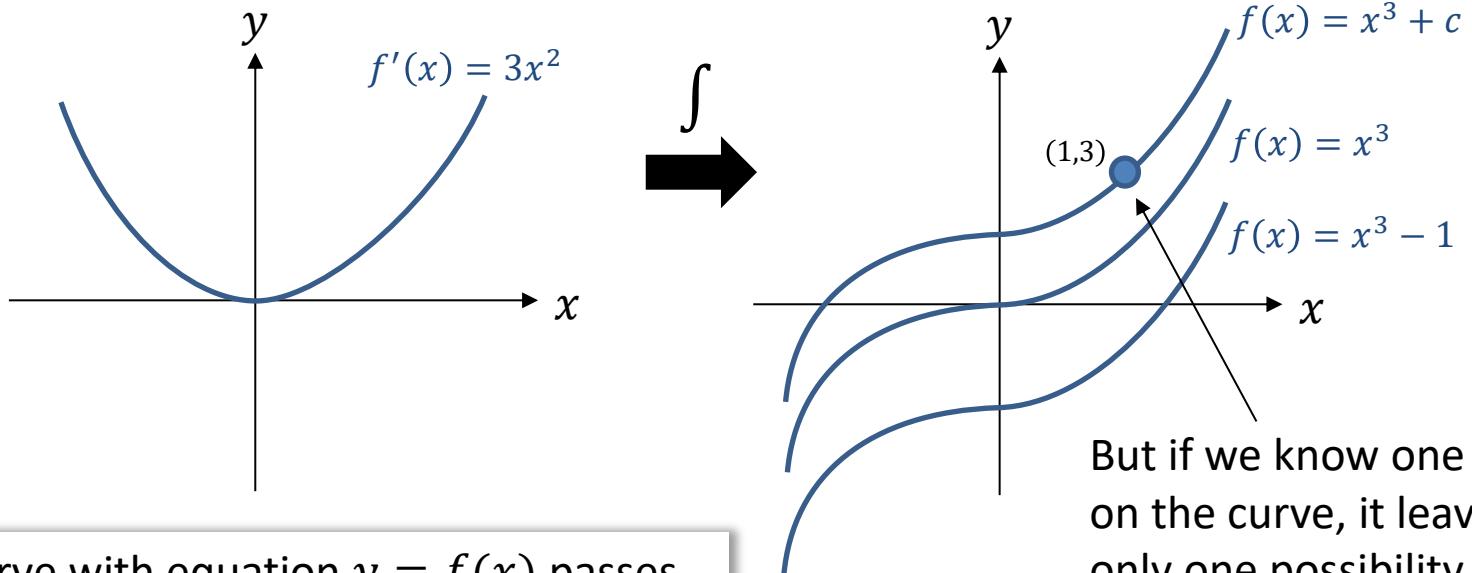

P1 Chapter 13: Integration

Definite Functions

Finding constant of integration

Recall that when we integrate, we get a constant of integration, which could be any real value. This means **we don't know what the exact original function was.**



The curve with equation $y = f(x)$ passes through $(1,3)$. Given that $f'(x) = 3x^2$, find the equation of the curve.

$$\begin{aligned}f'(x) &= 3x^2 \\ \therefore f(x) &= x^3 + c\end{aligned}$$

Using the point $(1,3)$: $3 = 1^3 + c \quad \therefore c = 2$
 $f(x) = x^3 + 2$

But if we know one point on the curve, it leaves only one possibility.

Test Your Understanding

Edexcel C1 May 2014 Q10

A curve with equation $y = f(x)$ passes through the point $(4, 25)$.

Given that

$$f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \quad x > 0$$

- (a) find $f(x)$, simplifying each term. (5)
- (b) Find an equation of the normal to the curve at the point $(4, 25)$.

Give your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(5)

To keep you occupied if
you finish (a) quickly!



? a

? b

Test Your Understanding

Edexcel C1 May 2014 Q10

A curve with equation $y = f(x)$ passes through the point $(4, 25)$.

Given that

$$f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \quad x > 0$$

- (a) find $f(x)$, simplifying each term.

(5)

- (b) Find an equation of the normal to the curve at the point $(4, 25)$.

Give your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(5)

To keep you occupied if
you finish (a) quickly!

