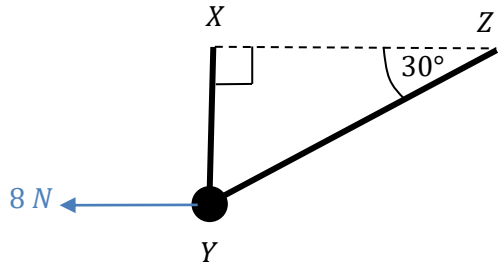
M2 Chapter 7: Application of Forces

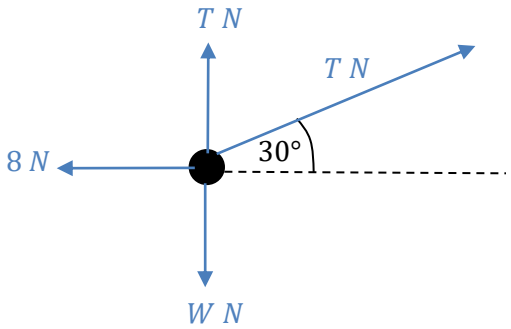
Modelling with Statics

Modelling with Statics

We can apply this to problems involving tension, weight and pulleys.

[Textbook] A smooth bead Y is threaded on a light inextensible string. The ends of the string are attached to two fixed points, X and Y, on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 8 N acting parallel to ZX. The bead Y is vertically below X and $\angle XZY = 30°$ as shown in the diagram.
Find the tension in the string and the weight of the bead.





As the bead is smooth, the two parts of the string can be considered as a single piece of string, and therefore the tension is the same throughout.

$$R(\rightarrow): \quad T\cos 30° = 8 \quad \therefore \quad T = 9.24 \text{ (3sf)}$$
$$R(\uparrow): \quad W = T + T\sin 30°$$
$$= 13.9 \text{ (3sf)}$$

Further Example

[Textbook] A mass of 3kg rests on the surface of a smooth plane which is inclined at an angle of $45°$ to the horizontal. The mass is attached to a cable which passes up the plane along the line of greatest slope and then passes over a smooth pulley at the top of the plane. The cable carries a mass of 1kg freely suspended at the other end. The masses are modelled as particles, and the cable as a light inextensible string. There is a force of P N acting horizontally on the 3kg mass and the system is in equilibrium.

Calculate (a) the magnitude of P (b) the normal reaction between the mass and the plane (c) State how you have used the assumption that the pulley is smooth in your calculations.

? Force Diagram

a

For 1kg mass:

$R(\uparrow)$: ?

For 3kg mass:

$R(\nearrow)$: ?

b

$R(\nwarrow)$: ?

c

?

Further Example

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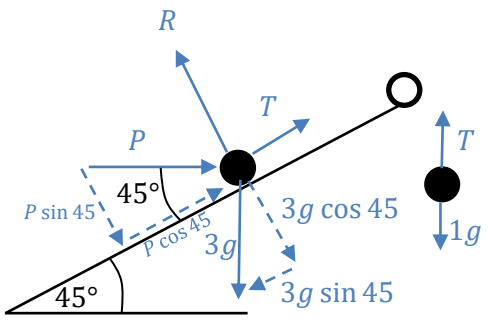

a For 1kg mass:

$R(\uparrow)$: $T = g$

For 3kg mass:

$R(\nearrow)$: $T + P\cos 45° = 3g\sin 45°$

$$\therefore P = \frac{3g\sin 45° - g}{\cos 45°} = 3g - g\sqrt{2} = 16 \text{ (2sf)}$$

b $R(\nwarrow)$: $R = 3g\cos 45° + P\sin 45°$

$$= 6g\frac{\sqrt{2}}{2} - g = 32 \text{ (2sf)}$$

c Pulley is smooth so tension in string will be same on both sides of the pulley.

