
P2 Chapter 6: CoAngle Trigonometry

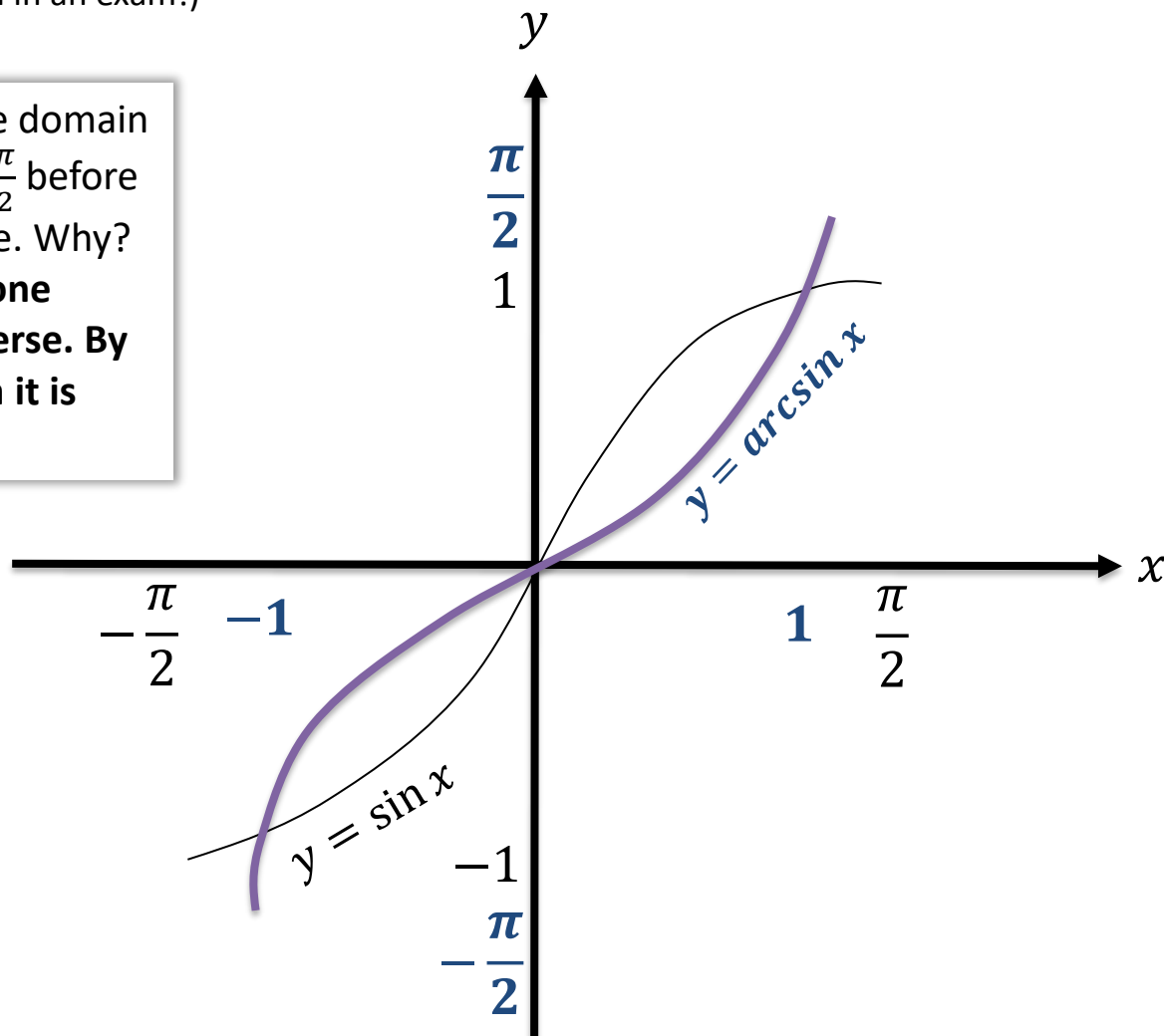
Arc Functions

Inverse Trig Functions

You need to know how to sketch $y = \arcsin x$, $y = \arccos x$, $y = \arctan x$.

(Yes, you could be asked in an exam!)

