P2 Chapter 2: Graphing Functions

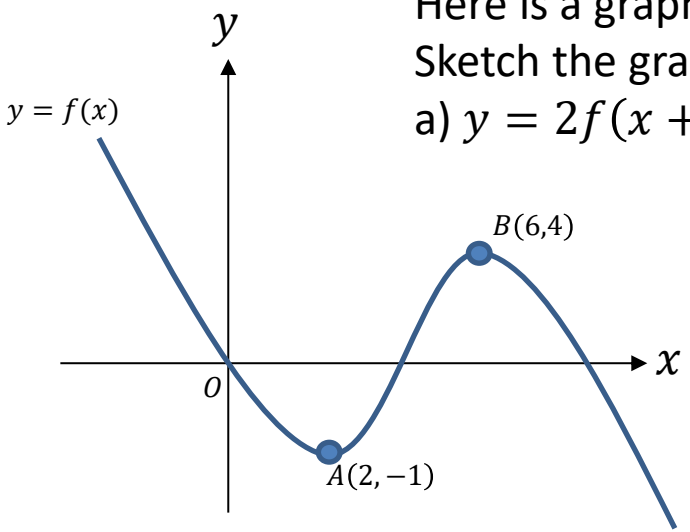
Combining Transformations

Combining Transformations

RECAP:

	Affects which axis?	What we expect or opposite?
Change **inside** $f(\ )$	x	Opposite
Change **outside** $f(\ )$	y	What we expect

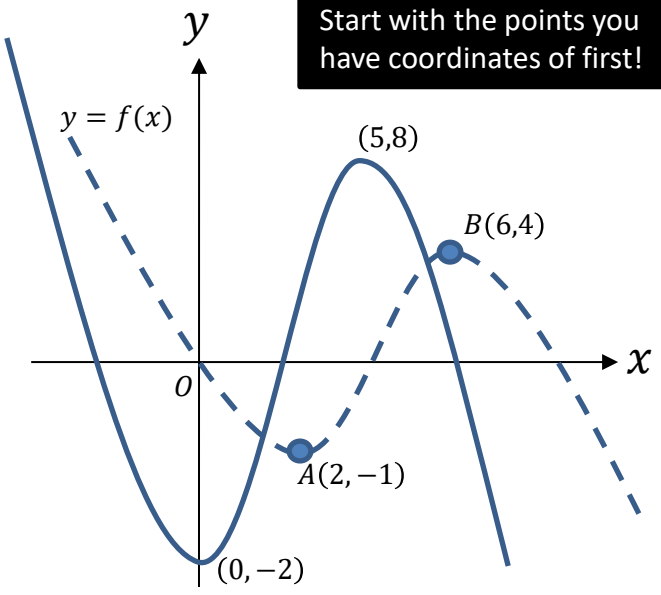
There is nothing new here relative to Year 1, except that you might have to do more than one transformation…



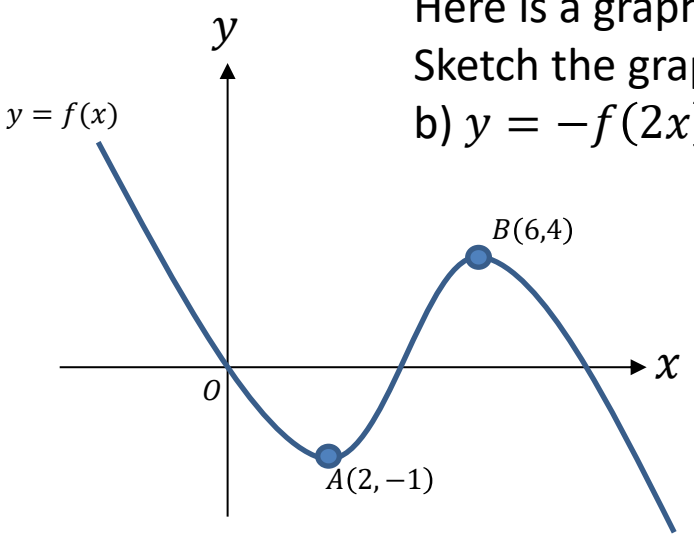
Here is a graph of $y = f(x)$.
Sketch the graph of:
a) $y = 2f(x + 2)$

Subtract 2 from x values.
y values multiplied by 2.

