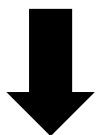

P1 Chapter 4: Transforming Graphs

Translation of Graphs

Transformations of Functions

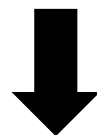
Suppose $f(x) = x^2$



Sketch $y = f(x)$:



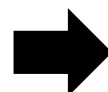
Then $f(x + 2) =$?



Sketch $y = f(x + 2)$



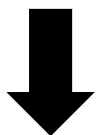
What do you notice about the relationship between the graphs of $y = f(x)$ and $y = f(x + 2)$?



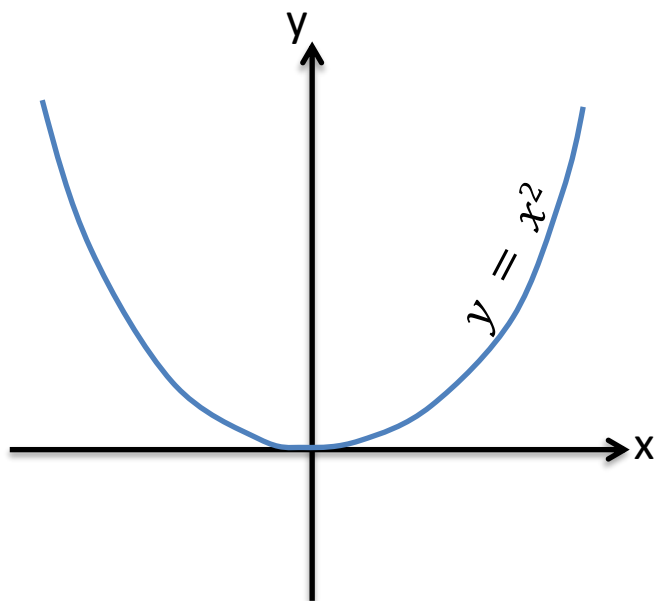
?

Transformations of Functions

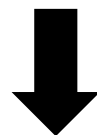
Suppose $f(x) = x^2$



Sketch $y = f(x)$:

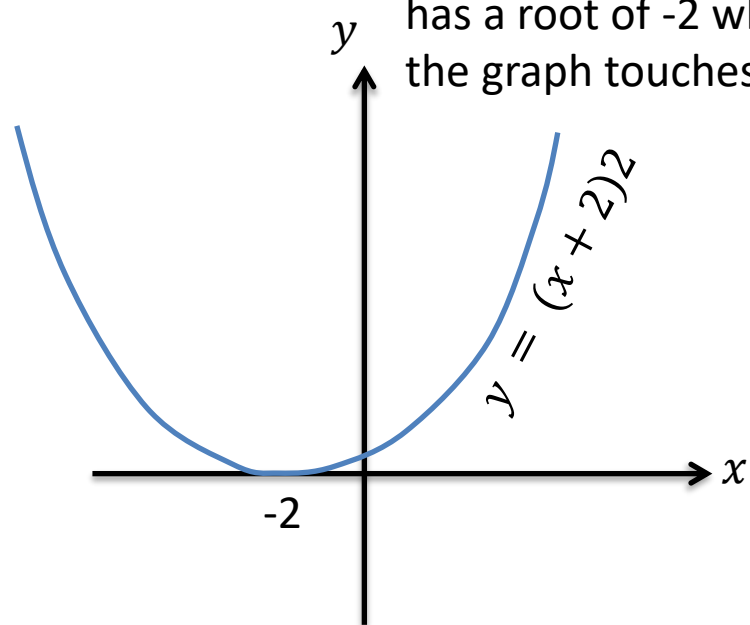


Then $f(x + 2) =$?

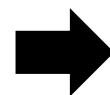


Sketch $y = f(x + 2)$

We know $y = (x + 2)^2$ has a root of -2 where the graph touches.




What do you notice about the relationship between the graphs of $y = f(x)$ and $y = f(x + 2)$?