We have to restrict the domain of $\sin x$ to $-\frac{\pi}{2} \leq x < \frac{\pi}{2}$ before we can find the inverse. Why? **Because only one-to-one functions have an inverse. By restricting the domain it is now one-to-one.**



Inverse Trig Functions

$$y = \arccos x$$

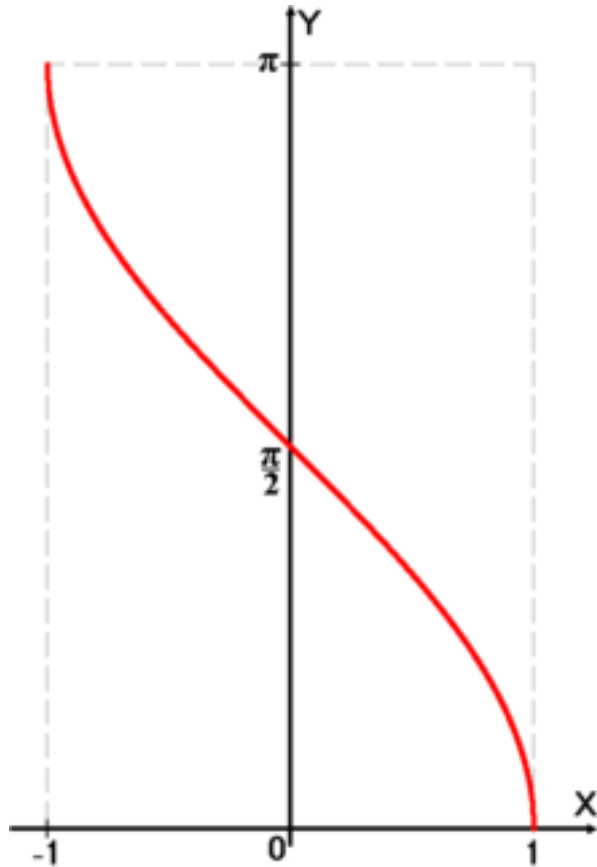
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$$y = \arctan x$$

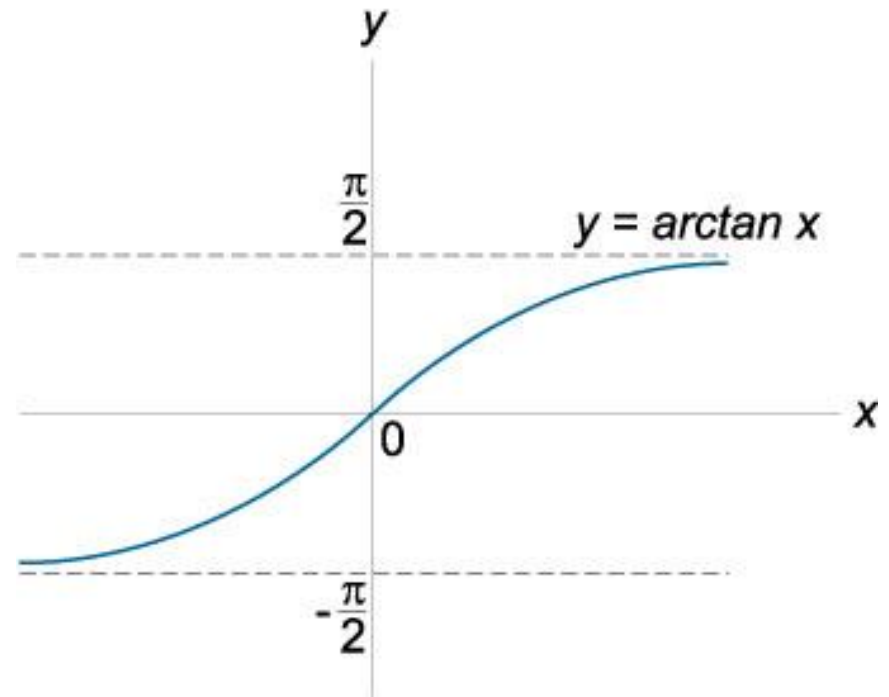
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Inverse Trig Functions

$$y = \arccos x$$



$$y = \arctan x$$



Note that this graph has asymptotes.

Evaluating inverse trig functions

[Textbook] Work out, in radians, the values of:

- a) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$
- b) $\arccos(-1)$
- c) $\arctan(\sqrt{3})$

You can simply use the $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ buttons on your calculator.

If you don't have a calculator, just use the *sin*, *cos*, *tan* graphs backwards.

?

?

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Evaluating inverse trig functions

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$$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

$$\arccos(-1) = \pi$$

$$\arctan(\sqrt{3}) = \frac{\pi}{3}$$

You can simply use the $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ buttons on your calculator.

