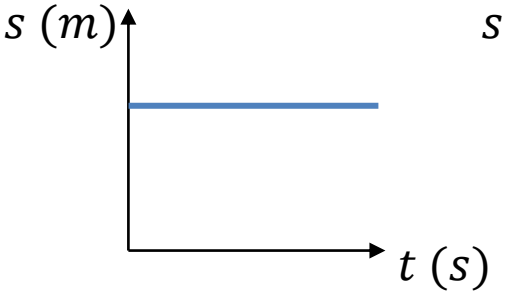
M1 Chapter 9: Constant Acceleration

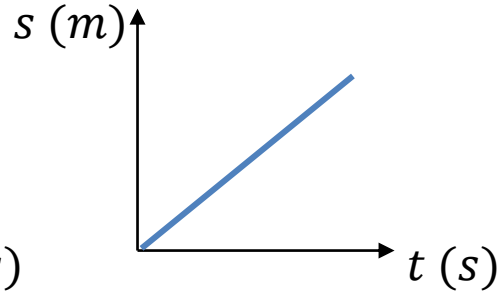
Motion Graphs

RECAP :: Displacement-Time Graphs

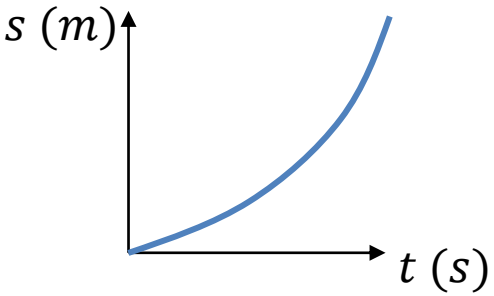
Describe the motion of each object:



Object is stationary.



Object is moving with constant velocity.



Object is accelerating.

Velocity is the rate of change of displacement
(i.e. gradient of displacement-time graph)

$$\textbf{Average Velocity} = \frac{\textbf{Displacement from starting point}}{\textbf{Time taken}}$$

$$\textbf{Average Speed} = \frac{\textbf{Total distance travelled}}{\textbf{Time taken}}$$

The distinction is important. If you went out then some time later travelled back home, your average velocity is 0 because your eventual displacement is 0!

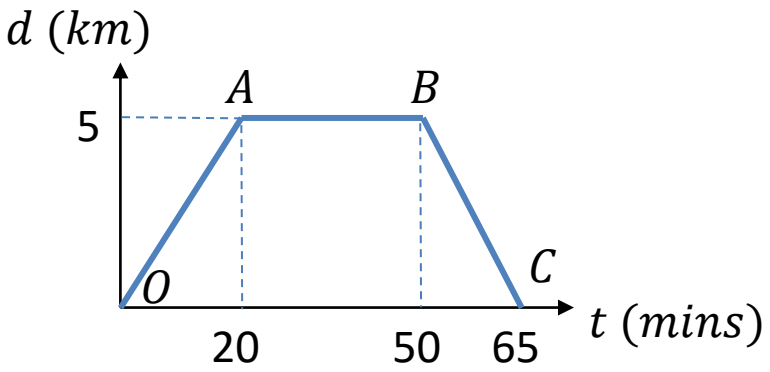
Example

[Textbook] A cyclist rides in a straight line for 20 minutes. She waits for half an hour, then returns in a straight line to her starting point in 15 minutes. This is a displacement-time graph for her journey.

(a) Work out the average velocity for each stage of the journey in km h^{-1}.

(b) Write down the average velocity for the whole journey.

(c) Work out average speed for the whole journey.



a ?

b ?

c ?

Example

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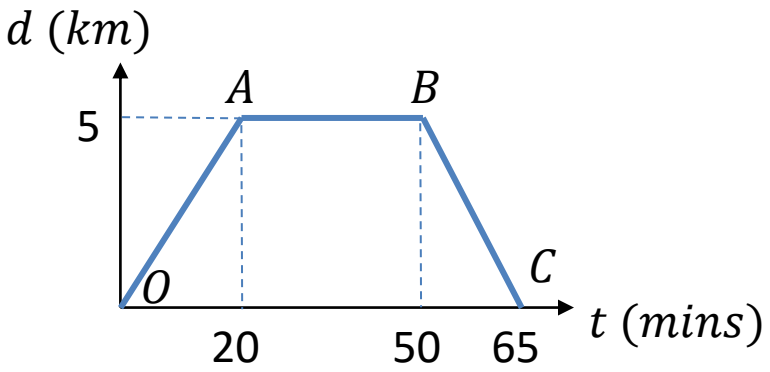