The graph has been translated by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, i.e. we have subtracted 2 from each x value.

Transformations of Functions

This is all you need to remember when considering how transforming your function transforms your graph...




	Affects which axis?	What we expect or opposite?
Change inside $f()$?	?
Change outside $f()$?	?

Therefore...

$y = f(x - 3)$	➡	?
$y = f(x) + 4$	➡	?
$y = f(5x)$	➡	?
$y = 2f(x)$	➡	?

Transformations of Functions

This is all you need to remember when considering how transforming your function transforms your graph...



	Affects which axis?	What we expect or opposite?
Change inside $f()$	x	Opposite
Change outside $f()$	y	What we expect

Therefore...

$$\begin{aligned} y = f(x - 3) &\longrightarrow \text{Translation by } \begin{pmatrix} 3 \\ 0 \end{pmatrix} \\ y = f(x) + 4 &\longrightarrow \text{Translation by } \begin{pmatrix} 0 \\ 4 \end{pmatrix} \\ y = f(5x) &\longrightarrow \text{Stretch in } x\text{-direction by scale factor } \frac{1}{5} \\ y = 2f(x) &\longrightarrow \text{Stretch in } y\text{-direction by scale factor } 2 \end{aligned}$$

Sketching transformed graphs

Sketch $y = x^2 + 3$

?

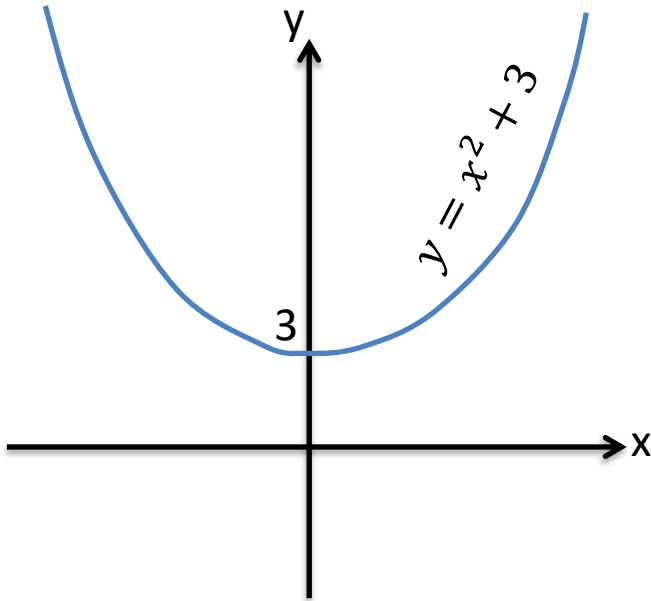
Sketch $y = \frac{2}{x+1}$

?