Start with the points you have coordinates of first!



Combining Transformations



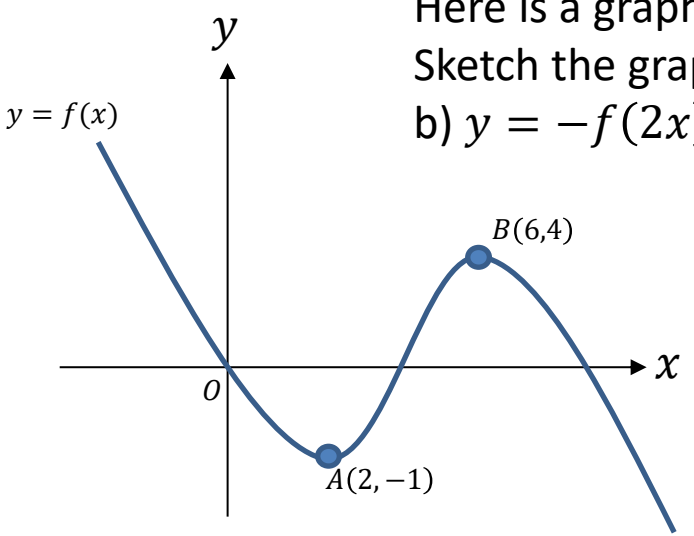
Here is a graph of $y = f(x)$.
Sketch the graph of:
b) $y = -f(2x)$

? b

c) $y = |f(-x)|$

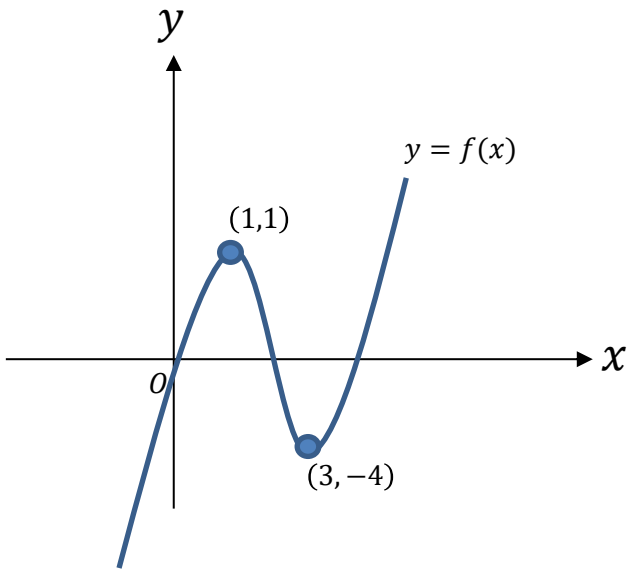
? c

Combining Transformations

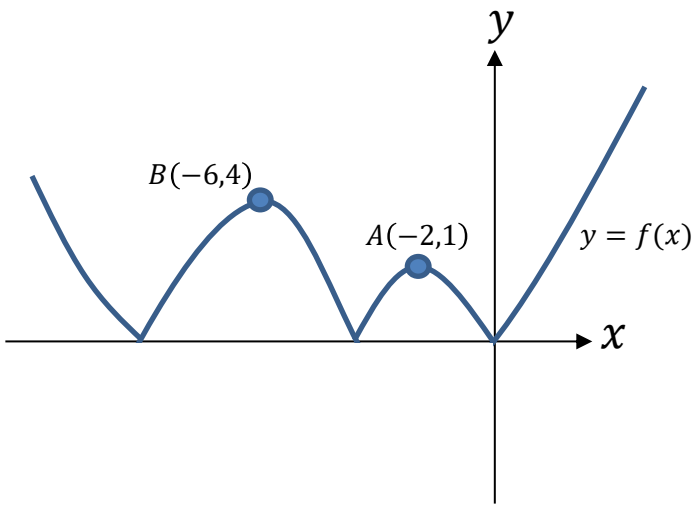


Here is a graph of $y = f(x)$.
Sketch the graph of:
b) $y = -f(2x)$

Halve x values.
y values negated.



c) $y = |f(-x)|$



x values negated.
Negative y flipped up.