a

OA: $\frac{5}{20} = 0.25\ km\ min^{-1} = 15 km\ h^{-1}$

AB: $0\ km\ h^{-1}$

BC: $-\frac{5}{15} = -\frac{1}{3}\ km\ min^{-1} = 20\ km\ h^{-1}$

b 0 (as displacement is 0)

c Total distance: 10km. Total time: 65 mins

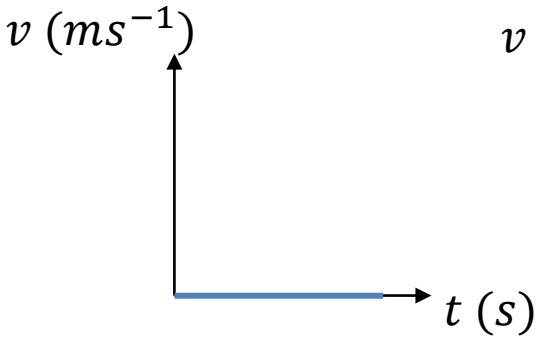
$Avg\ speed = \frac{10}{65} = \frac{2}{13}\ km\ min^{-1} = 9.23\ km\ h^{-1}\ (3sf)$

Exercise 9.1 Displacement-time graphs.

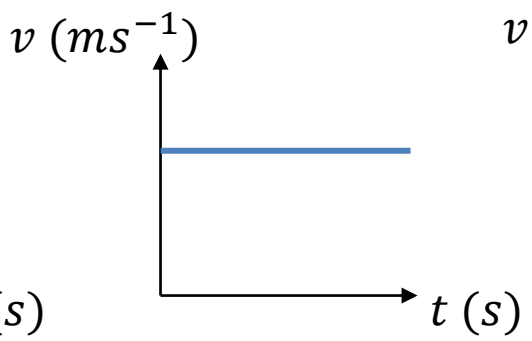
Pearson Stats/Mechanics Year 1
Pages 58-59

RECAP :: Velocity-Time Graphs

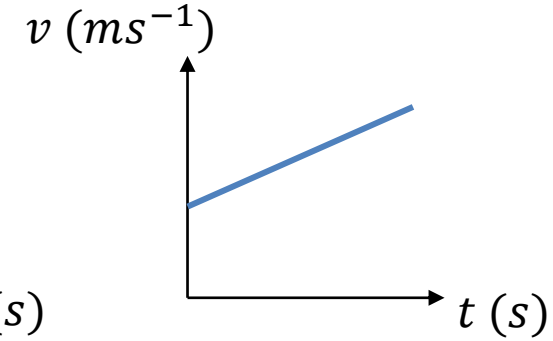
Describe the motion of each object:



Object is stationary.



Object is moving with constant velocity (as change in velocity is 0).



Object has constant acceleration (as velocity is increasing at constant rate).

Acceleration is the rate of change of velocity
(i.e. gradient of velocity-time graph)

The **area** under a velocity-time graph gives the **distance travelled.**

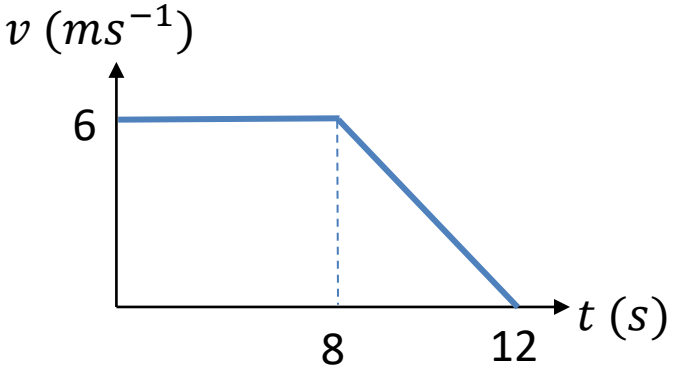
Fro Note: We'll see later in Chapter 11 that when we differentiate displacement we get velocity, and therefore integrating velocity gives displacement. But we know that integrating finds the area under the graph.

Examples

[Textbook] The figure shows a velocity-time graph illustrating the motion of a cyclist moving along a straight road for a period of 12 seconds. For the first 8 seconds, she moves at a constant speed of 6 m s^{-1}. She then decelerates at a constant rate, stopping after a further 4 seconds.

(a) Find the displacement from the starting point of the cyclist after this 12 second period.

