M2 Chapter 4: Moments

Tilting

Tilting

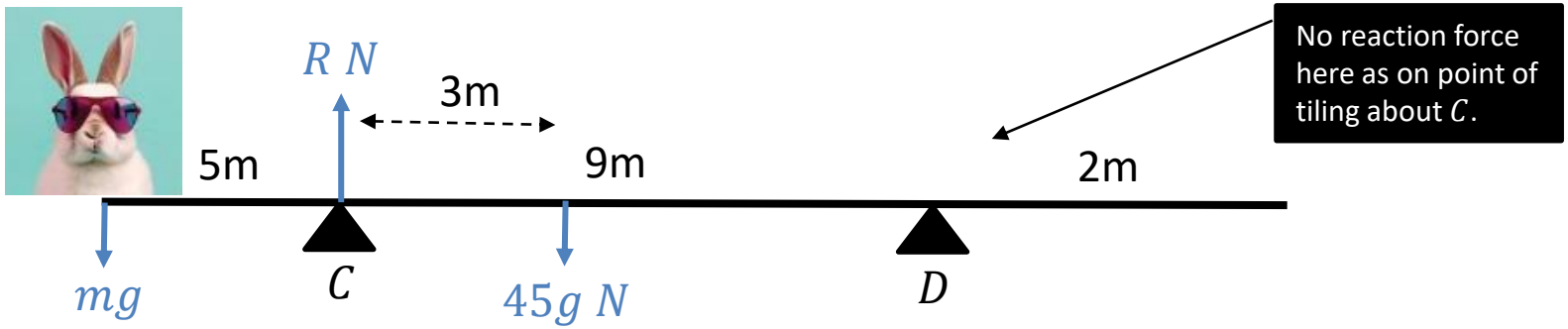
Buggs

Eating too many carrots causes Buggs to gradually increase the weight he is applying at one end of a beam, until the rod is on the verge of tilting about the support A. What can we say about the forces at B?

The rod is about to lift off pivot B, so there must be no reaction force from this support being exerted on the rod.

✎ When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or tension in any other wire/string) is zero.

Example

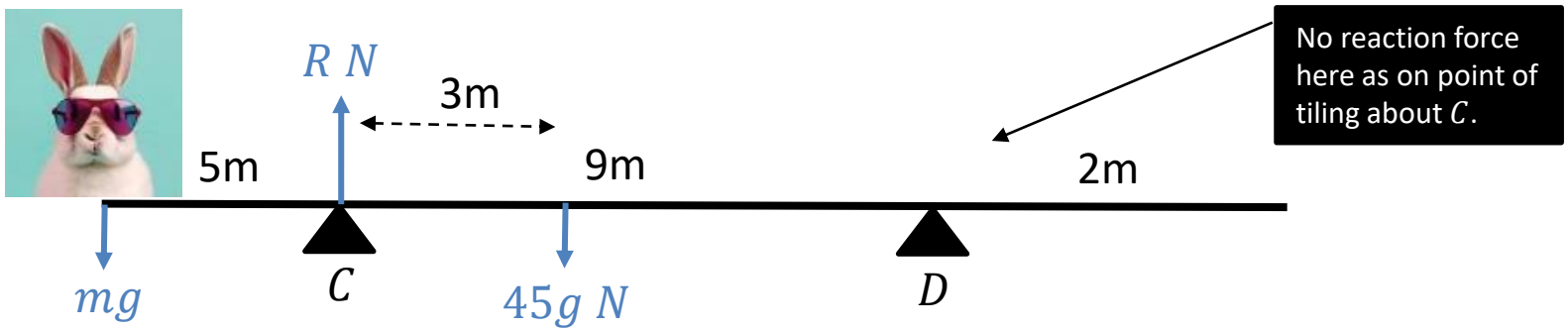


A uniform beam AB, of mass 45kg and length 16m, rests horizontally on supports C and D where $AC = 5$ m and $CD = 9$ m.
When Buggs hops onto the point A, the beam is on the point of tilting about C.
Determine Buggs' mass.

Best to take moments about C as we don't require the value of R. In general, take moments about points that avoids variables you don't want.

?

Example



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Take moments about C:

$$mg \times 5 = 3 \times 45g$$
$$5mg = 135g$$
$$m = \frac{135g}{5g} = 27$$

Buggs is now 27kg, and has eaten too many carrots.

