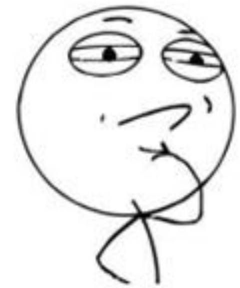
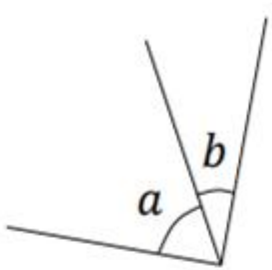
P2 Chapter 6: CoAngle Trigonometry

CoAngle Functions

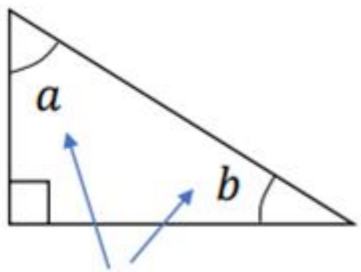
Co-angles

Have you ever wondered why "cosine" contains the word "sine"?

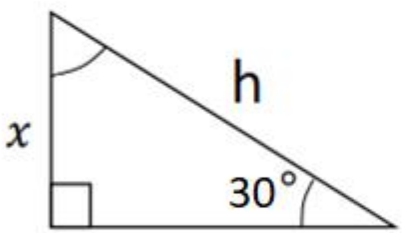




Complementary Angles
add to 90°



Therefore these angles are complementary.



Q: a) What is the value of sin(30) ? $\frac{1}{2}$

b) What is the value of cosine(60) ? $\frac{1}{2}$

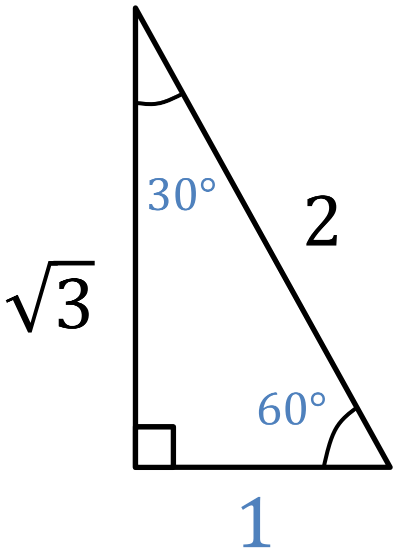
Co-Sine

✎ **cosine** (θ) **=** **sin**$(90-\theta)$**.**
(co-angle)

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$$

$$\sin(30°) = \frac{1}{2}$$

$$\sin(60°) = \frac{\sqrt{3}}{2}$$



$$\text{cosine}(30°) = \sin(60°) = \frac{\sqrt{3}}{2}$$

$$\text{cosine}(60°) = \sin(30°) = \frac{1}{2}$$

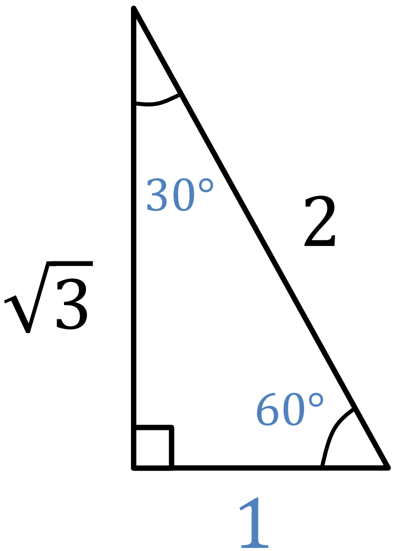
Co-Tangent

✎ **cotangent**(θ) **= tangent** $(90-\theta)$.
(co-angle)

$$\text{tangent}(\theta) = \frac{\text{opp}}{\text{adj}}$$

$$\text{tangent}(30°) = \frac{1}{\sqrt{3}}$$

$$\text{tangent}(60°) = \frac{\sqrt{3}}{1}$$



1) cotangent(60°) = ?

2) cotangent(30°) = ?

3) In terms of opp and adj, what is the formula: $\text{cotangent}(\theta) =$?

4) What is the formula relating cotangent to tangent: ?

