P1 Chapter 12: Differentiation

Chord Approximations

Gradients of Curved Graphs

a) For axes $0 \leq x \leq 5$ and $0 \leq y \leq 20$ plot:

$$y = x^2$$

b) From your plot, by using your best judgement to drawing a tangent, estimate the gradient at $x = 1$.

c) Can we do another estimate that doesn't involve human judgement (something a computer could do, and iterate to make better)?

?

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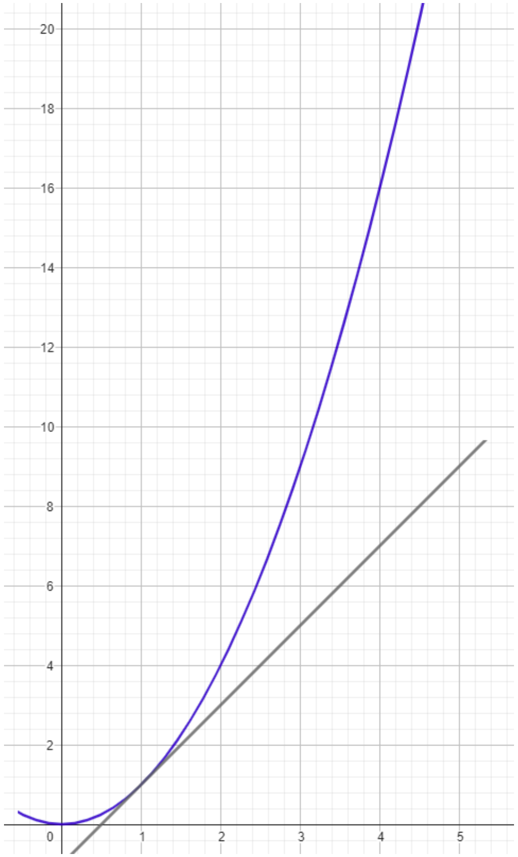


b) From your plot, by using your best judgement to drawing a tangent, estimate the gradient at $x = 1$.

c) Can we do another estimate that doesn't involve human judgement (something a computer could do, and iterate to make better)?

Gradients of Improved Chords

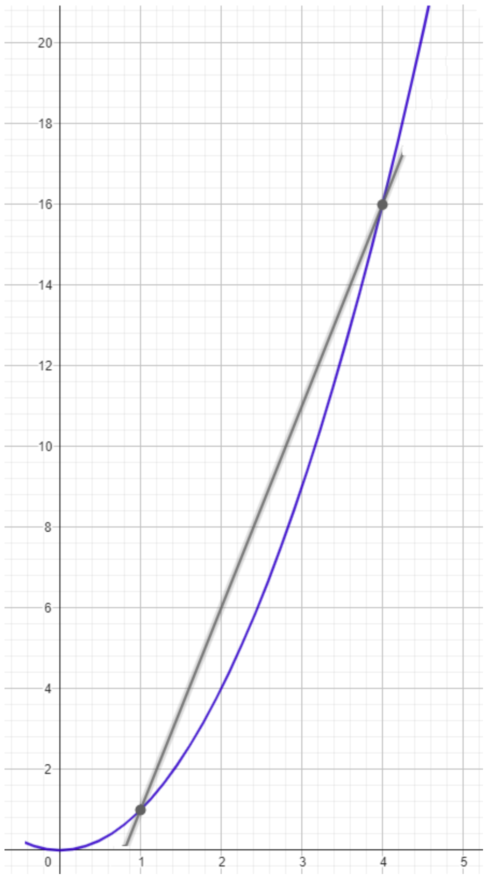
a) Draw a chord from $(1, 1)$ to $(4, 16)$ and calculate the gradient of this chord.

b) How close (in %) is the gradient of this chord to your estimate?

c) Draw a chord from $(1, 1)$ to $(3, 9)$ and calculate the gradient of that chord.

d) How close (in %) is the gradient of that chord to your estimate?

e) Is the second chord an improved estimate over the first?



Chords of x^2

a) Repeat the process by halving the run Δx each time.

b) Copy and complete the table using calculated values for rise $\Delta y = y - 1$ and run $\Delta x = x - 1$.

