M1 Chapter 9: Constant Acceleration

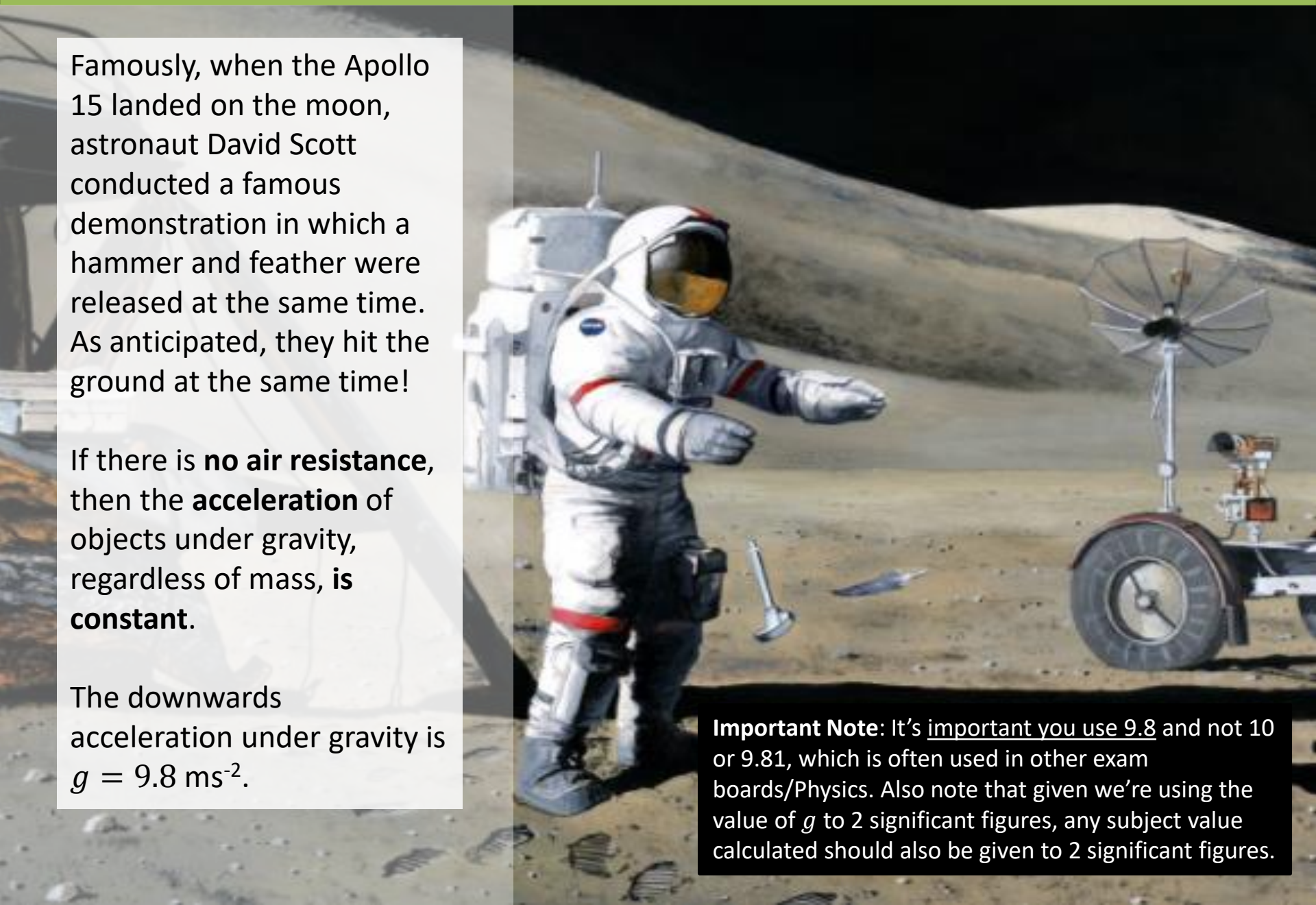
Vertical Motion Under Gravity

Vertical Motion Under Gravity

Famously, when the Apollo 15 landed on the moon, astronaut David Scott conducted a famous demonstration in which a hammer and feather were released at the same time. As anticipated, they hit the ground at the same time!

If there is **no air resistance**, then the **acceleration** of objects under gravity, regardless of mass, **is constant**.

The downwards acceleration under gravity is $g = 9.8\ \text{ms}^{-2}$.



Important Note: It's important you use 9.8 and not 10 or 9.81, which is often used in other exam boards/Physics. Also note that given we're using the value of g to 2 significant figures, any subject value calculated should also be given to 2 significant figures.

Example

[Textbook] A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find:
(a) the time the book takes to reach the floor,
(b) the speed with which the book strikes the floor.

a

$s =$? $u =$? $a =$?
$t =$?

?

b

?

At this stage, it's hugely important you consider **what direction is considered as 'positive'** and then be consistent in your suvat table. If 'up' was positive, then $a = -9.8$. If 'down' is positive, then $a = +9.8$. The direction does not matter provided that you are consistent with each letter of *suvat*.

