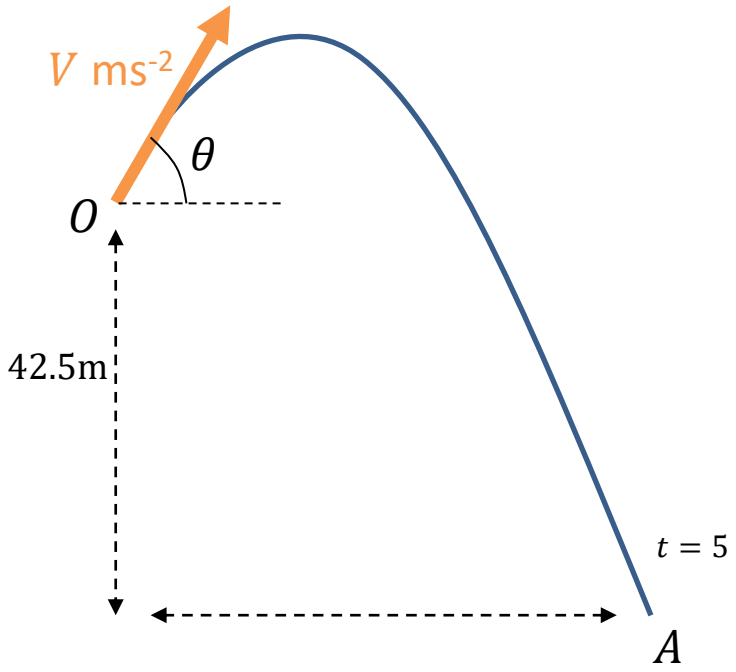
M2 Chapter 6: Projectiles

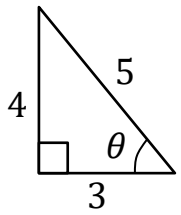
Projection at any Angle

Projected from above ground

[Textbook] A particle is projected from a point O with speed V ms^{-1} and at an angle of elevation of θ, where $\tan\theta = \frac{4}{3}$. The point O is 42.5m above a horizontal plane. The particle strikes the horizontal plane at a point A, 5 s after it is projected.
(a) Show that $V = 20$. (b) Find the distance between O and A.



a



$\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{3}{5}$

$R(\uparrow)$: $s = -42.5, u = \frac{4}{5}V, v = , a = -9.8, t = 5$

$$s = ut + \frac{1}{2}at^2$$

$$-42.5 = \frac{4}{5}V \times 5 - 4.9 \times 25$$

$$\ldots \quad \Rightarrow \quad V = 20$$

b

Let horizontal distance be x m:

$$x = 20\cos\theta \times 5 = \frac{3}{5}(20) \times 5 = 60 \text{ m}$$

c

Using Pythagoras' theorem:

$$OA = \sqrt{42.5^2 + 60^2} = 73.527\ldots$$

Distance is 74 m to 2sf.

Time above a given point

[Textbook] A particle is projected from a point O with speed 35 ms^{-1} and at an angle of elevation of $30°$. The particle moves freely under gravity. Find the length of time for which the particle is 15 m or more above O.

The key is to find the two times at which the particle is 15m above ground. The time above 15m will then be the difference between these times.

$R(\uparrow)$: $s = 15$, $u = 35\sin 30° = 75$, $v =$, $a = -9.8$, $t =$?

$$s = ut + \frac{1}{2}at^2$$

$$15 = 17.5t - 4.9t^2$$

$$4.9t^2 - 17.5\,t + 15 = 0$$

Use the quadratic solver on your calculator.

$$t = \frac{10}{7}, \frac{15}{7}$$

Time above 15m: $\frac{15}{7} - \frac{10}{7} = \frac{5}{7} = 0.71$ s (2sf)

Note: The textbook implies you can leave your answer as an exact value of $\frac{5}{7}$. But we have used an approximate value of g, to 2 significant figures, so it would not be appropriate to do so.