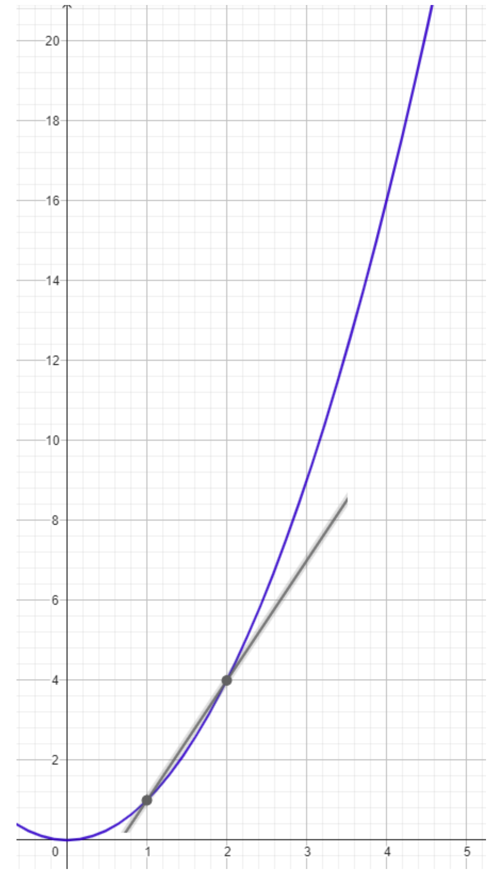
x	y	run	rise	gradient
4	16	3	15	5
3				
2				
1.5				
1.25				

?

?

c) What do you notice about the gradient?

d) What *is* the gradient to 1 decimal place?



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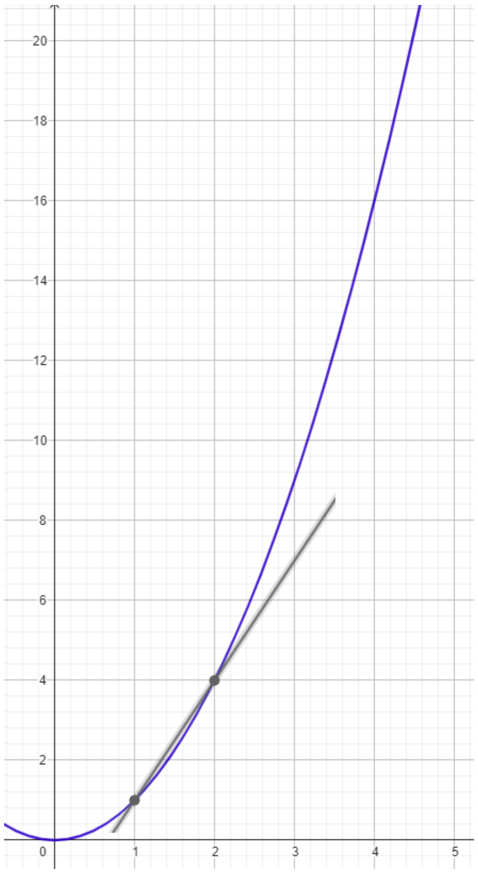
x	y	run	rise	gradient
4	16	3	15	5
3	9	2	8	4
2	4	1	3	3
1.5	2.25	0.5	1.25	2.5
1.25	1.5625	0.25	0.5625	2.25
1.125	1.265625	0.125	0.265625	2.125
1.0625	1.128906	0.0625	0.128906	2.0625
1.03125	1.063477	0.03125	0.063477	2.03125
1.015625	1.031494	0.015625	0.031494	2.015625



c) What do you notice about the gradient?

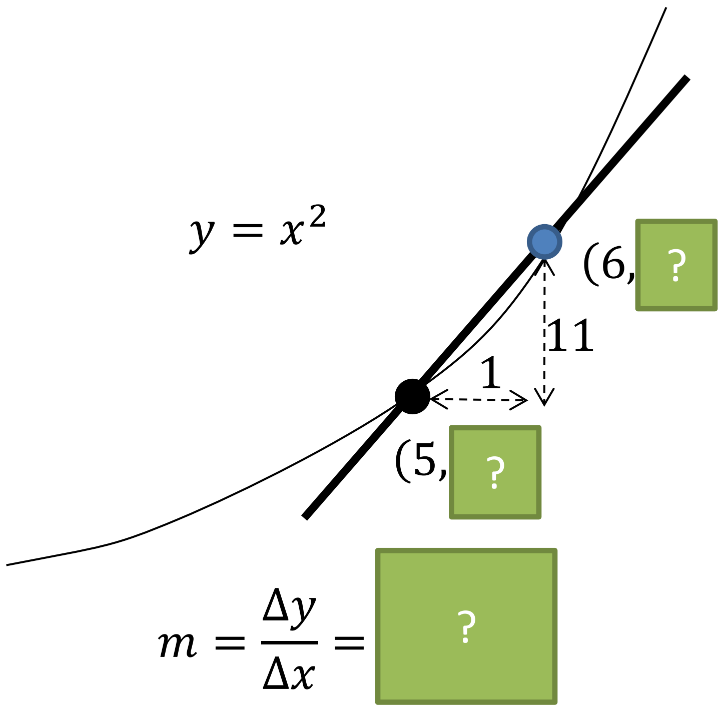
d) What *is* the gradient to 1 decimal place?

