Stats1 Chapter 7: Hypothesis Testing

Chapter Practice

Key Points

Summary of key points

1 The event A and B can be written as $A \cap B$. The '$\cap$' symbol is the symbol for **intersection**.

The event A or B can be written as $A \cup B$. The '$\cup$' symbol is the symbol for **union**.

The event not A can be written as A'. This is also called the **complement** of A.

2 The probability that B occurs given that A has already occurred is written as $P(B|A)$.

For independent events, $P(A|B) = P(A|B') = P(A)$, and $P(B|A) = P(B|A') = P(B)$.

3 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

4 $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$ so $(B \cap A) = P(B|A) \times P(A)$

1 A and B are two events such that $P(A) = 0.4$ and $P(B) = 0.35$. If $P(A \cap B) = 0.2$, find:

   a $P(A \cup B)$ **(1 mark)**

   b $P(A' \cap B')$ **(1 mark)**

   c $P(B|A)$ **(2 marks)**

   d $P(A'|B)$ **(2 marks)**

2 J, K and L are three events such that $P(J) = 0.25$, $P(K) = 0.45$ and $P(L) = 0.15$. Given that K and L are independent, J and L are mutually exclusive and $P(J \cap K) = 0.1$

   a draw a Venn diagram to illustrate this situation.   **(2 marks)**

   b Find:

      i $P(J \cup K)$ **(1 mark)**

      ii $P(J' \cap L')$ **(1 mark)**

      iii $P(J|K)$ **(2 marks)**

      iv $P(K|J' \cap L')$ **(2 marks)**

3 Of 60 students in a high-school sixth form, 35 study French and 45 study Spanish. If 27 students study both, find the probability that a student chosen at random:

   a studies only one subject   **(1 mark)**

   b studies French given that they study Spanish   **(2 marks)**

   c studies Spanish given that they do not study French.   **(2 marks)**

   It is found that 75% of the students who study just French wear glasses and half of the students who study just Spanish wear glasses. Find the probability that a student chosen at random:

   d studies one language and wears glasses   **(2 marks)**

   e wears glasses given that they study one language.   **(2 marks)**

4 A bag contains 6 red balls and 9 green balls. A ball is chosen at random from that bag, its colour noted and the ball placed to one side. A second ball is chosen at random and its colour noted.

  a Draw a tree diagram to illustrate this situation. **(2 marks)**

  b Find the probability that:

    i both balls are green **(1 mark)**

    ii the balls are different colours. **(2 marks)**

  Further balls are drawn from the bag and not replaced. Find the probability that:

  c the third ball is red **(2 marks)**

  d it takes just four selections to get four green balls. **(2 marks)**

5 In a tennis match, the probability that Anne wins the first set against Colin is 0.7. If Anne wins the first set, the probability that she wins the second set is 0.8. If Anne loses the first set, the probability that she wins the second set is 0.4. A match is won when one player wins two sets.

  a Find the probability that the game is over after two sets. **(2 marks)**

  b Find the probability that Anne wins given that the game is over after two sets. **(2 marks)**

  If the game is tied at one set all, a tiebreaker is played and the probability of Anne winning it is 0.55.

  c Find the probability of Anne winning the entire match. **(3 marks)**

6 The colours of the paws of 75 kittens are recorded. 26 kittens have all black paws and 14 kittens have all white paws. 15 have a combination of black and white paws. One kitten is chosen at random. Find the probability that the kitten has:

  a neither white nor black paws **(1 mark)**

  b a combination of black and white paws given that they have some black paws. **(2 marks)**

  Two kittens are now chosen. Find the probability that:

  c both kittens have all black paws **(2 marks)**

  d both kittens have some white paws. **(2 marks)**

7 Two events A and B are such that $P(A) = 0.4$ and $P(A \cap B) = 0.12$. If A and B are independent, find:

a $P(B)$ **(1 mark)**

b $P(A' \cap B')$ **(1 mark)**

A third event C has $P(C) = 0.4$. Given that A and C are mutually exclusive and $P(B \cap C) = 0.1$,

c draw a Venn diagram to illustrate this situation.   **(2 marks)**

d Find:

  i $P(B|C)$ **(2 marks)**

  ii $P(A \cap (B' \cup C))$ **(2 marks)**

8 In a football match, the probability that team A scores first is 0.6, and the probability that team B scores first is 0.35.

