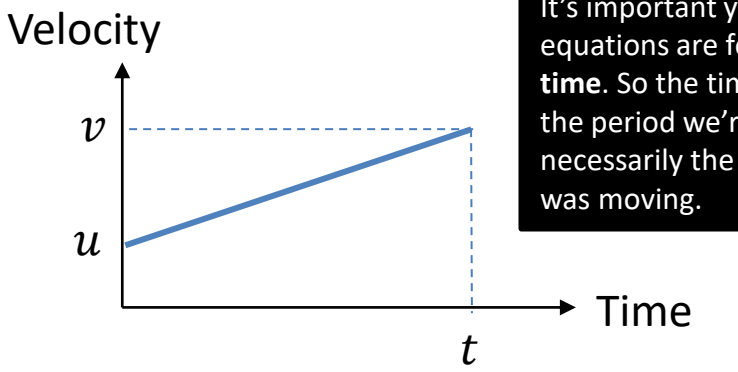
M1 Chapter 9: Constant Acceleration

Two Base Formulae

“suvat” Equations (part 1)

When there is **constant acceleration**, there are a variety of formulae which relate the following 5 quantities:

- s: displacement
- u: initial velocity
- v: final velocity
- a: acceleration
- t: time



It’s important you recognise these equations are for **a specific interval of time**. So the time t is the duration of the period we’re considering, not necessarily the time since the object was moving.

Each “suvat” equation we will see involves 4 of the 5 quantities. In a problem we’ll know 3 of the quantities and we wish to find an unknown 4th quantity. We need to select the appropriate equation.

Express the gradient of the graph (acceleration a) in terms of {u, v, t}:

$a = \frac{v-u}{t} \quad \rightarrow \quad v = u + at$

You are expected to be able to **prove** each “suvat” question using the above graph.

Express the area under the graph (displacement s) in terms of {u, v, t}:

$s = \left(\frac{u+v}{2}\right) t$

Memorisation Tip: This formula is effectively “distance = average speed × time” which you knew from GCSE.

Examples

[Textbook] A cyclist is travelling along a straight road. She accelerates at a constant rate from a velocity of 4 ms^{-1} to a velocity of 7.5 ms-1 in 40 seconds. Find:

(a) the distance she travels in these 40 seconds

(b) her acceleration in these 40 seconds.

a

?

b

?

Exam Pro: Write a *suvat*-table indicating the quantity you need to find, and in particular, the quantity you DO NOT need for the given calculation.

Examples

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(b) her acceleration in these 40 seconds.

a $s = ?, u = 4, v = 7.5,$
$a = ?, t = 40$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{4+7.5}{2}\right) \times 40$$
$$= 230\ m$$

b $v = u + at$
$7.5 = 4 + 40a$
$a = 0.0875$

Exam Pro: Write a *suvat*-table indicating the quantity you need to find, and in particular, the quantity you DO NOT need for the given calculation.

Examples

[Textbook] A particle moves in a straight line from a point A to a point B with a constant deceleration 1.5 ms^{-2}. The velocity of the particle at A is 8 ms^{-1} and the velocity of the particle at B is 2 ms^{-1}. Find:

(a) the time taken for the particle to move from A to B.

(b) the distance from A to B.

After reaching B the particle continues to move along the straight line with constant deceleration 1.5 ms^{-2}. The particle is at the point C 6 seconds after passing through the point A. Find:

(c) the velocity of the particle at C.

(d) The distance from A to C.

a

?

b

?

As stated before, think about what period of time we're considering.

c

?

d

?

Examples

[Textbook] A particle moves in a straight line from a point A to a point B with a constant deceleration 1.5 ms^{-2}. The velocity of the particle at A is 8 ms^{-1} and the velocity of the particle at B is 2 ms^{-1}. Find:
(a) the time taken for the particle to move from A to B.
(b) the distance from A to B.

