P1 Chapter 1: Algebraic Expressions

Chapter Practice

Key Points

1 You can use the laws of indices to simplify powers of the **same base**.

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$

2 Factorising is the opposite of expanding brackets.

3 A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.

4 $x^2 - y^2 = (x + y)(x - y)$

5 You can use the laws of indices with any rational power.

- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$

6 You can manipulate surds using these rules:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

7 The rules to rationalise denominators are:

- Fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.
- Fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the numerator and denominator by $a - \sqrt{b}$.
- Fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the numerator and denominator by $a + \sqrt{b}$.

Chapter Exercises

1 Simplify:

a $y^3 \times y^5$ **b** $3x^2 \times 2x^5$ **c** $(4x^2)^3 \div 2x^5$ **d** $4b^2 \times 3b^3 \times b^4$

2 Expand and simplify if possible:

a $(x + 3)(x - 5)$ **b** $(2x - 7)(3x + 1)$ **c** $(2x + 5)(3x - y + 2)$

3 Expand and simplify if possible:

a $x(x + 4)(x - 1)$ **b** $(x + 2)(x - 3)(x + 7)$ **c** $(2x + 3)(x - 2)(3x - 1)$

4 Expand the brackets:

a $3(5y + 4)$ **b** $5x^2(3 - 5x + 2x^2)$ **c** $5x(2x + 3) - 2x(1 - 3x)$ **d** $3x^2(1 + 3x) - 2x(3x - 2)$

5 Factorise these expressions completely:

a $3x^2 + 4x$ **b** $4y^2 + 10y$ **c** $x^2 + xy + xy^2$ **d** $8xy^2 + 10x^2y$

6 Factorise:

a $x^2 + 3x + 2$ **b** $3x^2 + 6x$ **c** $x^2 - 2x - 35$ **d** $2x^2 - x - 3$

e $5x^2 - 13x - 6$ **f** $6 - 5x - x^2$

7 Factorise:

a $2x^3 + 6x$ **b** $x^3 - 36x$ **c** $2x^3 + 7x^2 - 15x$

8 Simplify:

a $9x^3 \div 3x^{-3}$ **b** $(4^{\frac{3}{2}})^{\frac{1}{3}}$ **c** $3x^{-2} \times 2x^4$ **d** $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

Chapter Exercises

9 Evaluate:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **b** $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

10 Simplify:

a $\frac{3}{\sqrt{63}}$ **b** $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

11 **a** Find the value of $35x^2 + 2x - 48$ when $x = 25$.

b By factorising the expression, show that your answer to part **a** can be written as the product of two prime factors.

12 Expand and simplify if possible:

a $\sqrt{2}(3 + \sqrt{5})$ **b** $(2 - \sqrt{5})(5 + \sqrt{3})$ **c** $(6 - \sqrt{2})(4 - \sqrt{7})$

13 Rationalise the denominator and simplify:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2} - 1}$ **c** $\frac{3}{\sqrt{3} - 2}$ **d** $\frac{\sqrt{23} - \sqrt{37}}{\sqrt{23} + \sqrt{37}}$ **e** $\frac{1}{(2 + \sqrt{3})^2}$ **f** $\frac{1}{(4 - \sqrt{7})^2}$

14 **a** Given that $x^3 - x^2 - 17x - 15 = (x + 3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c.

b Hence, fully factorise $x^3 - x^2 - 17x - 15$.

15 Given that $y = \frac{1}{64}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(1 mark)**

b $4y^{-1}$ **(1 mark)**

16 Show that $\frac{5}{\sqrt{75} - \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. **(5 marks)**

Chapter Exercises

17 Expand and simplify $(\sqrt{11} - 5)(5 - \sqrt{11})$. **(2 marks)**

18 Factorise completely $x - 64x^3$. **(3 marks)**

19 Express 27^{2x+1} in the form 3^y, stating y in terms of x. **(2 marks)**

20 Solve the equation $8 + x\sqrt{12} = \dfrac{8x}{\sqrt{3}}$

Give your answer in the form $a\sqrt{b}$ where a and b are integers. **(4 marks)**

