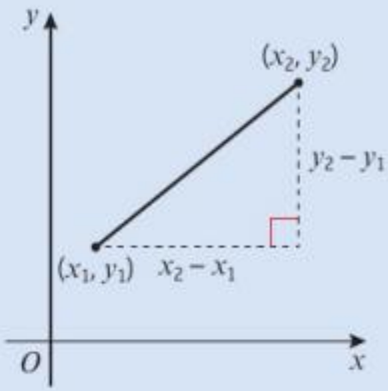
P1 Chapter 5: Linear Graphs

Chapter Practice

Key Points

1. The gradient m of the line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) can be calculated using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



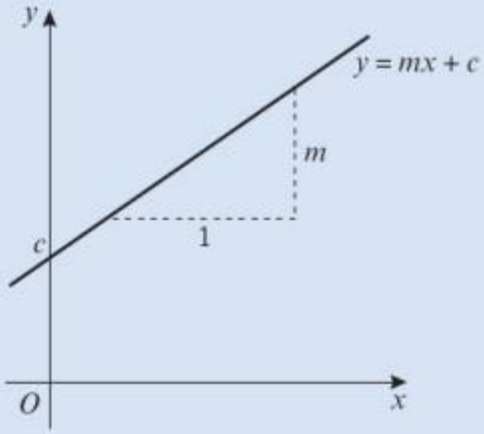
2.
- The equation of a straight line can be written in the form

  $y = mx + c,$

  where m is the gradient and $(0, c)$ is the y-intercept.
- The equation of a straight line can also be written in the form

  $ax + by + c = 0,$

  where a, b and c are integers.



3. The equation of a line with gradient m that passes through the point with coordinates (x_1, y_1) can be written as $y - y_1 = m(x - x_1)$.

4. Parallel lines have the same gradient.

Key Points

5 If a line has a gradient m, a line perpendicular to it has a gradient of $-\frac{1}{m}$

6 If two lines are perpendicular, the product of their gradients is −1.

7 You can find the distance d between (x_1, y_1) and (x_2, y_2) by using the formula
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

8 The point of intersection of two lines can be found using simultaneous equations.

9 Two quantities are in direct proportion when they increase at the same rate.
The graph of these quantities is a straight line through the origin.

10 A mathematical model is an attempt to represent a real-life situation using mathematical concepts. It is often necessary to make assumptions about the real-life problems in order to create a model.

Chapter Exercises

1 The straight line passing through the point $P(2, 1)$ and the point $Q(k, 11)$ has gradient $-\frac{5}{12}$

a Find the equation of the line in terms of x and y only. **(2 marks)**

b Determine the value of k. **(2 marks)**

2 The points A and B have coordinates $(k, 1)$ and $(8, 2k - 1)$ respectively, where k is a constant.
Given that the gradient of AB is $\frac{1}{3}$

a show that $k = 2$ **(2 marks)**

b find an equation for the line through A and B. **(3 marks)**

3 The line L_1 has gradient $\frac{1}{7}$ and passes through the point $A(2, 2)$. The line L_2 has gradient -1 and passes through the point $B(4, 8)$. The lines L_1 and L_2 intersect at the point C.

a Find an equation for L_1 and an equation for L_2. **(4 marks)**

b Determine the coordinates of C. **(2 marks)**

4 **a** Find an equation of the line l which passes through the points $A(1, 0)$ and $B(5, 6)$. **(2 marks)**

The line m with equation $2x + 3y = 15$ meets l at the point C.

b Determine the coordinates of C. **(2 marks)**

5 The line L passes through the points $A(1, 3)$ and $B(-19, -19)$.
Find an equation of L in the form $ax + by + c = 0$. where a, b and c are integers. **(3 marks)**

6 The straight line l_1 passes through the points A and B with coordinats $(2, 2)$ and $(6, 0)$ respectively.

a Find an equation of l_1. **(3 marks)**

The straight line l_2 passes through the point C with coordinate $(-9, 0)$ and has gradient $\frac{1}{4}$.

