P1 Chapter 4: Transforming Graphs

Chapter Practice

Key Points

1. If p is a root of the function $f(x)$, then the graph of $y = f(x)$ touches or crosses the x-axis at the point $(p, 0)$.
2. The graphs of $y = \frac{k}{x}$ and $y = \frac{k}{x^2}$, where k is a real constant, have asymptotes at $x = 0$ and $y = 0$.
3. The x-coordinate(s) at the points of intersection of the curves with equations $y = f(x)$ and $y = g(x)$ are the solution(s) to the equation $f(x) = g(x)$.
4. The graph of $y = f(x) + a$ is a translation of the graph $y = f(x)$ by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$.
5. The graph of $y = f(x + a)$ is a translation of the graph $y = f(x)$ by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$.
6. When you translate a function, any asymptotes are also translated.
7. The graph of $y = af(x)$ is a stretch of the graph $y = f(x)$ by a scale factor of a in the vertical direction.
8. The graph of $y = f(ax)$ is a stretch of the graph $y = f(x)$ by a scale factor of $\frac{1}{a}$ in the horizontal direction.
9. The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in the x-axis.
10. The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ in the y-axis.

Chapter Exercises

1 a On the same axes sketch the graphs of $y = x^2(x - 2)$ and $y = 2x - x^2$.

b By solving a suitable equation find the points of intersection of the two graphs.

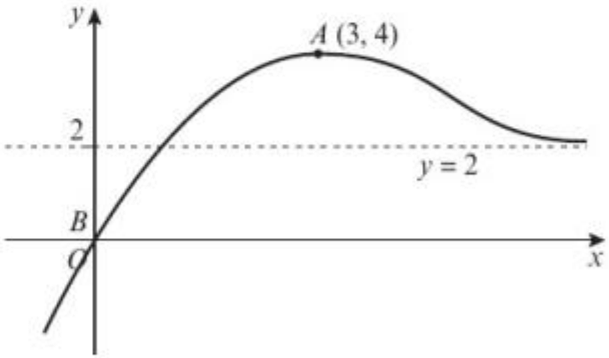
2 a On the same axes sketch the curves with equations $y = \frac{6}{x}$ and $y = 1 + x$.

b The curves intersect at the points A and B. Find the coordinates of A and B.

c The curve C with equation $y = x^2 + px + q$, where p and q are integers, passes through A and B. Find the values of p and q.

d Add C to your sketch.

3 The diagram shows a sketch of the curve $y = f(x)$. The point $B(0, 0)$ lies on the curve and the point $A(3, 4)$ is a maximum point. The line $y = 2$ is an asymptote.

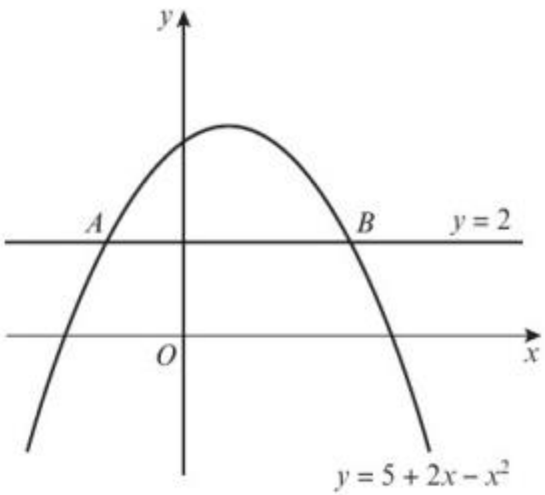


Sketch the following and in each case give the coordinates of the new positions of A and B and state the equation of the asymptote:

a $f(2x)$	b $\frac{1}{2}f(x)$	c $f(x) - 2$
d $f(x + 3)$	e $f(x - 3)$	f $f(x) + 1$

4 The diagram shows the curve with equation $y = 5 + 2x - x^2$ and the line with equation $y = 2$. The curve and the line intersect at the points A and B.