Test Your Understanding

Edexcel M2(Old) May 2012 Q7

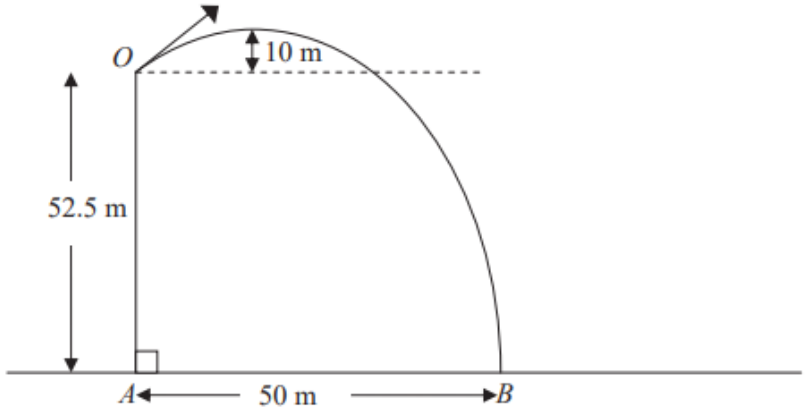


Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}.

(3)

(b) Find the speed of projection.

(9)

(c) Find the time after projection when the stone is moving parallel to OB.

(5)

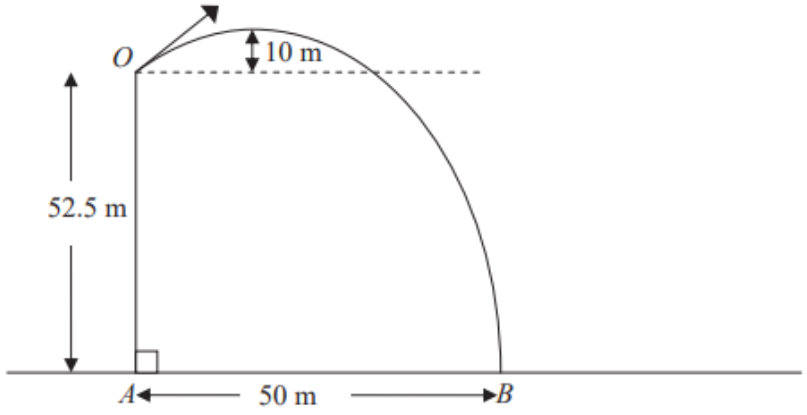
a ?

b ?

c ?

Test Your Understanding

Edexcel M2(Old) May 2012 Q7



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(5)

a

$0^2 = u_V{}^2 - 2 \times 9.8 \times 10$ M1

$u_V = 14$ * A1 A1

b

$(\uparrow),\ -52.5 = 14t - \frac{1}{2}gt^2$ M1 A1 A1

$49t^2 - 140t - 525 = 0$ DM

$(t-5)(49t+105) = 0 \quad t = 5$ A1

$(\rightarrow),\ 50 = 5u_H$ M1

$u_H = 10$ A1

$u = \sqrt{10^2 + 14^2}$ M1

$= \sqrt{296}\ ;\ 17.2 \text{ m s}^{-1}$ A1

c

$\tan OBA = \frac{52.5}{50} = 1.05$

$v_V = 1.05 \times 10 = 10.5$

$(\uparrow),\ -10.5 = 14 - gt$

$t = 2.5$

Exercise 6C

Pearson Stats/Mechanics Year 2
Pages 51-52

A ball is projected from ground level at an angle of θ. Prove that when the ball hits the ground, the distance the ball has travelled along the ground is maximised when $\theta = 45°$.
(Year 2 differentiation knowledge required)

?

Exercise 6C

Pearson Stats/Mechanics Year 2
Pages 51-52

A ball is projected from ground level at an angle of θ. Prove that when the ball hits the ground, the distance the ball has travelled along the ground is maximised when $\theta = 45°$.
(Year 2 differentiation knowledge required)

Let speed be u_0 and horizontal distance be x.

$R(\uparrow)$: $s = 0, u = u_0 \sin\theta, \cancel{v = \quad}, a = -g, t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = u_0(\sin\theta)t - \frac{1}{2}gt^2 = t\left(u_0(\sin\theta) - \frac{1}{2}gt\right)$$

$$t = 0 \text{ or } t = \frac{2(\sin\theta)u_0}{g}$$

$R(\rightarrow)$: $x = u_0 cos\theta \times \frac{2(\sin\theta)u_0}{g} = \frac{2(u_0)^2}{g}\sin\theta\cos\theta$

$$x = \frac{(u_0)^2}{g}\sin 2\theta$$

$$\frac{dx}{d\theta} = \frac{2(u_0)^2}{g}\cos 2\theta = 0 \quad \Rightarrow \quad \cos 2\theta = 0 \quad \Rightarrow \quad 2\theta = 90° \quad \Rightarrow \quad \theta = 45°$$

We want to maximise x as θ varies, i.e. $\frac{dx}{d\theta} = 0$

Homework Exercise

1 A ball is projected from a point A on level ground with speed $24\ \text{m s}^{-1}$. The ball is projected at an angle θ to the horizontal where $\sin\theta = \frac{4}{5}$. The ball moves freely under gravity until it strikes the ground at a point B. Find:

a the time of flight of the ball

b the distance from A to B.