Sketching transformed graphs

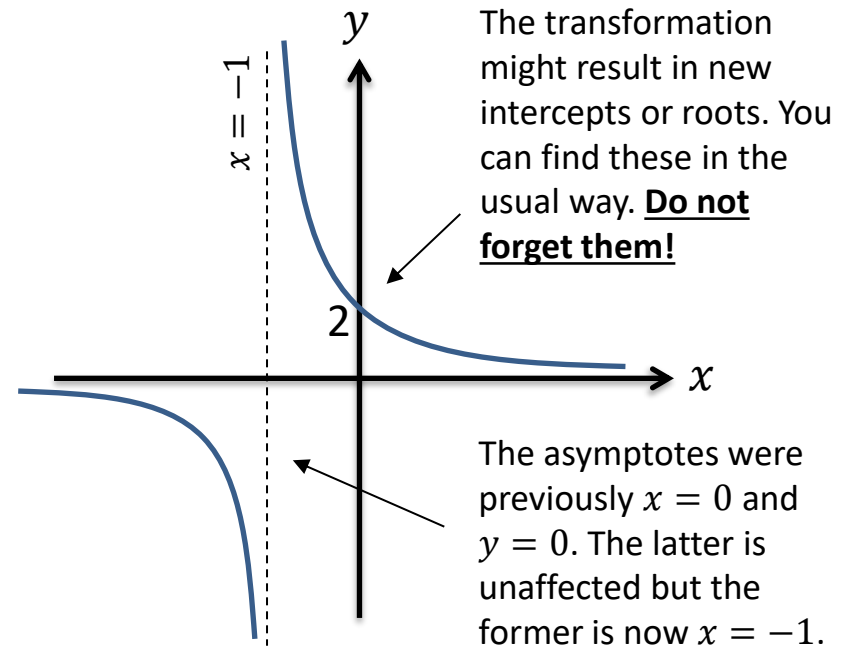
Sketch $y = x^2 + 3$

If $y = x^2$, the +3 is 'outside' the squared function, so translation of $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$. Imagine a sketch of $y = x^2$ and then do the translation, ensuring you adjust any intercepts with the axes.



Sketch $y = \frac{2}{x+1}$

This looks like a reciprocal function $y = \frac{2}{x}$. The change of +1 is *inside* the reciprocal function, so we have a translation to the left by 1.



Draw asymptotes using a dotted line and write its equation on it.

Example

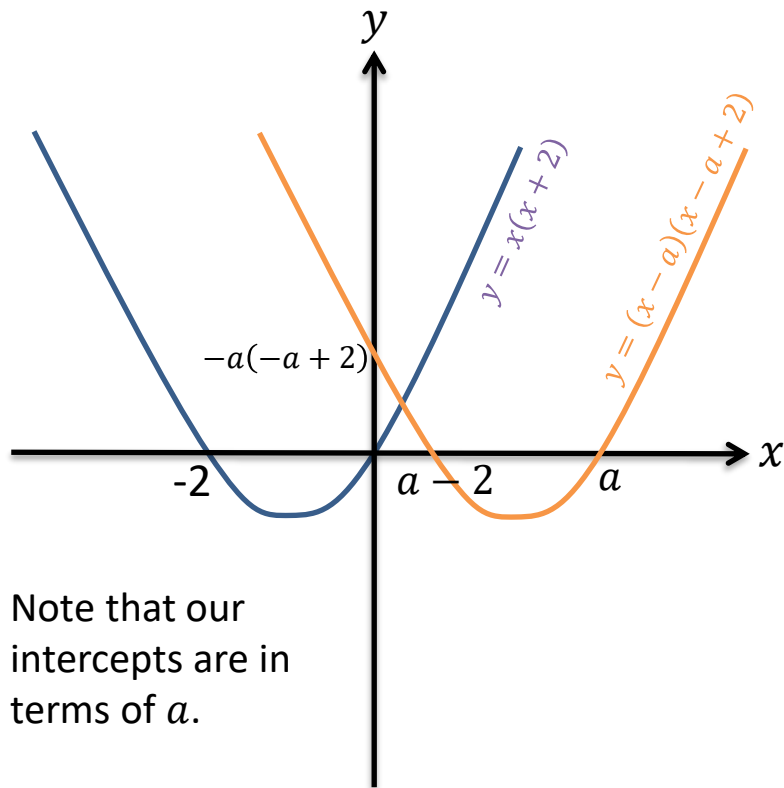
Sketch $y = x(x + 2)$. On the same axes, sketch $y = (x - a)(x - a + 2)$, where $a > 2$.

?

Example

Sketch $y = x(x + 2)$. On the same axes, sketch $y = (x - a)(x - a + 2)$, where $a > 2$.

The **input** x has been replaced with $x - a$, i.e. a change inside the function. We translate right by a . The significance of $a > 2$ is that the original root of -2 will now be positive.



Note that our intercepts are in terms of a .

