P1 Chapter 3: Inequalities

Linear Simultaneous Equations

Simultaneous Equations

Recap!

Solve the simultaneous equations:

$$3x + y = 8$$
$$2x - 3y = 9$$

We can either use substitution (i.e. making x or y the subject of one equation, and substituting it into the other) or elimination, but the latter is easier for linear equations.

$$\mathbf{9x + 3y = 24}$$
$$\mathbf{2x - 3y = 9}$$

Adding the two equations to 'eliminate' y:

$$\mathbf{11x = 33 \quad \rightarrow \quad x = 3}$$

Substituting into first equation:

$$\mathbf{27 + 3y = 24 \quad \rightarrow \quad y = -1}$$

Solve the simultaneous equations:

$$7x + 2y = 3$$
$$3x + 5y = 12$$

Test Your Understanding

Solve the simultaneous equations:

$$3x + 9 = 21$$
$$y = x + 1$$

$$3x^2 + (x + 1)^2 = 21$$

Solutions sets

The solution(s) to an equation may be:

A single value:	$2x + 1 = 5$
Multiple values:	$x^2 + 3x + 2 = 0$
An infinitely large set of values:	$x > 3$
No (real) values!	$x^2 = -1$
Every value!	$x^2 + x = x(x + 1)$

The point is that you shouldn't think of the solution to an equation/inequality as an 'answer', but a **set** of values, which might just be a set of 1 value (known as a singleton set), a set of no values (i.e. the empty set ∅), or an infinite set (in the last example above, this was $\mathbb{R}$)

✏ The solutions to an equation are known as the **solution set**.

Solutions sets

For simultaneous equations, the same is true, except each ‘solution’ in the solution set is an assignment to **multiple** variables.
All equations have to be satisfied **at the same time, i.e. ‘simultaneously’**.

Scenario	**Example**	**Solution Set**
A single solution:	?	?
Two solutions:	?	?
No solutions:	?	?
Infinitely large set of solutions:	?	?

Solutions sets

For simultaneous equations, the same is true, except each ‘solution’ in the solution set is an assignment to **multiple** variables.
All equations have to be satisfied **at the same time, i.e. ‘simultaneously’**.

Scenario	Example	Solution Set
A single solution:	$x + y = 9$ $x - y = 1$	**Solution 1:** $\boldsymbol{x = 5, y = 4}$ To be precise here, the solution set is of size 1, but this solution is an assignment to multiple variables, i.e. a pair of values.
Two solutions:	$x^2 + y^2 = 10$ $x + y = 4$	**Solution 1:** $\boldsymbol{x = 3, y = 1}$ **Solution 2:** $\boldsymbol{x = 1, y = 3}$ This time we have two solutions, each an x, y pair.
No solutions:	$x + y = 1$ $x + y = 3$	The solution set is empty, i.e. ∅, as both equation can’t be satisfied at the same time.
Infinitely large set of solutions:	$x + y = 1$ $2x + 2y = 2$	**Solution 1:** $\boldsymbol{x = 0, y = 1}$ **Solution 2:** $\boldsymbol{x = 1, y = 0}$ **Solution 3:** $\boldsymbol{x = 2, y = -1}$ **Solution 4:** $\boldsymbol{x = 0.5, y = 0.5}$ **…** Infinite possibilities!

Exercise 3.1

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Extension

1 *[MAT 2012 1G]* There are *positive* real numbers x and y which solve the equations

$$2x + ky = 4,$$
$$x + y = k$$

for:

A) All values of k;
B) No values of k;
C) $k = 2$ only;
D) Only $k > -2$

?

2 *[STEP 2010 Q1]* Given that

$$5x^2 + 2y^2 - 6xy + 4x - 4y$$
$$\equiv a(x - y + 2)^2 + b(cx + y)^2 + d$$

a) Find the values of a, b, c, d.

b) Solve the simultaneous equations:

$$5x^2 + 2y^2 - 6xy + 4x - 4y = 9,$$
$$6x^2 + 3y^2 - 8xy + 8x - 8y = 14$$

(Hint: Can we use the same method in (a) to rewrite the second equation?)

