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# P2 Chapter 6: CoAngle Trigonometry

## Arc Functions

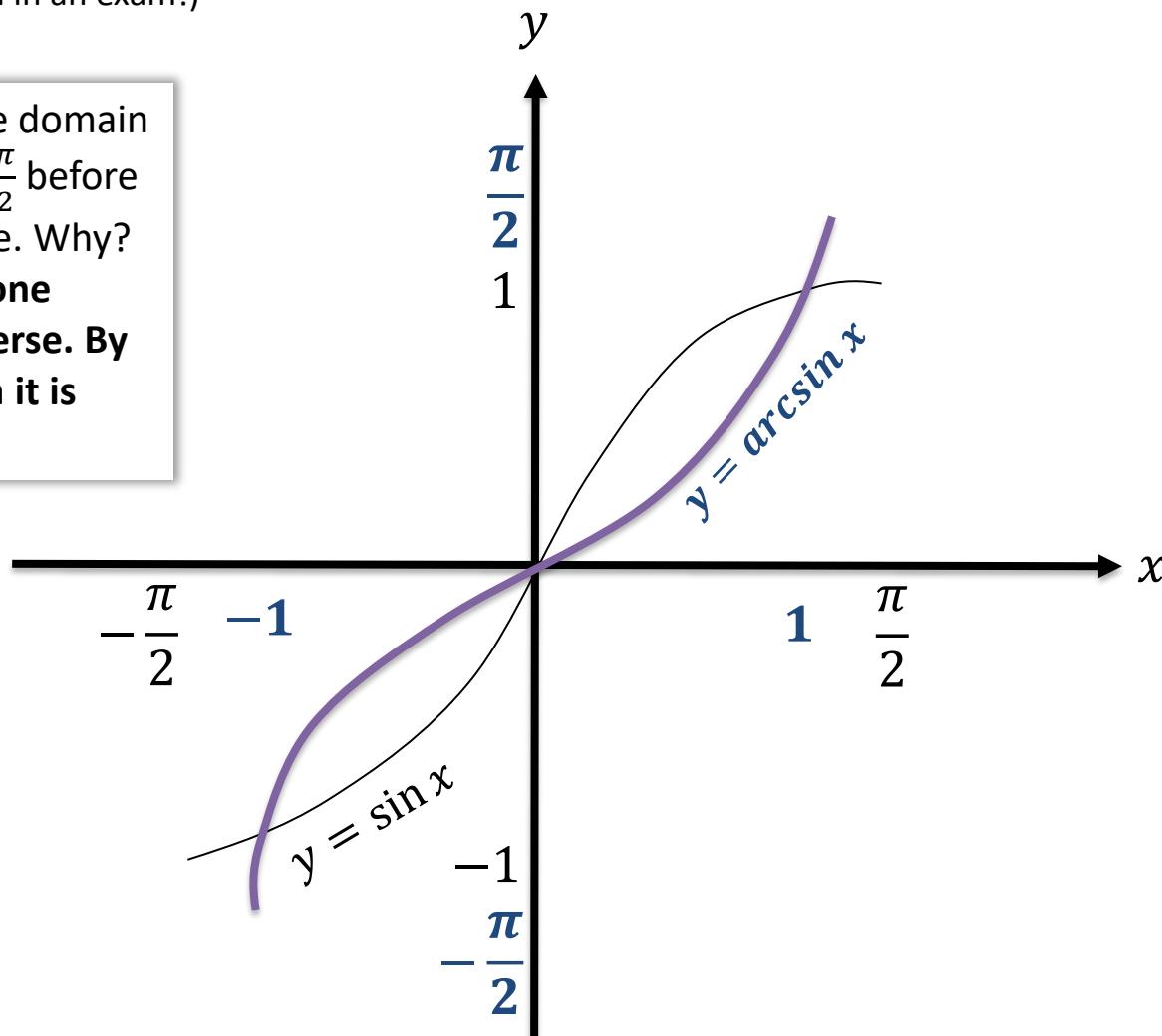
# Inverse Trig Functions

You need to know how to sketch  $y = \arcsin x$ ,  $y = \arccos x$ ,  $y = \arctan x$ .

(Yes, you could be asked in an exam!)

We have to restrict the domain of  $\sin x$  to  $-\frac{\pi}{2} \leq x < \frac{\pi}{2}$  before we can find the inverse. Why?