Exercise 4.5

Pearson Pure Mathematics Year 1/AS

Page 30

Homework Exercise

1 Apply the following transformations to the curves with equations $y = f(x)$ where:

i $f(x) = x^2$ **ii** $f(x) = x^3$ **iii** $f(x) = \frac{1}{x}$

In each case state the coordinates of points where the curves cross the axes and in **iii** state the equations of the asymptotes.

a $f(x + 2)$ **b** $f(x) + 2$ **c** $f(x - 1)$
d $f(x) - 1$ **e** $f(x) - 3$ **f** $f(x - 3)$

2 a Sketch the curve $y = f(x)$ where $f(x) = (x - 1)(x + 2)$.

b On separate diagrams sketch the graphs of **i** $y = f(x + 2)$ **ii** $y = f(x) + 2$.

c Find the equations of the curves $y = f(x + 2)$ and $y = f(x) + 2$, in terms of x , and use these equations to find the coordinates of the points where your graphs in part **b** cross the y -axis.

3 a Sketch the graph of $y = f(x)$ where $f(x) = x^2(1 - x)$.

b Sketch the curve with equation $y = f(x + 1)$.

c By finding the equation $f(x + 1)$ in terms of x , find the coordinates of the point in part **b** where the curve crosses the y -axis.

4 a Sketch the graph of $y = f(x)$ where $f(x) = x(x - 2)^2$.

b Sketch the curves with equations $y = f(x) + 2$ and $y = f(x + 2)$.

c Find the coordinates of the points where the graph of $y = f(x + 2)$ crosses the axes.

Homework Exercise

- 5 a** Sketch the graph of $y = f(x)$ where $f(x) = x(x - 4)$.
- b** Sketch the curves with equations $y = f(x + 2)$ and $y = f(x) + 4$.
- c** Find the equations of the curves in part **b** in terms of x and hence find the coordinates of the points where the curves cross the axes.
- 6 a** Sketch the graph of $y = f(x)$ where $f(x) = x^2(x - 1)(x - 2)$.
- b** Sketch the curves with equations $y = f(x + 2)$ and $y = f(x) - 1$.
- 7** The point $P(4, -1)$ lies on the curve with equation $y = f(x)$.
- a** State the coordinates that point P is transformed to on the curve with equation $y = f(x - 2)$. **(1 mark)**
- b** State the coordinates that point P is transformed to on the curve with equation $y = f(x) + 3$. **(1 mark)**
- 8** The graph of $y = f(x)$ where $f(x) = \frac{1}{x}$ is translated so that the asymptotes are at $x = 4$ and $y = 0$. Write down the equation for the transformed function in the form $y = \frac{1}{x + a}$ **(3 marks)**
- 9 a** Sketch the graph of $y = x^3 - 5x^2 + 6x$, marking clearly the points of intersection with the axes.
- b** Hence sketch $y = (x - 2)^3 - 5(x - 2)^2 + 6(x - 2)$.

Homework Exercise

- 10 a Sketch the graph of $y = x^2(x - 3)(x + 2)$, marking clearly the points of intersection with the axes.
b Hence sketch $y = (x + 2)^2(x - 1)(x + 4)$.

- 11 a Sketch the graph of $y = x^3 + 4x^2 + 4x$. (6 marks)

- b The point with coordinates $(-1, 0)$ lies on the curve with equation $y = (x + a)^3 + 4(x + a)^2 + 4(x + a)$ where a is a constant. Find the two possible values of a . (3 marks)

Problem-solving

Look at your sketch and picture the curve sliding to the left or right.

