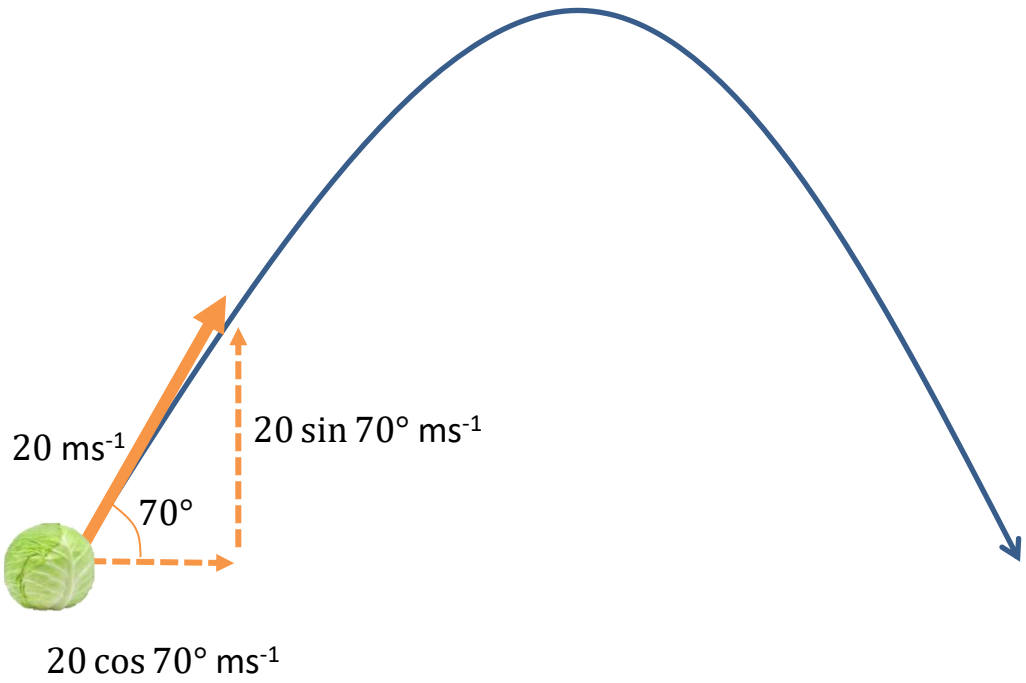
M2 Chapter 6: Projectiles

Horizontal Vertical Components

Components of velocity

Just as **we split forces into its horizontal and vertical components**, in order to consider forces in the horizontal and vertical directions respectively, we can do **exactly the same with velocity**!



When the object is at its highest point:
The vertical velocity is 0.

We know that the scalar form of velocity is **speed**, and thus we just find the **magnitude** of the velocity vector:

$$\begin{pmatrix} 12 \\ 5 \end{pmatrix} ms^{-1}$$

$$\Rightarrow \sqrt{\mathbf{12^2 + 5^2}}$$

$$= \mathbf{13\ ms^{-1}}$$

The Standard Example

[Textbook] A particle P is projected from a point O on a horizontal plane with speed 28 ms^{-1} and with angle of elevation $30°$. After projection, the particle moves freely under gravity until it strikes the plane at a point A.
Find:

(a) the greatest height above the plane reached by P
(b) the time of flight of P
(c) the distance OA

a

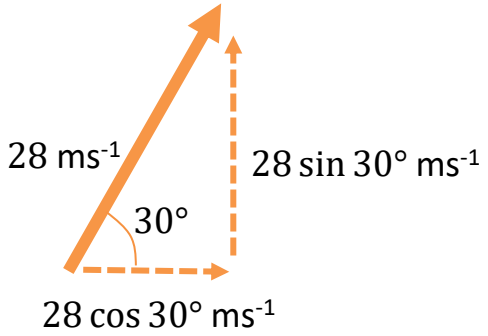
?

b

?

c

?



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a

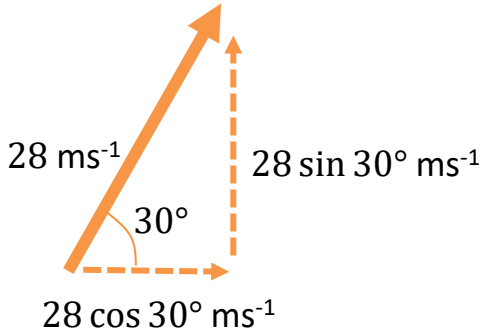
$R(\uparrow)$: $s = ?, u = 28\sin 30°, v = 0, a = -9.8, t =$ ~~—~~

$v^2 = u^2 + 2as$

$0 = 14^2 + 2 \times (-9.8) \times s$

$s = 10$

Greatest height is 10 m.



b

$R(\uparrow)$: $s = 0, u = 28\sin 30°,$ ~~$v =$ —~~ $, a = -9.8, t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 14t - 4.9t^2 = t(14 - 4.9t)$

$t = 0 \text{ or } \frac{14}{4.9} = 2.857\ldots$

Vertically the particle has returned to its original position, so displacement is 0.