b Find an equation of l_2. **(2 marks)**

7 The straight line l passes through $A(1, 3\sqrt{3})$ and $B(2 + \sqrt{3}, 3 + 4\sqrt{3})$.

Show that l meets the x-axis at the point $C(-2, 0)$. **(5 marks)**

8 The points A and B have coordinates $(-4, 6)$ and $(2, 8)$ respectively. A line p is drawn through B perpendicular to AB to meet the y-axis at the point C.

a Find an equation of the line p. **(3 marks)**

b Determine the coordinates of C. **(1 mark)**

9 The line l has equation $2x - y - 1 = 0$.

The line m passes through the point $A(0, 4)$ and is perpendicular to the line l.

a Find an equation of m. **(2 marks)**

b Show that the lines l and m intersect at the point $P(2, 3)$. **(2 marks)**

The line n passes through the point $B(3, 0)$ and is parallel to the line m.

c Find the coordinates of the point of intersection of the lines l and n. **(3 marks)**

10 The line l_1 passes through the points A and B with coordinates $(0, -2)$ and $(6, 7)$ respectively.
The line l_2 has equation $x + y = 8$ and cuts the y-axis at the point C.
The line l_1 and l_2 intersect at D.
Find the area of triangle ACD. **(6 marks)**

11 The points A and B have coordinates $(2, 16)$ and $(12, -4)$ respectively.
A straight line l_1 passes through A and B.

a Find an equation for l_1 in the form $ax + by = c$. **(2 marks)**

The line l_2 passes through the point C with coordinates $(-1, 1)$ and has gradient $\frac{1}{3}$

b Find an equation for l_2. **(2 marks)**

12 The points $A(-1, -2)$, $B(7, 2)$ and $C(k, 4)$, where k is a constant, are the vertices of $\triangle ABC$. Angle ABC is a right angle.

a Find the gradient of AB. **(1 mark)**

b Calculate the value of k. **(2 marks)**

c Find an equation of the straight line passing through B and C. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers **(2 marks)**

d Calculate the area of $\triangle ABC$. **(2 marks)**

13 **a** Find an equation of the straight line passing through the points with coordinates $(-1, 5)$ and $(4, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3 marks)**

The line crosses the x-axis at the point A and the y-axis at the point B, and O is the origin.

b Find the area of $\triangle AOB$. **(3 marks)**

14 The straight line l_1 has equation $4y + x = 0$.

The straight line l_2 has equation $y = 2x - 3$.

a On the same axes, sketch the graphs of l_1 and l_2. Show clearly the coordinates of all points at which the graphs meet the coordinate axes. **(2 marks)**

The lines l_1 and l_2 intersect at the point A.

b Calculate, as exact fractions, the coordinates of A. **(2 marks)**

c Find an equation of the line through A which is perpendicular to l_1. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(2 marks)**

15 The points A and B have coordinates (4, 6) and (12, 2) respectively.
The straight line l_1 passes through A and B.

a Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers. **(3 marks)**

The straight line l_2 passes through the origin and has gradient $-\frac{2}{3}$

b Write down an equation for l_2. **(1 mark)**

The lines l_1 and l_2 intersect at the point C.

c Find the coordinates of C. **(2 marks)**

d Show that the lines OA and OC are perpendicular, where O is the origin. **(2 marks)**

e Work out the lengths of OA and OC. Write your answers in the form $k\sqrt{13}$. **(2 marks)**

f Hence calculate the area of $\triangle OAC$. **(2 marks)**

16 **a** Use the distance formula to find the distance between $(4a, a)$ and $(-3a, 2a)$.