21 A rectangle has a length of $(1 + \sqrt{3})$ cm and area of $\sqrt{12}$ cm^2.
Calculate the width of the rectangle in cm.
Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

22 Show that $\dfrac{(2 - \sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$. **(2 marks)**

23 Given that $243\sqrt{3} = 3^a$, find the value of a. **(3 marks)**

24 Given that $\dfrac{4x^3 + x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$, write down the value of a and the value of b. **(2 marks)**

Challenge

a Simplify $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$.

b Hence show that $\dfrac{1}{\sqrt{1} + \sqrt{2}} + \dfrac{1}{\sqrt{2} + \sqrt{3}} + \dfrac{1}{\sqrt{3} + \sqrt{4}} + \ldots + \dfrac{1}{\sqrt{24} + \sqrt{25}} = 4$

Chapter Answers

1 **a** y^8 **b** $6x^7$ **c** $32x$ **d** $12b^9$

2 **a** $x^2 - 2x - 15$ **b** $6x^2 - 19x - 7$

c $6x^2 - 2xy + 19x - 5y + 10$

3 **a** $x^3 + 3x^2 - 4x$ **b** $x^3 + 6x^2 - 13x - 42$

c $6x^3 - 5x^2 - 17x + 6$

4 **a** $15y + 12$ **b** $15x^2 - 25x^3 + 10x^4$

c $16x^2 + 13x$ **d** $9x^3 - 3x^2 + 4x$

5 **a** $x(3x + 4)$ **b** $2y(2y + 5)$

c $x(x + y + y^2)$ **d** $2xy(4y + 5x)$

6 **a** $(x + 1)(x + 2)$ **b** $3x(x + 2)$

c $(x - 7)(x + 5)$ **d** $(2x - 3)(x + 1)$

e $(5x + 2)(x - 3)$ **f** $(1 - x)(6 + x)$

7 **a** $2x(x^2 + 3)$ **b** $x(x + 6)(x - 6)$

c $x(2x - 3)(x + 5)$

8 **a** $3x^6$ **b** 2 **c** $6x^2$ **d** $\frac{1}{2}x^{-\frac{1}{3}}$

9 **a** $\frac{4}{9}$ **b** $\pm\frac{3375}{4913}$

10 **a** $\frac{\sqrt{7}}{7}$ **b** $4\sqrt{5}$

11 **a** 21 877

b $(5x + 6)(7x - 8)$

When $x = 25$, $5x + 6 = 131$ and $7x - 8 = 167$; both 131 and 167 are prime numbers.

12 **a** $3\sqrt{2} + \sqrt{10}$ **b** $10 + 2\sqrt{3} - 5\sqrt{5} - \sqrt{15}$

c $24 - 6\sqrt{7} - 4\sqrt{2} + \sqrt{14}$

13 **a** $\frac{\sqrt{3}}{3}$ **b** $\sqrt{2} + 1$ **c** $-3\sqrt{3} - 6$

d $\frac{30 - \sqrt{851}}{-7}$ **e** $7 - 4\sqrt{3}$ **f** $\frac{23 + 8\sqrt{7}}{81}$

14 **a** $b = -4$ and $c = -5$ **b** $(x + 3)(x - 5)(x + 1)$

15 **a** $\frac{1}{4}x$ **b** $256x^{-3}$

16 $\frac{5}{\sqrt{75} - \sqrt{50}} = \frac{1}{\sqrt{3} - \sqrt{2}} = \sqrt{3} + \sqrt{2}$

17 $-36 + 10\sqrt{11}$

18 $x(1 + 8x)(1 - 8x)$

19 $y = 6x + 3$

20 $4\sqrt{3}$

21 $3 - \sqrt{3}$ cm

22 $\frac{4 - 4x^{\frac{1}{2}} + x^1}{x^{\frac{1}{2}}} = 4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$

23 $\frac{11}{2}$

24 $4x^{\frac{5}{2}} + x^2$, $a = \frac{5}{2}$ $b = 2$

Challenge

a $a - b$

b $\frac{(\sqrt{1} - \sqrt{2}) + (\sqrt{2} - \sqrt{3}) + \ldots + (\sqrt{24} - \sqrt{25})}{-1} = \sqrt{25} - \sqrt{1} = 4$