Time of flight is 2.9 s (2sf)

c

$28\cos 30° \times 2.857\ldots = 69.282\ldots = 69$ m (2sf)

Exercise 6.2

Pearson Stats/Mechanics Year 2
Pages 50-51

Homework Exercise

In this exercise **i** and **j** are unit vectors acting in a vertical plane, horizontally and vertically respectively.

Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ unless otherwise stated.

1 A particle is projected from a point on a horizontal plane with an initial velocity of $25\,\text{m s}^{-1}$ at an angle of $40°$ above the horizontal.

a Find the horizontal and vertical components of the initial velocity.

b Express the initial velocity as a vector in the form $p\mathbf{i} + q\mathbf{j}\,\text{m s}^{-1}$.

2 A particle is projected from a cliff top with an initial velocity of $18\,\text{m s}^{-1}$ at an angle of $20°$ below the horizontal.

a Find the horizontal and vertical components of the initial velocity.

b Express the initial velocity as a vector in the form $p\mathbf{i} + q\mathbf{j}\,\text{m s}^{-1}$.

3 A particle is projected from a point on level ground with an initial velocity of $35\,\text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$.

a Find the horizontal and vertical components of the initial velocity.

b Express the initial velocity as a vector in terms of **i** and **j**.

4 A particle is projected from the top of a building with an initial velocity of $28\,\text{m s}^{-1}$ at an angle θ below the horizontal, where $\tan\theta = \frac{7}{24}$.

a Find the horizontal and vertical components of the initial velocity.

b Express the initial velocity as a vector in terms of **i** and **j**.

Homework Exercise

5 A particle is projected with initial velocity $\mathbf{U} = (6\mathbf{i} + 9\mathbf{j})\,\text{m s}^{-1}$. Find the initial speed of the particle and its angle of projection.

6 A particle is projected with initial velocity $\mathbf{U} = (4\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-1}$. Find the initial speed of the particle and its angle of projection.

7 A particle is projected with initial velocity $\mathbf{U} = 3k\mathbf{i} + 2k\mathbf{j}\,\text{m s}^{-1}$.

a Find the angle of projection.

Given the initial speed is $3\sqrt{13}\,\text{m s}^{-1}$,

b find the value of k.

8 A particle is projected with speed $35\,\text{m s}^{-1}$ at an angle of elevation of $60°$. Find the time the particle takes to reach its greatest height.

9 A ball is projected from a point 5 m above horizontal ground with speed $18\,\text{m s}^{-1}$ at an angle of elevation of $40°$. Find the height of the ball above the ground 2 s after projection.

10 A stone is projected from a point above horizontal ground with speed $32\,\text{m s}^{-1}$, at an angle of $10°$ below the horizontal. The stone takes 2.5 s to reach the ground. Find:

a the height of the point of projection above the ground

b the distance from the point on the ground vertically below the point of projection to the point where the stone reaches the ground.

Homework Exercise

11 A projectile is launched from a point on horizontal ground with speed $150\,\text{m s}^{-1}$ at an angle of $10°$ above the horizontal. Find:

a the time the projectile takes to reach its highest point above the ground

b the range of the projectile.

12 A particle is projected from a point O on a horizontal plane with speed $20\,\text{m s}^{-1}$ at an angle of elevation of $45°$. The particle moves freely under gravity until it strikes the ground at a point X. Find:

a the greatest height above the plane reached by the particle

b the distance OX.

Homework Answers

1 **a** $u_x = 19.2\,\text{m s}^{-1}$, $u_y = 16.1\,\text{m s}^{-1}$
b $(19.2\mathbf{i} + 16.1\mathbf{j})\,\text{m s}^{-1}$

2 **a** $u_x = 16.9\,\text{m s}^{-1}$, $u_y = -6.2\,\text{m s}^{-1}$
b $(16.9\mathbf{i} - 6.2\mathbf{j})\,\text{m s}^{-1}$

3 **a** $u_x = 32.3\,\text{m s}^{-1}$, $u_y = 13.5\,\text{m s}^{-1}$
b $(32.3\mathbf{i} + 13.5\mathbf{j})\,\text{m s}^{-1}$

4 **a** $u_x = 26.9\,\text{m s}^{-1}$, $u_y = -7.8\,\text{m s}^{-1}$
b $(26.9\mathbf{i} - 7.8\mathbf{j})\,\text{m s}^{-1}$

5 $10.8\,\text{m s}^{-1}$, $56.3°$

6 $6.4\,\text{m s}^{-1}$, $51.3°$ below the horizontal

7 **a** $33.7°$ **b** $k = 3$ or $k = -3$

8 3.1 (2 s.f.)

9 8.5 m (2 s.f.)

10 **a** 44 m (2 s.f.) **b** 79 m

11 **a** 2.7 s (2 s.f.) **b** 790 m (2 s.f.)

12 **a** 10 m (2 s.f.) **b** 41 m (2 s.f.)