P1 Chapter 5: Linear Graphs

Linear Equations

Equations using two points/point + gradient

Find the equation of the line that goes through $(3,5)$ and has gradient 2.

How would you have done this at **GCSE**?

?

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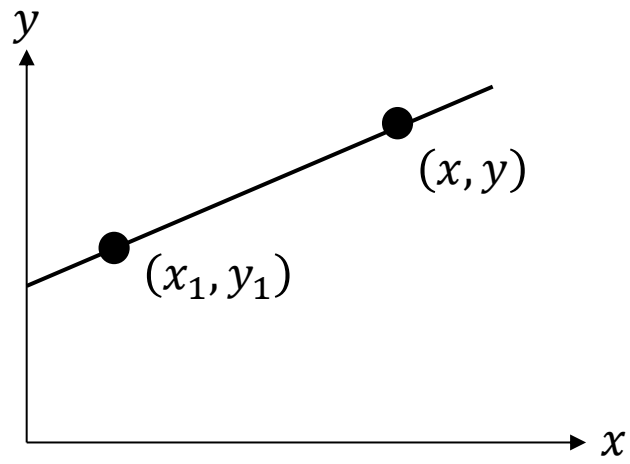
If gradient is 2, then equation of the line is of the form:

$$y = 2x + c$$

Since $(3, 5)$ is on the line, it must satisfy this equation.
Therefore:

$$5 = 2(3) + c$$
$$c = -1$$
$$\therefore y = 2x - 1$$

A new way…



Note: Note that x_1 and y_1 are constants while x and y are variables. The latter are variables because as these 'vary', we get different points on the line.

Suppose that (x_1, y_1) is some fixed point on the line that we specify (e.g. $(3,5)$). Suppose that (x, y) represents a generic point on the line, which is allowed to change as we consider different points on this line.

Then:

$$m = \text{?}$$

Thus:

The equation of a line that has gradient m and passes through a point (x_1, y_1) is:

$$y - y_1 = m(x - x_1)$$

Let's revisit:

Find the equation of the line that goes through $(3,5)$ and has gradient 2.

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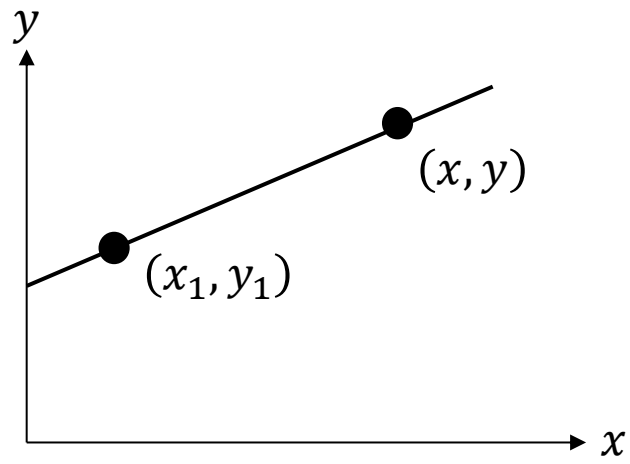

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Then:

$$m = \frac{y - y_1}{x - x_1}$$

Thus:

✎ The equation of a line that has gradient m and passes through a point (x_1, y_1) is:
$$y - y_1 = m(x - x_1)$$

Let's revisit:

Find the equation of the line that goes through $(3,5)$ and has gradient 2.

$$y - 5 = 2(x - 3)$$

No manipulation required! Just plug in numbers and we're done.
Again, note that x and y are variables so need to remain as such.

Quickfire Questions

In a nutshell: You can use this formula whenever you have (a) a gradient and (b) any point on the line.

Gradient	Point	(Unsimplified) Equation
3	$(1,2)$	?
5	$(3,0)$	?
2	$(-3,4)$	?
$\frac{1}{2}$	$(1,-5)$	?
9	$(-4,-4)$	?

Important Fro Side Note: I've found that many students shun this formula and just use the GCSE method. Please persist with it – it'll be much easier when fractions are involved. Further Mathematicians, don't even think about using the GCSE method, because you'll encounter massive headaches when you consider algebraic points. Trust me on this one!