Find the x-coordinates of A and B. **(4 marks)**



Chapter Exercises

5 $f(x) = x^2(x - 1)(x - 3)$.

a Sketch the graph of $y = f(x)$. **(2 marks)**

b On the same axes, draw the line $y = 2 - x$. **(2 marks)**

c State the number of real solutions to the equation $x^2(x - 1)(x - 3) = 2 - x$. **(1 mark)**

d Write down the coordinates of the point where the graph with equation $y = f(x) + 2$ crosses the y-axis. **(1 mark)**

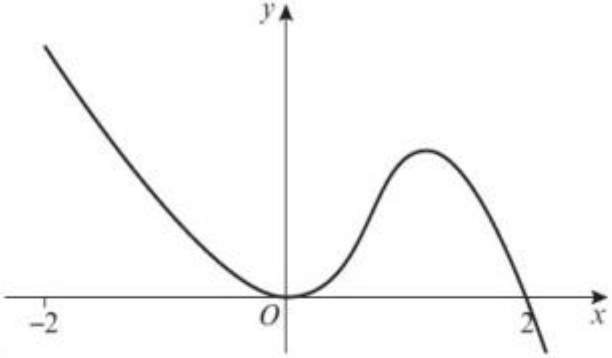
6 The figure shows a sketch of the curve with equation $y = f(x)$.

On separate axes sketch the curves with equations:

a $y = f(-x)$ **(2 marks)**

b $y = -f(x)$ **(2 marks)**

Mark on each sketch the x-coordinate of any point, or points, where the curve touches or crosses the x-axis.



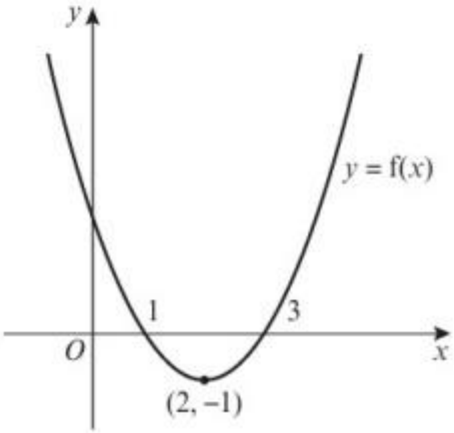
7 The diagram shows the graph of the quadratic function $f(x)$. The graph meets the x-axis at (1, 0) and (3, 0) and the minimum point is (2, −1).

a Find the equation of the graph in the form $y = ax^2 + bx + c$ **(2 marks)**

b On separate axes, sketch the graphs of

i $y = f(x + 2)$ ii $y = (2x)$. **(2 marks)**

c On each graph label the coordinates of the points at which the graph meets the x-axis and label the coordinates of the minimum point.



8 $f(x) = (x-1)(x-2)(x+1)$.

a State the coordinates of the point at which the graph $y = f(x)$ intersects the y-axis. **(1 mark)**

b The graph of $y = af(x)$ intersects the y-axis at $(0, -4)$. Find the value of a. **(1 mark)**

c The graph of $y = f(x + b)$ passes through the origin. Find three possible values of b. **(3 marks)**

9 The point $P(4, 3)$ lies on a curve $y = f(x)$.

a State the coordinates of the point to which P is transformed on the curve with equation:

i $y = f(3x)$ **ii** $\frac{1}{2}y = f(x)$ **iii** $y = f(x - 5)$ **iv** $-y = f(x)$ **v** $2(y + 2) = f(x)$

b P is transformed to point $(2, 3)$. Write down two possible transformations of $f(x)$.

c P is transformed to point $(8, 6)$. Write down a possible transformation of $f(x)$ if

i $f(x)$ is translated only **ii** $f(x)$ is stretched only.

10 The curve C_1 has equation $y = -\frac{a}{x^2}$ where a is a positive constant. The curve C_2 has the equation $y = x^2(3x + b)$ where b is a positive constant.

a Sketch C_1 and C_2 on the same set of axes, showing clearly the coordinates of any point where the curves touch or cross the axes. **(4 marks)**

b Using your sketch state, giving reasons, the number of solutions to the equation $x^4(3x + b) + a = 0$. **(2 marks)**

11 a Factorise completely $x^3 - 6x^2 + 9x$. **(2 marks)**

b Sketch the curve of $y = x^3 - 6x^2 + 9x$ showing clearly the coordinates of the points where the curve touches or crosses the axes. **(4 marks)**

c The point with coordinates $(-4, 0)$ lies on the curve with equation $y = (x - k)^3 - 6(x - k)^2 + 9(x - k)$ where k is a constant.
Find the two possible values of k. **(3 marks)**

Chapter Exercises

12 $f(x) = x(x-2)^2$

Sketch on separate axes the graphs of:

a $y = f(x)$ **(2 marks)**

b $y = f(x+3)$ **(2 marks)**

Show on each sketch the coordinates of the points where each graph crosses or meets the axes.

