# P1 Chapter 13: Integration

## Indefinite Integrals

# Integration notation

The following notation could be used to differentiate an expression:

The $dx$ here means differentiating "with respect to $x$".

$$\frac{d}{dx}(5x^2) = 10x$$

There is similarly notation for integrating an expression:

$$\int 10x \ dx = 5x^2 + c$$

"Integrate…"

"…this expression"

"…with respect to $x$"
(the $dx$ is needed just as it was needed in the differentiation notation at the top of this slide)

This is known as **indefinite integration**, in contrast to definite integration, which we'll see later in the chapter.

It is called 'indefinite' because the exact expression is unknown (due to the $+c$).

# Examples

**Fro Note**: The brackets are required if there's multiple terms.

Find $\int (x^{-\frac{3}{2}} + 2)\, dx$

$$= -2x^{-\frac{1}{2}} + 2x + c$$

Find $\int (6t^2 - 1)\, dt$

Note the $dt$ instead of $dx$.

$$= 2t^3 - t + c$$

Find $\int (px^3 + q)\, dx$ where $p$ and $q$ are constants.

$$= \frac{1}{4}px^4 + qx + c$$

**Textbook (Minor) Error**: "*any other letters must be treated as constants*". Similar to the error in the differentiation chapter, it should read "*any other letters, which are either constants or variables independent of $x$, can be treated as numbers*". In $\int xy\, dx$, if $y$ is a variable, we can only treat $y$ as a constant if it is not dependent on $x$, i.e. there is not some equation relating $y$ to $x$.

# Test Your Understanding

Edexcel C1 May 2014(R) Q4b

Given that $y = 2x^5 + \frac{6}{\sqrt{x}}$, $x > 0$, find in their simplest form

(b) $\int y\,dx$ **(3)**

?

# Test Your Understanding

**Edexcel C1 May 2014(R) Q4b**

Given that $y = 2x^5 + \frac{6}{\sqrt{x}}$, $x > 0$, find in their simplest form

(b) $\int y\,dx$ **(3)**

$$y = 2x^5 + 6x^{-\frac{1}{2}}$$

$$\int y\,dx = \frac{1}{3}x^6 + 12x^{\frac{1}{2}} + c$$

# Exercise 13.2

Pearson Pure Mathematics Year 1/AS
Page 105

# Homework Exercise

**1** Find the following integrals:

**a** $\int x^3 \, dx$ **b** $\int x^7 \, dx$ **c** $\int 3x^{-4} \, dx$ **d** $\int 5x^2 \, dx$

**2** Find the following integrals:

**a** $\int (x^4 + 2x^3) dx$ **b** $\int (2x^3 - x^2 + 5x) dx$ **c** $\int (5x^{\frac{3}{2}} - 3x^2) dx$

**3** Find the following integrals:

**a** $\int (4x^{-2} + 3x^{-\frac{1}{2}}) dx$ **b** $\int (6x^{-2} - x^{\frac{1}{2}}) dx$ **c** $\int (2x^{-\frac{3}{2}} + x^2 - x^{-\frac{1}{2}}) dx$

**4** Find the following integrals:

**a** $\int (4x^3 - 3x^{-4} + r) dx$

**b** $\int (x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}}) dx$

**c** $\int (px^4 + 2t + 3x^{-2}) dx$

**Hint** In **Q4** part **c** you are integrating with respect to $x$, so treat $p$ and $t$ as constants.

**5** Find the following integrals:

**a** $\int (3t^2 - t^{-2}) dt$ **b** $\int (2t^2 - 3t^{-\frac{3}{2}} + 1) dt$ **c** $\int (pt^3 + q^2 + px^3) dt$

**6** Find the following integrals:

**a** $\int \frac{(2x^3 + 3)}{x^2} dx$ **b** $\int (2x + 3)^2 \, dx$ **c** $\int (2x + 3)\sqrt{x} \, dx$

# Homework Exercise

**7** Find $\int f(x)\,dx$ when $f(x)$ is given by the following:

**a** $\left(x + \frac{1}{x}\right)^2$ **b** $(\sqrt{x} + 2)^2$ **c** $\left(\frac{1}{\sqrt{x}} + 2\sqrt{x}\right)$

**8** Find the following integrals:

**a** $\int\left(x^{\frac{2}{3}} + \frac{4}{x^3}\right)dx$ **b** $\int\left(\frac{2+x}{x^3} + 3\right)dx$ **c** $\int(x^2 + 3)(x - 1)dx$

**d** $\int\frac{(2x+1)^2}{\sqrt{x}}dx$ **e** $\int\left(3 + \frac{\sqrt{x} + 6x^3}{x}\right)dx$ **f** $\int\sqrt{x}(\sqrt{x} + 3)^2\,dx$

**9** Find the following integrals:

**a** $\int\left(\frac{A}{x^2} - 3\right)dx$ **b** $\int\left(\sqrt{Px} + \frac{2}{x^3}\right)dx$ **c** $\int\left(\frac{p}{x^2} + q\sqrt{x} + r\right)dx$