Quickfire Questions

In a nutshell: You can use this formula whenever you have (a) a gradient and (b) any point on the line.

Gradient	Point	(Unsimplified) Equation
3	$(1,2)$	$\boldsymbol{y-2=3(x-1)}$
5	$(3,0)$	$\boldsymbol{y=5(x-3)}$
2	$(-3,4)$	$\boldsymbol{y-4=2(x+3)}$
$\frac{1}{2}$	$(1,-5)$	$\boldsymbol{y+5=\frac{1}{2}(x-1)}$
9	$(-4,-4)$	$\boldsymbol{y+4=9(x+4)}$

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Using 2 points

Find the equation of the line that goes through $(4,5)$ and $(6,2)$, giving your equation in the form $ax + by + c = 0$.

?

Test Your Understanding:
Find the equation of the line that goes through $(-1,9)$ and $(4,5)$, giving your equation in the form $ax + by + c = 0$.

?

Using 2 points

Find the equation of the line that goes through $(4,5)$ and $(6,2)$, giving your equation in the form $ax + by + c = 0$.

We need to work out the gradient first before we can use $y - y_1 = m(x - x_1)$:

$$m = -\frac{3}{2}$$

Using $(4,5)$ (we could have also used $(6,2)$):

$$y - 5 = -\frac{3}{2}(x - 4)$$
$$2y - 10 = -3(x - 4)$$
$$2y - 10 = -3x + 12$$
$$3x + 2y - 22 = 0$$

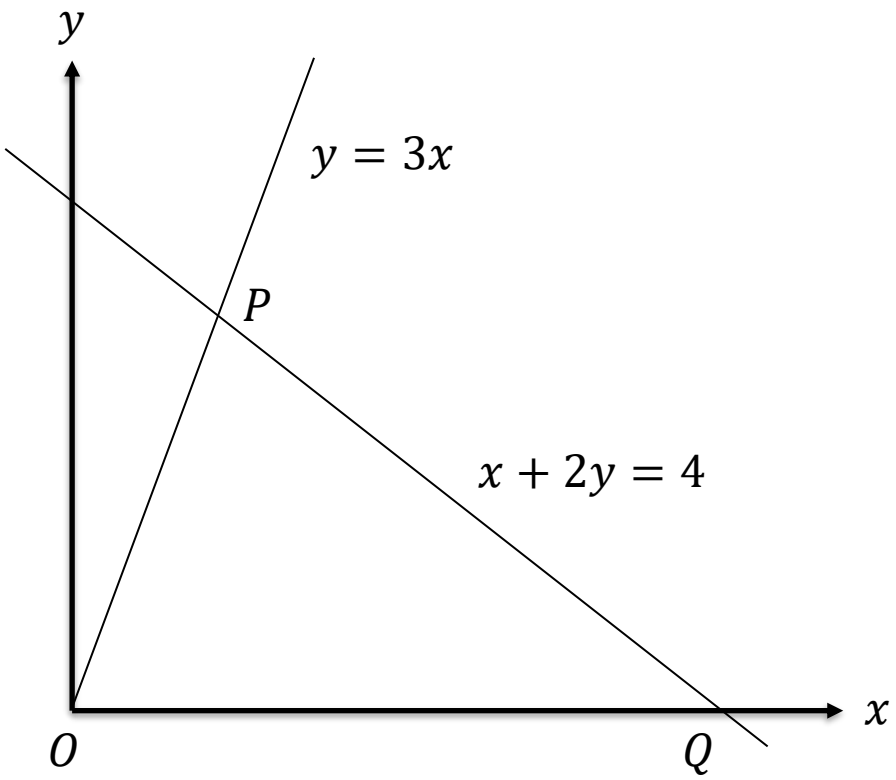
Test Your Understanding:
Find the equation of the line that goes through $(-1,9)$ and $(4,5)$, giving your equation in the form $ax + by + c = 0$.

$$m = -\frac{4}{5}$$

Using $(-1,9)$:

$$y - 9 = -\frac{4}{5}(x + 1)$$
$$5y - 45 = -4(x + 1)$$
$$5y - 45 = -4x - 4$$
$$4x + 5y - 41 = 0$$

Intersection of lines



The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point P.