2 A particle is projected with speed $21\ \text{m s}^{-1}$ at an angle of elevation α. Given that the greatest height reached above the point of projection is 15 m, find the value of α, giving your answer to the nearest degree.

3 A particle P is projected from the origin with velocity $(12\mathbf{i} + 24\mathbf{j})\ \text{m s}^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors respectively. The particle moves freely under gravity. Find:

a the position vector of P after 3 s

b the speed of P after 3 s.

4 A stone is thrown with speed $30\ \text{m s}^{-1}$ from a window which is 20 m above horizontal ground. The stone hits the ground 3.5 s later. Find:

a the angle of projection of the stone

b the horizontal distance from the window to the point where the stone hits the ground.

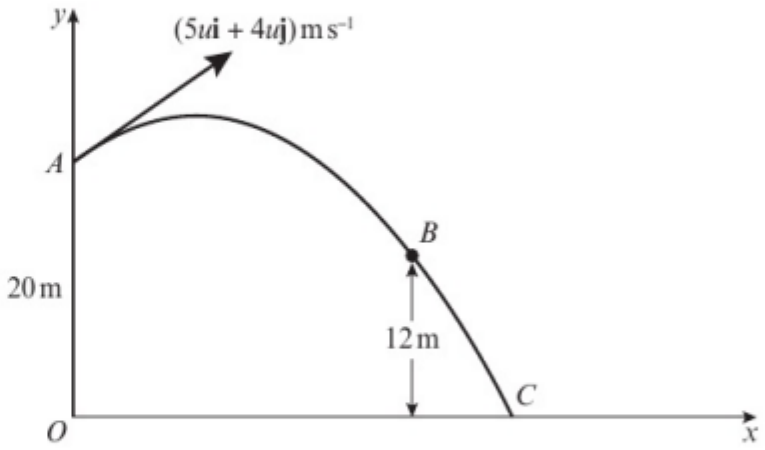
Homework Exercise

5 A ball is thrown from a point O on horizontal ground with speed $U\,\text{m s}^{-1}$ at an angle of elevation of θ, where $\tan\theta = \frac{3}{4}$. The ball strikes a vertical wall which is 20 m from O at a point which is 3 m above the ground. Find:

a the value of U **(6 marks)**

b the time from the instant the ball is thrown to the instant that it strikes the wall. **(2 marks)**

6 A particle P is projected from a point A with position vector $20\mathbf{j}$ m with respect to a fixed origin O. The velocity of projection is $(5u\mathbf{i} + 4u\mathbf{j})\,\text{m s}^{-1}$.
The particle moves freely under gravity, passing through a point B, which has position vector $(k\mathbf{i} + 12\mathbf{j})$ m, where k is a constant, before reaching the point C on the x-axis, as shown in the diagram.
The particle takes 4 s to move from A to B. Find:



a the value of u **(4 marks)**

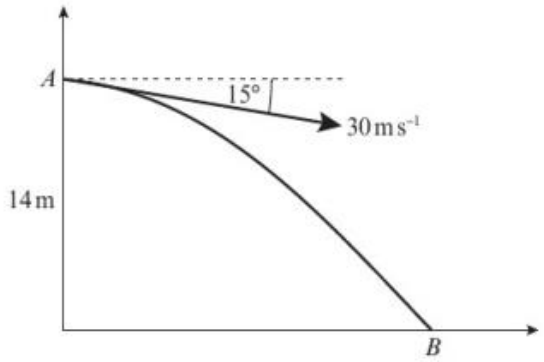
b the value of k **(2 marks)**

c the angle the velocity of P makes with the x-axis as it reaches C. **(6 marks)**

Watch out When finding a square root involving use of $g = 9.8\,\text{m s}^{-2}$ to work out an answer, an exact surd answer is **not** acceptable.

