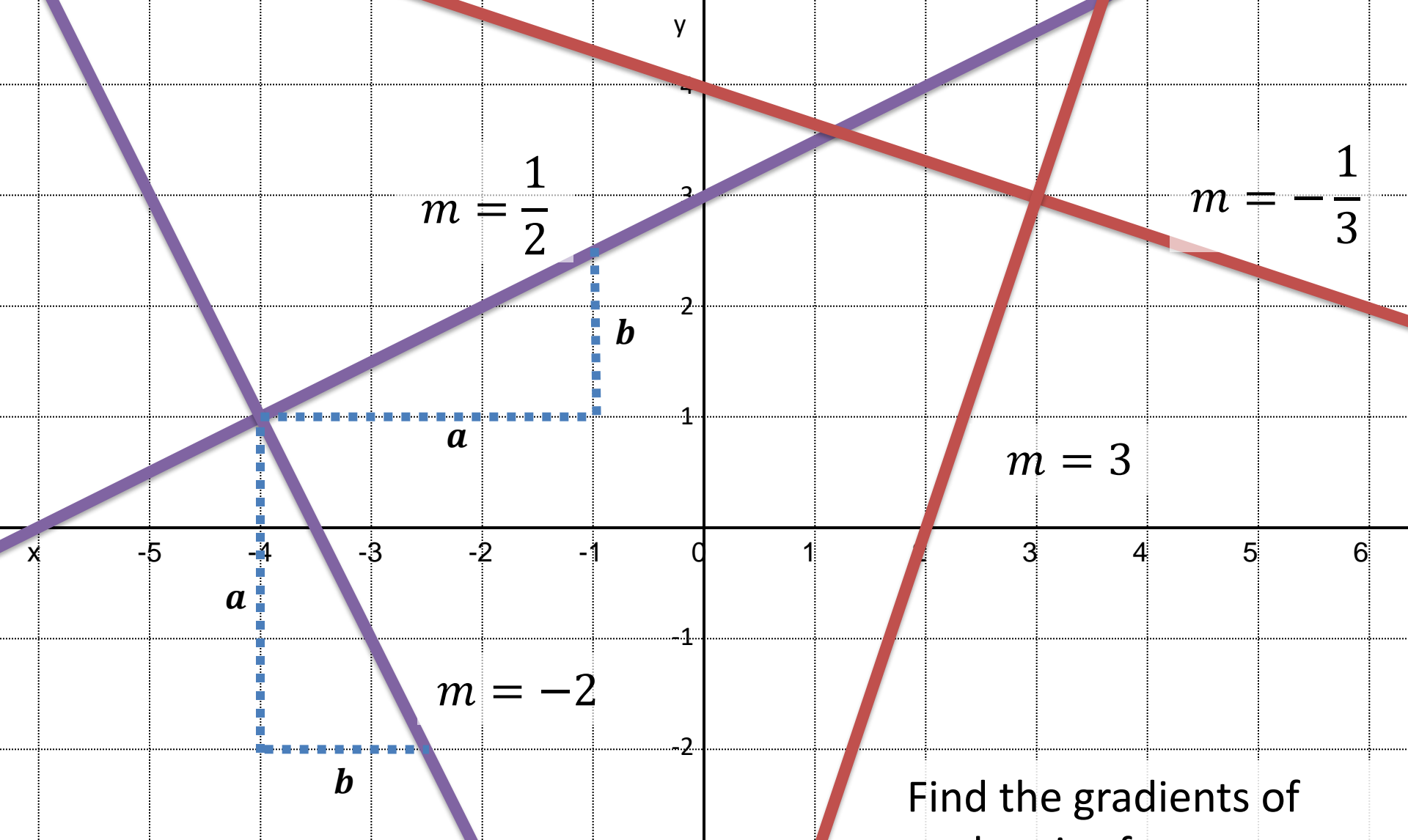

P1 Chapter 5: Linear Graphs

Parallel and Perpendicular



Using the changes above, we can see the gradient of one line is $m = \frac{b}{a}$ and the other $m = -\frac{a}{b}$. One is the 'negative reciprocal' of the other.

Find the gradients of each pair of **perpendicular** lines. What do you notice?

Perpendicular Lines



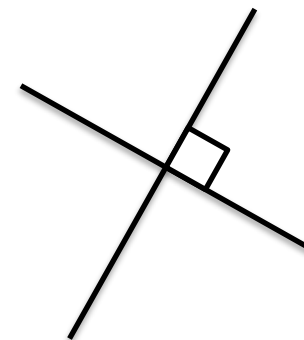
The gradients of parallel lines are equal.

