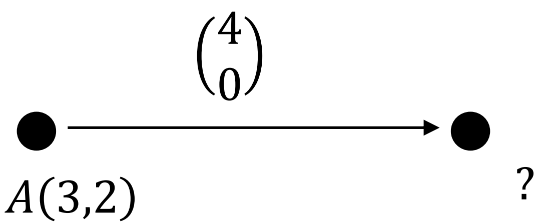
P1 Chapter 11: Vectors

Position Vectors

Position Vectors

Suppose we started at a point $(3,2)$ and translated by the vector $\begin{pmatrix}4\\0\end{pmatrix}$:

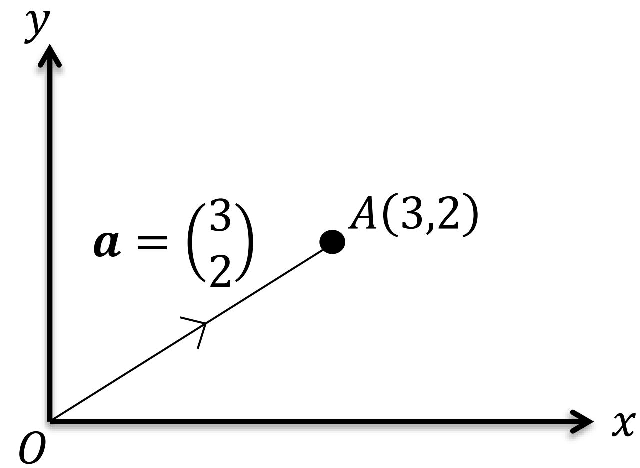


You might think we can do something like:

$$(3,2)+\begin{pmatrix}4\\0\end{pmatrix}=(7,2)$$

But only vectors can be added to other vectors. **If we treated the point $(3,2)$ as a vector**, then this solves the problem:

$$\begin{pmatrix}3\\2\end{pmatrix}+\begin{pmatrix}4\\0\end{pmatrix}=\begin{pmatrix}7\\2\end{pmatrix}$$



A vector used to represent a position is unsurprisingly known as a **position vector**.
A position can be thought of as a translation from the origin, as per above. It enables us to use positions in all sorts of vector (and matrix!) calculations.

✎ The position vector of a point A is the vector $\overrightarrow{OA}$, where O is the origin. $\overrightarrow{OA}$ is usually written as $\boldsymbol{a}$.

Example

The points A and B have coordinates $(3,4)$ and $(11,2)$ respectively.
Find, in terms of i and j:

a) The position vector of A
b) The position vector of B
c) The vector $\overrightarrow{AB}$

a $\overrightarrow{OA} = 3\boldsymbol{i} + 4\boldsymbol{j}$

b $\overrightarrow{OB} = 11\boldsymbol{i} + 2\boldsymbol{j}$

c $\overrightarrow{AB} = 8\boldsymbol{i} - 2\boldsymbol{j}$

You can see this by inspection of the change in x and the change in y:

$$+8 \quad -2$$

$$A(3,4) \quad \rightarrow \quad B(11,2)$$

More formally:

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= \binom{11}{2} - \binom{3}{4} = \binom{8}{-2}$$

Further Example

$\overrightarrow{OA} = 5i - 2j$ and $\overrightarrow{AB} = 3i + 4j$. Find:

a) The position vector of B.

b) The exact value of $\left|\overrightarrow{OB}\right|$ in simplified surd form.

a $\overrightarrow{OB} =$?

b $\left|\overrightarrow{OB}\right| =$?

Either a quick sketch will help you see this, or thinking of $\overrightarrow{OA}$ as the original position and $\overrightarrow{AB}$ as the translation.

