P1 Chapter 1: Algebra

Index Laws

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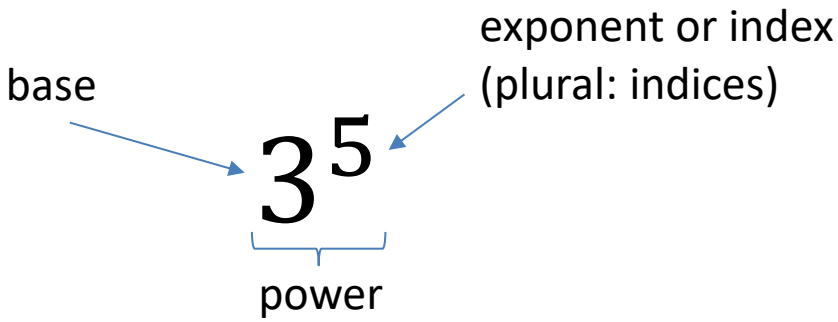

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$(ab)^n = a^n b^n$$

Some also refer to the 5 as the 'power'. But this would lead to unfortunate ambiguous phrases like "when we multiply two powers together, we add the powers" instead of "when we multiply two powers together, we add the indices".

Simplify $(a^3)^2 \times 2a^2$

$$= a^6 \times 2a^2 = 2a^8$$

Simplify $(4x^3y)^3$

$$= 64x^9y^3$$

Simplify $2x^2(3 + 5x) - x(4 - x^2)$

$$= 6x^2 + 10x^3 - 4x + x^3$$
$$= 11x^3 + 6x^2 - 4x$$

Tip: A common student error is to get the sign wrong of $+x^3$

Simplify $\frac{x^3-2x}{3x^2}$

$$= \frac{x^3}{3x^2} - \frac{2x}{3x^2}$$
$$= \frac{1}{3}x - \frac{2}{3x}$$

Tip: While $\frac{a+b}{c}$ can be split into $\frac{a}{c} + \frac{b}{c}$, a common student error is to think that $\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$

Test Your Understanding

1 Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

?

2 Simplify $\frac{2x+x^5}{4x^3}$

?

3 Expand and simplify

$$2x(3-x^2)-4x^3(3-x)$$

?

4 Simplify $2^x \times 3^x$

?

Test Your Understanding

1 Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

$= (2a^3)^2 \times 3a$
$= 4a^6 \times 3a$
$= 12a^7$

2 Simplify $\frac{2x+x^5}{4x^3}$

$= \frac{1}{2}x^{-2} + \frac{1}{4}x^2$

or $\frac{1}{2x^2} + \frac{x^2}{4}$

3 Expand and simplify

$2x(3-x^2) - 4x^3(3-x)$

$= 6x - 2x^3 - 12x^3 + 4x^4$
$= 4x^4 - 14x^3 + 6x$

4 Simplify $2^x \times 3^x$

$= 6^x$

Fro Note: This is using $(ab)^n = a^n b^n$ law backwards.

Exercise 1.1

Pearson Pure Mathematics Year 1/AS
Pages 1

1 *[MAT 2006 1A]*

Which of the following numbers is largest?

- $\left(\left(2^3\right)^2\right)^3$
- $\left(2^3\right)^{\left(2^3\right)}$
- $2^{\left(\left(3^2\right)^3\right)}$
- $2^{\left(3^{\left(2^3\right)}\right)}$

?

2 *[MAT 2012 1B]*

Let $N = 2^k \times 4^m \times 8^n$ where k, m, n are positive whole numbers.

Then N will definitely be a square number whenever:

- k is even;
- $k + n$ is odd;
- k is odd but $m + n$ is even;
- $k + n$ is even.

?

Exercise 1.1

Pearson Pure Mathematics Year 1/AS
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1 *[MAT 2006 1A]*

Which of the following numbers is largest?

- $\left((2^3)^2\right)^3$
- $(2^3)^{(2^3)}$
- $2^{\left((3^2)^3\right)}$
- $2^{\left(3^{(2^3)}\right)}$

Solution: $2^{\left(3^{(2^3)}\right)}$

2 *[MAT 2012 1B]*

Let $N = 2^k \times 4^m \times 8^n$ where k, m, n are positive whole numbers.

