P1 Chapter 10: Trigonometry Equations

Simple Trig Equations

Solving Trigonometric Equations

Remember those trigonometric angle laws (on the right) earlier this chapter? They're about to become **super freakin' useful**!

Reminder of 'trig laws':

- $\sin(x) = \sin(180 - x)$
- $\cos(x) = \cos(360 - x)$
- sin, cos repeat every $360°$ but tan every $180°$

Solve $\sin\theta = \frac{1}{2}$ in the interval $0 \leq \theta \leq 360°$.

?

Calculator Note:
When you do sin^{-1}, cos^{-1} and tan^{-1} on a calculator, it gives you only one value, known as the **principal value**.

Solve $5\tan\theta = 10$ in the interval $-180° \leq \theta < 180°$

?

Tip: Look out for the solution range required. $-180 \leq \theta < 180°$ is a particularly common one.

tan repeats every $180°$, so can add/subtract $180°$ as we please.

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- $\cos(x) = \cos(360 - x)$
- sin, cos repeat every 360° but tan every 180°

Solve $\sin\theta = \frac{1}{2}$ in the interval $0 \leq \theta \leq 360°$.

$$\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

or $\theta = 180° - 30° = 150°$

Calculator Note:
When you do sin^{-1}, cos^{-1} and tan^{-1} on a calculator, it gives you only one value, known as the **principal value**.

Solve $5\tan\theta = 10$ in the interval $-180° \leq \theta < 180°$

$$\tan\theta = \frac{10}{5} = 2$$

$\theta = \tan^{-1}(2) = 63.4° \ \ (1dp)$

or $\theta = 63.4° - 180° = -116.6° \ (1dp)$

Tip: Look out for the solution range required. $-180 \leq \theta < 180°$ is a particularly common one.

tan repeats every 180°, so can add/subtract 180° as we please.

Slightly Harder Ones...

Solve $\sin\theta = -\frac{1}{2}$ in the interval $0 \leq \theta \leq 360°$.

?

Solve $\sin\theta = \sqrt{3}\cos\theta$ in the interval $0 \leq \theta \leq 360°$.

?

Hint: The problem here is that we have two different trig functions. Is there anything we can divide both sides by so we only have one trig function?

Slightly Harder Ones…

Solve $\sin\theta = -\frac{1}{2}$ in the interval $0 \leq \theta \leq 360°$.

$\theta = \sin^{-1}\left(-\frac{1}{2}\right) = -30°$

or $\theta = 180° - (-30°) = 210°$

or $\theta = -30° + 360° = 330°$

This is not in range. In general you should have 2 solutions per $360°$ (except when at a peak or trough of the trig graph)

Note that we've had to use a second law, i.e. that sin repeats every $360°$.

Solve $\sin\theta = \sqrt{3}\cos\theta$ in the interval $0 \leq \theta \leq 360°$.

$$\frac{\sin\theta}{\cos\theta} = \sqrt{3}$$

$$\tan\theta = \sqrt{3}$$

$\theta = \tan^{-1}(\sqrt{3}) = 60°$

or $\theta = 60° + 180° = 240°$

Hint: The problem here is that we have two different trig functions. Is there anything we can divide both sides by so we only have one trig function?

Test Your Understanding

Solve $2\cos\theta = \sqrt{3}$ in the interval $0 \leq \theta \leq 360°$.

?

Solve $\sqrt{3}\sin\theta = \cos\theta$ in the interval $-180° \leq \theta \leq 180°$.

?

Test Your Understanding

Solve $2\cos\theta = \sqrt{3}$ in the interval $0 \leq \theta \leq 360°$.

$$\cos\theta = \frac{\sqrt{3}}{2}$$
$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$$
or $\theta = 360° - 30° = 330°$

Solve $\sqrt{3}\sin\theta = \cos\theta$ in the interval $-180° \leq \theta \leq 180°$.

$$\tan\theta = \frac{1}{\sqrt{3}}$$
$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$
or $\theta = 30° - 180° = -150°$