If two lines are perpendicular, then the gradient of one is the **negative reciprocal** of the other.

$$m_1 = -\frac{1}{m_2}$$

To show that two lines are perpendicular:

$$m_1 m_2 = -1$$



Gradient	Gradient of Perpendicular Line
2	?
-3	?
$\frac{1}{4}$?
5	?
$-\frac{2}{7}$?
$\frac{7}{5}$?

Perpendicular Lines



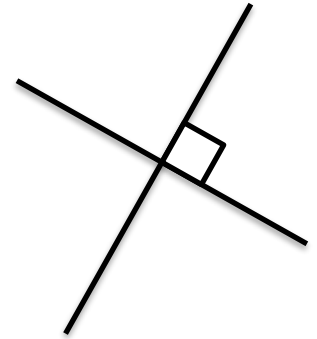
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$$m_1 = -\frac{1}{m_2}$$

To show that two lines are perpendicular:

$$m_1 m_2 = -1$$



Gradient	Gradient of Perpendicular Line
2	$-\frac{1}{2}$
-3	$\frac{1}{3}$
$\frac{1}{4}$	-4
5	$-\frac{1}{5}$
$-\frac{2}{7}$	$\frac{7}{2}$
$\frac{7}{5}$	$-\frac{5}{7}$

Example Problems

- 1 A line goes through the point $(9,10)$ and is perpendicular to another line with equation $y = 3x + 2$. What is the equation of the line?

?

- 2 A line L_1 goes through the points $A(1,3)$ and $B(3, -1)$. A second line L_2 is perpendicular to L_1 and passes through point B. Where does L_2 cross the x-axis?

?

- 3 Are the following lines parallel, perpendicular, or neither?

$$y = \frac{1}{2}x$$
$$2x - y + 4 = 0$$

?

Example Problems

- 1 A line goes through the point (9,10) and is perpendicular to another line with equation $y = 3x + 2$. What is the equation of the line?

$$y - 10 = -\frac{1}{3}(x - 9)$$

- 2 A line L_1 goes through the points $A(1,3)$ and $B(3,-1)$. A second line L_2 is perpendicular to L_1 and passes through point B. Where does L_2 cross the x-axis?

$$(5, 0)$$

- 3 Are the following lines parallel, perpendicular, or neither?

$$y = \frac{1}{2}x$$

$$2x - y + 4 = 0$$

Neither. Gradients are $\frac{1}{2}$ and 2. But $\frac{1}{2} \times 2 = 1$, not -1, so not perpendicular.

Test Your Understanding

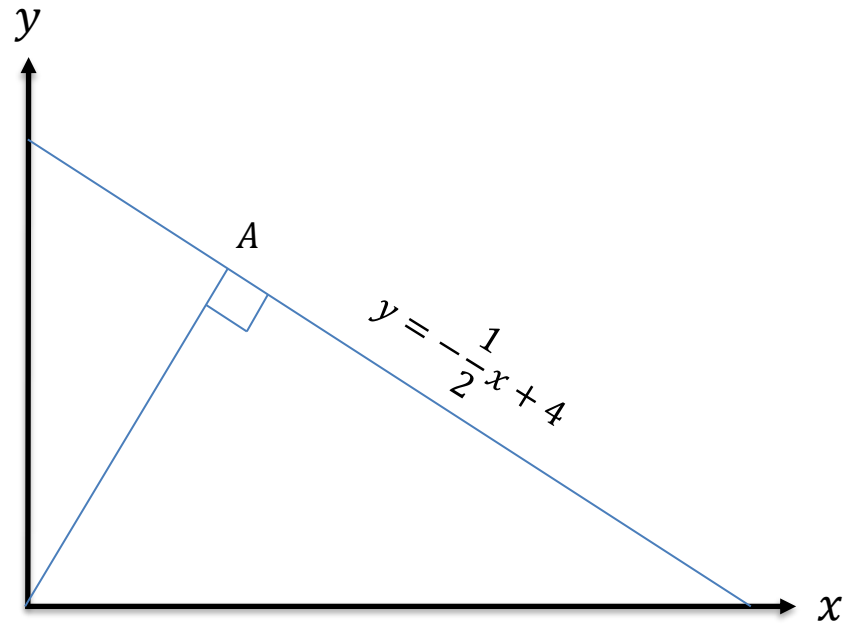
1

A line goes through the point $(4,7)$ and is perpendicular to another line with equation $y = 2x + 2$. What is the equation of the line? Put your answer in the form $ax + by + c = 0$, where a, b, c are integers.