(b) Work out the rate at which the cyclist decelerates.



a ?

b ?

In case you've forgotten:

Area of trapezium
= average of parallel sides
× height between them

You're welcome.

Examples

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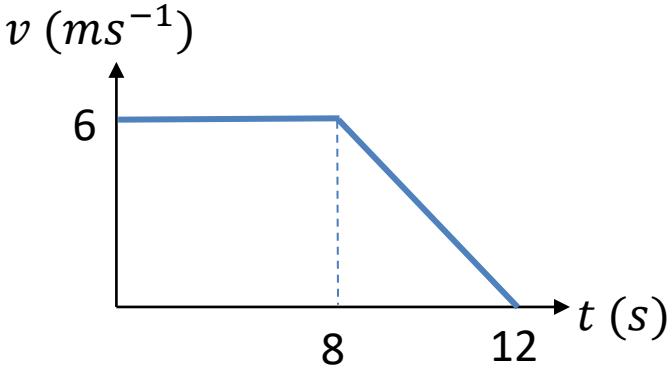


a Area of trapezium:

$$\frac{8+12}{2} \times 6 = 60\ m$$

b Using the gradient:

$$-\frac{6}{4} = -1.5\ ms^{-2}$$

In case you've forgotten:

Area of trapezium
= average of parallel sides
× height between them

You're welcome.

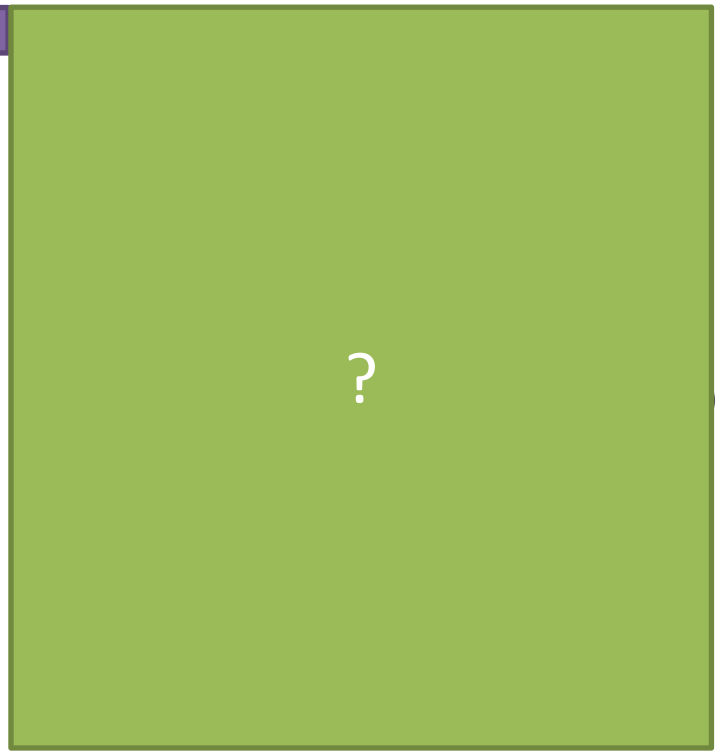
Algebraic Example

[Textbook] A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 ms^{-1} in T seconds. The particle then travels at a constant velocity of 8 ms^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

(a) Sketch a velocity-time graph to illustrate the motion of the particle. Give then the total displacement of the particle is 600m.

(b) find the value of T.

a



b

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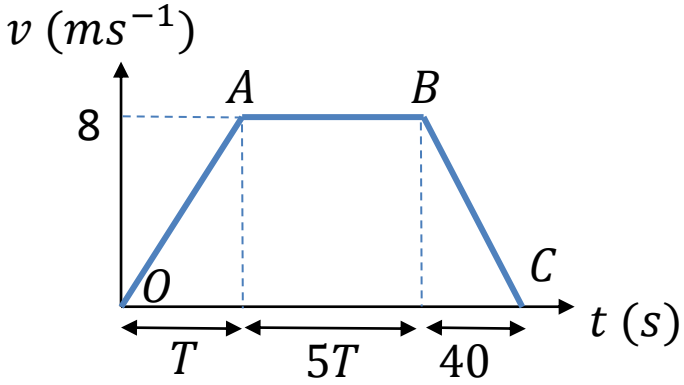

Fro Tip: Sometimes it's easier to indicate the period of time that has passed (using arrows) rather than the time at the end of the interval.

b

Using area:

$$\frac{5T + (6T + 40)}{2} \times 8 = 600$$
$$44T + 160 = 600$$
$$T = 10$$

Test Your Understanding

Edexcel M1 May 2013

A car is travelling along a straight horizontal road. The car takes 120 s to travel between two sets of traffic lights which are 2145 m apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for 30 s until its speed is 22 m s^{-1}. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

(a) Sketch, in the space below, a speed-time graph for the motion of the car between the two sets of traffic lights.

