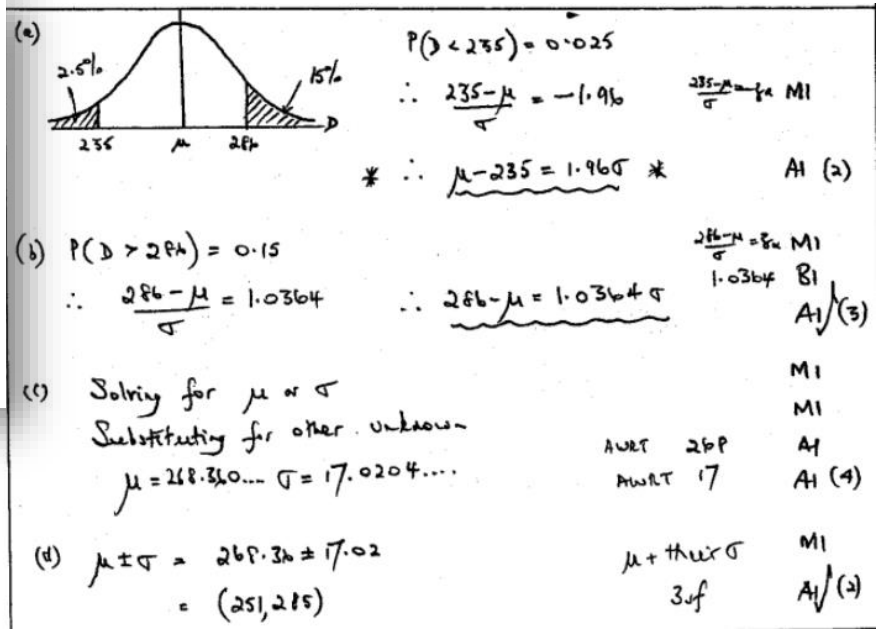

Conditional Probabilities

This is not in the textbook. But given the recent Chapter 2 on Conditional Probabilities and the fact that the type of question below occurred frequently in S1 papers, it seems worthwhile to cover!

Edexcel S1 May 2014(R) Q4

The time, in minutes, taken to fly from London to Malaga has a normal distribution with mean 150 minutes and standard deviation 10 minutes.

The time, X minutes, taken to fly from London to another city has a normal distribution with mean μ minutes.

Given that $P(X < \mu - 15) = 0.35$

(c) find $P(X > \mu + 15 \mid X > \mu - 15)$. **(3)**

?

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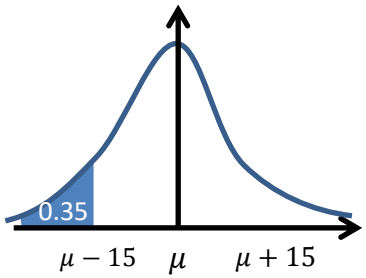

$$P(X > \mu + 15 \mid X > \mu - 15)$$
$$= \frac{P(X > \mu + 15)}{P(X > \mu - 15)}$$
$$= \frac{0.35}{0.65} = \frac{7}{13}$$

The intersection of "above $\mu + 15$" and "above $\mu - 15$" is just "above $\mu + 15$" because the stronger statements takes precedent. If I said "my age is above 20, and above 30", this is equivalent to saying just "my age is above 30".

Test Your Understanding

Edexcel S1 Jan 2013 Q4a,c

The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(*a*) Find P($L > 127$).

(3)

Alice is about to go on a 6 hour journey. Given that it is 127 hours since Alice last charged her phone,

(*c*) find the probability that her phone will not need charging before her journey is completed.

(4)

(a) ?

(c) ?

Test Your Understanding

Edexcel S1 Jan 2013 Q4a,c

The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

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(c) find the probability that her phone will not need charging before her journey is completed.

(4)

(a)

$$\frac{127-100}{15}$$ M1

So $P(L > 127) = P(Z > 1.8)$ or $1 - P(Z < 1.8)$ o.e. A1

$= 1 - 0.9641 = \underline{\mathbf{0.0359}}$ (awrt $\underline{\mathbf{0.0359}}$) A1

(3)

(c) Require $P(L > 133 \mid L > 127)$ M1

$$= \left[\frac{P(L>133)}{P(L>127)}\right] = \frac{P(Z>2.2)}{P(L>127)}$$ dM1

$$= \left[\frac{1-0.9861}{1-0.9641}\right] = \frac{0.0139}{[0.0359]}$$ A1

$= 0.3871...$ $=$ awrt $\underline{\mathbf{0.39}}$ A1

(4)

Exercise 3.5

Pearson Stats/Mechanics Year 2
Pages 26-27

Homework Exercise

1 The random variable $X \sim N(\mu, 5^2)$ and $P(X < 18) = 0.9032$.
Find the value of μ.

2 The random variable $X \sim N(11, \sigma^2)$ and $P(X > 20) = 0.01$.
Find the value of σ.

3 The random variable $Y \sim N(\mu, 40)$ and $P(Y < 25) = 0.15$.
Find the value of μ.

4 The random variable $Y \sim N(50, \sigma^2)$ and $P(Y > 40) = 0.6554$.
Find the value of σ.

5 The random variable $X \sim N(\mu, \sigma^2)$.
Given that $P(X < 17) = 0.8159$ and $P(X < 25) = 0.9970$, find the value of μ and the value of σ.