?

2

Determine the point A .



?

Test Your Understanding

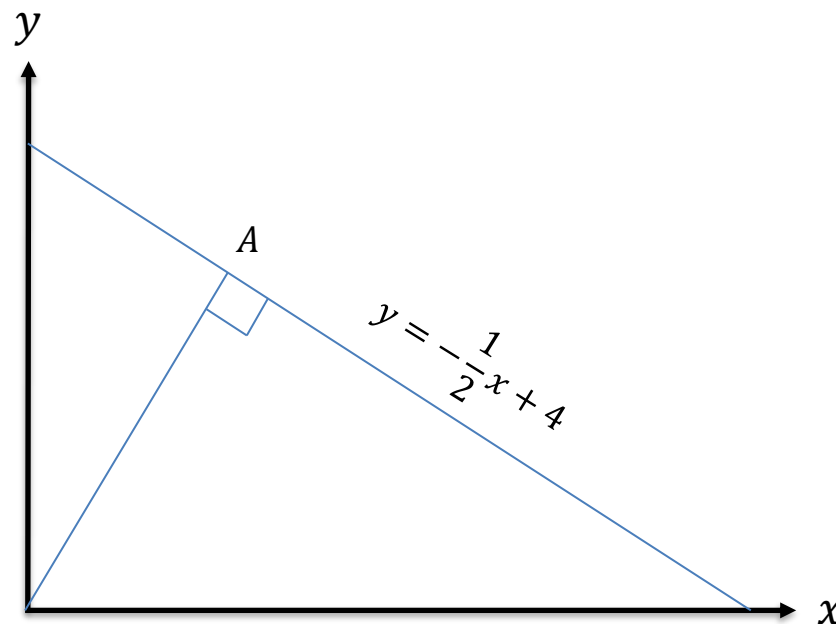
1

A line goes through the point (4,7) and is perpendicular to another line with equation $y = 2x + 2$. What is the equation of the line? Put your answer in the form $ax + by + c = 0$, where a, b, c are integers.

$$\begin{aligned}y &= -\frac{1}{2}x + 9 \\2y &= -x + 18 \\x + 2y - 18 &= 0\end{aligned}$$

2

Determine the point A.



Equation of other line: $y = 2x$

$$\begin{aligned}2x &= -\frac{1}{2}x + 4 \\4x &= -x + 8 \\5x &= 8 \rightarrow x = \frac{8}{5} \\y &= 2 \times \frac{8}{5} = \frac{16}{5}\end{aligned}$$

Exercise 5.3

Pearson Pure Mathematics Year 1/AS

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Extension Problems

- 1** [MAT 2004 1D] What is the reflection of the point $(3,4)$ in the line $3x + 4y = 50$?

?

- 2** [MAT 2014 1D] The reflection of the point $(1,0)$ in the line $y = mx$ has coordinates: (in terms of m)

?

- 3** [STEP I 2004 Q6] The three points A, B, C have coordinates $(p_1, q_1), (p_2, q_2)$ and (p_3, q_3) , respectively. Find the point of intersection of the line joining A to the midpoint of BC , and the line joining B to the midpoint of AC . Verify that this point lies on the line joining C to the midpoint of AB . The point H has coordinates $(p_1 + p_2 + p_3, q_1 + q_2 + q_3)$. Show that if the line AH intersects the line BC at right angles, then $p_2^2 + q_2^2 = p_3^2 + q_3^2$, and write down a similar result if the line BH intersects the line AC at right angles.

Deduce that if AH is perpendicular to BC and also BH is perpendicular to AC , then CH is perpendicular to AB .

Exercise 5.3

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Extension Problems

1 [MAT 2004 1D]

What is the reflection of the point $(3, 4)$ in the line $3x + 4y = 50$?

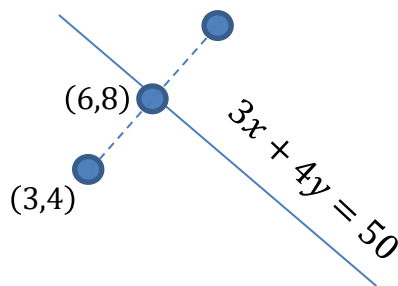
Gradient of $3x + 4y = 50$ is $-\frac{3}{4}$.