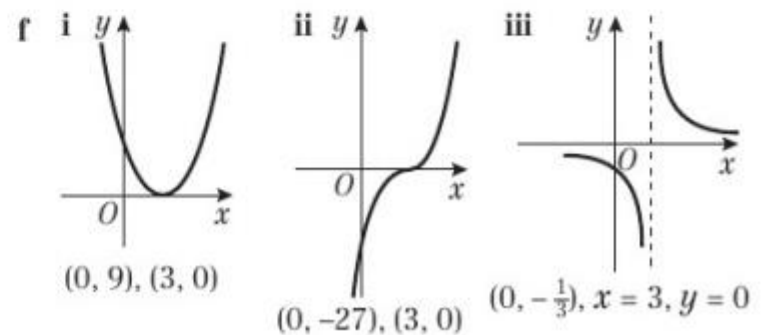
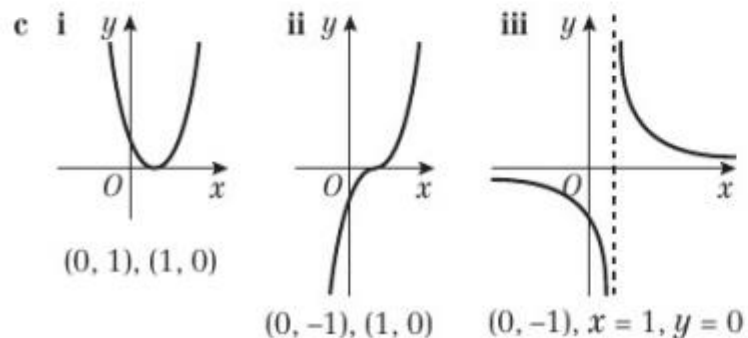
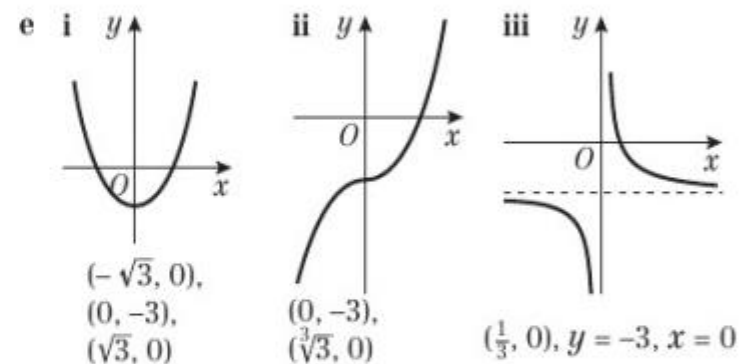
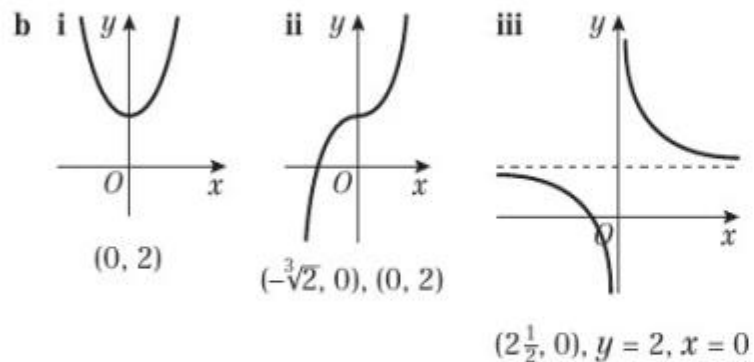
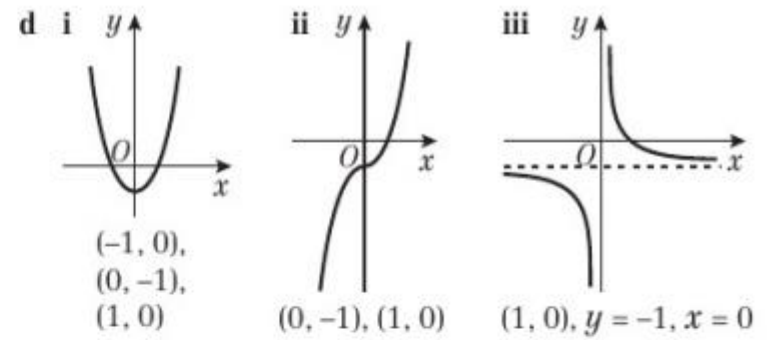
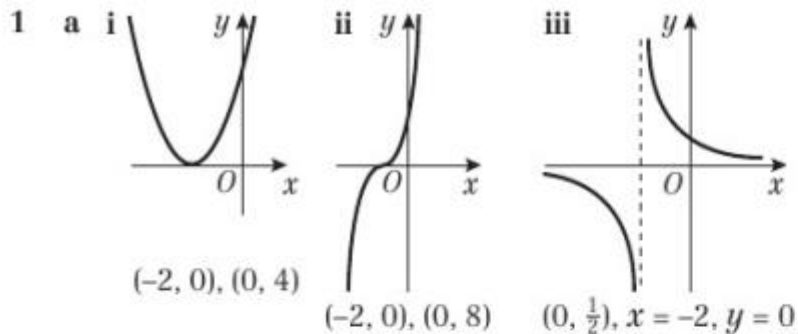
- 12 a Sketch the graph of $y = x(x + 1)(x + 3)^2$. (4 marks)

