
Stats Yr2 Chapter 2: Probability Theory

Set Notation

Chapter Overview

1:: Set Notation

How sets are used to describe a sample space/event and how notation like $A \cap B$ is used to combine sets.

2:: Conditional Probability in Venn Diagrams

The notation $P(A|B)$ means “the probability of A given that B happened”. How we can find such probabilities using a Venn Diagram.

3:: Formula for Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

4:: Tree Diagrams

“I have 3 red and 4 green balls in a bag. I take one ball out the bag, keep it, then take another. **Given that** the second ball was green, determine the probability the first was red.”

Teacher Notes: All of this is from the old S1. The chapter was effectively split into two: all the non-conditional probability content in Year 1 and the rest in Year 2. Set notation was not used in Year 1.

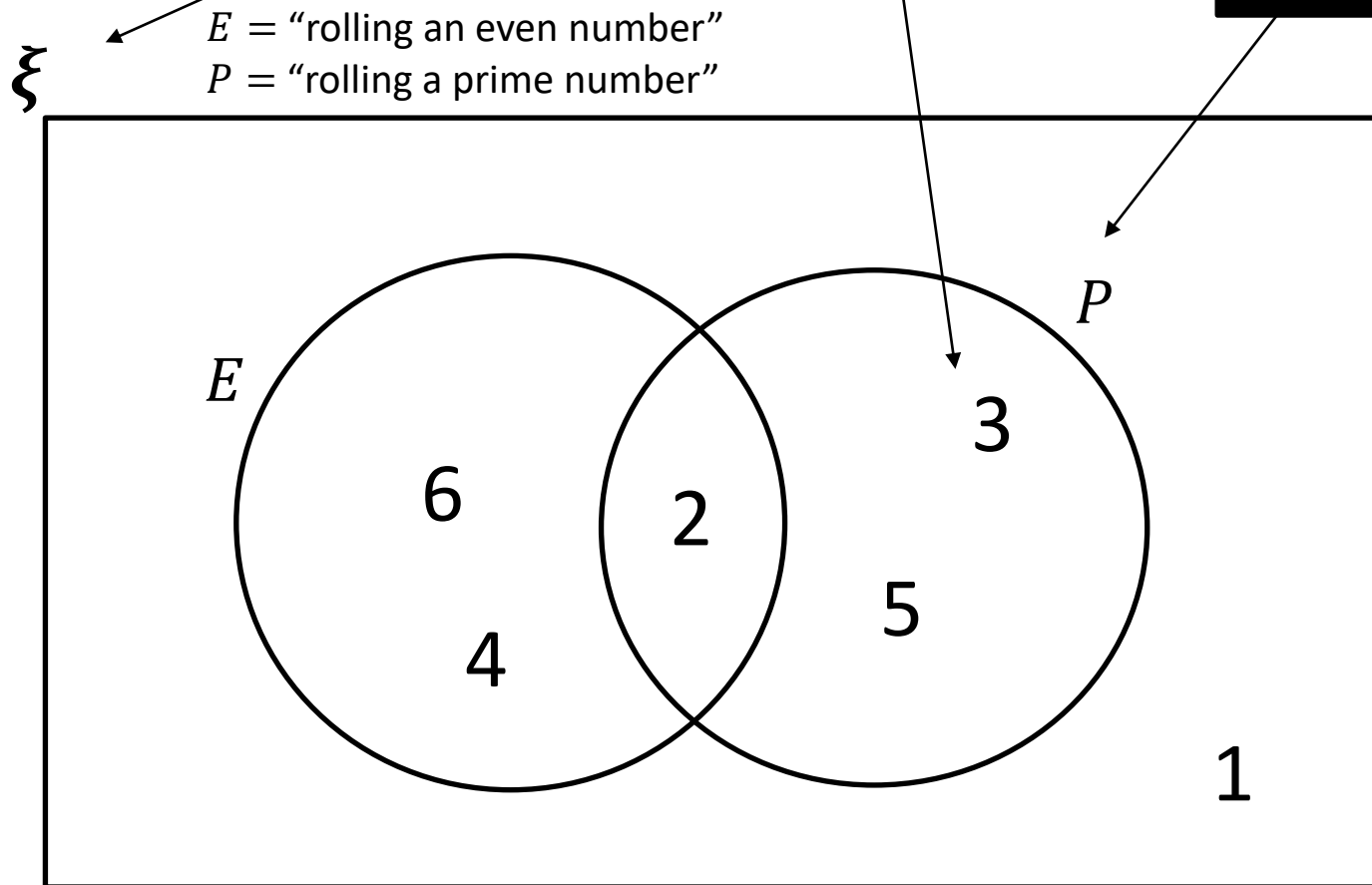
RECAP :: Using sets for sample spaces and events

In general, sets are used to represent **collections of items**.

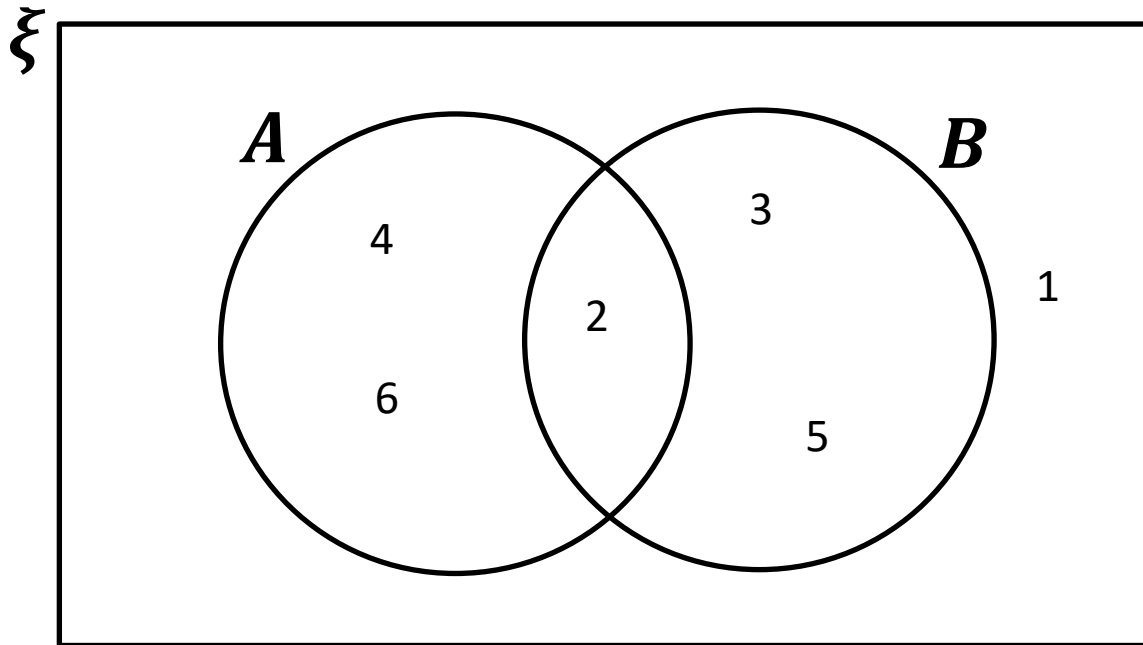
A **sample space** is **set of all possible outcomes**. We use ξ (Greek 'Xi'), or sometimes just S , to represent this set. We use a rectangle in a Venn Diagram.

Each number represents an **outcome**.

In probability, an **event** is a **set of one or more outcomes**. These are the circles in the Venn Diagram. We use capital letters for the variables representing sets.



