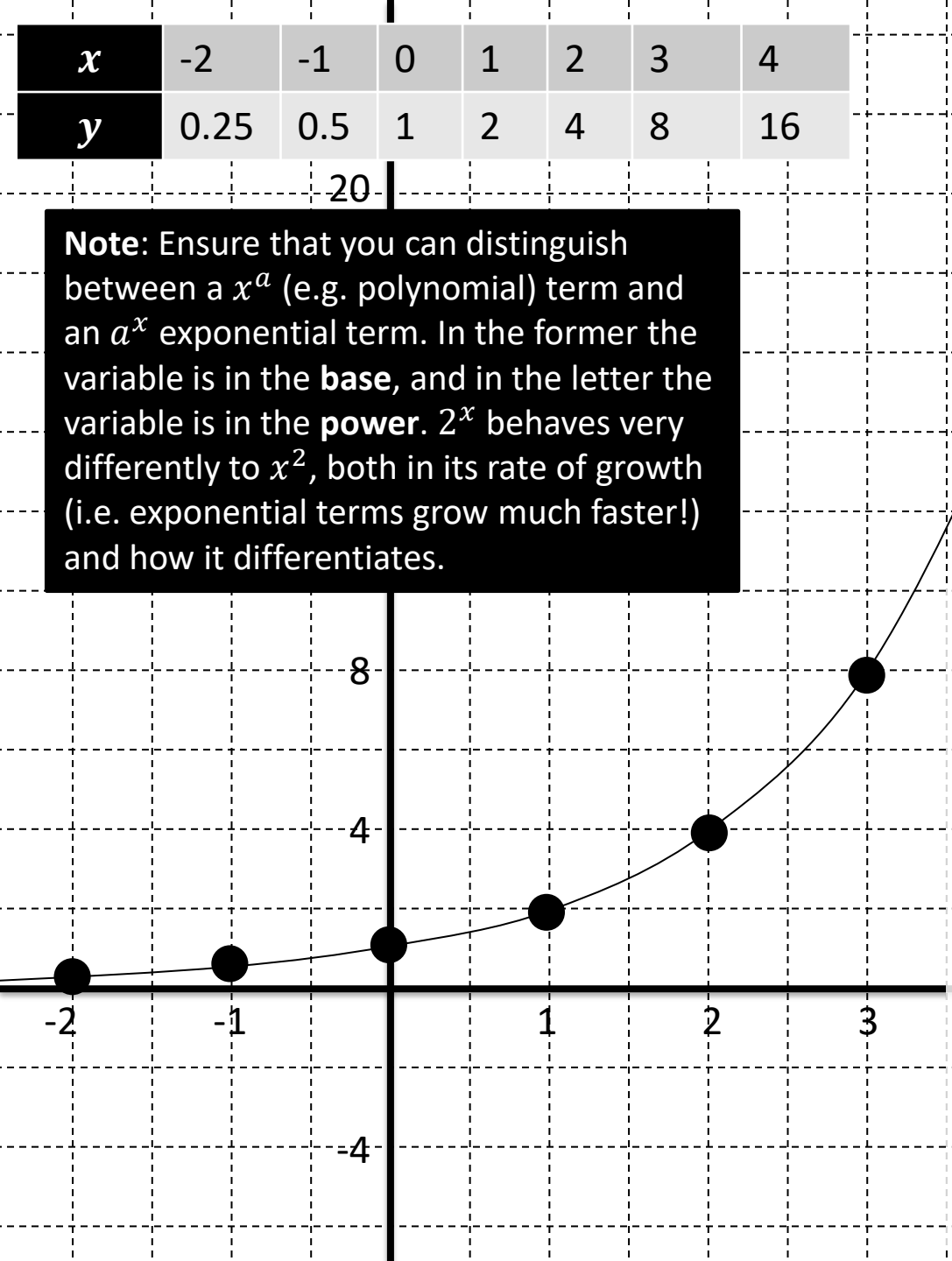
P1 Chapter 14: Logarithms

Exponential Functions

$y = 2^x$

x	-2	-1	0	1	2	3	4
y	0.25	0.5	1	2	4	8	16



Note: Ensure that you can distinguish between a x^a (e.g. polynomial) term and an a^x exponential term. In the former the variable is in the **base**, and in the letter the variable is in the **power**. 2^x behaves very differently to x^2, both in its rate of growth (i.e. exponential terms grow much faster!) and how it differentiates.

Why are exponential functions important?

Each of the common graphs have a **key property** that makes them **useful for modelling**.

For reciprocal graphs $y = \frac{k}{x}$, as x doubles, y halves. This means we'd use it to represent variables which are inversely proportional.

Linear graphs are used when we're **adding the same amount** each time.
In contrast, exponential graphs are used when we're **multiplying by the same amount** each time. For example, we might use $1000(1.05^t)$ to model our savings with interest, where each year we have 1.05 times as much, i.e. with 5% added interest.

Contrasting exponential graphs

On the same axes sketch $y = 3^x, y = 2^x, y = 1.5^x$

?

On the same axes sketch $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$

?