A good rule of thumb is to make the direction of motion (initial velocity) positive. If unknown, make right and up the positive directions.

Here the entire motion is down so g=+9.8 is convenient.

For up and down projectiles g=-9.8 is convenient.

As per previous slides, quote only to 2 or 3 significant figures. <u>You may be penalised</u> if you quote more!

Example

[Textbook] A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find:
(a) the time the book takes to reach the floor,
(b) the speed with which the book strikes the floor.

a

$s = 1.4, \quad u = 0, \quad a = +9.8, \quad t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$1.4 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$t = 0.53$$

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Here the entire motion is down so g=+9.8 is convenient.

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b

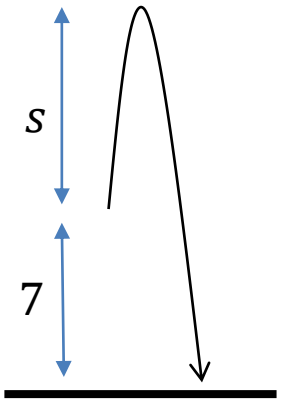
$$v^2 = u^2 + 2as$$

$$v = \sqrt{0^2 + 2 \times 9.8 \times 1.4} = 5.2$$

As per previous slides, quote only to 2 or 3 significant figures. <u>You may be penalised</u> if you quote more!

Further Example

[Textbook] A ball is projected vertically upwards, from a point X which is 7m above the ground, with speed 21 ms^{-1}. Find
(a) the greatest height above the ground reached by the ball,
(b) the time of flight of the ball



a ? $s =?,$ $u =$? $v =$? $a =$?

?

b $s =$? $u =$? $a =$? $t =?$

?

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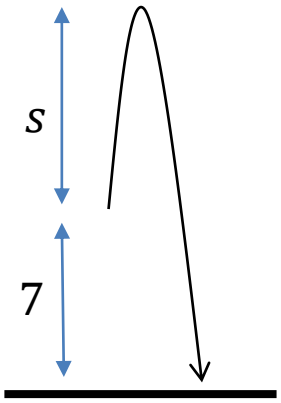


a ↑ $s = ?, \quad u = 21, \quad v = 0, \quad a = -9.8,$

$$v^2 = u^2 + 2as$$
$$0 = 21^2 + (2 \times -9.8 \times s)$$
$$s = 22.5$$

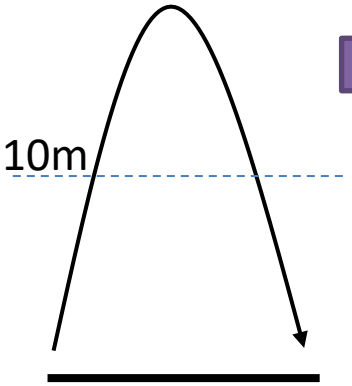
Therefore greatest height is $22.5 + 7 = 29.5$ m

b ↑ $s = -7, \quad u = 21, \quad a = -9.8, \quad t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$-7 = 21t - 4.9t^2$$
...
$$t = 4.5965 \; or \; -0.3108$$
Therefore time of flight is 4.6 s (2sf)

A further common type of question…

A ball is projected vertically upwards from ground level at a speed of 20 ms^{-1}.
Determine the amount of time the ball is at least 10m above ground level.



a ? $s =$? $u =$? $a =$? $t =$?

?

Froculator Tip: Be sure to use the quadratic solver on your calculator (within 'Equation' mode on the ClassWiz).

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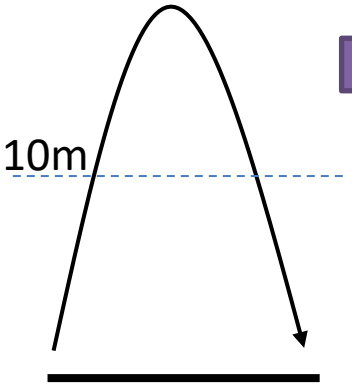

a

$s = 10, \quad u = 20, \quad a = -9.8, \quad t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$10 = 20t - 4.9t^2$$

$$t = 0.5834 \; or \; t = 3.4983$$

Therefore time above 10m:

$3.4983 - 0.5834 = 2.9$ s (2sf)

Froculator Tip: Be sure to use the quadratic solver on your calculator (within 'Equation' mode on the ClassWiz).

Test Your Understanding

Edexcel M1 May 2013(R) Q4

At time $t = 0$, two balls A and B are projected vertically upwards. The ball A is projected vertically upwards with speed 2 m s^{-1} from a point 50 m above the horizontal ground. The ball B is projected vertically upwards from the ground with speed 20 m s^{-1}. At time $t = T$ seconds, the two balls are at the same vertical height, h metres, above the ground. The balls are modelled as particles moving freely under gravity. Find

(a) the value of T,

(5)

(b) the value of h.

(2)

(a)

?

(b)

?

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(a) the value of T,

(5)

(b) the value of h.