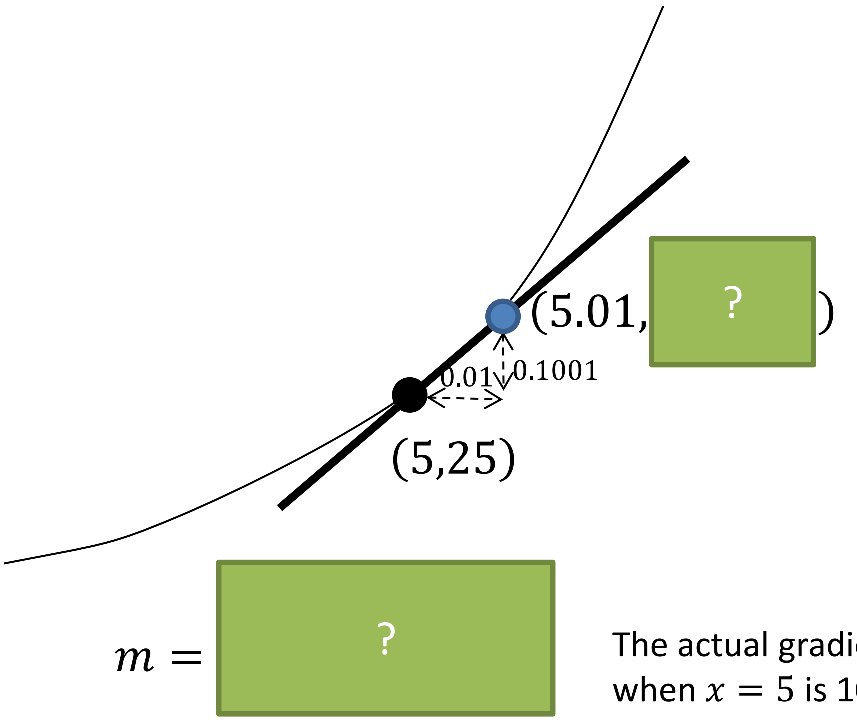
Approximating Gradients with Chords

The question is then: Is there a method to work out the gradient function without having to draw lots of tangents and hoping that we can spot the rule?

To approximate the gradient on the curve $y = x^2$ when $x = 5$, we could pick a point on the curve just slightly to the right, then find the gradient between the two points:

As the second point gets closer and closer, the gradient becomes a better approximation of the true gradient:





The actual gradient when $x = 5$ is 10, so this approximation is damn close!

