M2 Chapter 4: Moments

Centre of Mass

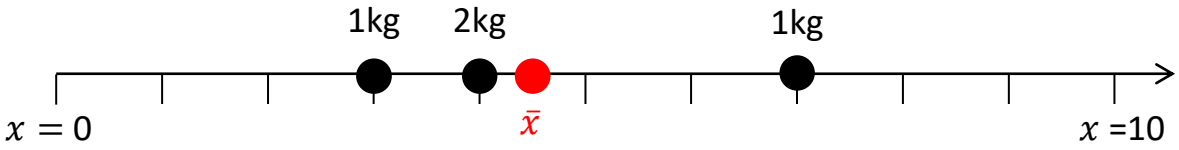
Centre of Mass of a set of particles on a straight line

If n particles of mass $m_1, m_2, \ldots, m_n$ are positioned at points with x co-ordinates $x_1, x_2, \ldots, x_n$ then the centre of mass[1], $\bar{x}$ is the position of an equivalent total mass $M = \sum_{i=1}^{n} m_i$, :

$$\mathrm{M}\,\bar{x} = \sum_{i=1}^{n} m_i x_i$$

In other words, it is the total mass times the ***average position of each unit of mass***.

[example] A system of 3 particles with masses 1kg, 2kg and 1kg are placed along the x-axis at the points 3, 4 and 7 respectively. Find the x coordinate of the centre of mass.



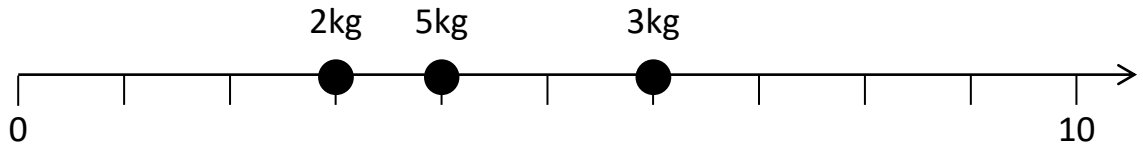
$$\sum m_i x_i = \bar{x} \sum m_i \quad \Rightarrow \quad (1 \times 3) + [1 \times 4 + 1 \times 4] + (1 \times 7) = \bar{x} \times (1 + 2 + 1)$$

$$\therefore \quad 18 = 4\bar{x} \quad \Rightarrow \quad \bar{x} = 4.5$$

note: $[1 \times 4 + 1 \times 4] = 2 \times 4$

An example

[textbook] A system of 3 particles with masses 2kg, 5kg and 3kg are placed along the x-axis at the points (3,0), (4,0) and (6,0) respectively. Find the centre of mass of the system.



?

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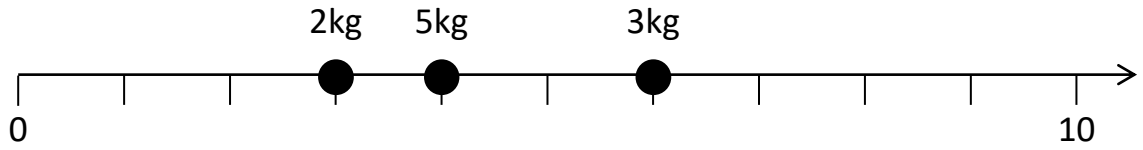


$$\sum m_i x_i = \bar{x} \sum m_i$$

$$2 \times 3 + 5 \times 4 + 3 \times 6 = \bar{x} \times (2 + 5 + 3)$$

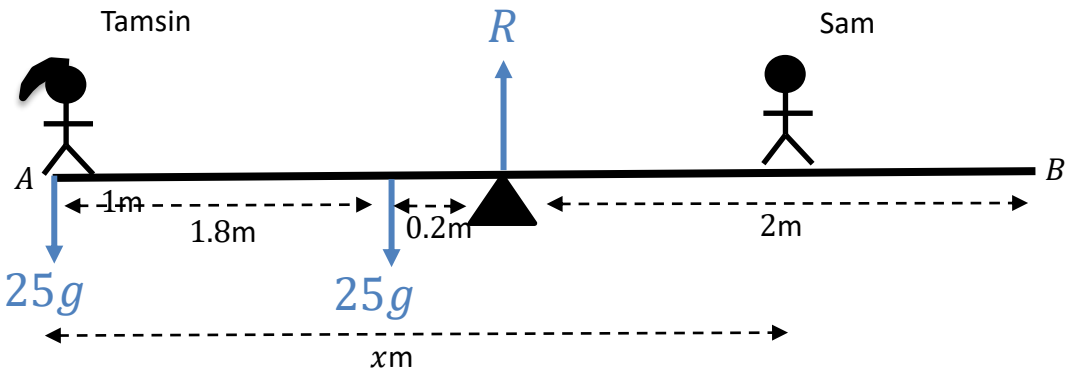
$$\bar{x} = \frac{44}{10} = 4.4$$

$\Rightarrow$ centre of mass is $(4.4, 0)$

Centres of Mass

We have assumed that the rod is uniform, that is, its mass is equally distributed across the rod, such that the centre of mass is the centre.
But this may not be the case, and for **non-uniform** rods we may wish to find where the centre of mass lies, or we will be told where it lies.