If you don't have a calculator, just use the *sin*, *cos*, *tan* graphs backwards.

One Final Problem...

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8. (ii) Given that

$$y = \arccos x, \quad -1 \leq x \leq 1 \quad \text{and} \quad 0 \leq y \leq \pi,$$

(a) express $\arcsin x$ in terms of y .

(2)

(b) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π .

(1)

?

Fewer than 10%
of candidates got
this part right.

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(1)

$$y = \arccos x$$

$$x = \cos y = \sin\left(\frac{\pi}{2} - y\right)$$

$$\arcsin x = \frac{\pi}{2} - y$$

$$\arccos x + \arcsin x$$

$$= y + \frac{\pi}{2} - y = \frac{\pi}{2}$$

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Exercise 6.5

Pearson Pure Mathematics Year 2/AS

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Homework Exercise

In this exercise, all angles are given in radians.

1 Without using a calculator, work out, giving your answer in terms of π :

- | | | | |
|---|---|--|---|
| a $\arccos(0)$ | b $\arcsin(1)$ | c $\arctan(-1)$ | d $\arcsin(-\frac{1}{2})$ |
| e $\arccos(-\frac{1}{\sqrt{2}})$ | f $\arctan(-\frac{1}{\sqrt{3}})$ | g $\arcsin(\sin \frac{\pi}{3})$ | h $\arcsin(\sin \frac{2\pi}{3})$ |

2 Find:

- | | | |
|---|---|-------------------------------------|
| a $\arcsin(\frac{1}{2}) + \arcsin(-\frac{1}{2})$ | b $\arccos(\frac{1}{2}) - \arccos(-\frac{1}{2})$ | c $\arctan(1) - \arctan(-1)$ |
|---|---|-------------------------------------|

3 Without using a calculator, work out the values of:

- | | |
|--------------------------------------|--|
| a $\sin(\arcsin \frac{1}{2})$ | b $\sin(\arcsin(-\frac{1}{2}))$ |
| c $\tan(\arctan(-1))$ | d $\cos(\arccos 0)$ |

4 Without using a calculator, work out the exact values of:

- | | | |
|---------------------------------------|--|---|
| a $\sin(\arccos(\frac{1}{2}))$ | b $\cos(\arcsin(-\frac{1}{2}))$ | c $\tan(\arccos(-\frac{\sqrt{2}}{2}))$ |
| d $\sec(\arctan(\sqrt{3}))$ | e $\operatorname{cosec}(\arcsin(-1))$ | f $\sin(2\arcsin(\frac{\sqrt{2}}{2}))$ |

Homework Exercise

5 Given that $\arcsin k = \alpha$, where $0 < k < 1$ and α is in radians, write down, in terms of α , the first two positive values of x satisfying the equation $\sin x = k$.

6 Given that x satisfies $\arcsin x = k$, where $0 < k < \frac{\pi}{2}$,

a state the range of possible values of x

(1 mark)

b express, in terms of x ,

i $\cos k$ **ii** $\tan k$

(4 marks)

Given, instead, that $-\frac{\pi}{2} < k < 0$,

c how, if at all, are your answers to part **b** affected?

(2 marks)

7 Sketch the graphs of:

a $y = \frac{\pi}{2} + 2 \arcsin x$

b $y = \pi - \arctan x$

c $y = \arccos(2x + 1)$

d $y = -2 \arcsin(-x)$

Homework Exercise

- 8 The function f is defined as $f: x \mapsto \arcsin x$, $-1 \leq x \leq 1$, and the function g is such that $g(x) = f(2x)$.
- a Sketch the graph of $y = f(x)$ and state the range of f . (3 marks)
 - b Sketch the graph of $y = g(x)$. (2 marks)
 - c Define g in the form $g: x \mapsto \dots$ and give the domain of g . (3 marks)
 - d Define g^{-1} in the form $g^{-1}: x \mapsto \dots$ (2 marks)
- 9 a Prove that for $0 \leq x \leq 1$, $\arccos x = \arcsin \sqrt{1 - x^2}$ (4 marks)
- b Give a reason why this result is not true for $-1 \leq x \leq 0$. (2 marks)

Challenge

- a Sketch the graph of $y = \sec x$, with the restricted domain $0 \leq x \leq \pi$, $x \neq \frac{\pi}{2}$
- b Given that $\operatorname{arcsec} x$ is the inverse function of $\sec x$, $0 \leq x \leq \pi$, $x \neq \frac{\pi}{2}$, sketch the graph of $y = \operatorname{arcsec} x$ and state the range of $\operatorname{arcsec} x$.

