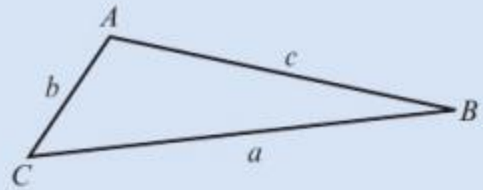
P1 Chapter 9: Trigonometric Ratios

Chapter Practice

Key Points

1 This version of the cosine rule is used to find a missing side if you know two sides and the angle between them:

$$a^2 = b^2 + c^2 - 2bc \cos A$$



2 This version of the cosine rule is used to find an angle if you know all three sides:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

3 This version of the sine rule is used to find the length of a missing side:

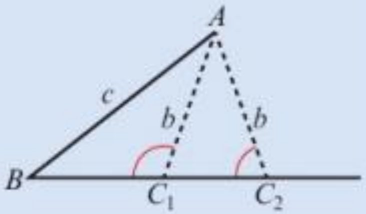
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

4 This version of the sine rule is used to find a missing angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

5 The sine rule sometimes produces two possible solutions for a missing angle:

$$\sin \theta = \sin (180° - \theta)$$



6 Area of a triangle $= \frac{1}{2}ab \sin C$.

Key Points

7 The graphs of sine, cosine and tangent are **periodic**. They repeat themselves after a certain interval.

- The graph of $y = \sin\theta$: repeats every 360° and crosses the x-axis at …, –180°, 0, 180°, 360°, …
- has a maximum value of 1 and a minimum value of –1.
- The graph of $y = \cos\theta$: repeats every 360° and crosses the x-axis at …, –90°, 90°, 270°, 450°, …
- has a maximum value of 1 and a minimum value of –1
- The graph of $y = \tan\theta$: repeats every 180° and crosses the x-axis at … –180°, 0°, 180°, 360°, …
- has no maximum or minimum value
- has vertical asymptotes at $x = -90°$, $x = 90°$, $x = 270°$, …

Chapter Exercises

Give non-exact answers to 3 significant figures.

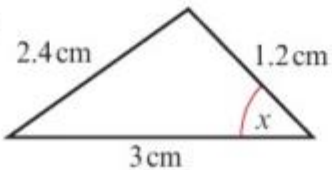
1 Triangle ABC has area $10\,\text{cm}^2$. $AB = 6$ cm, $BC = 8$ cm and $\angle ABC$ is obtuse. Find:

a the size of $\angle ABC$

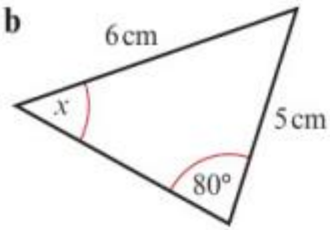
b the length of AC

2 In each triangle below, find the size of x and the area of the triangle.

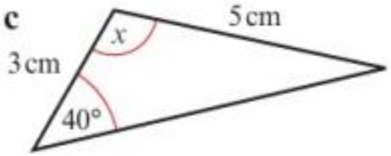
a



b



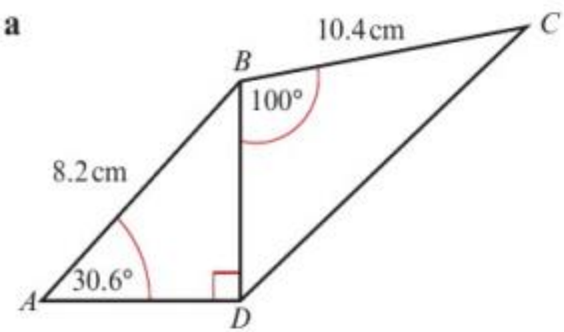
c



3 The sides of a triangle are 3 cm, 5 cm and 7 cm respectively. Show that the largest angle is 120°, and find the area of the triangle.