\therefore Equation of line passing through $(3, 4)$ perpendicular to this line:

$$y - 4 = \frac{4}{3}(x - 3)$$

Find intersection of two lines to get closest point of $(3, 4)$ to original line:
 $(6, 8)$

Thus reflected point is $(9, 12)$



2 [MAT 2014 1D] The reflection of the point $(1, 0)$ in the line $y = mx$ has coordinates: (in terms of m)

Using a similar method:

$$\left(\frac{1 - m^2}{1 + m^2}, \frac{2m}{1 + m^2} \right)$$

3 [STEP I 2004 Q6] The three points A, B, C have coordinates $(p_1, q_1), (p_2, q_2)$ and (p_3, q_3) , respectively. Find the point of intersection of the line joining A to the midpoint of BC , and the line joining B to the midpoint of AC . Verify that this point lies on the line joining C to the midpoint of AB . The point H has coordinates $(p_1 + p_2 + p_3, q_1 + q_2 + q_3)$. Show that if the line AH intersects the line BC at right angles, then $p_2^2 + q_2^2 = p_3^2 + q_3^2$, and write down a similar result if the line BH intersects the line AC at right angles.

Deduce that if AH is perpendicular to BC and also BH is perpendicular to AC , then CH is perpendicular to AB .

Exercise 5.3

Pearson Pure Mathematics Year 1/AS

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Homework Exercise

1 Work out whether each pair of lines is parallel.

a $y = 5x - 2$

$$15x - 3y + 9 = 0$$

b $7x + 14y - 1 = 0$

$$y = \frac{1}{2}x + 9$$

c $4x - 3y - 8 = 0$

$$3x - 4y - 8 = 0$$

2 The line r passes through the points $(1, 4)$ and $(6, 8)$ and the line s passes through the points $(5, -3)$ and $(20, 9)$. Show that the lines r and s are parallel.

3 The coordinates of a quadrilateral $ABCD$ are $A(-6, 2)$, $B(4, 8)$, $C(6, 1)$ and $D(-9, -8)$. Show that the quadrilateral is a trapezium.

Hint A trapezium has exactly one pair of parallel sides.

4 A line is parallel to the line $y = 5x + 8$ and its y -intercept is $(0, 3)$. Write down the equation of the line.

Hint The line will have gradient 5.

5 A line is parallel to the line $y = -\frac{2}{5}x + 1$ and its y -intercept is $(0, -4)$. Work out the equation of the line. Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

6 A line is parallel to the line $3x + 6y + 11 = 0$ and its intercept on the y -axis is $(0, 7)$. Write down the equation of the line.

7 A line is parallel to the line $2x - 3y - 1 = 0$ and it passes through the point $(0, 0)$. Write down the equation of the line.

8 Find an equation of the line that passes through the point $(-2, 7)$ and is parallel to the line $y = 4x + 1$. Write your answer in the form $ax + by + c = 0$.

Homework Answers

1 **a** Parallel **b** Not parallel **c** Not parallel

2 $r: y = \frac{4}{5}x + 3.2$, $s: y = \frac{4}{5}x - 7$

Gradients equal therefore lines are parallel.

3 Gradient of $AB = \frac{3}{5}$, gradient of $BC = -\frac{7}{2}$, gradient of $CD = \frac{3}{5}$, gradient of $AD = \frac{10}{3}$. The quadrilateral has a pair of parallel sides, so it is a trapezium.

4 $y = 5x + 3$

5 $2x + 5y + 20 = 0$

6 $y = -\frac{1}{2}x + 7$

7 $y = \frac{2}{3}x$

8 $4x - y + 15 = 0$

Homework Exercise

1 Work out whether these pairs of lines are parallel, perpendicular or neither:

a $y = 4x + 2$

$$y = -\frac{1}{4}x - 7$$

b $y = \frac{2}{3}x - 1$

$$y = \frac{2}{3}x - 11$$

c $y = \frac{1}{5}x + 9$

$$y = 5x + 9$$

d $y = -3x + 2$

$$y = \frac{1}{3}x - 7$$

e $y = \frac{3}{5}x + 4$

$$y = -\frac{5}{3}x - 1$$

f $y = \frac{5}{7}x$

$$y = \frac{5}{7}x - 3$$

g $y = 5x - 3$

$$5x - y + 4 = 0$$

h $5x - y - 1 = 0$

$$y = -\frac{1}{5}x$$

i $y = -\frac{3}{2}x + 8$

$$2x - 3y - 9 = 0$$

j $4x - 5y + 1 = 0$

$$8x - 10y - 2 = 0$$

k $3x + 2y - 12 = 0$

$$2x + 3y - 6 = 0$$

l $5x - y + 2 = 0$

$$2x + 10y - 4 = 0$$

2 A line is perpendicular to the line $y = 6x - 9$ and passes through the point $(0, 1)$. Find an equation of the line.

3 A line is perpendicular to the line $3x + 8y - 11 = 0$ and passes through the point $(0, -8)$. Find an equation of the line.

