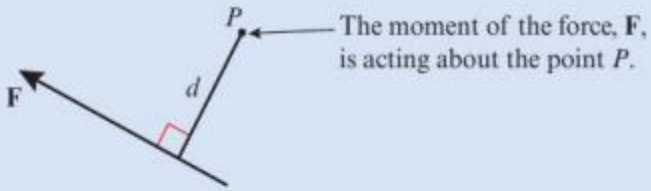
M2 Chapter 5: Moments

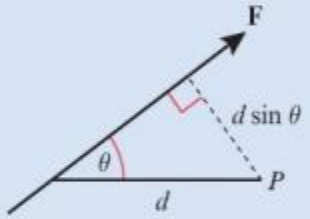
Chapter Practice

Key Points

1 Moment of **F** about $P = |\mathbf{F}| \times d$ clockwise



2 Moment of **F** about $P = |\mathbf{F}| \times d\sin\theta$ clockwise



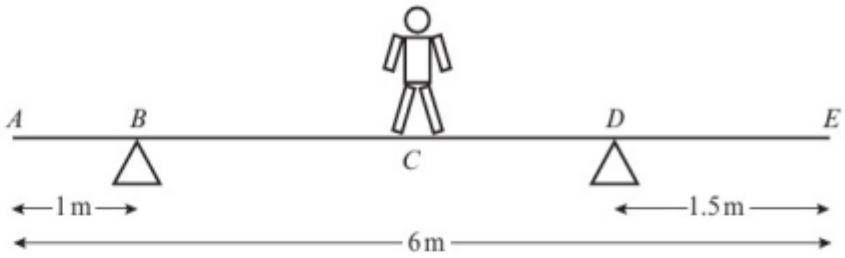
3 The sum of the moments acting on a body is called the resultant moment.

4 When a rigid body is in equilibrium the resultant force in any direction is 0 N and the resultant moment about any point is 0 N m.

5 When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or the tension in any other wire or string) is zero.

Chapter Exercises

1 A plank AE, of length 6 m and weight 100 N, rests in a horizontal position on supports at B and D, where $AB = 1$ m and $DE = 1.5$ m. A child of weight 145 N stands at C, the midpoint of AE, as shown in the diagram. The child is modelled as a particle and the plank as a uniform rod. The child and the plank are in equilibrium. Calculate:



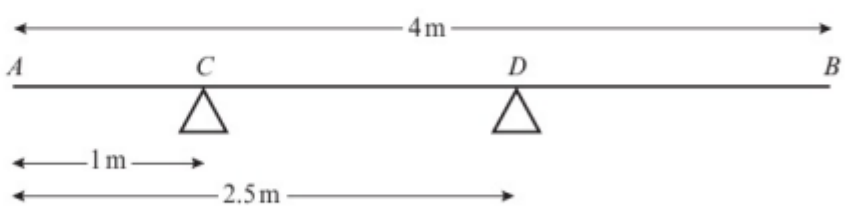
a the magnitude of the force exerted by the support on the plank at B **(3 marks)**

b the magnitude of the force exerted by the support on the plank at D. **(2 marks)**

The child now stands at a different point F on the plank. The plank is in equilibrium and on the point of tilting about D.

c Calculate the distance DF. **(4 marks)**

2 A uniform rod AB has length 4 m and weight 150 N. The rod rests in equilibrium in a horizontal position, smoothly supported at points C and D, where $AC = 1$ m and $AD = 2.5$ m as shown in the diagram. A particle of weight W N is attached to the rod at a point E where $AE = x$ metres. The rod remains in equilibrium and the magnitude of the reaction at C is now equal to the magnitude of the reaction at D.

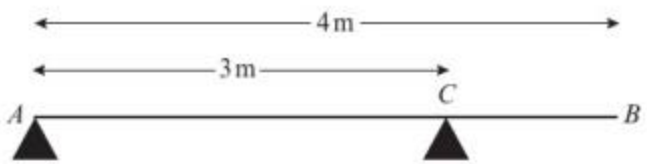


a Show that $W = \dfrac{150}{7 - 4x}$ **(6 marks)**

b Hence deduce the range of possible values of x. **(3 marks)**

Chapter Exercises

3 A uniform plank AB has mass 40 kg and length 4 m. It is supported in a horizontal position by two smooth pivots. One pivot is at the end A and the other is at the point C where $AC = 3$ m, as shown in the diagram.



