# P1 Chapter 9: Trigonometric Ratios

## Graph Transformation

# Transforming Trigonometric Graphs

There is no new theory here: just use your knowledge of transforming graphs, i.e. whether the transformation occurs 'inside' the function (i.e. input modified) or 'outside' the function (i.e. output modified).

Sketch $y = 4\sin x$, $0 \leq x \leq 360°$

?

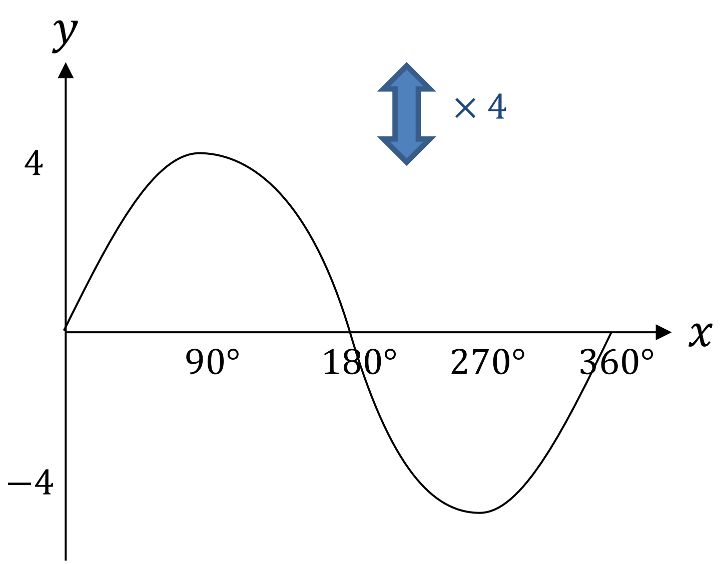
Sketch $y = \cos(x + 45°)$, $0 \leq x \leq 360°$

?

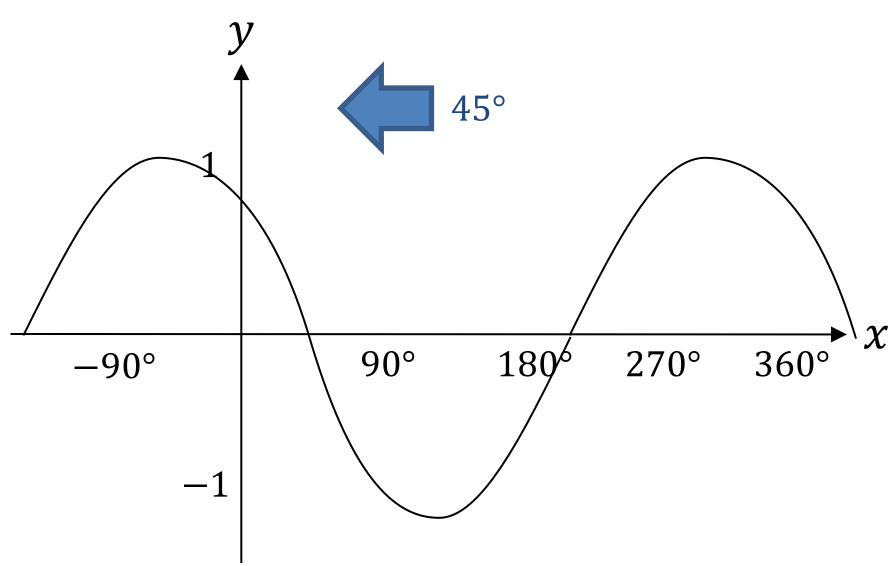
# Transforming Trigonometric Graphs

There is no new theory here: just use your knowledge of transforming graphs, i.e. whether the transformation occurs 'inside' the function (i.e. input modified) or 'outside' the function (i.e. output modified).

