P2 Chapter 6: CoAngle Trigonometry
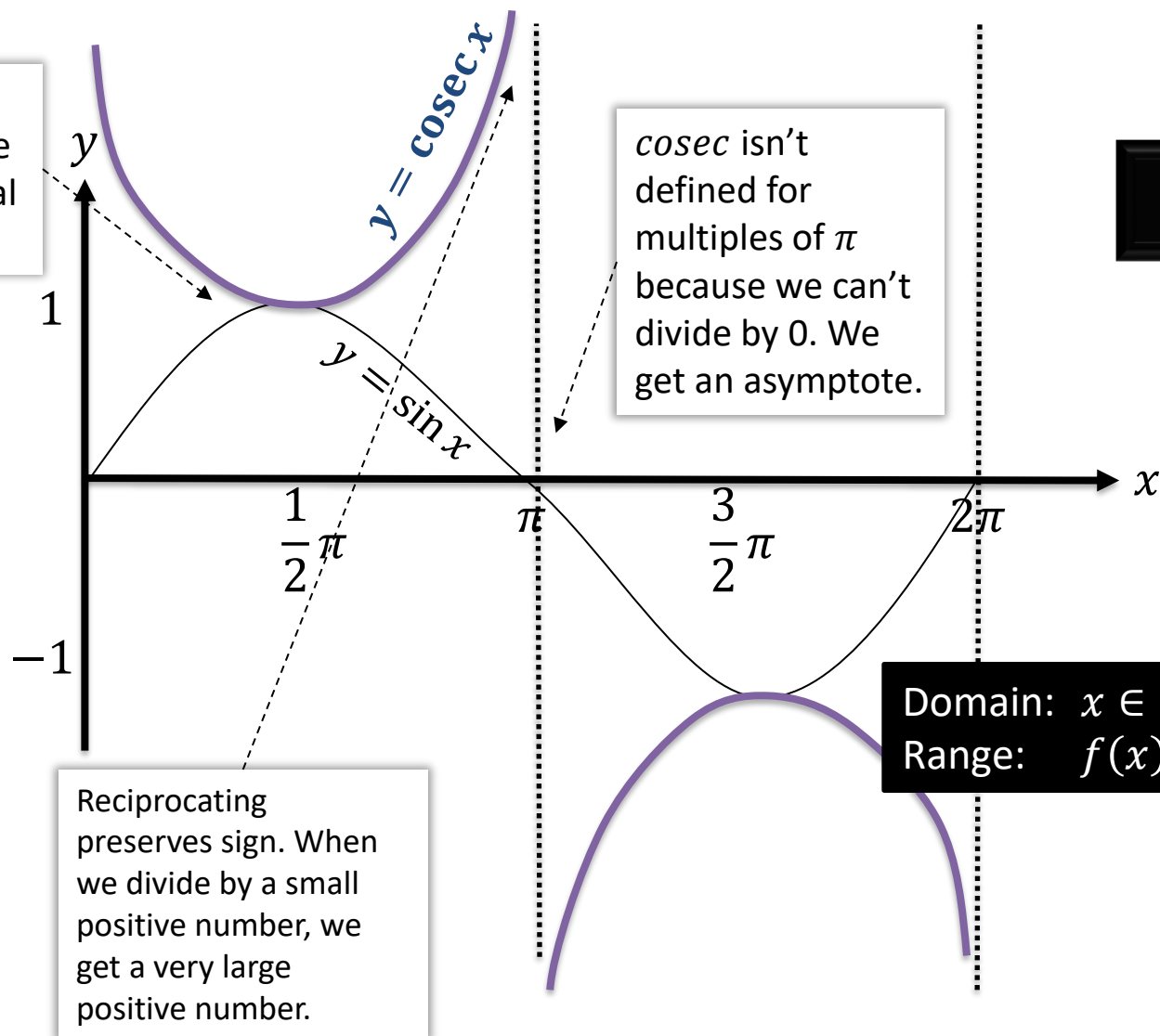
Co-angle Graphs
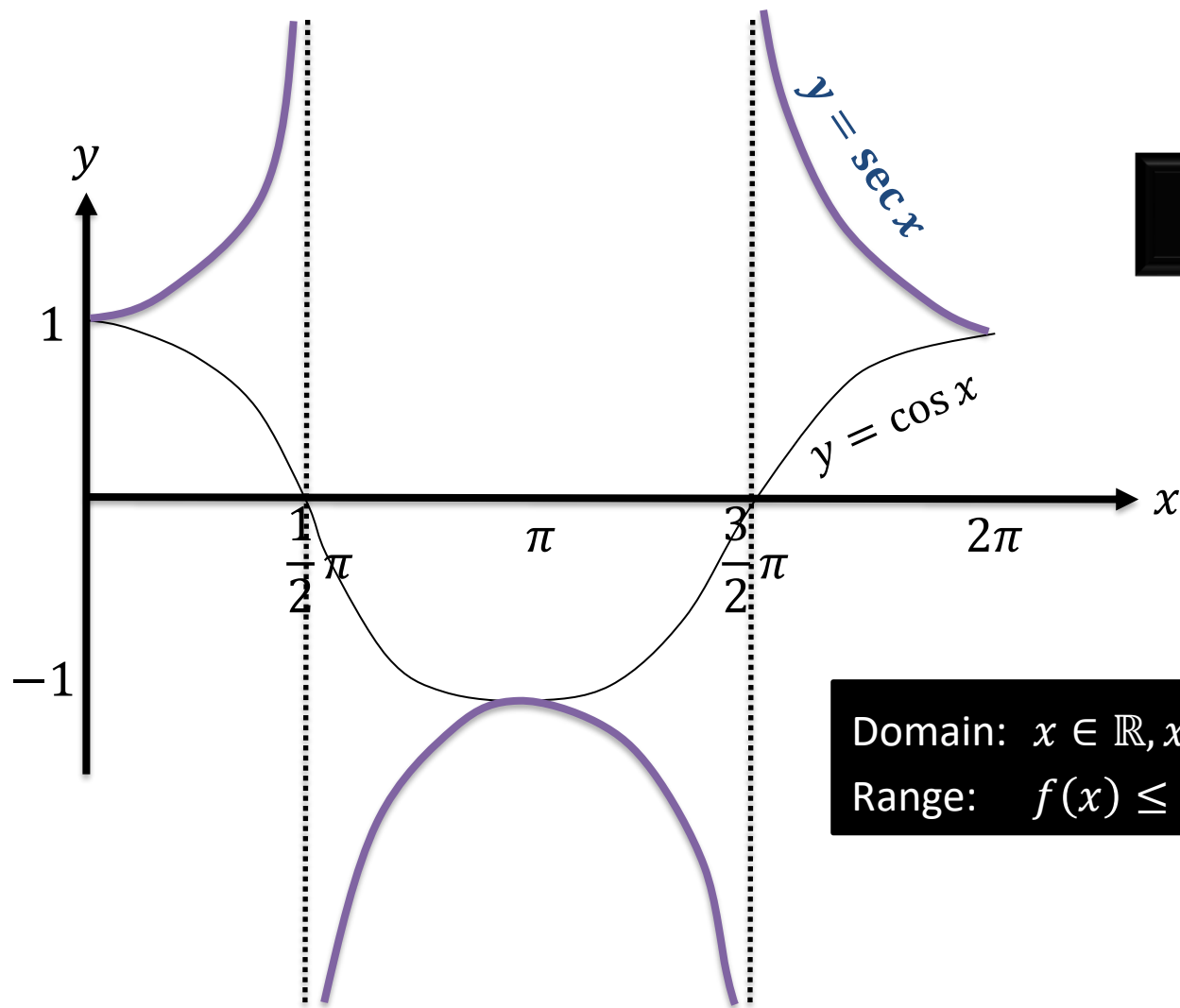
Sketches