**Because only one-to-one functions have an inverse. By restricting the domain it is now one-to-one.**



# Inverse Trig Functions

$$y = \arccos x$$

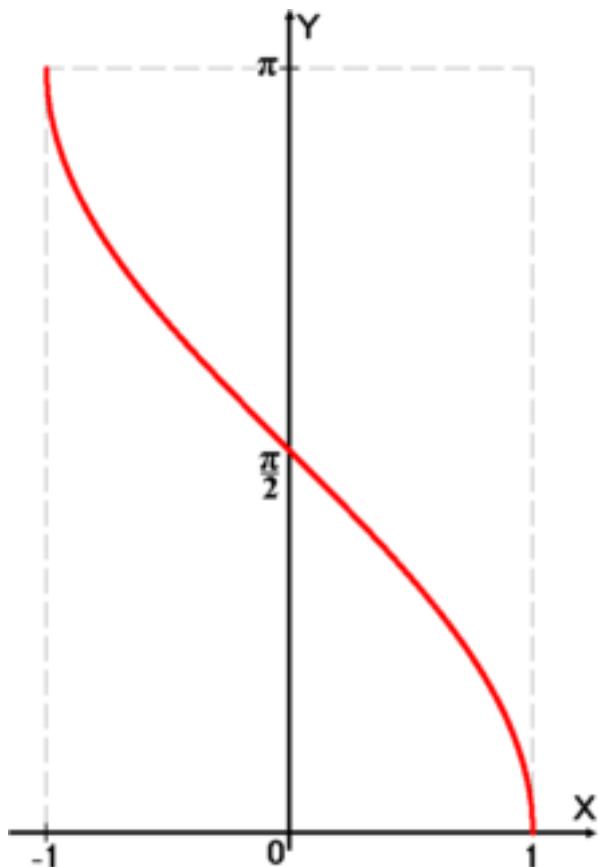
$$y = \arctan x$$

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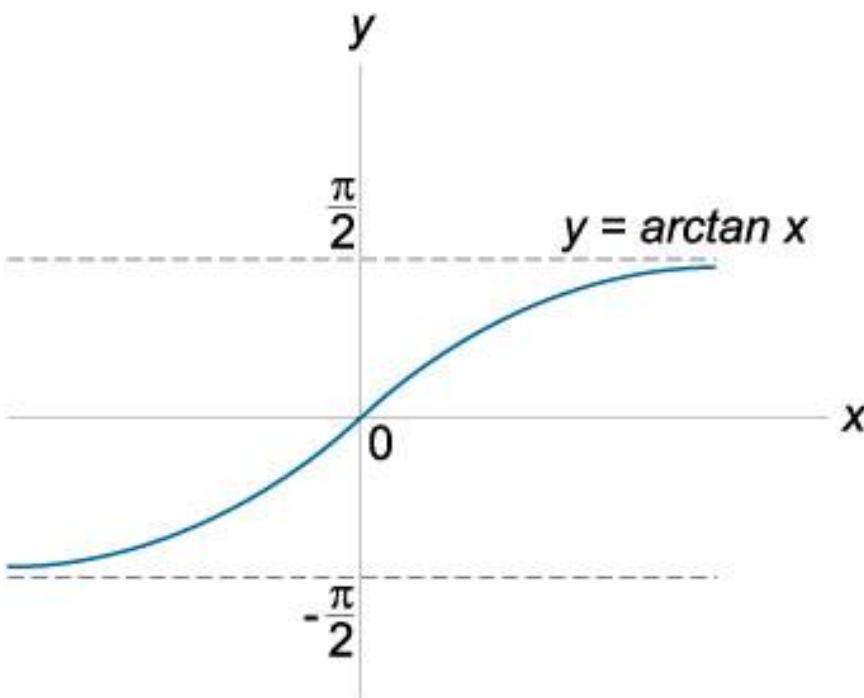
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# Inverse Trig Functions

$$y = \arccos x$$



$$y = \arctan x$$



Note that this graph has asymptotes.

# Evaluating inverse trig functions

[Textbook] Work out, in radians, the values of:

- a)  $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$
- b)  $\arccos(-1)$
- c)  $\arctan(\sqrt{3})$

You can simply use the  $\sin^{-1} x$ ,  $\cos^{-1} x$  and  $\tan^{-1} x$  buttons on your calculator.

If you don't have a calculator, just use the  $\sin, \cos, \tan$  graphs backwards.

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$$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

$$\arccos(-1) = \pi$$

$$\arctan(\sqrt{3}) = \frac{\pi}{3}$$

# One Final Problem...

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8. (ii) Given that

$$y = \arccos x, \quad -1 \leq x \leq 1 \quad \text{and} \quad 0 \leq y \leq \pi,$$

- (a) express  $\arcsin x$  in terms of  $y$ .