Sketch $y = 4\sin x,\ 0 \leq x \leq 360°$




Sketch $y = \cos(x + 45°),\ 0 \leq x \leq 360°$



# Transforming Trigonometric Graphs

Sketch $y = -\tan x$, $0 \leq x \leq 360°$

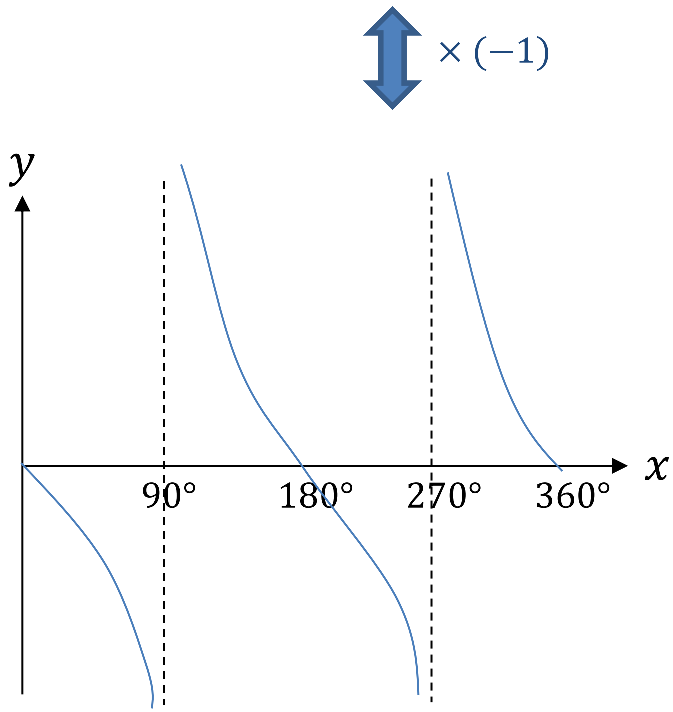
?

Sketch $y = \sin\left(\frac{x}{2}\right)$, $0 \leq x \leq 360°$

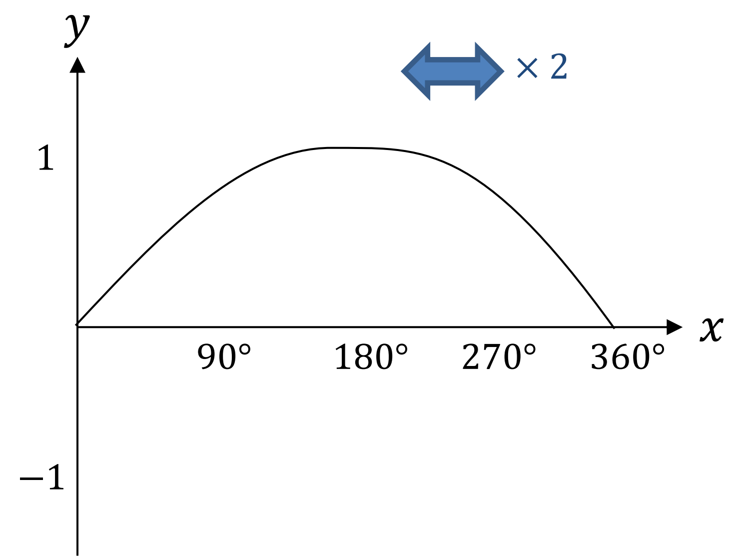
?

# Transforming Trigonometric Graphs

Sketch $y = -\tan x,\ 0 \leq x \leq 360°$




Sketch $y = \sin\left(\frac{x}{2}\right),\ 0 \leq x \leq 360°$



# Exercise 9.6

## Pearson Pure Mathematics Year 1/AS
## Page 75

**Extension**

1 *[MAT 2013 1B]* The graph of $y = \sin x$ is reflected first in the line $x = \pi$ and then in the line $y = 2$. The resulting graph has equation:

A) $y = \cos x$
B) $y = 2 + \sin x$
C) $y = 4 + \sin x$
D) $y = 2 - \cos x$

?

2 *[MAT 2011 1D]* What fraction of the interval $0 \leq x \leq 360°$ is one (or both) of the inequalities:

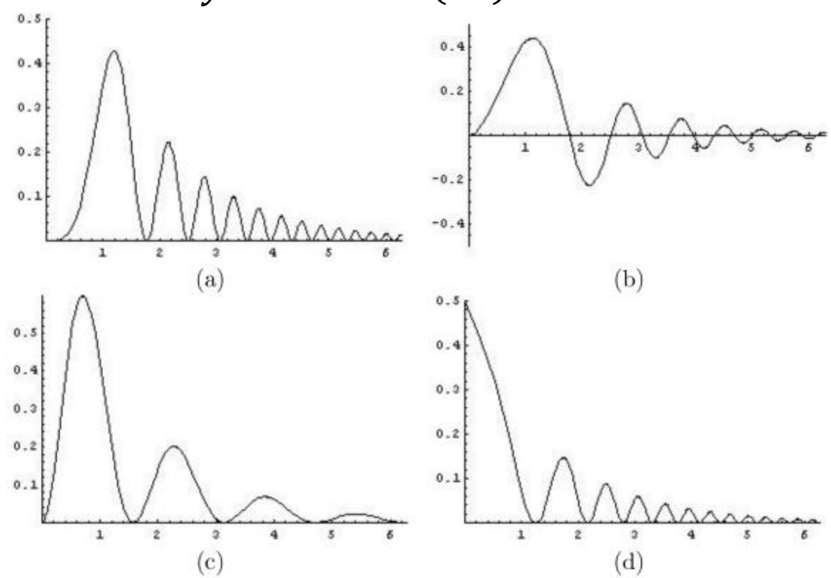
$$\sin x \geq \frac{1}{2}, \qquad \sin 2x \geq \frac{1}{2}$$

true?