To draw a graph of $y = cosec\ x$, start with a graph of $y = \sin x$, then consider what happens when we reciprocate each y value.

It touches here because the reciprocal of 1 is 1.

$y = cosec\ x$

$y = \sin x$

$cosec$ isn't defined for multiples of π because we can't divide by 0. We get an asymptote.

Click to sketch
$y = cosec\ x$

Reciprocating preserves sign. When we divide by a small positive number, we get a very large positive number.

Domain: $x \in \mathbb{R}, x \neq \pm\pi, \pm 2\pi, ...$
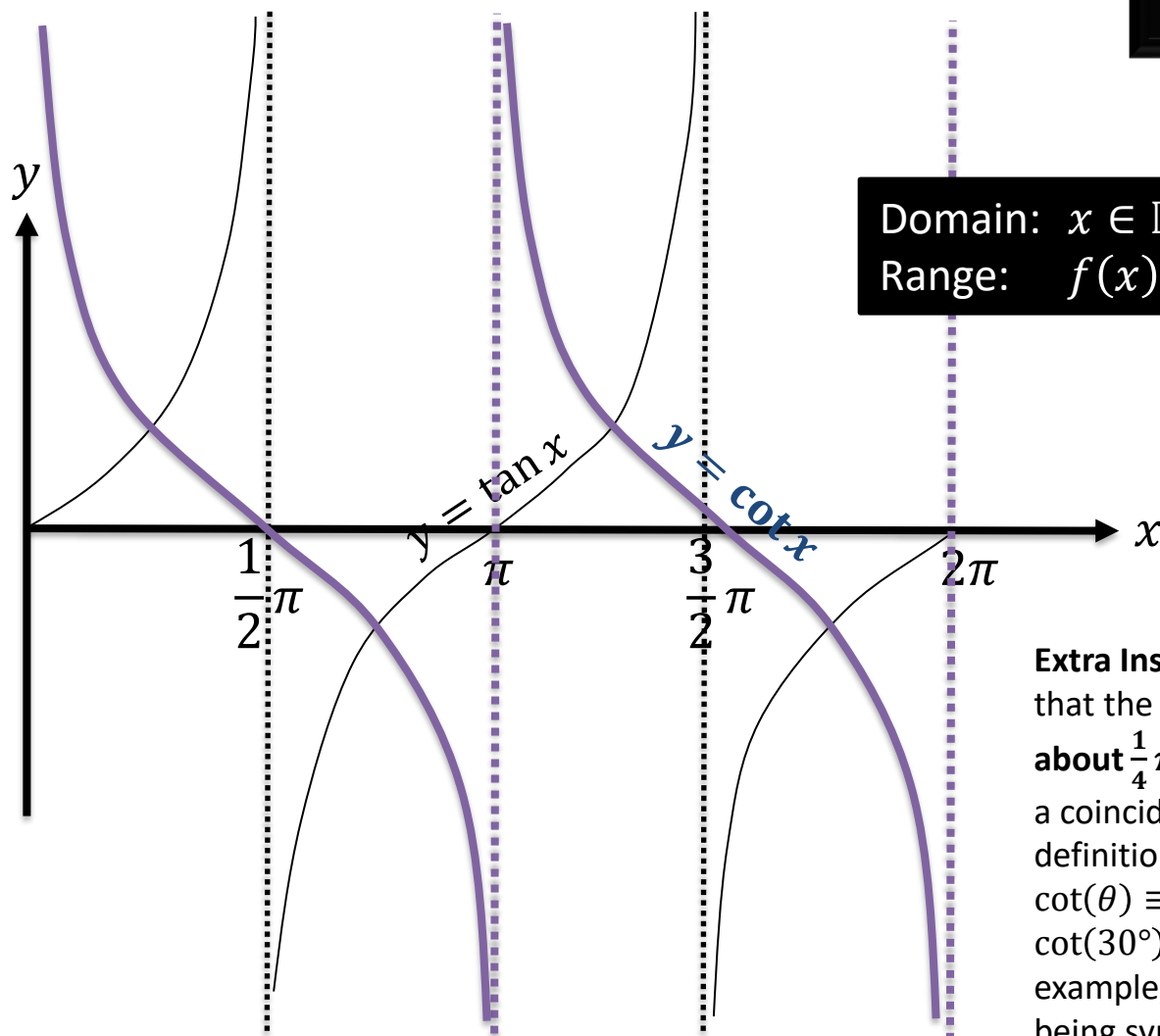Range: $f(x) \leq 1\ or\ f(x) \geq 1$

