P1 Chapter 3: Inequalities

Quadratic Simultaneous Equations

Simultaneous Equations

Solve the simultaneous equations:

$$x + 2y = 3$$
$$x^2 + 3xy = 10$$

We can't use elimination this time as nothing would cancel.

We instead:

(1) Rearrange linear equation to make x or y the subject.

(2) Substitute into quadratic equation and solve.

$$\boldsymbol{x = 3 - 2y}$$

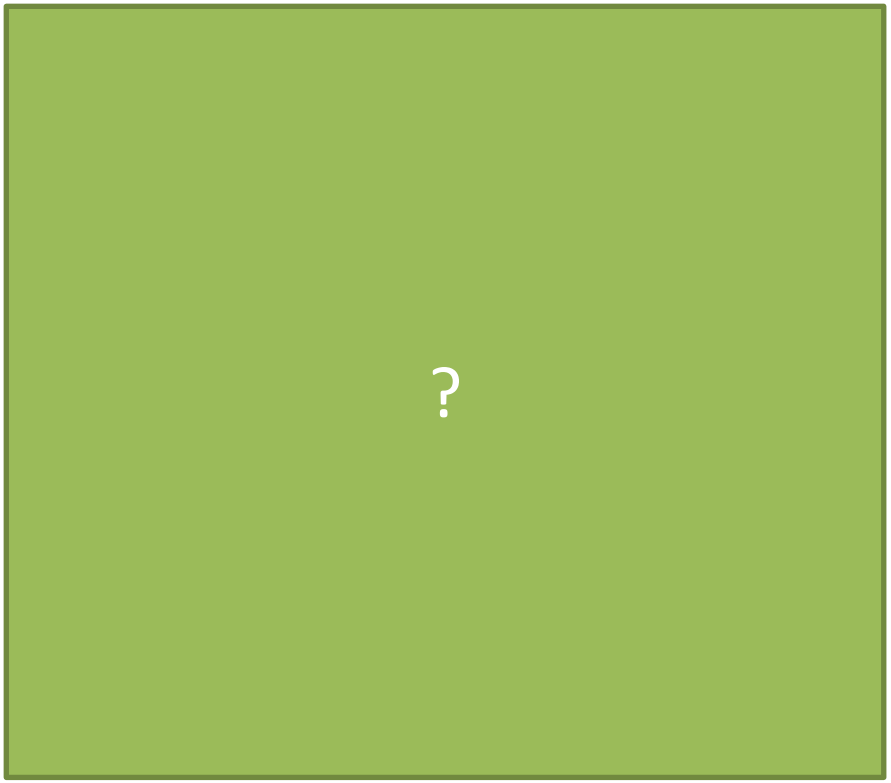
Substitute into other equation:

$$\boldsymbol{(3 - 2y)^2 + 3y(3 - 2y) = 10}$$
$$\boldsymbol{\ldots 2y^2 + 3y + 1 = 0}$$
$$\boldsymbol{(2y + 1)(y + 1) = 0}$$
$$\boldsymbol{y = -\frac{1}{2} \quad \rightarrow \quad x = 4}$$
$$\boldsymbol{y = -1 \quad \rightarrow \quad x = 5}$$

Test Your Understanding

Solve the simultaneous equations:

$$3x^2 + y^2 = 21$$
$$y = x + 1$$



Test Your Understanding

Solve the simultaneous equations:

$$3x^2 + y^2 = 21$$
$$y = x + 1$$

$$3x^2 + (x+1)^2 = 21$$
$$3x^2 + x^2 + 2x + 1 = 21$$
$$4x^2 + 2x - 20 = 0$$
$$2x^2 + x - 10 = 0$$
$$(2x + 5)(x - 2) = 0$$
$$x = -\frac{5}{2} \; or \; x = 2$$
$$y = -\frac{3}{2} \; or \; y = 3$$

