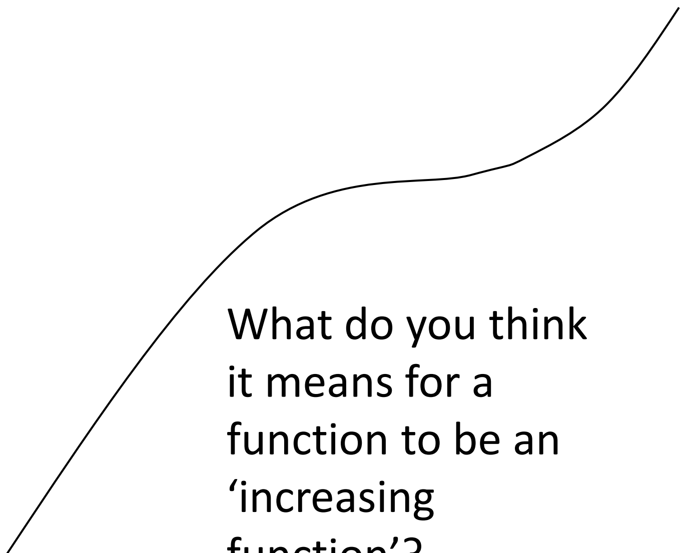
P1 Chapter 12: Differentiation

Increasing and Decreasing

Increasing and Decreasing Functions

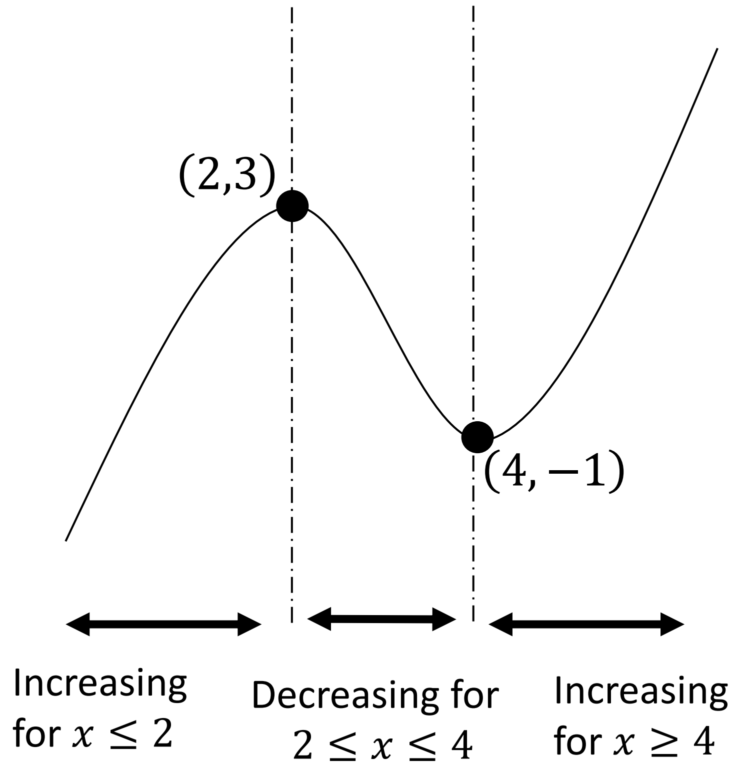


What do you think it means for a function to be an 'increasing function'?

✏ An increasing function is one whose gradient is always at least 0.
$f'(x) \geq 0$ for all x.

It would be '**<u>strictly</u> increasing**' if $f(x) > 0$ for all x, i.e. is not allowed to go horizontal.

A function can also be increasing and decreasing in certain intervals.



We could also write "$f(x)$ is decreasing in the interval $[2,4]$"
$[a, b]$ represents all the real numbers between a and b inclusive, i.e:

$$[a, b] = \{x : a \leq x \leq b\}$$

Examples

Show that the function $f(x) = x^3 + 6x^2 + 21x + 2$ is increasing for all real values of x.

?

Find the interval on which the function $f(x) = x^3 + 3x^2 - 9x$ is decreasing.

?

Fro Tip: To show a quadratic is always positive, complete the square, then indicate the squared term is always at least 0.