Combining events/sets



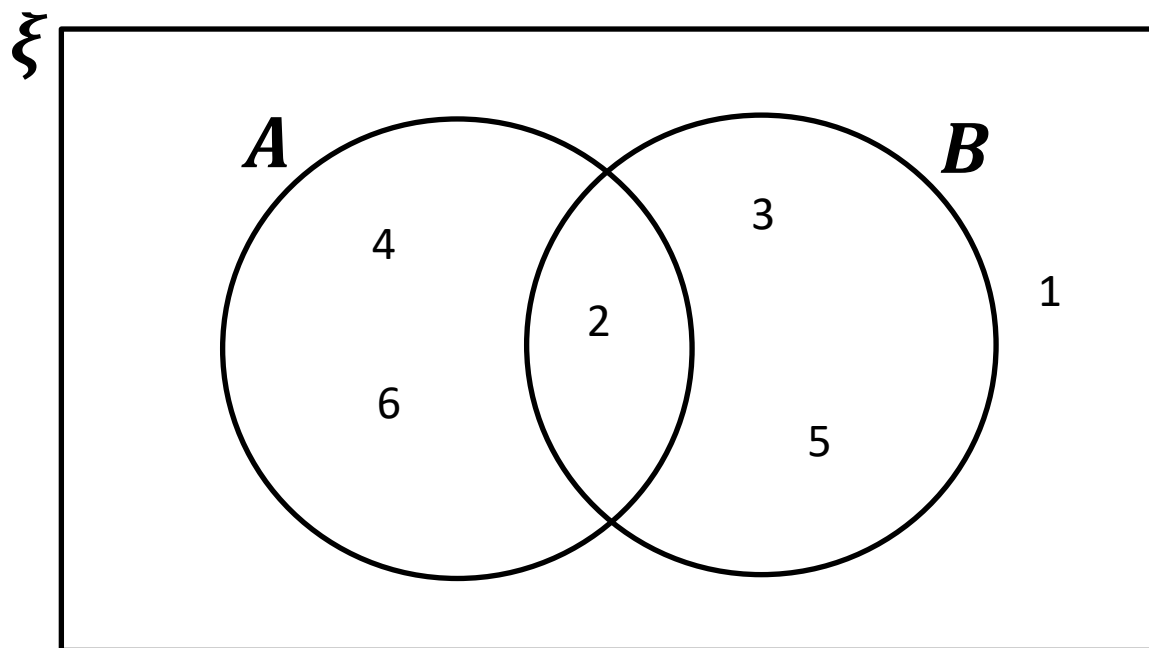
ξ = the whole sample space (1 to 6)

A = even number on a die thrown

B = prime number on a die thrown

	What does it mean in this context?	What is the resulting set of outcomes?
A'	?	?
$A \cup B$?	?
$A \cap B$?	?

Some fundamentals



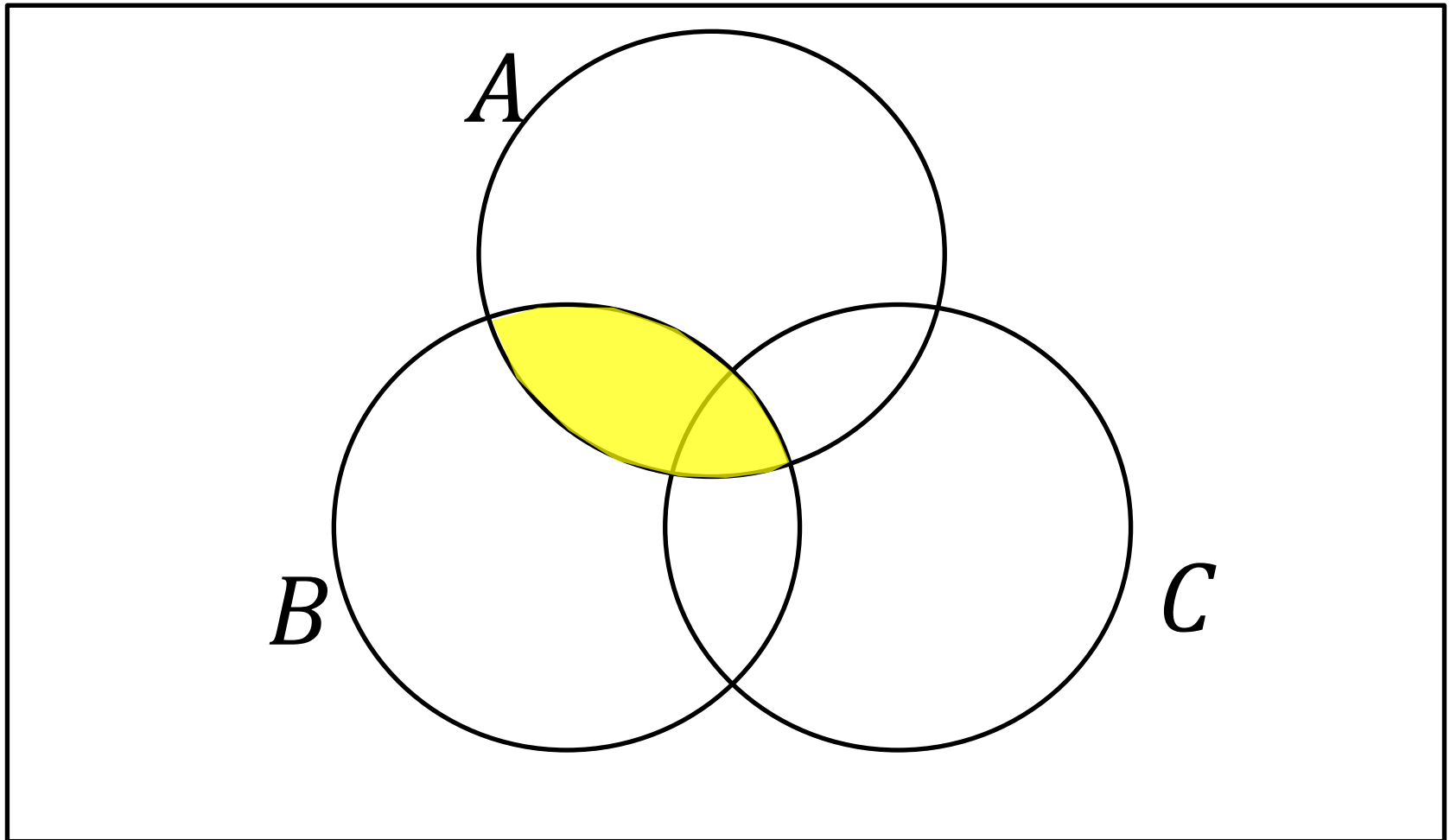
S = the whole sample space (1 to 6)

A = even number on a die thrown

B = prime number on a die thrown

	What does it mean in this context?	What is the resulting set of outcomes?
$A \cap B'$?	?
$(A \cup B)'$?	?
$(A \cap B)'$?	?

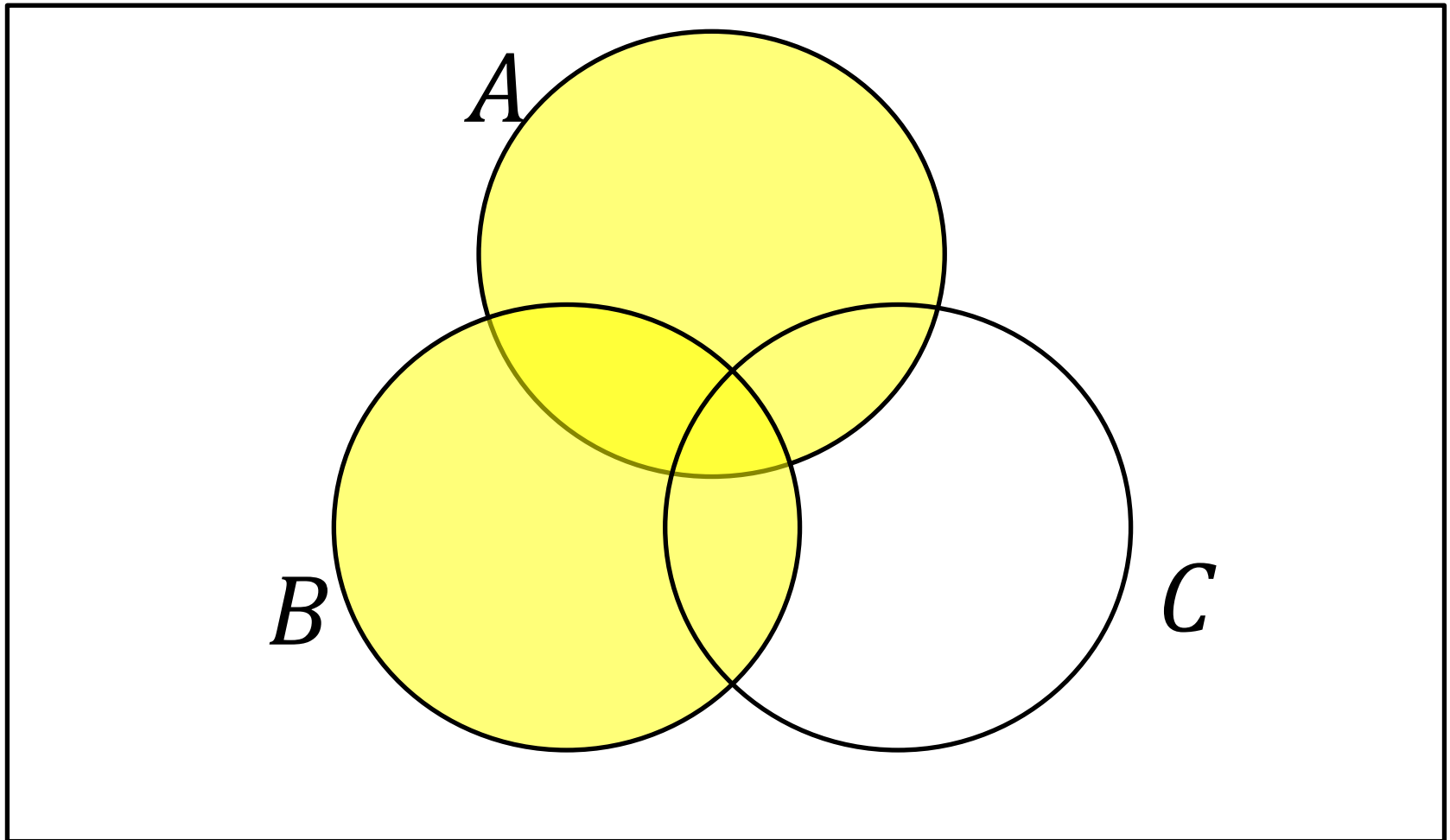
What area is indicated?



