P1 Chapter 4: Transforming Graphs

Quartic Graphs

Recap

If we sketched $y = (x - a)(x - b)^2(x - c)^3$ what happens on the x-axis at:

$x = a$: ?

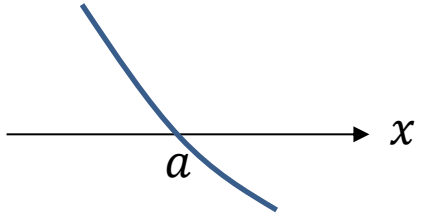
$x = b$: ?

$x = c$: ?

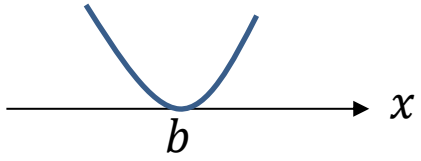
Recap

If we sketched $y = (x - a)(x - b)^2(x - c)^3$ what happens on the x-axis at:

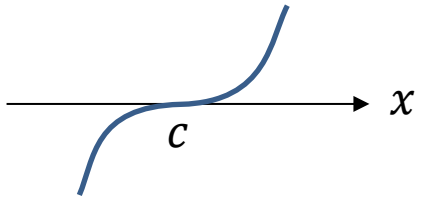
$x = a$: The line **crosses** the axis.



$x = b$: The line **touches** the axis.



$x = c$: **Point of inflection** on the axis.

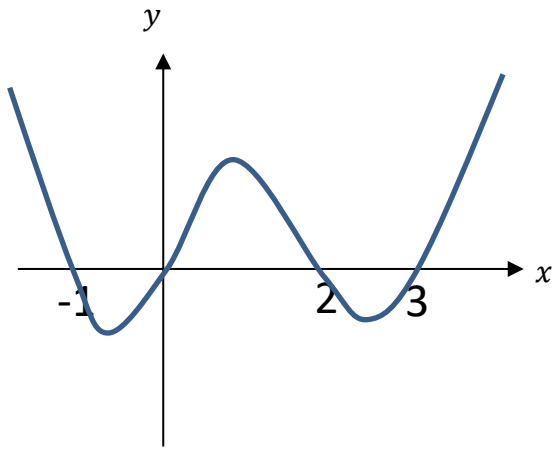


Quartics

If you understand the principle of sketching polynomials in general, then sketching quartics shouldn't feel like anything new.
Recall that if the x^4 term is positive, the 'tails' both go upwards, otherwise downwards.

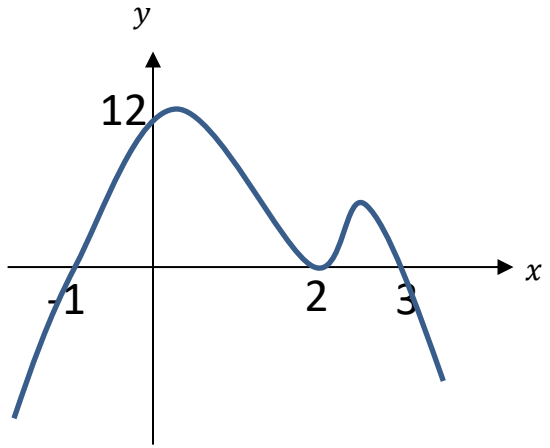
Sketch the curve with equation
$y = x(x+1)(x-2)(x-3)$

Shape: **Tails upwards**
Roots: **-1, 0, 2, 3**
y-intercept: **0**



Sketch the curve with equation
$y = (x-2)^2(x+1)(3-x)$

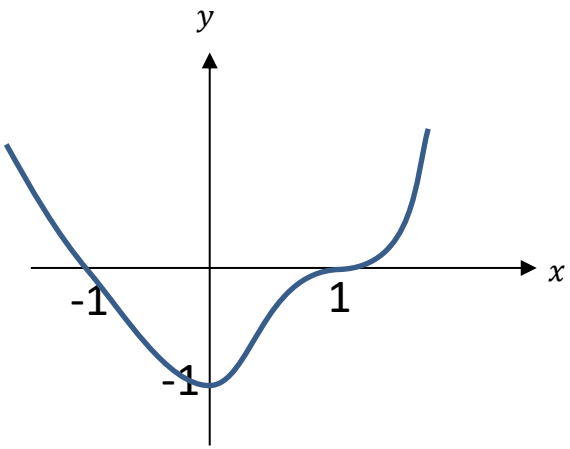
Shape: **Tails downwards**
Roots: **-1, 2, 3**
2 is repated.
y-intercept: $\mathbf{4 \times 1 \times 3 = 12}$