?

3 *[MAT 2007 1G]* On which of the axes is a sketch of the graph

$$y = 2^{-x} \sin^2(x^2)$$




?

# Exercise 9.6

## Pearson Pure Mathematics Year 1/AS
## Page 75

### Extension

**1** *[MAT 2013 1B]* The graph of $y = \sin x$ is reflected first in the line $x = \pi$ and then in the line $y = 2$. The resulting graph has equation:

A) $y = \cos x$
B) $y = 2 + \sin x$
C) $y = 4 + \sin x$
D) $y = 2 - \cos x$

**Solution: C**

**2** *[MAT 2011 1D]* What fraction of the interval $0 \leq x \leq 360°$ is one (or both) of the inequalities:

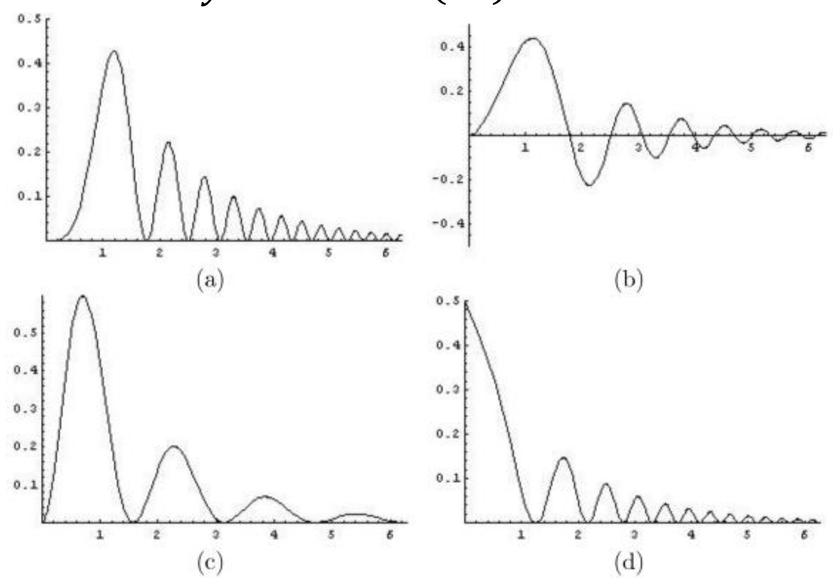
$$\sin x \geq \frac{1}{2}, \qquad \sin 2x \geq \frac{1}{2}$$

true?

**Solution: $\frac{13}{24}$ (this is clear if you draw the graphs $y = \sin x$ and $y = \sin 2x$ on the same axes)**

**3** *[MAT 2007 1G]* On which of the axes is a sketch of the graph

$$y = 2^{-x} \sin^2(x^2)$$




**$2^{-x}$ and $sin^2$ are always positive eliminating (b). When $x = 0, y = 0$ eliminating (d). Multiplying by $2^{-x}$ causes the amplitude of the peaks to go down as $x$ increases. The $x^2$ increases more rapidly as $x$ increases, hence reducing the wavelength (i.e. distance between peaks). The answer is therefore (a).**

# Homework Exercise

**1** Write down **i** the maximum value, and **ii** the minimum value, of the following expressions, and in each case give the smallest positive (or zero) value of $x$ for which it occurs.

**a** $\cos x$ **b** $4 \sin x$ **c** $\cos(-x)$

**d** $3 + \sin x$ **e** $-\sin x$ **f** $\sin 3x$

**2** Sketch, on the same set of axes, in the interval $0 \leqslant \theta \leqslant 360°$, the graphs of $\cos\theta$ and $\cos 3\theta$.

**3** Sketch, on separate sets of axes, the graphs of the following, in the interval $0 \leqslant \theta \leqslant 360°$. Give the coordinates of points of intersection with the axes, and of maximum and minimum points where appropriate.

**a** $y = -\cos\theta$ **b** $y = \frac{1}{3}\sin\theta$ **c** $y = \sin\frac{1}{3}\theta$ **d** $y = \tan(\theta - 45°)$

**4** Sketch, on separate sets of axes, the graphs of the following, in the interval $-180° \leqslant \theta \leqslant 180°$. Give the coordinates of points of intersection with the axes, and of maximum and minimum points where appropriate.

**a** $y = -2\sin\theta$ **b** $y = \tan(\theta + 180°)$ **c** $y = \cos 4\theta$ **d** $y = \sin(-\theta)$

**5** Sketch, on separate sets of axes, the graphs of the following in the interval $-360° \leqslant \theta \leqslant 360°$. In each case give the periodicity of the function.

