P1 Chapter 3: Inequalities

Quadratic Inequalities

Solving Quadratic Inequalities

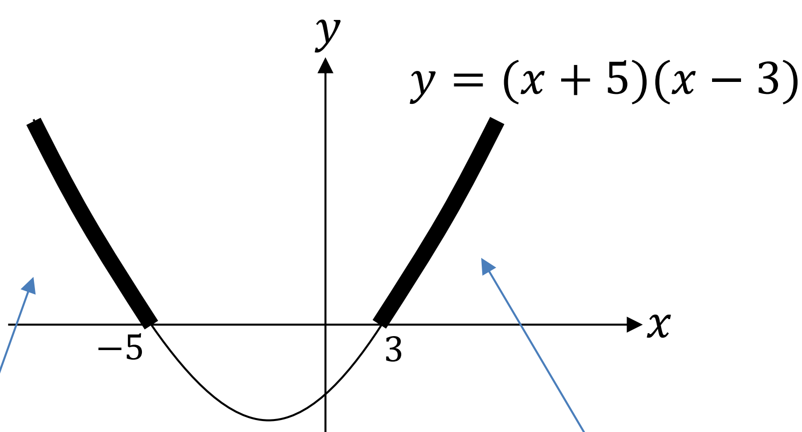
Solve $x^2 + 2x - 15 > 0$

Step 1: Get 0 on one side
(already done!)

Step 2: Factorise

$$(x+5)(x-3) > 0$$

Step 3: Sketch and reason



Since we sketched $y = (x+5)(x-3)$ we're interested where $y > 0$, i.e. the parts of the line where the y value is positive.

What can you say about the x values of points in this region?

$\boldsymbol{x < -5}$

What can you say about the x values of points in this region?

$\boldsymbol{x > 3}$

$$\{x : x < -5\} \cup \{x : x > 3\}$$

Note: If the y value is 'strictly' greater than 0, i.e. > 0, then the x value is strictly less than -5. So the < vs ≤ must match the original question.

Solving Quadratic Inequalities

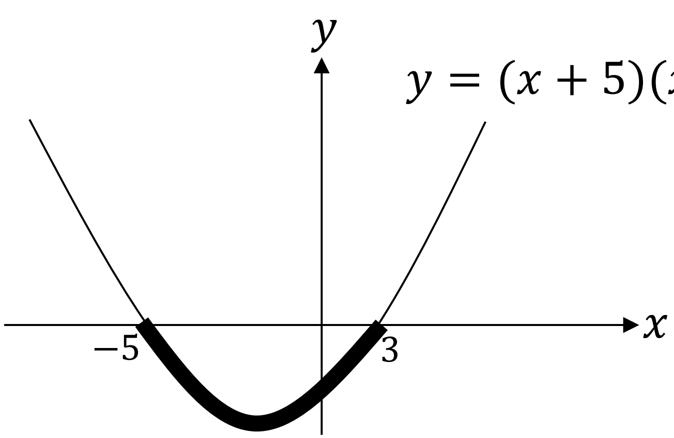
Solve $x^2 + 2x - 15 \leq 0$

Step 1: Get 0 on one side
(already done!)

$$(x+5)(x-3) \leq 0$$

Step 2: Factorise

Step 3: Sketch and reason



Again, what can we say about the x value of any point in this region?

$$\{x : -5 \leq x \leq 3\}$$

Bro Note: As discussed previously, we need $\leq$ rather than $<$ to be consistent with the original inequality.

Further Examples

Solve $x^2 + 5x \geq -4$

?

Solve $x^2 < 9$

?

Note: The most common error I've seen students make with quadratic inequalities is to skip the 'sketch step'. Sod's Law states that even though you have a 50% chance of getting it right without a sketch (presuming you've factorised correctly).

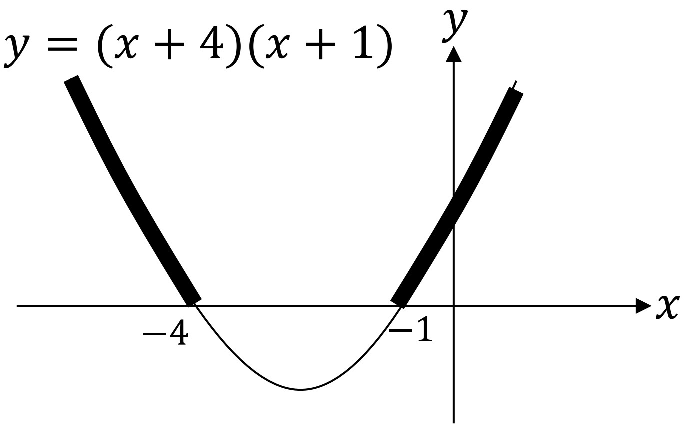
Use of Technology:
Use the quadratic inequality solver on my ClassWiz. Just go to Menu → Inequalities, then choose 'order 2' (i.e. quadratic)