6 The random variable $Y \sim N(\mu, \sigma^2)$.
Given that $P(Y < 25) = 0.10$ and $P(Y > 35) = 0.005$, find the value of μ and the value of σ.

7 The random variable $X \sim N(\mu, \sigma^2)$.
Given that $P(X > 15) = 0.20$ and $P(X < 9) = 0.20$,
find the value of μ and the value of σ.

Hint Draw a diagram and use symmetry to find μ.

8 The random variable $X \sim N(\mu, \sigma^2)$.
The lower quartile of X is 25 and the upper quartile of X is 45.
Find the value of μ and the value of σ.

Homework Exercise

9 The random variable $X \sim N(0, \sigma^2)$.
Given that $P(-4 < X < 4) = 0.6$, find the value of σ.

10 The random variable $X \sim N(2.68, \sigma^2)$.
Given that $P(X > 2a) = 0.2$ and $P(X < a) = 0.4$, find the value of σ and the value of a.

11 An automated pottery wheel is used to make bowls. The diameter of the bowls, D mm, is normally distributed with mean μ and standard deviation 5 mm. Given that 75% of bowls are greater than 200 mm in diameter, find:

a the value of μ **(2 marks)**

b $P(204 < D < 206)$ **(1 mark)**

Three bowls are chosen at random.

c Find the probability that all of the bowls are greater than 205 mm in diameter. **(3 marks)**

12 A loom makes table cloths with an average thickness of 2.5 mm. The thickness, T mm, can be modelled using a normal distribution. Given that 65% of table cloths are less than 2.55 mm thick, find:

a the standard deviation of the thickness **(2 marks)**

b the proportion of table cloths with thickness between 2.4 mm and 2.6 mm. **(1 mark)**

A table cloth can be sold if the thickness is between 2.4 mm and 2.6 mm. A sample of 20 table cloths is taken.

c Find the probability that at least 15 table cloths can be sold. **(3 marks)**

Homework Exercise

13 The masses of the penguins on an island are found to be normally distributed with mean μ, and standard deviation σ. Given that 10% of the penguins have a mass less than 18 kg and 5% of the penguins have a mass greater than 30 kg,

a sketch a diagram to represent this information **(2 marks)**

b find the value of μ and the value of σ. **(6 marks)**

10 penguins are chosen at random.

c Find the probability that at least 4 of them have a mass greater than 25 kg. **(4 marks)**

14 The length of an adult Dachshund is found to be normally distributed with mean μ and standard deviation σ. Given that 20% of Dachshunds have a length less than 16 inches and 10% have a length greater than 18 inches, find:

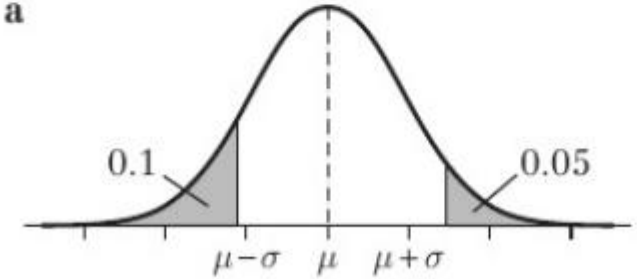
a the value of μ and the value of σ **(6 marks)**

b the interquartile range. **(2 marks)**

Homework Answers

For Chapter 3, student answers may differ slightly from those shown here when calculators are used rather than table values.

1 11.5
2 3.87
3 31.6
4 25
5 $\mu = 13.1, \sigma = 4.32.$
6 $\mu = 28.3, \sigma = 2.59.$
7 $\mu = 12, \sigma = 3.56.$
8 $\mu = 35, \sigma = 14.8$ or $\sigma = 14.9.$
9 4.75
10 $\sigma = 1.99, a = 2.18.$
11 **a** 203.37 mm **b** 0.1504 **c** 0.0516
12 **a** 0.1299 mm **b** 0.5587 **c** 0.0644
13 **a**



b $\mu = 23.26, \sigma = 4.100$ **c** 0.4469
14 **a** $\mu = 16.79, \sigma = 0.9421$ **b** 1.27