[Textbook] Sam and Tamsin are sitting on a non-uniform plan AB of mass 25kg and length 4m. The plank is pivoted at M, the midpoint of AB. The centre of mass of AB is at C where AC is 1.8. Sam has mass 35 kg. Tamsin has mass 25 kg and sits at A.
Where must Sam sit for the plank to be horizontal?



Moments about M:
$(25g \times 2) + (25g \times 0.2) = 35g(x - 2)$
...
$x = 3.57$

i.e. Sam should sit 3.57m from end A.

Test Your Understanding

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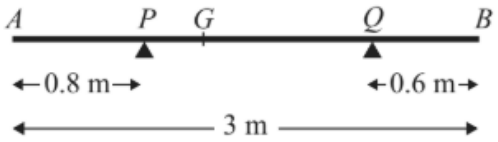

Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q, where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G. Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod,

(3)

(b) the distance AG.

(4)

? Diagram

(a) ?

(b) ?

Test Your Understanding

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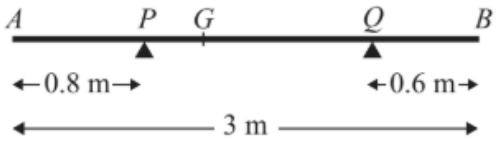

Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q, where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G. Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

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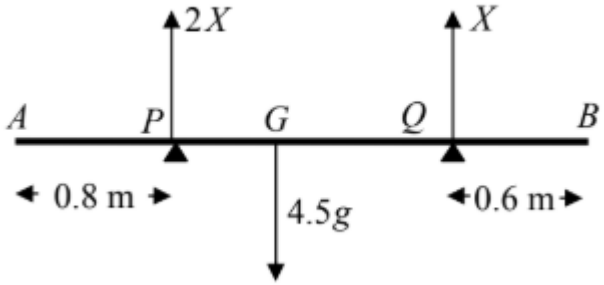

(a) $\uparrow \quad 2X + X = 4.5g$ | M1 A1

Leading to $X = \frac{3g}{2}$ or 14.7 or 15 (N) | A1 **(3)**

(b) $\mathrm{M}(A) \quad 4.5g \times AG = (2X) \times 0.8 + X \times 2.4$ | M1 A2 **ft** (1,0)

$AG = \frac{4}{3}$ (m), 1.3, 1.33,... | A1 **(4)**

[7]

Exercise 4.4

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Homework Exercise

1 A non-uniform rod AB, of length 4 m and weight 6 N, rests horizontally on two supports, A and B. Given that the centre of mass of the rod is 2.4 m from the end A, find the reactions at the two supports.

2 A non-uniform bar AB of length 5 m is supported horizontally on supports, A and B. The reactions at these supports are $3g$ N and $7g$ N respectively.

a State the weight of the bar.

b Find the distance of the centre of mass of the bar from A.

3 A non-uniform plank AB, of length 4 m and weight 120 N, is pivoted at its midpoint. The plank is in equilibrium in a horizontal position with a child of weight 200 N sitting at A and a child of weight 300 N sitting at B. By modelling the plank as a rod and the two children as particles, find the distance of the centre of mass of the plank from A.

4 A non-uniform rod AB, of length 5 m and mass 15 kg, rests horizontally suspended from the ceiling by two vertical strings attached to C and D, where $AC = 1$ m and $AD = 3.5$ m.

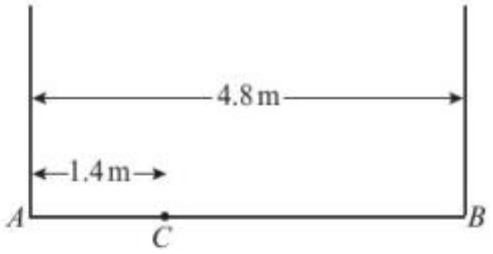
a Given that the centre of mass is at E where $AE = 3$ m, find the magnitudes of the tensions in the strings.

When a particle of mass 9 kg is attached to the rod at F the tension in the string at D is twice the tension in the string at C.

b Find the distance AF.

Homework Exercise

5 A plank AB has mass 24 kg and length 4.8 m. A load of mass 15 kg is attached to the plank at the point C, where $AC = 1.4$ m. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes, one attached at A and the other attached at B, as shown in the diagram. The plank is modelled as a uniform rod, the load as a particle and the ropes as light inextensible strings.

