
M1 Chapter 10: Forces and Motion

Force Vectors

Forces as Vectors

Forces have direction, and therefore we can naturally write them as vectors, either in \mathbf{i} - \mathbf{j} notation or as column vectors.

 You can find the resultant of two or more forces given as vectors by adding the vectors.

[Textbook] The forces $2\mathbf{i} + 3\mathbf{j}$, $4\mathbf{i} - \mathbf{j}$, $-3\mathbf{i} + 2\mathbf{j}$ and $a\mathbf{i} + b\mathbf{j}$ act on an object which is in equilibrium. Find the values of a and b .

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3+a \\ 4+b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\therefore a = -3, \quad b = -4$

If in equilibrium, resultant force is 0.

[Textbook] The vector \mathbf{i} is due east and \mathbf{j} due north. A particle begins at rest at the origin. It is acted on by three forces $(2\mathbf{i} + \mathbf{j})$ N, $(3\mathbf{i} - 2\mathbf{j})$ N and $(-\mathbf{i} + 4\mathbf{j})$ N.

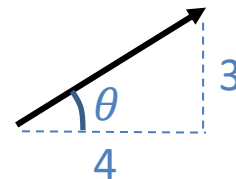
(a) Find the resultant force in the form $p\mathbf{i} + q\mathbf{j}$.

(b) Work out the magnitude and bearing of the resultant force.

a

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 4\mathbf{i} + 3\mathbf{j}$$

b



The magnitude of the force is the magnitude of the vector:

$$\left| \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right| = \sqrt{4^2 + 3^2} = 5 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{3}{4} \right) = 36.9^\circ$$

$$\therefore \text{Bearing} = 90 - 36.9 = 053.1^\circ$$

Test Your Understanding

Edexcel M1 Jan 2012 Q3

Three forces \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j}) \text{ N}$$

$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j}) \text{ N}$$

$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j}) \text{ N}$$

where p and q are constants.

Given that P is in equilibrium,

- (a) find the value of p and the value of q . (3)

The force \mathbf{F}_3 is now removed. The resultant of \mathbf{F}_1 and \mathbf{F}_2 is \mathbf{R} . Find

- (b) the magnitude of \mathbf{R} , (2)
(c) the angle, to the nearest degree, that the direction of \mathbf{R} makes with \mathbf{j} . (3)

(a)

?

(b)

?

(c)

?

Edexcel M1 May 2009 Q2

A particle is acted upon by two forces \mathbf{F}_1 and \mathbf{F}_2 , given by

$$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j}) \text{ N},$$

$$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j}) \text{ N}, \text{ where } p \text{ is a positive constant.}$$

- (a) Find the angle between \mathbf{F}_2 and \mathbf{j} . (2)

The resultant of \mathbf{F}_1 and \mathbf{F}_2 is \mathbf{R} . Given that \mathbf{R} is parallel to \mathbf{i} ,

- (b) find the value of p . (4)

Tip: If a vector is parallel to say $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, then it could be any multiple of it, i.e. $k \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(a)

?

(b)

?

Test Your Understanding

Edexcel M1 Jan 2012 Q3

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$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j}) \text{ N}$$

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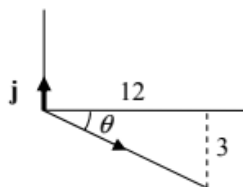
(b) the magnitude of \mathbf{R} , (2)

(c) the angle, to the nearest degree, that the direction of \mathbf{R} makes with \mathbf{j} . (3)

$$\begin{array}{l} \text{(a)} \quad 7 + 5 + p = 0 \quad \text{or} \quad -9 + 6 + q = 0 \\ \quad \quad \quad p = -12 \\ \quad \quad \quad q = 3 \end{array}$$

$$\begin{array}{l} \text{(b)} \quad \mathbf{R} = 12\mathbf{i} - 3\mathbf{j} \\ \quad \quad |\mathbf{R}| = \sqrt{(12^2 + (-3)^2)} = \sqrt{153} \text{ or } 3\sqrt{17} \text{ or } 12.4 \text{ or better (N)} \end{array}$$

$$\begin{array}{l} \text{(c)} \quad \tan \theta = \frac{3}{12} \\ \quad \quad \theta = 14.03^\circ \dots \\ \quad \quad \text{Angle with } \mathbf{j} \text{ is } 104^\circ, \text{ to the nearest degree} \end{array}$$