Exercise 10.4

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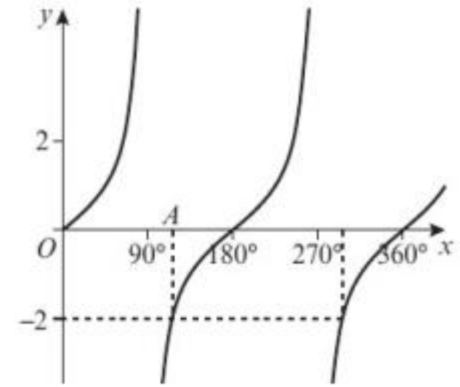
Homework Exercise

1 The diagram shows a sketch of $y = \tan x$.

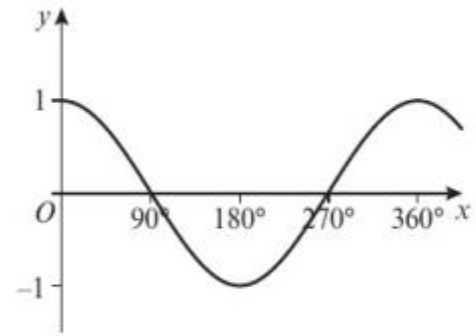


a Use your calculator to find the principal solution to the equation $\tan x = -2$.

Hint The principal solution is marked A on the diagram.

b Use the graph and your answer to part **a** to find solutions to the equation $\tan x = -2$ in the range $0 \leqslant x \leqslant 360°$.

2 The diagram shows a sketch of $y = \cos x$.



a Use your calculator to find the principal solution to the equation $\cos x = 0.4$.

b Use the graph and your answer to part **a** to find solutions to the equation $\cos x = \pm 0.4$ in the range $0 \leqslant x \leqslant 360°$.

3 Solve the following equations for θ, in the interval $0 < \theta \leqslant 360°$:

a $\sin\theta = -1$ **b** $\tan\theta = \sqrt{3}$ **c** $\cos\theta = \frac{1}{2}$

d $\sin\theta = \sin 15°$ **e** $\cos\theta = -\cos 40°$ **f** $\tan\theta = -1$

g $\cos\theta = 0$ **h** $\sin\theta = -0.766$

Hint Give your answers exactly where possible, or round to 3 significant figures.

Homework Exercise

4 Solve the following equations for θ, in the interval $0 < \theta \leqslant 360°$:

a $7\sin\theta = 5$ b $2\cos\theta = -\sqrt{2}$ c $3\cos\theta = -2$ d $4\sin\theta = -3$

e $7\tan\theta = 1$ f $8\tan\theta = 15$ g $3\tan\theta = -11$ h $3\cos\theta = \sqrt{5}$

5 Solve the following equations for θ, in the interval $0 < \theta \leqslant 360°$:

a $\sqrt{3}\sin\theta = \cos\theta$ b $\sin\theta + \cos\theta = 0$ c $3\sin\theta = 4\cos\theta$

d $2\sin\theta - 3\cos\theta = 0$ e $\sqrt{2}\sin\theta = 2\cos\theta$ f $\sqrt{5}\sin\theta + \sqrt{2}\cos\theta = 0$

6 Solve the following equations for x, giving your answers to 3 significant figures where appropriate, in the intervals indicated:

a $\sin x = -\frac{\sqrt{3}}{2}, -180° \leqslant x \leqslant 540°$ b $2\sin x = -0.3, -180° \leqslant x \leqslant 180°$

c $\cos x = -0.809, -180° \leqslant x \leqslant 180°$ d $\cos x = 0.84, -360° < x < 0°$

e $\tan x = -\frac{\sqrt{3}}{3}, 0 \leqslant x \leqslant 720°$ f $\tan x = 2.90, 80° \leqslant x \leqslant 440°$

7 A teacher asks two students to solve the equation $2\cos x = 3\sin x$ for $-180° \leqslant x \leqslant 180°$. The attempts are shown:

Student A:

$\tan x = \frac{3}{2}$

$x = 56.3°$ or $x = -123.7°$

Student B:

$4\cos^2 x = 9\sin^2 x$

$4(1 - \sin^2 x) = 9\sin^2 x$

$4 = 13\sin^2 x$

$\sin x = \pm\sqrt{\frac{4}{13}}$, $x = \pm 33.7°$ or $x = \pm 146.3°$

a Identify the mistake made by Student A. **(1 mark)**

b Identify the mistake made by Student B and explain the effect it has on their solution. **(2 marks)**

c Write down the correct answers to the question. **(1 mark)**

Homework Exercise

8 **a** Sketch the graphs of $y = 2\sin x$ and $y = \cos x$ on the same set of axes ($0 \leqslant x \leqslant 360°$).

b Write down how many solutions there are in the given range for the equation $2\sin x = \cos x$.

c Solve the equation $2\sin x = \cos x$ algebraically, giving your answers in exact form.