(2)

(b) Find the value of T.

(3)

A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, a m s^{-2}, and passes the car at the point A which is 990 m from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed 22 m s^{-1}.

(c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A.

(4)

(d) Find the value of a.

(2)

You won't likely have the knowledge for (d) yet…

(b) ?

(c) ?

(d) ?

(a) ?

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Edexcel M1 May 2013

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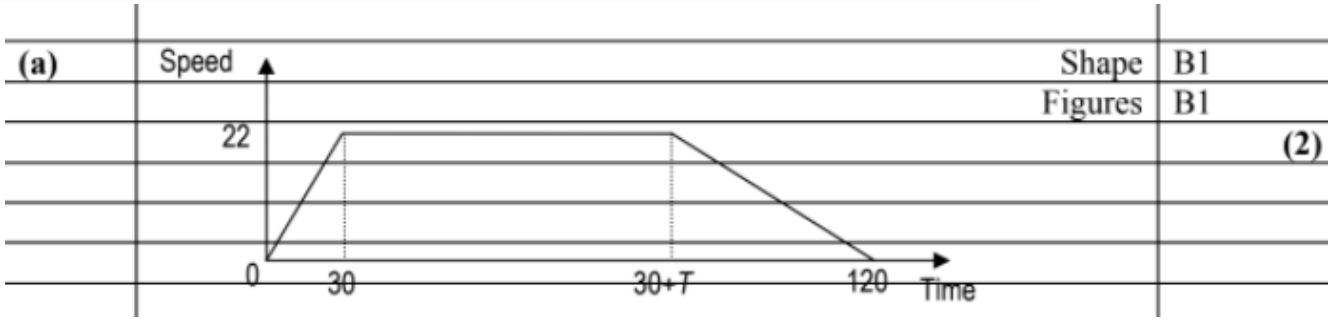
(4)

(d) Find the value of a.

(2)

You won't likely have the knowledge for (d) yet…

(b)	$\frac{(120+T)22}{2} = 2145$	M1 A1
	$T = 75$	A1
		(3)
(c)	$\frac{(t+t-30)22}{2} = 990$	M1 A1
	$t = 60$	A1
	Answer $= 60 - 10 = 50$	A1
		(4)
(d)	$990 = 0.5a50^2$	M1
	$a = 0.79, 0.792, 99/125$ oe	A1
		(2)
		[11]



Exercise 9B

Pearson Stats/Mechanics Year 1
Pages 135-136

Edexcel M1 May 2012 Q4

A car is moving on a straight horizontal road. At time $t = 0$, the car is moving with speed 20 m s^{-1} and is at the point A. The car maintains the speed of 20 m s^{-1} for 25 s. The car then moves with constant deceleration 0.4 m s^{-2}, reducing its speed from 20 m s^{-1} to 8 m s^{-1}. The car then moves with constant speed 8 m s^{-1} for 60 s. The car then moves with constant acceleration until it is moving with speed 20 m s^{-1} at the point B.

(a) Sketch a speed-time graph to represent the motion of the car from A to B.

(3)

(b) Find the time for which the car is decelerating.

(2)

Given that the distance from A to B is 1960 m,

(c) find the time taken for the car to move from A to B.

(8)

For (b), it may be helpful to know that:
final velocity = initial velocity
+ (time × acceleration)

(a) ?

(b) ?

(c) ?

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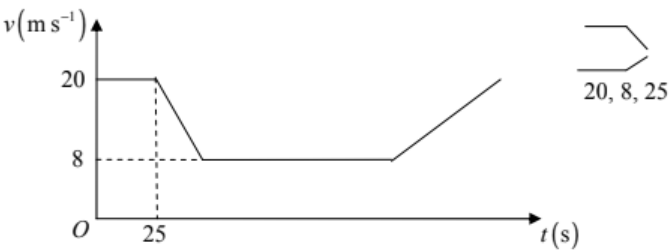


20, 8, 25

B1
B1
B1 **(3)**

(b) $v = u + at \Rightarrow 8 = 20 - 0.4t$

$t = 30$ (s)