Further Examples

Solve $x^2 + 5x \geq -4$

$$x^2 + 5x + 4 \geq 0$$

$$(x+4)(x+1) \geq 0$$

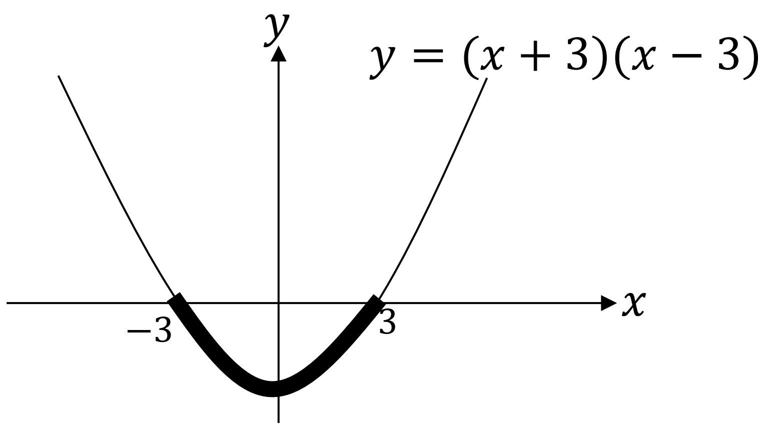


$$x \leq -4 \text{ or } x \geq -1$$

Solve $x^2 < 9$

$$x^2 - 9 < 0$$

$$(x+3)(x-3) < 0$$



$$-3 < x < 3$$

Fro Note: The most common error I've seen students make with quadratic inequalities is to skip the 'sketch step'. Sod's Law states that even though you have a 50% chance of getting it right without a sketch (presuming you've factorised correctly), you <u>will</u> get it wrong.

Use of Technology:
Use the quadratic inequality solver on my ClassWiz. Just go to Menu → Inequalities, then choose 'order 2' (i.e. quadratic)

Test Your Understanding

Edexcel C1 June 2008 Q8

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(*a*) show that $q^2 + 8q < 0$.

(2)

(*b*) Hence find the set of possible values of q.

(3)

(*a*) ?

(*b*) ?

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Edexcel C1 June 2008 Q8

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(*a*) show that $q^2 + 8q < 0$. **(2)**

(*b*) Hence find the set of possible values of q. **(3)**

(*a*)	[No real roots implies $b^2 - 4ac < 0$.] $b^2 - 4ac = q^2 - 4 \times 2q \times (-1)$	M1
	So $q^2 - 4 \times 2q \times (-1) < 0$ i.e. $q^2 + 8q < 0$ (*)	A1 cso (2)
(*b*)	$q(q+8) = 0$ or $(q \pm 4)^2 \pm 16 = 0$	M1
	$(q) = 0$ or -8 (2 cvs)	A1
	$-8 < q < 0$ <u>or</u> $q \in (-8, 0)$ <u>or</u> $q < 0$ and $q > -8$	A1 ft (3)

Note: What often confuses students is that the original equation has no solutions, but the inequality $q^2 + 8q < 0$ *did* have solutions. But think carefully what we've done: We've found the solutions for q that result in the original equation not having any solutions for x. These are different variables, so have different solutions sets, even if the solution set of q influences the solution set of x.

Exercise 3.5

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Homework Exercise

1 Find the set of values of x for which:

a $x^2 - 11x + 24 < 0$
b $12 - x - x^2 > 0$
c $x^2 - 3x - 10 > 0$
d $x^2 + 7x + 12 \geqslant 0$
e $7 + 13x - 2x^2 > 0$
f $10 + x - 2x^2 < 0$
g $4x^2 - 8x + 3 \leqslant 0$
h $-2 + 7x - 3x^2 < 0$
i $x^2 - 9 < 0$
j $6x^2 + 11x - 10 > 0$
k $x^2 - 5x > 0$
l $2x^2 + 3x \leqslant 0$