Test Your Understanding

Edexcel M1(Old) May 2013(R) Q2

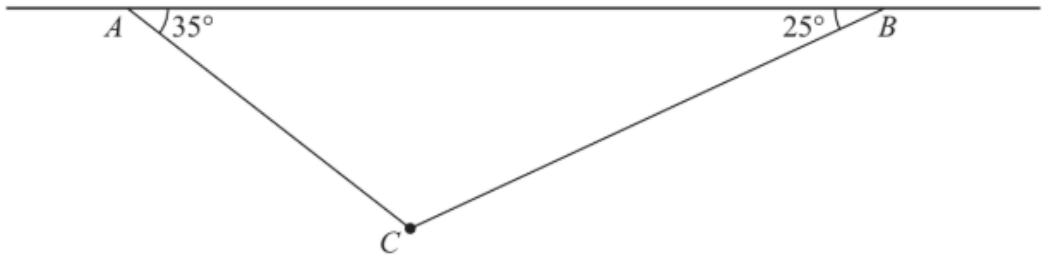

Figure 1

A particle of weight 8 N is attached at C to the ends of two light inextensible strings AC and BC. The other ends, A and B, are attached to a fixed horizontal ceiling. The particle hangs at rest in equilibrium, with the strings in a vertical plane. The string AC is inclined at 35° to the horizontal and the string BC is inclined at 25° to the horizontal, as shown in Figure 1. Find

(i) the tension in the string AC,

(ii) the tension in the string BC.

The particle can't move along the string, so we have two separate strings with separate tensions. Introduce suitable variables for the tensions of each, e.g. T_1 and T_2.

?

Test Your Understanding

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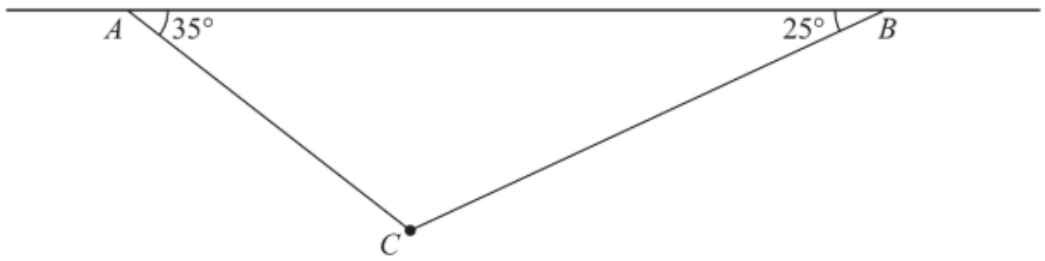

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The particle can't move along the string, so we have two separate strings with separate tensions. Introduce suitable variables for the tensions of each, e.g. T_1 and T_2.

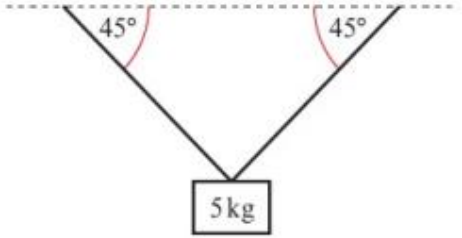
Resolve horizontally: $T_A \cos 35° = T_B \cos 25°$	M1A1
Resolve vertically: $T_A \sin 35° + T_B \sin 25° = 8$	M1A1
Equation in one unknown: $T_B \dfrac{\cos 25°}{\cos 35°} \sin 35° + T_B \sin 25° = 8$ or $T_A \sin 35° + T_A \dfrac{\cos 35°}{\cos 25°} \sin 25° = 8$	**DM1**A1
$T_A = 8.4, 8.37, 8.372$ (N) or better	A1
$T_B = 7.6, 7.57, 7.567$ (N) or better	A1

Exercise 7.2

Pearson Stats/Mechanics Year 2
Pages 57-58

Homework Exercise

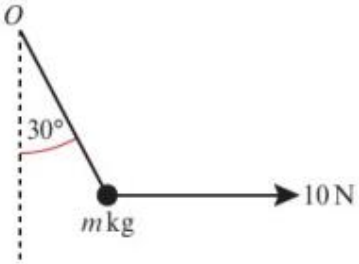
1 A picture of mass 5 kg is suspended by two light inextensible strings, each inclined at 45° to the horizontal as shown. By modelling the picture as a particle find the tension in the strings when the system is in equilibrium.