Quartics

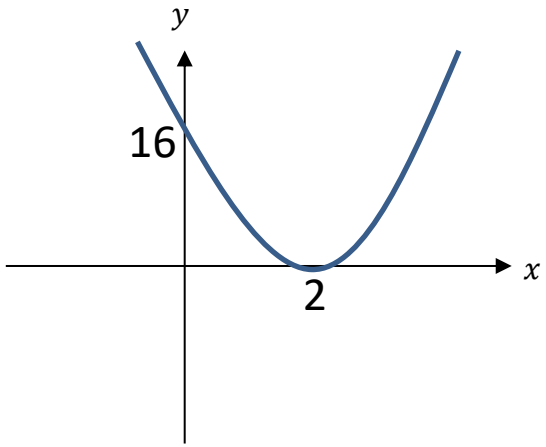
Sketch the curve with equation
$y = (x + 1)(x - 1)^3$

-1 root only appears once so line crosses at $x = -1$
+1 root triple repeated so point of inflection at $x = 1$



Sketch the curve with equation
$y = (x - 2)^4$

2 is a quadruple repeated root!
Because the line effectively crosses the axis 4 times all at -2, it ends up in the opposite direction, and hence looks like a 'touch' point.



Test Your Understanding

Sketch the curve with equation

$$y = x^2(x+1)(x-1)$$

?

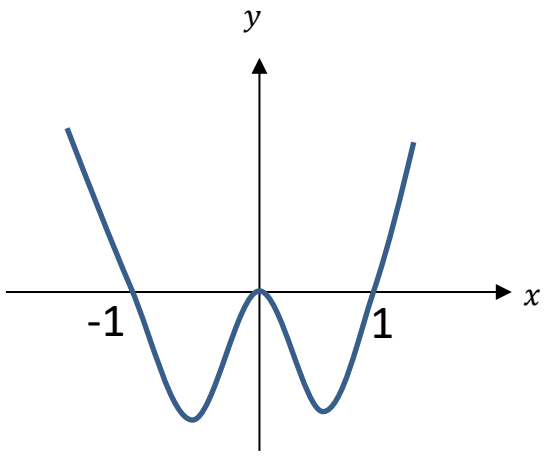
Sketch the curve with equation

$$y = -(x+1)(x-3)^3$$

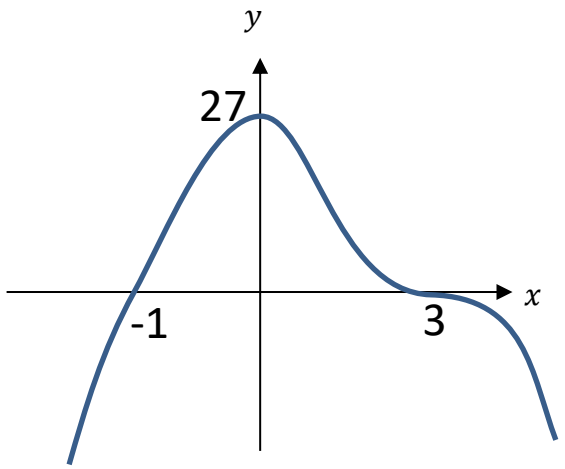
?

Test Your Understanding

Sketch the curve with equation
$y = x^2(x+1)(x-1)$



Sketch the curve with equation
$y = -(x+1)(x-3)^3$



Exercise 4.2

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Extension

1 *[STEP I 2012 Q2a]*

a. Sketch $y = x^4 - 6x^2 + 9$

b. For what values of b does the equation $y = x^4 - 6x^2 + b$ have the following number of distinct roots (i) 0, (ii) 1, (iii) 2, (iv) 3, (v) 4.

a)

b) By changing b, we shift the graph up and down. Then we can see that:

i) ?
ii) ?
iii) ?
iv) ?
v) ?