(a) $f(x) = \int \left(\frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1 \right) dx$

$$x^n \rightarrow x^{n+1} \Rightarrow f(x) = \frac{3}{8} \times \frac{x^3}{3} - 10 \times \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + x (+c)$$

Substitute $x = 4, y = 25 \Rightarrow 25 = 8 - 40 + 4 + c \Rightarrow c =$

$$(f(x)) = \frac{x^3}{8} - 20x^{\frac{1}{2}} + x + 53$$

(b) $y - 25 = -\frac{1}{2}(x - 4)$

M1, A1, A1

M1

A1

Exercise 13.3

Pearson Pure Mathematics Year 1/AS

Page 105

Homework Exercise

- 1 Find the equation of the curve with the given derivative of y with respect to x that passes through the given point:

a $\frac{dy}{dx} = 3x^2 + 2x;$ point (2, 10)

b $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3;$ point (1, 4)

c $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2;$ point (4, 11)

d $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x;$ point (4, 0)

e $\frac{dy}{dx} = (x + 2)^2;$ point (1, 7)

f $\frac{dy}{dx} = \frac{x^2 + 3}{\sqrt{x}};$ point (0, 1)

- 2 The curve C , with equation $y = f(x)$, passes through the point (1, 2) and $f'(x) = 2x^3 - \frac{1}{x^2}$. Find the equation of C in the form $y = f(x)$.

- 3 The gradient of a particular curve is given by $\frac{dy}{dx} = \frac{\sqrt{x} + 3}{x^2}$. Given that the curve passes through the point (9, 0), find an equation of the curve.

- 4 The curve with equation $y = f(x)$ passes through the point (-1, 0). Given that $f'(x) = 9x^2 + 4x - 3$, find $f(x)$. (5 marks)

5 $\frac{dy}{dx} = 3x^{-\frac{1}{2}} - 2x\sqrt{x}, x > 0.$

Given that $y = 10$ at $x = 4$, find y in terms of x , giving each term in its simplest form. (7 marks)

Homework Exercise

- 6 Given that $\frac{6x + 5x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 5x^q$,
- a write down the value of p and the value of q . (2 marks)

Given that $\frac{dy}{dx} = \frac{6x + 5x^{\frac{3}{2}}}{\sqrt{x}}$ and that $y = 100$ when $x = 9$,

- b find y in terms of x , simplifying the coefficient of each term. (5 marks)

- 7 The displacement of a particle at time t is given by the function $f(t)$, where $f(0) = 0$.

Given that the velocity of the particle is given by $f'(t) = 10 - 5t$,

- a find $f(t)$
- b determine the displacement of the particle when $t = 3$.
- Problem-solving**
- You don't need any specific knowledge of mechanics to answer this question. You are told that the displacement of the particle at time t is given by $f(t)$.
- 8 The height, in metres, of an arrow fired horizontally from the top of a castle is modelled by the function $f(t)$, where $f(0) = 35$. Given that $f'(t) = -9.8t$,
- a find $f(t)$.
- b determine the height of the arrow when $t = 1.5$.
- c write down the height of the castle according to this model.
- d estimate the time it will take the arrow to hit the ground.
- e state one assumption used in your calculation.

Homework Exercise

Challenge

- 1** A set of curves, where each curve passes through the origin, has equations $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x)$... where $f'_n(x) = f_{n-1}(x)$ and $f_1(x) = x^2$.
 - a** Find $f_2(x)$, $f_3(x)$.
 - b** Suggest an expression for $f_n(x)$.
- 2** A set of curves, with equations $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x)$, ... all pass through the point $(0, 1)$ and they are related by the property $f'_n(x) = f_{n-1}(x)$ and $f_1(x) = 1$. Find $f_2(x)$, $f_3(x)$, $f_4(x)$.

Homework Answers

1 a $y = x^3 + x^2 - 2$ b $y = x^4 - \frac{1}{x^2} + 3x + 1$

c $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + \frac{1}{3}$ d $y = 6\sqrt{x} - \frac{1}{2}x^2 - 4$

e $y = \frac{1}{3}x^3 + 2x^2 + 4x + \frac{2}{3}$ f $y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + 1$

2 $f(x) = \frac{1}{2}x^4 + \frac{1}{x} + \frac{1}{2}$

3 $y = 1 - \frac{2}{\sqrt{x}} - \frac{3}{x}$

4 $f(x) = 3x^3 + 2x^2 - 3x - 2$

5 $y = 6x^{\frac{1}{2}} - \frac{4x^{\frac{5}{2}}}{5} + \frac{118}{5}$

6 a $p = \frac{1}{2}, q = 1$ b $y = 4x^{\frac{3}{2}} + \frac{5x^2}{2} - \frac{421}{2}$

7 a $f(t) = 10t - \frac{5t^2}{2}$ b $7\frac{1}{2}$

8 a $f(t) = -4.9t^2 + 35$ b 23.975 m
c 35 m d 2.67 seconds
e e.g. the ground is flat

Challenge

1 $f_2(x) = \frac{x^3}{3}; f_4(x) = \frac{x^4}{12}$ b $\frac{x^{n+1}}{3 \times 4 \times 5 \times \dots \times (n+1)}$

2 $f_2(x) = x + 1; f_3(x) = \frac{1}{2}x^2 + x + 1; f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$

Homework Answers