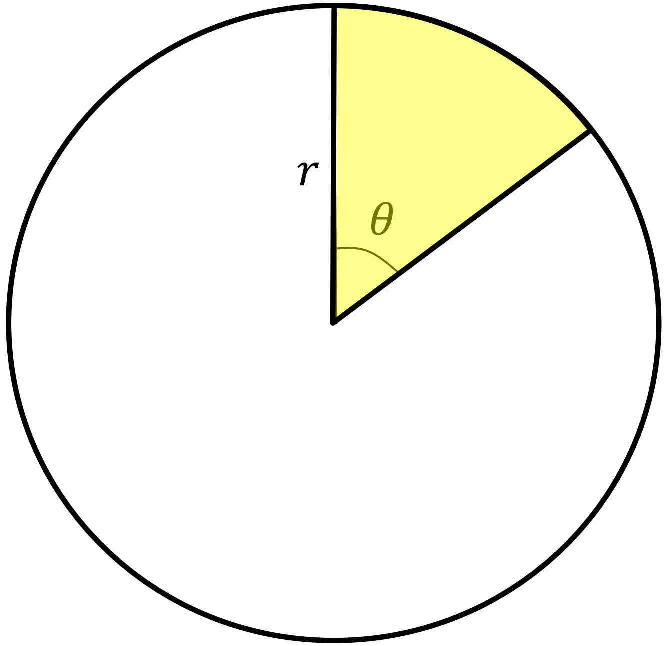
P2 Chapter 5: Radians

Sectors and Segments

Sector Area



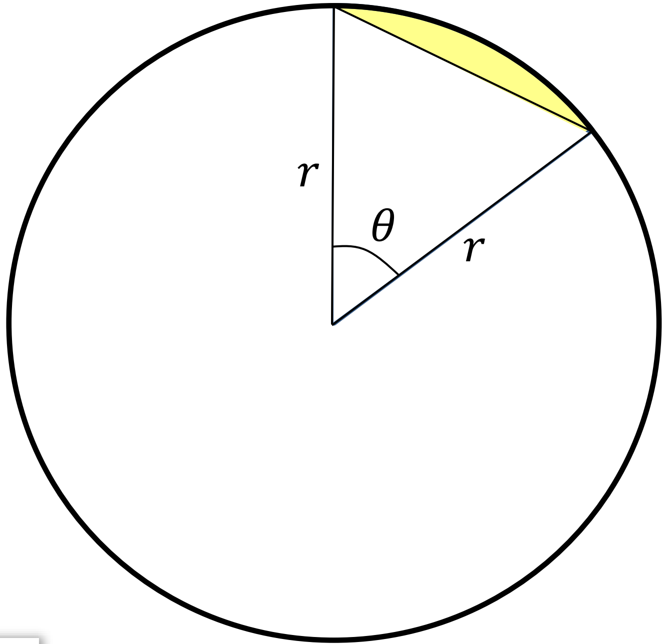
Area using Degrees

$$A = \frac{\theta}{360} \times \pi r^2$$

Area using Radians

$$A = \frac{1}{2} r^2 \theta$$

Segment Area



A segment is the region bound between a chord and the circumference.

This is just a sector with a triangle cut out.

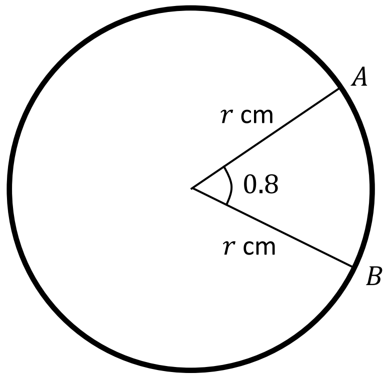
Area using radians:

$$A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$
$$= \frac{1}{2}r^2(\theta - \sin\theta)$$

Recall that the area of a triangle is $\frac{1}{2}ab\sin C$ where C is the 'included angle' (i.e. between a and b)

Examples

[Textbook] In the diagram, the area of the minor sector AOB is 28.9 cm^2. Given that $\angle AOB = 0.8$ radians, calculate the value of r.