Sketches

Click to sketch
$y = \sec x$

Domain: $x \in \mathbb{R}, x \neq \pm\frac{1}{2}\pi, \pm\frac{3}{2}\pi, \dots$
Range: $f(x) \leq 1 \ or \ f(x) \geq 1$

Sketches

y

$y = \tan x$

$y = \cot x$

$\frac{1}{2}\pi$ π $\frac{3}{2}\pi$ 2π x

Domain: $x \in \mathbb{R}, x \neq \pi, 2\pi, \ldots$
Range: $f(x) \in \mathbb{R}$

Extra Insight: We might spot that the graph is **symmetrical about** $\frac{1}{4}\pi, \frac{3}{4}\pi$, etc. This is not a coincidence: the 'proper' definition of $\cot(\theta) \equiv \tan(90° - \theta)$, so $\cot(30°) = \tan(60°)$ for example, with 30° and 60° being symmetrical about 45°.
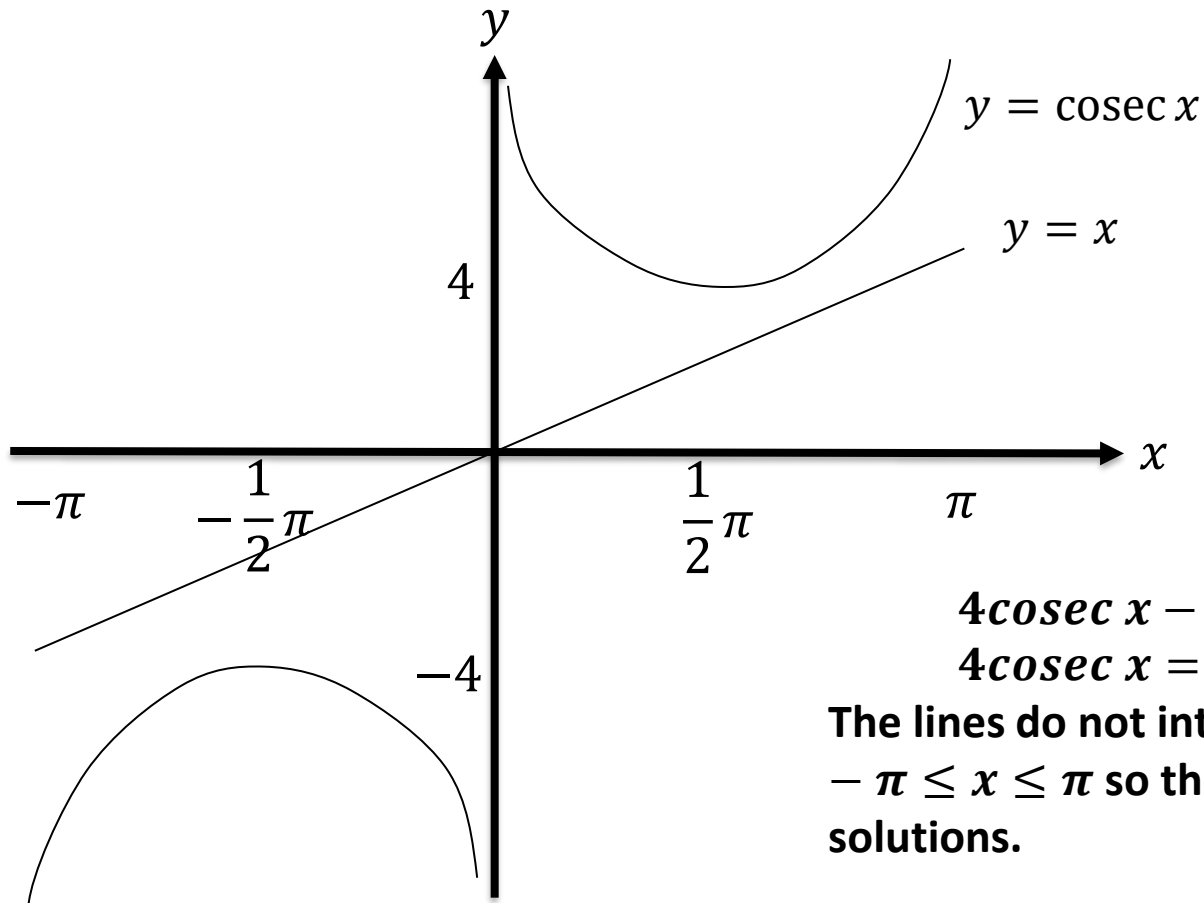
Example

a) Sketch the graph of $y = 4\operatorname{cosec} x, \quad -\pi \leq x \leq \pi$.
b) On the same axes, sketch the line $y = x$.
c) State the number of solutions to the equation $4\operatorname{cosec} x - x = 0, \quad -\pi \leq x \leq \pi$

?