- b Find the possible values of b such that the point $(2, 0)$ lies on the curve with equation $y = (x + b)(x + b + 1)(x + b + 3)^2$. (3 marks)

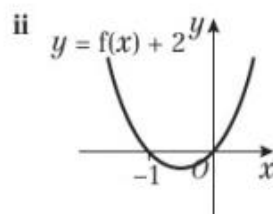
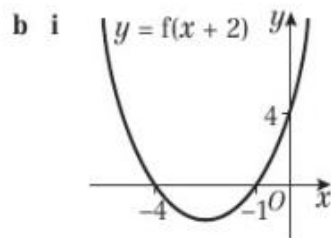
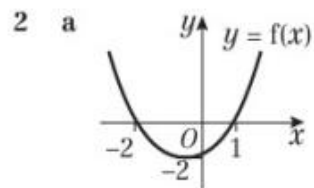
Challenge

- 1 Sketch the graph of $y = (x - 3)^3 + 2$ and determine the coordinates of the point of inflection. → Section 12.9
- 2 The point $Q(-5, -7)$ lies on the curve with equation $y = f(x)$.
 - a State the coordinates that point Q is transformed to on the curve with equation $y = f(x + 2) - 5$.
 - b The coordinates of the point Q on a transformed curve are $(-3, -6)$. Write down the transformation in the form $y = f(x + a) - b$.

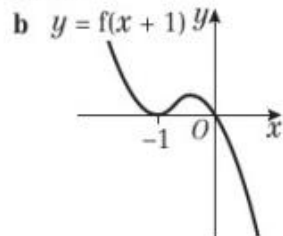
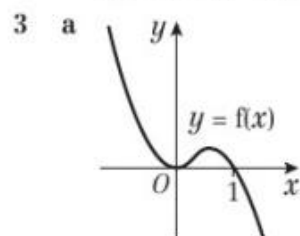
Homework Answers



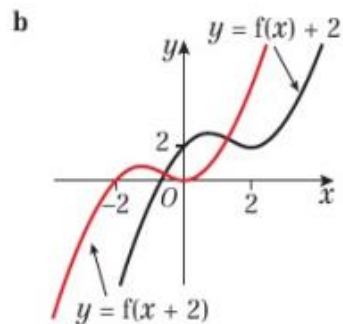
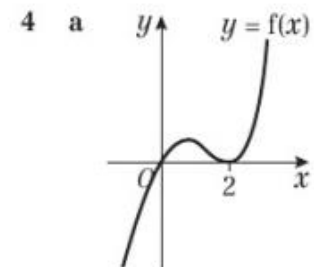
Homework Answers



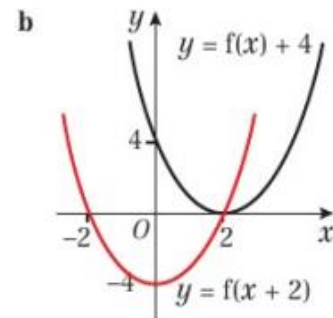
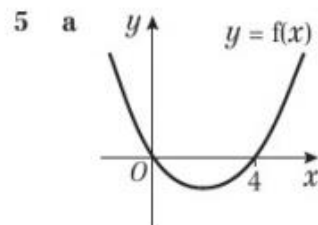
c $f(x+2) = (x+1)(x+4)$; $(0, 4)$
 $f(x) + 2 = (x+1)(x+2) + 2$; $(0, 0)$