Test Your Understanding

C4 June 2011 Q3

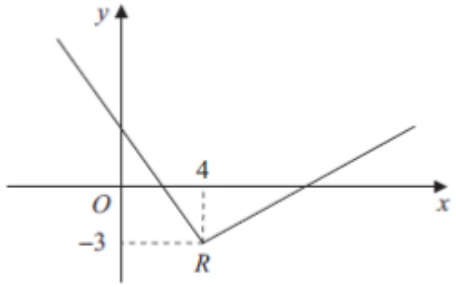


Figure 1

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R(4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x + 4)$, **(3)**

(b) $y = |f(-x)|$. **(3)**

On each diagram, show the coordinates of the point corresponding to R.

(a) ? a

(b) ? b

Test Your Understanding

C4 June 2011 Q3

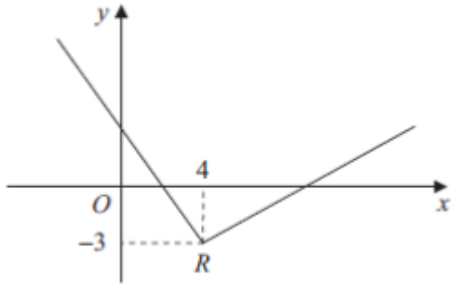


Figure 1

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R(4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 2f(x + 4)$, **(3)**

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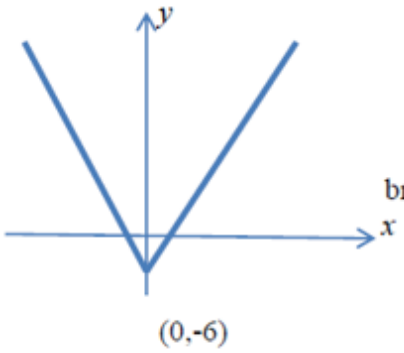
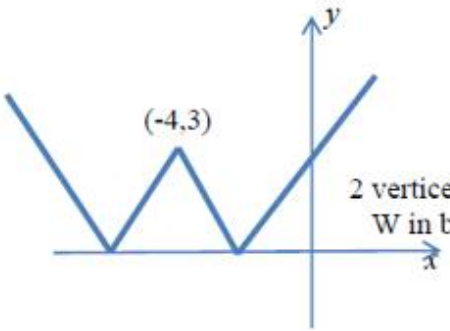
On each diagram, show the coordinates of the point corresponding to *R*.

(a)



V shape	B1
vertex on *y* axis &both branches of graph cross x axis	B1
'*y*' co-ordinate of R is -6	B1

(b)



W shape	B1
2 vertices on the negative *x* axis. W in both quad 1 & quad 2.	B1dep
R'=(-4,3)	B1

What if two x changes or two y changes?

$$y = 2f(x) + 1$$

?

$$y = f(2x + 1)$$

You will <u>not</u> get multiple x transformations in your exam, but theoretically…

?

Sketch $y = \ln(1 - 2x)$

?

What if two x changes or two y changes?

$$y = 2f(x) + 1$$

The y values are multiplied by 2, and then 1 is added.

$$y = f(2x + 1)$$

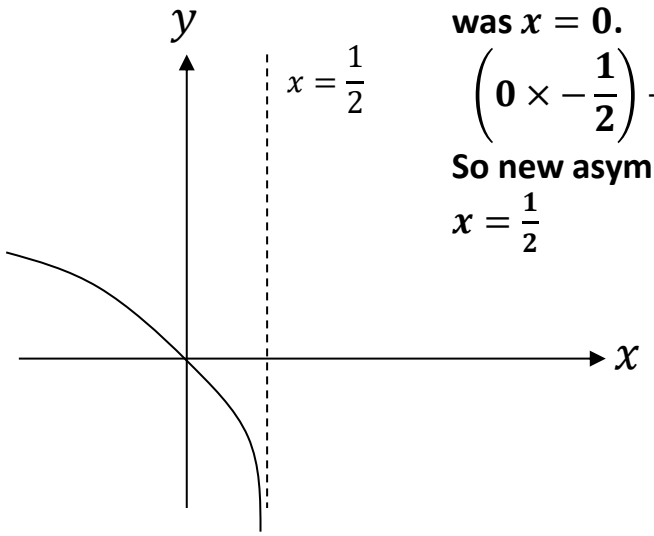
You will not get multiple x transformations in your exam, but theoretically…

The easiest way is the think of the **inverse function** of $2x + 1$, i.e. $\frac{x-1}{2}$.
This gives us the changes to the x values, and in the correct order! In this case, we would -1 from the x values (translation 1 left) and then halve the x values (stretch on x-axis of scale factor $\frac{1}{2}$)

Sketch $y = \ln(1 - 2x)$

Inverse of $1 - 2x$ is $\frac{1-x}{2} = -\frac{1}{2}x + \frac{1}{2}$

So multiply x values by $-\frac{1}{2}$ and then add $\frac{1}{2}$.



