M2 Chapter 6: Projectiles

Chapter Practice

Key Points

1 The **horizontal** motion of a projectile is modelled as having **constant velocity** ($a = 0$). You can use the formula $s = vt$.

2 The **vertical** motion of a projectile is modelled as having **constant acceleration** due to gravity ($a = g$).

3 When a particle is projected with initial velocity U, at an angle α above the horizontal:

- The **horizontal component** of the initial velocity is $U\cos\alpha$
- The **vertical component** of the initial velocity is $U\sin\alpha$

4 A projectile reaches its point of greatest height when the vertical component of its velocity is equal to 0.

5 For a particle which is projected from a point on a horizontal plane with an initial velocity U at an angle α above the horizontal, and that moves freely under gravity:

- Time of flight = $\dfrac{2U\sin\alpha}{g}$
- Time to reach greatest height = $\dfrac{U\sin\alpha}{g}$
- Range on horizontal plane = $\dfrac{U^2\sin 2\alpha}{g}$
- Equation of trajectory: $y = x\tan\alpha - gx^2\dfrac{(1+\tan^2\alpha)}{2U^2}$

where y is the vertical height of the particle, x is the horizontal distance from the point of projection, and g is the acceleration due to gravity.

Chapter Exercises

Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ unless otherwise stated.

1 A particle P is projected from a point O on a horizontal plane with speed $42\,\text{m s}^{-1}$ and with angle of elevation 45°. After projection, the particle moves freely under gravity until it strikes the plane. Find:

a the greatest height above the plane reached by P

b the time of flight of P.

2 A stone is thrown horizontally with speed $21\,\text{m s}^{-1}$ from a point P on the edge of a cliff h metres above sea level. The stone lands in the sea at a point Q, where the horizontal distance of Q from the cliff is 56 m.

Calculate the value of h.

3 A ball is thrown from a window above a horizontal lawn. The velocity of projection is $15\,\text{m s}^{-1}$ and the angle of elevation is α, where $\tan\alpha = \frac{4}{3}$. The ball takes 4 s to reach the lawn. Find:

a the horizontal distance between the point of projection and the point where the ball hits the lawn **(3 marks)**

b the vertical height above the lawn from which the ball was thrown. **(3 marks)**

4 A projectile is fired with velocity $40\,\text{m s}^{-1}$ at an angle of elevation of 30° from a point A on horizontal ground. The projectile moves freely under gravity until it reaches the ground at the point B. Find:

a the distance AB **(5 marks)**

b the speed of the projectile at the first instant when it is 15 m above the ground. **(5 marks)**

Chapter Exercises

5 A particle P is projected from a point on a horizontal plane with speed U at an angle of elevation θ.

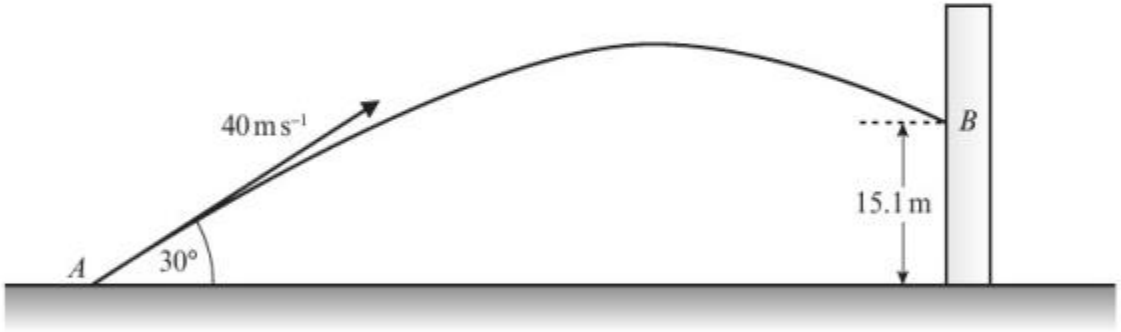
a Show that the range of the projectile is $\frac{U^2 \sin 2\theta}{g}$. **(6 marks)**

b Hence find, as θ varies, the maximum range of the projectile. **(2 marks)**

c Given that the range of the projectile is $\frac{2U^2}{3g}$, find the two possible value of θ.