Hence find the distance between the following pairs of points:

b (4, 1) and (−3, 2) **c** (12, 3) and (−9, 6) **d** (−20, −5) and (15, −10)

17 A is the point (−1, 5). Let (x, y) be any point on the line $y = 3x$.

a Write an equation in terms of x for the distance between (x, y) and $A(-1, 5)$. **(3 marks)**

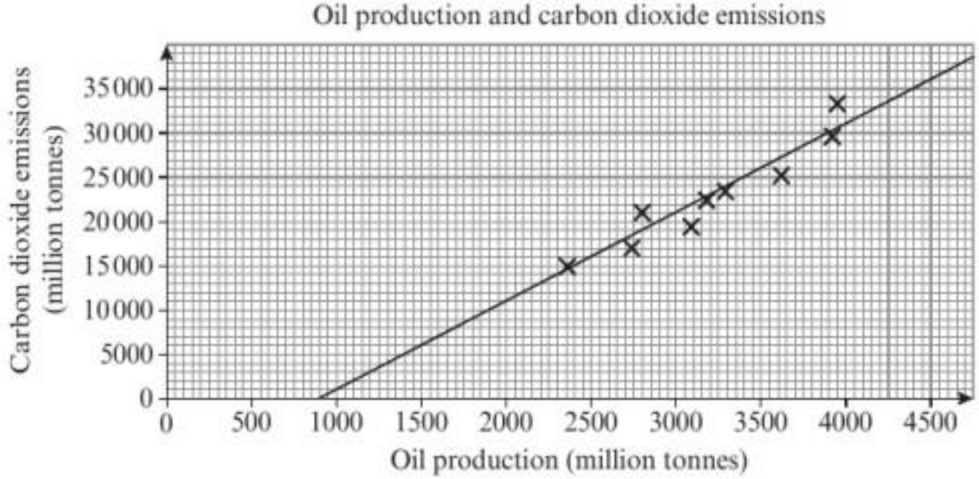
b Find the coordinates of the two points, B and C, on the line $y = 3x$ which are a distance of $\sqrt{74}$ from (−1, 5). **(3 marks)**

c Find the equation of the line l_1 that is perpendicular to $y = 3x$ and goes through the point (−1, 5). **(2 marks)**

d Find the coordinates of the point of intersection between l_1 and $y = 3x$. **(2 marks)**

e Find the area of triangle ABC. **(2 marks)**

18 The scatter graph shows the oil production P and carbon dioxide emissions C for various years since 1970. A line of best fit has been added to the scatter graph.



a Use two points on the line to calculate its gradient. **(1 mark)**

b Formulate a linear model linking oil production P and carbon dioxide emissions C, giving the relationship in the form $C = aP + b$. **(2 marks)**

c Interpret the value of a in your model. **(1 mark)**

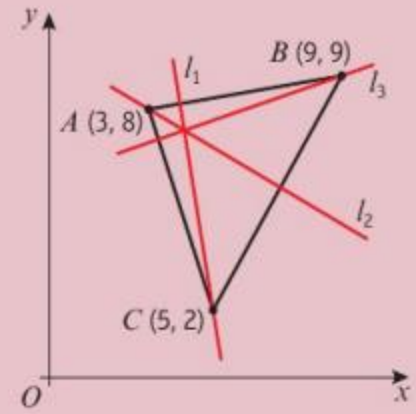
d With reference to your value of b, comment on the validity of the model for small values of P. **(1 mark)**

Chapter Exercises

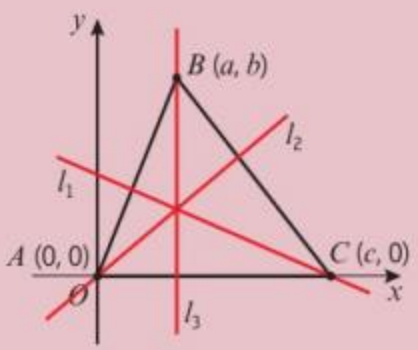
Challenge

1 Find the area of the triangle with vertices $A(-2, -2)$, $B(13, 8)$ and $C(-4, 14)$.

2 A triangle has vertices $A(3, 8)$, $B(9, 9)$ and $C(5, 2)$ as shown in the diagram.
The line l_1 is perpendicular to AB and passes through C.
The line l_2 is perpendicular to BC and passes through A.
The line l_3 is perpendicular to AC and passes through B.
Show that the lines l_1, l_2 and l_3 meet at a point and find the coordinates of that point.