Original asymptote was $x = 0$.

$$\left(0 \times -\frac{1}{2}\right) + \frac{1}{2} = \frac{1}{2}$$

So new asymptote is $x = \frac{1}{2}$

Exercise 2.6

Pearson Pure Mathematics Year 2/AS

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Homework Exercise

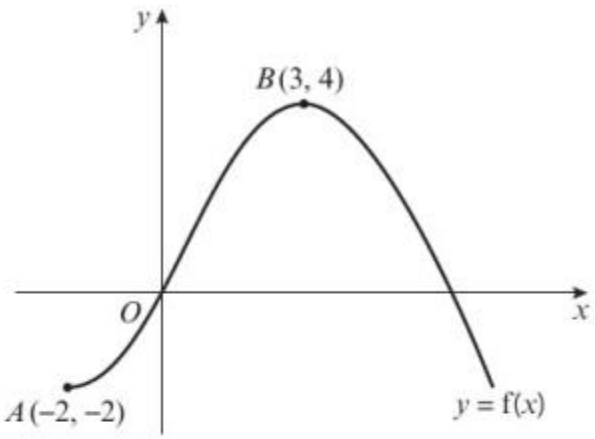
1 The diagram shows a sketch of the graph $y = f(x)$.
The curve passes through the origin O, the point $A(-2, -2)$ and the point $B(3, 4)$.
On separate axes, sketch the graphs of:

a $y = 3f(x) + 2$ **b** $y = f(x - 2) - 5$

c $y = \frac{1}{2}f(x + 1)$ **d** $y = -f(2x)$

e $y = |f(x)|$ **f** $y = |f(-x)|$

In each case find the coordinates of the images of the points O, A and B.



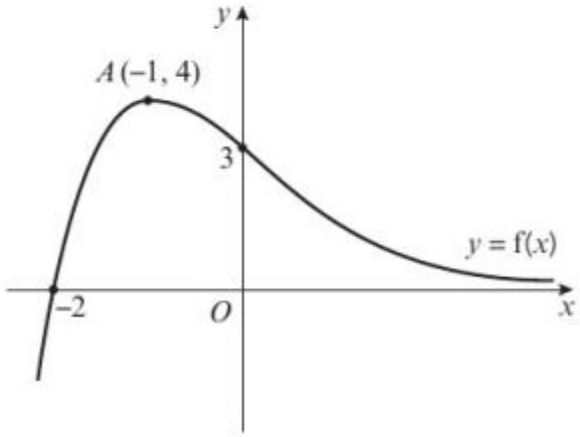
2 The diagram shows a sketch of the graph $y = f(x)$.
The curve has a maximum at the point $A(-1, 4)$ and crosses the axes at the points $(0, 3)$ and $(-2, 0)$.
On separate axes, sketch the graphs of:

a $y = 3f(x - 2)$ **b** $y = \frac{1}{2}f\left(\frac{1}{2}x\right)$

c $y = -f(x) + 4$ **d** $y = -2f(x + 1)$

e $y = 2f(|x|)$

For each graph, find, where possible, the coordinates of the maximum or minimum and the coordinates of the intersection points with the axes.



