M1 Chapter 9: Constant Acceleration

Three Derived Formulae

“suvat” equations (Part 2)

The other “suvat” equations can be derived using $v = u + at$ and $s = \left(\frac{u+v}{2}\right)t$.

Eliminating t:

$$t = \frac{v-u}{a}$$

$$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$= \cdots$$

$$v^2 = u^2 + 2as$$

Eliminating v:

$$s = \left(\frac{u+u+at}{2}\right)t$$

$$= \left(\frac{2u+at}{2}\right)t$$

$$s = ut + \frac{1}{2}at^2$$

Note: Because this is quadratic in t, we typically end up with two different possible times.

Eliminating u:

$$u = v - at$$

$$s = \left(\frac{v-at+v}{2}\right)t$$

$$= \left(\frac{2v-at}{2}\right)t$$

$$s = vt - \frac{1}{2}at^2$$

Note: Exam questions that use this *suvat* formula are rare.

Examples

[Textbook] A particle is moving along a straight line from A to B with constant acceleration 5 ms^{-2}. The velocity of the particle is 3 ms^{-1} in the direction $\overrightarrow{AB}$. The velocity of the particle at B is 18 ms^{-1} in the same direction. Find the distance from A to B.

- $v = u + at$
- $s = \left(\frac{u+v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$

$s = ?, u = 3, v = 18, a = 5$

$v^2 = u^2 + 2as$
$18^2 = 3^2 + 2 \times 5 \times s$
$s = 31.5$

[Textbook] A particle is moving in a straight horizontal line with constant deceleration 4 ms^{-2}. At time $t = 0$ the particle passes through a point O with speed 13 ms^{-1} travelling towards a point A, where $OA = 20$ m. Find:
(a) the times when the particle passes through A
(b) the value of t when the particle returns to O.

a

?

b

?

The $t = 0$ solution is expected because the particle started at O.

Examples

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(a) the times when the particle passes through A
(b) the value of t when the particle returns to O.

a

$s = 20, u = 13, a = -4, t = ?$
$s = ut + \frac{1}{2}at^2$
$20 = 13t - \frac{1}{2} \times 4t^2$
$2t^2 - 13t + 20 = 0$
$(2t - 5)(t - 4) = 0$
$t = \frac{5}{2}, t = 4$

b

$s = 0, u = 13, a = -4, t = ?$
$s = ut + \frac{1}{2}at^2$
$0 = 13t - 2t^2$
$= t(13 - 2t)$
$t = 0, or\ t = \frac{13}{2}$
Particle returns after 6.5 s

The $t = 0$ solution is expected because the particle started at O.

Test Your Understanding

Edexcel M1 May 2013 Q4

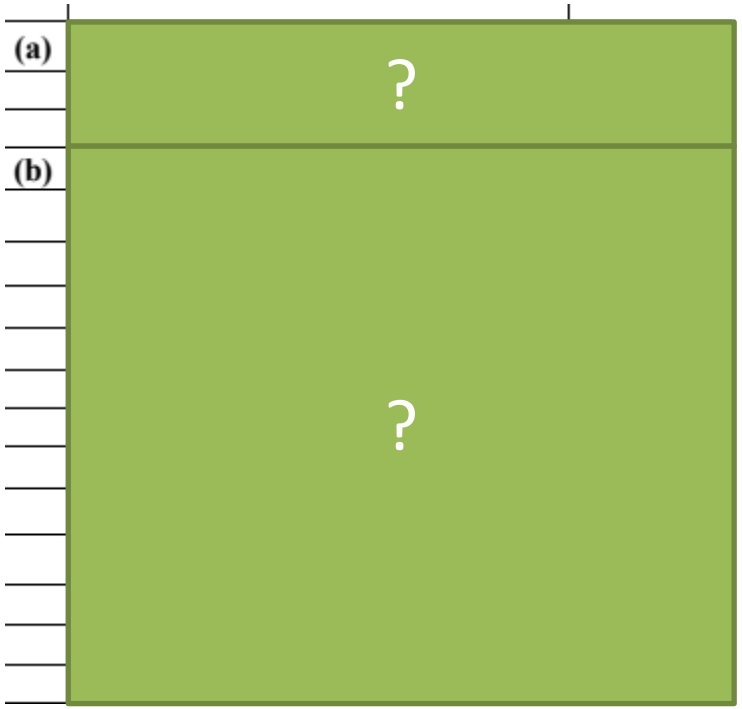
A lorry is moving along a straight horizontal road with constant acceleration. The lorry passes a point A with speed u m s^{-1}, ($u < 34$), and 10 seconds later passes a point B with speed 34 m s^{-1}. Given that $AB = 240$ m, find

(a) the value of u,

(3)

(b) the time taken for the lorry to move from A to the mid-point of AB.