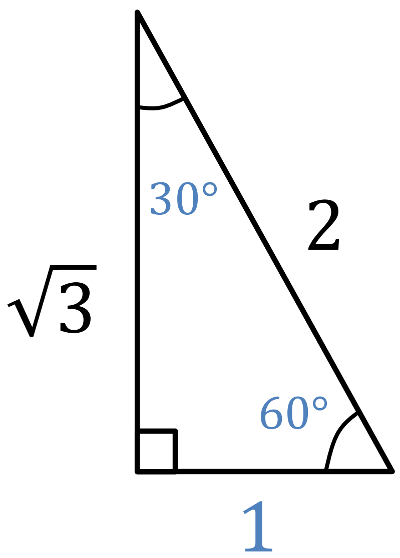
Co-Tangent

✎ **cotangent**(θ) **= tangent** $(90 - \theta)$**.**
(co-angle)

$$\text{tangent}(\theta) = \frac{\text{opp}}{\text{adj}}$$

$$\text{tangent}(30°) = \frac{1}{\sqrt{3}}$$

$$\text{tangent}(60°) = \frac{\sqrt{3}}{1}$$



1) $\text{cotangent}(60°) = \text{tangent}(30°) = \frac{1}{\sqrt{3}}$

2) $\text{cotangent}(30°) = \text{tangent}(60°) = \frac{\sqrt{3}}{1}$

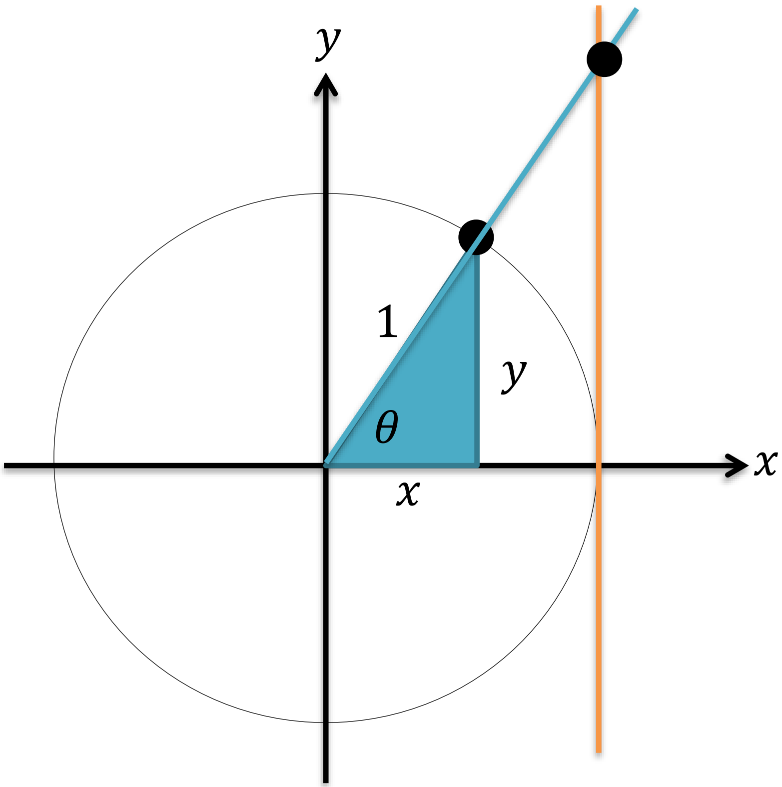
3) In terms of opp and adj, what is the formula: $\text{cotangent}(\theta) = \frac{\text{adj}}{\text{opp}}$