**10** Given that $f(x) = \frac{6}{x^2} + 4\sqrt{x} - 3x + 2$, $x > 0$, find $\int f(x)dx$. **(5 marks)**

**11** Find $\int\left(8x^3 + 6x - \frac{3}{\sqrt{x}}\right)dx$, giving each term in its simplest form. **(4 marks)**

**12** **a** Show that $(2 + 5\sqrt{x})^2$ can be written as $4 + k\sqrt{x} + 25x$, where $k$ is a constant to be found. **(2 marks)**

**b** Hence find $\int(2 + 5\sqrt{x})^2\,dx$. **(3 marks)**

# Homework Exercise

**13** Given that $y = 3x^5 - \frac{4}{\sqrt{x}}$, $x > 0$, find $\int y \, dx$ in its simplest form. **(3 marks)**

**14** $\int \left( \frac{p}{2x^2} + pq \right) dx = \frac{2}{x} + 10x + c$ **(5 marks)**

Find the value of $p$ and the value of $q$.

**Problem-solving**

Integrate the expression on the left-hand side, treating $p$ and $q$ as constants, then compare the result with the right-hand side.

**15** $f(x) = (2 - x)^{10}$

Given that $x$ is small, and so terms in $x^3$ and higher powers of $x$ can be ignored:

**a** find an approximation for $f(x)$ in the form $A + Bx + Cx^2$ **(3 marks)**

**b** find an approximation for $\int f(x) dx$. **(3 marks)**

**Hint** Find the first three terms of the binomial expansion of $(2 - x)^{10}$. ← Section 8.3

# Homework Answers

**1** **a** $\frac{x^4}{4} + c$ **b** $\frac{x^8}{8} + c$

**c** $-x^{-3} + c$ **d** $\frac{5x^3}{3} + c$

**2** **a** $\frac{1}{5}x^5 + \frac{1}{2}x^4 + c$ **b** $\frac{x^4}{2} - \frac{x^3}{3} + \frac{5x^2}{2} + c$

**c** $2x^{\frac{5}{2}} - x^3 + c$

**3** **a** $-4x^{-1} + 6x^{\frac{1}{2}} + c$ **b** $-6x^{-1} - \frac{2}{5}x^{\frac{5}{2}} + c$

**c** $-4x^{-\frac{1}{2}} + \frac{x^3}{3} - 2x^{\frac{1}{2}} + c$

**4** **a** $x^4 + x^{-3} + rx + c$ **b** $\frac{1}{2}x^2 + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$

**c** $\frac{px^5}{5} + 2tx - 3x^{-1} + c$

**5** **a** $t^3 + t^{-1} + c$ **b** $\frac{2}{3}t^3 + 6t^{-\frac{1}{2}} + t + c$

**c** $\frac{p}{4}t^4 + q^2t + px^3t + c$

**6** **a** $2x - \frac{3}{x} + c$ **b** $\frac{4}{3}x^3 + 6x^2 + 9x + c$

**c** $\frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$

**7** **a** $\frac{1}{3}x^3 + 2x - \frac{1}{x} + c$ **b** $\frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$

**c** $2x^{\frac{1}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$

**8** **a** $\frac{3}{5}x^{\frac{5}{3}} - \frac{2}{x^2} + c$ **b** $-\frac{1}{x^2} - \frac{1}{x} + 3x + c$

**c** $\frac{1}{x}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 - 3x + c$ **d** $\frac{8}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

**e** $3x + 2x^{\frac{1}{2}} + 2x^3 + c$ **f** $\frac{2}{5}x^{\frac{5}{2}} + 3x^2 + 6x^{\frac{3}{2}} + c$

**9** **a** $-\frac{A}{x} - 3x + c$ **b** $\frac{2}{3}\sqrt{P}\,x^{\frac{3}{2}} - \frac{1}{x^2} + c$

**c** $-\frac{p}{x} + \frac{2qx^{\frac{3}{2}}}{3} + rx + c$

**10** $-\frac{6}{x} + \frac{8x^{\frac{3}{2}}}{3} - \frac{3x^2}{2} + 2x + c$

**11** $2x^4 + 3x^2 - 6x^{\frac{1}{2}} + c$

**12** **a** $(2 + 5\sqrt{x})^2 = 4 + 10\sqrt{x} + 10\sqrt{x} + 25x = 4 + 20\sqrt{x} + 25x$

**b** $4x + \frac{40x^{\frac{3}{2}}}{3} + \frac{25x^2}{2} + c$

**13** $\frac{x^6}{2} - 8x^{\frac{1}{2}} + c$

**14** $p = -4,\ q = -2.5$

**15** **a** $1024 - 5120x + 11\,520x^2$

**b** $1024x - 2560x^2 + 3840x^3 + c$