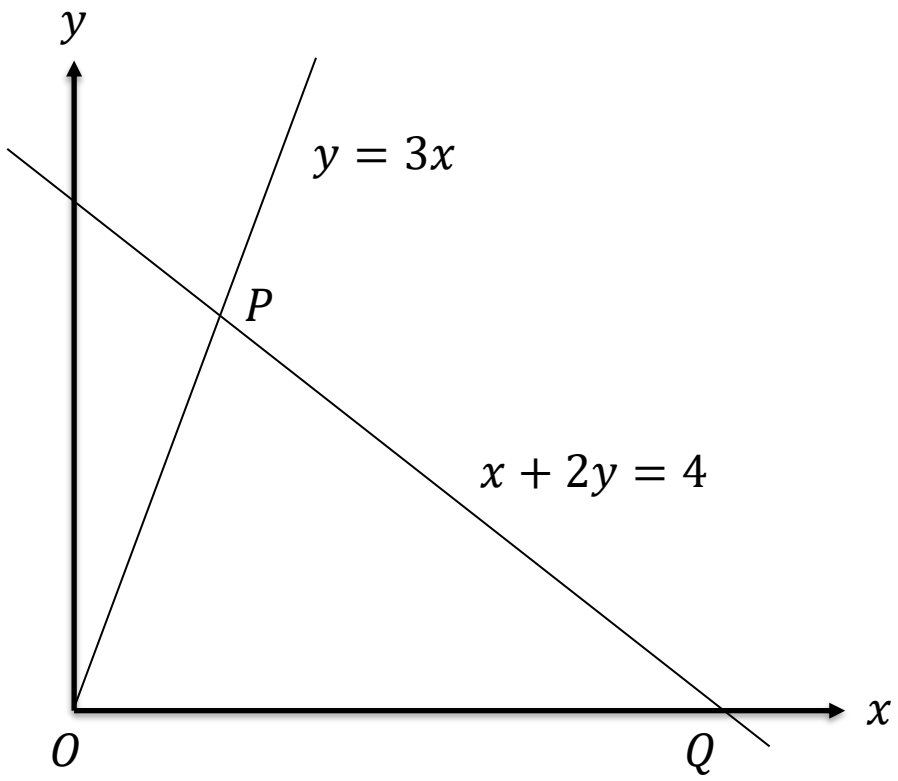
a) Determine the coordinates of P.

?

b) The line $x + 2y = 4$ intersects the x-axis at the point Q. Determine the coordinate of Q.

?

Intersection of lines



The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point P.

a) Determine the coordinates of P.

We know from Chapter 4 we can just solve two equations simultaneously.

$$\boldsymbol{x + 2(3x) = 4}$$
$$\boldsymbol{7x = 4}$$
$$\boldsymbol{x = \frac{4}{7}}$$
$$\boldsymbol{y = 3\left(\frac{4}{7}\right) = \frac{12}{7}}$$

b) The line $x + 2y = 4$ intersects the x-axis at the point Q. Determine the coordinate of Q.

When $\boldsymbol{y = 0, x = 4}$

Area $\boldsymbol{= \frac{1}{2} \times 4 \times \frac{12}{7} = \frac{24}{7}}$

Test Your Understanding

C1 Edexcel May 2013 Q6

The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(a) Find an equation for L_1 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(b) Find the coordinates of the point of intersection of L_1 and L_2 .

(3)

(a)

?

(b)

?