(2)

(a)

Use of $s = ut + \frac{1}{2}at^2$ — M1

$-2t + \frac{1}{2}gt^2$ (+ or – 50) — A1

$20t - \frac{1}{2}gt^2$ (+ or – 50) — A1

$50 = -2T + \frac{1}{2}gT^2 + 20T - \frac{1}{2}gT^2 = 18T$ — M1

$T = \frac{50}{18} = 2.777.... = 2.8$ or better — A1

(5)

(b)

$h = 20 \times T - 4.9 \times T^2 = 17.74.... \approx 17.7$ (18 to 2 s.f.) — M1A1

(use of 2.8 gives 17.584)

(2)

[7]

Exercise 9.5 Vertical projectiles

Pearson Stats/Mechanics Year 1
Pages 64-65

Homework Exercise

1 A cliff diver jumps from a point 28 m above the surface of the water. Modelling the diver as a particle moving freely under gravity with initial velocity 0, find:

a the time taken for the diver to hit the water

b the speed of the diver when he hits the water.

2 A particle is projected vertically upwards with speed $20\,\text{m s}^{-1}$ from a point on the ground. Find the time of flight of the particle.

3 A ball is thrown vertically downward from the top of a tower with speed $18\,\text{m s}^{-1}$. It reaches the ground in 1.6 s. Find the height of the tower.

4 A pebble is catapulted vertically upwards with speed $24\,\text{m s}^{-1}$. Find:

a the greatest height above the point of projection reached by the pebble

b the time taken to reach this height.

5 A ball is projected upwards from a point which is 4 m above the ground with speed $18\,\text{m s}^{-1}$. Find:

a the speed of the ball when it is 15 m above its point of projection

b the speed with which the ball hits the ground.

6 A particle P is projected vertically downwards from a point 80 m above the ground with speed $4\,\text{m s}^{-1}$. Find:

a the speed with which P hits the ground

b the time P takes to reach the ground.

7 A particle P is projected vertically upwards from a point X. Five seconds later P is moving downwards with speed $10\,\text{m s}^{-1}$. Find:

a the speed of projection of P

b the greatest height above X attained by P during its motion.

Homework Exercise

8 A ball is thrown vertically upwards with speed 21 m s^{-1}. It hits the ground 4.5 s later. Find the height above the ground from which the ball was thrown.

9 A stone is thrown vertically upward from a point which is 3 m above the ground, with speed 16 m s^{-1}. Find:

a the time of flight of the stone

b the total distance travelled by the stone.

10 A particle is projected vertically upwards with speed 24.5 m s^{-1}. Find the total time for which it is 21 m or more above its point of projection.

11 A particle is projected vertically upwards from a point O with speed u m s^{-1}. Two seconds later it is still moving upwards and its speed is $\frac{1}{3}u$ m s^{-1}. Find:

a the value of u **(3 marks)**

b the time from the instant that the particle leaves O to the instant that it returns to O. **(4 marks)**

Problem-solving

Use $v = u + at$ and substitute $v = \frac{1}{3}u$.

12 A ball A is thrown vertically downwards with speed 5 m s^{-1} from the top of a tower block 46 m above the ground. At the same time as A is thrown downwards, another ball B is thrown vertically upwards from the ground with speed 18 m s^{-1}. The balls collide. Find the distance of the point where A and B collide from the point where A was thrown. **(5 marks)**

13 A ball is released from rest at a point which is 10 m above a wooden floor. Each time the ball strikes the floor, it rebounds with three-quarters of the speed with which it strikes the floor. Find the greatest height above the floor reached by the ball

a the first time it rebounds from the floor **(3 marks)**

b the second time it rebounds from the floor. **(4 marks)**

Problem-solving

Consider each bounce as a separate motion.

Homework Exercise

Challenge

1 A particle P is projected vertically upwards from a point O with speed $12\ \text{m s}^{-1}$. One second after P has been projected from O, another particle Q is projected vertically upwards from O with speed $20\ \text{m s}^{-1}$. Find: **a** the time between the instant that P is projected from O and the instant when P and Q collide, **b** the distance of the point where P and Q collide from O.

2 A stone is dropped from the top of a building and two seconds later another stone is thrown vertically downwards at a speed of $25\ \text{m s}^{-1}$. Both stones reach the ground at the same time. Find the height of the building.

Homework Answers

1 **a** 2.4 s **b** 23.4 m s^{-1}
2 4.1 s (2 s.f.)
3 41 m (2 s.f.)
4 **a** 29 m (2 s.f.) **b** 2.4 s (2 s.f.)
5 **a** 5.5 m s^{-1} (2 s.f.) **b** 20 m s^{-1} (2 s.f.)
6 **a** 40 m s^{-1} (2 s.f.) **b** 3.7 s (2 s.f.)
7 **a** 39 m s^{-1} **b** 78 m (2 s.f.)
8 4.7 m (2 s.f.)
9 **a** 3.4 s (2 s.f.) **b** 29 m (2 s.f.)
10 2.8 s (2 s.f.)
11 **a** $u = 29$ (2 s.f.) **b** 6 s
12 30 m (2 s.f.)
13 **a** 5.6 m (2 s.f.) **b** 3.2 m (2 s.f.)

Challenge
1 **a** 1.4 s (2 s.f.) **b** 7.2 m (2 s.f.)
2 155 m (3 s.f.)