Edexcel M1 May 2009 Q2

A particle is acted upon by two forces \mathbf{F}_1 and \mathbf{F}_2 , given by

$$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j}) \text{ N},$$

$$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j}) \text{ N}, \text{ where } p \text{ is a positive constant.}$$

(a) Find the angle between \mathbf{F}_2 and \mathbf{j} . (2)

The resultant of \mathbf{F}_1 and \mathbf{F}_2 is \mathbf{R} . Given that \mathbf{R} is parallel to \mathbf{i} ,

(b) find the value of p . (4)

Tip: If a vector is parallel to say $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, then it could be any multiple of it, i.e. $k \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\begin{array}{l} \text{(a)} \quad \tan \theta = \frac{p}{2p} \Rightarrow \theta = 26.6^\circ \\ \text{(b)} \quad \mathbf{R} = (\mathbf{i} - 3\mathbf{j}) + (p\mathbf{i} + 2p\mathbf{j}) = (1+p)\mathbf{i} + (-3+2p)\mathbf{j} \\ \quad \quad \mathbf{R} \text{ is parallel to } \mathbf{i} \Rightarrow (-3+2p) = 0 \\ \quad \quad \quad \Rightarrow p = \frac{3}{2} \end{array}$$

Classwork Exercise 10.2

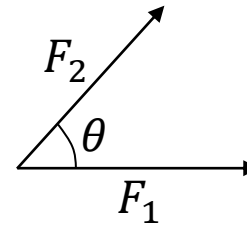
Pearson Stats/Mechanics Year 1 Exercise Book

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Extension

A force F_1 acts in the direction of i and a force F_2 acts at an angle of θ to i , as shown. Show that the resultant force has magnitude

$$\sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$



?

Homework Exercise

- 1 In each part of the question a particle is acted upon by the forces given. Work out the resultant force acting on the particle.

a $(-\mathbf{i} + 3\mathbf{j})$ N and $(4\mathbf{i} - \mathbf{j})$ N **b** $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ N and $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$ N
c $(\mathbf{i} + \mathbf{j})$ N, $(5\mathbf{i} - 3\mathbf{j})$ N and $(-2\mathbf{i} - \mathbf{j})$ N **d** $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ N, $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ N and $\begin{pmatrix} -2 \\ -7 \end{pmatrix}$ N

- 2 An object is in equilibrium at O under the action of three forces \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 . Find \mathbf{F}_3 in these cases.

a $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$ and $\mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})$ **b** $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$ and $\mathbf{F}_2 = (2\mathbf{i} + 3\mathbf{j})$

- 3 The forces $\begin{pmatrix} a \\ 2b \end{pmatrix}$ N, $\begin{pmatrix} -2a \\ -b \end{pmatrix}$ N and $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ N act on an object which is in equilibrium.

Find the values of a and b .

- 4 For each force find:

i the magnitude of the force

ii the angle the force makes with \mathbf{i}

a $(3\mathbf{i} + 4\mathbf{j})$ N

b $(5\mathbf{i} - \mathbf{j})$ N

c $(-2\mathbf{i} + 3\mathbf{j})$ N

d $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ N

Homework Exercise

- 5 In this question, \mathbf{i} represents the unit vector due east, and \mathbf{j} represents the unit vector due north. A particle is acted upon by forces of:

a $(-2\mathbf{i} + \mathbf{j})$ N, $(5\mathbf{i} + 2\mathbf{j})$ N and $(-\mathbf{i} - 4\mathbf{j})$ N b $(-2\mathbf{i} + \mathbf{j})$ N, $(2\mathbf{i} - 3\mathbf{j})$ N and $(3\mathbf{i} + 6\mathbf{j})$ N

Work out:

- i the resultant vector
 - ii the magnitude of the resultant vector
 - iii the bearing of the resultant vector.
- 6 The forces $(a\mathbf{i} - b\mathbf{j})$ N, $(b\mathbf{i} + a\mathbf{j})$ N and $(-4\mathbf{i} - 2\mathbf{j})$ N act on an object which is in equilibrium. Find the values of a and b .