2 Find the set of values of x for which:

a $x^2 < 10 - 3x$
b $11 < x^2 + 10$
c $x(3 - 2x) > 1$
d $x(x + 11) < 3(1 - x^2)$

3 Use set notation to describe the set of values of x for which:

a $x^2 - 7x + 10 < 0$ and $3x + 5 < 17$
b $x^2 - x - 6 > 0$ and $10 - 2x < 5$
c $4x^2 - 3x - 1 < 0$ and $4(x + 2) < 15 - (x + 7)$
d $2x^2 - x - 1 < 0$ and $14 < 3x - 2$
e $x^2 - x - 12 > 0$ and $3x + 17 > 2$
f $x^2 - 2x - 3 < 0$ and $x^2 - 3x + 2 > 0$

4 Given that $x \neq 0$, find the set of values of x for which:

a $\frac{2}{x} < 1$
b $5 > \frac{4}{x}$
c $\frac{1}{x} + 3 > 2$
d $6 + \frac{5}{x} > \frac{8}{x}$
e $25 > \frac{1}{x^2}$
f $\frac{6}{x^2} + \frac{7}{x} \leqslant 3$

Homework Exercise

5 **a** Find the range of values of k for which the equation $x^2 - kx + (k + 3) = 0$ has no real roots.

b Find the range of values of p for which the roots of the equation $px^2 + px - 2 = 0$ are real.

Hint The quadratic equation $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geqslant 0$. ← Section 2.5

6 Find the set of values of x for which $x^2 - 5x - 14 > 0$. **(4 marks)**

7 Find the set of values of x for which

a $2(3x - 1) < 4 - 3x$ **(2 marks)**

b $2x^2 - 5x - 3 < 0$ **(4 marks)**

c **both** $2(3x - 1) < 4 - 3x$ **and** $2x^2 - 5x - 3 < 0$. **(2 marks)**

8 Given that $x \neq 3$, find the set of values for which $\dfrac{5}{x - 3} < 2$. **(6 marks)**

Problem-solving

Multiply both sides of the inequality by $(x - 3)^2$.

9 The equation $kx^2 - 2kx + 3 = 0$, where k is a constant, has no real roots. Prove that k satisfies the inequality $0 \leqslant k < 3$. **(4 marks)**

Homework Answers

1 **a** $3 < x < 8$ **b** $-4 < x < 3$
c $x < -2, x > 5$ **d** $x \leqslant -4, x \geqslant -3$
e $-\frac{1}{2} < x < 7$ **f** $x < -2, x > 2\frac{1}{2}$
g $\frac{1}{2} \leqslant x \leqslant 1\frac{1}{2}$ **h** $x < \frac{1}{3}, x > 2$
i $-3 < x < 3$ **j** $x < -2\frac{1}{2}, x > \frac{2}{3}$
k $x < 0, x > 5$ **l** $-1\frac{1}{2} \leqslant x \leqslant 0$

2 **a** $-5 < x < 2$ **b** $x < -1, x > 1$
c $\frac{1}{2} < x < 1$ **d** $-3 < x < \frac{1}{4}$

3 **a** $\{x: 2 < x < 4\}$ **b** $\{x: x > 3\}$
c $\{x: -\frac{1}{4} < x < 0\}$ **d** No values
e $\{x: -5 < x < -3\} \cup \{x: x > 4\}$
f $\{x: -1 < x < 1\} \cup \{x: 2 < x < 3\}$

4 **a** $x < 0$ or $x > 2$ **b** $x < 0$ or $x > 0.8$
c $x < -1$ or $x > 0$ **d** $x < 0$ or $x > 0.5$
e $x < -\frac{1}{5}$ or $x > \frac{1}{5}$ **f** $x \leqslant -\frac{2}{3}$ or $x \geqslant 3$

5 **a** $-2 < k < 6$ **b** $p \leqslant -8$ or $p \geqslant 0$

6 $\{x: x < -2\} \cup \{x: x > 7\}$

7 **a** $\{x: x < \frac{2}{3}\}$ **b** $\{x: -\frac{1}{2} < x < 3\}$
c $\{x: -\frac{1}{2} < x < \frac{2}{3}\}$

8 $x < 3$ or $x > 5.5$

9 No real roots $b^2 - 4ac < 0$ $(-2k)^2 - 4 \times k \times 3 < 0$
$4k^2 - 12k = 0$ when $k = 0$ and $k = 3$
solution $0 \leqslant k < 3$
note when $k = 0$ equation gives $3 = 0$