**a** $y = \sin\frac{1}{2}\theta$ **b** $y = -\frac{1}{2}\cos\theta$ **c** $y = \tan(\theta - 90°)$ **d** $y = \tan 2\theta$

**6** **a** By considering the graphs of the functions, or otherwise, verify that:

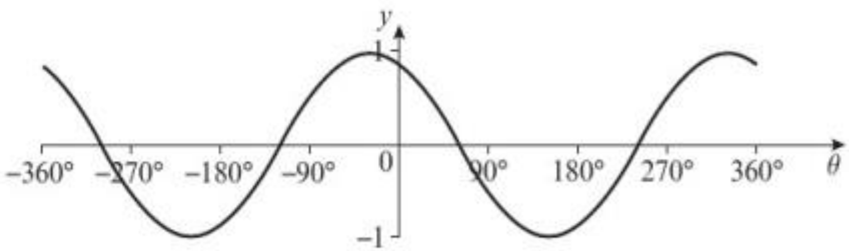
**i** $\cos\theta = \cos(-\theta)$ **ii** $\sin\theta = -\sin(-\theta)$ **iii** $\sin(\theta - 90°) = -\cos\theta$.

**b** Use the results in **a ii** and **iii** to show that $\sin(90° - \theta) = \cos\theta$.

**c** In Example 14 you saw that $\cos(\theta - 90°) = \sin\theta$.
Use this result with part **a i** to show that $\cos(90° - \theta) = \sin\theta$.

# Homework Exercise

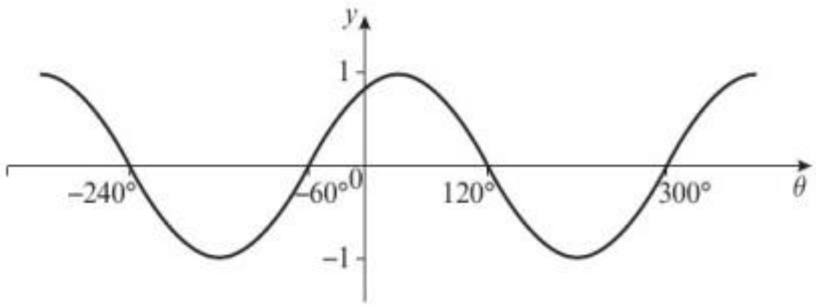
**7** The graph shows the curve $y = \cos(x + 30°)$, $-360° \leqslant x \leqslant 360°$.




**a** Write down the coordinates of the points where the curve crosses the $x$-axis. **(2 marks)**

**b** Find the coordinates of the point where the curve crosses the $y$-axis. **(1 mark)**

**8** The graph shows the curve with equation $y = \sin(x + k)$, $-360° \leqslant x \leqslant 360°$, where $k$ is a constant.




**a** Find one possible value for $k$. **(2 marks)**

**b** Is there more than one possible answer to part **a**? Give a reason for your answer. **(2 marks)**

**9** The variation in the depth of water in a rock pool can be modelled using the function $y = \sin(30t)°$, where $t$ is the time in hours and $0 \leqslant t \leqslant 6$.

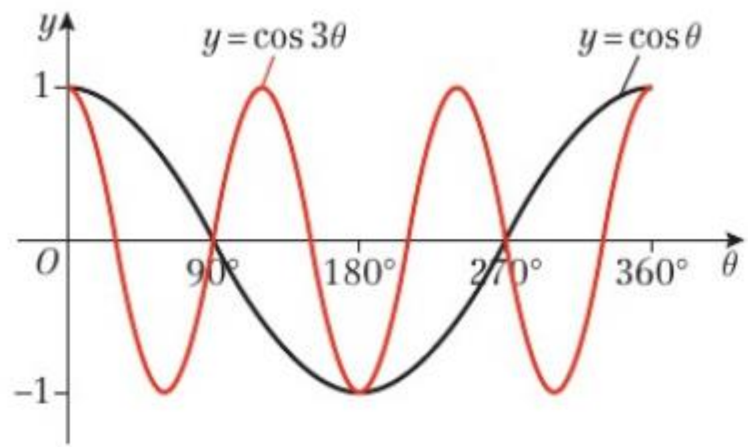
**a** Sketch the function for the given interval. **(2 marks)**