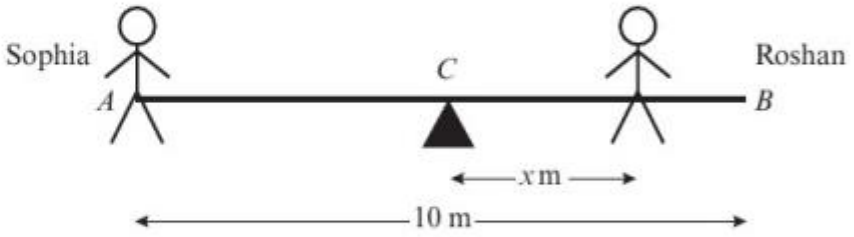


a Find the tension in the rope attached at B. **(4 marks)**

The plank is now modelled as a non-uniform rod. With the new model, the tension in the rope attached at A is 25 N greater than the tension in the rope attached at B.

b Find the distance of the centre of mass of the plank from A. **(6 marks)**

6 A seesaw in a playground consists of a beam AB of length 10 m which is supported by a smooth pivot at its centre C. Sophia has mass 30 kg and sits on the end A. Roshan has mass 50 kg and sits at a distance x metres from C, as shown in the diagram. The beam is initially modelled as a uniform rod. Using this model,



a find the value of x for which the seesaw can rest in equilibrium in a horizontal position. **(3 marks)**

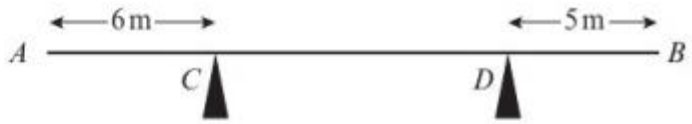
b State what is implied by the modelling assumption that the beam is uniform. **(1 mark)**

Roshan finds he must sit at a distance 4 m from C for the seesaw to rest horizontally in equilibrium. The beam is now modelled as a non-uniform rod of mass 25 kg. Using this model,

c find the distance of the centre of mass of the beam from C. **(4 marks)**

Homework Exercise

7 A non-uniform rod AB, of length 25 m and weight 80 N, rests horizontally in equilibrium on supports C and D as shown in the diagram. The centre of mass of the rod is 10 m from A.

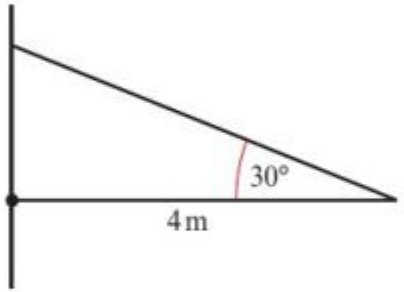


A particle of weight W newtons is attached to the rod at a point E, where E is x metres from A.

The rod remains in equilibrium and the magnitude of the reaction at C is five times the magnitude of the reaction at D.

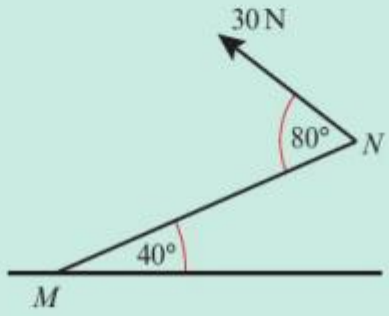
Show that $W = \dfrac{400}{25 - 3x}$ **(5 marks)**

8 A non-uniform rod of weight 100 N and length 4 m is freely hinged to a vertical wall, and held in place by a cable attached at an angle of 30° to the end of the rod. Given that the tension in the cable is 80 N and that the rod is held in horizontal equilibrium, find the distance of the centre of mass of the rod from the wall. **(8 marks)**



Challenge

A non-uniform beam of weight 120 N and length 5 m is smoothly pivoted at a point M and is held at an angle of 40° by a cable attached at point N. Given that the tension in the cable is 30 N and it makes an angle of 80° with the beam, find the distance of the centre of mass of the beam from M.



Homework Answers

1 $R_A = 2.4\,\text{N}, R_B = 3.6\,\text{N}$

2 **a** $10g\,\text{N}$ **b** 3.5 m from A

3 $\frac{1}{3}$ m from A

4 **a** 29.4 N, 118 N **b** 2.11 m

5 **a** 160 N **b** 2.77 m

6 **a** 3 m

b Centre of mass lies at the midpoint of the seesaw.

c 2 m towards Sophia.

7 $R_C = 5R_D$

$R_C + R_D = 80 + W$

$R_D = \frac{80 + W}{6}$

Taking moments about A: $6R_C + 20R_D = 80 \times 10 + xW$

$50R_D = 800 + xW$

$25W - 3xW = 400$

$W = \frac{400}{25 - 3x}$

8 1.6 m

Challenge

1.61 m