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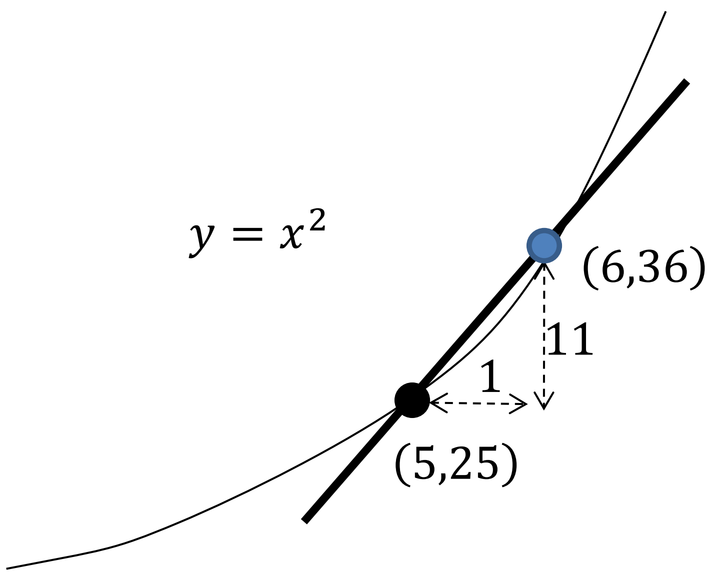
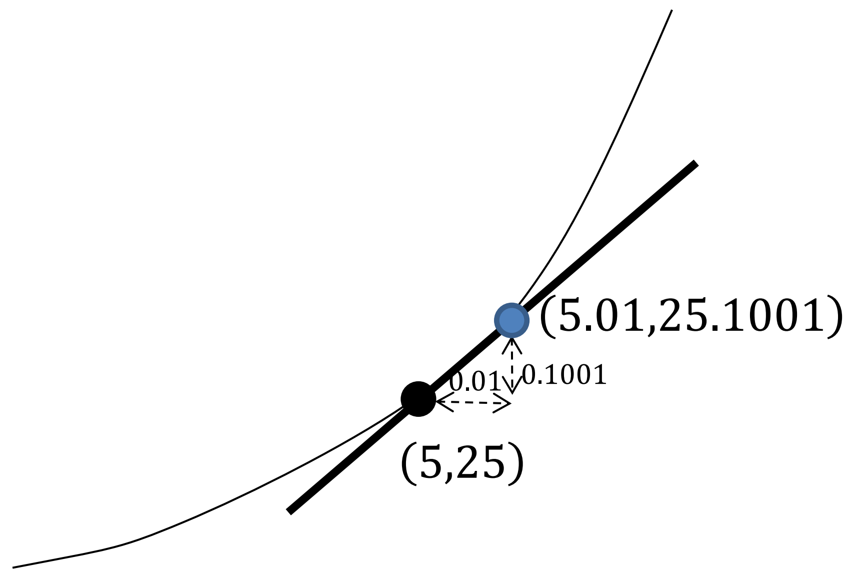


$$m = \frac{\Delta y}{\Delta x} = \frac{11}{1} = 11$$

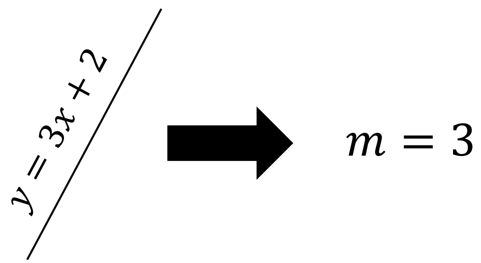


$$m = \frac{0.1001}{0.01} = 10.01$$

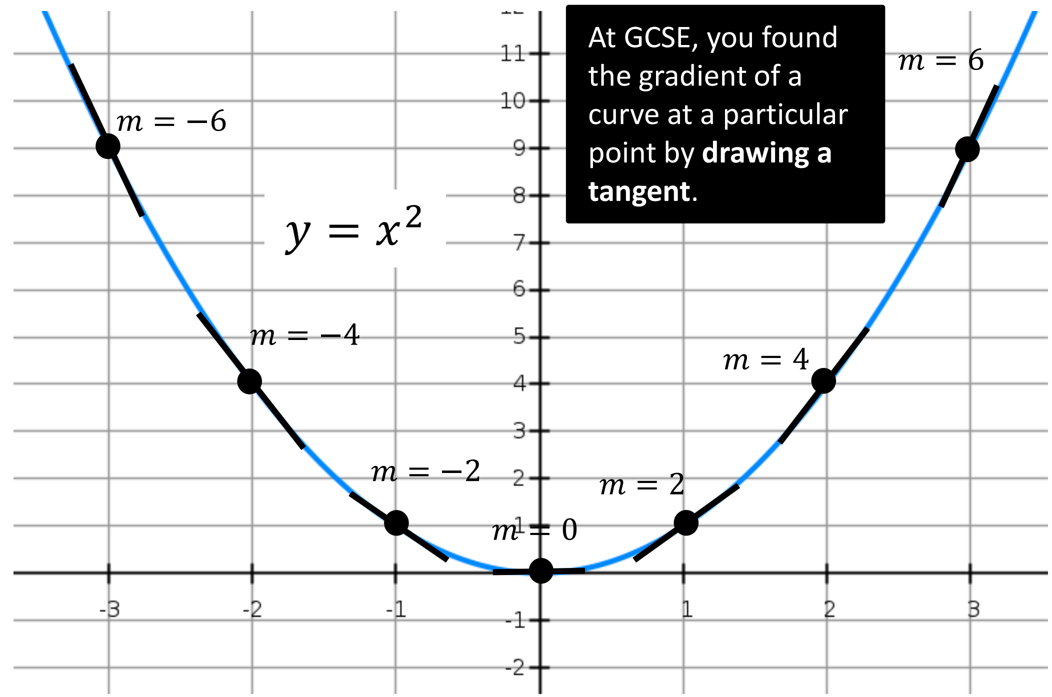
The actual gradient when $x = 5$ is 10, so this approximation is damn close!