(6)



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(6)

(a)	$240 = \frac{1}{2}(u+34)10$	M1 A1
	$u = 14$	A1
		(3)
(b)	$34 = 14 + 10a \Rightarrow a = 2$	M1 A1
	$120 = 14t + \frac{1}{2} \times 2 \times t^2$	M1 A1
	$t^2 + 14t - 120 = 0$	
	Solving, $t = -20$ or 6	**DM**1
	$t = 6$	A1
	OR	
	$34 = 14 + 10a \Rightarrow a = 2$	M1 A1
	$v^2 = 14^2 + 2 \times 2 \times 120 \Rightarrow v = 26$	
	AND $26 = 14 + 2t$	M1 A1
	$t = 6$	**DM**1 A1
		(6)

Exercise 9.4 The second two formulae

Pearson Stats/Mechanics Year 1
Pages 62-63

Homework Exercise

1 A particle is moving in a straight line with constant acceleration $2.5\,\text{m s}^{-2}$. It passes a point A with velocity $3\,\text{m s}^{-1}$ and later passes through a point B, where $AB = 8\,\text{m}$. Find the velocity of the particle as it passes through B.

2 A car is accelerating at a constant rate along a straight horizontal road. Travelling at $8\,\text{m s}^{-1}$, it passes a pillar box and 6 s later it passes a sign. The distance between the pillar box and the sign is 60 m. Find the acceleration of the car.

3 A cyclist travelling at $12\,\text{m s}^{-1}$ applies her brakes and comes to rest after travelling 36 m in a straight line. Assuming that the brakes cause the cyclist to decelerate uniformly, find the deceleration.

4 A train is moving along a straight horizontal track with constant acceleration. The train passes a signal with a velocity of $54\,\text{km h}^{-1}$ and a second signal with a velocity of $72\,\text{km h}^{-1}$. The distance between the two signals is 500 m. Find, in m s^{-2}, the acceleration of the train.

5 A particle moves along a straight line, with constant acceleration, from a point A to a point B where $AB = 48\,\text{m}$. At A the particle has velocity $4\,\text{m s}^{-1}$ and at B it has velocity $16\,\text{m s}^{-1}$. Find:

a the acceleration of the particle

b the time the particle takes to move from A to B.

6 A particle moves along a straight line with constant acceleration $3\,\text{m s}^{-2}$. The particle moves 38 m in 4 s. Find:

a the initial velocity of the particle

b the final velocity of the particle.

7 The driver of a car is travelling at $18\,\text{m s}^{-1}$ along a straight road when she sees an obstruction ahead. She applies the brakes and the brakes cause the car to slow down to rest with a constant deceleration of $3\,\text{m s}^{-2}$. Find:

a the distance travelled as the car decelerates

b the time it takes for the car to decelerate from $18\,\text{m s}^{-1}$ to rest.

Homework Exercise

8 A stone is sliding across a frozen lake in a straight line. The initial speed of the stone is $12\,\text{m s}^{-1}$. The friction between the stone and the ice causes the stone to slow down at a constant rate of $0.8\,\text{m s}^{-2}$. Find:

a the distance moved by the stone before coming to rest

b the speed of the stone at the instant when it has travelled half of this distance.

9 A particle is moving along a straight line OA with constant acceleration $2.5\,\text{m s}^{-2}$. At time $t = 0$, the particle passes through O with speed $8\,\text{m s}^{-1}$ and is moving in the direction OA. The distance OA is 40 m. Find:

a the time taken for the particle to move from O to A

b the speed of the particle at A. Give your answers to one decimal place.

10 A particle travels with uniform deceleration $2\,\text{m s}^{-2}$ in a horizontal line. The points A and B lie on the line and $AB = 32$ m. At time $t = 0$, the particle passes through A with velocity $12\,\text{m s}^{-1}$ in the direction $\overrightarrow{AB}$. Find:

a the values of t when the particle is at B

b the velocity of the particle for each of these values of t.

