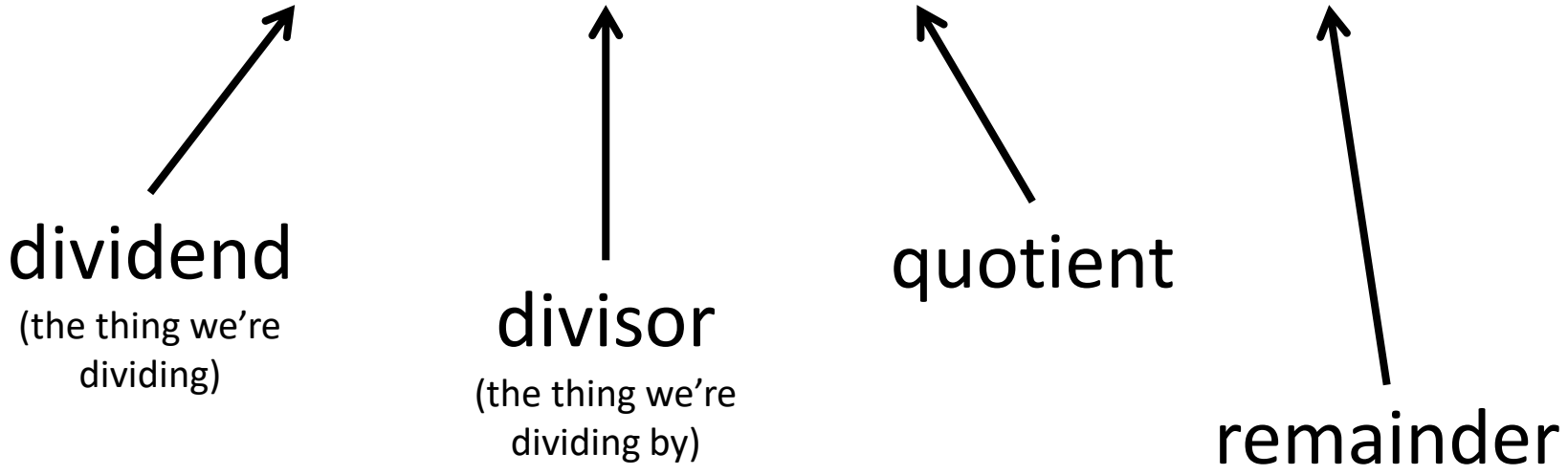
P1 Chapter 7: Algebraic Methods

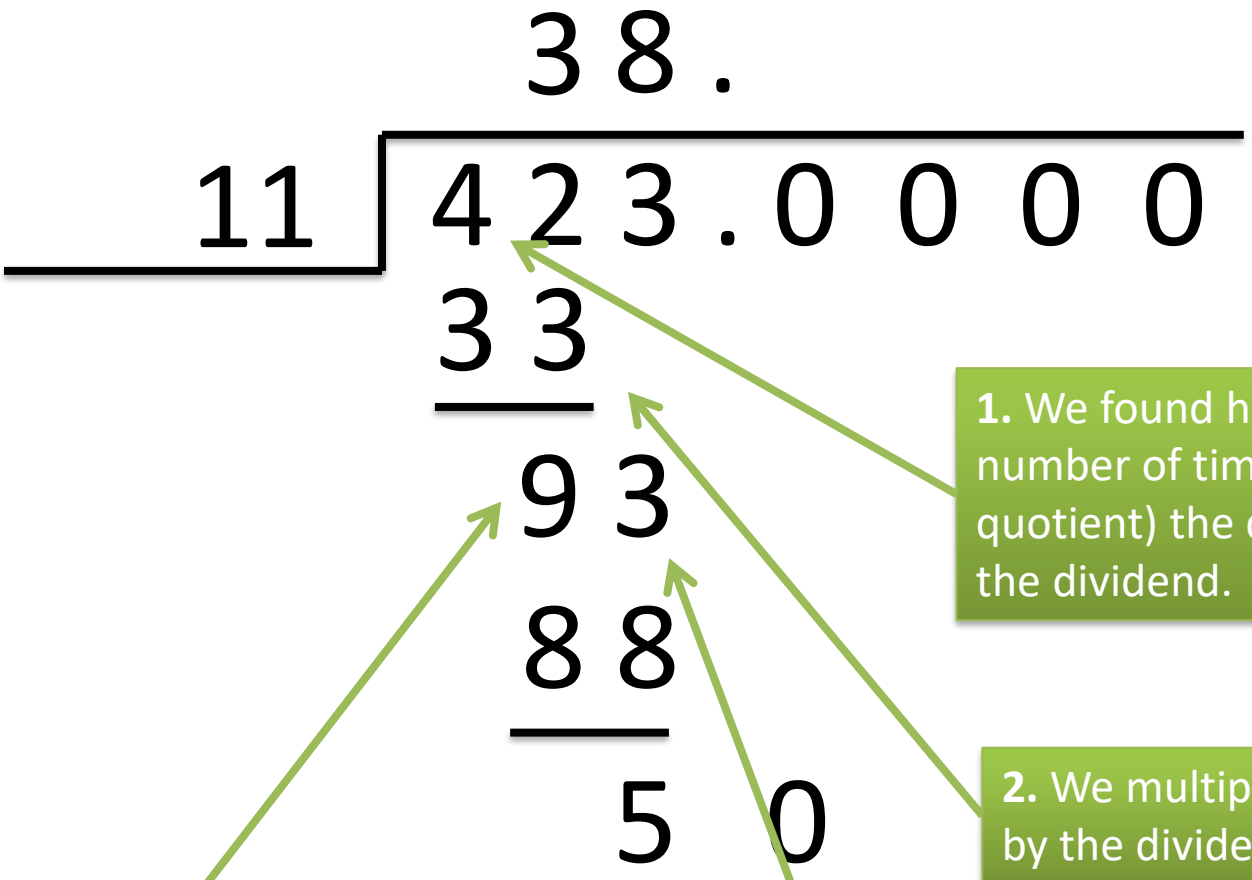
Polynomial Long Division

Terminology

$$11 \div 4 = 2\ r\ 3$$



Normal Long Division



1. We found how many whole number of times (i.e. the quotient) the divisor went into the dividend.

2. We multiplied the quotient by the dividend.

3. …in order to find the remainder.

4. Find we 'brought down' the next number.

How many times does x go into $6x^3$?

Now repeat! How many times does x go into $-2x^2$?

$$6x^2 - 2x + 3$$

$$x + 5 \;\overline{)\; 6x^3 + 28x^2 - 7x + 15}$$

Multiply $6x^2$ by $(x + 5)$. The first term should match with above.

$$6x^3 + 30x^2$$

Subtract and carry down next term.

$$-2x^2 - 7x$$

$$-2x^2 - 10x$$

$$3x + 15$$

$$3x + 15$$

This is the remainder.

$$0$$

Tip:
You can check your solution by expanding:
$(x + 5)(6x^2 - 2x + 3)$
But if you know you should get no remainder, ending with 0 at the bottom is a good sign!

Tip:
Be **very** careful subtracting negatives. $-7x - (-10x) = 3x$

Further Example

Find the remainder when $3x^3 - 2x + 4$ is divided by $x - 1$.

$$
\begin{array}{r|l}
 & 3x^2 + 3x + 1 \\
\hline
x - 1 & 3x^3 + 0x^2 - 2x + 4 \\
 & \underline{3x^3 - 3x^2} \\
 & \quad 3x^2 - 2x \\