Gradient Function

For a straight line, the gradient is **constant**:



However, for a curve **the gradient varies**. We can no longer have a single value for the gradient; **we ideally want an expression in terms of x** that gives us the gradient for any value of x (unsurprisingly known as the **<u>gradient function</u>**).



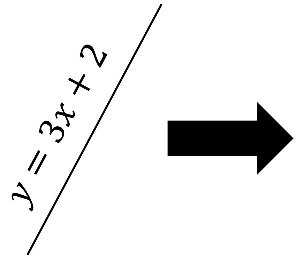
x	-3	-2	-1	0	1	2	3
Gradient	?	?	?	?	?	?	?

By looking at the relationship between x and the gradient at that point, can you come up with an expression, in terms of x for the gradient?

?

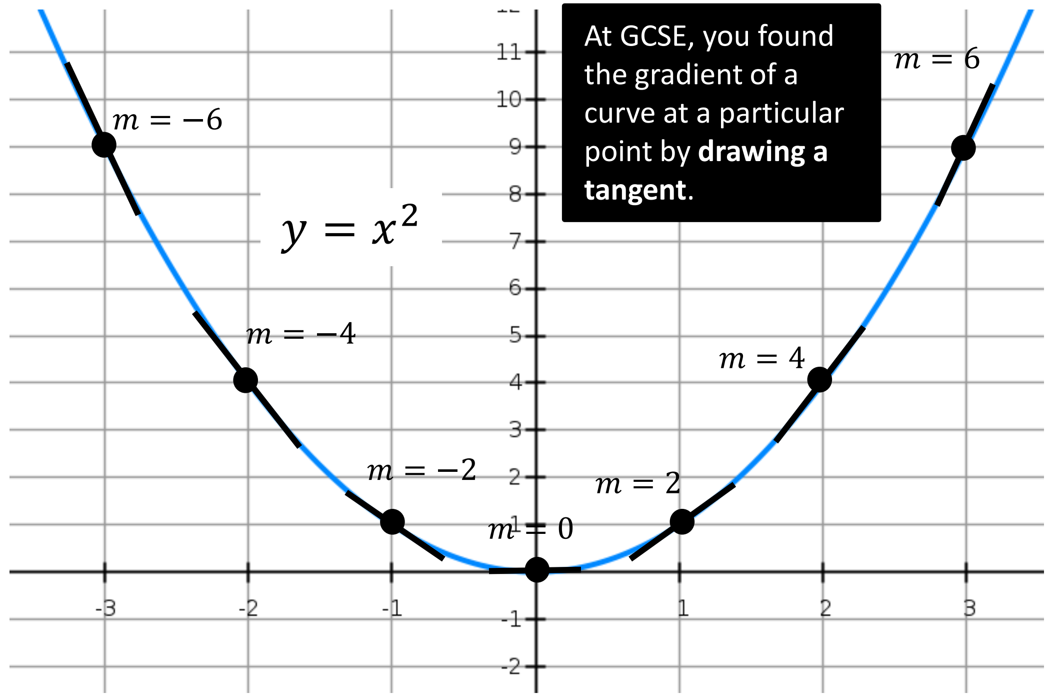
Gradient Function

For a straight line, the gradient is **constant**:



$m = 3$

However, for a curve **the gradient varies**. We can no longer have a single value for the gradient; **we ideally want an expression in terms of x** that gives us the gradient for any value of x (unsurprisingly known as the **<u>gradient function</u>**).



x	-3	-2	-1	0	1	2	3
Gradient	-6	-4	-2	0	2	4	6

By looking at the relationship between x and the gradient at that point, can you come up with an expression, in terms of x for the gradient?