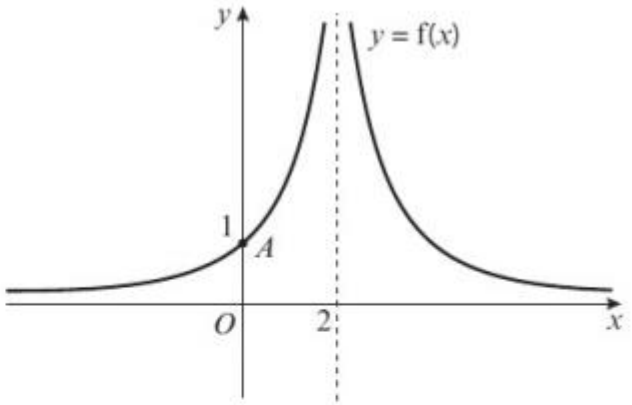
Homework Exercise

3 The diagram shows a sketch of the graph $y = f(x)$.
The lines $x = 2$ and $y = 0$ (the x-axis) are asymptotes to the curve.
On separate axes, sketch the graphs of:

a $y = 3f(x) - 1$ **b** $y = f(x + 2) + 4$

c $y = -f(2x)$ **d** $y = f(|x|)$

For each part, state the equations of the asymptotes and the new coordinates of the point A.



4 The function g is defined by

$g: x \mapsto (x - 2)^2 - 9, x \in \mathbb{R}$.

a Draw a sketch of the graph of $y = g(x)$, labelling the turning point and the x- and y-intercepts. **(3 marks)**

b Write down the coordinates of the turning point when the curve is transformed as follows:

i $2g(x - 4)$ **(2 marks)**

ii $g(2x)$ **(2 marks)**

iii $|g(x)|$ **(2 marks)**

c Sketch the curve with equation $y = g(|x|)$. On your sketch show the coordinates of all turning points and all x- and y-intercepts. **(4 marks)**

Homework Exercise

5 $h(x) = 2\sin x$, $-180° \leqslant x \leqslant 180°$.

a Sketch the graph of $y = h(x)$.

b Write down the coordinates of the minimum, A, and the maximum, B.

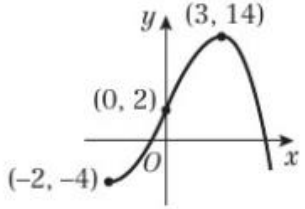
c Sketch the graphs of:

i $h(x - 90°) + 1$ **ii** $\frac{1}{4}h\left(\frac{1}{2}x\right)$ **iii** $\frac{1}{2}|h(-x)|$

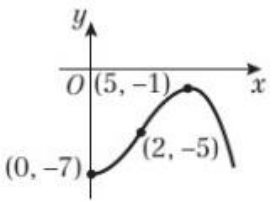
In each case find the coordinates of the images of the points O, A and B.

Homework Answers

1 **a**



b



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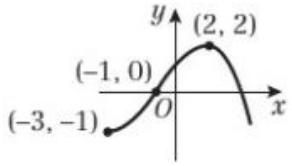

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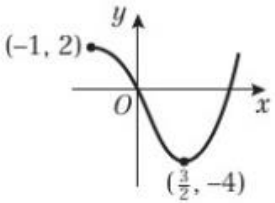

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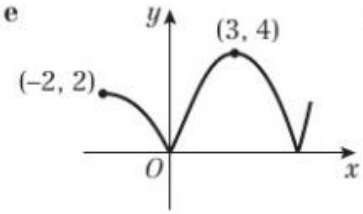
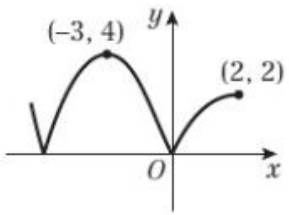
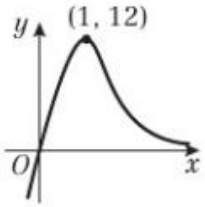


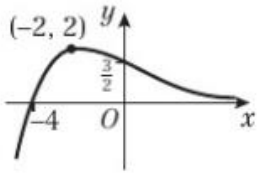
f



2 **a**



b



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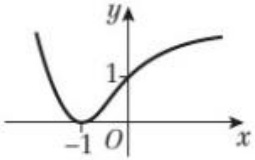

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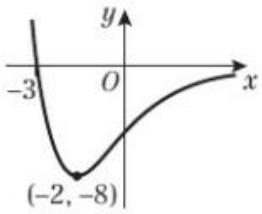