Suspended System Example

[Textbook] A non-uniform rod AB, of length 10 m and weight 40 N, is suspended from a pair of light cables attached to C and D where $AC = 3$ m and $BD = 2$ m.
When a weight of 25 N is hung from A the rod is on the point of rotating.
Find the distance of the centre of mass of the rod from A.

? Diagram

? Working

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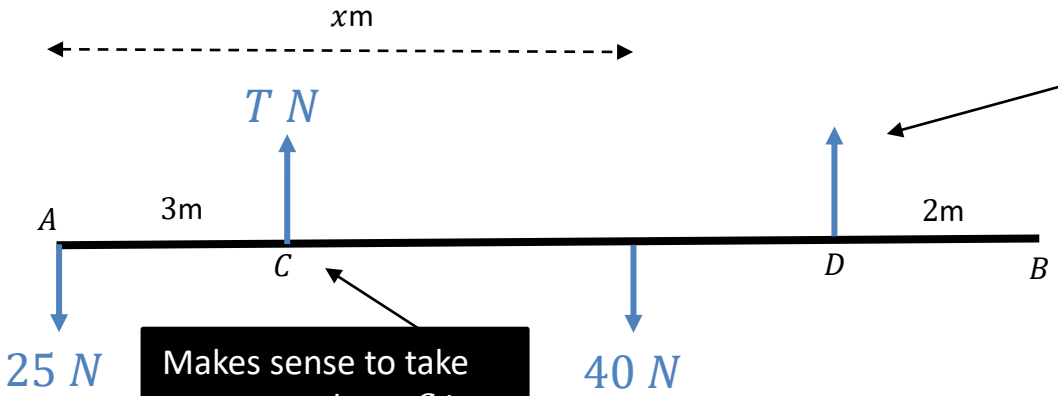

No tension here as rod is on verge of tilting about pivot C.

Makes sense to take moments about C is we don't require the value of T.

Taking moments about C:

$$25 \times 3 = 40(x - 3)$$

…

$$x = 4.875$$

So centre of mass from A is 4.875 m

Test Your Understanding

Edexcel M1(Old) May 2013 Q6

A beam AB has length 15 m. The beam rests horizontally in equilibrium on two smooth supports at the points P and Q, where $AP = 2$ m and $QB = 3$ m. When a child of mass 50 kg stands on the beam at A, the beam remains in equilibrium and is on the point of tilting about P. When the same child of mass 50 kg stands on the beam at B, the beam remains in equilibrium and is on the point of tilting about Q. The child is modelled as a particle and the beam is modelled as a non-uniform rod.

(a) (i) Find the mass of the beam.

(ii) Find the distance of the centre of mass of the beam from A.

(8)

When the child stands at the point X on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,

(b) find AX.

(6)

a

?

(i)

(ii)

b

?

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(a) (i) Find the mass of the beam.

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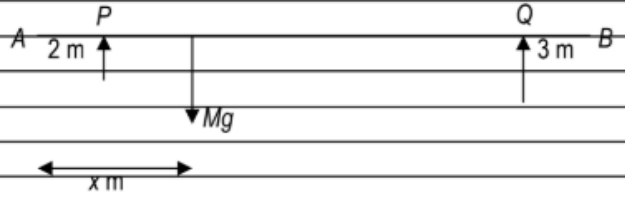
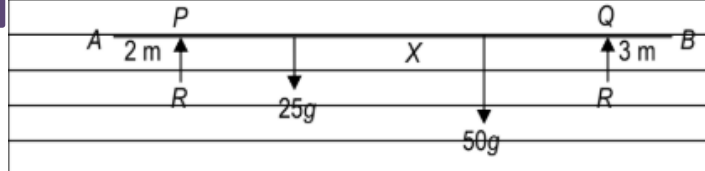


	M(P), $50g \times 2 = Mg \times (x-2)$	M1 A1
	M(Q), $50g \times 3 = Mg \times (12-x)$	M1 A1
(i)	$M = 25$ (kg)	**DM**1 A1
(ii)	$x = 6$ (m)	**DM**1 A1
		(8)

b



$(\uparrow) R + R = 25g + 50g$	M1 A1 **ft**
M(A), $2R + 12R = 25g \times 6 + 50g \times AX$	M1 A1 **ft**
$AX = 7.5$ (m)	**DM**1 A1
	(6)

Exercise 4.5

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Homework Exercise

1 A uniform rod AB has length 4 m and mass 8 kg. It is resting in a horizontal position on supports at points C and D where $AC = 1$ m and $AD = 2.5$ m. A particle of mass m kg is placed at point E where $AE = 3.3$ m. Given that the rod is about to tilt about D, calculate the value of m.