Homework Exercise

7 A stone is thrown from a point A with speed $30\,\text{m s}^{-1}$ at an angle of $15°$ below the horizontal. The point A is $14\,\text{m}$ above horizontal ground. The stone strikes the ground at the point B, as shown in the diagram. Find:



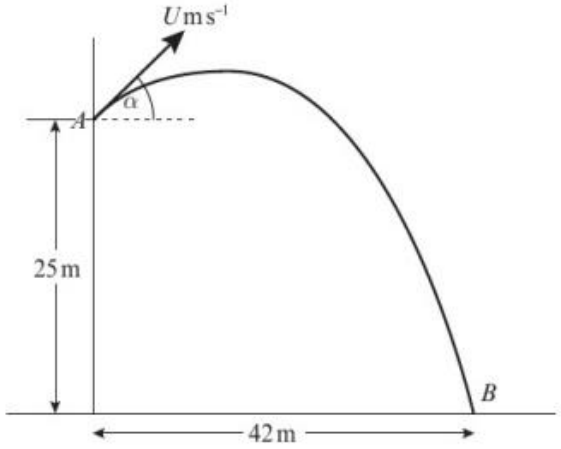
a the time the stone takes to travel from A to B **(6 marks)**

b the distance AB. **(2 marks)**

8 A particle is projected from a point on level ground with speed $U\,\text{m s}^{-1}$ and angle of elevation α. The maximum height reached by the particle is $42\,\text{m}$ above the ground and the particle hits the ground $196\,\text{m}$ from its point of projection.

Find the value of α and the value of U. **(9 marks)**

9 In this question use $g = 10\,\text{m s}^{-2}$.
An object is projected with speed $U\,\text{m s}^{-1}$ from a point A at the top of a vertical building. The point A is $25\,\text{m}$ above the ground. The object is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$. The object hits the ground at the point B, which is at a horizontal distance of $42\,\text{m}$ from the foot of the building, as shown in the diagram. The object is modelled as a particle moving freely under gravity.



Find:

a the value of U **(6 marks)**

b the time taken by the object to travel from A to B **(2 marks)**

c the speed of the object when it is $12.4\,\text{m}$ above the ground, giving your answer to 2 significant figures. **(5 marks)**

Homework Exercise

10 An object is projected from a fixed origin O with velocity $(4\mathbf{i} + 5\mathbf{j})\,\text{m s}^{-1}$. The particle moves freely under gravity and passes through the point P with position vector $k(\mathbf{i} - \mathbf{j})\,\text{m}$, where k is a positive constant.

a Find the value of k. **(6 marks)**

b Find:

i the speed of the object at the instant when it passes through P

ii the direction of motion of the object at the instant when it passes through P. **(7 marks)**

11 A basketball player is standing on the floor 10 m from the basket. The height of the basket is 3.05 m, and he shoots the ball from a height of 2 m, at an angle of 40° above the horizontal.

The basketball can be modelled as a particle moving in a vertical plane. Given that the ball passes through the basket,

a find the speed with which the basketball is thrown. **(6 marks)**

b State two factors that can be ignored by modelling the basketball as a particle. **(2 marks)**

Challenge

A vertical tower is 85 m high. A stone is projected at a speed of $20\,\text{m s}^{-1}$ from the top of a tower at an angle of α below the horizontal. At the same time, a second stone is projected horizontally at a speed of $12\,\text{m s}^{-1}$ from a window in the tower 45 m above the ground.

Given that the two stones move freely under gravity in the same vertical plane, and that they collide in mid-air, show that the time that elapses between the moment they are projected and the moment they collide is 2.5 s.

Homework Answers

1 **a** 3.9 s (2 s.f.) **b** 56 m (2 s.f.)
55° (nearest degree)
a $(36\mathbf{i} + 27.9\mathbf{j})$ m **b** 13 m s^{-1} (2 s.f.)
2 **a** 22° (2 s.f.) **b** 97 m (2 s.f.)
3 **a** 16 (2 s.f.) **b** 1.6 s (2 s.f.)
4 **a** 4.4 **b** 88 **c** 50° (2 s.f.)
5 **a** 1.1 s (2 s.f.) **b** 34 m (2 s.f.)
6 $a = 40.6°$ (nearest 0.1°)
7 $U = 44$ (2 s.f.)
8 **a** 15.6 m s^{-1} **b** 2.92 s **c** 22.3 m s^{-1}
9 **a** $k = 7.35$
10 **b** **i** 13.6 m s^{-1} **ii** 72.9°
11 **a** 10.7 m s^{-1} **b** e.g. weight of the ball; air resistance

Challenge

R(→): $s = 12t$ and $s = (20\cos\alpha)t$
so $\cos\alpha = 0.6$ and $\sin\alpha = 0.8$
R(↑): $s = -4.9t^2 + 40$ and $s = (20\sin\alpha)t - 4.9t^2$
So $t = \frac{40}{20\sin\alpha} = \frac{40}{16} = 2.5$ seconds