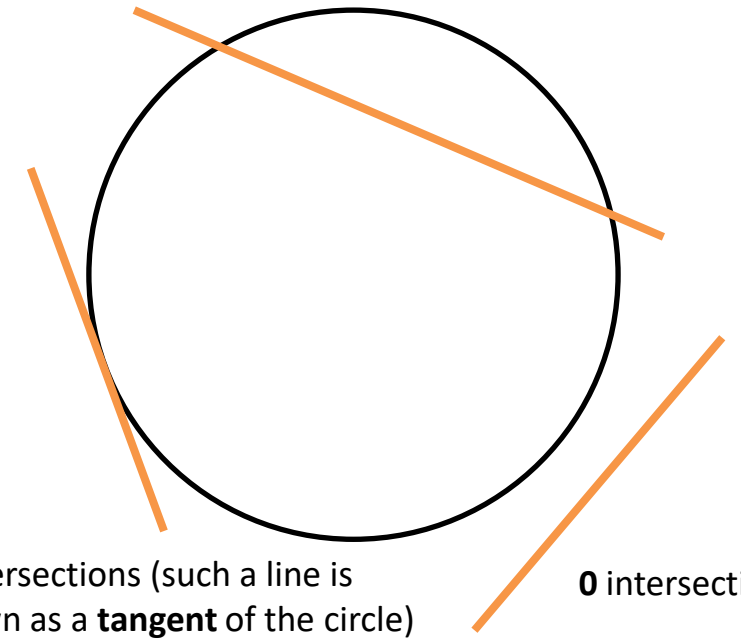
P1 Chapter 6: Circles

Straight Line Intersections

Intersections of Lines and Circles

Recall that to consider the **intersection of two lines**, we attempt to solve them **simultaneously** by substitution, potentially using the **discriminant** to show that there are no solutions (and hence no points of intersection).

2 intersections (such a line is known as a **secant** of the circle)



1 intersections (such a line is known as a **tangent** of the circle)

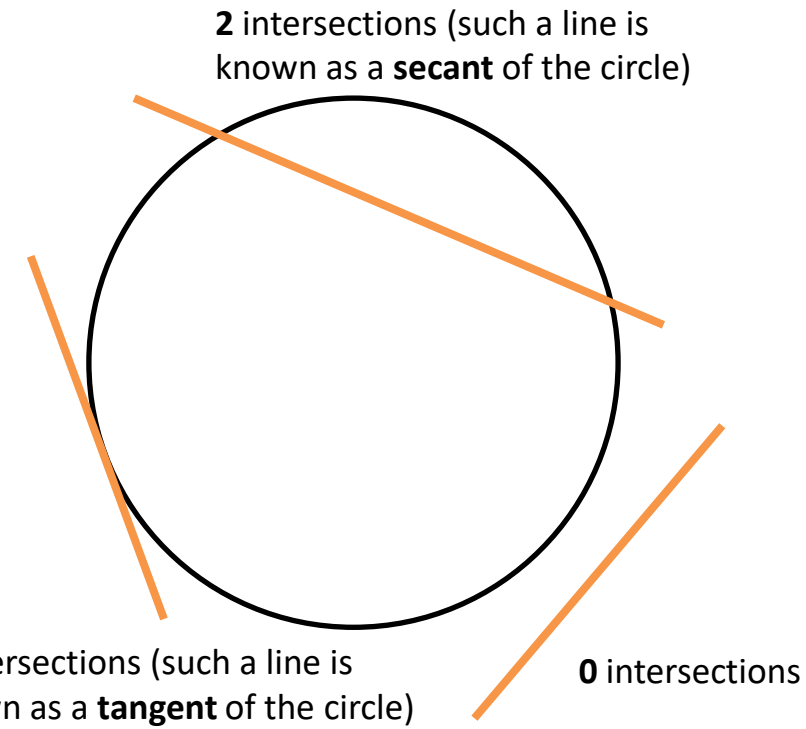
0 intersections

Show that the line $y = x + 3$ never intersects the circle with equation $x^2 + y^2 = 1$.

?

Intersections of Lines and Circles

Recall that to consider the **intersection of two lines**, we attempt to solve them **simultaneously** by substitution, potentially using the **discriminant** to show that there are no solutions (and hence no points of intersection).



Show that the line $y = x + 3$ never intersects the circle with equation $x^2 + y^2 = 1$.

Using substitution:

$$x^2 + (x + 3)^2 = 1$$
$$x^2 + x^2 + 6x + 9 = 1$$
$$2x^2 + 6x + 8 = 0$$
$$x^2 + 3x + 4 = 0$$

Discriminant:

$$a = 1, b = 3, c = 4$$
$$9 - 16 = -7$$

$-7 < 0$ therefore no solutions.

Test Your Understanding

Find the points of intersection where the line $y = x + 6$ meets $x^2 + (y - 3)^2 = 29$.

?

Using an algebraic (and not geometric) method, determine the k such that the line $y = x + k$ **touches** the circle with equation $x^2 + y^2 = 1$.

?

Test Your Understanding

Find the points of intersection where the line $y = x + 6$ meets $x^2 + (y - 3)^2 = 29$.