Exercise 4.2

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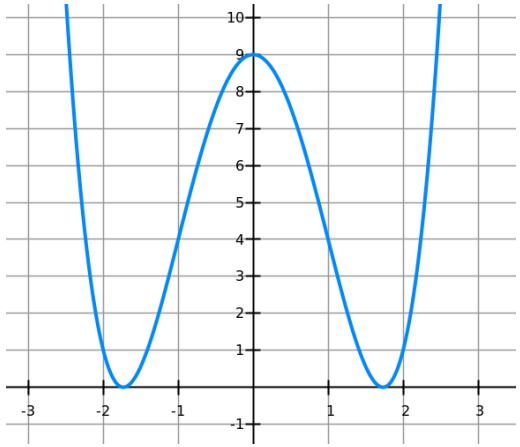
Extension

1 *[STEP I 2012 Q2a]*

a. Sketch $y = x^4 - 6x^2 + 9$

b. For what values of b does the equation $y = x^4 - 6x^2 + b$ have the following number of distinct roots (i) 0, (ii) 1, (iii) 2, (iv) 3, (v) 4.

a) By factorising, $y = (x^2 - 3)^2$. This is a quartic, where y is always positive, and has repeated roots at $x = \pm\sqrt{3}$:



b) By changing b, we shift the graph up and down. Then we can see that:

i) 0 roots: When $b > 9$
ii) 1 root: Not possible.
iii) 2 roots: When $b = 9$ or $b < 0$
iv) 3 roots: $b = 0$
v) 4 roots: $0 < b < 9$

Exercise 1.1

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Homework Exercise

1 Sketch the following curves and indicate clearly the points of intersection with the axes:

a $y = (x + 1)(x + 2)(x + 3)(x + 4)$ **b** $y = x(x - 1)(x + 3)(x - 2)$

c $y = x(x + 1)^2(x + 2)$ **d** $y = (2x - 1)(x + 2)(x - 1)(x - 2)$

e $y = x^2(4x + 1)(4x - 1)$ **f** $y = -(x - 4)^2(x - 2)^2$

g $y = (x - 3)^2(x + 1)^2$ **h** $y = (x + 2)^3(x - 3)$

i $y = -(2x - 1)^3(x + 5)$ **j** $y = (x + 4)^4$

Hint In part **f** the coefficient of x^4 will be negative.

2 Sketch the following curves and indicate clearly the points of intersection with the axes:

a $y = (x + 2)(x - 1)(x^2 - 3x + 2)$ **b** $y = (x + 3)^2(x^2 - 5x + 6)$

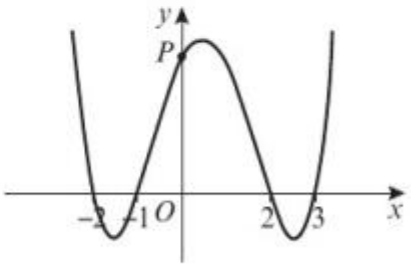
c $y = (x - 4)^2(x^2 - 11x + 30)$ **d** $y = (x^2 - 4x - 32)(x^2 + 5x - 36)$

Hint Factorise the quadratic factor first.

3 The graph of $y = x^4 + bx^3 + cx^2 + dx + e$ is shown opposite, where b, c, d and e are real constants.

a Find the coordinates of point P. **(2 marks)**

b Find the values of b, c, d and e. **(3 marks)**



4 Sketch the graph of $y = (x + 5)(x - 4)(x^2 + 5x + 14)$. **(3 marks)**

Problem-solving

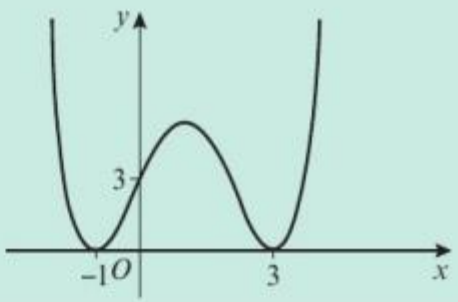
Consider the discriminant of the quadratic factor.

Homework Exercise

Challenge

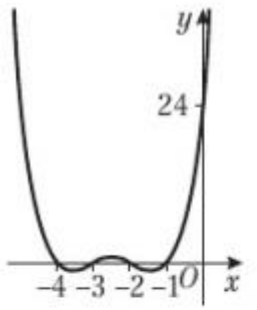
The graph of $y = ax^4 + bx^3 + cx^2 + dx + e$ is shown, where a, b, c, d and e are real constants.

Find the values of a, b, c, d and e.



