M1 Chapter 10: Forces and Motion

Connected Particles

Starter

5. A particle P of mass 0.5 kg is moving under the action of a single force $(3\mathbf{i} - 2\mathbf{j})$ N.

(a) Show that the magnitude of the acceleration of P is $2\sqrt{13}$ m s^{-2}.

(4)

At time $t = 0$, the velocity of P is $(\mathbf{i} + 3\mathbf{j})$ m s^{-1}.

(b) Find the velocity of P at time $t = 2$ seconds.

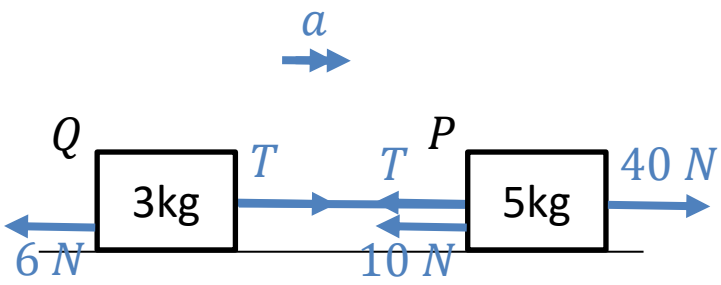
(3)

Connected Particles

General approach: create a free body force diagram for each part separately.
If their connection is light and inextensible this means the tension T in the connection is equal everywhere and opposite at the two ends.
If (and only if) the objects move in the same direction, you can treat the connected bodies as a single object to save a little algebra (what happens to T ? how does this differ from the general approach?).

[Textbook] Two particles, P and Q, of masses 5kg and 3kg respectively, are connected by a light inextensible string. Particle P is pulled by a horizontal force of magnitude 40N along a rough horizontal plane. Particle P experiences a frictional force of 10N and particle Q experiences a frictional force of 6N.

(a) Find the acceleration of the particles.
(b) Find the tension in the string.
(c) Explain how the modelling assumptions that the string is light and inextensible have been used.



Key Point 1: The tension in a given piece of string is the same in any part of the string (at a given time). The tension acts away from each particle in the direction of the string.

Key Point 2: Recall that N is used for the normal support force, acting perpendicular to the plane. The forces vertically are all balance so do not affect the motion and we can ignore them.

a If we considered the two particles as one, we can ignore the tensions (they cancel as they are in opposite directions):

$\boldsymbol{R(\rightarrow): 40 - 10 - 6 = 8a}$

$\boldsymbol{8a = 24}$

$\boldsymbol{a = 3\ ms^{-2}}$

If we considered them as separate particles (not recommended):

At Q, $R(\rightarrow)$: $T - 6 = 3a$

At P, $R(\rightarrow)$: $40 - T - 10 = 5a$

Adding the equations to eliminate T: $24 = 8a$ as before.

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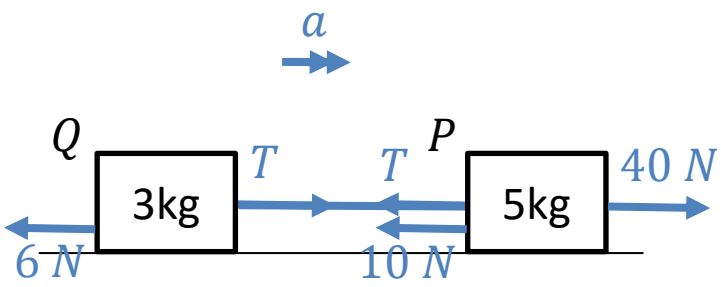

b

?

c

?

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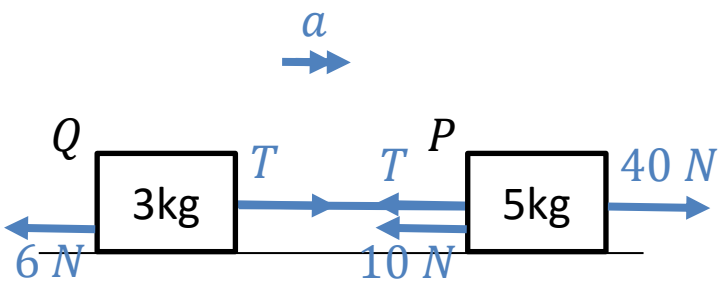

b For P: $R(\rightarrow)$: $40 - T - 10 = 5 \times 3$

$T = 15$ N

c Inextensible ⇒ acceleration of objects the same

Light ⇒ tension is the same throughout the length of the string and the mass of the string is negligible

Classwork Exercise 10.6

Pearson Stats/Mechanics Year 1 Exercise Book
Pages 72-73
Questions 1-3 (not 4) 5-9

Homework Exercise

1 Two particles P and Q of masses 8 kg and 2 kg respectively, are connected by a light inextensible string. The particles are on a smooth horizontal plane. A horizontal force of magnitude F is applied to P in a direction away from Q and when the string is taut the particles move with acceleration $0.4\,\text{m s}^{-2}$.

a Find the value of F.

b Find the tension in the string.

c Explain how the modelling assumptions that the string is light and inextensible are used.

