P2 Chapter 7: Trigonometric Equations

Identity Modelling

Modelling

[June 2013 (Withdrawn) Q8]

(a) Express $9\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the exact value of R and give the value of α to 4 decimal places. **(3)**

(b) (i) State the maximum value of $9\cos\theta - 2\sin\theta$

(ii) Find the value of θ, for $0 < \theta < 2\pi$, at which this maximum occurs. **(3)**

Ruth models the height H above the ground of a passenger on a Ferris wheel by the equation

$$H = 10 - 9\cos\left(\frac{\pi t}{5}\right) + 2\sin\left(\frac{\pi t}{5}\right)$$

where H is measured in metres and t is the time in minutes after the wheel starts turning.

(c) Calculate the maximum value of H predicted by this model, and the value of t, when this maximum first occurs. Give your answers to 2 decimal places. **(4)**

(d) Determine the time for the Ferris wheel to complete two revolutions. **(2)**

When trigonometric equations are in the form $R\sin(ax \pm b)$ or $R\cos(ax \pm b)$, they can be used to model various things which have an oscillating behaviour, e.g. tides, the swing of a pendulum and sound waves.

(a)

? a

(b) (i)

(ii)

? b

? c

? d

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(a) $9\cos\theta - 2\sin\theta = R\cos(\theta+\alpha)$		
$R = \sqrt{(9^2+2^2} = \sqrt{85}$	B1	
$\alpha = \arctan(\frac{2}{9}) = 0.21866... = \text{awrt } 0.2187$	M1A1	
		(3)
(b) (i) $\sqrt{85}$	B1√	
(ii) $\theta + \alpha = 2\pi \Rightarrow \theta = \text{awrt } 6.06 \text{ 2dp}$	M1A1	
		(3)

(c) Seeing (or implied by their working) $H = 10 - R\cos(\frac{\pi t}{5} + \alpha)$ for their R and α	M1	
$H_{max} = 10 + \textit{their } R = 10 + \sqrt{85}$ (=19.22m)	A1√	
Maximum occurs when $\cos(\frac{\pi t}{5} + \alpha) = -1$ or $(\frac{\pi t}{5} + \alpha) = \pi$	M1	
t= awrt 4.65	A1	
		(4)
(d) Setting and solving $\frac{\pi t}{5} = 2\pi$ (for 1 cycle) or $\frac{\pi t}{5} = 4\pi$ (for 2 cycles)	M1	
Two revolutions = 20 minutes	A1	
		(2)

Test Your Understanding

[June 2010 Q7] 2. (*a*) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places. **(3)**

(*b*) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ, for $0 \leq \theta < \pi$, at which this maximum occurs. **(3)**

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin\left(\frac{4\pi t}{25}\right) - 1.5 \cos\left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(*c*) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs. **(3)**

(*d*) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres. **(6)**

Tip: Reflect carefully on the substitution you use to allow (bii) to match your identity in (a). θ =?

(a) ? a

(b) (i) ? bi

(ii) ? bii

(c) ? c

(d) ? d

Test Your Understanding

[June 2010 Q7] 2. (*a*) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places. **(3)**

(*b*) (i) Find the maximum value of $2\sin\theta - 1.5\cos\theta$.

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Tip: Reflect carefully on the substitution you use to allow (bii) to match your identity in (a). $\theta = ?$

(a)	$R = \sqrt{6.25}$ or 2.5	B1	
	$\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = $ awrt 0.6435	M1A1	
			(3)
(b) (i)	Max Value $= 2.5$	B1 $\sqrt{}$	
(ii)	$\underline{\sin(\theta - 0.6435) = 1}$ or $\underline{\theta - \text{their } \alpha = \frac{\pi}{2}}$; $\Rightarrow \theta = $ awrt 2.21	M1;A1 $\sqrt{}$	
			(3)
(c)	$H_{Max} = 8.5$ (m)	B1 $\sqrt{}$	
	$\underline{\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1}$ or $\underline{\frac{4\pi t}{25} = \text{their (b) answer}}$; $\Rightarrow t = $ awrt 4.41	M1;A1	
			(3)
(d)	$\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$; $\Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$	M1;M1	
	$\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41	A1	
	Either $t = $ awrt 2.1 or awrt 6.7	A1	
	So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \{\pi - 0.411517\ldots \text{ or } 2.730076\ldots^{c}\}$	ddM1	
	Times $= \{14:06, 18:43\}$	A1	(6)
			[15]

Exercise 7.7

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Homework Exercise

1 The height, h, of a buoy on a boating lake can be modelled by $h = 0.25 \sin(1800t)°$, where h is the height in metres above the buoy's resting position and t is the time in minutes.

a State the maximum height the buoy reaches above its resting position according to this model.

b Calculate the time, to the nearest tenth of a second, at which the buoy is first at a height of 0.1 metres.

c Calculate the time interval between successive minimum heights of the buoy.

2 The angle of displacement of a pendulum, θ, at time t seconds after it is released is modelled as $\theta = 0.03 \cos(25t)$, where all angles are measured in radians.

a State the maximum displacement of the pendulum according to this model.

b Calculate the angle of displacement of the pendulum after 0.2 seconds.

c Find the time taken for the pendulum to return to its starting position.

d Find all the times in the first half second of motion that the pendulum has a displacement of 0.01 radians.