13 Given that $f(x) = \frac{1}{x}, x \neq 0$,

a Sketch the graph of $y = f(x) - 2$ and state the equations of the asymptotes. **(3 marks)**

b Find the coordinates of the point where the curve $y = f(x) - 2$ cuts a coordinate axis. **(2 marks)**

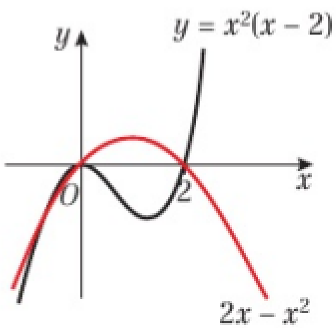
c Sketch the graph of $y = f(x+3)$. **(2 marks)**

d State the equations of the asymptotes and the coordinates of the point where the curve cuts a coordinate axis. **(2 marks)**

Challenge

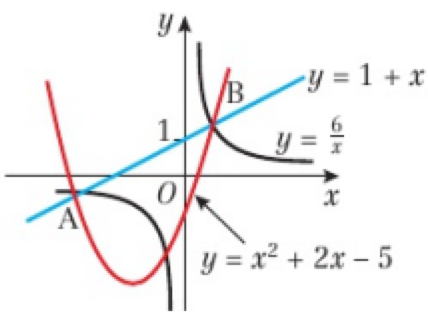
The point $R(6, -4)$ lies on the curve with equation $y = f(x)$. State the coordinates that point R is transformed to on the curve with equation $y = f(x+c) - d$.

1 **a**



b $x = 0, -1, 2$; points $(0, 0)$, $(2, 0)$, $(-1, -3)$

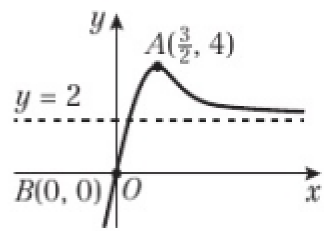
2 **a**



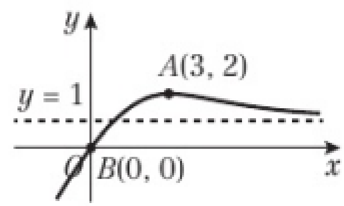
b $A(-3, -2)$, $B(2, 3)$

c $y = x^2 + 2x - 5$

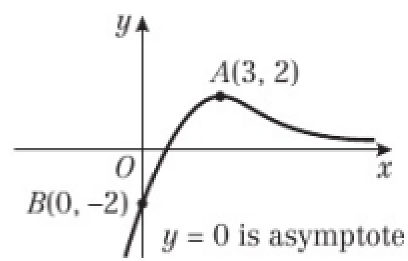
3 **a**



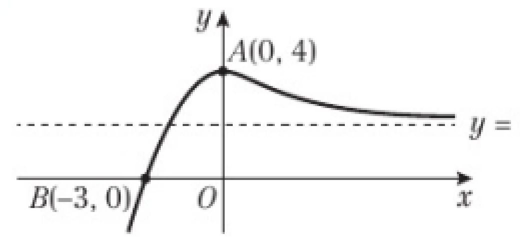
b



c



d



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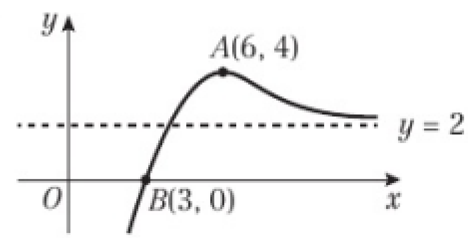

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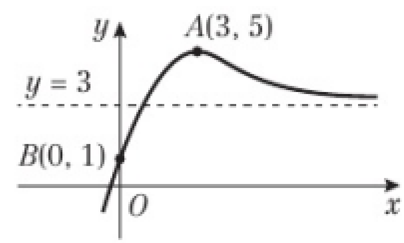
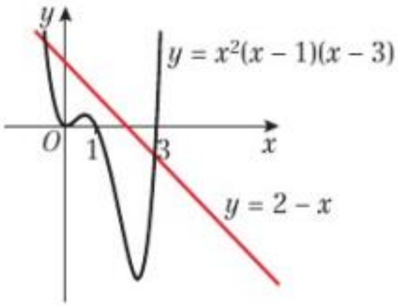