**b** If $t = 0$ represents midday, during what times will the rock pool be at least half full? **(3 marks)**

# Homework Answers

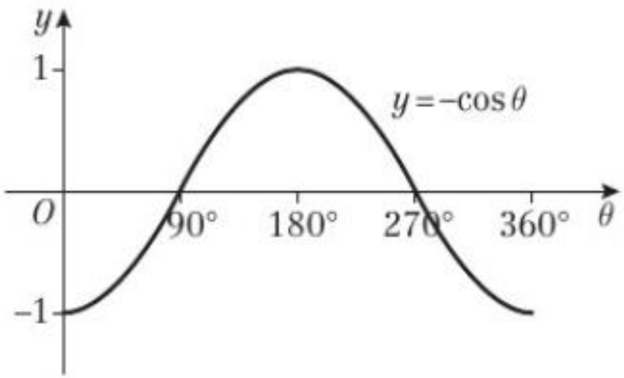
**1** **a** **i** $1, x = 0°$ **ii** $-1, x = 180°$
**b** **i** $4, x = 90°$ **ii** $-4, x = 270°$
**c** **i** $1, x = 0°$ **ii** $-1, x = 180°$
**d** **i** $4, x = 90°$ **ii** $2, x = 270°$
**e** **i** $1, x = 270°$ **ii** $-1, x = 90°$
**f** **i** $1, x = 30°$ **ii** $-1, x = 90°$

**2**



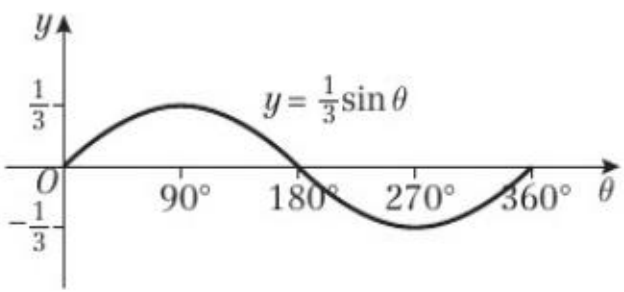
# Homework Answers

3 **a** The graph of $y = -\cos\theta$ is the graph of $y = \cos\theta$ reflected in the $\theta$-axis



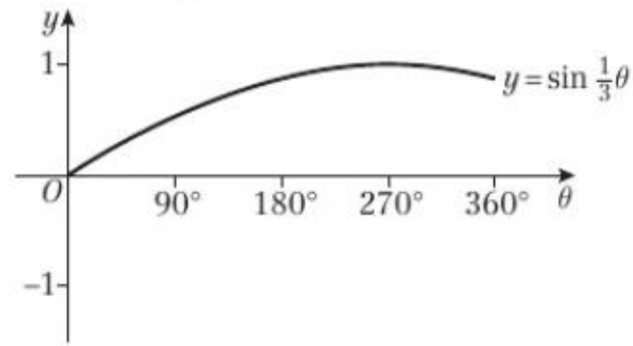

Meets $\theta$-axis at (90°, 0), (270°, 0)
Meets $y$-axis at (0°, −1)
Maximum at (180°, 1)
Minimum at (0°, −1) and (360°, −1)

**b** The graph of $y = \frac{1}{3}\sin\theta$ is the graph of $y = \sin\theta$ stretched by a scale factor $\frac{1}{3}$ in the $y$ direction.



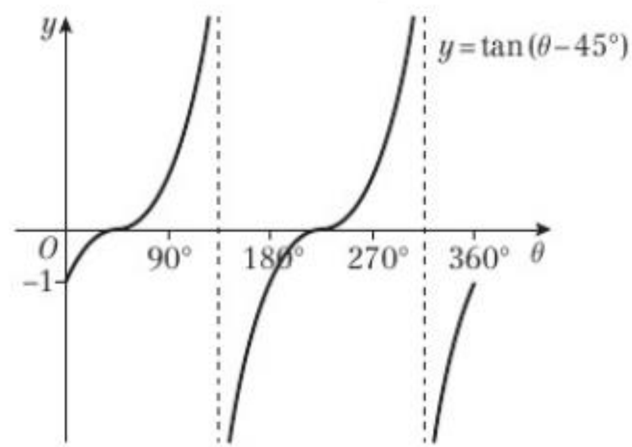

Meets $\theta$-axis at (0°, 0), (180°, 0), (360°, 0)
Meets $y$-axis at (0°, 0)
Maximum at (90°, $\frac{1}{3}$)
Minimum at (270°, $-\frac{1}{3}$)

**c** The graph of $y = \sin\frac{1}{3}\theta$ is the graph of $y = \sin\theta$ stretched by a scale factor 3 in the $\theta$ direction.