? a

? b

Exercise 3.1

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1 *[MAT 2012 1G]* There are *positive* real numbers x and y which solve the equations

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for:

A) All values of k;
B) No values of k;
C) $k = 2$ only;
D) Only $k > -2$

If $k = 2$ then $2x + 2y = 4$ and $x + y = 2$ which are equivalent. This would give an infinite solution set, thus the answer is C.

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a) Find the values of a, b, c, d.

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(Hint: Can we use the same method in (a) to rewrite the second equation?)

a) Expanding RHS:

$$\left(a + bc^2\right)x^2 + (a + b)y^2 + (-2a + 2bc)xy + 4ax - 4ay + (4a + d)$$

Comparing coefficients: $a = 1, b = 1, c = -2, d = -4$

b) $(x - y + 2)^2 + (-2x + y)^2 - 4 = 9$

Using method in (a): $2(x - y + 2)^2 + (-2x + y)^2 - 8 = 14$

Subtracting yields $y - 2x = \pm 2$ and $x - y + 2 = \pm 3$

We have to consider each of 4 possibilities.

Final solution set: $x = -3, y = -4$ *or* $x = 1, y = 0$ or $x = 3, y = 8$ *or* $x = 7, y = 12$

Homework Exercise

1 Solve these simultaneous equations by elimination:

a $2x - y = 6$
$4x + 3y = 22$

b $7x + 3y = 16$
$2x + 9y = 29$

c $5x + 2y = 6$
$3x - 10y = 26$

d $2x - y = 12$
$6x + 2y = 21$

e $3x - 2y = -6$
$6x + 3y = 2$

f $3x + 8y = 33$
$6x = 3 + 5y$

2 Solve these simultaneous equations by substitution:

a $x + 3y = 11$
$4x - 7y = 6$

b $4x - 3y = 40$
$2x + y = 5$

c $3x - y = 7$
$10x + 3y = -2$

d $2y = 2x - 3$
$3y = x - 1$

3 Solve these simultaneous equations:

a $3x - 2y + 5 = 0$
$5(x + y) = 6(x + 1)$

b $\dfrac{x - 2y}{3} = 4$
$2x + 3y + 4 = 0$

c $3y = 5(x - 2)$
$3(x - 1) + y + 4 = 0$

Hint First rearrange both equations into the same form e.g. $ax + by = c$.

4 $3x + ky = 8$
$x - 2ky = 5$
are simultaneous equations where k is a constant.

a Show that $x = 3$. **(3 marks)**

b Given that $y = \frac{1}{2}$ determine the value of k. **(1 mark)**

Problem-solving
k is a constant, so it has the same value in both equations.

5 $2x - py = 5$
$4x + 5y + q = 0$
are simultaneous equations where p and q are constants.
The solution to this pair of simultaneous equations is $x = q$, $y = -1$.
Find the value of p and the value of q. **(5 marks)**

Homework Answers

1 **a** $x = 4, y = 2$ **b** $x = 1, y = 3$
c $x = 2, y = -2$ **d** $x = 4\frac{1}{2}, y = -3$
e $x = -\frac{2}{3}, y = 2$ **f** $x = 3, y = 3$

2 **a** $x = 5, y = 2$ **b** $x = 5\frac{1}{2}, y = -6$
c $x = 1, y = -4$ **d** $x = 1\frac{3}{4}, y = \frac{1}{4}$

3 **a** $x = -1, y = 1$ **b** $x = 4, y = -4$
c $x = 0.5, y = -2.5$

4 **a** $3x + ky = 8$ (1); $x - 2ky = 5$ (2)
(1) $\times$ 2: $6x + 2ky = 16$ (3)
(2) + (3) $7x = 21$ so $x = 3$
b -2

5 $p = 3, q = 1$