 & \quad \underline{3x^2 - 3x} \\
 & \qquad x + 4 \\
 & \qquad \underline{x - 1} \\
 & \qquad 5
\end{array}
$$

Note: Ensure that any missing terms in the polynomial are filled in, so that the powers decrease by 1 each time.

The remainder is 5.

Test Your Understanding

Find the remainder when $2x^3 - 5x^2 - 16x + 10$ is divided by $x - 4$.

?

Test Your Understanding

Find the remainder when $2x^3 - 5x^2 - 16x + 10$ is divided by $x - 4$.

$$
\begin{array}{r}
2x^2 + 3x - 4 \\
x - 4 \overline{\big)\ 2x^3 - 5x^2 - 16x + 10} \\
\underline{2x^3 - 8x^2} \\
3x^2 - 16x \\
\underline{3x^2 - 12x} \\
-4x + 10 \\
\underline{-4x + 16} \\
-6
\end{array}
$$

The remainder is -6.

Further Test Your Understanding

Divide $8x^3 - 1$ by $2x - 1$.

?

Further Test Your Understanding

Divide $8x^3 - 1$ by $2x - 1$.

$$
\begin{array}{r|l}
 & 4x^2 + 2x + 1 \\
\hline
2x - 1 & 8x^3 + 0x^2 + 0x - 1 \\
 & \underline{8x^3 - 4x^2} \\
 & \quad 4x^2 + 0x \\
 & \quad \underline{4x^2 - 2x} \\
 & \qquad\quad 2x - 1 \\
 & \qquad\quad \underline{2x - 1} \\
 & \qquad\quad\quad 0
\end{array}
$$

Fro Fun Fact: There is a well-known factorisation for the '*difference of two cubes*':

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

This is <u>NOT</u> in the A Level syllabus.