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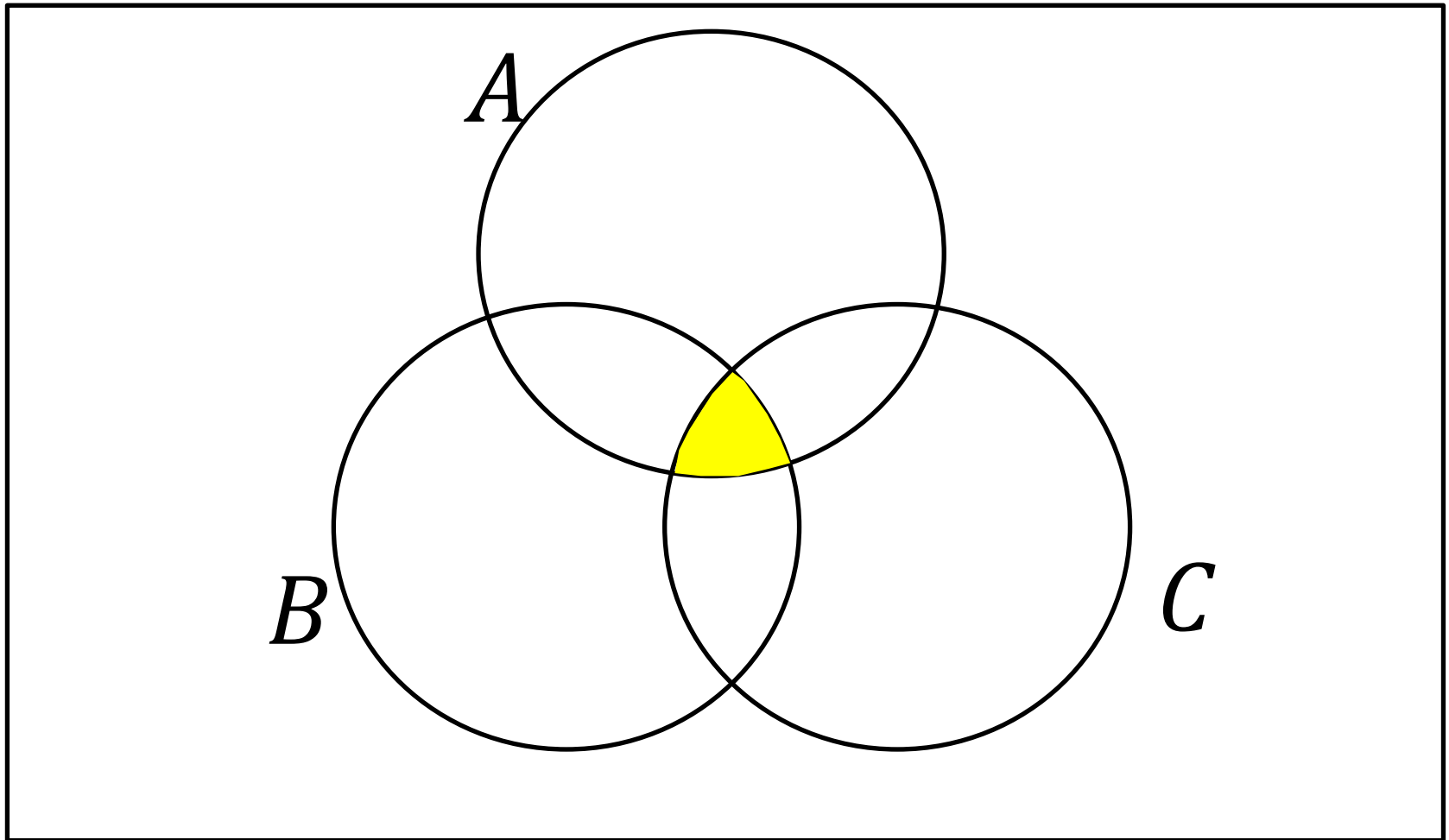
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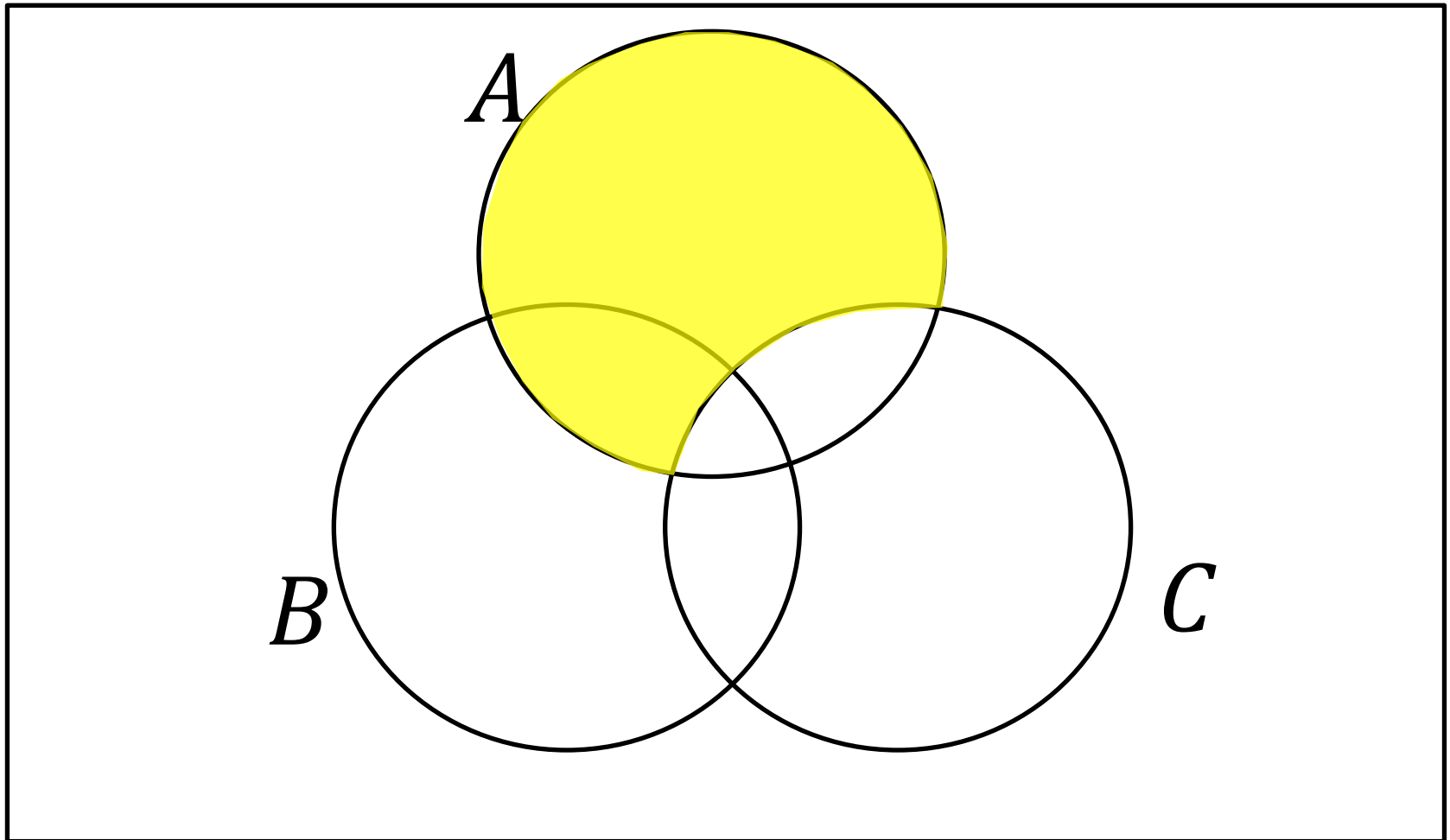
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