Problem-solving

Use the \mathbf{i} components and the \mathbf{j} components to set up and solve two simultaneous equations.

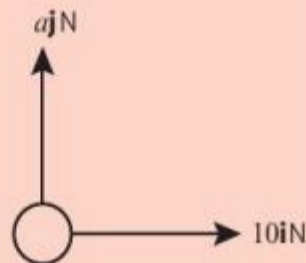
- 7 The forces $(2a\mathbf{i} + 2b\mathbf{j})$ N, $(-5b\mathbf{i} + 3a\mathbf{j})$ N and $(-11\mathbf{i} - 7\mathbf{j})$ N act on an object which is in equilibrium. Find the values of a and b .
- 8 Three forces \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 act on a particle. $\mathbf{F}_1 = (-3\mathbf{i} + 7\mathbf{j})$ N, $\mathbf{F}_2 = (\mathbf{i} - \mathbf{j})$ N and $\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})$ N.
- a Given that this particle is in equilibrium, determine the value of p and the value of q .
The resultant of the forces \mathbf{F}_1 and \mathbf{F}_2 is \mathbf{R} .
 - b Calculate, in N, the magnitude of \mathbf{R} .
 - c Calculate, to the nearest degree, the angle between the line of action of \mathbf{R} and the vector \mathbf{j} .

Homework Exercise

- 9 A particle is acted upon by two forces \mathbf{F}_1 and \mathbf{F}_2 , given by $\mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j}) \text{ N}$ and $\mathbf{F}_2 = (a\mathbf{i} + 2a\mathbf{j}) \text{ N}$, where a is a positive constant.
- a Find the angle between \mathbf{F}_2 and \mathbf{i} . (2 marks)
- The resultant of \mathbf{F}_1 and \mathbf{F}_2 is \mathbf{R} .
- b Given that \mathbf{R} is parallel to $13\mathbf{i} + 10\mathbf{j}$, find the value of a . (4 marks)
- 10 Three forces \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 acting on a particle P are given by the vectors $\mathbf{F}_1 = \begin{pmatrix} -7 \\ -4 \end{pmatrix} \text{ N}$, $\mathbf{F}_2 = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \text{ N}$ and $\mathbf{F}_3 = \begin{pmatrix} a \\ b \end{pmatrix} \text{ N}$, where a and b are constants.
- Given that P is in equilibrium,
- a find the value of a and the value of b . (3 marks)
- b The force \mathbf{F}_1 is now removed. The resultant of \mathbf{F}_2 and \mathbf{F}_3 is \mathbf{R} . Find:
- i the magnitude of \mathbf{R} (2 marks)
- ii the angle, to the nearest degree, that the direction of \mathbf{R} makes with the horizontal. (3 marks)

Challenge

An object is acted upon by a horizontal force of $10\mathbf{i} \text{ N}$ and a vertical force $a\mathbf{j} \text{ N}$ as shown in the diagram. The resultant of the two forces acts in the direction 60° to the horizontal. Work out the value of a and the magnitude of the resultant force.



Homework Answers

1 **a** $(3\mathbf{i} + 2\mathbf{j})N$ **b** $\begin{pmatrix} 2 \\ -3 \end{pmatrix}N$

c $(4\mathbf{i} - 3\mathbf{j})N$ **d** $\begin{pmatrix} 3 \\ -3 \end{pmatrix}N$

b $-5\mathbf{i} + \mathbf{j}$

b $-5\mathbf{i} + \mathbf{j}$

ii 53.1°

ii 11.3°

ii 123.7°

ii 135°

ii $\sqrt{5}$ N iii 116.6°

ii 5 N **iii** 36.9°

iii 116.6°

iii 36.9°

b $\sqrt{40} \text{ N}$ **c** 18°

b 3.5

b i $\sqrt{65}$ N ii 30°

Challenge

 $a = 17.3$ (3 s.f.), magnitude of resultant force = 20 N