Problem-solving

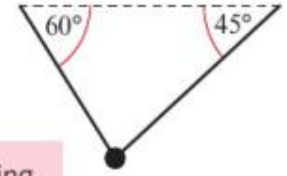
This is a three-force problem involving an object in static equilibrium, so you could use a triangle of forces.

2 A particle of mass m kg is suspended by a single light inextensible string. The string is inclined at an angle of 30° to the vertical and the other end of the string is attached to a fixed point O. Equilibrium is maintained by a horizontal force of magnitude 10 N which acts on the particle, as shown in the diagram. Find:
a the tension in the string **b** the value of m.



3 A particle of weight 12 N is suspended by a light inextensible string from a fixed point O. A horizontal force of 8 N is applied to the particle and the particle remains in equilibrium with the string at an angle θ to the vertical. Find:
a the angle θ **b** the tension in the string.

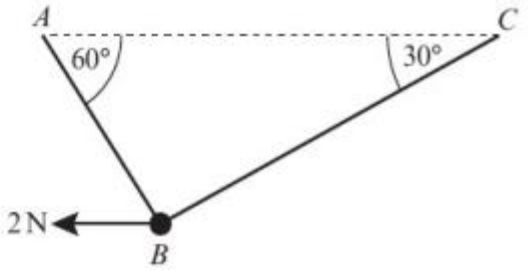
4 A particle of mass 6 kg hangs in equilibrium, suspended by two light inextensible strings, inclined at 60° and 45° to the horizontal, as shown. Find the tension in each of the strings.



Hint The particle is attached **separately** to each string, so the tension in the two strings can be different.

Homework Exercise

5 A smooth bead B is threaded on a light inextensible string. The ends of the string are attached to two fixed points, A and C, on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 2 N acting parallel to CA. The sections of string make angles of 60° and 30° with the horizontal. Find:



a the tension in the string **(3 marks)**

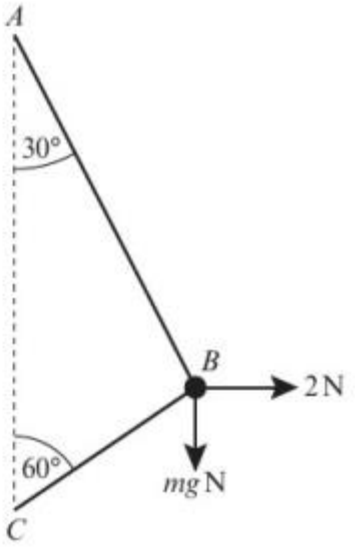
b the mass of the bead. **(4 marks)**

c State how you have used the modelling assumption that the bead is smooth in your calculations. **(1 mark)**

6 A smooth bead B is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C where A is vertically above C. The bead is held in equilibrium by a horizontal force of magnitude 2 N. The sections AB and BC of the string make angles of 30° and 60° with the vertical respectively. Find:

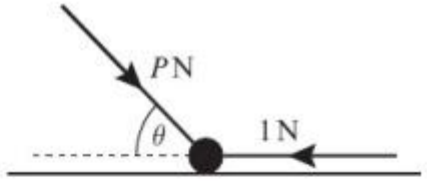
a the tension in the string **(3 marks)**

b the mass of the bead, giving your answer to the nearest gram. **(4 marks)**



Homework Exercise

7 A particle of weight 2 N rests on a smooth horizontal surface and remains in equilibrium under the action of the two external forces shown in the diagram. One is a horizontal force of magnitude 1 N and the other is a force of magnitude P N which acts at an angle θ to the horizontal, where $\tan\theta = \frac{12}{5}$. Find:



a the value of P **(3 marks)**

b the normal reaction between the particle and the surface. **(2 marks)**

8 A particle A of mass m kg rests on a smooth horizontal table. The particle is attached by a light inextensible string to another particle B of mass $2m$ kg, which hangs over the edge of the table. The string passes over a smooth pulley, which is fixed at the edge of the table so that the string is horizontal between A and the pulley and then is vertical between the pulley and B.
A horizontal force F N applied to A maintains equilibrium. The normal reaction between A and the table is R N.