4 In each of the figures below calculate the total area.

a



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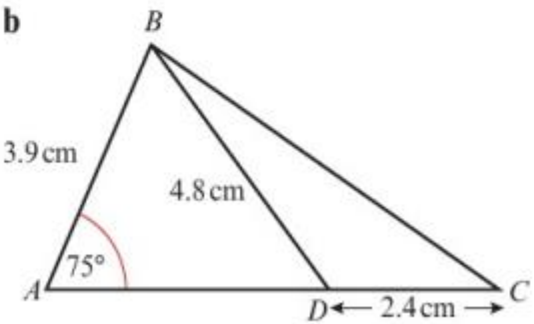


5 In $\triangle ABC$, $AB = 10$ cm, $BC = a\sqrt{3}$ cm, $AC = 5\sqrt{13}$ cm and $\angle ABC = 150°$. Calculate:

a the value of a

b the exact area of $\triangle ABC$.

Chapter Exercises

6 In a triangle, the largest side has length 2 cm and one of the other sides has length $\sqrt{2}$ cm. Given that the area of the triangle is 1 cm^2, show that the triangle is right-angled and isosceles.

7 The three points A, B and C, with coordinates $A(0, 1)$, $B(3, 4)$ and $C(1, 3)$ respectively, are joined to form a triangle.

a Show that $\cos \angle ACB = -\frac{4}{5}$ **(5 marks)**

b Calculate the area of $\triangle ABC$. **(2 marks)**

8 The longest side of a triangle has length $(2x - 1)$ cm. The other sides have lengths $(x - 1)$ cm and $(x + 1)$ cm. Given that the largest angle is 120°, work out

a the value of x **(5 marks)**

b the area of the triangle. **(3 marks)**

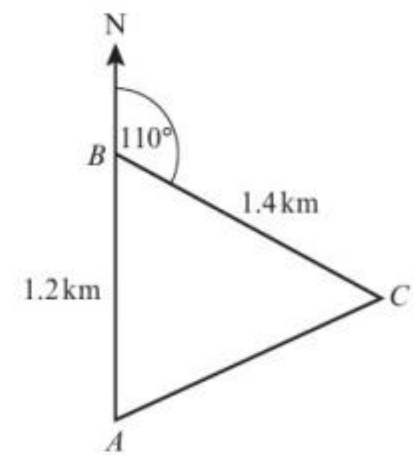
9 A park is in the shape of a triangle ABC as shown.

A park keeper walks due north from his hut at A until he reaches point B. He then walks on a bearing of 110° to point C.

a Find how far he is from his hut when at point C. Give your answer in km to 3 s.f. **(3 marks)**

b Work out the bearing of the hut from point C. Give your answer to the nearest degree. **(3 marks)**

c Work out the area of the park. **(3 marks)**



10 A windmill has four identical triangular sails made from wood. If each triangle has sides of length 12 m, 15 m and 20 m, work out the total area of wood needed. **(5 marks)**

11 Two points, A and B are on level ground. A church tower at point C has an angle of elevation from A of 15° and an angle of elevation from B of 32°. A and B are both on the same side of C, and A, B and C lie on the same straight line. The distance $AB = 75$ m.
Find the height of the church tower. **(4 marks)**

12 Describe geometrically the transformations which map:

a the graph of $y = \tan x$ onto the graph of $\tan \frac{1}{2}x$

b the graph of $y = \tan \frac{1}{2}x$ onto the graph of $3 + \tan \frac{1}{2}x$

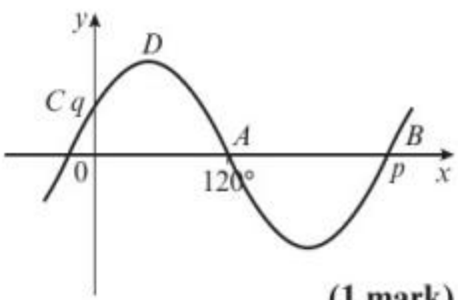
c the graph of $y = \cos x$ onto the graph of $-\cos x$

d the graph of $y = \sin (x - 10)$ onto the graph of $\sin (x + 10)$.

13 a Sketch on the same set of axes, in the interval $0 \leqslant x \leqslant 180°$, the graphs of $y = \tan (x - 45°)$ and $y = -2\cos x$, showing the coordinates of points of intersection with the axes. **(6 marks)**

b Deduce the number of solutions of the equation $\tan (x - 45°) + 2\cos x = 0$, in the interval $0 \leqslant x \leqslant 180°$. **(2 marks)**

14 The diagram shows part of the graph of $y = \mathrm{f}(x)$.
It crosses the x-axis at $A(120°, 0)$ and $B(p, 0)$.
It crosses the y-axis at $C(0, q)$ and has a maximum value at D, as shown.