3 The price, P, of stock in pounds during a 9-hour trading window can be modelled by $P = 17.4 + 2 \sin(0.7t - 3)$, where t is the time in hours after the stock market opens, and angles are measured in radians.

a State the beginning and end price of the stock.

b Calculate the maximum price of the stock and the time when it occurs.

c A day trader wants to sell the stock when it firsts shows a profit of £0.40 above the day's starting price. At what time should the trader sell the stock?

Homework Exercise

4 The temperature of an oven can be modelled by the equation $T = 225 - 0.3\sin(2x - 3)$, where T is the temperate in Celsius and x is the time in minutes after the oven first reaches the desired temperature, and angles are measured in radians.

a State the minimum temperature of the oven.

b Find the times during the first 10 minutes when the oven is at a minimum temperature.

c Calculate the time when the oven first reaches a temperature of 225.2 °C.

5 a Express $0.3\sin\theta - 0.4\cos\theta$ in the form $R\sin(\theta - \alpha)°$, where R > 0 and $0 < \alpha < 90°$. Give the value of α to 2 decimal places. **(4 marks)**

b i Find the maximum value of $0.3\sin\theta - 0.4\cos\theta$. **(2 marks)**

ii Find the value of θ, for $0 < \theta < 180$ at which the maximum occurs. **(1 mark)**

Jack models the temperature in his house, T°C, on a particular day by the equation

$$T = 23 + 0.3\sin(18x)° - 0.4\cos(18x)°,\ x \geqslant 0$$

where x is the number of minutes since the thermostat was adjusted.

c Calculate the minimum value of T predicted by this model, and the value of x, to 2 decimal places, when this minimum occurs. **(3 marks)**

d Calculate, to the nearest minute, the times in the first hour when the temperature is predicted, by this model, to be exactly 23 °C. **(4 marks)**

Homework Exercise

6 **a** Express $65\cos\theta - 20\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α correct to 4 decimal places. **(4 marks)**

A city wants to build a large circular wheel as a tourist attraction. The height of a tourist on the circular wheel is modelled by the equation

$$H = 70 - 65\cos 0.2t + 20\sin 0.2t$$

where H is the height of the tourist above the ground in metres, t is the number of minutes after boarding and the angles are given in radians. Find:

b the maximum height of the wheel **(2 marks)**

c the time for one complete revolution **(2 marks)**

d the number of minutes the tourist will be over 100 m above the ground in each revolution. **(4 marks)**

7 **a** Express $200\sin\theta - 150\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 4 decimal places. **(4 marks)**

The electric field strength, E V/m, in a microwave of width 25 cm can be modelled using the equation

$$E = 1700 + 200\sin\left(\frac{4\pi x}{25}\right) - 150\cos\left(\frac{4\pi x}{25}\right)$$

where x is the distance in cm from the left hand edge of the microwave oven.

b **i** Calculate the maximum value of E predicted by this model.

ii Find the values of x, for $0 \leqslant x < 25$, where this maximum occurs. **(3 marks)**

c Food in the microwave will heat best when the electric field strength at the centre of the food is above 1800 V/m. Find the range of possible locations for the centre of the food. **(5 marks)**

Homework Exercise

Challenge

Look at the model for the electric field strength in a microwave oven given in question **7** above. For food of the same type and mass, the energy transferred by the oven is proportional to the square of the electric field strength. Given that a square of chocolate placed at a point of maximum field strength takes 20 seconds to melt,

a estimate the range of locations within the oven that an identical square of chocolate will take longer than 30 seconds to melt.

b State two limitations of the model.

Homework Answers

1 **a** 0.25 m **b** 0.013 minutes, 0.8 seconds
c 0.2 minutes or 12 seconds

2 **a** 0.03 radians **b** 0.0085 radians
c 0.251 seconds
d 0.0492, 0.2021, 0.3006, 0.4534 seconds

3 **a** £17.12, £17.08
b £19.40, 6.53 hours or 6 h 32 min
c After 4.37 hours (4 h 22 min after market opens)

4 **a** 224.7°C
b 2 m 17 s, 5 m 26 s, 8 m 34 s
c 17.6 seconds.

5 **a** $R = 0.5$, $\alpha = 53.13°$
b **i** 0.5 **ii** $\theta = 143.1°$
c Minimum value is 22.5, occurs at 17.95 minutes
d 2.95, 12.95, 22.95, 32.95, 42.95, 52.95 minutes

6 **a** R = 68.0074, $\alpha = 0.2985$ **b** 138.0 m
c 31.4 minutes **d** 11.1 minutes

7 **a** $R = 250$, $\alpha = 0.6435$
b **i** 1950 V/m **ii** $x = 4.41$ cm, $x = 16.91$ cm
c $2.10 \leqslant x \leqslant 6.71$, $14.60 \leqslant x \leqslant 19.21$

Challenge

a $0\text{ cm} \leqslant x < 0.39\text{ cm}$, $8.42\text{ cm} < x < 12.89\text{ cm}$, $20.92\text{ cm} < x < 25\text{ cm}$

b Identifying the exact point of maximum field strength, microwave oven would not work exactly the same every time it is used.