11 A particle is moving along the x-axis with constant deceleration $5\,\text{m s}^{-2}$. At time $t = 0$, the particle passes through the origin O with velocity $12\,\text{m s}^{-1}$ in the positive direction. At time t seconds the particle passes through the point A with x-coordinate 8. Find:

Problem-solving

The particle will pass through A twice. Use $s = ut + \frac{1}{2}at^2$ to set up and solve a quadratic equation.

a the values of t **(3 marks)**

b the velocity of the particle as it passes through the point with x-coordinate -8. **(3 marks)**

Homework Exercise

12 A particle P is moving on the x-axis with constant deceleration $4\,\text{m s}^{-2}$. At time $t = 0$, P passes through the origin O with velocity $14\,\text{m s}^{-1}$ in the positive direction. The point A lies on the axis and $OA = 22.5\,\text{m}$. Find:

a the difference between the times when P passes through A **(4 marks)**

b the total distance travelled by P during the interval between these times. **(3 marks)**

13 A car is travelling along a straight horizontal road with constant acceleration. The car passes over three consecutive points A, B and C where $AB = 100\,\text{m}$ and $BC = 300\,\text{m}$. The speed of the car at B is $14\,\text{m s}^{-1}$ and the speed of the car at C is $20\,\text{m s}^{-1}$. Find:

a the acceleration of the car **(3 marks)**

b the time take for the car to travel from A to C. **(3 marks)**

14 Two particles P and Q are moving along the same straight horizontal line with constant accelerations $2\,\text{m s}^{-2}$ and $3.6\,\text{m s}^{-2}$ respectively. At time $t = 0$, P passes through a point A with speed $4\,\text{m s}^{-1}$. One second later Q passes through A with speed $3\,\text{m s}^{-1}$, moving in the same direction as P.

a Write down expressions for the displacements of P and Q from A, in terms of t, where t seconds is the time after P has passed through A. **(2 marks)**

b Find the value of t where the particles meet. **(3 marks)**

c Find the distance of A from the point where the particles meet. **(3 marks)**

Problem-solving

When P and Q meet, their displacements from A are equal.

15 In an orienteering competition, a competitor moves in a straight line past three checkpoints, P, Q and R, where $PQ = 2.4\,\text{km}$ and $QR = 11.5\,\text{km}$. The competitor is modelled as a particle moving with constant acceleration. She takes 1 hour to travel from P to Q and 1.5 hours to travel from Q to R. Find:

a the acceleration of the competitor

b her speed at the instant she passes P. **(7 marks)**

Homework Answers

1 $7\,\text{m s}^{-1}$

2 $\frac{2}{3}\,\text{m s}^{-2}$

3 $2\,\text{m s}^{-2}$

4 $0.175\,\text{m s}^{-2}$

5 **a** $2.5\,\text{m s}^{-2}$ **b** $4.8\,\text{s}$

6 **a** $3.5\,\text{m s}^{-1}$ **b** $15.5\,\text{m s}^{-1}$

7 **a** $54\,\text{m}$ **b** $6\,\text{s}$

8 **a** $90\,\text{m}$ **b** $8.49\,\text{m s}^{-1}$ (3 s.f.)

9 **a** $3.3\,\text{s}$ (1 d.p.) **b** $16.2\,\text{m s}^{-1}$ (1 d.p.)

10 **a** $t = 4$ or $t = 8$

b $t = 4$: $4\,\text{m s}^{-1}$ in direction $\overrightarrow{AB}$, $t = 8$: $4\,\text{m s}^{-1}$ in direction $\overrightarrow{BA}$.

11 **a** $t = 0.8$ or $t = 4$

b $15.0\,\text{m s}^{-1}$ (3 s.f.)

12 **a** $2\,\text{s}$ **b** $4\,\text{m}$

13 **a** $0.34\,\text{m s}^{-1}$ **b** $25.5\,\text{s}$ (3 s.f.)

14 **a** P: $(4t + t^2)\,\text{m}$ Q: $[3(t - 1) + 1.8(t - 1)^2]\,\text{m}$

b $t = 6$ **c** $60\,\text{m}$

15 **a** $4.21\,\text{km h}^{-2}$ **b** $0.295\,\text{km h}^{-1}$