Given that $\mathrm{f}(x) = \sin (x + k)$, where $k > 0$, write down

a the value of p **(1 mark)**

b the coordinates of D **(1 mark)**

c the smallest value of k **(1 mark)**

d the value of q. **(1 mark)**

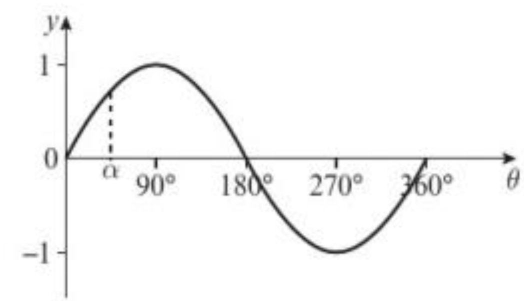
15 Consider the function $f(x) = \sin px$, $p \in \mathbb{R}$, $0 \leqslant x \leqslant 360°$.

The closest point to the origin that the graph of $f(x)$ crosses the x-axis has x-coordinate $36°$.

a Determine the value of p and sketch the graph of $y = f(x)$. **(5 marks)**

b Write down the period of $f(x)$. **(1 mark)**

16 The graph below shows $y = \sin\theta$, $0 \leqslant \theta \leqslant 360°$, with one value of θ $(\theta = \alpha)$ marked on the axis.



a Copy the graph and mark on the θ-axis the positions of $180° - \alpha$, $180° + \alpha$, and $360° - \alpha$.

b Verify that:

$\sin\alpha = \sin(180° - \alpha) = -\sin(180° + \alpha) = -\sin(360° - \alpha)$.

17 **a** Sketch on separate sets of axes the graphs of $y = \cos\theta$ $(0 \leqslant \theta \leqslant 360°)$ and $y = \tan\theta$ $(0 \leqslant \theta \leqslant 360°)$, and on each θ-axis mark the point $(\alpha, 0)$ as in question **16**.

b Verify that:

i $\cos\alpha = -\cos(180° - \alpha) = -\cos(180° + \alpha) = \cos(360° - \alpha)$

ii $\tan\alpha = -\tan(180° - \alpha) = \tan(180° + \alpha) = -\tan(360° - \alpha)$

18 A series of sand dunes has a cross-section which can be modelled using a sine curve of the form $y = \sin(60x)°$ where x is the length of the series of dunes in metres.

a Draw the graph of $y = \sin(60x)°$ for $0 \leqslant x \leqslant 24°$. **(3 marks)**

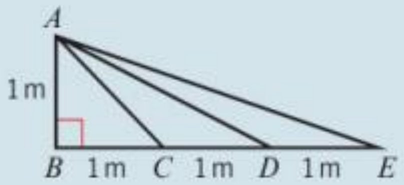
b Write down the number of sand dunes in this model. **(1 mark)**

c Give one reason why this may not be a realistic model. **(1 mark)**

Chapter Exercises

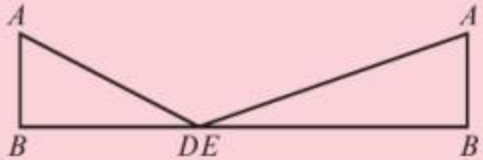
Challenge

In this diagram $AB = BC = CD = DE = 1$ m.



Prove that $\angle AEB + \angle ADB = \angle ACB$.

Hint Try drawing triangles ADB and AEB back to back.