a Suggest a reason why these probabilities do not add up to 1.   **(1 mark)**

The probability that team A scores first and wins the match is 0.48.

b Find the probability that team A scores first and does not win the match.   **(3 marks)**

If team B scores first, the probability that team A will win the match is 0.3.

c Given that team A won the match, find the probability that they did not score first.   **(3 marks)**
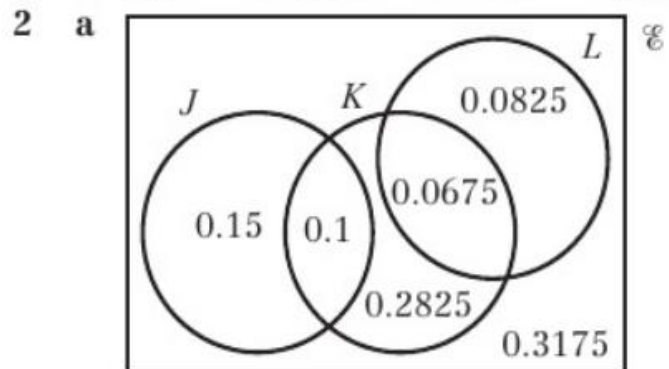
Challenge

$P(A) = 0.6$ and $P(B) = 0.2$

a Given that $P(A \cap B') = p$, find the range of possible values of p.
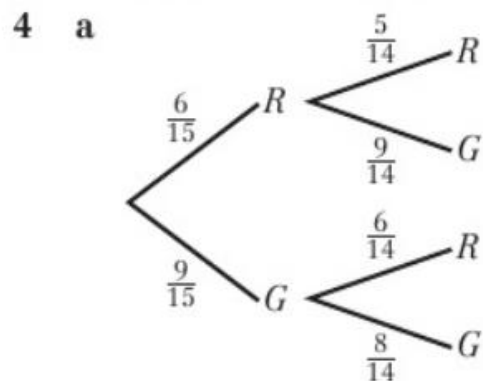
$P(C) = 0.7$ and $P(A \cap B \cap C) = 0.1$

b Given $P(A \cap B' \cap C) = q$, find the range of possible values of q.

1 **a** 0.55 **b** 0.45 **c** 0.5 **d** 0.429 (3 s.f.)

2 **a**

b **i** 0.6 **ii** 0.6 **iii** 0.222 (3 s.f.) **iv** 0.471 (3 s.f.)

3 **a** 0.433 (3 s.f.) **b** 0.6 **c** 0.72

d 0.25 **e** 0.577 (3 s.f.)

4 **a**

b **i** $\frac{12}{35}$ **ii** $\frac{18}{35}$

c $\frac{2}{5}$ **d** $\frac{6}{65}$

5 **a** 0.74 **b** 0.757 (3 s.f.) **c** 0.703

6 **a** $\frac{4}{15}$ **b** $\frac{15}{41}$

c 0.117 (3 s.f.) **d** 0.146 (3 s.f.)

7 **a** 0.3 **b** 0.42

c
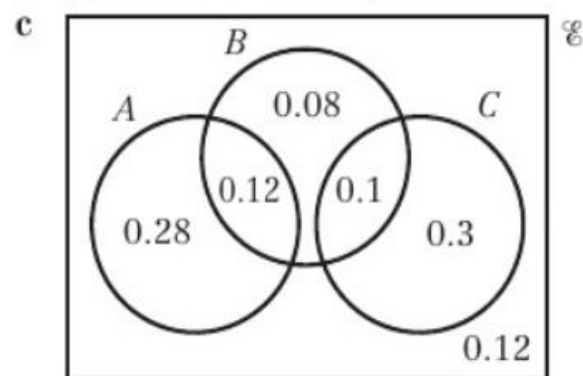
d **i** 0.25 **ii** 0.28

8 **a** In some football matches, neither team scores.

b 0.12 **c** 0.179 (3 s.f.)

Challenge

a $0.4 \leqslant p \leqslant 0.6$ **b** $0.2 \leqslant q \leqslant 0.5$