Exercise 3.2

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Homework Exercise

1 Solve the simultaneous equations:

a $x + y = 11$
$xy = 30$

b $2x + y = 1$
$x^2 + y^2 = 1$

c $y = 3x$
$2y^2 - xy = 15$

d $3a + b = 8$
$3a^2 + b^2 = 28$

e $2u + v = 7$
$uv = 6$

f $3x + 2y = 7$
$x^2 + y = 8$

2 Solve the simultaneous equations:

a $2x + 2y = 7$
$x^2 - 4y^2 = 8$

b $x + y = 9$
$x^2 - 3xy + 2y^2 = 0$

c $5y - 4x = 1$
$x^2 - y^2 + 5x = 41$

3 Solve the simultaneous equations, giving your answers in their simplest surd form:

a $x - y = 6$
$xy = 4$

b $2x + 3y = 13$
$x^2 + y^2 = 78$

Watch out Use brackets when you are substituting an expression into an equation.

4 Solve the simultaneous equations:

$x + y = 3$
$x^2 - 3y = 1$

(6 marks)

Homework Exercise

5 a By eliminating y from the equations

$$y = 2 - 4x$$
$$3x^2 + xy + 11 = 0$$

show that $x^2 - 2x - 11 = 0$. **(2 marks)**

b Hence, or otherwise, solve the simultaneous equations

$$y = 2 - 4x$$
$$3x^2 + xy + 11 = 0$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers. **(5 marks)**

6 One pair of solutions for the simultaneous equations

$$y = kx - 5$$
$$4x^2 - xy = 6$$

is $(1, p)$ where k and p are constants.

a Find the values of k and p.

b Find the second pair of solutions for the simultaneous equations.

Problem-solving

If $(1, p)$ is a solution, then $x = 1$, $y = p$ satisfies both equations.

Challenge

$$y - x = k$$
$$x^2 + y^2 = 4$$

Given that the simultaneous equations have exactly one pair of solutions, show that

$$k = \pm 2\sqrt{2}$$

Homework Answers

1 **a** $x = 5, y = 6$ or $x = 6, y = 5$

b $x = 0, y = 1$ or $x = \frac{4}{5}, y = -\frac{3}{5}$

c $x = -1, y = -3$ or $x = 1, y = 3$

d $a = 1, b = 5$ or $a = 3, b = -1$

e $u = 1\frac{1}{2}, v = 4$ or $u = 2, v = 3$

f $x = -1\frac{1}{2}, y = 5\frac{3}{4}$ or $x = 3, y = -1$

2 **a** $x = 3, y = \frac{1}{2}$ or $x = 6\frac{1}{3}, y = -2\frac{5}{6}$

b $x = 4\frac{1}{2}, y = 4\frac{1}{2}$ or $x = 6, y = 3$

c $x = -19, y = -15$ or $x = 6, y = 5$

3 **a** $x = 3 + \sqrt{13}, y = -3 + \sqrt{13}$ or $x = 3 - \sqrt{13}$, $y = -3 - \sqrt{13}$

b $x = 2 - 3\sqrt{5}, y = 3 + 2\sqrt{5}$ or $x = 2 + 3\sqrt{5}, y = 3 - 2\sqrt{5}$

4 $x = -5, y = 8$ or $x = 2, y = 1$

5 **a** $3x^2 + x(2 - 4x) + 11 = 0$

$3x^2 + 2x - 4x^2 + 11 = 0$

$x^2 - 2x - 11 = 0$

b $x = 1 + 2\sqrt{3}, y = -2 - 8\sqrt{3}$

$x = 1 - 2\sqrt{3}, y = -2 + 8\sqrt{3}$

6 **a** $k = 3, p = -2$

b $x = -6, y = -23$ or $x = 1, y = -2$

Challenge

$y = x + k$

$x^2 + (x + k)^2 = 4$

$x^2 + x^2 + 2kx + k^2 - 4 = 0$

$2x^2 + 2kx + k^2 - 4 = 0$ for one solution $b^2 - 4ac = 0$

$4k^2 - 4 \times 2(k^2 - 4) = 0$

$4k^2 - 8k^2 + 32 = 0$ $4k^2 = 32$ $k^2 = 8$ $k = \pm 2\sqrt{2}$