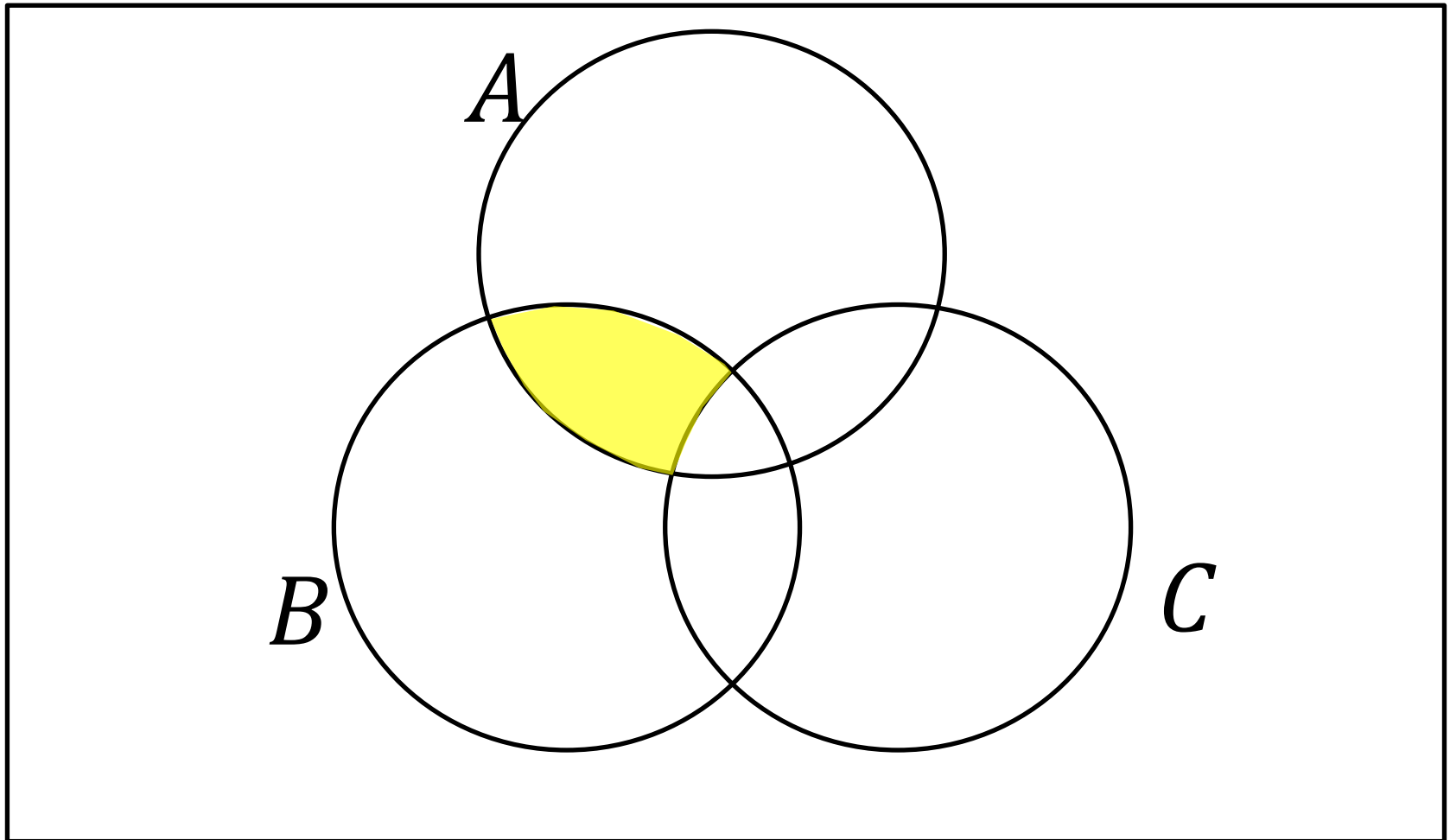
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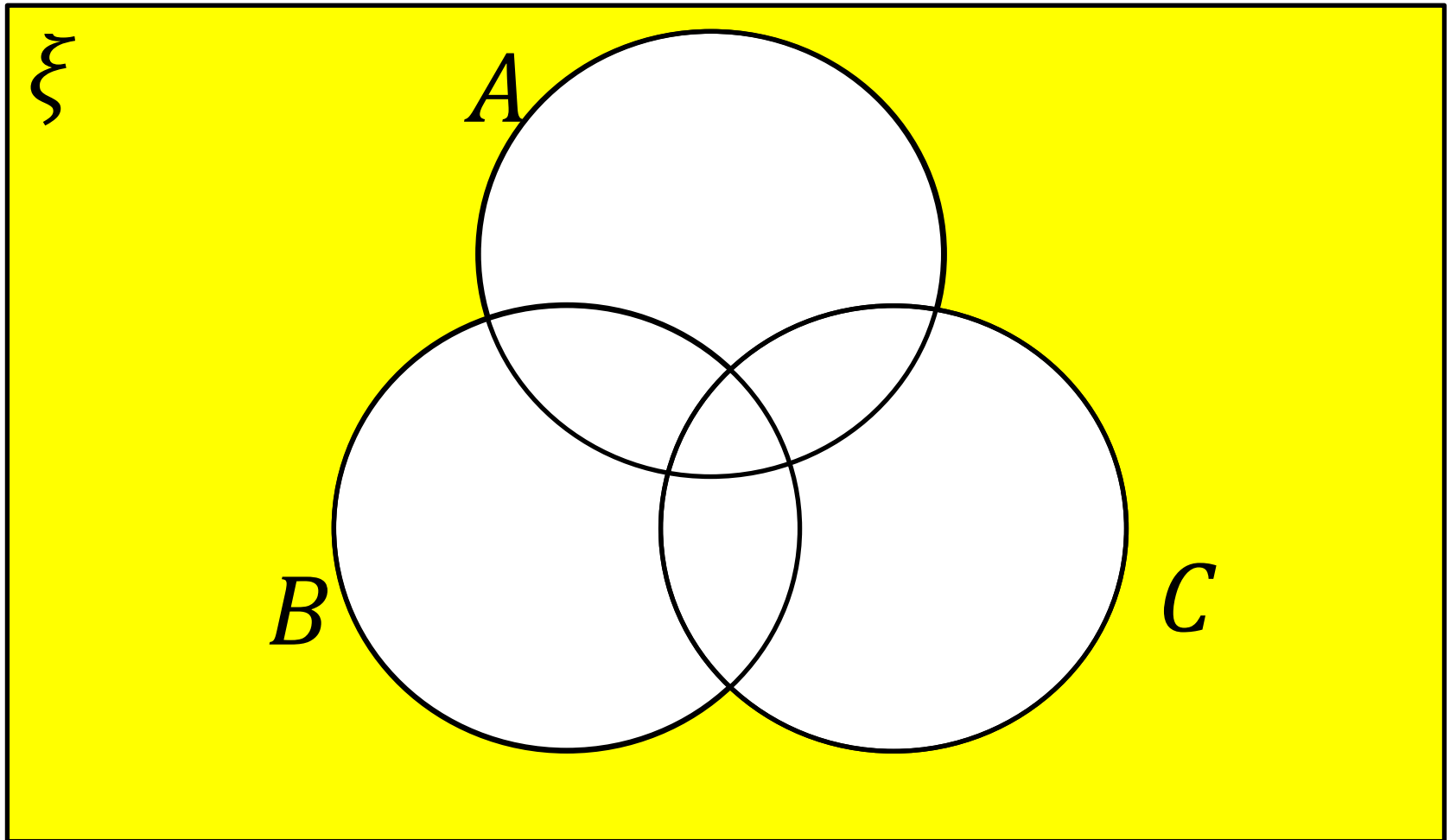
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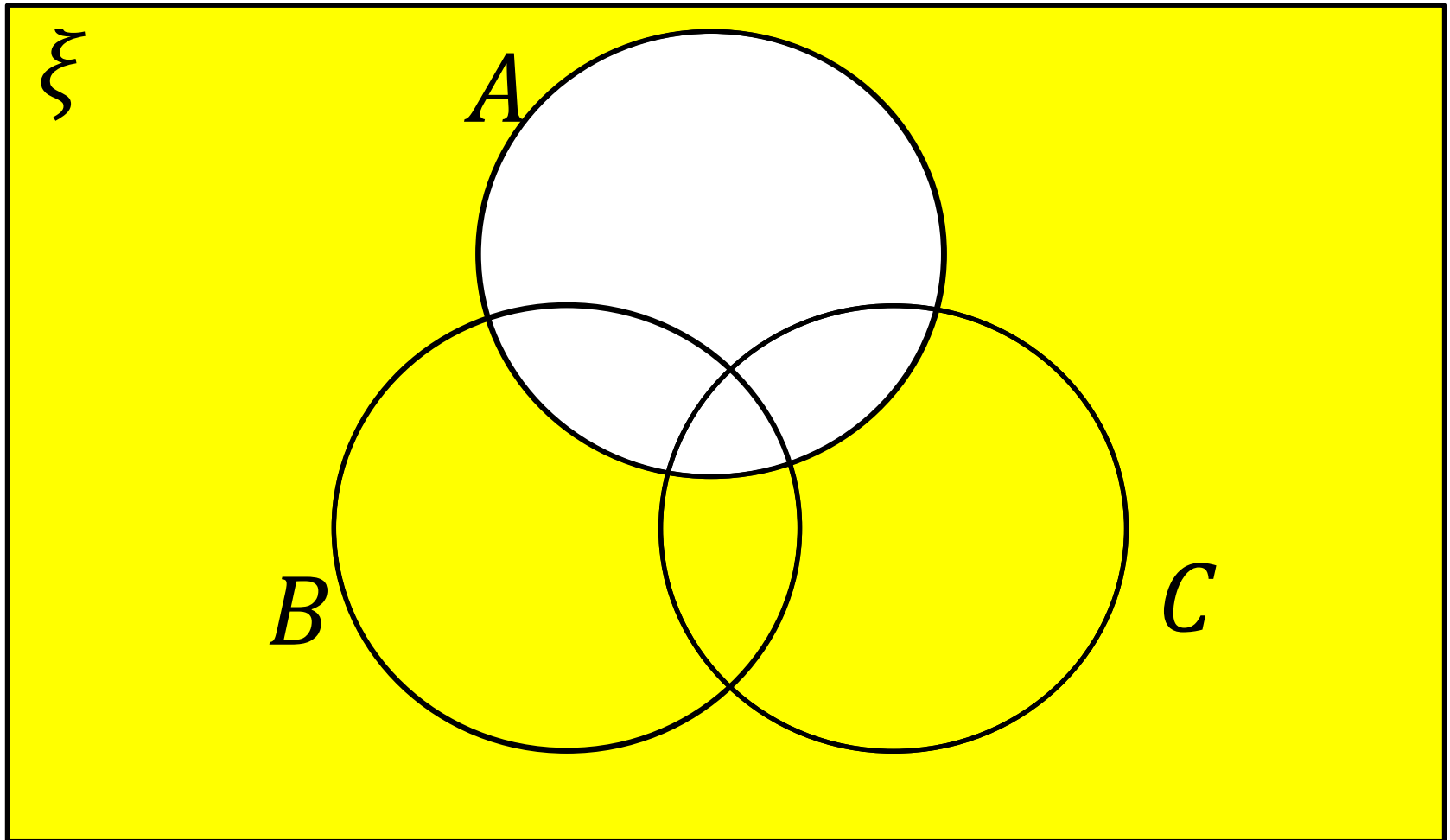