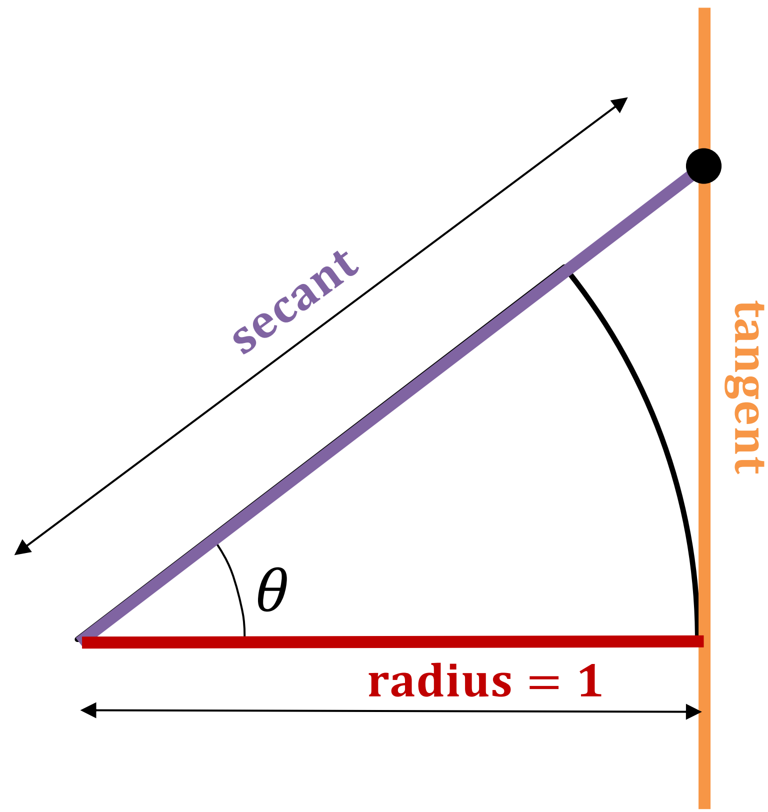
4) What is the formula relating cotangent to tangent: $\cot(x) = \frac{1}{\tan(x)}$

Secant and The Unit Circle

Draw a tangent on the unit circle and extend the radius out to the tangent.

This extended line is known as the *secant* and makes the *hypotenuse of a larger similar triangle* with the tangent as the opposite length.





1) Write $\cos(\theta)$ with the secant as a hypotenuse: $\cos(\theta) = \frac{1}{\text{secant}}$

2) Make $\sec(\theta) \equiv \text{secant}$ the subject of the formula: $\sec(\theta) = \frac{1}{\cos}$

Reciprocal Trigonometric Functions

Summary to learn:

$$\sec(x) = \frac{1}{\cos(x)}$$

Short for "**secant**"
Pronounced "sehk" in shortened form or "sea-Kant" in full.

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

There is also a "cosec" to go with sine
Short for "**cosecant**"

$$\cot(x) = \frac{1}{\tan(x)}$$

Short for "**cotangent**"
In shortened form, rhymes with "pot".

Calculations

Calculate these exact values:

$\cot\frac{\pi}{4} =$?

$\sec\frac{\pi}{4} =$?

$\operatorname{cosec}\frac{\pi}{3} =$?

$\cot\frac{\pi}{6} =$?

$\operatorname{cosec}\frac{5\pi}{6} =$?

$\cot\frac{\pi}{3} =$?

$\sec\frac{\pi}{6} =$?

$\operatorname{cosec}\frac{\pi}{2} =$?

$\sec\frac{5\pi}{3} =$?

Calculations

Calculate these exact values:

$$\cot\frac{\pi}{4} = \frac{1}{\boldsymbol{tan}\,\boldsymbol{\frac{\pi}{4}}} = \mathbf{\frac{1}{1}} = \mathbf{1}$$

$$\sec\frac{\pi}{4} = \frac{\mathbf{1}}{\mathbf{cos}\,\boldsymbol{\frac{\pi}{4}}} = \frac{\mathbf{1}}{\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}} = \sqrt{\mathbf{2}}$$

$$\operatorname{cosec}\frac{\pi}{3} = \frac{\mathbf{1}}{\mathbf{sin}\,\boldsymbol{\frac{\pi}{3}}} = \frac{\mathbf{2}}{\sqrt{\mathbf{3}}}$$

$$\cot\frac{\pi}{6} = \sqrt{\mathbf{3}}$$

$$\operatorname{cosec}\frac{5\pi}{6} = \mathbf{cosec}\,\boldsymbol{\frac{\pi}{6}} = \mathbf{2}$$

$$\cot\frac{\pi}{3} = \sqrt{\mathbf{3}}$$

$$\sec\frac{\pi}{6} = \frac{\mathbf{2}}{\sqrt{\mathbf{3}}}$$

$$\operatorname{cosec}\frac{\pi}{2} = \mathbf{1}$$

$$\sec\frac{5\pi}{3} = \boldsymbol{sec}\,\boldsymbol{\frac{\pi}{3}} = \mathbf{2}$$