M1
A1 **(2)**

(c)

$1960 = (25 \times 20) + (30 \times 8) + (\frac{1}{2} \times 30 \times 12) + (60 \times 8) + 8 \times t + \frac{1}{2} \times t \times 12$ M1A3 **ft** (2 ,1, 0)

$1960 = 500 + 240 + 180 + 480 + 14t$ DM1 A1

$T = 115 + 40$ DM1

$= 155$ A1

(8)

Exercise 9.2

Pearson Stats/Mechanics Year 1
Pages 59-60

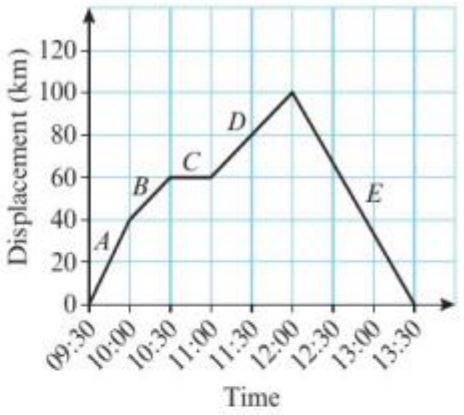
Homework Exercise

1 This is a displacement–time graph for a car travelling along a straight road. The journey is divided into 5 stages labelled A to E.

a Work out the average velocity for each stage of the journey.

b State the average velocity for the whole journey.

c Work out the average speed for the whole journey.



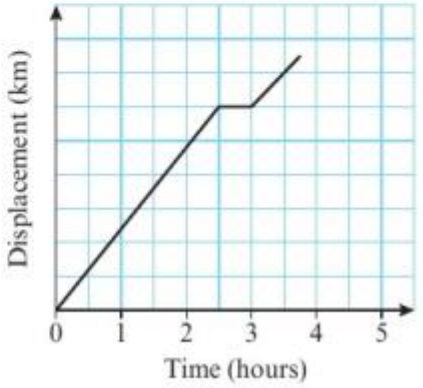
2 Khalid drives from his home to a hotel. He drives for $2\frac{1}{2}$ hours at an average velocity of 60 km h^{-1}. He then stops for lunch before continuing to his hotel. The diagram shows a displacement–time graph for Khalid's journey.

a Work out the displacement of the hotel from Khalid's home.

b Work out Khalid's average velocity for his whole journey.

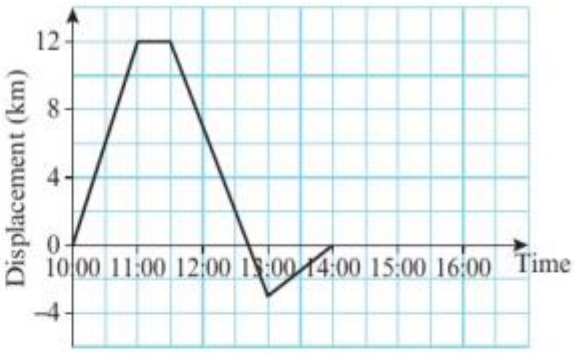
Problem-solving

You need to work out the scale on the vertical axis.



Homework Exercise

3 Sarah left home at 10:00 and cycled north in a straight line. The diagram shows a displacement–time graph for her journey.



a Work out Sarah's velocity between 10:00 and 11:00.

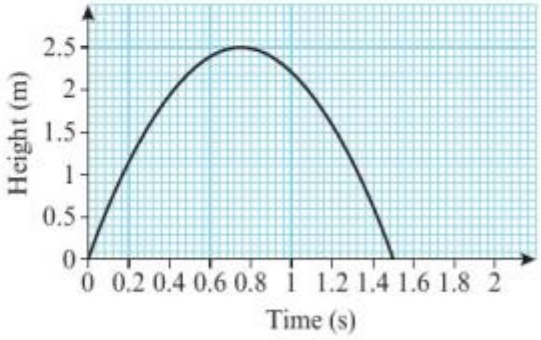
On her return journey, Sarah continued past her home before returning.

b Estimate the time that Sarah passed her home.

c Work out Sarah's velocity for each of the last two stages of her journey.

d Calculate Sarah's average speed for her entire journey.

