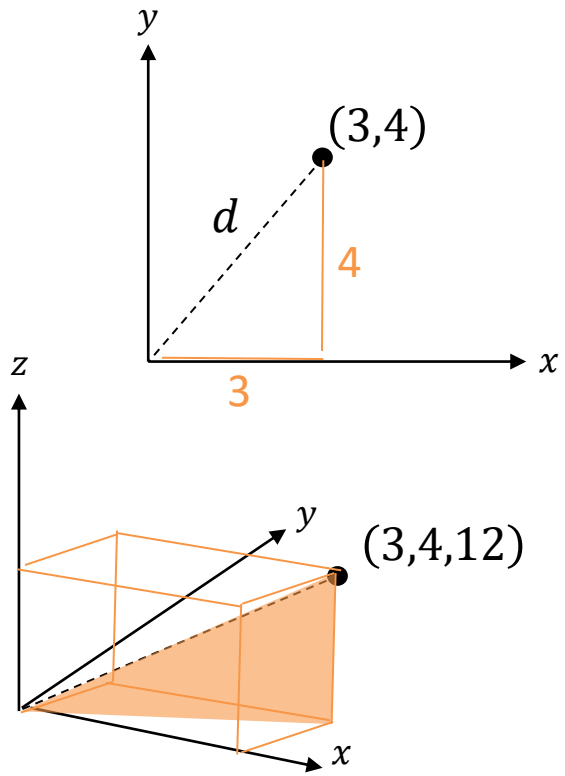
P1 Chapter 11: 3D Vectors

3D Coordinates

Distance from the origin and magnitude of a vector



In 2D, how did we find the distance from a point to the origin?

Using Pythagoras:

$$\boldsymbol{d = \sqrt{3^2 + 4^2} = 5}$$

How about in 3D then?

You may be familiar with this method from GCSE.

Using Pythagoras on the base of the cuboid:

$$\boldsymbol{\sqrt{3^2 + 4^2} = 5}$$

Then using the highlighted triangle:

$$\boldsymbol{\sqrt{5^2 + 12^2} = 13}$$

We could have similarly done this is one go using:

$$\boldsymbol{\sqrt{3^2 + 4^2 + 12^2} = 13}$$

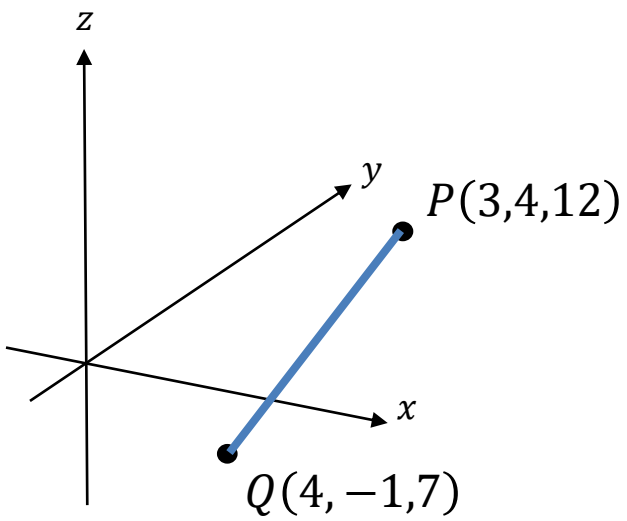
From earlier you will be familiar with the magnitude $|\boldsymbol{a}|$ of a vector $\boldsymbol{a}$ being its length.
We can see from above that this nicely extends to 3D:

The magnitude of a vector $\boldsymbol{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$:

$$|\boldsymbol{a}| = \sqrt{x^2 + y^2 + z^2}$$

And the distance of (x, y, z) from the origin is $\sqrt{x^2 + y^2 + z^2}$

Distance between two 3D points



How do we find the distance between P and Q?

?

The distance between two points is:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

Δx means "change in x"

Quickfire Questions:

Distance of $(4,0,-2)$ from the origin:

?

$\left|\begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}\right| =$?

Distance between $(0,4,3)$ and $(5,2,3)$.

?

Distance between $(1,1,1)$ and $(2,1,0)$.