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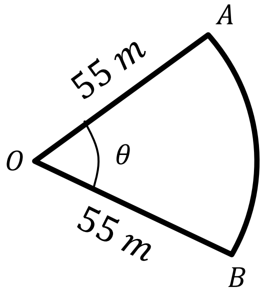
[Textbook] A plot of land is in the shape of a sector of a circle of radius 55 m. The length of fencing that is erected along the edge of the plot to enclose the land is 176 m. Calculate the area of the plot of land.



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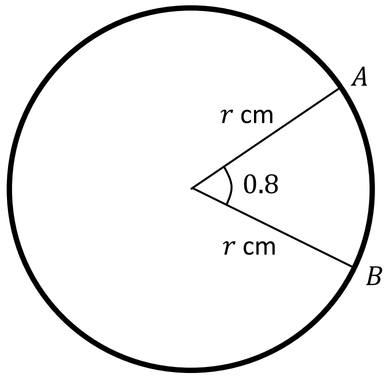

$$28.9 = \frac{1}{2}r^2 \times 0.8$$
$$28.9 = 0.4r^2$$
$$r^2 = \frac{28.9}{0.4} = 72.25$$
$$r = \sqrt{72.25} = 8.5$$

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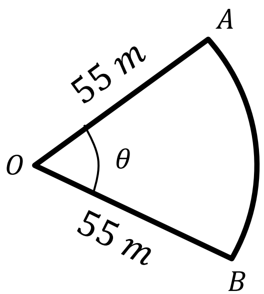


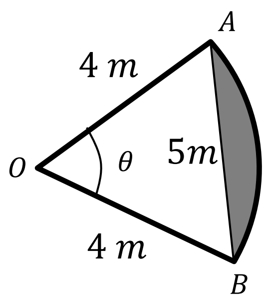
Arc $AB = 176 - 55 - 55 = 66\ m$

$$66 = 5\theta$$
$$\therefore \theta = 1.2\ rad$$

Area $= \frac{1}{2} \times 55^2 \times 1.2 = 1815$ m^2

Segment Examples

[Textbook] In the diagram above, OAB is a sector of a circle, radius 4m. The chord AB is 5m long. Find the area of the shaded segment.



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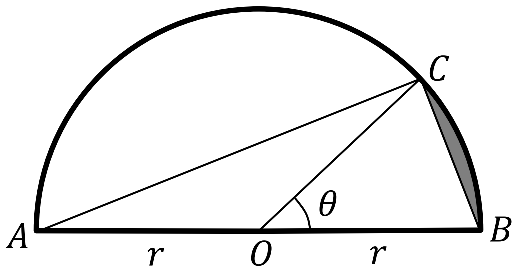
[Textbook] In the diagram, AB is the diameter of a circle of radius $r$cm, and $\angle BOC = \theta$ radians. Given that the area of ΔAOC is three times that of the shaded segment, show that $3\theta - 4\sin\theta = 0$.



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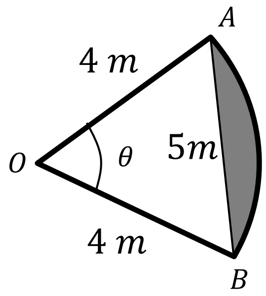

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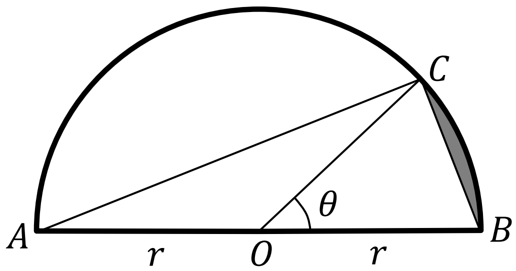

Using cosine rule:

$$5^2 = 4^2 + 4^2 - (2 \times 4 \times 4 \times \cos\theta)$$

$$25 = 32 - 32\cos\theta$$

$$32\cos\theta = 32 - 25 = 7$$

$$\theta = \cos^{-1}\left(\frac{7}{32}\right) = 1.3502\ldots$$

Area of shaded segment:

$$\frac{1}{2} \times 4^2(1.3502\ldots - \sin 1.3502\ldots)$$

$= 3.00$ m^2.