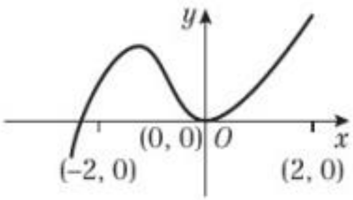
4 **a** $x = -1$ at A, $x = 3$ at B

5 **a, b**

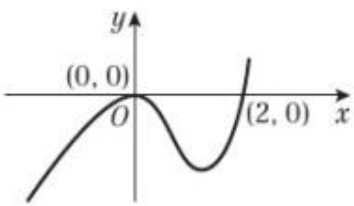


c 2 **d** (0, 2)

6 **a**

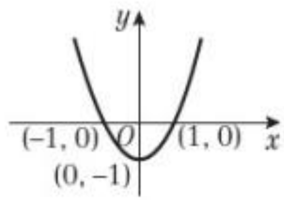


b

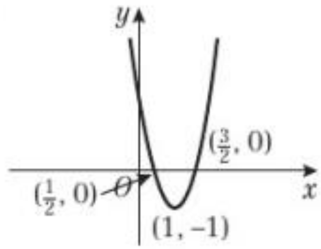


7 **a** $y = x^2 - 4x + 3$

b **i**



ii



8 **a** (0, 2)

b −2

c −1, 1, 2

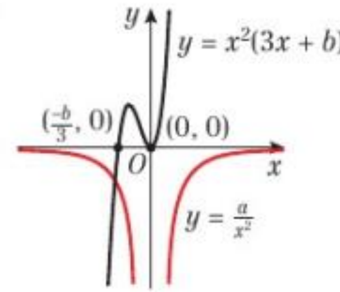
9 **a** **i** $(\frac{4}{3}, 3)$ **ii** (4, 6) **iii** (9, 3)

iv (4, −3) **v** $(4, -\frac{1}{2})$

b $f(2x)$, $f(x + 2)$

c **i** $f(x - 4) + 3$ **ii** $2f(\frac{1}{2}x)$

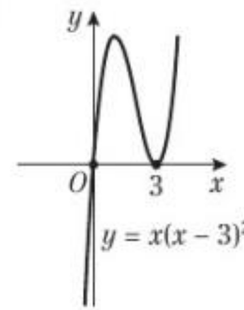
10 **a**



b 1; only one intersection of the two curves

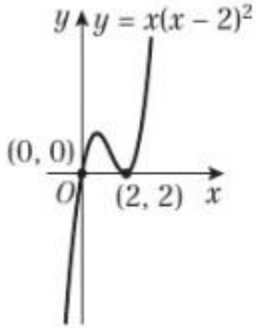
11 **a** $x(x - 3)^2$

b

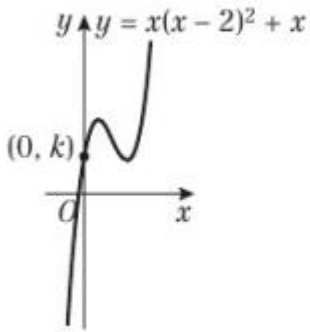


c −4 and −7

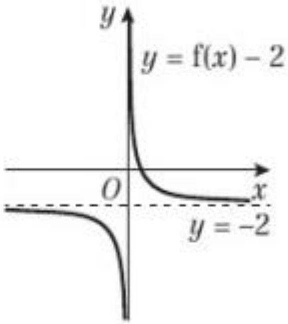
12 a



b

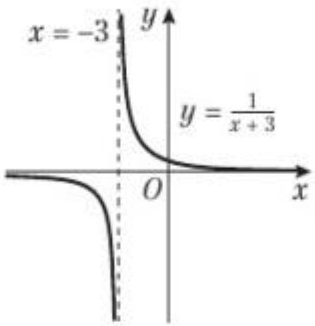


13 a Asymptotes at $x = 0$ and $y = -2$



b $(\frac{1}{2}, 0)$

c



d Asymptotes at $y = 0$ and $x = -3$; intersection at $(0, \frac{1}{3})$

Challenge
$(6 - c, -4 - d)$