Only meets axis at origin
Maximum at (270°, 1)

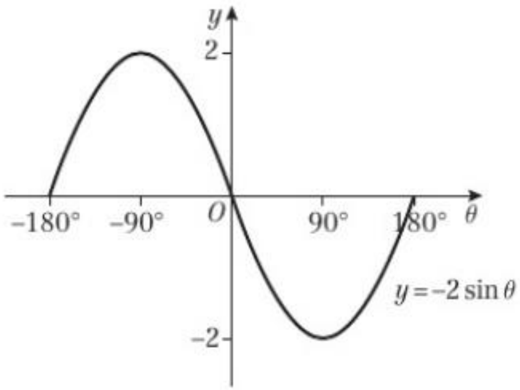
**d** The graph of $y = \tan(\theta - 45°)$ is the graph of $\tan\theta$ translated by 45° in the positive $\theta$ direction.




Meets $\theta$-axis at (45°, 0), (225°, 0)
Meets $y$-axis at (0°, −1)
(Asymptotes at $\theta = 135°$ and $\theta = 315°$)

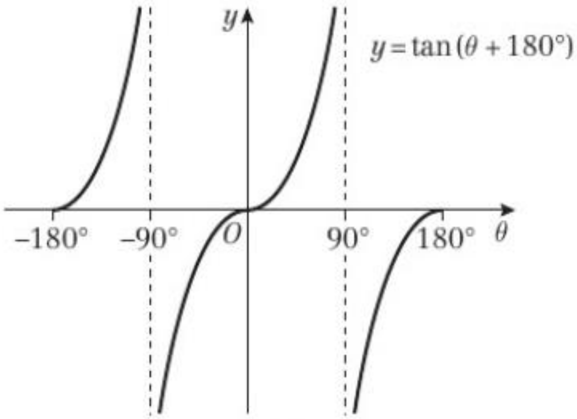
# Homework Answers

4 a This is the graph of $y = \sin\theta$ stretched by scale factor $-2$ in the $y$-direction (i.e. reflected in the $\theta$-axis and scaled by 2 in the $y$-direction).



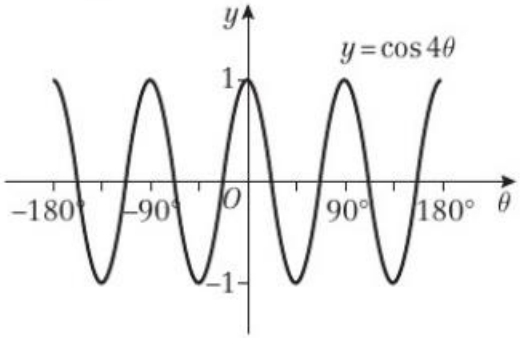

Meets $\theta$-axis at (−180°, 0), (0, 0), (180, 0)
Maximum at (−90°, 2)
Minimum at (90°, −2).

b This is the graph of $y = \tan\theta$ translated by 180° in the negative $\theta$ direction.



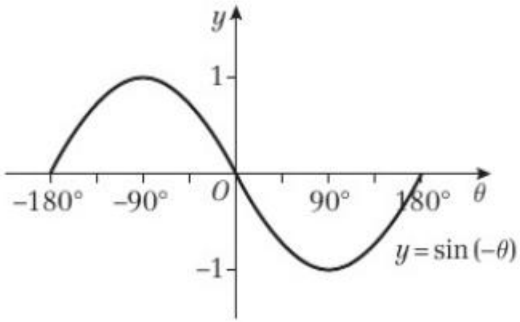

As $\tan\theta$ has a period of 180°
$\tan(\theta + 180°) = \tan\theta$
Meets $\theta$-axis at (−180°, 0), (0, 0), (180°, 0)
Meets $y$-axis at (0, 0)

c This is the graph of $y = \cos\theta$ stretched by scale factor $\frac{1}{4}$ horizontally.




Meets $\theta$-axis at $(-157\frac{1}{2}°, 0)$, $(-112\frac{1}{2}°, 0)$, $(-67\frac{1}{2}°, 0)$, $(-22\frac{1}{2}°, 0)$, $(22\frac{1}{2}°, 0)$, $(67\frac{1}{2}°, 0)$, $(112\frac{1}{2}°, 0)$, $(157\frac{1}{2}°, 0)$
Meets $y$-axis at (0, 1)
Maxima at (−180°, 1), (−90°, 1), (0, 1), (90°, 1), (180°, 1)
Minima at (−135°, −1), (−45°, −1), (45°, −1), (135°, −1)

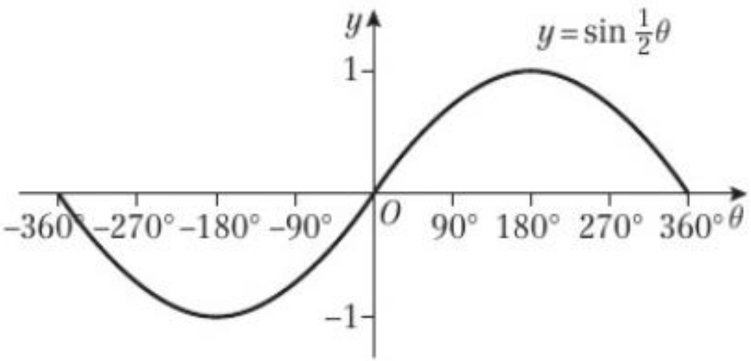
d This is the graph of $y = \sin\theta$ reflected in the $y$-axis. (This is the same as $y = -\sin\theta$.)