$x^2 + (x + 6 - 3)^2 = 29$
$x^2 + x^2 + 6x + 9 = 29$
$2x^2 + 6x - 20 = 0$
$x^2 + 3x - 10 = 0$
$(x + 5)(x - 2) = 0$
$x = -5 \; or \; x = 2$
$y = 1 \; or \; y = 8$
$(-5, 1), (2,8)$

Using an algebraic (and not geometric) method, determine the k such that the line $y = x + k$ **touches** the circle with equation $x^2 + y^2 = 1$.

$x^2 + (x + k)^2 = 1$
$x^2 + x^2 + 2kx + k^2 = 1$
$2x^2 + 2kx + k^2 - 1 = 0$

If the line touches the circle, one point of intersection and therefore one solution.

Discriminant:

$a = 2, b = 2k, c = k^2 - 1$
$4k^2 - 4(2)(k^2 - 1) = 0$
$4k^2 - 8k^2 + 8 = 0$
$4k^2 - 8 = 0$
$k = \pm\sqrt{2}$

Exercise 6.3

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Homework Exercise

1 Find the coordinates of the points where the circle $(x - 1)^2 + (y - 3)^2 = 45$ meets the x-axis.

Hint Substitute $y = 0$ into the equation.

2 Find the coordinates of the points where the circle $(x - 2)^2 + (y + 3)^2 = 29$ meets the y-axis.

3 The line $y = x + 4$ meets the circle $(x - 3)^2 + (y - 5)^2 = 34$ at A and B.
Find the coordinates of A and B.

4 Find the coordinates of the points where the line $x + y + 5 = 0$ meets the circle $x^2 + 6x + y^2 + 10y - 31 = 0$.

5 Show that the line $x - y - 10 = 0$ does not meet the circle $x^2 - 4x + y^2 = 21$.

Problem-solving

Attempt to solve the equations simultaneously. Use the discriminant to show that the resulting quadratic equation has no solutions.

6 **a** Show that the line $x + y = 11$ meets the circle with equation $x^2 + (y - 3)^2 = 32$ at only one point. **(4 marks)**

b Find the coordinates of the point of intersection. **(1 mark)**

7 The line $y = 2x - 2$ meets the circle $(x - 2)^2 + (y - 2)^2 = 20$ at A and B.

a Find the coordinates of A and B. **(5 marks)**

b Show that AB is a diameter of the circle. **(2 marks)**

8 The line $x + y = a$ meets the circle $(x - p)^2 + (y - 6)^2 = 20$ at $(3, 10)$, where a and p are constants.

a Work out the value of a. **(1 mark)**

b Work out the two possible values of p. **(5 marks)**

Homework Exercise

8 The line $x + y = a$ meets the circle $(x - p)^2 + (y - 6)^2 = 20$ at (3, 10), where a and p are constants.

a Work out the value of a. **(1 mark)**

b Work out the two possible values of p. **(5 marks)**

9 The circle with equation $(x - 4)^2 + (y + 7)^2 = 50$ meets the straight line with equation $x - y - 5 = 0$ at points A and B.

a Find the coordinates of the points A and B. **(5 marks)**

b Find the equation of the perpendicular bisector of line segment AB. **(3 marks)**

c Show that the perpendicular bisector of AB passes through the centre of the circle. **(1 mark)**

d Find the area of triangle OAB. **(2 marks)**

10 The line with equation $y = kx$ intersects the circle with equation $x^2 - 10x + y^2 - 12y + 57 = 0$ at two distinct points. Find a range of possible values of k. Round your answer to 2 decimal places.

a Show that $21k^2 - 60k + 32 < 0$. **(5 marks)**

b Hence determine the range of possible values for k. **(3 marks)**

11 The line with equation $y = 4x - 1$ does not intersect the circle with equation $x^2 + 2x + y^2 = k$. Find the range of possible values of k.

12 The line with equation $y = 2x + 5$ meets the circle with equation $x^2 + kx + y^2 = 4$ at exactly one point. Find two possible values of k. **(7 marks)**

Problem-solving

If you are solving a problem where there are 0, 1 or 2 solutions (or points of intersection), you might be able to use the discriminant.

Homework Answers

1 $(7, 0), (-5, 0)$

2 $(0, 2), (0, -8)$

3 $(6, 10), (2, -2)$

4 $(4, -9), (-7, 2)$

5 $2x^2 - 24x + 79 = 0$ has no real solutions, therefore lines do not intersect

6 **a** $b^2 - 4ac = 64 - 4 \times 1 \times 16 = 0$. So there is only one point of intersection.

b $(4, 7)$

7 **a** $(0, -2), (4, 6)$ **b** midpoint of AB is $(2, 2)$

8 **a** 13 **b** $p = 1$ or 5

9 **a** $A(5, 0)$ and $B(-3, -8)$ (or vice-versa)

b $y = -x - 3$

c $(4, -7)$ is a solution to $y = -x - 3$.

d 20

10 **a** Substitute $y = kx$ to give

$(k^2 + 1)x^2 - (12k + 10)x + 57 = 0$

$b^2 - 4ac > 0$, $-84k^2 + 240k - 128 > 0$,

$21k^2 - 60k + 32 < 0$

b $0.71 < k < 2.15$

Exact answer is $\frac{10}{7} - \frac{2\sqrt{57}}{21} < k < \frac{10}{7} + \frac{2\sqrt{57}}{2}$

11 $k < \frac{8}{17}$

12 $k = -20 \pm 2\sqrt{105}$