4 A ball is thrown vertically up in the air and falls to the ground. This is a displacement–time graph for the motion of the ball.



a Find the maximum height of the ball and the time at which it reaches that height.

b Write down the velocity of the ball when it reaches its highest point.

c Describe the motion of the ball:

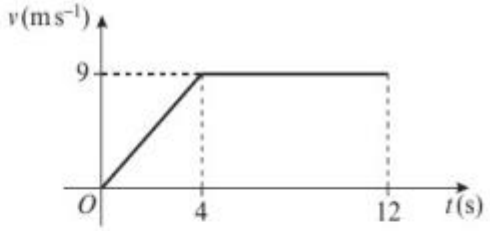
i from the time it is thrown to the time it reaches its highest point

ii after reaching its highest point.

Hint To describe the motion you should state the direction of travel of the ball and whether it is accelerating or decelerating.

Homework Exercise

5 The diagram shows the velocity–time graph of the motion of an athlete running along a straight track. For the first 4 s, he accelerates uniformly from rest to a velocity of $9\,\text{m s}^{-1}$.
This velocity is then maintained for a further 8 s. Find:



a the rate at which the athlete accelerates

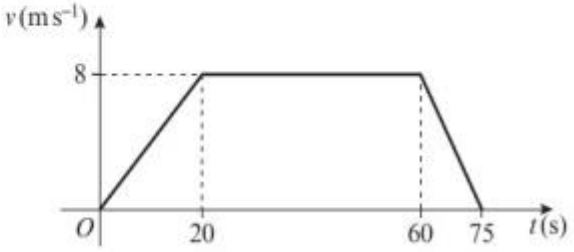
b the displacement from the starting point of the athlete after 12 s.

6 A car is moving along a straight road. When $t = 0$ s, the car passes a point A with velocity $10\,\text{m s}^{-1}$ and this velocity is maintained until $t = 30$ s. The driver then applies the brakes and the car decelerates uniformly, coming to rest at the point B when $t = 42$ s.

a Sketch a velocity–time graph to illustrate the motion of the car.

b Find the distance from A to B.

7 The diagram shows the velocity–time graph of the motion of a cyclist riding along a straight road. She accelerates uniformly from rest to $8\,\text{m s}^{-1}$ in 20 s. She then travels at a constant velocity of $8\,\text{m s}^{-1}$ for 40 s. She then decelerates uniformly to rest in 15 s. Find:



a the acceleration of the cyclist in the first 20 s of motion **(2 marks)**

b the deceleration of the cyclist in the last 15 s of motion **(2 marks)**

c the displacement from the starting point of the cyclist after 75 s. **(2 marks)**

Homework Exercise

8 A motorcyclist starts from rest at a point S on a straight race track. He moves with constant acceleration for 15 s, reaching a velocity of $30\,\text{m s}^{-1}$. He then travels at a constant velocity of $30\,\text{m s}^{-1}$ for T seconds. Finally he decelerates at a constant rate coming to rest at a point F, 25 s after he begins to decelerate.

a Sketch a velocity–time graph to illustrate the motion. **(3 marks)**

Given that the distance between S and F is 2.4 km:

b calculate the time the motorcyclist takes to travel from S to F. **(3 marks)**

9 A train starts from a station X and moves with constant acceleration of $0.6\,\text{m s}^{-2}$ for 20 s. The velocity it has reached after 20 s is then maintained for T seconds. The train then decelerates from this velocity to rest in a further 40 s, stopping at a station Y.

a Sketch a velocity–time graph to illustrate the motion of the train. **(3 marks)**

Given that the distance between the stations is 4.2 km, find:

b the value of T **(3 marks)**

c the distance travelled by the train while it is moving with constant velocity. **(2 marks)**

10 A particle moves along a straight line. The particle accelerates from rest to a velocity of $10\,\text{m s}^{-1}$ in 15 s. The particle then moves at a constant velocity of $10\,\text{m s}^{-1}$ for a period of time. The particle then decelerates uniformly to rest. The period of time for which the particle is travelling at a constant velocity is 4 times the period of time for which it is decelerating.