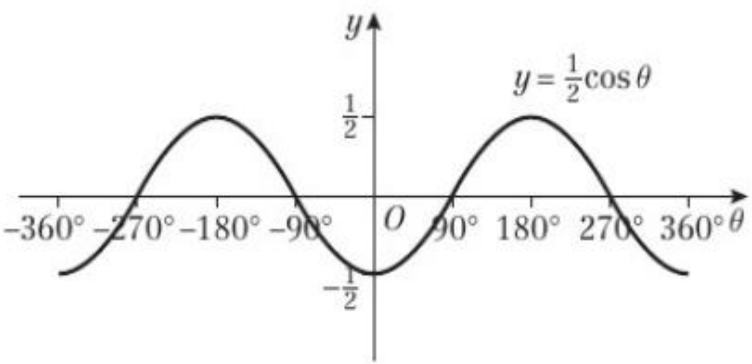

Meets $\theta$-axis at (−180°, 0), (0°, 0), (180°, 0)
Maximum at (−90°, 1)
Minimum at (90°, −1)

# Homework Answers

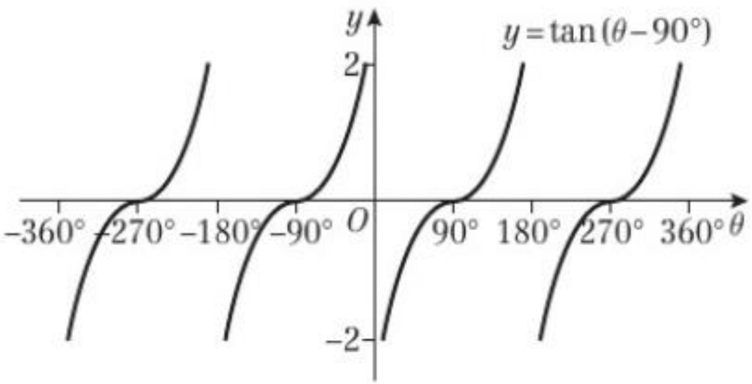
**5** **a** Period = 720°



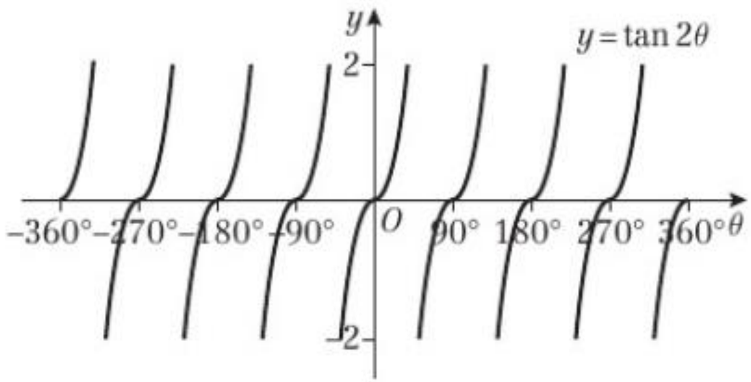

**b** Period = 360°




**c** Period = 180°

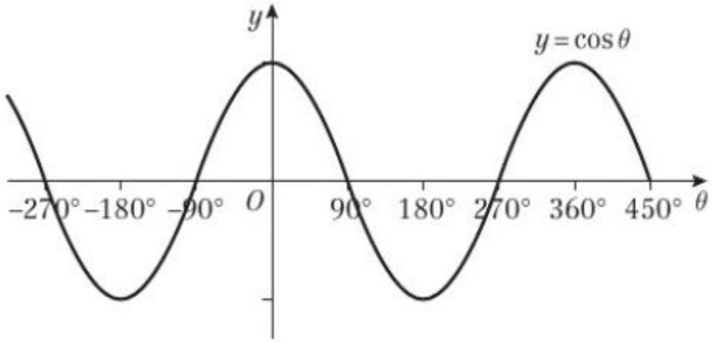



**d** Period = 90°

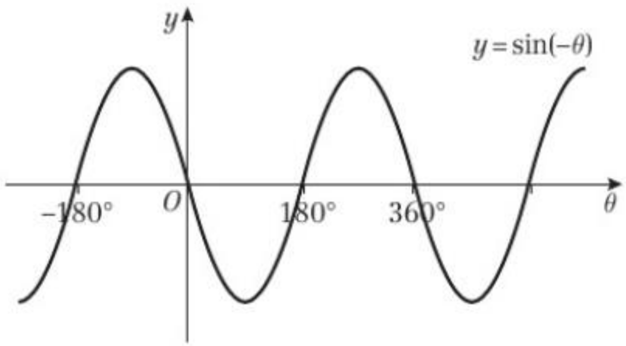


# Homework Answers

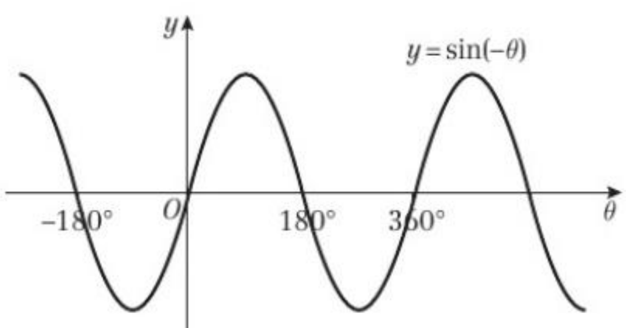
**6** **a** **i** $y = \cos(-\theta)$ is a reflection of $y = \cos\theta$ in the $y$-axis, which is the same curve, so $\cos\theta = \cos(-\theta)$.



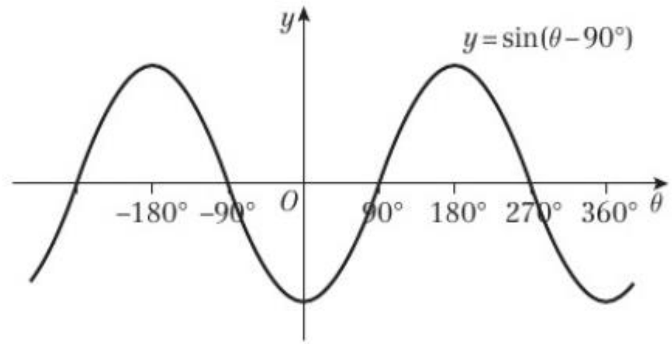

**ii** $y = \sin(-\theta)$ is a reflection of $y = \sin\theta$ in the $y$-axis.




$y = -\sin(-\theta)$ is a reflection of $y = \sin(-\theta)$ in the $\theta$-axis, which is the graph of $y = \sin\theta$, so $-\sin(-\theta) = \sin\theta$.