Example

a) Sketch the graph of $y = 4cosec\ x, \ -\pi \le x \le \pi$.
b) On the same axes, sketch the line $y = x$.
c) State the number of solutions to the equation $4cosec\ x - x = 0, \ -\pi \le x \le \pi$

$y = \text{cosec}\ x$

$y = x$

$4\boldsymbol{cosec}\ \boldsymbol{x} - \boldsymbol{x} = \boldsymbol{0}$
$4\boldsymbol{cosec}\ \boldsymbol{x} = \boldsymbol{x}$
The lines do not intersect for $-\pi \le x \le \pi$ **so there are no solutions.**
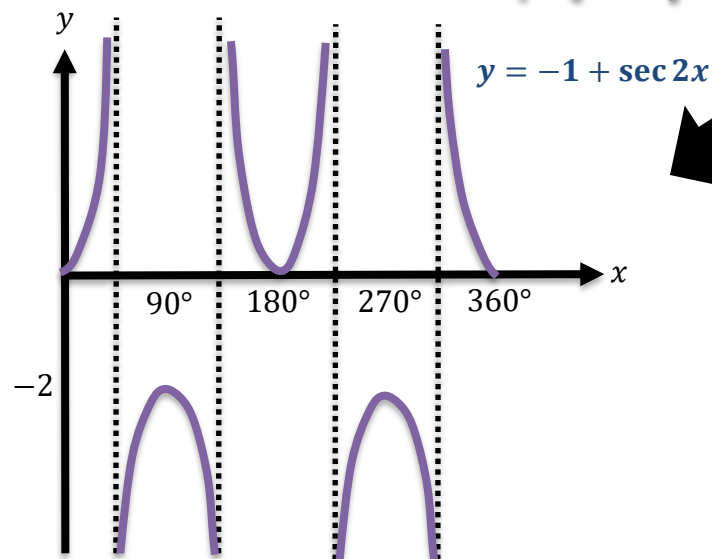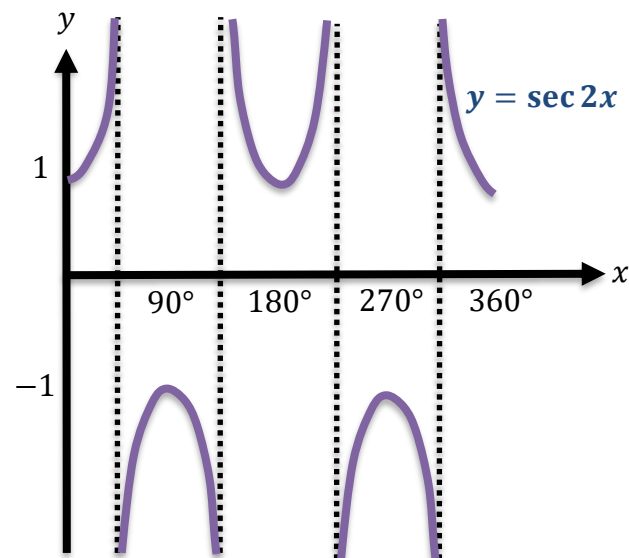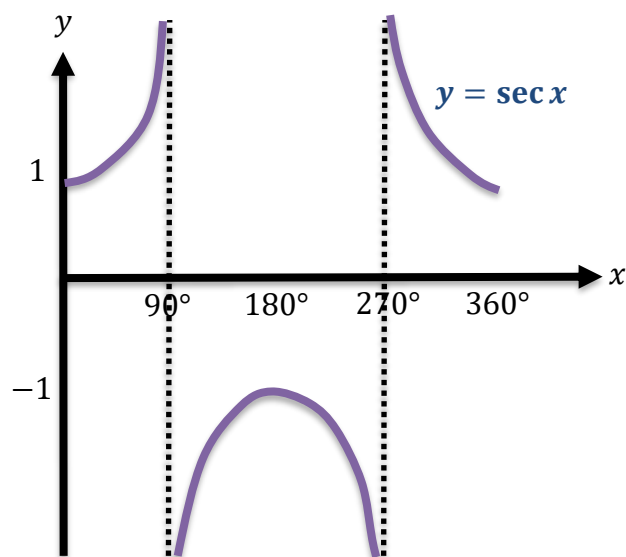
Sketch $y = -1 + \sec 2x$ in the interval $0 \leq x < 360°$.

?

Test Your Understanding

Sketch $y = -1 + \sec 2x$ in the interval $0 \leq x < 360°$.

$y = \sec x$

$y = \sec 2x$

$y = -1 + \sec 2x$

Draw the transformations stage by stage, unless you feel comfortable doing multiple transformations at once.

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1 Sketch, in the interval $-540° \leqslant \theta \leqslant 540°$, the graphs of:

   a $y = \sec\theta$ **b** $y = \operatorname{cosec}\theta$ **c** $y = \cot\theta$

2 a Sketch, on the same set of axes, in the interval $-\pi \leqslant x \leqslant \pi$, the graphs of $y = \cot x$ and $y = -x$.

   b Deduce the number of solutions of the equation $\cot x + x = 0$ in the interval $-\pi \leqslant x \leqslant \pi$.

3 a Sketch, on the same set of axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $y = \sec\theta$ and $y = -\cos\theta$.

   b Explain how your graphs show that $\sec\theta = -\cos\theta$ has no solutions.

4 a Sketch, on the same set of axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $y = \cot\theta$ and $y = \sin 2\theta$.

   b Deduce the number of solutions of the equation $\cot\theta = \sin 2\theta$ in the interval $0 \leqslant \theta \leqslant 360°$.

5 a Sketch on separate axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $y = \tan\theta$ and $y = \cot(\theta + 90°)$.

   b Hence, state a relationship between $\tan\theta$ and $\cot(\theta + 90°)$.

6 a Describe the relationships between the graphs of:

    i $y = \tan\left(\theta + \dfrac{\pi}{2}\right)$ and $y = \tan\theta$ **ii** $y = \cot(-\theta)$ and $y = \cot\theta$

    iii $y = \operatorname{cosec}\left(\theta + \dfrac{\pi}{4}\right)$ and $y = \operatorname{cosec}\theta$ **iv** $y = \sec\left(\theta - \dfrac{\pi}{4}\right)$ and $y = \sec\theta$

  b By considering the graphs of $y = \tan\left(\theta + \dfrac{\pi}{2}\right)$, $y = \cot(-\theta)$, $y = \operatorname{cosec}\left(\theta + \dfrac{\pi}{4}\right)$ and $y = \sec\left(\theta - \dfrac{\pi}{4}\right)$, state which pairs of functions are equal.