Further Example

$\overrightarrow{OA} = 5i - 2j$ and $\overrightarrow{AB} = 3i + 4j$. Find:
a) The position vector of B.
b) The exact value of $\left|\overrightarrow{OB}\right|$ in simplified surd form.

a $\quad \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$

Either a quick sketch will help you see this, or thinking of $\overrightarrow{OA}$ as the original position and $\overrightarrow{AB}$ as the translation.

b $\quad \left|\overrightarrow{OB}\right| = \sqrt{8^2 + 2^2} = 2\sqrt{17}$

Exercise 11.4

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Homework Exercise

1 The points A, B and C have coordinates $(3, -1)$, $(4, 5)$ and $(-2, 6)$ respectively, and O is the origin.

Find, in terms of $\mathbf{i}$ and $\mathbf{j}$:

a **i** the position vectors of A, B and C **ii** $\overrightarrow{AB}$ **iii** $\overrightarrow{AC}$

b Find, in surd form: **i** $|\overrightarrow{OC}|$ **ii** $|\overrightarrow{AB}|$ **iii** $|\overrightarrow{AC}|$

2 $\overrightarrow{OP} = 4\mathbf{i} - 3\mathbf{j}$, $\overrightarrow{OQ} = 3\mathbf{i} + 2\mathbf{j}$

a Find $\overrightarrow{PQ}$

b Find, in surd form: **i** $|\overrightarrow{OP}|$ **ii** $|\overrightarrow{OQ}|$ **iii** $|\overrightarrow{PQ}|$

3 $\overrightarrow{OQ} = 4\mathbf{i} - 3\mathbf{j}$, $\overrightarrow{PQ} = 5\mathbf{i} + 6\mathbf{j}$

a Find $\overrightarrow{OP}$

b Find, in surd form: **i** $|\overrightarrow{OP}|$ **ii** $|\overrightarrow{OQ}|$ **iii** $|\overrightarrow{PQ}|$

4 $OABCDE$ is a regular hexagon. The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where O is the origin.

Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the position vectors of

a C **b** D **c** E.

Homework Exercise

5 The position vectors of 3 vertices of a parallelogram are $\begin{pmatrix}4\\2\end{pmatrix}$, $\begin{pmatrix}3\\5\end{pmatrix}$ and $\begin{pmatrix}8\\6\end{pmatrix}$.
Find the possible position vectors of the fourth vertex.

Problem-solving

Use a sketch to check that you have considered all the possible positions for the fourth vertex.

6 Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $6\mathbf{i} + 3\mathbf{j}$,

a find the vector $\overrightarrow{AB}$. **(2 marks)**

b find $|\overrightarrow{AB}|$ giving your answer as a simplified surd. **(2 marks)**

7 The point A lies on the circle with equation $x^2 + y^2 = 9$. Given that $\overrightarrow{OA} = 2k\mathbf{i} + k\mathbf{j}$, find the exact value of k. **(3 marks)**

Challenge

The point B lies on the line with equation $2y = 12 - 3x$. Given that $|OB| = \sqrt{13}$, find possible expressions for $\overrightarrow{OB}$ in the form $p\mathbf{i} + q\mathbf{j}$.

Homework Answers

1 **a** **i** $\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j}$, $\overrightarrow{OB} = 4\mathbf{i} + 5\mathbf{j}$, $\overrightarrow{OC} = -2\mathbf{i} + 6\mathbf{j}$

ii $\mathbf{i} + 6\mathbf{j}$ **iii** $-5\mathbf{i} + 7\mathbf{j}$

b **i** $\sqrt{40} = 2\sqrt{10}$ **ii** $\sqrt{37}$ **iii** $\sqrt{74}$

2 **a** $-\mathbf{i} + 5\mathbf{j}$ or $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$

b **i** 5 **ii** $\sqrt{13}$ **iii** $\sqrt{26}$

3 **a** $-\mathbf{i} - 9\mathbf{j}$ or $\begin{pmatrix} -1 \\ -9 \end{pmatrix}$

b **i** $\sqrt{82}$ **ii** 5 **iii** $\sqrt{61}$

4 **a** $-2\mathbf{a} + 2\mathbf{b}$ **b** $-3\mathbf{a} + 2\mathbf{b}$ **c** $-2\mathbf{a} + \mathbf{b}$

5 $\begin{pmatrix} 7 \\ 9 \end{pmatrix}$ or $\begin{pmatrix} 9 \\ 3 \end{pmatrix}$

6 **a** $2\mathbf{i} + 8\mathbf{j}$ **b** $2\sqrt{17}$

7 $\dfrac{3\sqrt{5}}{5}$

Challenge

$\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{j}$ or $\overrightarrow{OB} = \frac{46}{13}\mathbf{i} + \frac{9}{13}\mathbf{j}$