**iii** $y = \sin(\theta - 90°)$ is the graph of $y = \sin\theta$ translated by 90° to the right, which is the graph of $y = -\cos\theta$, so $\sin(\theta - 90°) = -\cos\theta$.




**b** $\sin(90° - \theta)$
$= -\sin(-(90° - \theta)) = -\sin(\theta - 90°)$
using (a) (ii)
$= -(-\cos\theta)$ using (a) (iii)
$= \cos\theta$

**c** Using (a)(i) $\cos(90° - \theta) = \cos(-(90° - \theta))$ $= \cos(\theta - 90°)$, but $\cos(\theta - 90°) = \sin\theta$, so $\cos(90° - \theta) = \sin\theta$

# Homework Answers

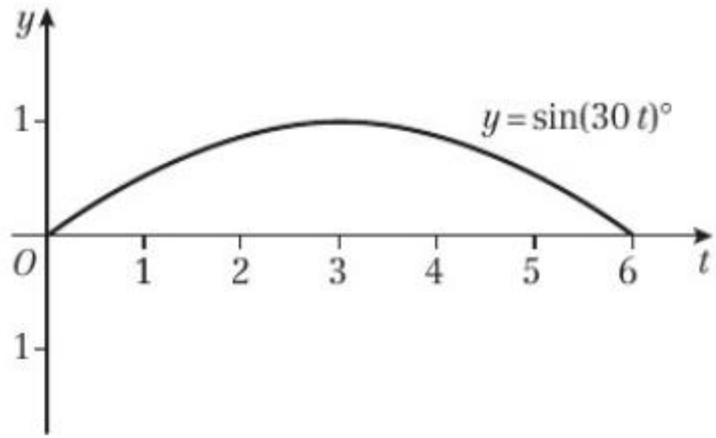
**7** **a** $(-300°, 0), (-120°, 0), (60°, 0), (240°, 0)$

**b** $\left(0°, \frac{\sqrt{3}}{2}\right)$

**8** **a** $y = \sin(x + 60°)$

**b** Yes – could also be a translation of the cos graph, e.g. $y = \cos(x - 30°)$

**9** **a**




**b** Between 1 pm and 5 pm