Area of segment $= \frac{1}{2}r^2(\theta - \sin\theta)$

Area of $\Delta AOC = \frac{1}{2}r^2\sin(\pi - \theta)$

$$= \frac{1}{2}r^2 sin\theta$$

Recall that $\sin(\theta) = \sin(\pi - \theta)$

$$\therefore \frac{1}{2}r^2\sin\theta = 3 \times \frac{1}{2}r^2(\theta - \sin\theta)$$

$$\sin\theta = 3(\theta - \sin\theta)$$

$$\therefore 3\theta - 4\sin\theta = 0$$

Exercise 5.3

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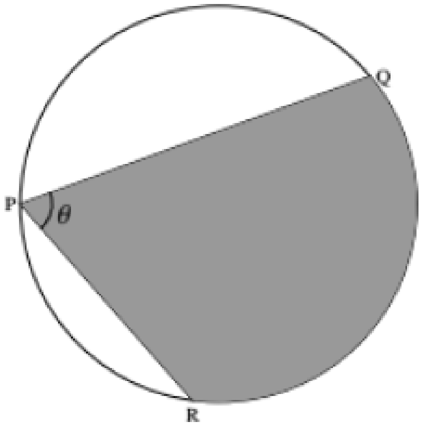
Extension



[MAT 2012 1J]
If two chords QP and RP on a circle of radius 1 meet in an angle θ at P, for example as drawn in the diagram on the left, then find the largest possible area of the shaded region RPQ, giving your answer in terms of θ.

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Exercise 5.3

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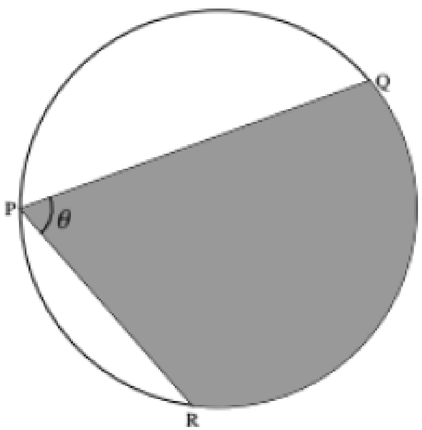
Extension

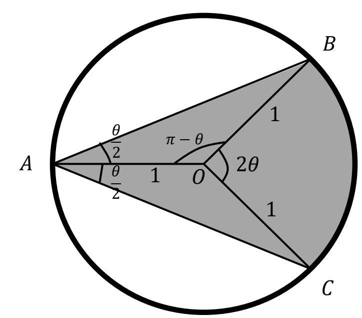


[MAT 2012 1J]
If two chords QP and RP on a circle of radius 1 meet in an angle θ at P, for example as drawn in the diagram on the left, then find the largest possible area of the shaded region RPQ, giving your answer in terms of θ.

For a fixed θ the largest area is obtained when the angle bisector of PQ and PR is the diameter of the circle. This can be broken up into two isosceles triangles and a sector as shown.
$\angle BOC = 2\theta$ as angle at centre is twice angle at circumference.

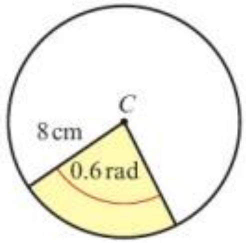
$$\angle AOB = \pi - \frac{\theta}{2} - \frac{\theta}{2} = \pi - \theta$$

Area of $AOB = \frac{1}{2} \times 1^2 \times \sin(\pi - \theta) = \frac{1}{2}\sin(\theta)$
Area of sector $BOC = \frac{1}{2} \times 1^2 \times 2\theta = \theta$.
Total shaded area $= 2 \times \frac{1}{2}\sin\theta + \theta = \theta + \sin\theta$.