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	$(-1, 3)$, $(11, 12)$		
(a)	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-3}{11-(-1)}, = \frac{3}{4}$	M1:Correct method for the gradient A1: Any correct fraction or decimal	M1,A1
	$y-3 = \frac{3}{4}(x+1)$ or $y-12 = \frac{3}{4}(x-11)$ or $y = \frac{3}{4}x + c$ with attempt at substitution to find c	Correct straight line method using either of the given points and a numerical gradient.	M1
	$4y - 3x - 15 = 0$	Or equivalent with integer coefficients (= 0 **is** required)	A1
	This A1 should only be awarded in (a)		
(b)	Solves their equation from part (a) and L_2 simultaneously to eliminate one variable	Must reach as far as an equation in x only or in y only. (Allow slips in the algebra)	M1
	$x = 3$ or $y = 6$	One of $x = 3$ or $y = 6$	A1
	Both $x = 3$ **and** $y = 6$	Values can be un-simplified fractions.	A1
	Fully correct answers with no working can score 3/3 in (b)		
			(3)

Exercise 5.2

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Homework Exercise

1 Find the equation of the line with gradient m that passes through the point (x_1, y_1) when:

a $m = 2$ and $(x_1, y_1) = (2, 5)$

b $m = 3$ and $(x_1, y_1) = (-2, 1)$

c $m = -1$ and $(x_1, y_1) = (3, -6)$

d $m = -4$ and $(x_1, y_1) = (-2, -3)$

e $m = \frac{1}{2}$ and $(x_1, y_1) = (-4, 10)$

f $m = -\frac{2}{3}$ and $(x_1, y_1) = (-6, -1)$

g $m = 2$ and $(x_1, y_1) = (a, 2a)$

h $m = -\frac{1}{2}$ and $(x_1, y_1) = (-2b, 3b)$

2 Find the equations of the lines that pass through these pairs of points:

a (2, 4) and (3, 8)

b (0, 2) and (3, 5)

c (−2, 0) and (2, 8)

d (5, −3) and (7, 5)

e (3, −1) and (7, 3)

f (−4, −1) and (6, 4)

g (−1, −5) and (−3, 3)

h (−4, −1) and (−3, −9)

i $(\frac{1}{3}, \frac{2}{5})$ and $(\frac{2}{3}, \frac{4}{5})$

j $(-\frac{3}{4}, \frac{1}{7})$ and $(\frac{1}{4}, \frac{3}{7})$

Hint In each case find the gradient m then use $y - y_1 = m(x - x_1)$.

3 Find the equation of the line l which passes through the points $A(7, 2)$ and $B(9, -8)$. Give your answer in the form $ax + by + c = 0$. **(3 marks)**

4 The vertices of the triangle ABC have coordinates $A(3, 5)$, $B(-2, 0)$ and $C(4, -1)$. Find the equations of the sides of the triangle.

Homework Exercise

5 The straight line l passes through $(a, 4)$ and $(3a, 3)$. An equation of l is $x + 6y + c = 0$.
Find the value of a and the value of c. **(3 marks)**

6 The straight line l passes through $(7a, 5)$ and $(3a, 3)$.
An equation of l is $x + by - 12 = 0$.
Find the value of a and the value of b. **(3 marks)**

Problem-solving

It is often easier to find unknown values in the order they are given in the question. Find the value of a first then find the value of c.

Challenge

Consider the line passing through points (x_1, y_1) and (x_2, y_2).

a Write down the formula for the gradient, m, of the line.

b Show that the general equation of the line can be written in the form $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

c Use the equation from part **b** to find the equation of the line passing through the points $(-8, 4)$ and $(-1, 7)$.

Homework Answers

1 **a** $y = 2x + 1$ **b** $y = 3x + 7$ **c** $y = -x - 3$
d $y = -4x - 11$ **e** $y = \frac{1}{2}x + 12$ **f** $y = -\frac{2}{3}x - 5$
g $y = 2x$ **h** $y = -\frac{1}{2}x + 2b$

2 **a** $y = 4x - 4$ **b** $y = x + 2$ **c** $y = 2x + 4$
d $y = 4x - 23$ **e** $y = x - 4$ **f** $y = \frac{1}{2}x + 1$
g $y = -4x - 9$ **h** $y = -8x - 33$ **i** $y = \frac{6}{5}x$
j $y = \frac{2}{7}x + \frac{5}{14}$