(2)

- (b) Hence evaluate  $\arccos x + \arcsin x$ . Give your answer in terms of  $\pi$ .

(1)

Fewer than 10%  
of candidates got  
this part right.

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(1)

$$y = \arccos x$$

$$x = \cos y = \sin\left(\frac{\pi}{2} - y\right)$$

$$\arcsin x = \frac{\pi}{2} - y$$

Fewer than 10%  
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$$\begin{aligned} & \arccos x + \arcsin x \\ &= y + \frac{\pi}{2} - y = \frac{\pi}{2} \end{aligned}$$

# Exercise 6.5

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# Homework Exercise

In this exercise, all angles are given in radians.

1 Without using a calculator, work out, giving your answer in terms of  $\pi$ :

a  $\arccos(0)$

b  $\arcsin(1)$

c  $\arctan(-1)$

d  $\arcsin\left(-\frac{1}{2}\right)$

e  $\arccos\left(-\frac{1}{\sqrt{2}}\right)$

f  $\arctan\left(-\frac{1}{\sqrt{3}}\right)$

g  $\arcsin\left(\sin \frac{\pi}{3}\right)$

h  $\arcsin\left(\sin \frac{2\pi}{3}\right)$

2 Find:

a  $\arcsin\left(\frac{1}{2}\right) + \arcsin\left(-\frac{1}{2}\right)$

b  $\arccos\left(\frac{1}{2}\right) - \arccos\left(-\frac{1}{2}\right)$

c  $\arctan(1) - \arctan(-1)$

3 Without using a calculator, work out the values of:

a  $\sin\left(\arcsin \frac{1}{2}\right)$

b  $\sin\left(\arcsin\left(-\frac{1}{2}\right)\right)$

c  $\tan(\arctan(-1))$

d  $\cos(\arccos 0)$

4 Without using a calculator, work out the exact values of:

a  $\sin\left(\arccos\left(\frac{1}{2}\right)\right)$

b  $\cos\left(\arcsin\left(-\frac{1}{2}\right)\right)$

c  $\tan\left(\arccos\left(-\frac{\sqrt{2}}{2}\right)\right)$

d  $\sec(\arctan(\sqrt{3}))$

e  $\operatorname{cosec}(\arcsin(-1))$

f  $\sin\left(2\arcsin\left(\frac{\sqrt{2}}{2}\right)\right)$

# Homework Exercise

- 5 Given that  $\arcsin k = \alpha$ , where  $0 < k < 1$  and  $\alpha$  is in radians, write down, in terms of  $\alpha$ , the first two positive values of  $x$  satisfying the equation  $\sin x = k$ .
- 6 Given that  $x$  satisfies  $\arcsin x = k$ , where  $0 < k < \frac{\pi}{2}$ ,
- state the range of possible values of  $x$  (1 mark)
  - express, in terms of  $x$ ,
    - $\cos k$
    - $\tan k$(4 marks)
- Given, instead, that  $-\frac{\pi}{2} < k < 0$ ,
- how, if at all, are your answers to part b affected? (2 marks)
- 7 Sketch the graphs of:
- $y = \frac{\pi}{2} + 2 \arcsin x$
  - $y = \pi - \arctan x$
  - $y = \arccos(2x + 1)$
  - $y = -2 \arcsin(-x)$

# Homework Exercise

- 8** The function  $f$  is defined as  $f : x \mapsto \arcsin x$ ,  $-1 \leq x \leq 1$ , and the function  $g$  is such that  $g(x) = f(2x)$ .
- Sketch the graph of  $y = f(x)$  and state the range of  $f$ . (3 marks)
  - Sketch the graph of  $y = g(x)$ . (2 marks)
  - Define  $g$  in the form  $g : x \mapsto \dots$  and give the domain of  $g$ . (3 marks)
  - Define  $g^{-1}$  in the form  $g^{-1} : x \mapsto \dots$  (2 marks)
- 9 a** Prove that for  $0 \leq x \leq 1$ ,  $\arccos x = \arcsin \sqrt{1 - x^2}$  (4 marks)
- b** Give a reason why this result is not true for  $-1 \leq x \leq 0$ . (2 marks)

## Challenge

- Sketch the graph of  $y = \sec x$ , with the restricted domain  $0 \leq x \leq \pi$ ,  $x \neq \frac{\pi}{2}$
- Given that  $\text{arcsec } x$  is the inverse function of  $\sec x$ ,  $0 \leq x \leq \pi$ ,  $x \neq \frac{\pi}{2}$ , sketch the graph of  $y = \text{arcsec } x$  and state the range of  $\text{arcsec } x$ .