Homework Answers

1 a $\frac{\pi}{2}$ b $\frac{\pi}{2}$ c $-\frac{\pi}{4}$ d $-\frac{\pi}{6}$

 e $\frac{3\pi}{4}$ f $-\frac{\pi}{6}$ g $\frac{\pi}{3}$ h $\frac{\pi}{3}$

2 a 0 b $-\frac{\pi}{3}$ c $\frac{\pi}{2}$

3 a $\frac{1}{2}$ b $-\frac{1}{2}$ c -1 d 0

4 a $\frac{\sqrt{3}}{2}$ b $\frac{\sqrt{3}}{2}$ c -1 d 2

 e -1 f 1

5 $\alpha, \pi - \alpha$

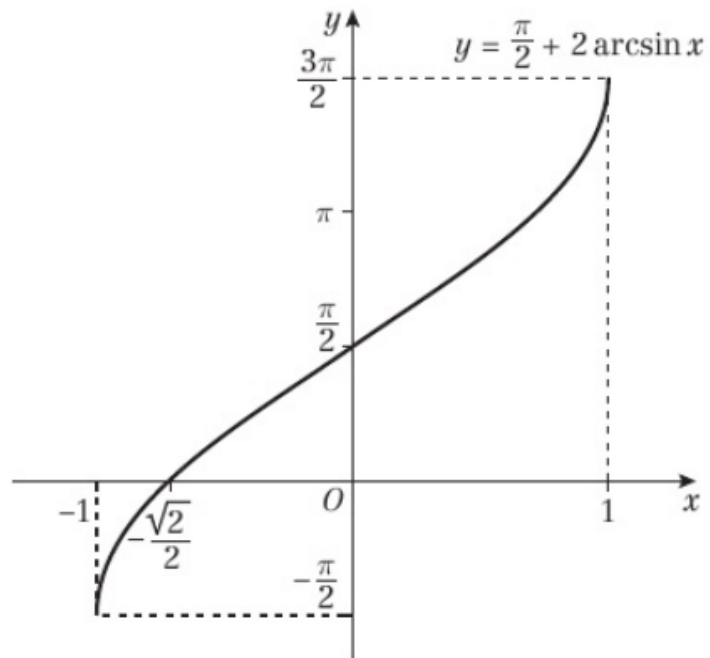
6 a $0 < x < 1$

 b i $\sqrt{1 - x^2}$ ii $\frac{x}{\sqrt{1 - x^2}}$

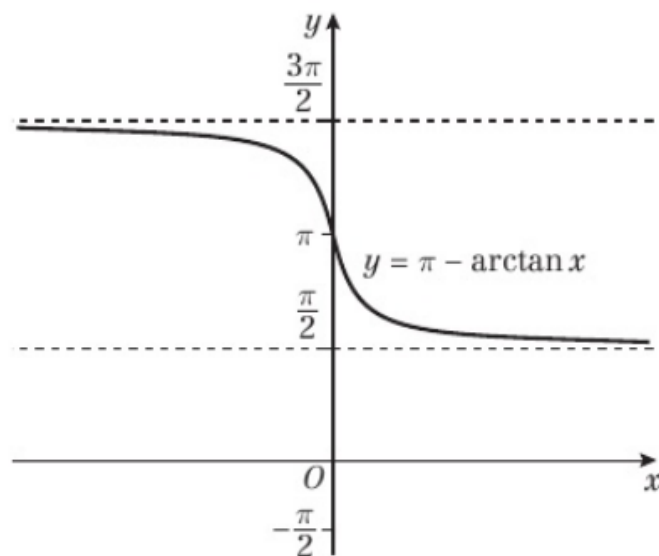
 c i no change ii no change

Chapter Answers

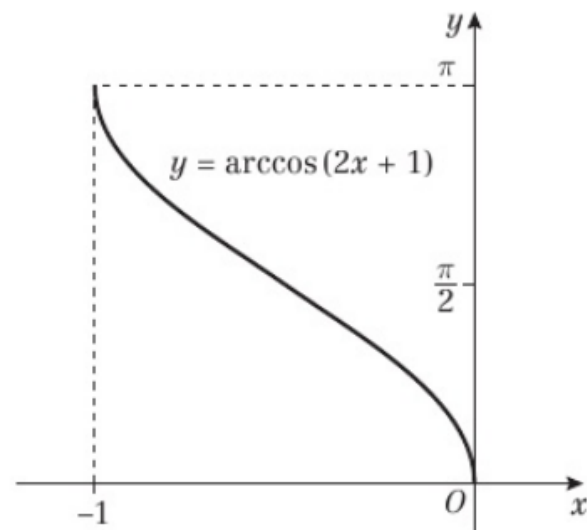
7 a



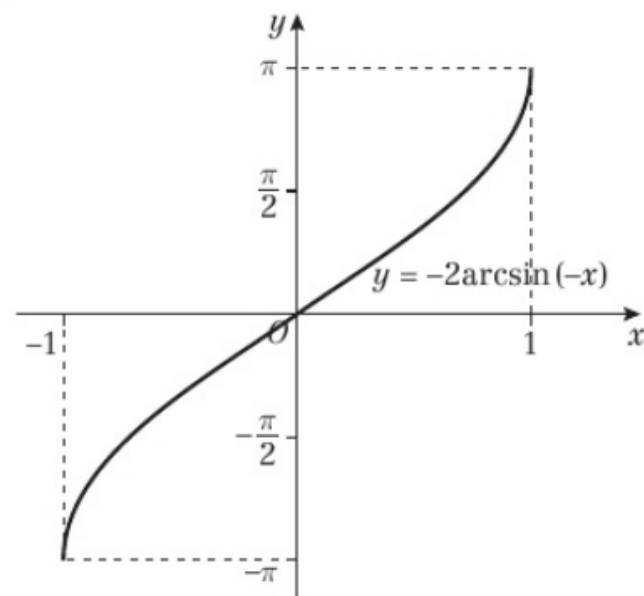
b



c

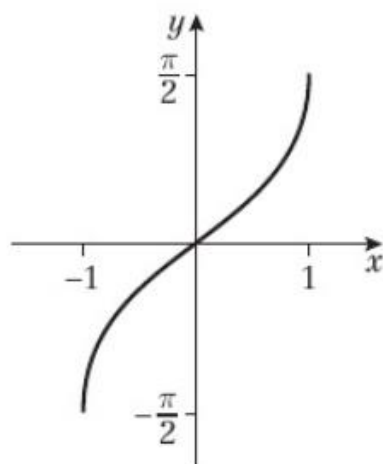


d

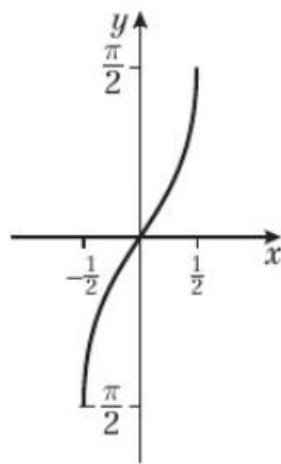


Chapter Answers

8 a



b



$$\text{Range: } -\frac{\pi}{2} \leq f(x) \leq \frac{\pi}{2}$$