?

or
alternatively...

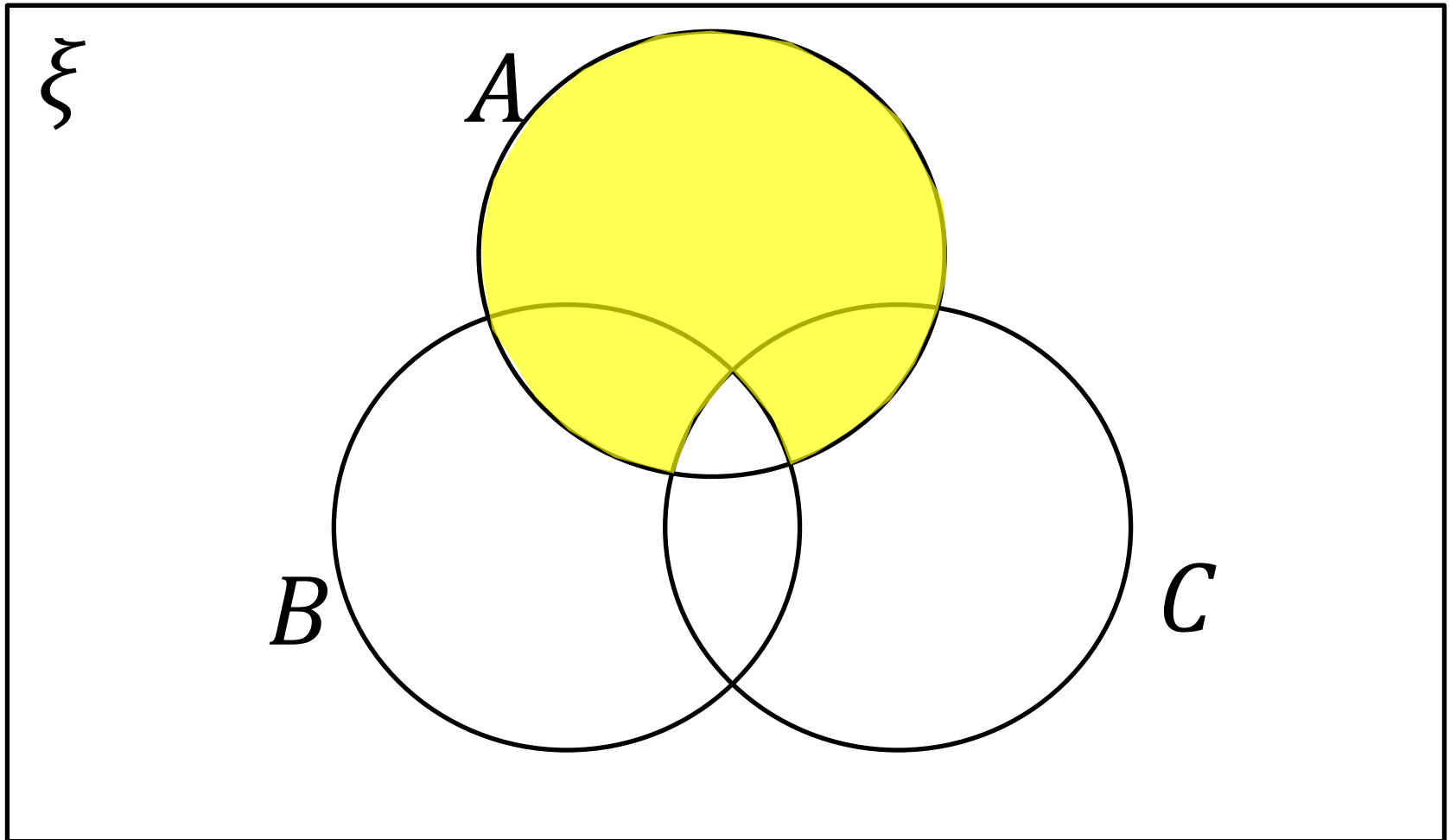
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What area is indicated?