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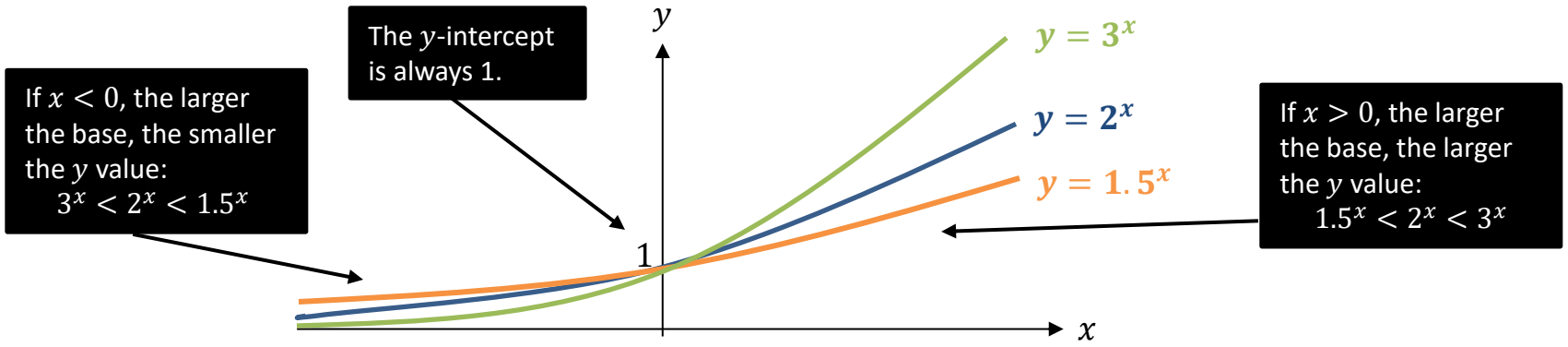
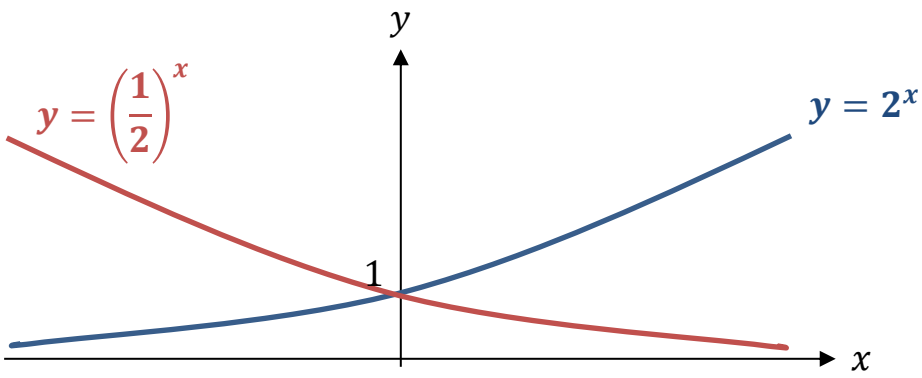


On the same axes sketch $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$



Three important notes:

- $y = 2^x$ is said to be "**exponential growing**" whereas $y = \left(\frac{1}{2}\right)^x$ is said to be "**exponentially decaying**", because it's getting smaller (halving) each time x increases by 1.
- $y = \left(\frac{1}{2}\right)^x$ is a reflection of $y = 2^x$ in the line $x = 0$.
  Proof:
  If $f(x) = 2^x, f(-x) = 2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x$
- $\left(\frac{1}{2}\right)^x$ would usually be written 2^{-x}.
  You should therefore in general be able to recognise and sketch the graph $y = a^{-x}$.

Graph Transformations

Sketch $y = 2^{x+3}$

?

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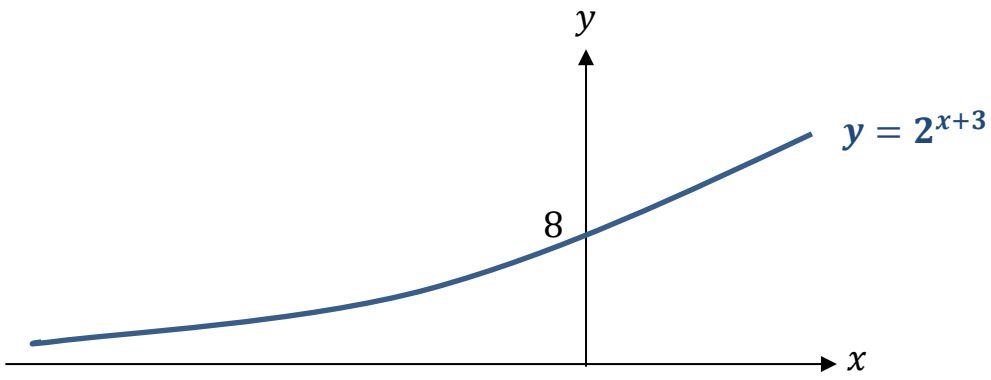

The 'change' to 2^x is 'inside the function' (i.e. input x is replaced with $x + 3$).
So a translation to the left by 3.