Exercise 7.1

Pearson Pure Mathematics Year 1/AS
Pages 1

Homework Exercise

1 Write each polynomial in the form $(x \pm p)(ax^2 + bx + c)$ by dividing:

a $x^3 + 6x^2 + 8x + 3$ by $(x + 1)$

b $x^3 + 10x^2 + 25x + 4$ by $(x + 4)$

c $x^3 - x^2 + x + 14$ by $(x + 2)$

d $x^3 + x^2 - 7x - 15$ by $(x - 3)$

e $x^3 - 8x^2 + 13x + 10$ by $(x - 5)$

f $x^3 - 5x^2 - 6x - 56$ by $(x - 7)$

2 Write each polynomial in the form $(x \pm p)(ax^2 + bx + c)$ by dividing:

a $6x^3 + 27x^2 + 14x + 8$ by $(x + 4)$

b $4x^3 + 9x^2 - 3x - 10$ by $(x + 2)$

c $2x^3 + 4x^2 - 9x - 9$ by $(x + 3)$

d $2x^3 - 15x^2 + 14x + 24$ by $(x - 6)$

e $-5x^3 - 27x^2 + 23x + 30$ by $(x + 6)$

f $-4x^3 + 9x^2 - 3x + 2$ by $(x - 2)$

3 Divide:

a $x^4 + 5x^3 + 2x^2 - 7x + 2$ by $(x + 2)$

b $4x^4 + 14x^3 + 3x^2 - 14x - 15$ by $(x + 3)$

c $-3x^4 + 9x^3 - 10x^2 + x + 14$ by $(x - 2)$

d $-5x^5 + 7x^4 + 2x^3 - 7x^2 + 10x - 7$ by $(x - 1)$

4 Divide:

a $3x^4 + 8x^3 - 11x^2 + 2x + 8$ by $(3x + 2)$

b $4x^4 - 3x^3 + 11x^2 - x - 1$ by $(4x + 1)$

c $4x^4 - 6x^3 + 10x^2 - 11x - 6$ by $(2x - 3)$

d $6x^5 + 13x^4 - 4x^3 - 9x^2 + 21x + 18$ by $(2x + 3)$

e $6x^5 - 8x^4 + 11x^3 + 9x^2 - 25x + 7$ by $(3x - 1)$

f $8x^5 - 26x^4 + 11x^3 + 22x^2 - 40x + 25$ by $(2x - 5)$

g $25x^4 + 75x^3 + 6x^2 - 28x - 6$ by $(5x + 3)$

h $21x^5 + 29x^4 - 10x^3 + 42x - 12$ by $(7x - 2)$

5 Divide:

a $x^3 + x + 10$ by $(x + 2)$

b $2x^3 - 17x + 3$ by $(x + 3)$

c $-3x^3 + 50x - 8$ by $(x - 4)$

6 Divide:

a $x^3 + x^2 - 36$ by $(x - 3)$

b $2x^3 + 9x^2 + 25$ by $(x + 5)$

c $-3x^3 + 11x^2 - 20$ by $(x - 2)$

Hint Include $0x$ when you write out $f(x)$.

Homework Exercise

7 Show that $x^3 + 2x^2 - 5x - 10 = (x + 2)(x^2 - 5)$

8 Find the remainder when:

a $x^3 + 4x^2 - 3x + 2$ is divided by $(x + 5)$

b $3x^3 - 20x^2 + 10x + 5$ is divided by $(x - 6)$

c $-2x^3 + 3x^2 + 12x + 20$ is divided by $(x - 4)$

9 Show that when $3x^3 - 2x^2 + 4$ is divided by $(x - 1)$ the remainder is 5.

10 Show that when $3x^4 - 8x^3 + 10x^2 - 3x - 25$ is divided by $(x + 1)$ the remainder is -1.

11 Show that $(x + 4)$ is a factor of $5x^3 - 73x + 28$.

12 Simplify $\dfrac{3x^3 - 8x - 8}{x - 2}$

Hint Divide $3x^3 - 8x - 8$ by $(x - 2)$.

13 Divide $x^3 - 1$ by $(x - 1)$.

Hint Write $x^3 - 1$ as $x^3 + 0x^2 + 0x - 1$.