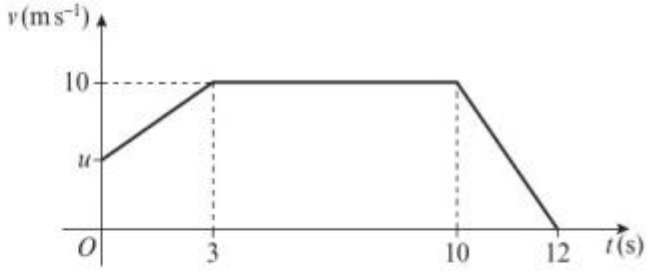
a Sketch a velocity–time graph to illustrate the motion of the particle. **(3 marks)**

Given that the displacement from the starting point of the particle after it comes to rest is 480 m

b find the total time for which the particle is moving. **(3 marks)**

Homework Exercise

11 A particle moves 100 m in a straight line. The diagram is a sketch of a velocity–time graph of the motion of the particle. The particle starts with velocity u m s^{-1} and accelerates to a velocity of 10 m s^{-1} in 3 s. The velocity of 10 m s^{-1} is maintained for 7 s and then the particle decelerates to rest in a further 2 s. Find:



a the value of u **(3 marks)**

b the acceleration of the particle in the first 3 s of motion. **(3 marks)**

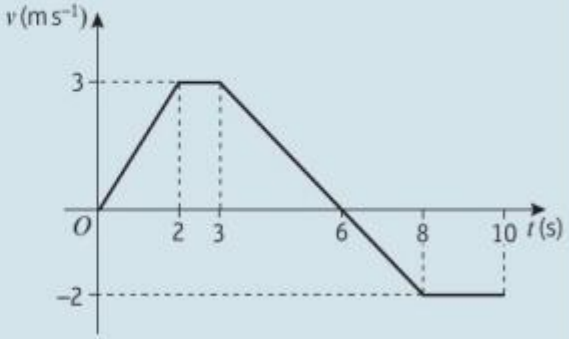
12 A motorcyclist M leaves a road junction at time $t = 0$ s. She accelerates from rest at a rate of 3 m s^{-2} for 8 s and then maintains the velocity she has reached. A car C leaves the same road junction as M at time $t = 0$ s. The car accelerates from rest to 30 m s^{-1} in 20 s and then maintains the velocity of 30 m s^{-1}. C passes M as they both pass a pedestrian.

a On the same diagram, sketch velocity–time graphs to illustrate the motion of M and C. **(3 marks)**

b Find the distance of the pedestrian from the road junction. **(3 marks)**

Challenge

The graph shows the velocity of an object travelling in a straight line during a 10-second time interval.



a After how long did the object change direction?

b Work out the total distance travelled by the object.

c Work out the displacement from the starting point of the object after:

i 6 seconds **ii** 10 seconds.

Homework Answers

1 **a** A 80 km h^{-1}, B 40 km h^{-1}, C 0 km h^{-1}, D km h^{-1}, E −66.7 km h^{-1}

b 0 km h^{-1} **c** 50 km h^{-1}

2 **a** 187.5 km **b** 50 km h^{-1}

3 **a** 12 km h^{-1} **b** 12:45

c −10 km h^{-1}, 3 km h^{-1} **d** 7.5 km h^{-1}

4 **a** 2.5 m, 0.75 s

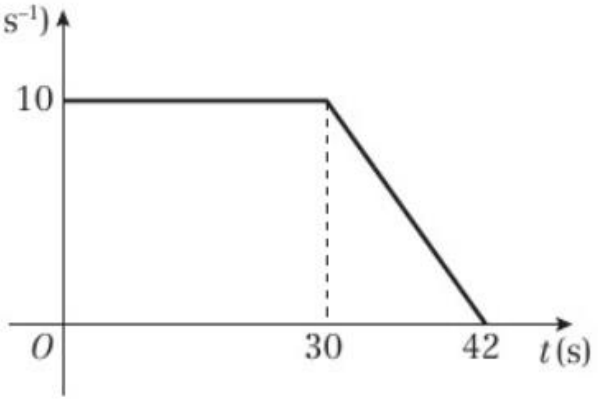
b 0 m s^{-1}

c **i** The velocity of the ball is positive (upwards). The ball is decelerating until it reaches 0 at the highest point.

ii The velocity of the ball is negative (downwards), and the ball is accelerating.