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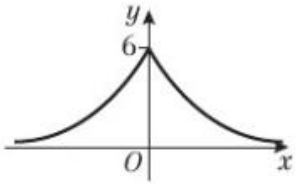
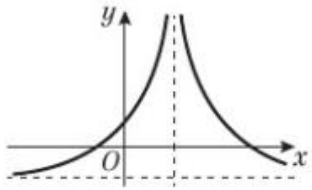
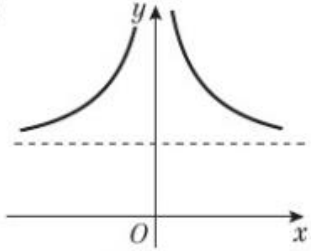


3 **a**



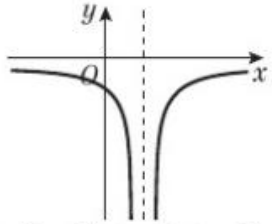
$A = (0, 2), x = 2, y = -1$

b



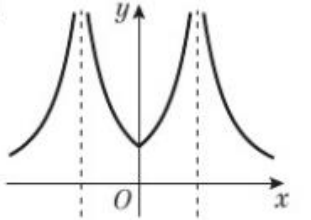
$A = (-2, 5), x = 0, y = 4$

c



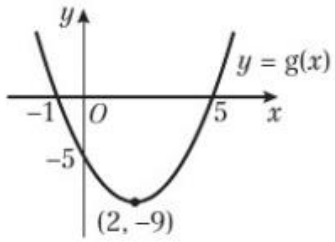
$A = (0, -1), x = 1, y = 0$

d



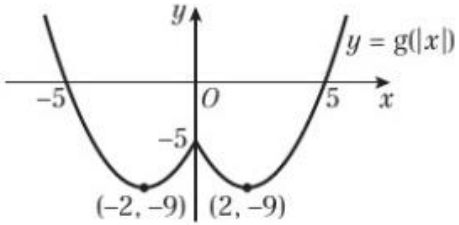
$A = (0, 1), x = 2, x = -2, y = 0$

4 **a**



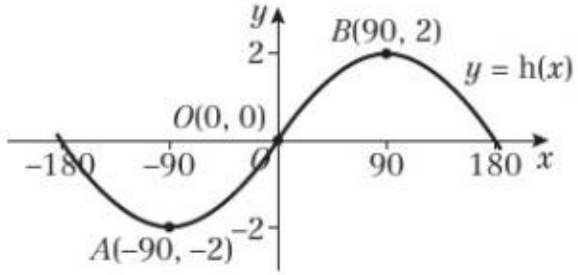
b **i** (6, –18) **ii** (1, –9) **iii** (2, 9)

c



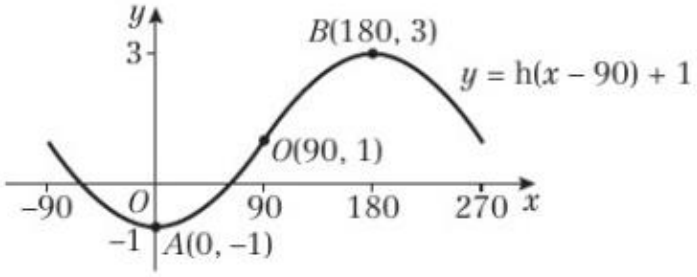
Homework Answers

5 a



b $A(-90, -2)$ and $B(90, 2)$

c i



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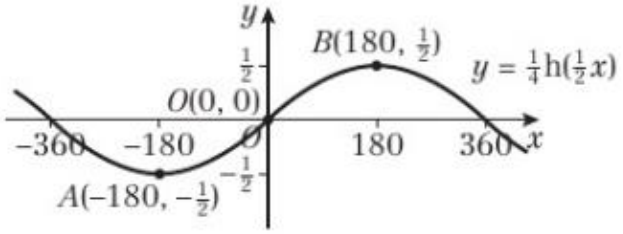
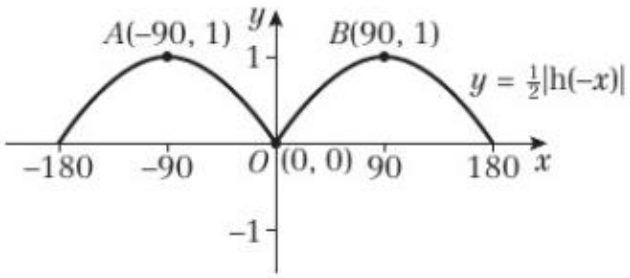
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