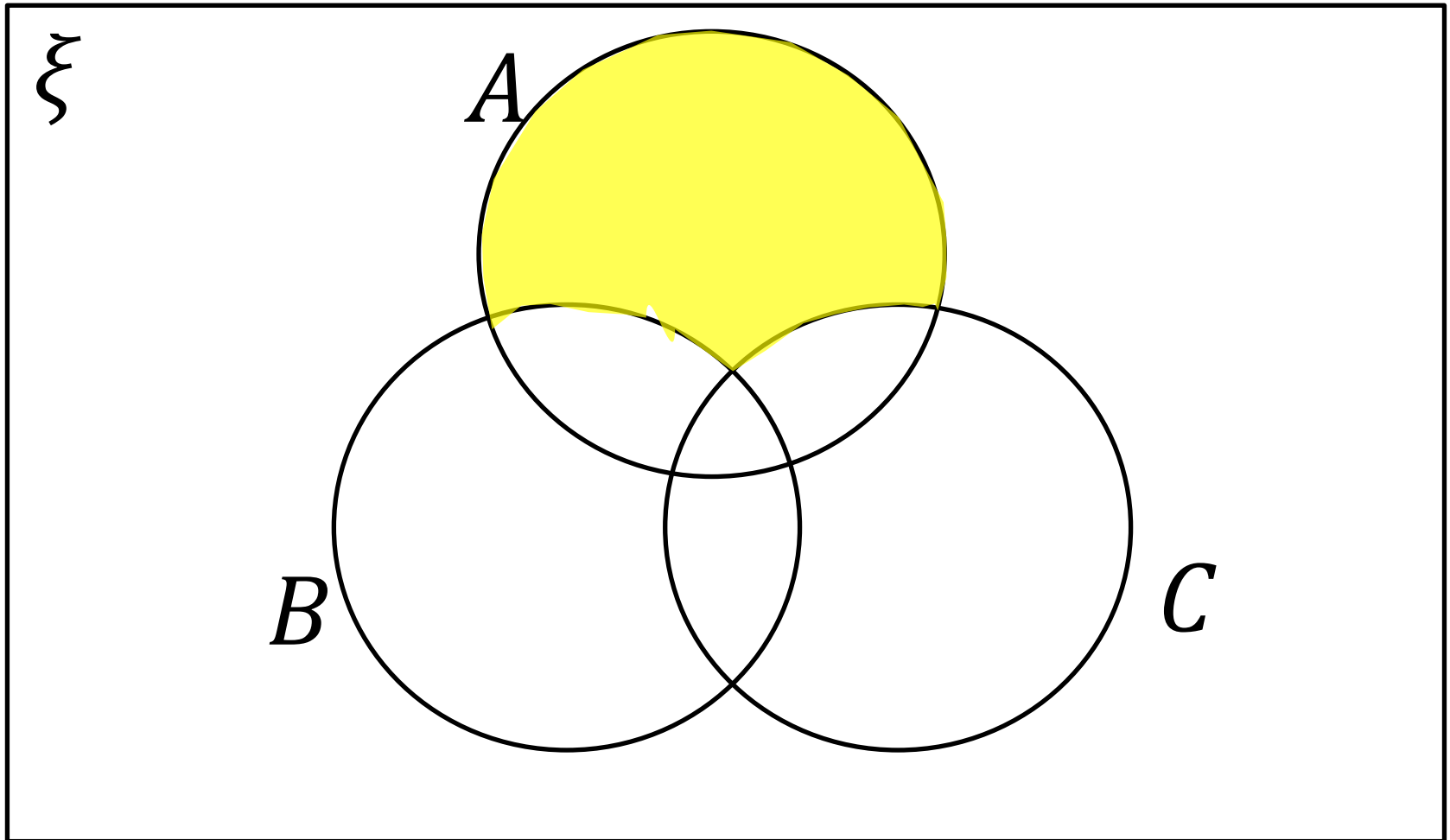
?

What area is indicated?



?

What area is indicated?



?

Examples

Venn Diagram can either contain:

- (a) The **specific outcomes** in each set
- (b) The number of items in the set (i.e. **frequencies**)
- (c) The **probability** of being in that set.

← This will usually be stated or made obvious from the context.

[Textbook] A card is selected at random from a pack of 52 playing cards. Let A be the event that the card is an ace and D the event that the card is a diamond. Find:

- a) $P(A \cap D)$ b) $P(A \cup D)$ c) $P(A')$ d) $P(A' \cap D)$

? Venn Diagram

? a

? b

? c

? d

← Think "A and D".

For union, I visualise this 'figure-of-8' shape:



Examples

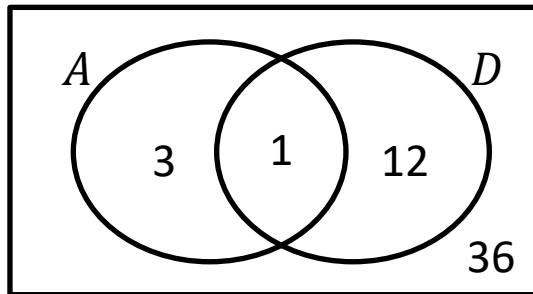
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- a) $P(A \cap D)$ b) $P(A \cup D)$ c) $P(A')$ d) $P(A' \cap D)$



$$\begin{aligned}P(A \cap D) &= \frac{1}{52} \\P(A \cup D) &= \frac{16}{52} \\P(A') &= \frac{48}{52} \\P(A' \cap D) &= \frac{12}{52}\end{aligned}$$

← Think "A and D".

← For union, visualise this 'figure-of-8' shape:



Examples

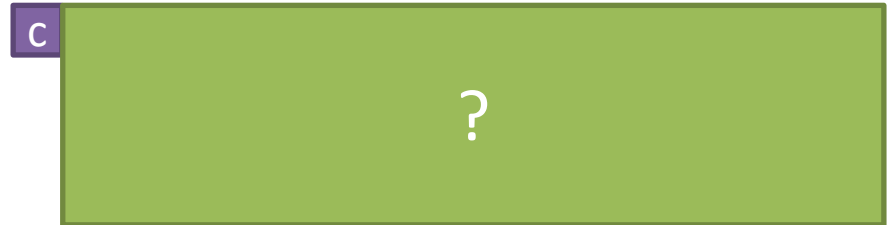
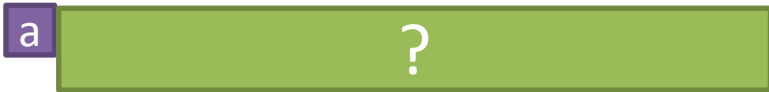
[Textbook] Given that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cap B) = 0.25$,

a. Explain why events A and B are not independent.

Given also that $P(C) = 0.2$, that events A and C are mutually exclusive and that events B and C are independent,

b. Draw a Venn diagram to illustrate the events A , B and C , showing the probabilities for each region.

c. Find $P((A \cap B') \cup C)$



Examples

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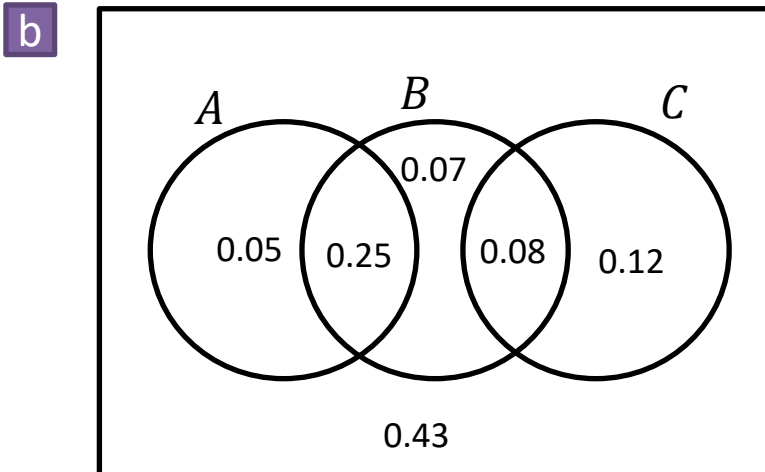
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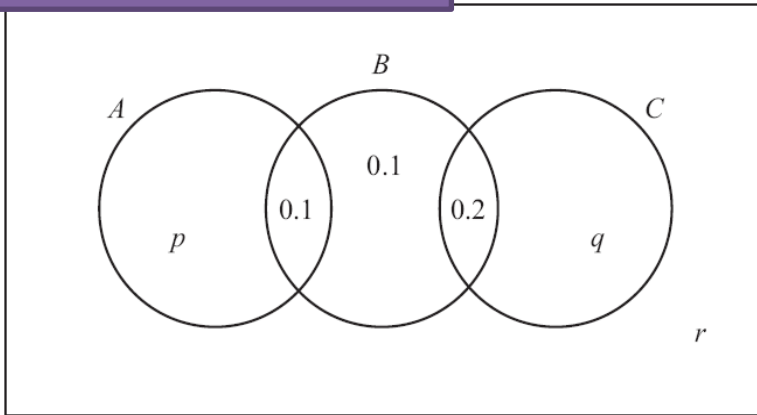
a $P(A) \times P(B) = 0.3 \times 0.4 = 0.12$

c $P(A \cap B') = 0.05$
 $P((A \cap B') \cup C) = 0.05 + 0.2 = 0.25$

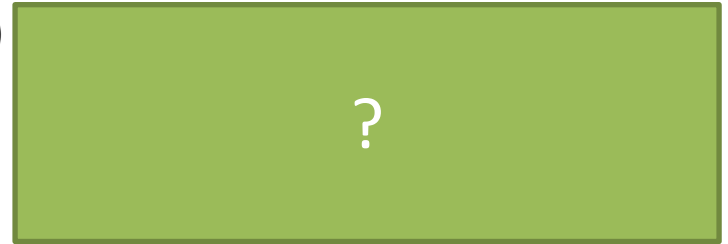


Test Your Understanding

May 2013 (R) Q6



(a)