3 A triangle has vertices $A(0, 0)$, $B(a, b)$ and $C(c, 0)$ as shown in the diagram.
The line l_1 is perpendicular to AB and passes through C.
The line l_2 is perpendicular to BC and passes through A.
The line l_3 is perpendicular to AC and passes through B.
Find the coordinates of the point of intersection of l_1, l_2 and l_3.



Chapter Answers

1 **a** $y = -\frac{5}{12}x + \frac{11}{6}$ **b** -22

2 **a** $\frac{2k-2}{8-k} = \frac{1}{3}$ therefore $7k = 14$, $k = 2$

b $y = \frac{1}{3}x + \frac{1}{3}$

3 **a** $L_1 = y = \frac{1}{7}x + \frac{12}{7}$, $L_2 = y = -x + 12$

b $(9, 3)$

4 **a** $y = \frac{3}{2}x - \frac{3}{2}$ **b** $(3, 3)$

5 $11x - 10y + 19 = 0$

6 **a** $y = -\frac{1}{2}x + 3$ **b** $y = \frac{1}{4}x + \frac{9}{4}$

7 Gradient $= \frac{3 + 4\sqrt{3} - 3\sqrt{3}}{2 + \sqrt{3} - 1} = \frac{3 + \sqrt{3}}{1 + \sqrt{3}} = \sqrt{3}$

$y = \sqrt{3}x + c$ and $A(1, 3\sqrt{3})$, so $c = 2\sqrt{3}$

Equation of line is $y = \sqrt{3}x + 2\sqrt{3}$

When $y = 0$, $x = -2$, so the line meets the x-axis at $(-2, 0)$

8 **a** $y = -3x + 14$ **b** $(0, 14)$

9 **a** $y = -\frac{1}{2}x + 4$ **b** Students own work.

c $(1, 1)$. Note: equation of line n: $y = -\frac{1}{2}x + \frac{3}{2}$

10 20

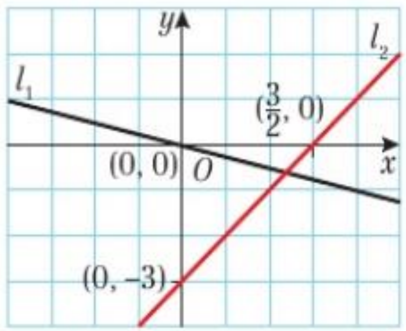
11 **a** $2x + y = 20$ **b** $y = \frac{1}{3}x + \frac{4}{3}$

12 **a** $\frac{1}{2}$ **b** 6 **c** $2x + y - 16 = 0$

d 10

13 **a** $7x + 5y - 18 = 0$ **b** $\frac{162}{35}$

14 **a**



b $\left(\frac{4}{3}, -\frac{1}{3}\right)$ **c** $12x - 3y - 17 = 0$

15 **a** $x + 2y - 16 = 0$
b $y = -\frac{2}{3}x$
c $C(-48, 32)$
d Slope of OA is $\frac{3}{2}$. Slope of OC is $-\frac{2}{3}$. Lines are perpendicular.
e $OA = 2\sqrt{13}$ and $OC = 16\sqrt{13}$
f Area = 208

16 **a** $d = \sqrt{50a^2} = 5a\sqrt{2}$ **b** $5\sqrt{2}$
c $15\sqrt{2}$ **d** $25\sqrt{2}$

17 **a** $d = \sqrt{10x^2 - 28x + 26}$
b $B\left(-\frac{6}{5}, -\frac{18}{5}\right)$ and $C(4, 12)$
c $y = -\frac{1}{3}x + \frac{14}{5}$
d $\left(\frac{7}{5}, \frac{21}{5}\right)$
e 20.8

18 **a** gradient = 10.5
b $C = 10.5P - 10751$
c When the oil production increases by 1 million tonnes, the carbon dioxide emissions increase by 10.5 million tonnes.
d The model is not valid for small values of P, as it is not possible to have a negative amount of carbon dioxide emissions. It is always dangerous to extrapolate beyond the range on the model in this way.

Challenge

1 130

2 $\left(\frac{78}{19}, \frac{140}{19}\right)$

3 $\left(a, \frac{a(c-a)}{b}\right)$