Examples

Show that the function $f(x) = x^3 + 6x^2 + 21x + 2$ is increasing for all real values of x.

$$f'(x) = 3x^2 + 12x + 21$$
$$f'(x) = 3(x^2 + 4x + 7)$$
$$= 3(x+2)^2 + 9$$

$(x+2)^2 \geq 0$ for all real x,
$\therefore 3(x+2)^2 + 9 \geq 0$ for all real x
$\therefore f(x)$ is an increasing function for all x.

Fro Tip: To show a quadratic is always positive, complete the square, then indicate the squared term is always at least 0.

Find the interval on which the function $f(x) = x^3 + 3x^2 - 9x$ is decreasing.

$$f(x) = x^3 + 3x^2 - 9x$$
$$f'(x) = 3x^2 + 6x - 9$$
$$f'(x) \leq 0$$
$$3x^2 + 6x - 9 \leq 0$$
$$x^2 + 2x - 3 \leq 0$$
$$(x+3)(x-1) \leq 0$$
$$-3 \leq x \leq 1$$

So $f(x)$ is decreasing in the interval $[-3,1]$

Test Your Understanding

Show that the function $f(x) = x^3 + 16x - 2$ is increasing for all real values of x.

?

Find the interval on which the function $f(x) = x^3 + 6x^2 - 135x$ is decreasing.

?

Test Your Understanding

Show that the function $f(x) = x^3 + 16x - 2$ is increasing for all real values of x.

$$f'(x) = 3x^2 + 16$$

$x^2 \geq 0$ for all real x

$\therefore 3x^2 + 16 \geq 0$ for all real x.

Therefore $f(x)$ is an increasing function for all real x.

Find the interval on which the function $f(x) = x^3 + 6x^2 - 135x$ is decreasing.

$f'(x) = 3x^2 + 12x - 135$

$f'(x) \leq 0$

$\therefore 3x^2 + 12x - 135 \leq 0$

$x^2 + 4x - 45 \leq 0$

$(x + 9)(x - 5) \leq 0$

$-9 \leq x \leq 5$

So $f(x)$ is decreasing in the interval $[-9,5]$

Exercise 12.7

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Homework Exercise

1 Find the values of x for which $f(x)$ is an increasing function, given that $f(x)$ equals:

a $3x^2 + 8x + 2$ **b** $4x - 3x^2$ **c** $5 - 8x - 2x^2$ **d** $2x^3 - 15x^2 + 36x$

e $3 + 3x - 3x^2 + x^3$ **f** $5x^3 + 12x$ **g** $x^4 + 2x^2$ **h** $x^4 - 8x^3$

2 Find the values of x for which $f(x)$ is a decreasing function, given that $f(x)$ equals:

a $x^2 - 9x$ **b** $5x - x^2$ **c** $4 - 2x - x^2$ **d** $2x^3 - 3x^2 - 12x$

e $1 - 27x + x^3$ **f** $x + \frac{25}{x}$ **g** $x^{\frac{1}{2}} + 9x^{-\frac{1}{2}}$ **h** $x^2(x + 3)$

3 Show that the function $f(x) = 4 - x(2x^2 + 3)$ is decreasing for all $x \in \mathbb{R}$. **(3 marks)**

4 **a** Given that the function $f(x) = x^2 + px$ is increasing on the interval $[-1, 1]$, find one possible value for p. **(2 marks)**

b State with justification whether this is the only possible value for p. **(1 mark)**

Homework Exercise

1 **a** $x \geqslant -\frac{4}{3}$ **b** $x \leqslant \frac{2}{3}$ **c** $x \leqslant -2$

d $x \leqslant 2, x \geqslant 3$ **e** $x \in \mathbb{R}$ **f** $x \in \mathbb{R}$

g $x \geqslant 0$ **h** $x \geqslant 6$

2 **a** $x \leqslant 4.5$ **b** $x \geqslant 2.5$ **c** $x \geqslant -1$

d $-1 \leqslant x \leqslant 2$ **e** $-3 \leqslant x \leqslant 3$ **f** $-5 \leqslant x \leqslant 5$

g $0 < x \leqslant 9$ **h** $-2 \leqslant x \leqslant 0$

3 $f'(x) = -6x^2 - 3$

$x^2 \geqslant 0$ for all $x \in \mathbb{R}$, so $-6x^2 - 3 \leq 0$ for all $x \in \mathbb{R}$.

$\therefore$ f is decreasing for all $x \in \mathbb{R}$.

4 **a** Any $p \geqslant 2$

b No. Can be any $p \geqslant 2$.