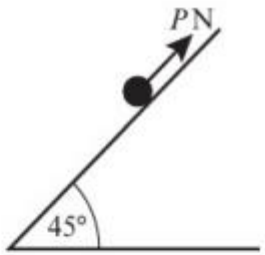
a Find the values of F and R in terms of m.

The pulley is now raised to a position above the edge of the table so that the string is inclined at 30° to the horizontal between A and the pulley. The string still hangs vertically between the pulley and B. A horizontal force F' N applied to A maintains equilibrium in this new situation. The normal reaction between A and the table is now R' N.

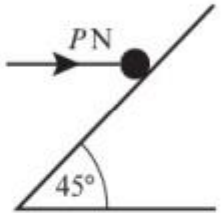
b Find, in terms of m, the values of F' and R'.

Homework Exercise

9 A particle of mass 2 kg rests on a smooth inclined plane, which makes an angle of 45° with the horizontal. The particle is maintained in equilibrium by a force P N acting up the line of greatest slope of the inclined plane, as shown in the diagram. Find the value of P.

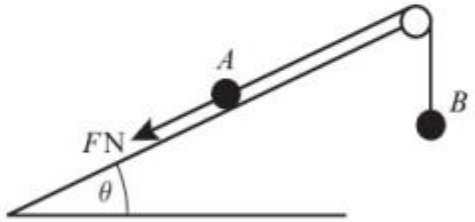


10 A particle of mass 4 kg is held in equilibrium on a smooth plane which is inclined at 45° to the horizontal by a horizontal force of magnitude P N, as shown in the diagram. Find the value of P.



11 A particle A of mass 2 kg rests in equilibrium on a smooth inclined plane. The plane makes an angle θ with the horizontal, where $\tan\theta = \frac{3}{4}$.

The particle is attached to one end of a light inextensible string which passes over a smooth pulley, as shown in the diagram. The other end of the string is attached to a particle B of mass 5 kg. Particle A is also acted upon by a force of magnitude F N down the plane, along a line of greatest slope.



Find:

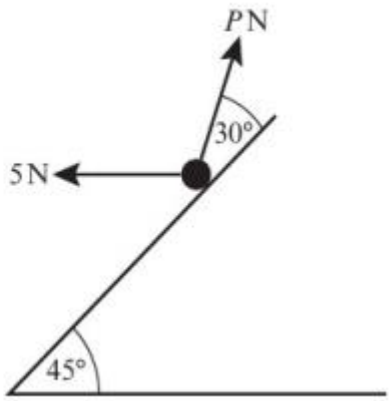
a the magnitude of the normal reaction between A and the plane **(5 marks)**

b the value of F. **(3 marks)**

c State how you have used the fact that the pulley is smooth in your calculations. **(1 mark)**

Homework Exercise

12 A particle of weight 20 N rests in equilibrium on a smooth inclined plane. It is maintained in equilibrium by the application of two external forces as shown in the diagram. One of the forces is a horizontal force of 5 N, the other is a force P N acting at an angle of 30° to the plane, as shown in the diagram. Find the magnitude of the normal reaction between the particle and the plane. **(8 marks)**



Homework Answers

1 35 N (2 s.f.)

2 **a** 20 N **b** 1.77

3 **a** 33.7° **b** 14.4

4 30 N and 43 N (2 s.f.)

5 **a** 5.46 N **b** 0.76 kg

c Assumption that there is no friction between the string and the bead.

6 **a** 1.46 N **b** 55 g

7 **a** 2.6 **b** 4.4 N

8 **a** $F = 19.6m$, $R = 9.8m$ **b** $F' = 17m$ (2 s.f.), $R' = 0$

9 13.9 N

10 39.2 N

11 **a** 15.7 N (3 s.f.)

b 37.2 N (3 s.f.)

c Assumption that there is no friction between the string and the pulley.

12 $R = 0.40$ N (2 s.f.)