A man of mass 80 kg stands on the plank which remains in equilibrium. The magnitude of the reaction at A is twice the magnitude of the reaction at C. The magnitude of the reaction at C is R N. The plank is modelled as a rod and the man is modelled as a particle.

a Find the value of R. **(2 marks)**

b Find the distance of the man from A. **(3 marks)**

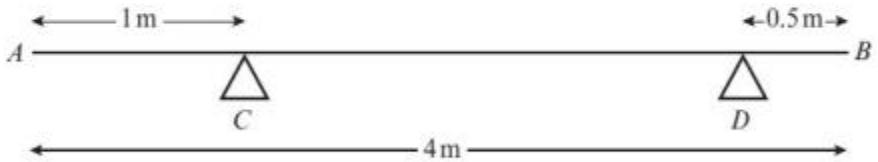
c State how you have used the modelling assumption that:

i the plank is uniform

ii the plank is a rod

iii the man is a particle. **(3 marks)**

4 A non-uniform rod AB has length 4 m and weight 150 N. The rod rests horizontally in equilibrium on two smooth supports C and D, where $AC = 1$ m and $DB = 0.5$ m, as shown in the diagram. The centre of mass of AB is x metres from A.



A particle of weight W N is placed on the rod at A. The rod remains in equilibrium and the magnitude of the reaction of C on the rod is 100 N.

a Show that $550 + 7W = 300x$. **(4 marks)**

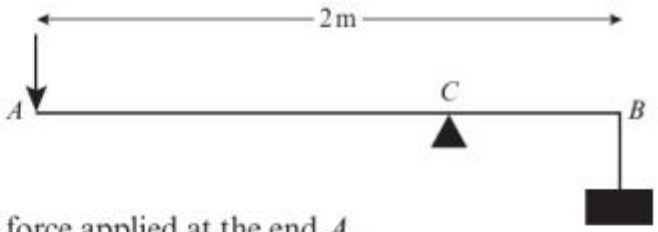
The particle is now removed from A and placed on the rod at B. The rod remains in equilibrium and the reaction of C on the rod now has magnitude 52 N.

b Obtain another equation connecting W and x. **(4 marks)**

c Calculate the value of x and the value of W. **(3 marks)**

Chapter Exercises

5 A lever consists of a uniform steel rod AB, of weight 100 N and length 2 m, which rests on a small smooth pivot at a point C. A load of weight 1700 N is suspended from the end B of the rod by a rope. The lever is held in equilibrium in a horizontal position by a vertical force applied at the end A, as shown in the diagram. The rope is modelled as a light string.



a Given that $BC = 0.25$ m find the magnitude of the force applied at A. **(4 marks)**

The position of the pivot is changed so that the rod remains in equilibrium when the force at A has magnitude 150 N.

b Find, to the nearest centimetre, the new distance of the pivot from B. **(4 marks)**

6 A plank AB has length 4 m. It lies on a horizontal platform, with the end A lying on the platform and the end B projecting over the edge, as shown in the diagram. The edge of the platform is at the point C.

Jack and Jill are experimenting with the plank. Jack has mass 48 kg and Jill has mass 36 kg. They discover that if Jack stands at B and Jill stands at A and $BC = 1.8$ m, the plank is in equilibrium and on the point of tilting about C.

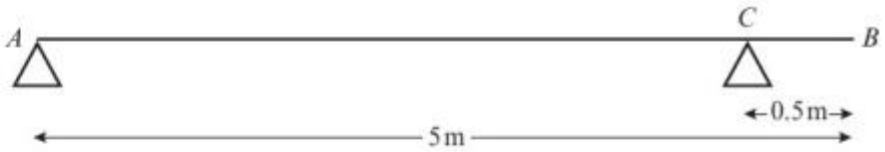
a By modelling the plank as a uniform rod, and Jack and Jill as particles, find the mass of the plank. **(4 marks)**

They now alter the position of the plank in relation to the platform so that, when Jill stands at B and Jack stands at A, the plank is again in equilibrium and on the point of tilting about C.

b Find the distance BC in this position. **(4 marks)**

Chapter Exercises

7 A plank of wood AB has mass 12 kg and length 5 m. It rests in a horizontal position on two smooth supports. One support is at the end A. The other is at the point C, 0.5 m from B, as shown in the diagram. A girl of mass 30 kg stands at B with the plank in equilibrium.