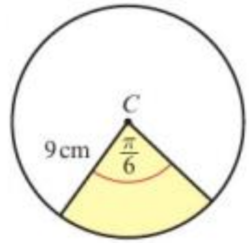


1 Find the shaded area in each of the following circles. Leave your answers in terms of π where appropriate.

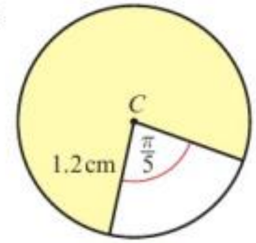
a



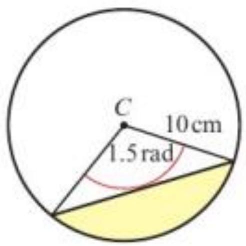
b



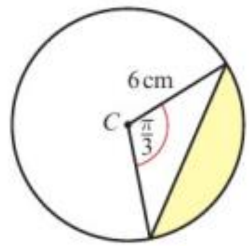
c



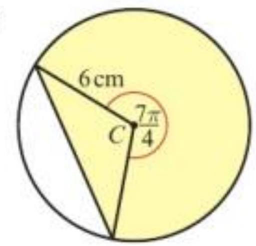
d



e

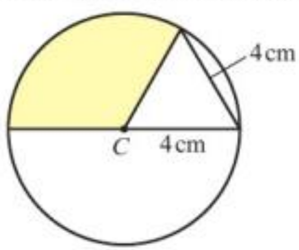


f



2 Find the shaded area in each of the following circles with centre C.

a



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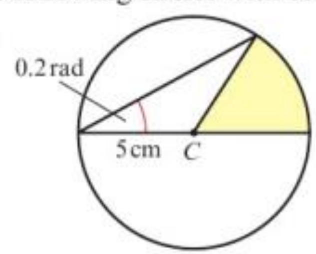
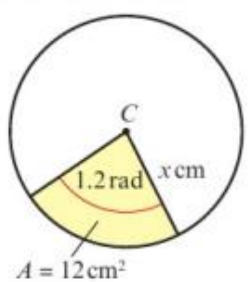


3 For the following circles with centre C, the area A of the shaded sector is given. Find the value of x in each case.

a



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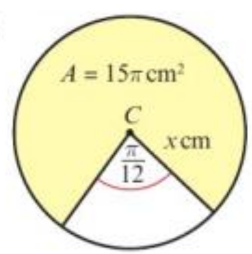
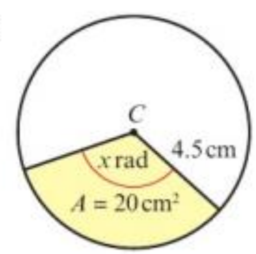


c



Homework Exercise

4 The arc AB of a circle, centre O and radius 6 cm, has length 4 cm.
Find the area of the minor sector AOB.

5 The chord AB of a circle, centre O and radius 10 cm, has length 18.65 cm and subtends an angle of θ radians at O.

a Show that $\cos\theta = -0.739$ (to 3 significant figures).

b Find the area of the minor sector AOB.

6 The area of a sector of a circle of radius 12 cm is 100 cm^2. Find the perimeter of the sector.

7 The arc AB of a circle, centre O and radius r cm, is such that $\angle AOB = 0.5$ radians.
Given that the perimeter of the minor sector AOB is 30 cm,

a calculate the value of r

b show that the area of the minor sector AOB is 36 cm^2

c calculate the area of the segment enclosed by the chord AB and the minor arc AB.

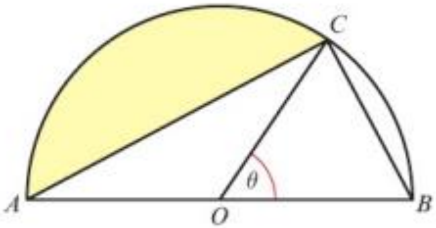
8 The arc AB of a circle, centre O and radius x cm, is such that angle $AOB = \frac{\pi}{12}$ radians.
Given that the arc length AB is l cm,

a show that the area of the sector can be written as $\frac{6l^2}{\pi}$

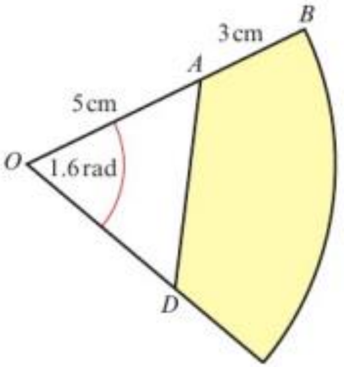
The area of the full circle is 3600π cm^2.

b Find the arc length of AB.

c Calculate the value of x.