The Venn diagram in Figure 1 shows three events A , B and C and the probabilities associated with each region of B . The constants p , q and r each represent probabilities associated with the three separate regions outside B .

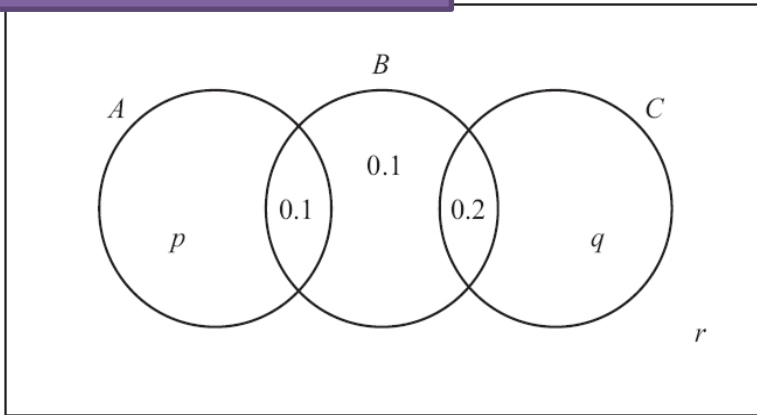
The events A and B are independent.

(a) Find the value of p .

(3)

Test Your Understanding

May 2013 (R) Q6



(a) Using the above:

$$0.1 = (p + 0.1) \times 0.4$$

$$p + 0.1 = 0.25$$

$$p = 0.15$$

The Venn diagram in Figure 1 shows three events A , B and C and the probabilities associated with each region of B . The constants p , q and r each represent probabilities associated with the three separate regions outside B .

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(a) Find the value of p .

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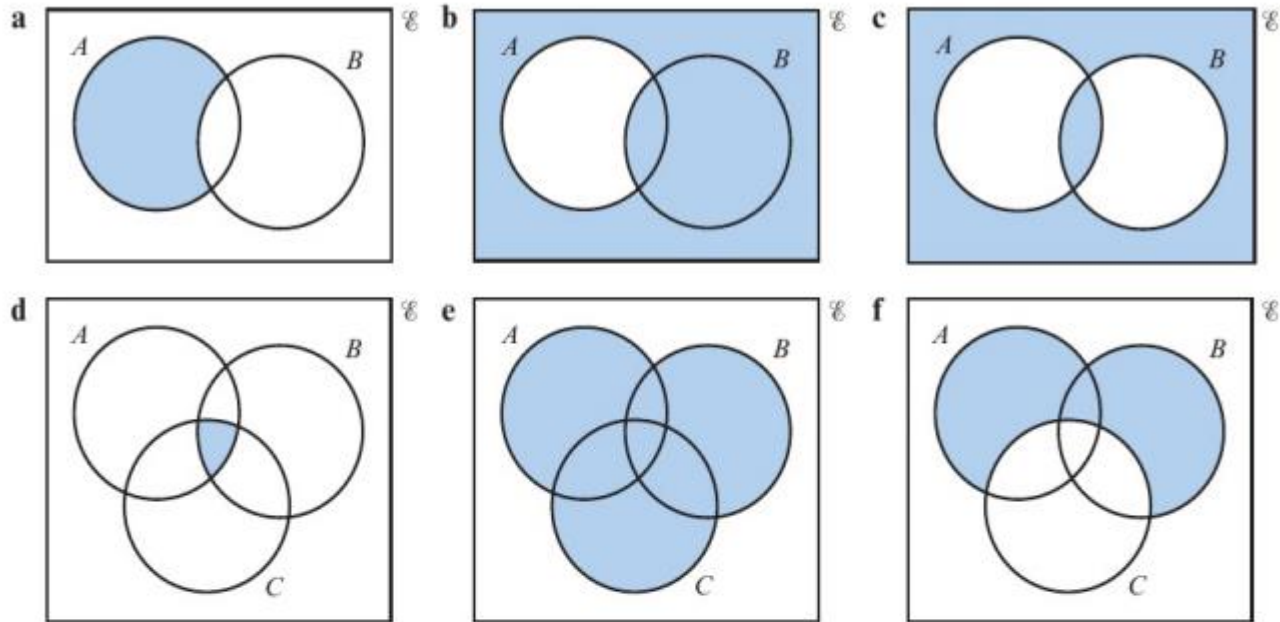
Exercise 2.1

Pearson Stats/Mechanics Year 2

Pages 9-11

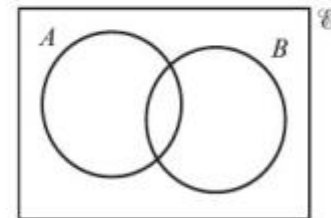
Homework Exercise

1 Use set notation to describe the shaded area in each of these Venn diagrams:



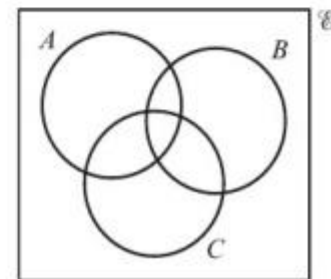
2 On copies of this Venn diagram, shade:

- a** $A \cup B'$
- b** $A' \cap B'$
- c** $(A \cap B)'$



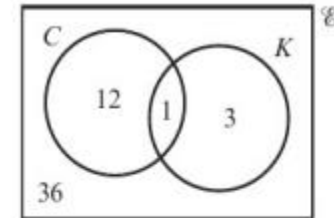
3 On copies of this Venn diagram, shade:

- a** $(A \cap B) \cup C$
- b** $(A' \cup B') \cap C$
- c** $(A \cap B \cap C)'$



Homework Exercise

- 4 A card is chosen at random from a pack of 52 playing cards. C is the event 'the card chosen is a club' and K is the event 'the card chosen is a King'. The Venn diagram shows the number of outcomes for each event.



Find:

- | | | |
|-----------------|-----------|-----------------|
| a $P(K)$ | b $P(C)$ | c $P(C \cap K)$ |
| d $P(C \cup K)$ | e $P(C')$ | f $(K' \cap C)$ |

- 5 A and B are two events. $P(A) = 0.5$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$.

Find:

- | | |
|------------------|------------------|
| a $P(A \cup B)$ | b $P(B')$ |
| c $P(A \cap B')$ | d $P(A \cup B')$ |

Hint

Draw a Venn diagram.

- 6 C and D are two events. $P(D) = 0.4$, $P(C \cap D) = 0.15$ and $P(C' \cap D') = 0.1$.

Find:

- | | | | |
|------------------|------------------|----------|-------------------|
| a $P(C' \cap D)$ | b $P(C \cap D')$ | c $P(C)$ | d $P(C' \cap D')$ |
|------------------|------------------|----------|-------------------|

- 7 The probability that a member of a sports club plays hockey (H) is 0.5 and the probability that they play cricket (C) is 0.4. The probability that they play both is 0.25.

a Draw a Venn diagram to illustrate these probabilities.

b Find:

- | | | |
|-----------------|-------------------|--------------------|
| i $P(H \cup C)$ | ii $P(H' \cap C)$ | iii $P(H \cup C')$ |
|-----------------|-------------------|--------------------|

Homework Exercise

- 8 A bag contains 50 counters numbered from 1 to 50. The counters are either red or blue. A counter is picked at random. The two events R and E are the events 'counter is red' and 'counter is even-numbered' respectively. Given that $n(R) = 17$, $n(E) = 30$ and $n(R \cup E) = 40$,

a draw a Venn diagram to illustrate the outcomes.

b Find:

- i $n(R \cap E)$
- ii $P(R' \cap E')$
- iii $P((R \cap E)')$

Watch out $n(R)$ represents the **number** of outcomes in the event R , whereas $P(R)$ represents the **probability** that the event R occurs.