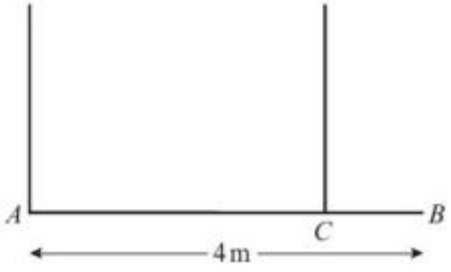


a By modelling the plank as a uniform rod and the girl as a particle, find the reaction on the plank at A. **(4 marks)**

The girl gets off the plank. A boulder of mass m kg is placed on the plank at A and a man of mass 93 kg stands on the plank at B. The plank remains in equilibrium and is on the point of tilting about C.

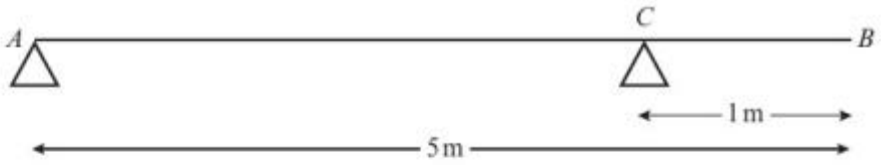
b By modelling the plank again as a uniform rod, and the man and the boulder as particles, find the value of m. **(5 marks)**

8 A plank AB has mass 50 kg and length 4 m. A load of mass 25 kg is attached to the plank at B. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes attached at A and C, as shown in the diagram. The plank is modelled as a uniform rod and the load as a particle. Given that the tension in the rope at C is four times the tension in the rope at A, calculate the distance CB. **(7 marks)**



Chapter Exercises

9 A beam AB has weight 200 N and length 5 m. The beam rests in equilibrium in a horizontal position on two smooth supports.



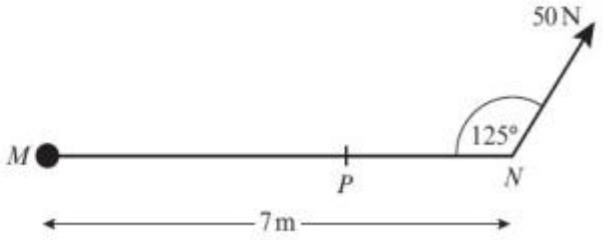
One support is at end A and the other is at a point C on the beam, where $BC = 1$ m, as shown in the diagram. The beam is modelled as a uniform rod.

a Find the reaction on the beam at C. **(4 marks)**

A woman of weight 500 N stands on the beam at the point D. The beam remains in equilibrium. The reactions on the beam at A and C are now equal.

b Find the distance AD. **(5 marks)**

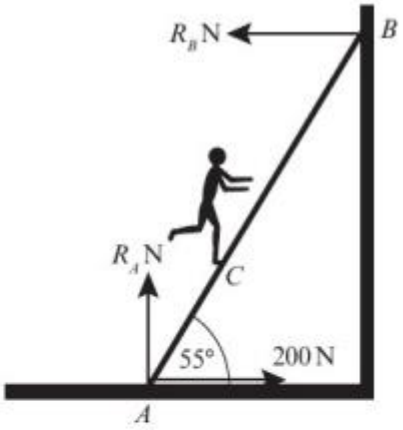
10 A non-uniform plank MN of length 7 m is attached to a pivot at M and is held in a horizontal position by a force of 50 N applied at N at an angle of 125° to the plank as shown in the diagram. The centre of mass of the plank is at the point P. Given that the plank is in equilibrium and has a mass of 6 kg, find the distance MP.