7 Sketch on separate axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of:

  a $y = \sec 2\theta$ **b** $y = -\operatorname{cosec}\theta$ **c** $y = 1 + \sec\theta$

  d $y = \operatorname{cosec}(\theta - 30°)$ **e** $y = 2\sec(\theta - 60°)$ **f** $y = \operatorname{cosec}(2\theta + 60°)$

  g $y = -\cot(2\theta)$ **h** $y = 1 - 2\sec\theta$

In each case show the coordinates of any maximum and minimum points, and of any points at which the curve meets the axes.

8 Write down the periods of the following functions. Give your answers in terms of π.

  a $\sec 3\theta$ **b** $\operatorname{cosec}\tfrac{1}{2}\theta$ **c** $2\cot\theta$ **d** $\sec(-\theta)$
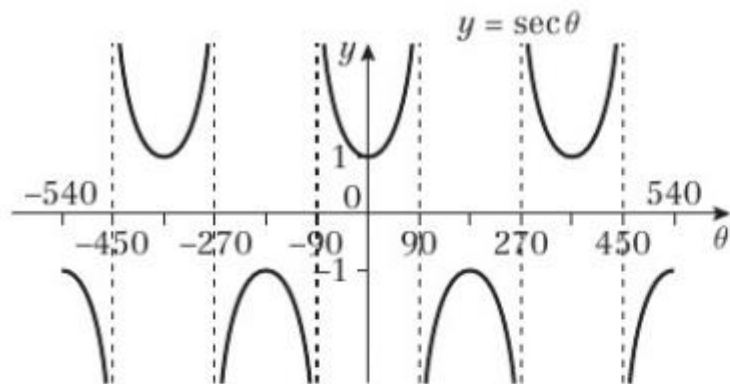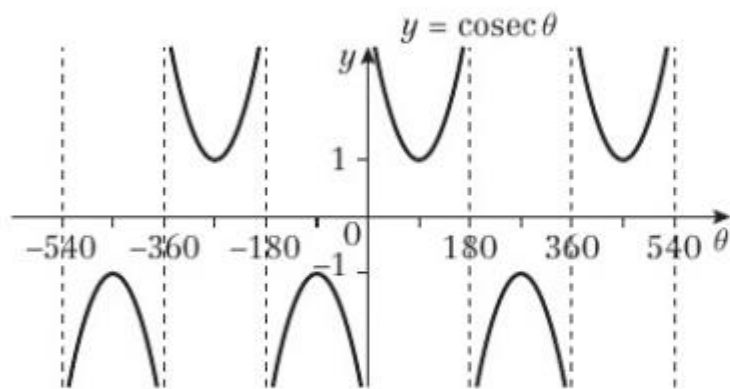
9 a Sketch, in the interval $-2\pi \leqslant x \leqslant 2\pi$, the graph of $y = 3 + 5 \operatorname{cosec} x$. **(3 marks)**

  b Hence deduce the range of values of k for which the equation $3 + 5 \operatorname{cosec} x = k$ has no solutions. **(2 marks)**

10 a Sketch the graph of $y = 1 + 2 \sec \theta$ in the interval $-\pi \leqslant \theta \leqslant 2\pi$. **(3 marks)**

  b Write down the θ-coordinates of points at which the gradient is zero. **(2 marks)**

  c Deduce the maximum and minimum values of $\dfrac{1}{1 + 2 \sec \theta}$, and give the smallest positive values of θ at which they occur. **(4 marks)**
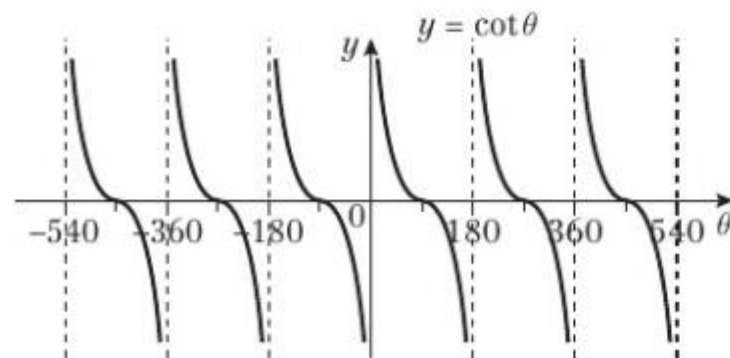
1 a

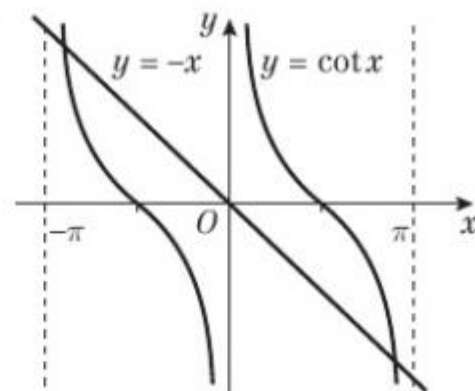
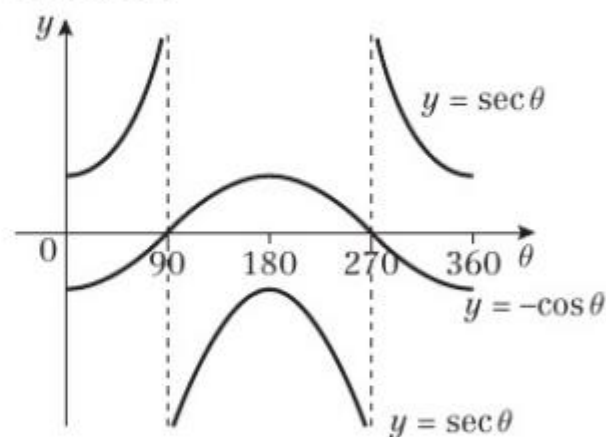$y = \sec\theta$