After reaching B the particle continues to move along the straight line with constant deceleration 1.5 ms^{-2}. The particle is at the point C 6 seconds after passing through the point A. Find:
(c) the velocity of the particle at C.
(d) The distance from A to C.

a $s = ?, u = 8, v = 2, a = -1.5, t = ?$

$$v = u + at$$
$$2 = 8 - 1.5t$$
$$t = 4 \text{ s}$$

b $s = \left(\frac{u+v}{2}\right)t = \left(\frac{8+2}{2}\right) \times 4 = 20$ m

As stated before, think about what period of time we're considering.

c $u = 8, a = -1.5, t = 6, v = ?$

$$v = u + at$$
$$= 8 + (-1.5) \times 6 = -1$$

$\therefore$ velocity is 1ms^{-1} in the direction $\overrightarrow{BA}$ (i.e. backwards)

d $s = \left(\frac{u+v}{2}\right)t = \left(\frac{8-1}{2}\right) \times 6 = 21$ m

Test Your Understanding

[Textbook] A car moves from traffic lights along a straight road with constant acceleration. The car starts from rest at the traffic lights and 30 second later the car passes a speed-trap where it is registered as travelling at 45 km h^{-1}. Find:

(a) the acceleration of the car

(b) the distance between the traffic lights and the speed-trap.

Tip: Ensure everything is in SI units first.

a

?

b

?

Test Your Understanding

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(a) the acceleration of the car

(b) the distance between the traffic lights and the speed-trap.

Tip: Ensure everything is in SI units first.

a

$$45 \times \frac{1000}{3600} = 12.5\, ms^{-1}$$

$$u = 0, v = 12.5, t = 30, a = ?, s = ?$$

$$v = u + at$$

$$12.5 = 0 + 30a \quad \rightarrow \quad a = \frac{5}{12}\, ms^{-2}$$

b

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{0+12.5}{2}\right) \times 30 = 187.5 \text{ m}$$

Exercise 9.3 First two formulae

Pearson Stats/Mechanics Year 1
Pages 60-61

Homework Exercise

1 A particle is moving in a straight line with constant acceleration $3\,\text{m s}^{-2}$. At time $t = 0$, the velocity of the particle is $2\,\text{m s}^{-1}$. Find the velocity of the particle at time $t = 6\,\text{s}$.

2 A car is approaching traffic lights. The car is travelling with velocity $10\,\text{m s}^{-1}$. The driver applies the brakes to the car and the car comes to rest with constant deceleration in 16 s. Modelling the car as a particle, find the deceleration of the car.

3 A car accelerates uniformly while travelling on a straight road. The car passes two signposts 360 m apart. The car takes 15 s to travel from one signpost to the other. When passing the second signpost, it has velocity $28\,\text{m s}^{-1}$. Find the velocity of the car at the first signpost.

4 A cyclist is moving along a straight road from A to B with constant acceleration $0.5\,\text{m s}^{-2}$. Her velocity at A is $3\,\text{m s}^{-1}$ and it takes her 12 seconds to cycle from A to B. Find:

a her velocity at B

b the distance from A to B.

5 A particle is moving along a straight line with constant acceleration from a point A to a point B, where $AB = 24\,\text{m}$. The particle takes 6 s to move from A to B and the velocity of the particle at B is $5\,\text{m s}^{-1}$. Find:

a the velocity of the particle at A

b the acceleration of the particle.

6 A particle moves in a straight line from a point A to a point B with constant deceleration $1.2\,\text{m s}^{-2}$. The particle takes 6 s to move from A to B. The speed of the particle at B is $2\,\text{m s}^{-1}$ and the direction of motion of the particle has not changed. Find:

a the speed of the particle at A

b the distance from A to B.

Homework Exercise

7 A train, travelling on a straight track, is slowing down with constant deceleration $0.6\,\text{m s}^{-2}$. The train passes one signal with speed $72\,\text{km h}^{-1}$ and a second signal 25 s later. Find:

a the velocity, in km h^{-1}, of the train as it passes the second signal

b the distance between the signals.

Hint Convert the speeds into m s^{-1} before substituting.

8 A particle moves in a straight line from a point A to a point B with a constant deceleration of $4\,\text{m s}^{-2}$. At A the particle has velocity $32\,\text{m s}^{-1}$ and the particle comes to rest at B. Find:

a the time taken for the particle to travel from A to B

b the distance between A and B.