9 Find all the values of θ, to 1 decimal place, in the interval $0 < \theta < 360°$ for which $\tan^2\theta = 9$. **(5 marks)**

Problem-solving

When you take square roots of both sides of an equation you need to consider both the positive and the negative square roots.

10 **a** Show that $4\sin^2 x - 3\cos^2 x = 2$ can be written as $7\sin^2 x = 5$. **(2 marks)**

b Hence solve, for $0 \leqslant x \leqslant 360°$, the equation $4\sin^2 x - 3\cos^2 x = 2$. Give your answers to 1 decimal place. **(7 marks)**

11 **a** Show that the equation $2\sin^2 x + 5\cos^2 x = 1$ can be written as $3\sin^2 x = 4$. **(2 marks)**

b Use your result in part **a** to explain why the equation $2\sin^2 x + 5\cos^2 x = 1$ has no solutions. **(1 marks)**

Homework Answers

1 **a** $-63.4°$ **b** $116.6°, 296.6°$

2 **a** $66.4°$ **b** $66.4°, 113.6°, 246.4°, 293.6°$

3 **a** $270°$ **b** $60°, 240°$
c $60°, 300°$ **d** $15°, 165°$
e $140°, 220°$ **f** $135°, 315°$
g $90°, 270°$ **h** $230°, 310°$

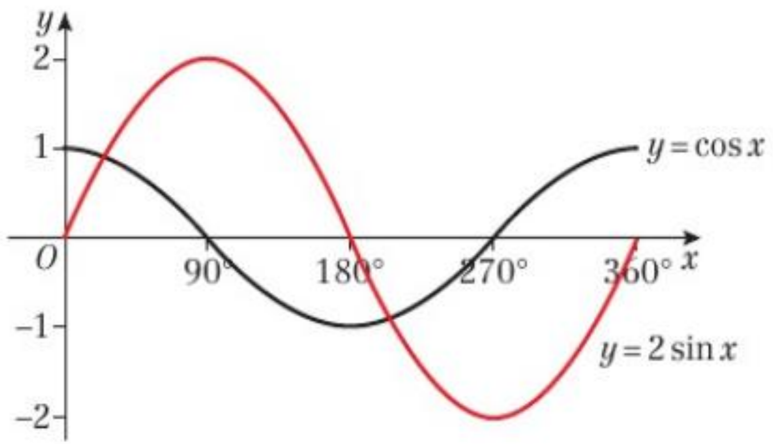
4 **a** $45.6°, 134.4°$ **b** $135°, 225°$
c $132°, 228°$ **d** $229°, 311°$
e $8.13°, 188°$ **f** $61.9°, 242°$
g $105°, 285°$ **h** $41.8°, 318°$

5 **a** $30°, 210°$ **b** $135°, 315°$
c $53.1°, 233°$ **d** $56.3°, 236°$
e $54.7°, 235°$ **f** $148°, 328°$

6 **a** $-120°, -60°, 240°, 300°$ **b** $-171°, -8.63°$
c $-144°, 144°$ **d** $-327°, -32.9°$
e $150°, 330°, 510°, 690°$ **f** $251°, 431°$

7 **a** $\tan x$ should be $\frac{2}{3}$
b Squaring both sides creates extra solutions
c $-146.3°, 33.7°$

8 **a**



b 2 **c** $26.6°, 206.6°$

9 $71.6°, 108.4°, 251.6°, 288.4°$

10 **a** $4\sin^2 x - 3(1 - \sin^2 x) = 2$.
Rearrange to get $7\sin^2 x = 5$
b $57.7°, 122.3°, 237.7°, 302.3°$

11 **a** $2\sin^2 x + 5(1 - \sin^2 x) = 1$.
Rearrange to get $3\sin^2 x = 4$
b $\sin x > 1$