3 $5x + y - 37 = 0$

4 $y = x + 2, y = -\frac{1}{6}x - \frac{1}{3}, y = -6x + 23$

5 $a = 3, c = -27$

6 $a = -4, b = 8$

Challenge

a $m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$

b $y - y_1 = \dfrac{(y_2 - y_1)}{(x^2 - y_1)}(x - x_1)$

$\dfrac{(y - y_1)}{(y_2 - y_1)} = \dfrac{(x - x_1)}{(x_2 - x_1)}$

c $y = \frac{3}{7}x + \frac{52}{7}$

Homework Exercise

1 The line $y = 4x - 8$ meets the x-axis at the point A. Find the equation of the line with gradient 3 that passes through the point A.

2 The line $y = -2x + 8$ meets the y-axis at the point B. Find the equation of the line with gradient 2 that passes through the point B.

3 The line $y = \frac{1}{2}x + 6$ meets the x-axis at the point C. Find the equation of the line with gradient $\frac{2}{3}$ that passes through the point C. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

4 The line $y = \frac{1}{4}x + 2$ meets the y-axis at the point B. The point C has coordinates $(-5, 3)$. Find the gradient of the line joining the points B and C.

5 The line that passes through the points $(2, -5)$ and $(-7, 4)$ meets the x-axis at the point P. Work out the coordinates of the point P.

Problem-solving

A sketch can help you check whether your answer looks right.

6 The line that passes through the points $(-3, -5)$ and $(4, 9)$ meets the y-axis at the point G. Work out the coordinates of the point G.

7 The line that passes through the points $(3, 2\frac{1}{2})$ and $(-1\frac{1}{2}, 4)$ meets the y-axis at the point J. Work out the coordinates of the point J.

8 The lines $y = x$ and $y = 2x - 5$ intersect at the point A. Find the equation of the line with gradient $\frac{2}{5}$ that passes through the point A.

Homework Exercise

9 The lines $y = 4x - 10$ and $y = x - 1$ intersect at the point T. Find the equation of the line with gradient $-\frac{2}{3}$ that passes through the point T. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

10 The line p has gradient $\frac{2}{3}$ and passes through the point $(6, -12)$. The line q has gradient -1 and passes through the point $(5, 5)$. The line p meets the y-axis at A and the line q meets the x-axis at B. Work out the gradient of the line joining the points A and B.

11 The line $y = -2x + 6$ meets the x-axis at the point P. The line $y = \frac{3}{2}x - 4$ meets the y-axis at the point Q. Find the equation of the line joining the points P and Q.

12 The line $y = 3x - 5$ meets the x-axis at the point M. The line $y = -\frac{2}{3}x + \frac{2}{3}$ meets the y-axis at the point N. Find the equation of the line joining the points M and N. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

13 The line $y = 2x - 10$ meets the x-axis at the point A. The line $y = -2x + 4$ meets the y-axis at the point B. Find the equation of the line joining the points A and B.

14 The line $y = 4x + 5$ meets the y-axis at the point C. The line $y = -3x - 15$ meets the x-axis at the point D. Find the equation of the line joining the points C and D. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

15 The lines $y = x - 5$ and $y = 3x - 13$ intersect at the point S. The point T has coordinates $(-4, 2)$. Find the equation of the line that passes through the points S and T.

16 The lines $y = -2x + 1$ and $y = x + 7$ intersect at the point L. The point M has coordinates $(-3, 1)$. Find the equation of the line that passes through the points L and M.

Homework Answers

1 $y = 3x - 6$

2 $y = 2x + 8$

3 $2x - 3y + 24 = 0$

4 $-\frac{1}{5}$

5 $(-3, 0)$

6 $(0, 1)$

7 $(0, 3\frac{1}{2})$

8 $y = \frac{2}{5}x + 3$

9 $2x + 3y - 12 = 0$

10 $\frac{8}{5}$

11 $y = \frac{4}{3}x - 4$

12 $6x + 15y - 10 = 0$

13 $y = -\frac{4}{5} + 4$

14 $x - y + 5 = 0$

15 $y = -\frac{3}{8}x + \frac{1}{2}$

16 $y = 4x + 13$