Then N will definitely be a square number whenever:

- k is even;
- $k + n$ is odd;
- k is odd but $m + n$ is even;
- $k + n$ is even.

$$N = 2^k \times 2^{2m} \times 2^{3n}$$
$$= 2^{k+2m+3n}$$

This is square if:

$k + 2m + 3n$ **is even**

$\Rightarrow k + 3n$ **is even**

$\Rightarrow k + n$ **is even**

Homework Exercise

1 Simplify these expressions:

a $x^3 \times x^4$ **b** $2x^3 \times 3x^2$ **c** $\frac{k^3}{k^2}$

d $\frac{4p^3}{2p}$ **e** $\frac{3x^3}{3x^2}$ **f** $(y^2)^5$

g $10x^5 \div 2x^3$ **h** $(p^3)^2 \div p^4$ **i** $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$ **k** $2a^4 \times 3a^5$ **l** $\frac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$ **n** $3x^3 \times 2x^2 \times 4x^6$ **o** $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$ **q** $2a^3 \div 3a^2 \times 6a^5$ **r** $3a^4 \times 2a^5 \times a^3$

2 Expand and simplify if possible:

a $9(x - 2)$ **b** $x(x + 9)$ **c** $-3y(4 - 3y)$

d $x(y + 5)$ **e** $-x(3x + 5)$ **f** $-5x(4x + 1)$

g $(4x + 5)x$ **h** $-3y(5 - 2y^2)$ **i** $-2x(5x - 4)$

j $(3x - 5)x^2$ **k** $3(x + 2) + (x - 7)$ **l** $5x - 6 - (3x - 2)$

m $4(c + 3d^2) - 3(2c + d^2)$ **n** $(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$

o $x(3x^2 - 2x + 5)$ **p** $7y^2(2 - 5y + 3y^2)$ **q** $-2y^2(5 - 7y + 3y^2)$

r $7(x - 2) + 3(x + 4) - 6(x - 2)$ **s** $5x - 3(4 - 2x) + 6$

t $3x^2 - x(3 - 4x) + 7$ **u** $4x(x + 3) - 2x(3x - 7)$ **v** $3x^2(2x + 1) - 5x^2(3x - 4)$

3 Simplify these fractions:

a $\frac{6x^4 + 10x^6}{2x}$ **b** $\frac{3x^5 - x^7}{x}$ **c** $\frac{2x^4 - 4x^2}{4x}$

d $\frac{8x^3 + 5x}{2x}$ **e** $\frac{7x^7 + 5x^2}{5x}$ **f** $\frac{9x^5 - 5x^3}{3x}$

Homework Answers

1

a x^7	**b** $6x^5$	**c** k	**d** $2p^2$
e x	**f** y^{10}	**g** $5x^2$	**h** p^2
i $2a^3$	**j** $2p$	**k** $6a^9$	**l** $3a^2b^3$
m $27x^8$	**n** $24x^{11}$	**o** $63a^{12}$	**p** $32y^6$
q $4a^6$	**r** $6a^{12}$		

2

a $9x - 18$		**b** $x^2 + 9x$	
c $-12y + 9y^2$		**d** $xy + 5x$	
e $-3x^2 - 5x$		**f** $-20x^2 - 5x$	
g $4x^2 + 5x$		**h** $-15y + 6y^3$	
i $-10x^2 + 8x$		**j** $3x^3 - 5x^2$	
k $4x - 1$		**l** $2x - 4$	
m $9d^2 - 2c$		**n** $13 - r^2$	
o $3x^3 - 2x^2 + 5x$		**p** $14y^2 - 35y^3 + 21y^4$	
q $-10y^2 + 14y^3 - 6y^4$		**r** $4x + 10$	
s $11x - 6$		**t** $7x^2 - 3x + 7$	
u $-2x^2 + 26x$		**v** $-9x^3 + 23x^2$	

3

a $3x^3 + 5x^5$	**b** $3x^4 - x^6$	**c** $\frac{x^3}{2} - x$
d $4x^2 + \frac{5}{2}$	**e** $\frac{7x^6}{5} + x$	**f** $3x^4 - \frac{5x^2}{3}$