- 9 A , B and C are three events with $P(A) = 0.55$, $P(B) = 0.35$ and $P(C) = 0.4$. $P(A \cap C) = 0.2$. Given that A and B are mutually exclusive and B and C are independent,

a draw a Venn diagram to illustrate the probabilities. (4 marks)

b Find:

- i $P(A' \cap B')$ (1 mark)
- ii $P(A \cup (B \cap C))$ (1 mark)
- iii $P((A \cap C)' \cup B')$ (1 mark)

- 10 A , B and C are three events with $P(A) = 0.25$, $P(B) = 0.4$, $P(C) = 0.45$ and $P(A \cap B \cap C) = 0.1$. Given that A and B are independent, B and C are independent, and $A \cap B' \cap C = \emptyset$,

Problem-solving

\emptyset is the empty set. $P(\emptyset) = 0$.

a draw a Venn diagram to illustrate the probabilities. (4 marks)

b Find:

- i $P(A' \cap (B' \cup C))$ (1 mark)
- ii $P((A \cup B) \cap C)$ (1 mark)

c State, with reasons, whether events A' and C independent. (2 marks)

Homework Exercise

- 11 Members of a school book club read either murder mysteries (M), ghost stories (G) or epic fiction (E). $P(M) = 0.5$, $P(G) = 0.4$ and $P(E) = 0.6$. Given that no one reads both ghost stories and epic fiction and that $P(M \cap G) = 0.3$,
- a draw a Venn diagram to illustrate these probabilities. (4 marks)
 - b Find:
 - i $P(M \cup G)$
 - ii $P((M \cap G) \cup (M \cap E))$ (2 marks)
 - c Are the events G' and M independent? You must justify your answer. (2 marks)
- 12 Given that events A and B are independent and that $P(A) = x$ and $P(B) = y$, find, in terms of x and y :
- a $P(A \cap B)$ (2 marks)
 - b $P(A \cup B)$ (2 marks)
 - c $P(A \cup B')$ (2 marks)

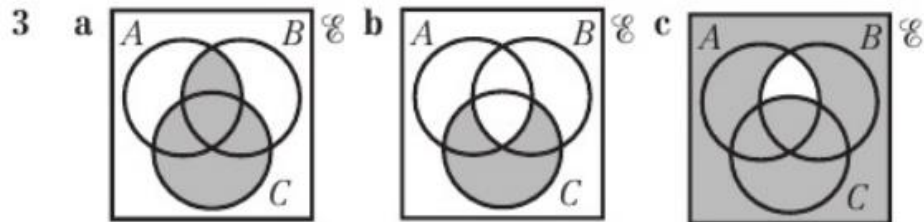
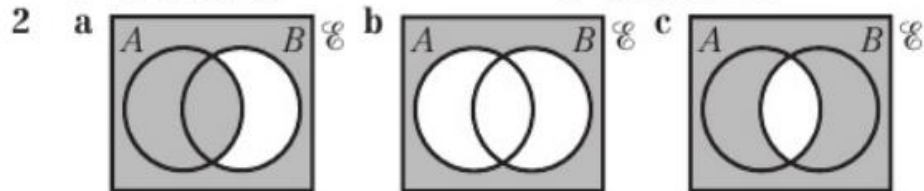
Challenge

Given that events A , B and C are all independent and that $P(A) = x$, $P(B) = y$ and $P(C) = z$, find, in terms of x , y and z :

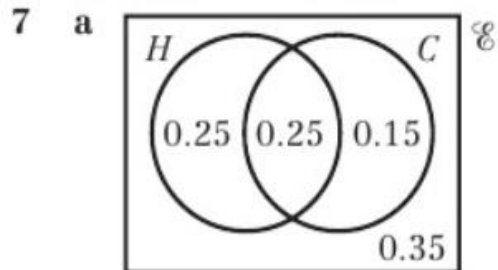
- a $P(A \cap B \cap C)$
- b $P(A \cup B \cup C)$
- c $P((A \cup B') \cap C)$

Homework Answers

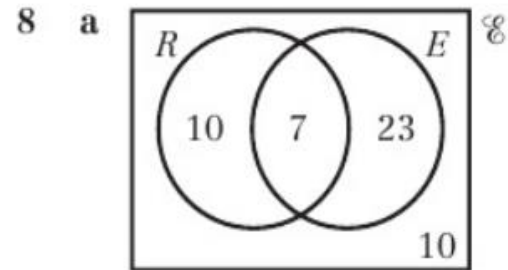
- 1 a $A \cap B'$ b $A' \cup B$
 c $(A \cap B) \cup (A' \cap B')$ d $A \cap B \cap C$
 e $A \cup B \cup C$ f $(A \cup B) \cap C'$



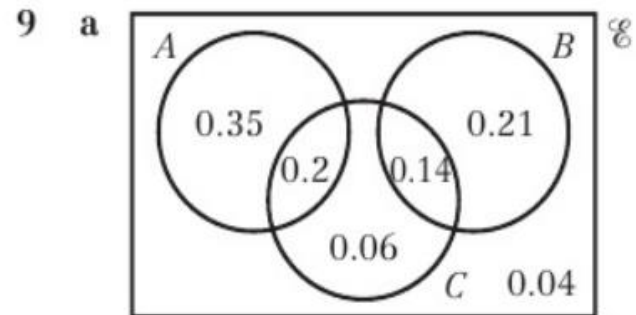
- 4 a 0.0769 b 0.25 c 0.0192
 d 0.308 e 0.75 f 0.231
 5 a 0.6 b 0.8 c 0.4 d 0.9
 6 a 0.25 b 0.5 c 0.65 d 0.1



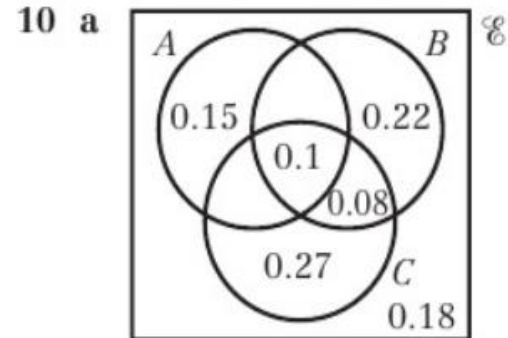
- b i 0.65 ii 0.15 iii 0.85



- b i 7 ii $\frac{1}{5}$ iii $\frac{43}{50}$



- b i 0.1 ii 0.76 iii 1

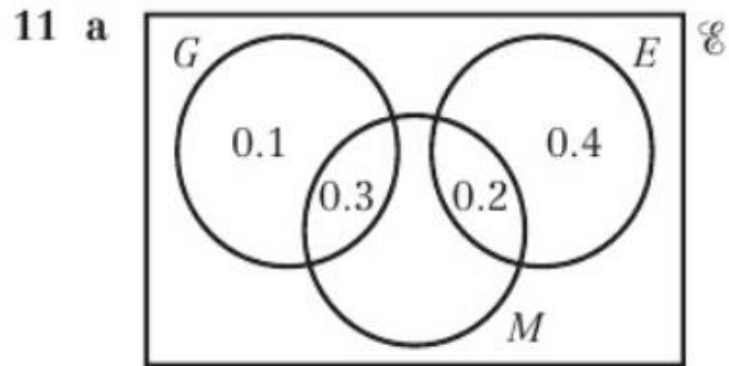


- b i 0.53 ii 0.18

c Not independent.

$$P(A' \cap C) = 0.35, P(A') \times P(C) = 0.75 \times 0.45 = 0.3375$$

Homework Answers



b i 0.6 ii 0.5

c Not independent.

$$P(G' \cap M) = 0.2, P(G') \times P(M) = 0.6 \times 0.5 = 0.3$$

12 a xy b $x + y + xy$ c $1 - y + xy$

Challenge

a xyz

b $x + y + z + xyz - xy - yz - xz$

c $z - yz + xyz$