Chapter Answers

1 **a** $155°$ **b** 13.7 cm

2 **a** $x = 49.5°$, area $= 1.37\text{ cm}^2$

b $x = 55.2°$, area $= 10.6\text{ cm}^2$

c $x = 117°$, area $= 6.66\text{ cm}^2$

3 6.50 cm^2

4 **a** 36.1 cm^2 **b** 12.0 cm^2

5 **a** 5 **b** $\dfrac{25\sqrt{3}}{2}\text{ cm}^2$

6 area $= \frac{1}{2}ab\sin C$

$1 = \frac{1}{2} \times 2\sqrt{2}\sin C$

$\dfrac{1}{\sqrt{2}} = \sin C \Rightarrow C = 45°$

Use the cosine rule to find the other side:

$x^2 - 2^2 + (\sqrt{2})^2 - 2 \times 2\sqrt{2}\cos C \Rightarrow x = \sqrt{2}\text{ cm}$

So the triangle is isosceles, with two $45°$ angles, thus is also right-angled.

7 **a** $AC = \sqrt{5}$, $AB = \sqrt{18}$, $BC = \sqrt{5}$

$$\cos\angle ACB = \frac{AC^2 + BC^2 - AB^2}{2 \times AC \times BC}$$

$$= \frac{5 + 5 - 18}{2 \times \sqrt{5} \times \sqrt{5}}$$

$$= -\frac{8}{10} = -\frac{4}{5}$$

b $1\frac{1}{2}\text{ cm}^2$

8 **a** 4 **b** $\dfrac{15\sqrt{3}}{4}(6.50)\text{ cm}^2$

9 **a** 1.50 km **b** $241°$ **c** 0.789 km^2

10 359 m^2

11 35.2 m

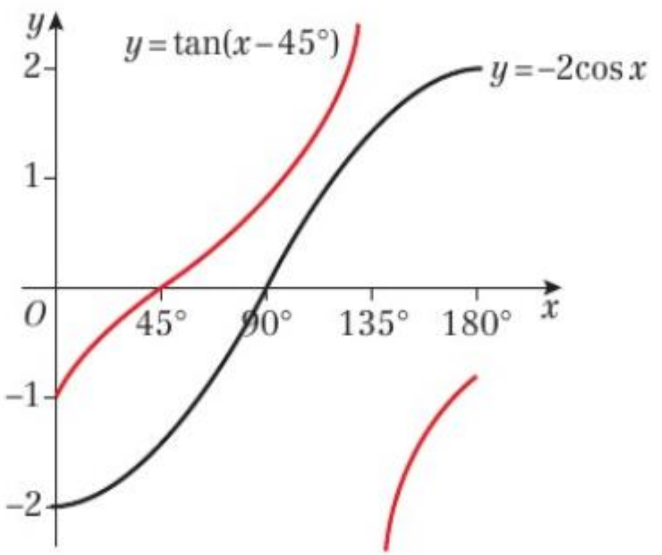
12 **a** A stretch of scale factor 2 in the x direction.

b A translation of $+3$ in the y direction.

c A reflection in the x-axis.

d A translation of -20 in the x direction.

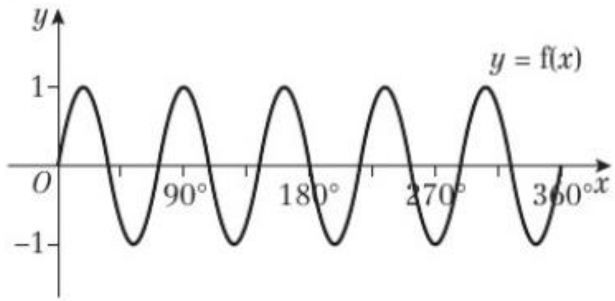
13 **a**



b There are no solutions.

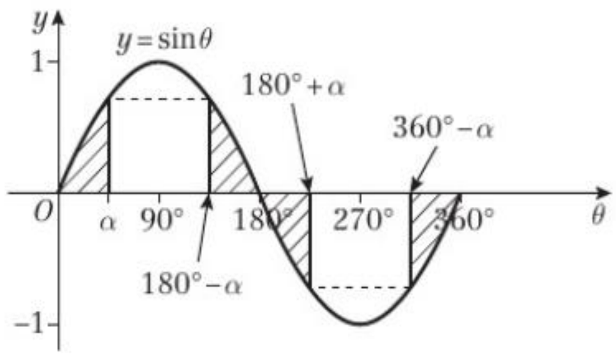
14 **a** 300 **b** $(30, 1)$ **c** 60 **d** $\dfrac{\sqrt{3}}{2}$

