P1 Chapter 1: Algebra

Fractional Indices

Negative and Fractional Indices

$$a^0 = 1$$
$$a^{\frac{1}{m}} = \sqrt[m]{a}$$
$$a^{\frac{n}{m}} = (\sqrt[m]{a})^n$$
$$a^{-m} = \frac{1}{a^m}$$

Note: $\sqrt{9}$ only means the <u>positive</u> square root of 9, i.e. 3 not -3.
Otherwise, what would be the point of the $\pm$ in the quadratic formula before the $\sqrt{b^2 - 4ac}$?

Prove that $x^{\frac{1}{2}} = \sqrt{x}$

?

Evaluate $27^{-\frac{1}{3}}$

?

Evaluate $32^{\frac{2}{5}}$

?

Simplify $\left(\frac{1}{9}x^6 y\right)^{\frac{1}{2}}$

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Evaluate $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

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If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form kb^n where k, n are constants.

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Prove that $x^{\frac{1}{2}} = \sqrt{x}$

$$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^1$$
But $\sqrt{x} \times \sqrt{x} = x$
$$\therefore x^{\frac{1}{2}} = \sqrt{x}$$

Evaluate $27^{-\frac{1}{3}}$

$$= \frac{1}{27^{\frac{1}{3}}} = \frac{1}{3}$$

Evaluate $32^{\frac{2}{5}}$

$$= 2^2 = 4$$

Simplify $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

$$= \frac{1}{3}x^3y^{\frac{1}{2}}$$

Evaluate $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

$$= \left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form kb^n where k, n are constants.

$$3b^{-2} = 3\left(\frac{1}{9}a^2\right)^{-2}$$
$$= 3(81a^{-4}) = 243a^{-4}$$

Writing a surd using indices

If $9\sqrt{3} = 3^k$, determine the value of k.

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The key here is to write everything as powers with a consistent base, in this case, 3.

$$9\sqrt{3} = 9 \times \sqrt{3}$$
$$= 3^2 \times 3^{\frac{1}{2}}$$
$$= 3^{\frac{5}{2}}$$
$$\therefore k = \frac{5}{2}$$

Note: In algebra we like to avoid mixed numbers. So don't write $3^{2\frac{1}{2}}$

Further Examples

[Edexcel IGCSE May14-4H]
Given that
$$\left(2^{\frac{1}{2}}\right)^n = \frac{2^x}{8^y}$$
Express n in terms of x and y.

?

[Edexcel IAL C12 Jan 2019 Q2c] Given that $y = 2^x$, express the following in terms of y.
$$\frac{1}{4^{2x-3}}$$
Write your expression in its simplest form.

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$$\frac{1}{4^{2x-3}}$$

Write your expression in its simplest form.

$$2^{\frac{1}{2}n} = \frac{2^x}{(2^3)^y}$$
$$= \frac{2^x}{2^{3y}} = 2^{x-3y}$$
$$\therefore \frac{1}{2}n = x - 3y$$
$$n = 2x - 6y$$

Write 8 as a power of 2 (putting a bracket around it), for consistency of base with the other powers.

$$\frac{1}{4^{2x-3}} = 4^{-(2x-3)}$$
$$= 4^{-2x+3}$$
$$= (2^2)^{-2x+3}$$
$$= 2^{-4x+6}$$
$$= 2^{-4x} \times 2^6$$
$$= (2^x)^{-4} \times 64$$
$$= 64y^{-4} \quad \text{or} \quad \frac{64}{y^4}$$

Test Your Understanding

Edexcel Paper 2 – May 2019

Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x. **(3)**

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$2^x \times 2^{2y} = 2^{-\frac{3}{2}}$	B1
$2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x+2y = -\frac{3}{2} \Rightarrow y = \ldots$	M1
E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1

Exercise 1.4

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[MAT 2007 1A]

Let r and s be integers. Then

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

is an integer if

- $r + s \leq 0$
- $s \leq 0$
- $r \leq 0$
- $r \geq s$

?

Exercise 1.4

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- $r + s \leq 0$
- $s \leq 0$
- $r \leq 0$
- $r \geq s$

$$= \frac{2^{r+s} \times 3^{r+s} \times 2^{2r-2s} \times 3^{r-s}}{2^{3r} \times 3^{2r+4s}}$$

$$= 2^{-s} \times 3^{-4s}$$

This is an integer only if $s \leq 0$.

Homework Exercise

1 Simplify:

a $x^3 \div x^{-2}$ **b** $x^5 \div x^7$ **c** $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$ **e** $(x^3)^{\frac{5}{3}}$ **f** $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$ **h** $5x^{\frac{7}{5}} \div x^{\frac{2}{5}}$ **i** $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$ **k** $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$ **l** $\dfrac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate:

a $25^{\frac{1}{2}}$ **b** $81^{\frac{3}{2}}$ **c** $27^{\frac{1}{3}}$

d 4^{-2} **e** $9^{-\frac{1}{2}}$ **f** $(-5)^{-3}$

g $\left(\frac{3}{4}\right)^0$ **h** $1296^{\frac{3}{4}}$ **i** $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

j $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ **k** $\left(\frac{6}{5}\right)^{-1}$ **l** $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$ **b** $\dfrac{5x^3 - 2x^2}{x^5}$ **c** $(125x^{12})^{\frac{1}{3}}$ **d** $\dfrac{x + 4x^3}{x^3}$

e $\dfrac{2x + x^2}{x^4}$ **f** $\left(\dfrac{4}{9}x^4\right)^{\frac{1}{2}}$ **g** $\dfrac{9x^2 - 15x^5}{3x^3}$ **h** $\dfrac{5x + 3x^2}{15x^3}$

4 a Find the value of $81^{\frac{1}{4}}$. **(1 mark)**

b Simplify $x(2x^{-\frac{1}{3}})^4$. **(2 marks)**

5 Given that $y = \frac{1}{8}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(2 marks)**

b $\frac{1}{2}y^{-2}$ **(2 marks)**

Homework Answers

1	**a**	x^5	**b**	x^{-2}	**c**	x^4	**d**	x^3
	e	x^5	**f**	$12x^0 = 12$	**g**	$3x^{\frac{1}{2}}$	**h**	$5x$
	i	$6x^{-1}$	**j**	$x^{\frac{5}{6}}$	**k**	$x^{\frac{17}{6}}$	**l**	$x^{\frac{1}{6}}$
2	**a**	5	**b**	729	**c**	3	**d**	$\frac{1}{16}$
	e	$\frac{1}{3}$	**f**	$\frac{-1}{125}$	**g**	1	**h**	216
	i	$\frac{125}{64}$	**j**	$\frac{9}{4}$	**k**	$\frac{5}{6}$	**l**	$\frac{64}{49}$
3	**a**	$8x^5$	**b**	$\frac{5}{x^2} - \frac{2}{x^3}$	**c**	$5x^4$		
	d	$\frac{1}{x^2} + 4$	**e**	$\frac{2}{x^3} + \frac{1}{x^2}$	**f**	$\frac{8}{27}x^6$		
	g	$\frac{3}{x} - 5x^2$	**h**	$\frac{1}{3x^2} + \frac{1}{5x}$				
4	**a**	3	**b**	$\frac{16}{\sqrt[3]{x}}$				
5	**a**	$\frac{x}{2}$	**b**	$\frac{32}{x^6}$				