Give your answers to the nearest 0.1°. **(3 marks)**

6



A golf ball is driven from a point A with a speed of 40 m s^{-1} at an angle of elevation of 30°. On its downward flight, the ball hits an advertising hoarding at a height 15.1 m above the level of A, as shown in the diagram above. Find:

a the time taken by the ball to reach its greatest height above A **(3 marks)**

b the time taken by the ball to travel from A to B **(6 marks)**

c the speed with which the ball hits the hoarding. **(5 marks)**

Chapter Exercises

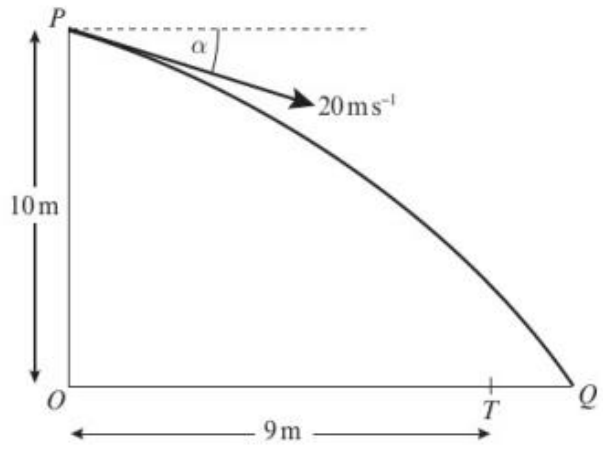
7 In this question use $g = 10\,\text{m s}^{-2}$.
A boy plays a game at a fairground. He needs to throw a ball through a hole in a vertical target to win a prize. The motion of the ball is modelled as that of a particle moving freely under gravity. The ball moves in a vertical plane which is perpendicular to the plane of the target. The boy throws the ball horizontally at the same height as the hole with a speed of $10\,\text{m s}^{-1}$. It hits the target at a point 20 cm below the hole.

a Find the horizontal distance from the point where the ball was thrown to the target. **(4 marks)**

The boy throws the ball again with the same speed and at the same distance from the target.

b Work out the possible angles above the horizontal the boy could throw the ball so that it passes through the hole. **(6 marks)**

8 In this question use $g = 10\,\text{m s}^{-2}$.
A stone is thrown from a point P at a target, which is on horizontal ground. The point P is 10 m above the point O on the ground. The stone is thrown from P with speed $20\,\text{m s}^{-1}$ at an angle of α below the horizontal, where $\tan\alpha = \frac{3}{4}$.



The stone is modelled as a particle and the target as a point T. The distance OT is 9 m. The stone misses the target and hits the ground at the point Q, where OTQ is a straight line, as shown in the diagram. Find:

a the time taken by the ball to travel from P to Q **(5 marks)**

b the distance TQ. **(4 marks)**

The point A is on the path of the ball vertically above T.

c Find the speed of the ball at A. **(5 marks)**

Chapter Exercises

9 A vertical mast is 32 m high. Two balls P and Q are projected simultaneously. Ball P is projected horizontally from the top of the mast with speed $18\,\text{m s}^{-1}$. Ball Q is projected from the bottom of the mast with speed $30\,\text{m s}^{-1}$ at an angle α above the horizontal. The balls move freely under gravity in the same vertical plane and collide in mid-air. By considering the horizontal motion of each ball,

a prove that $\cos\alpha = \frac{3}{5}$ **(4 marks)**

b Find the time which elapses between the instant when the balls are projected and the instant when they collide. **(4 marks)**

Challenge

A cruise ship is 250 m long, and is accelerating forwards in a straight line at a constant rate of $1.5\,\text{m s}^{-2}$. A golfer stands at the stern (back) of the cruise ship and hits a golf ball towards the bow (front). Given that the golfer hits the golf ball at an angle of elevation of 60°, and that the ball lands directly on the bow of the cruise ship, find the speed, v, with which the golfer hits the ball.

Chapter Answers

1 **a** 45 m **b** 6.1 s

2 $h = 35$ (2 s.f.)

3 **a** 36 m **b** 30 m (2 s.f.)

4 **a** 140 m (2 s.f.) **b** 36 ms^{-1} (2 s.f.)

5 **a** R(↑): $s = U\sin\theta\, t - \frac{g}{2}t^2$

When particle strikes plane, $s = 0 = t(U\sin\theta - \frac{g}{2}t)$

So $t = 0$ or $t = \frac{2U\sin\theta}{g}$

R(→): $s = Ut = U\cos\theta\left(\frac{2U\sin\theta}{g}\right) = \frac{U^2\sin 2\theta}{g}$

b $\frac{U^2}{g}$ **c** 20.9°, 69.1° (nearest 0.1°)

6 **a** 2.0 s (2 s.f.) **b** 3.1 s (2 s.f.) **c** 36 ms^{-1} (2 s.f.)

7 **a** 2 m **b** 5.77° or 84.2°

8 **a** 0.65 s **b** 1.5 m **c** 23.8 ms^{-1}

9 **a** Particle P: $x = 18t$, Particle Q: $x = 30\cos\alpha\, t$

When particles collide: $18t = 30\cos\alpha\, t \Rightarrow \cos\alpha = \frac{3}{5}$

b $\frac{4}{3}$ s

Challenge

62 ms^{-1}