Ensure you work out the new y-intercept.

Exercise 14.1

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Homework Exercise

1 **a** Draw an accurate graph of $y = (1.7)^x$, for $-4 \leqslant x \leqslant 4$.

b Use your graph to solve the equation $(1.7)^x = 4$.

2 **a** Draw an accurate graph of $y = (0.6)^x$, for $-4 \leqslant x \leqslant 4$.

b Use your graph to solve the equation $(0.6)^x = 2$.

3 Sketch the graph of $y = 1^x$.

4 For each of these statements, decide whether it is true or false, justifying your answer or offering a counter-example.

a The graph of $y = a^x$ passes through (0, 1) for all positive real numbers a.

b The function $f(x) = a^x$ is always an increasing function for $a > 0$.

c The graph of $y = a^x$, where a is a positive real number, never crosses the x-axis.

5 The function $f(x)$ is defined as $f(x) = 3^x$, $x \in \mathbb{R}$. On the same axes, sketch the graphs of:

a $y = f(x)$ **b** $y = 2f(x)$ **c** $y = f(x) - 4$ **d** $y = f\left(\frac{1}{2}x\right)$

Write down the coordinates of the point where each graph crosses the y-axis, and give the equations of any asymptotes.

Homework Exercise

6 The graph of $y = ka^x$ passes through the points (1, 6) and (4, 48). Find the values of the constants k and a.

Problem-solving

Substitute the coordinates into $y = ka^x$ to create two simultaneous equations. Use division to eliminate one of the two unknowns.

7 The graph of $y = pq^x$ passes through the points $(-3, 150)$ and $(2, 0.048)$.

a By drawing a sketch or otherwise, explain why $0 < q < 1$.

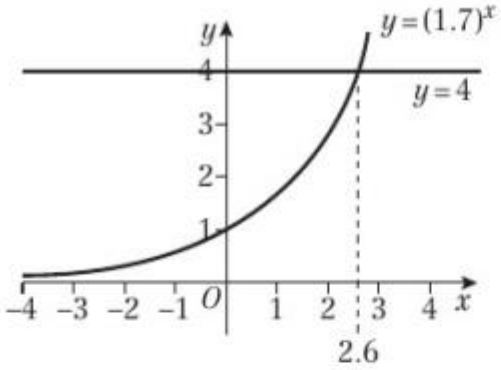
b Find the values of the constants p and q.

Challenge

Sketch the graph of $y = 2^{x-2} + 5$. Give the coordinates of the point where the graph crosses the y-axis.

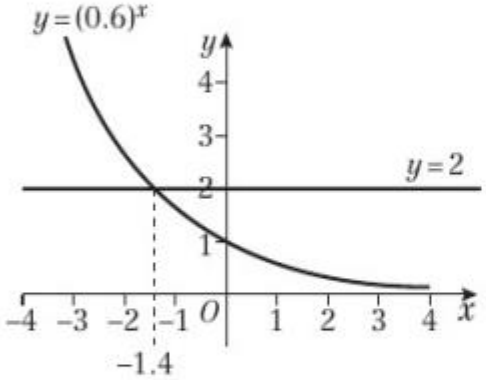
Homework Answers

1 **a**



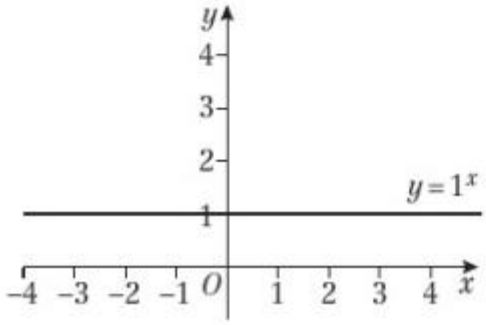
b $x \approx 2.6$

2 **a**



b $x \approx -1.4$

3

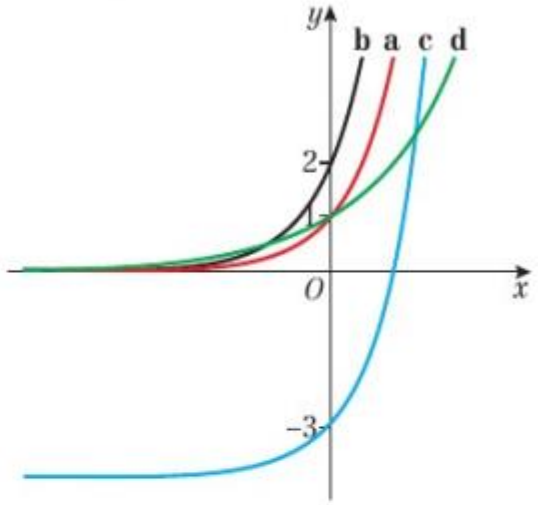


4 **a** True, because $a^0 = 1$ whenever a is positive

b False, for example when $a = \frac{1}{2}$

c True, because when a is positive, $a^x > 0$ for all values of x

5



6 $k = 3, a = 2$

7 **a** As x increases, y decreases

b $p = 1.2, q = 0.2$

Challenge