14 Divide $x^4 - 16$ by $(x + 2)$.

15 $f(x) = 10x^3 + 43x^2 - 2x - 10$

Find the remainder when $f(x)$ is divided by $(5x + 4)$. **(2 marks)**

Homework Exercise

16 $f(x) = 3x^3 - 14x^2 - 47x - 14$

a Find the remainder when $f(x)$ is divided by $(x - 3)$. **(2 marks)**

b Given that $(x + 2)$ is a factor of $f(x)$, factorise $f(x)$ completely. **(4 marks)**

Problem-solving

Write $f(x)$ in the form $(x + 2)(ax^2 + bx + c)$ then factorise the quadratic factor.

17 **a** Find the remainder when $x^3 + 6x^2 + 5x - 12$ is divided by

i $x - 2$,

ii $x + 3$. **(3 marks)**

b Hence, or otherwise, find all the solutions to the equation $x^3 + 6x^2 + 5x - 12 = 0$. **(4 marks)**

18 $f(x) = 2x^3 + 3x^2 - 8x + 3$

a Show that $f(x) = (2x - 1)(ax^2 + bx + c)$ where a, b and c are constants to be found. **(2 marks)**

b Hence factorise $f(x)$ completely. **(4 marks)**

c Write down all the real roots of the equation $f(x) = 0$. **(2 marks)**

19 $f(x) = 12x^3 + 5x^2 + 2x - 1$

a Show that $(4x - 1)$ is a factor of $f(x)$ and write $f(x)$ in the form $(4x - 1)(ax^2 + bx + c)$. **(6 marks)**

b Hence, show that the equation $12x^3 + 5x^2 + 2x - 1 = 0$ has exactly 1 real solution. **(2 marks)**

Homework Answers

1 **a** $(x + 1)(x^2 + 5x + 3)$ **b** $(x + 4)(x^2 + 6x + 1)$
c $(x + 2)(x^2 - 3x + 7)$ **d** $(x - 3)(x^2 + 4x + 5)$
e $(x - 5)(x^2 - 3x - 2)$ **f** $(x - 7)(x^2 + 2x + 8)$

2 **a** $(x + 4)(6x^2 + 3x + 2)$ **b** $(x + 2)(4x^2 + x - 5)$
c $(x + 3)(2x^2 - 2x - 3)$ **d** $(x - 6)(2x^2 - 3x - 4)$
e $(x + 6)(-5x^2 + 3x + 5)$ **f** $(x - 2)(-4x^2 + x - 1)$

3 **a** $x^3 + 3x^2 - 4x + 1$ **b** $4x^3 + 2x^2 - 3x - 5$
c $-3x^3 + 3x^2 - 4x - 7$ **d** $-5x^4 + 2x^3 + 4x^2 - 3x + 7$

4 **a** $x^3 + 2x^2 - 5x + 4$ **b** $x^3 - x^2 + 3x - 1$
c $2x^3 + 5x + 2$ **d** $3x^4 + 2x^3 - 5x^2 + 3x + 6$
e $2x^4 - 2x^3 + 3x^2 + 4x - 7$ **f** $4x^4 - 3x^3 - 2x^2 + 6x - 5$
g $5x^3 + 12x^2 - 6x - 2$ **h** $3x^4 + 5x^3 + 6$

5 **a** $x^2 - 2x + 5$ **b** $2x^2 - 6x + 1$
c $-3x^2 - 12x + 2$

6 **a** $x^2 + 4x + 12$ **b** $2x^2 - x + 5$
c $-3x^2 + 5x + 10$

7 f(−2) = −8 + 8 + 10 − 10 = 0, so $(x + 2)$ is a factor of $x^3 + 2x^2 - 5x - 10$. Divide $x^3 + 2x^2 - 5x - 10$ by $(x + 2)$ to give $(x^2 - 5)$. So $x^3 + 2x^2 - 5x - 10 = (x + 2)(x^2 - 5)$.

8 **a** −8 **b** −7 **c** −12

9 f(1) = 3 − 2 + 4 = 5

10 f(−1) = 3 + 8 + 10 + 3 − 25 = −1

11 $(x + 4)(5x^2 - 20x + 7)$

12 $3x^2 + 6x + 4$

13 $x^2 + x + 1$

14 $x^3 - 2x^2 + 4x - 8$

15 14

16 **a** −200 **b** $(x + 2)(x - 7)(3x + 1)$

17 **a** **i** 30 **ii** 0 **b** $x = -3, x = -4, x = 1$

18 **a** $a = 1, b = 2, c = -3$
b $f(x) = (2x - 1)(x + 3)(x - 1)$
c $x = 0.5, x = -3, x = 1$

19 **a** $a = 3, b = 2, c = 1$
b Quadratic has no real solutions so only $(4x - 1)$ is a solution