2 Two particles P and Q of masses 20 kg and m kg are connected by a light inextensible rod. The particles lie on a smooth horizontal plane. A horizontal force of 60 N is applied to Q in a direction towards P, causing the particles to move with acceleration $2\,\text{m s}^{-2}$.

a Find the mass, m, of Q.

b Find the thrust in the rod.

Hint For part **b** consider P on its own.

3 Two particles P and Q of masses 7 kg and 8 kg are connected by a light inextensible string. The particles are on a smooth horizontal plane. A horizontal force of 30 N is applied to Q in a direction away from P. When the string is taut the particles move with acceleration, $a\,\text{m s}^{-2}$.

a Find the acceleration, a, of the system.

b Find the tension in the string.

Homework Exercise

4 Two boxes A and B of masses 110 kg and 190 kg sit on the floor of a lift of mass 1700 kg. Box A rests on top of box B. The lift is supported by a light inextensible cable and is descending with constant acceleration $1.8\,\text{m s}^{-2}$.

a Find the tension in the cable.

b Find the force exerted by box B

i on box A **ii** on the floor of the lift.

5 A lorry of mass m kg is towing a trailer of mass $3m$ kg along a straight horizontal road. The lorry and trailer are connected by a light inextensible tow-bar. The lorry exerts a driving force of 50 000 N causing the lorry and trailer to accelerate at $5\,\text{m s}^{-2}$. The lorry and trailer experience resistances of 4000 N and 10 000 N respectively.

a Find the mass of the lorry and hence the mass of the trailer.

b Find the tension in the tow-bar.

c Explain how the modelling assumptions that the tow-bar is light and inextensible affect your calculations.

6 Two particles A and B of masses 10 kg and 5 kg respectively are connected by a light inextensible string. Particle B hangs directly below particle A. A force of 180 N is applied vertically upwards causing the particles to accelerate.

a Find the magnitude of the acceleration. **(3 marks)**

b Find the tension in the string. **(2 marks)**

Homework Exercise

7 Two particles A and B of masses 6 kg and m kg respectively are connected by a light inextensible string. Particle B hangs directly below particle A. A force of 118 N is applied vertically upwards causing the particles to accelerate at $2\,\text{m s}^{-2}$.

a Find the mass, m, of particle B. **(3 marks)**

b Find the tension in the string. **(2 marks)**

8 A train engine of mass 6400 kg is pulling a carriage of mass 1600 kg along a straight horizontal railway track. The engine is connected to the carriage by a shunt which is parallel to the direction of motion of the coupling. The shunt is modelled as a light rod. The engine provides a constant driving force of 12 000 N. The resistances to the motion of the engine and the carriage are modelled as constant forces of magnitude R N and 2000 N respectively.

Given that the acceleration of the engine and the carriage is $0.5\,\text{m s}^{-2}$:

a find the value of R **(3 marks)**

b show that the tension in the shunt is 2800 N. **(2 marks)**

9 A car of mass 900 kg pulls a trailer of mass 300 kg along a straight horizontal road using a light tow-bar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes 200 N and 100 N respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude 1200 N.

a Show that the acceleration of the car and trailer is $0.75\,\text{m s}^{-2}$. **(2 marks)**

b Find the magnitude of the tension in the tow-bar. **(3 marks)**

The car is moving along the road when the driver sees a set of traffic lights have turned red. He reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the car of magnitude F newtons and the car and trailer decelerate.

c Given that the resistances to motion are unchanged and the magnitude of the thrust in the towbar is 100 N, find the value of F. **(7 marks)**

Homework Answers

1 a 4 N

b 0.8 N

c Light ⇒ tension is the same throughout the length of the string and the mass of the string does not need to be considered. Inextensible ⇒ acceleration of masses is the same.

2 a 10 kg **b** 40 N

3 a $2\,m\,s^{-2}$ **b** 14 N

4 a 16 000 N

b i 880 N upwards

ii 2400 N downwards

5 a 1800 kg and 5400 kg

b 37 000 N

c Light ⇒ tension is the same throughout the length of the tow-bar and the mass of the tow-bar does not need to be considered. Inextensible ⇒ acceleration of lorry and trailer is the same.

6 a $2.2\,ms^{-2}$ **b** 60 N

7 a 4 kg **b** 47.2 N

8 a 6000 N

b For engine, $F = ma = 3200$ N

R(→) $12\,000 - 6000 - T = 3200$, $T = 2800$ N

9 a R(→) $1200 - 100 - 200 = 900$ N

$F = ma$, so $a = 900 \div (300 + 900) = 0.75\,m\,s^{-2}$

b 325 N **c** 500 N