5 **a** 2.25 m s^{-2} **b** 90 m

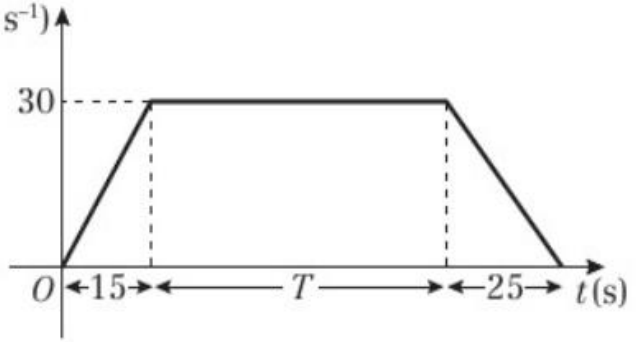
6 **a**



b 360 m

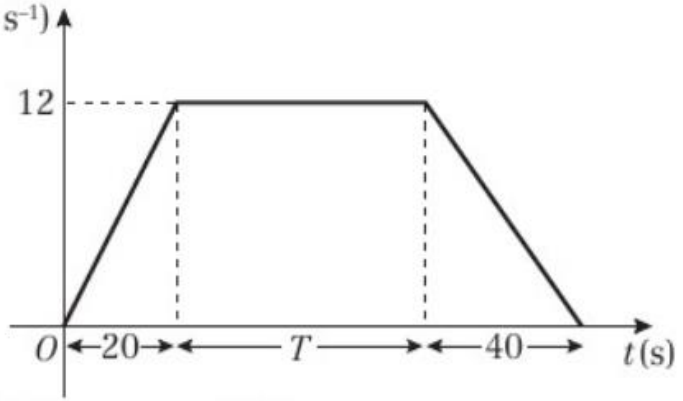
7 **a** 0.4 m s^{-2} **b** $\frac{8}{15}$ m s^{-2} or 0.53 m s^{-2} **c** 460 m

8 **a**



b 100 s

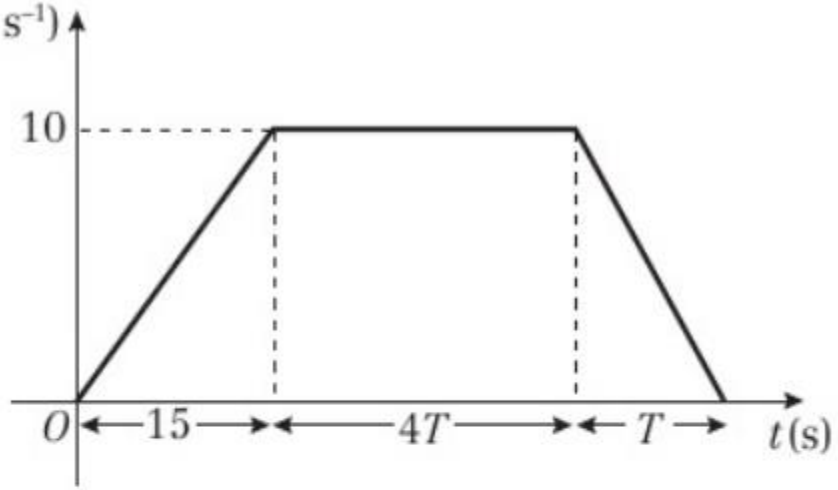
9 **a**



b $T = 320$ **c** 3840 m

Homework Answers

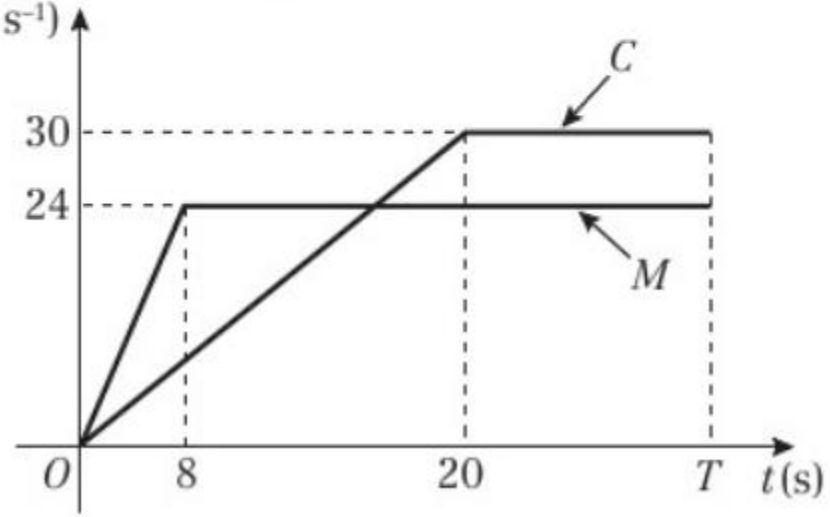
10 a



b 60 s

11 a $u = \frac{10}{3}$ **b** $\frac{20}{9}$ m s^{-2} = 2.22 m s^{-2}

12 a



b 720 m

Challenge

a 6 s **b** 16.5 m

c **i** 10.5 m **ii** 4.5 m