b

$y = \operatorname{cosec}\theta$

c

$y = \cot\theta$

2 a

$y = -x$ $y = \cot x$

b 2 solutions

3 a

$y = \sec\theta$

$y = -\cos\theta$

$y = \sec\theta$
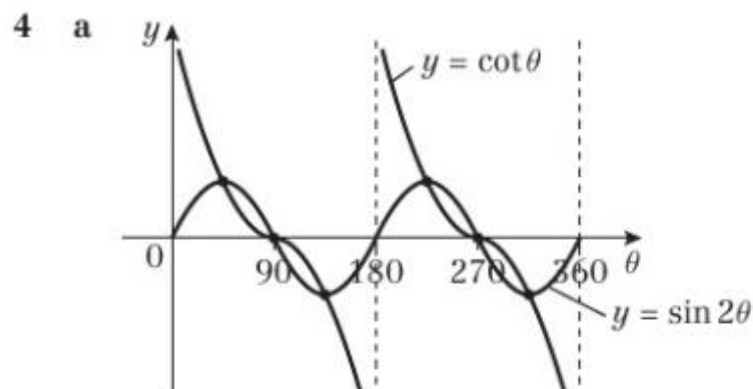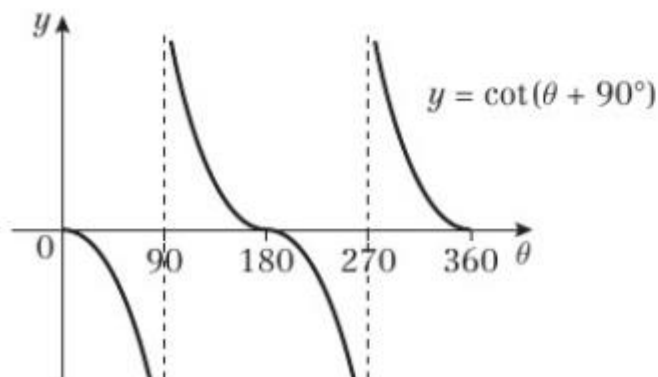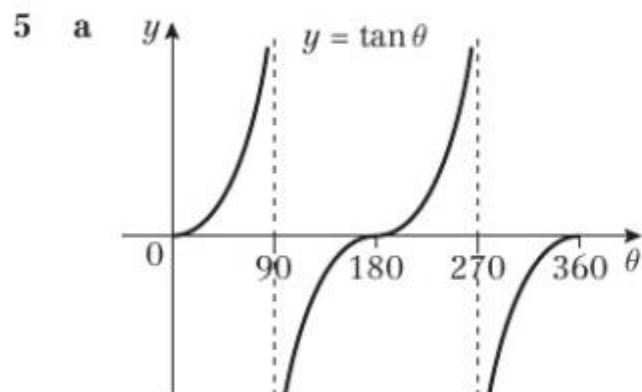
b The solutions of $\sec\theta = -\cos\theta$ are the θ values of the points of intersection of $y = \sec\theta$ and $y = -\cos\theta$. As they do not meet, there are no solutions.

4 a

Graph showing $y = \cot\theta$ and $y = \sin 2\theta$ with axis marked at 90, 180, 270, 360.

b 6

5 a

Graph showing $y = \tan\theta$ with axis marked at 90, 180, 270, 360.

Graph showing $y = \cot(\theta + 90°)$ with axis marked at 90, 180, 270, 360.

5 b $\cot(90° + \theta) = -\tan\theta$

6 a i The graph of $y = \tan\left(\theta + \frac{\pi}{2}\right)$ is the same as that of $y = \tan\theta$ translated by $\frac{\pi}{2}$ to the left.
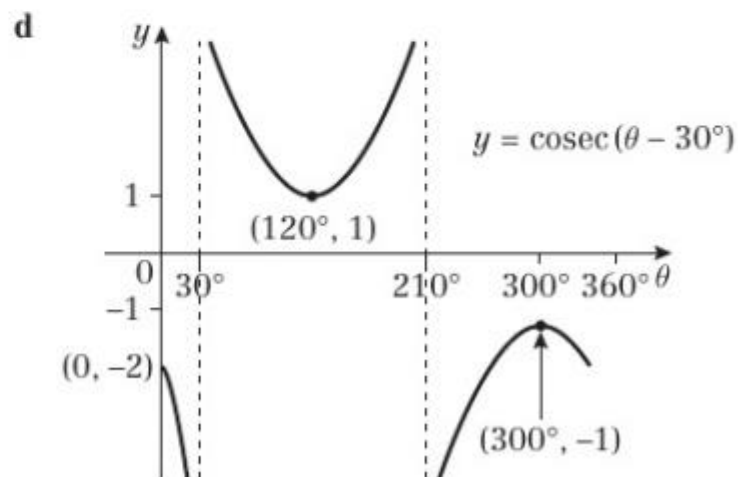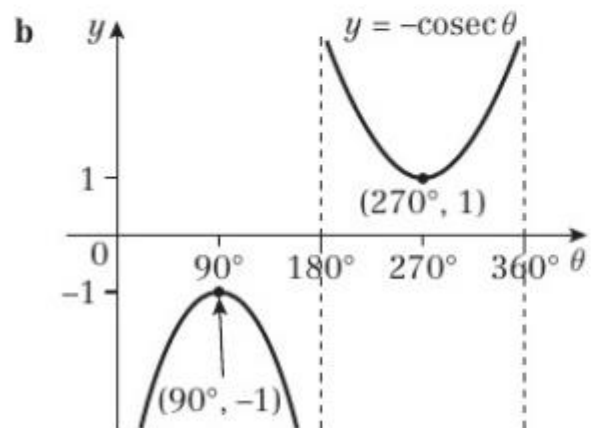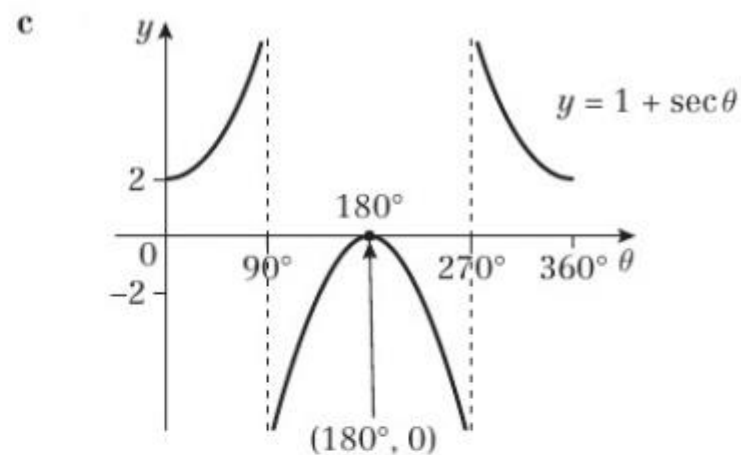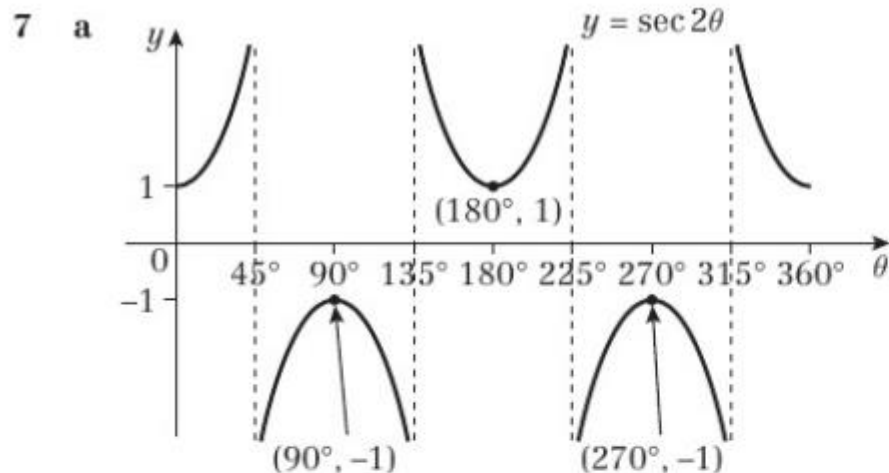
ii The graph of $y = \cot(-\theta)$ is the same as that of $y = \cot\theta$ reflected in the y-axis.