# Homework Answers

1    a  $\frac{\pi}{2}$                   b  $\frac{\pi}{2}$                   c  $-\frac{\pi}{4}$                   d  $-\frac{\pi}{6}$

e  $\frac{3\pi}{4}$                   f  $-\frac{\pi}{6}$                   g  $\frac{\pi}{3}$                   h  $\frac{\pi}{3}$

2    a 0                  b  $-\frac{\pi}{3}$                   c  $\frac{\pi}{2}$

3    a  $\frac{1}{2}$                   b  $-\frac{1}{2}$                   c -1                  d 0

4    a  $\frac{\sqrt{3}}{2}$                   b  $\frac{\sqrt{3}}{2}$                   c -1                  d 2

e -1                  f 1

5     $\alpha, \pi - \alpha$

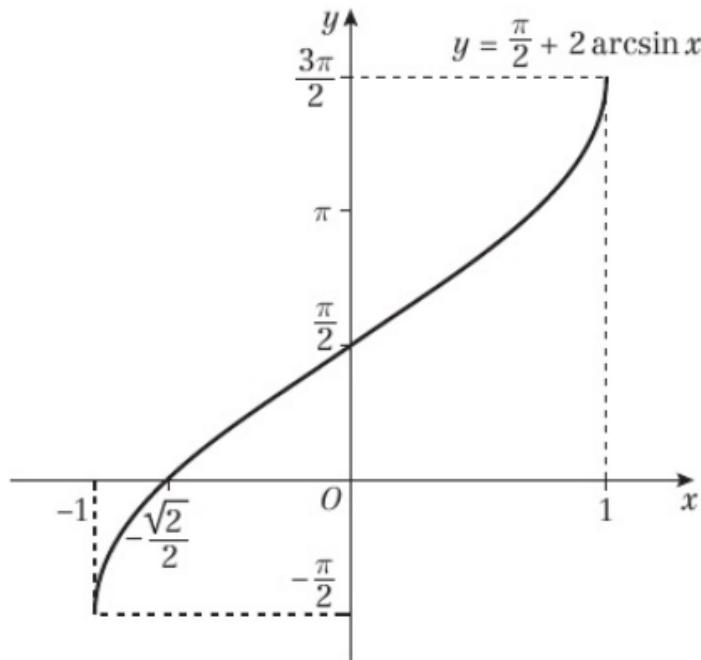
6    a  $0 < x < 1$

b i  $\sqrt{1 - x^2}$                   ii  $\frac{x}{\sqrt{1 - x^2}}$

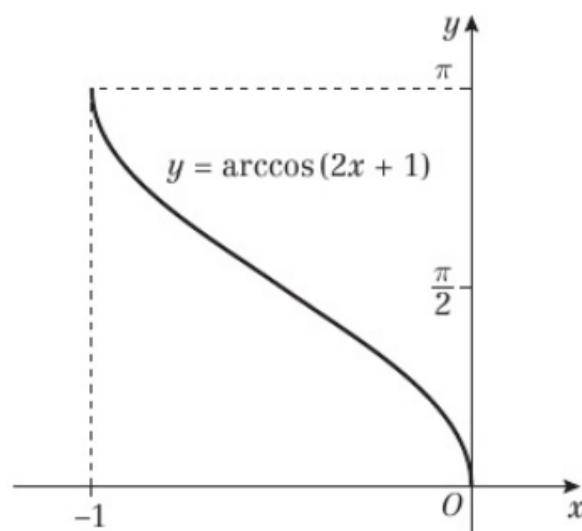
c i no change                  ii no change