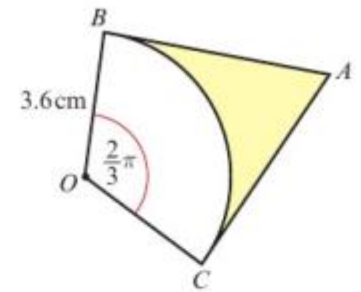
9 In the diagram, AB is the diameter of a circle of radius r cm and $\angle BOC = \theta$ radians.
Given that the area of ΔCOB is equal to that of the shaded segment, show that $\theta + 2\sin\theta = \pi$.



10 In the diagram, BC is the arc of a circle, centre O and radius 8 cm. The points A and D are such that $OA = OD = 5$ cm. Given that $\angle BOC = 1.6$ radians, calculate the area of the shaded region.

Homework Exercise

11 In the diagram, AB and AC are tangents to a circle, centre O and radius 3.6 cm. Calculate the area of the shaded region, given that $\angle BOC = \frac{2\pi}{3}$ radians.

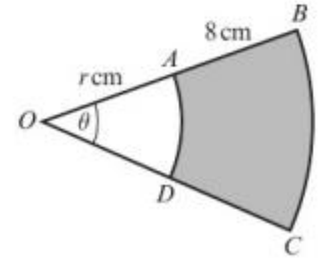


12 In the diagram, AD and BC are arcs of circles with centre O, such that $OA = OD = r$ cm, $AB = DC = 8$ cm and $\angle BOC = \theta$ radians.

a Given that the area of the shaded region is 48 cm², show that

$r = \frac{6}{\theta} - 4$ **(4 marks)**

b Given also that $r = 10\theta$, calculate the perimeter of the shaded region. **(6 marks)**

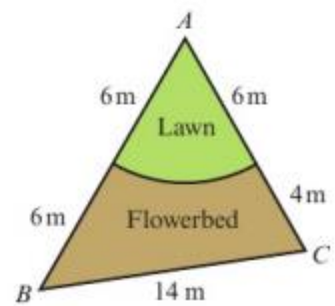


13 A sector of a circle of radius 28 cm has perimeter P cm and area A cm². Given that $A = 4P$, find the value of P.

14 The diagram shows a triangular plot of land. The sides AB, BC and CA have lengths 12 m, 14 m and 10 m respectively. The lawn is a sector of a circle, centre A and radius 6 m.

a Show that $\angle BAC = 1.37$ radians, correct to 3 significant figures.

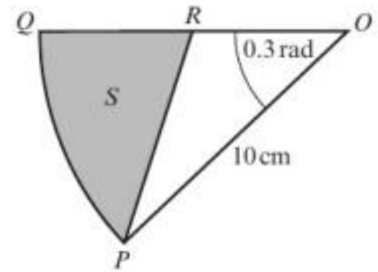
b Calculate the area of the flowerbed.



Homework Exercise

15 The diagram shows OPQ, a sector of a circle with centre O, radius 10 cm, with $\angle POQ = 0.3$ radians.

The point R is on OQ such that the ratio $OR : RQ$ is $1:3$.
The region S, shown shaded in the diagram, is bounded by QR, RP and the arc PQ.



Find:

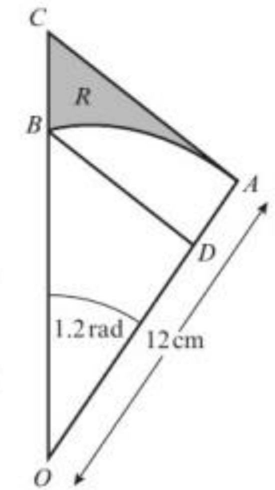
a the perimeter of S, giving your answer to 3 significant figures **(6 marks)**

b the area of S, giving your answer to 3 significant figures. **(6 marks)**

16 The diagram shows the sector OAB of a circle with centre O, radius 12 cm and angle 1.2 radians.

The line AC is a tangent to the circle with centre O, and OBC is a straight line.

The region R is bounded by the arc AB and the lines AC and CB.