iii The graph of $y = \operatorname{cosec}\left(\theta + \frac{\pi}{4}\right)$ is the same as that of $y = \operatorname{cosec}\theta$ translated by $\frac{\pi}{4}$ to the left.
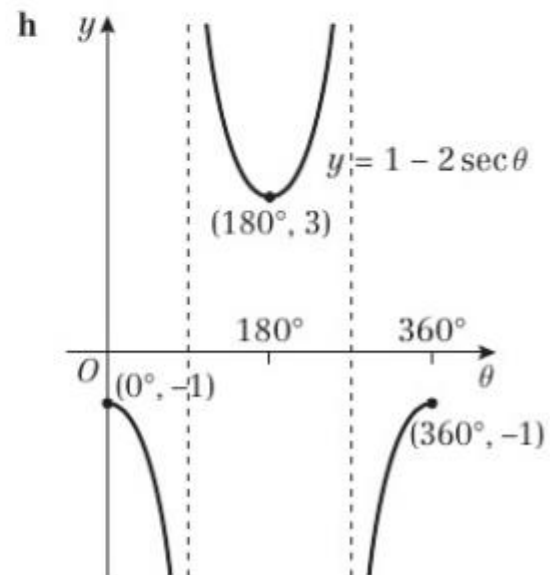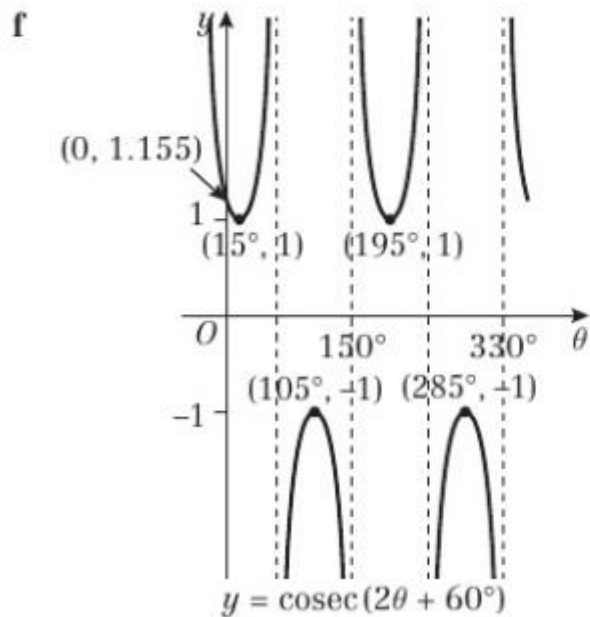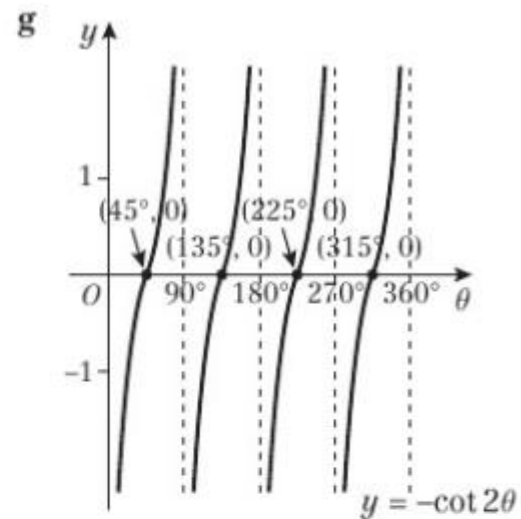
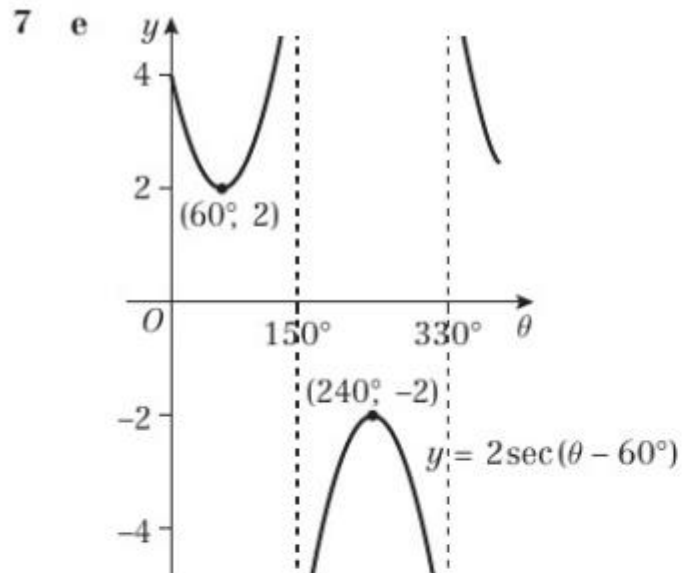**iv** The graph of $y = \sec\left(\theta - \frac{\pi}{4}\right)$ is the same as that of $y = \sec\theta$ translated by $\frac{\pi}{4}$ to the right.