Exercise 6.1

Pearson Pure Mathematics Year 2/AS
Pages 1

Homework Exercise

1 Without using your calculator, write down the sign of the following trigonometric ratios.

a sec 300° **b** cosec 190° **c** cot 110°

d cot 200° **e** sec 95°

2 Use your calculator to find, to 3 significant figures, the values of:

a sec 100° **b** cosec 260° **c** cosec 280°

d cot 550° **e** cot $\frac{4\pi}{3}$ **f** sec 2.4 rad

g cosec $\frac{11\pi}{10}$ **h** sec 6 rad

3 Find the exact values (in surd form where appropriate) of the following:

a cosec 90° **b** cot 135° **c** sec 180°

d sec 240° **e** cosec 300° **f** cot(−45°)

g sec 60° **h** cosec (−210°) **i** sec 225°

j cot $\frac{4\pi}{3}$ **k** sec $\frac{11\pi}{6}$ **l** cosec $\left(-\frac{3\pi}{4}\right)$

Homework Exercise

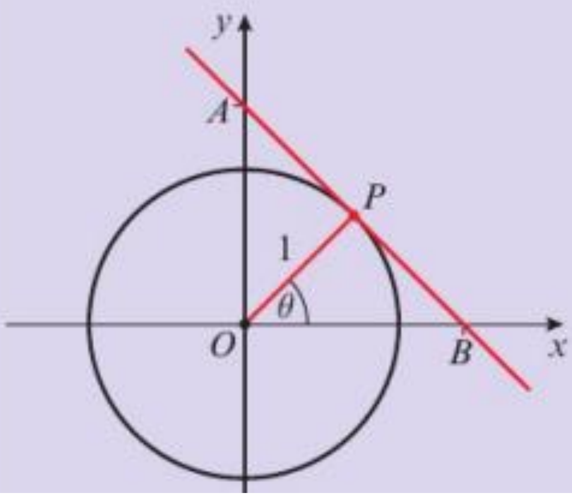
4 Prove that $\operatorname{cosec}(\pi - x) \equiv \operatorname{cosec} x$.

5 Show that $\cot 30° \sec 30° = 2$.

6 Show that $\operatorname{cosec}\frac{2\pi}{3} + \sec\frac{2\pi}{3} = a + b\sqrt{3}$ where a and b are real numbers to be found.

Challenge

The point P lies on the unit circle, centre O. The radius OP makes an acute angle of θ with the positive x-axis. The tangent to the circle at P intersects the coordinate axes at points A and B.



Prove that

a $OB = \sec\theta$

b $OA = \operatorname{cosec}\theta$

c $AP = \cot\theta$

Homework Answers

1 **a** +ve **b** −ve **c** −ve **d** +ve
e −ve

2 **a** −5.76 **b** −1.02 **c** −1.02 **d** 5.67
e 0.577 **f** −1.36 **g** −3.24 **h** 1.04

3 **a** 1 **b** −1 **c** −1 **d** −2
e $-\frac{2\sqrt{3}}{3}$ **f** −1 **g** 2 **h** 2
i $-\sqrt{2}$ **j** $\frac{\sqrt{3}}{3}$ **k** $\frac{2\sqrt{3}}{3}$ **l** $-\sqrt{2}$

4 $\operatorname{cosec}(\pi - x) = \frac{1}{\sin(\pi - x)} = \frac{1}{\sin x} = \operatorname{cosec} x$

5 $\cot 30° \sec 30° = \frac{1}{\tan 30°} \times \frac{1}{\cos 30°} = \frac{\sqrt{3}}{1} \times \frac{2}{\sqrt{3}} = 2$

6 $$\operatorname{cosec}\left(\frac{2\pi}{3}\right) + \sec\left(\frac{2\pi}{3}\right) = \frac{1}{\sin\left(\frac{2\pi}{3}\right)} + \frac{1}{\cos\left(\frac{2\pi}{3}\right)}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}} + \frac{1}{-\frac{1}{2}}$$

$$= -2 + \frac{2}{\sqrt{3}} = -2 + \frac{2}{3}\sqrt{3}$$

Challenge

a Using triangle OBP, $OB\cos\theta = 1$
$\Rightarrow OB = \frac{1}{\cos\theta} = \sec\theta$

b Using triangle OAP, $OA\sin\theta = 1$
$\Rightarrow OA = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$

c Using Pythagoras' theorem, $AP^2 = OA^2 - OP^2$
So $AP^2 = \operatorname{cosec}^2\theta - 1 = \frac{1}{\sin^2\theta} - 1$

$= \frac{1 - \sin^2\theta}{\sin^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$

Therefore $AP = \cot\theta$.