Homework Answers

1 a



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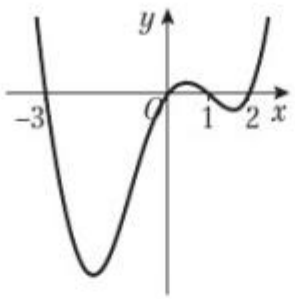
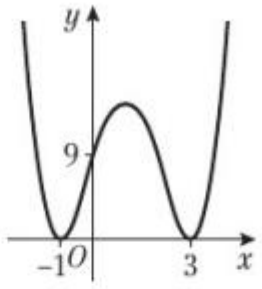


g



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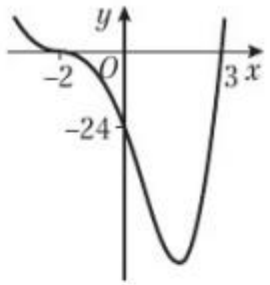

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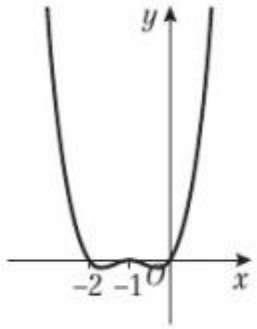

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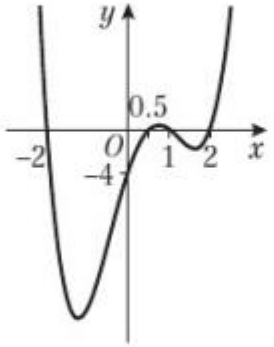

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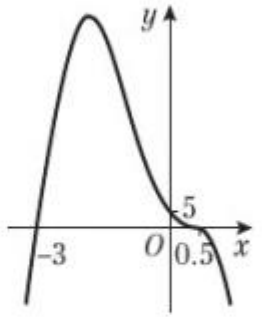

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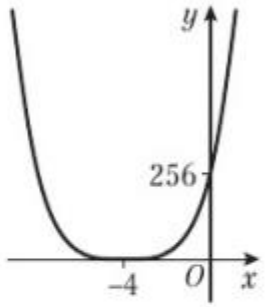

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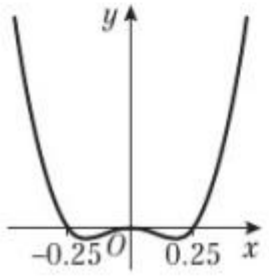
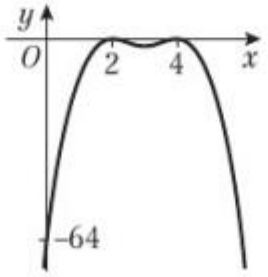
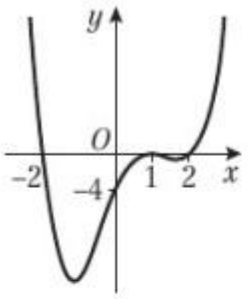


f

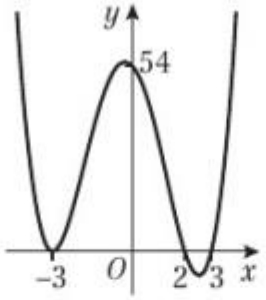


Homework Answers

2 **a**



b



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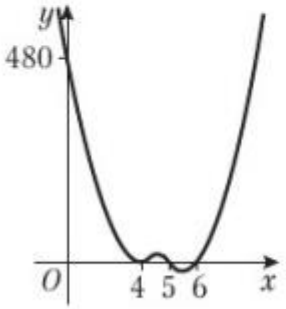
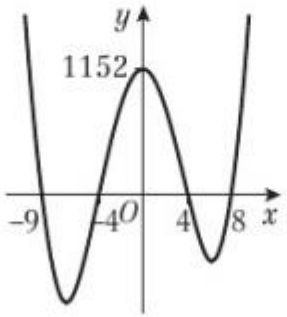


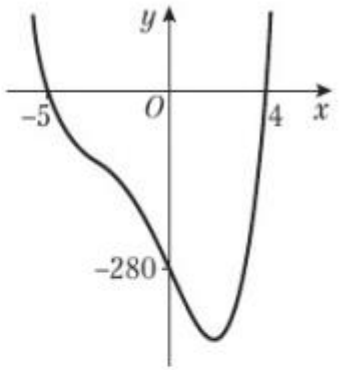
d



3 **a** (0, 12)

b $b = -2, c = -7, d = 8, e = 12$

4



Challenge

$a = \frac{1}{3}, b = -\frac{4}{3}, c = -\frac{2}{3}, d = 4, e = 3$