b $\tan\left(\theta + \frac{\pi}{2}\right) = \cot(-\theta)$; $\operatorname{cosec}\left(\theta + \frac{\pi}{4}\right) = \sec\left(\theta - \frac{\pi}{4}\right)$

7 a $y = \sec 2\theta$
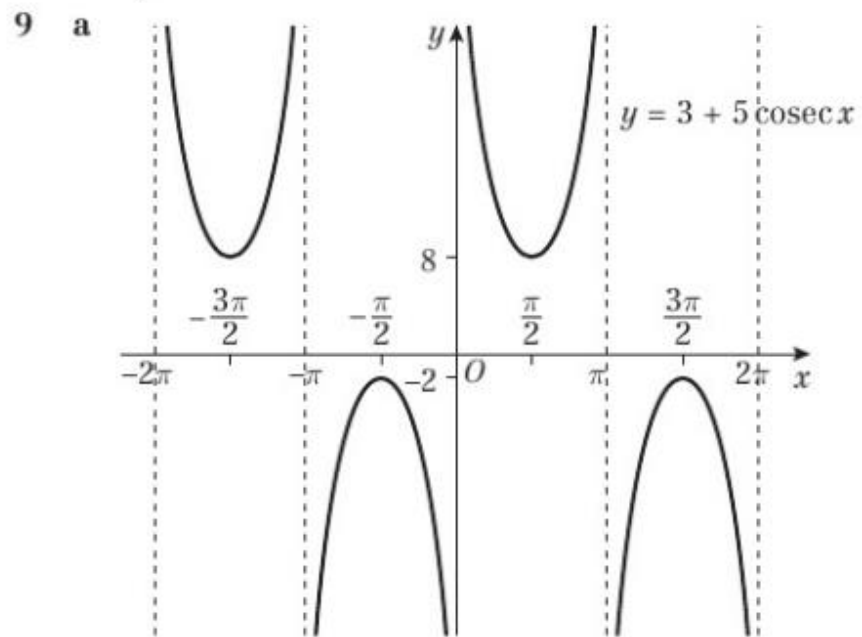
$(180°, 1)$
$(90°, -1)$
$(270°, -1)$

b $y = -\mathrm{cosec}\,\theta$

$(270°, 1)$
$(90°, -1)$

c $y = 1 + \sec\theta$

$180°$
$(180°, 0)$

d $y = \mathrm{cosec}\,(\theta - 30°)$

$(120°, 1)$
$(0, -2)$
$(300°, -1)$

7 e

$y = 2\sec(\theta - 60°)$

(60°, 2)
(240°, −2)

g

$y = -\cot 2\theta$

(45°, 0) (225°, 0)
(135°, 0) (315°, 0)

f

(0, 1.155)
(15°, 1) (195°, 1)
(105°, −1) (285°, −1)
$y = \csc(2\theta + 60°)$

h

$y = 1 - 2\sec\theta$
(180°, 3)
(0°, −1)
(360°, −1)

8   **a** $\dfrac{2\pi}{3}$ **b** 4π **c** π **d** 2π

9   **a**

$y = 3 + 5\operatorname{cosec} x$

b $-2 < k < 8$

10   **a**

$y = 1 + 2\sec\theta$
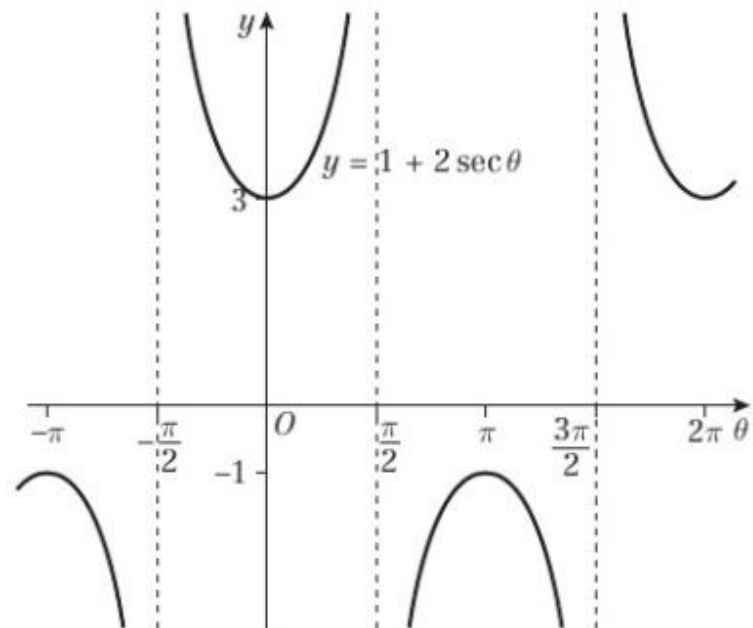
b $\theta = -\pi,\, 0,\, \pi,\, 2\pi$

c   Max $= \frac{1}{3}$, first occurs at $\theta = 2\pi$

    Min $= -1$, first occurs at $\theta = \pi$