# Chapter Answers

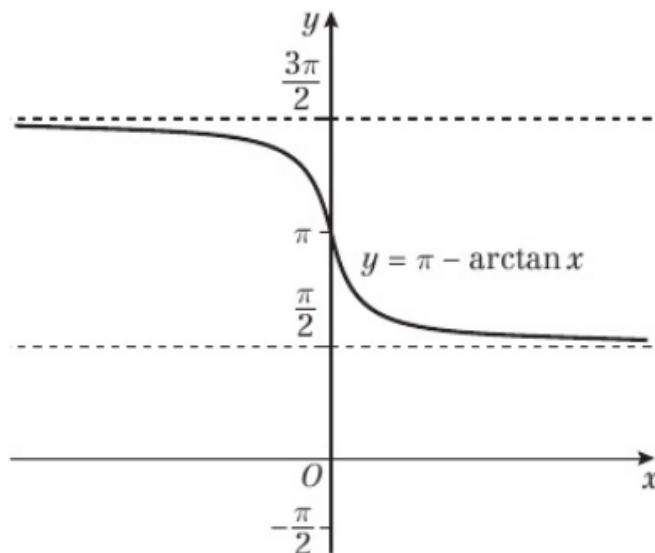
7 a



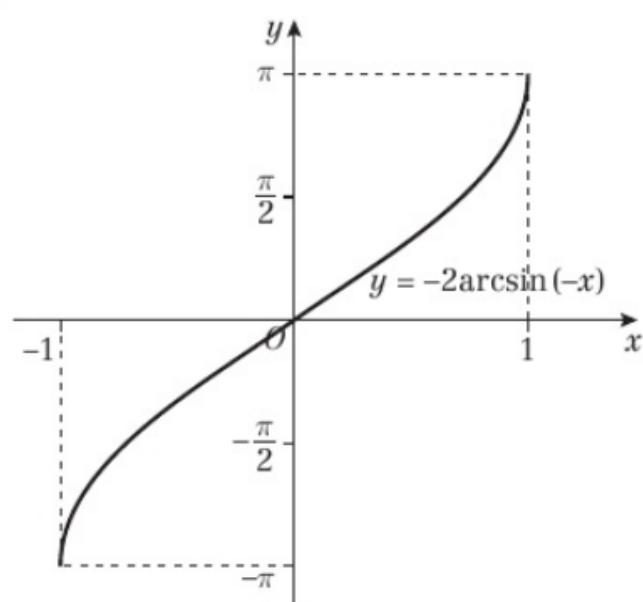
c



b

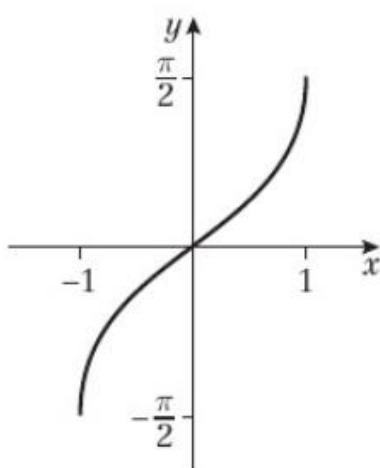


d



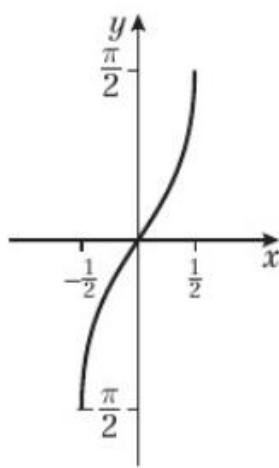
# Chapter Answers

8 a



$$\text{Range: } -\frac{\pi}{2} \leq f(x) \leq \frac{\pi}{2}$$

b



c g:  $x \rightarrow \arcsin 2x, -\frac{1}{2} \leq x \leq \frac{1}{2}$

d  $g^{-1}: x \rightarrow \frac{1}{2} \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

9 a Let  $y = \arccos x$ .  $x \in [0,1] \Rightarrow y \in [0, \frac{\pi}{2}]$

$$\cos y = x, \text{ so } \sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$$

$$(\text{Note, } \sin y \neq -\sqrt{1 - x^2} \text{ since } y \in [0, \frac{\pi}{2}], \text{ so } \sin y \geq 0)$$

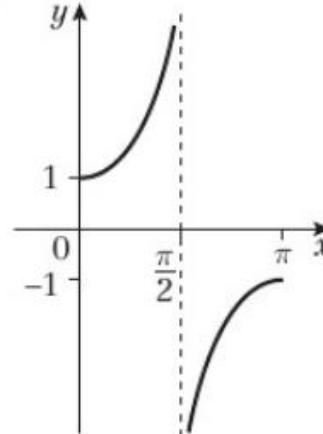
$$y = \arcsin \sqrt{1 - x^2}$$

Therefore,  $\arccos x = \arcsin \sqrt{1 - x^2}$  for  $x \in [0,1]$ .

b For  $x \in [-1,0]$ ,  $\arccos x \in (\frac{\pi}{2}, \pi)$ , but  $\arcsin$  only has range  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .

## Challenge

a



$$\text{Range: } 0 \leq \operatorname{arcsec} x \leq \pi, \operatorname{arcsec} x \neq \frac{\pi}{2}$$

b