$$Gradient\ Function = \mathbf{2x}$$

Exercise 12.1

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Homework Exercise

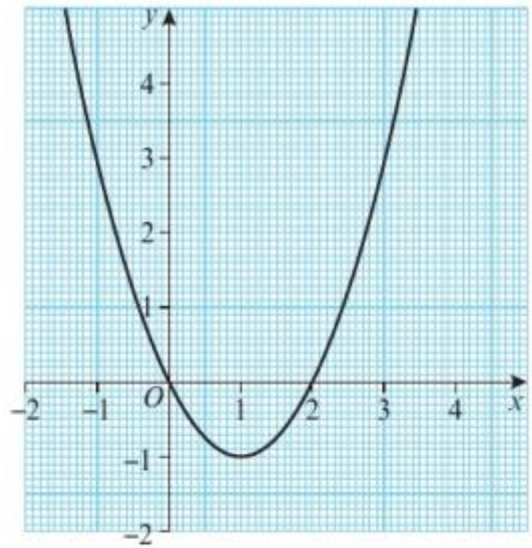
1 The diagram shows the curve with equation $y = x^2 - 2x$.

a Copy and complete this table showing estimates for the gradient of the curve.

x-coordinate	-1	0	1	2	3
Estimate for gradient of curve					

b Write a hypothesis about the gradient of the curve at the point where $x = p$.

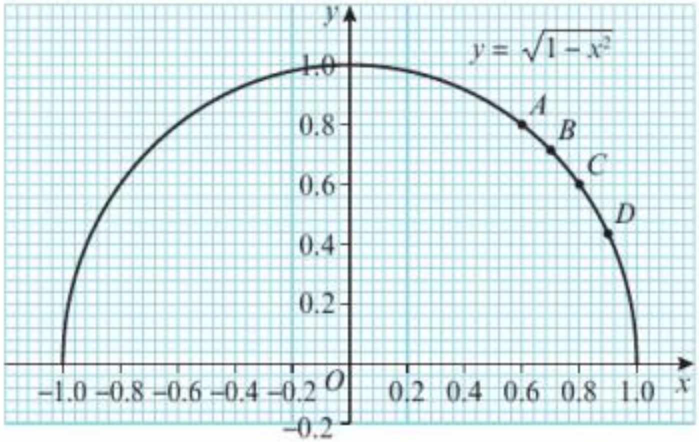
c Test your hypothesis by estimating the gradient of the graph at the point (1.5, -0.75).



Hint Place a ruler on the graph to approximate each tangent.

Homework Exercise

2 The diagram shows the curve with equation $y = \sqrt{1 - x^2}$.
The point A has coordinates (0.6, 0.8).
The points B, C and D lie on the curve with x-coordinates 0.7, 0.8 and 0.9 respectively.



a Verify that point A lies on the curve.

b Use a ruler to estimate the gradient of the curve at point A.

c Find the gradient of the line segments:

i AD

ii AC

iii AB

Hint Use algebra for part **c**.

d Comment on the relationship between your answers to parts **b** and **c**.

Homework Exercise

3 F is the point with coordinates (3, 9) on the curve with equation $y = x^2$.

a Find the gradients of the chords joining the point F to the points with coordinates:

i (4, 16) **ii** (3.5, 12.25) **iii** (3.1, 9.61)

iv (3.01, 9.0601) **v** $(3 + h, (3 + h)^2)$

b What do you deduce about the gradient of the tangent at the point (3, 9)?

4 G is the point with coordinates (4, 16) on the curve with equation $y = x^2$.

a Find the gradients of the chords joining the point G to the points with coordinates:

i (5, 25) **ii** (4.5, 20.25) **iii** (4.1, 16.81)

iv (4.01, 16.0801) **v** $(4 + h, (4 + h)^2)$

b What do you deduce about the gradient of the tangent at the point (4, 16)?

Homework Answers

1 **a**

x-coordinate	−1	0	1	2	3
Estimate for gradient of curve	−4	−2	0	2	4

b Gradient = $2p - 2$ **c** 1

2 **a** $\sqrt{1 - 0.6^2} = \sqrt{0.64} = 0.8$

b Gradient = −0.75

c **i** −1.21 (3 s.f.) **ii** −1 **iii** −0.859 (3 s.f.)

d As other point moves closer to A, gradient tends to −0.75.

3 **a** **i** 7 **ii** 6.5 **iii** 6.1

iv 6.01 **v** $h + 6$

b 6

4 **a** **i** 9 **ii** 8.5 **iii** 8.1

iv 8.01 **v** $8 + h$

b 8