9 A skier travelling in a straight line up a hill experiences a constant deceleration. At the bottom of the hill, the skier has a velocity of $16\,\text{m s}^{-1}$ and, after moving up the hill for 40 s, he comes to rest. Find:

a the deceleration of the skier **(2 marks)**

b the distance from the bottom of the hill to the point where the skier comes to rest. **(4 marks)**

10 A particle is moving in a straight line with constant acceleration. The points A, B and C lie on this line. The particle moves from A through B to C. The velocity of the particle at A is $2\,\text{m s}^{-1}$ and the velocity of the particle at B is $7\,\text{m s}^{-1}$. The particle takes 20 s to move from A to B.

a Find the acceleration of the particle. **(2 marks)**

The velocity of the particle is C is $11\,\text{m s}^{-1}$. Find:

b the time taken for the particle to move from B to C **(2 marks)**

c the distance between A and C. **(3 marks)**

Homework Exercise

11 A particle moves in a straight line from A to B with constant acceleration $1.5\,\text{m s}^{-2}$. It then moves along the same straight line from B to C with a different acceleration. The velocity of the particle at A is $1\,\text{m s}^{-1}$ and the velocity of the particle at C is $43\,\text{m s}^{-1}$. The particle takes 12 s to move from A to B and 10 s to move from B to C. Find:

a the velocity of the particle at B **(2 marks)**

b the acceleration of the particle as it moves from B to C **(2 marks)**

c the distance from A to C. **(3 marks)**

12 A cyclist travels with constant acceleration $x\,\text{m s}^{-2}$, in a straight line, from rest to $5\,\text{m s}^{-1}$ in 20 s. She then decelerates from $5\,\text{m s}^{-1}$ to rest with constant deceleration $\frac{1}{2}x\,\text{m s}^{-2}$. Find:

a the value of x **(2 marks)**

b the total distance she travelled. **(4 marks)**

Problem-solving

You could sketch a velocity–time graph of the cyclist's motion and use the area under the graph to find the total distance travelled.

13 A particle is moving with constant acceleration in a straight line. It passes through three points, A, B and C, with velocities $20\,\text{m s}^{-1}$, $30\,\text{m s}^{-1}$ and $45\,\text{m s}^{-1}$ respectively. The time taken to move from A to B is t_1 seconds and the time taken to move from B to C is t_2 seconds.

a Show that $\frac{t_1}{t_2} = \frac{2}{3}$. **(3 marks)**

Given also that the total time taken for the particle to move from A to C is 50 s:

b find the distance between A and B. **(5 marks)**

Homework Exercise

Challenge

A particle moves in a straight line from A to B with constant acceleration. The particle moves from A with velocity $3\ \text{m s}^{-1}$. It reaches point B with velocity $5\ \text{m s}^{-1}$ t seconds later.

One second after the first particle leaves point A, a second particle also starts to move in a straight line from A to B with constant acceleration. Its velocity at point A is $4\ \text{m s}^{-1}$ and it reaches point B with velocity $8\ \text{m s}^{-1}$ at the same time as the first particle.

Find:

a the value of t

b the distance between A and B.

Problem-solving

The time taken for the second particle to travel from A to B is $(t - 1)$ seconds.

Homework Answers

1 $20\,m\,s^{-1}$

2 $0.625\,m\,s^{-2}$

3 $20\,m\,s^{-1}$

4 **a** $9\,m\,s^{-1}$ **b** $72\,m$

5 **a** $3\,m\,s^{-1}$ **b** $\frac{1}{3}\,m\,s^{-2}$

6 **a** $9.2\,m\,s^{-1}$ **b** $33.6\,m$

7 **a** $18\,km\,h^{-1}$ **b** $312.5\,m$

8 **a** $8\,s$ **b** $128\,m$

9 **a** $0.4\,m\,s^{-2}$ **b** $320\,m$

10 **a** $0.25\,m\,s^{-2}$ **b** $16\,s$ **c** $234\,m$

11 **a** $19\,m\,s^{-1}$ **b** $2.4\,m\,s^{-2}$ **c** $430\,m$

12 **a** $x = 0.25$ **b** $150\,m$

13 **b** $500\,m$

Challenge

a $t = 3$ **b** $12\,m$