c $g: x \rightarrow \arcsin 2x, -\frac{1}{2} \leq x \leq \frac{1}{2}$

d $g^{-1}: x \rightarrow \frac{1}{2} \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

9 a Let $y = \arccos x$. $x \in [0, 1] \Rightarrow y \in \left[0, \frac{\pi}{2}\right]$

$$\cos y = x, \text{ so } \sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$$

(Note, $\sin y \neq -\sqrt{1 - x^2}$ since $y \in \left[0, \frac{\pi}{2}\right]$, so $\sin y \geq 0$)

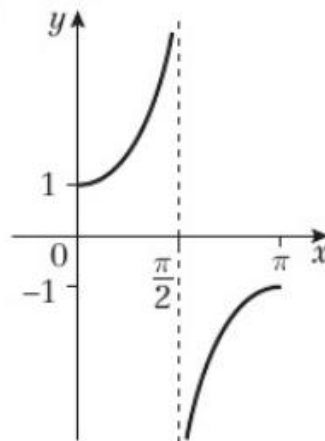
$$y = \arcsin \sqrt{1 - x^2}$$

Therefore, $\arccos x = \arcsin \sqrt{1 - x^2}$ for $x \in [0, 1]$.

b For $x \in [-1, 0]$, $\arccos x \in \left(\frac{\pi}{2}, \pi\right)$, but arcsin only has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Challenge

a



$$\text{Range: } 0 \leq \operatorname{arcsec} x \leq \pi, \operatorname{arcsec} x \neq \frac{\pi}{2}$$

b