2 A uniform bar AB, of length 6 m and weight 40 N, is resting in a horizontal position on supports at points C and D where $AC = 2$ m and $AD = 5$ m. When a particle of weight 30 N is attached to the bar at point E the bar is on the point of tilting about C. Calculate the distance AE.

3 A plank AB, of mass 12 kg and length 3 m, is in equilibrium in a horizontal position resting on supports at C and D where $AC = 0.7$ m and $DB = 1.1$ m. A boy of mass 32 kg stands on the plank at point E. The plank is about to tilt about D. By modelling the plank as a uniform rod and the boy as a particle, calculate the distance AE.

4 A uniform rod AB has length 5 m and weight 20 N. The rod is resting on supports at points C and D where $AC = 2$ m and $BD = 1$ m.

a Find the magnitudes of the reactions at C and D.

A particle of weight 12 N is placed on the rod at point A.

b Show that this causes the rod to tilt about C.

A second particle of weight 100 N is placed on the rod at E to hold it in equilibrium.

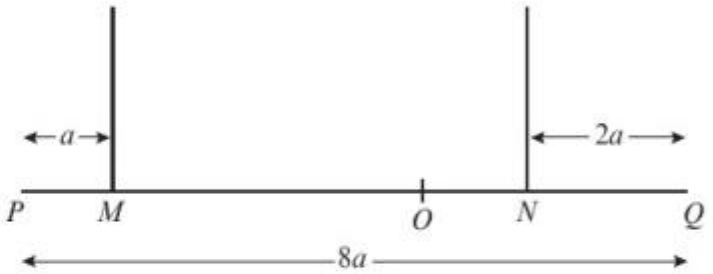
c Find the minimum and maximum possible distances of E from A.

Homework Exercise

5 A uniform plank of mass 100 kg and length 10 m rests horizontally on two smooth supports, A and B, as shown in the diagram. A man of mass 80 kg starts walking from one end of the plank, A, to the other end.

Find the distance he can walk past B before the plank starts to tip. **(4 marks)**

6 A non-uniform beam PQ, of mass m and length $8a$, hangs horizontally in equilibrium from two wires at M and N, where $PM = a$ and $QN = 2a$, as shown in the diagram. The centre of mass of the beam is at the point O. A particle of mass $\frac{3}{4}m$ is placed on the beam at Q and the beam is on the point of tipping about N.

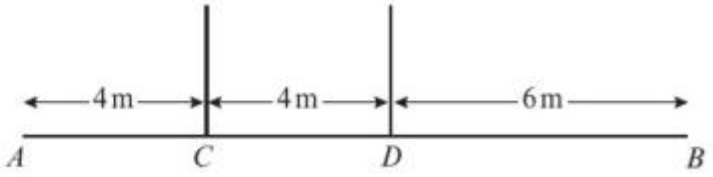


a Show that $ON = \frac{3}{2}a$. **(3 marks)**

The particle is removed and replaced at the midpoint of the beam and the beam remains in equilibrium.

b Find the magnitude of the tension in the wire attached at point N in terms of m. **(5 marks)**

7 A uniform beam AB, of weight W and length 14 m, hangs in equilibrium in a horizontal position from two vertical cables attached at points C and D where $AC = 4$ m and $BD = 6$ m.



A weight of 180 N is hung from A and the beam is about to tilt. The weight is removed and a different weight, V, is hung from B and the beam does not tilt. Find the maximum value of V. **(6 marks)**

Homework Answers

1 5

2 $\frac{2}{3}$m

3 2.05 m

4 **a** $C = 15\,\text{N}, D = 5\,\text{N}$ **b** $2 \times 12 \neq 20 \times 0.5$

c $2.14 \leqslant x \leqslant 4.78\,\text{m}$

5 2.5 m

6 **a** Taking moments about N:
$mg \times ON = \frac{3}{4}mg \times 2a$ so $ON = \frac{3}{2}a$

b $\frac{23}{20}mg\,\text{N}$

7 40 N