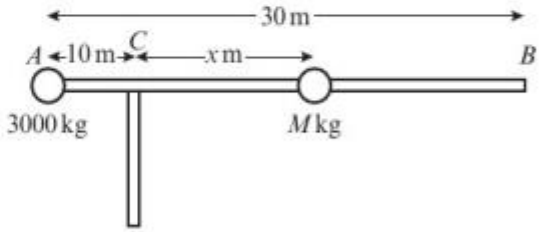
(4 marks)

Chapter Exercises

11 A ladder, AB, is leaning against a smooth vertical wall and on rough horizontal ground at an angle of 55° to the horizontal. The ladder has length 10 m and mass 20 kg. A man of mass 80 kg is standing at the point C on the ladder. Given that the magnitude of the frictional force at A is 200 N, find the distance AC. Model the ladder as a uniform rod and the man as a particle. **(5 marks)**



12 The beam of a crane is modelled as a uniform rod AB, of length 30 m and weight 4000 kg, resting in horizontal equilibrium. The beam is supported by a tower at C, where $AC = 10$ m. A counterbalance mass of weight 3000 kg is placed at A and a load of mass M is placed a variable distance x m from the supporting tower, where $x \geqslant 5$.



a Find an expression for M in terms of x. **(4 marks)**

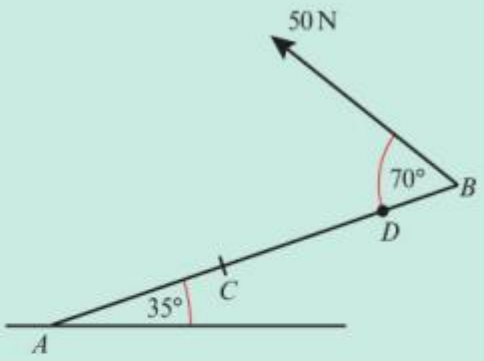
b Hence determine the maximum and minimum loads that can be lifted by the crane. **(2 marks)**

c Criticise this model in relation to the beam. **(1 mark)**

Chapter Exercises

Challenge

1 A non-uniform beam AB, of mass 10 kg and length 8 m, is pivoted at a point A. A particle of mass 2 kg is attached to the beam at a point D which is 1 m from B. The beam is held in equilibrium at an angle of 35° to the horizontal by a rope attached at point B. Given that the tension in the rope is 50 N and it makes an angle of 70° to the beam, find the distance of the centre of mass of the beam from A.

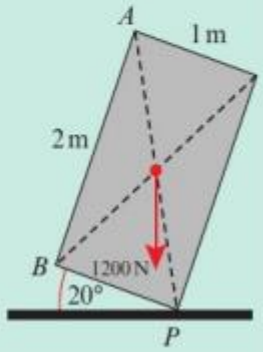


2 A builder is attempting to tip over a refrigerator. The refrigerator is modelled as a rectangular lamina of weight 1200 N. The centre of mass of the lamina is at the point of intersection of the diagonals of the rectangle, as shown in the diagram.

Given that the refrigerator is pivoting at vertex P and that the base of the refrigerator makes an angle of 20° to the floor, find the minimum force needed to tip the refrigerator if the force is applied:

a horizontally at A

b vertically at B.



Chapter Answers

1 **a** 105 N **b** 140 N
c 1.03 m to the right of D

2 **a** $(1 \times 150) + W(x - 1) = 1.5\left(\frac{150 + W}{2}\right)$
$150 + Wx - W = 112.5 + 0.75W$
$37.5 = 1.75W - Wx$
$150 = 7W - Wx$
$W = \frac{150}{7 - 4x}$
b $0 \leqslant x < \frac{7}{4}$

3 **a** 40 g **b** $x = \frac{1}{2}$
c **i** The weight acts at the centre of the plank.
ii The plank remains straight.
iii The man's weight acts at a single point.

4 **a** $2.5 \times 100 = 3.5W + 150(3.5 - x)$
$250 = 3.5W + 525 - 150x$
$150x = 3.5W + 275$
$300x = 7W + 550$
b $W = 790 - 300x$ **c** $x = 2.53$, $W = 30$

5 **a** 200 N **b** 21 cm

6 **a** 36 kg **b** 2.2 m

7 **a** 19.6 N **b** 5

8 $\frac{2}{3}$ m

9 **a** 125 N **b** 1.8 m

10 4.88 m

11 2.39 m

12 **a** $\frac{10000}{x}$ **b** $500\,\text{kg} \leqslant M \leqslant 2000\,\text{kg}$
c This model has the crane only able to lift weights of 500 kg at full extension, not very practical.

Challenge

1 3.28 m

2 **a** 69.1 N **b** 163 N