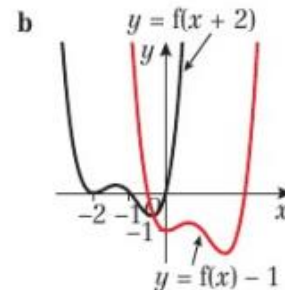
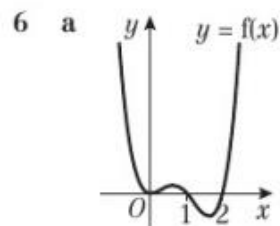
c $f(x+1) = -x(x+1)^2$; $(0, 0)$



c $f(x+2) = (x+2)x^2$; $(0, 0)$; $(-2, 0)$



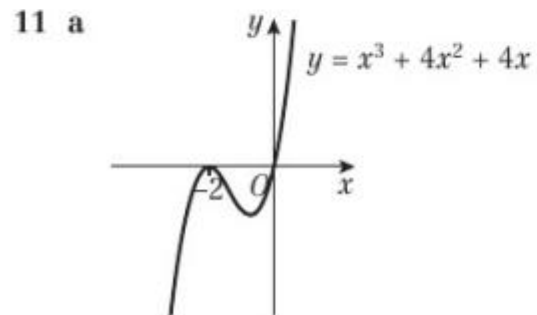
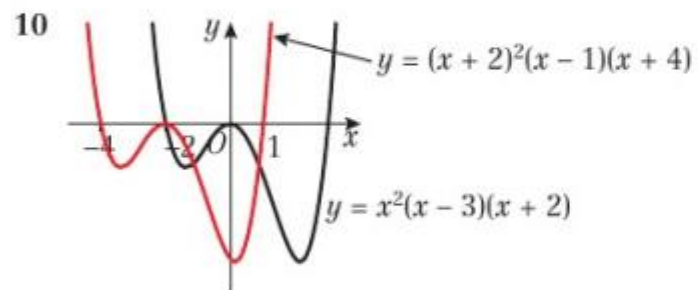
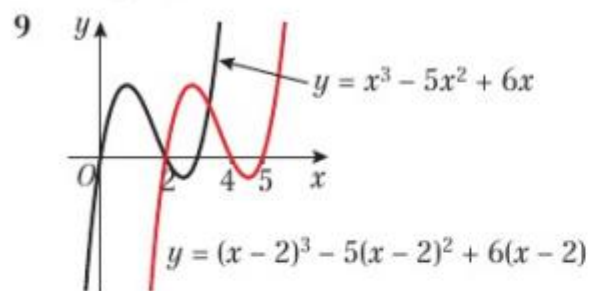
c $f(x+2) = (x+2)(x-2)$; $(2, 0)$; $(-2, 0)$
 $f(x) + 4 = (x-2)^2$; $(2, 0)$



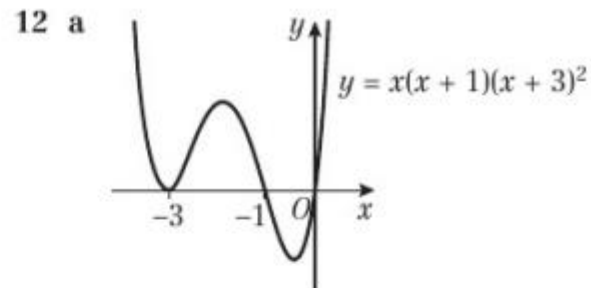
Homework Answers

7 a $(6, -1)$ b $(4, 2)$

8 $y = \frac{1}{x-4}$



b -1 or 1



b $-2, -3$ or -5

Challenge

1 $(3, 2)$

2 a $(-7, -12)$ b $f(x-2) + 1$