15 a $p = 5$



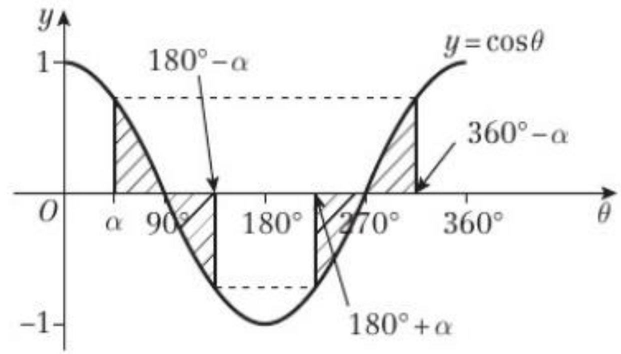
b 72°

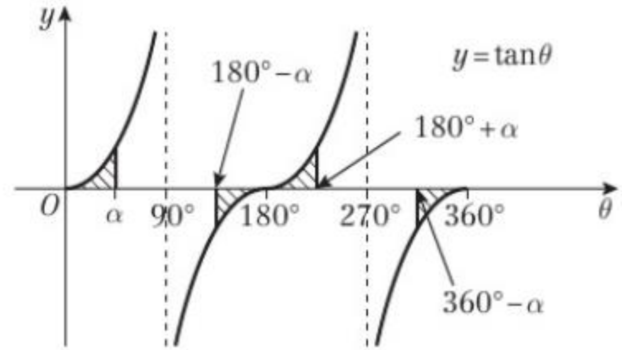
16 a The four shaded regions are congruent.



b $\sin\alpha$ and $\sin(180° - \alpha)$ have the same y value, (call it k)
so $\sin\alpha = \sin(180° - \alpha)$
$\sin(180° - \alpha)$ and $\sin(360° - \alpha)$ have the same y value, (which will be $-k$)
so $\sin\alpha = \sin(180° - \alpha)$
$= -\sin(180° + \alpha)$
$= -\sin(360° - \alpha)$

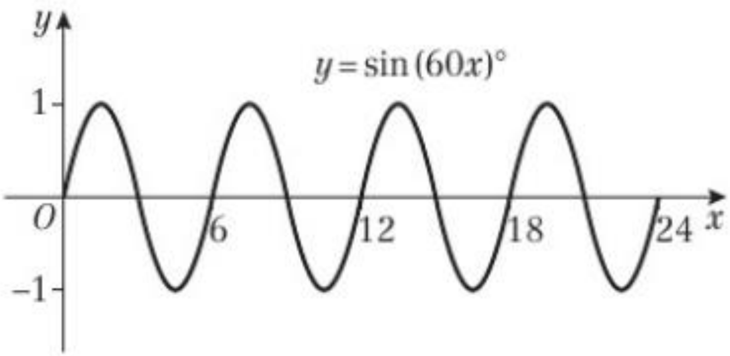
17 a





b i From the graph of $y = \cos\theta$, which shows four congruent shaded regions, if the y value at α is k, then y at $180° - \alpha$ is $-k$, y at $180° - \alpha = -k$ and y at $360° - \alpha = +k$
so $\cos\alpha = -\cos(180° - \alpha)$
$= -\cos(180° + \alpha)$
$= \cos(360° - \alpha)$

ii From the graph of $y = \tan\theta$, if the y value at α is k, then at $180° - \alpha$ it is $-k$, at $180° + \alpha$ it is $+k$ and at $360° - \alpha$ it is $-k$,
so $\tan\alpha = -\tan(180° - \alpha)$
$= +\tan(180° + \alpha)$
$= -\tan(360° - \alpha)$

18 a 

b 4

c The dunes may not all be the same height.

Challenge

Using the sine rule:

$$\sin(180° - \angle ADB - \angle AEB) = \frac{5\left(\frac{1}{\sqrt{5}}\right)}{\sqrt{10}} = \frac{1}{\sqrt{2}}$$

$180° - \angle ADB - \angle AEB = 135°$ (obtuse)

so $\angle ADB + \angle B = 45° = \angle ACB$