4 Find an equation of the line that passes through the point $(6, -2)$ and is perpendicular to the line $y = 3x + 5$.

5 Find an equation of the line that passes through the point $(-2, 5)$ and is perpendicular to the line $y = 3x + 6$.

6 Find an equation of the line that passes through the point $(3, 4)$ and is perpendicular to the line $4x - 6y + 7 = 0$.

Homework Exercise

- 7 Find an equation of the line that passes through the point $(5, -5)$ and is perpendicular to the line $y = \frac{2}{3}x + 5$. Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.
- 8 Find an equation of the line that passes through the point $(-2, -3)$ and is perpendicular to the line $y = -\frac{4}{7}x + 5$. Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.
- 9 The line l passes through the points $(-3, 0)$ and $(3, -2)$ and the line n passes through the points $(1, 8)$ and $(-1, 2)$. Show that the lines l and n are perpendicular.

Problem-solving

Don't do more work than you need to. You only need to find the gradients of both lines, not their equations.

- 10 The vertices of a quadrilateral $ABCD$ have coordinates $A(-1, 5)$, $B(7, 1)$, $C(5, -3)$ and $D(-3, 1)$. Show that the quadrilateral is a rectangle.

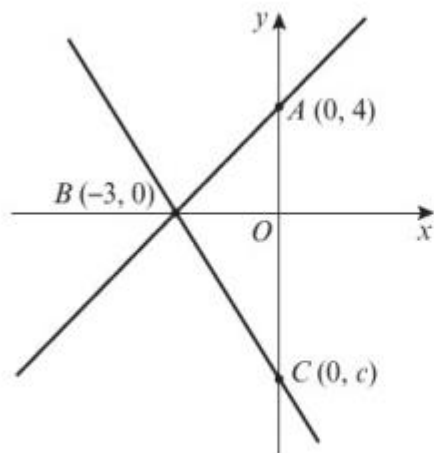
Hint

The sides of a rectangle are perpendicular.

- 11 A line l_1 has equation $5x + 11y - 7 = 0$ and crosses the x -axis at A . The line l_2 is perpendicular to l_1 and passes through A .
- a Find the coordinates of the point A . (1 mark)
- b Find the equation of the line l_2 . Write your answer in the form $ax + by + c = 0$. (3 marks)

Homework Exercise

- 12 The points A and C lie on the y -axis and the point B lies on the x -axis as shown in the diagram.



Problem-solving

Sketch graphs in coordinate geometry problems are not accurate, but you can use the graph to make sure that your answer makes sense. In this question c must be negative.

The line through points A and B is perpendicular to the line through points B and C .
Find the value of c .

(6 marks)

Homework Answers

- 1 **a** Perpendicular **b** Parallel
 c Neither **d** Perpendicular
 e Perpendicular **f** Parallel
 g Parallel **h** Perpendicular
 i Perpendicular **j** Parallel
 k Neither **l** Perpendicular
- 2 $y = -\frac{1}{6}x + 1$
- 3 $y = \frac{8}{3}x - 8$
- 4 $y = -\frac{1}{3}x$
- 5 $y = -\frac{1}{3}x + \frac{13}{3}$
- 6 $y = -\frac{3}{2}x + \frac{17}{2}$
- 7 $3x + 2y - 5 = 0$
- 8 $7x - 4y + 2 = 0$
- 9 $l: y = -\frac{1}{3}x - 1, n: y = 3x + 5$. Gradients are negative reciprocals, therefore lines perpendicular.
- 10 $AB: y = -\frac{1}{2}x + 4\frac{1}{2}, CD: y = -\frac{1}{2}x - \frac{1}{2}, AD: y = 2x + 7,$
 $BC: y = 2x - 13$. Two pairs of parallel sides and lines with gradients 2 and $-\frac{1}{2}$ are perpendicular, so $ABCD$ is a rectangle.
- 11 **a** $A(\frac{7}{5}, 0)$ **b** $55x - 25y - 77 = 0$
- 12 $-\frac{9}{4}$