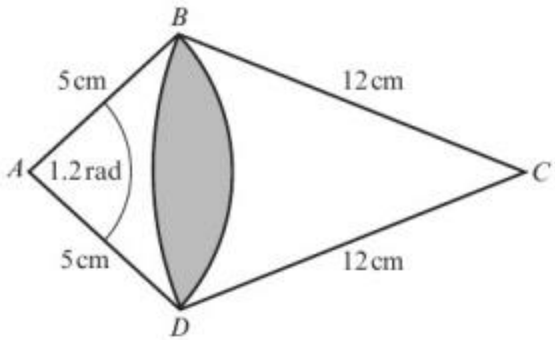
a Find the area of R, giving your answer to 2 decimal places. **(8 marks)**

The line BD is parallel to AC.

b Find the perimeter of DAB. **(5 marks)**

Homework Exercise

17



The diagram shows two intersecting sectors: ABD, with radius 5 cm and angle 1.2 radians, and CBD, with radius 12 cm.
Find the area of the overlapping section.

Challenge

Find an expression for the area of a sector of a circle with radius r and arc length l.

Homework Answers

1 **a** 19.2 cm^2 **b** $\frac{27}{4}\pi\text{ cm}^2$ **c** $\frac{162}{125}\pi\text{ cm}^2$

d 25.1 cm^2 **e** $6\pi - 9\sqrt{3}\text{ cm}^2$ **f** $\frac{63}{2}\pi + 9\sqrt{2}\text{ cm}^2$

2 **a** $\frac{16}{3}\pi\text{ cm}^2$ **b** 5 cm^2

3 **a** 4.47 **b** 3.96 **c** 1.98

4 12 cm^2

5 **a** $\cos\theta = \dfrac{10^2 + 10^2 - 18.65^2}{2 \times 10 \times 10} = -0.739\ldots$

b 120 cm^2

6 $40\frac{2}{3}\text{ cm}$

7 **a** 12

b $A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 12^2 \times 0.5 = 36\text{ cm}^2$

c 1.48 cm^2

8 **a** $l = r\theta = \dfrac{x\pi}{12}, x = \dfrac{12l}{\pi}$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}\left(\frac{12l}{\pi}\right)^2\frac{\pi}{12} = \frac{\pi}{24}\left(\frac{144l^2}{\pi^2}\right) = \frac{6l^2}{\pi}$$

b $5\pi\text{ cm}$ **c** 60

9 $\triangle COB = \frac{1}{2}r^2\sin\theta$

Shaded area $= \frac{1}{2}r^2(\pi - \theta) - \frac{1}{2}r^2\sin(\pi - \theta)$

$= \frac{1}{2}r^2\pi - \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

Since $\triangle$COB = shaded area,

$\frac{1}{2}r^2\sin\theta = \frac{1}{2}r^2\pi - \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$\sin\theta = \pi - \theta - \sin\theta$

$\theta + 2\sin\theta = \pi$

10 38.7 cm^2

11 8.88 cm^2

12 **a** $OAD = \frac{1}{2}r^2\theta$, $OBC = \frac{1}{2}(r + 8)^2\theta$

$ABCD = \frac{1}{2}(r + 8)^2\theta - \frac{1}{2}r^2\theta = 48$

$\frac{1}{2}(r^2 + 16r + 64)\theta - \frac{1}{2}r^2\theta = 48$

$(r^2 + 16r + 64)\theta - r^2\theta = 96$

$16r + 64 = \dfrac{96}{\theta} \Rightarrow r = \dfrac{6}{\theta} - 4$

b 28 cm

13 78.4 ($\theta = 0.8$)

14 **a** $14^2 = 12^2 + 10^2 - 2 \times 12 \times 10\cos A$

$196 = 144 + 100 - 240\cos A$

$-48 = -240\cos A$

$0.2 = \cos A$

$A = \cos^{-1}(0.2) = 1.369438406\ldots = 1.37$ (3 s.f.)

b 34.1 m^2

15 **a** 18.1 cm **b** 11.3 cm^2

16 **a** 98.79 cm^2 **b** 33.24 cm

17 4.62 cm^2

Challenge

Area $= \frac{1}{2}r^2\theta$, arc length, $l = r\theta$

Area $= \frac{1}{2}rl$