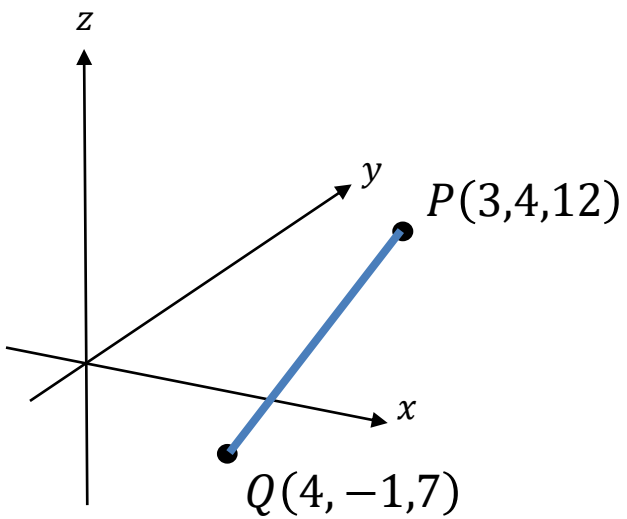
?

Distance between $(-5,2,0)$ and $(-2,-3,-3)$.

?

Tip: Because we're squaring, it doesn't matter whether the change is negative or positive.

Distance between two 3D points



How do we find the distance between P and Q?
It's just the magnitude/length of the vector between them.
i.e.

$$|\overrightarrow{PQ}| = \left|\begin{pmatrix} 1 \\ -5 \\ -5 \end{pmatrix}\right|$$
$$= \sqrt{1^2 + (-5)^2 + (-5)^2} = \sqrt{51}$$

The distance between two points is:
$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$
Δx means "change in x"

Quickfire Questions:

Distance of $(4,0,-2)$ from the origin:
$$\sqrt{4^2 + 0^2 + (-2)^2} = \sqrt{20}$$

$$\left|\begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}\right| = \sqrt{5^2 + 4^2 + (-1)^2} = \sqrt{42}$$

Distance between $(0,4,3)$ and $(5,2,3)$.
$$d = \sqrt{5^2 + (-2)^2 + 0^2} = \sqrt{29}$$

Distance between $(1,1,1)$ and $(2,1,0)$.
$$d = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$$

Distance between $(-5,2,0)$ and $(-2,-3,-3)$.
$$d = \sqrt{3^2 + 5^2 + 3^2} = \sqrt{43}$$

Tip: Because we're squaring, it doesn't matter whether the change is negative or positive.

Test Your Understanding So Far…

[Textbook] Find the distance from the origin to the point $P(7,7,7)$.

?

[Textbook] The coordinates of A and B are $(5,3,-8)$ and $(1,k,-3)$ respectively. Given that the distance from A to B is $3\sqrt{10}$ units, find the possible values of k.

?

Exercise 12.1

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Homework Exercise

1 Find the distance from the origin to the point $P(2, 8, -4)$.

2 Find the distance from the origin to the point $P(7, 7, 7)$.

3 Find the distance between A and B when they have the following coordinates:

a $A(3, 0, 5)$ and $B(1, -1, 8)$

b $A(8, 11, 8)$ and $B(-3, 1, 6)$

c $A(3, 5, -2)$ and $B(3, 10, 3)$

d $A(-1, -2, 5)$ and $B(4, -1, 3)$

4 The coordinates of A and B are $(7, -1, 2)$ and $(k, 0, 4)$ respectively.
Given that the distance from A to B is 3 units, find the possible values of k.

5 The coordinates of A and B are $(5, 3, -8)$ and $(1, k, -3)$ respectively.
Given that the distance from A to B is $3\sqrt{10}$ units, find the possible values of k.

Challenge

a The points $A(1, 3, -2)$, $B(1, 3, 4)$ and $C(7, -3, 4)$ are three vertices of a solid cube. Write down the coordinates of the remaining five vertices.

An ant walks from A to C along the surface of the cube.

b Determine the length of the shortest possible route the ant can take.

Homework Answers

1 $2\sqrt{21}$ **2** $7\sqrt{3}$

3 **a** $\sqrt{14}$ **b** 15 **c** $5\sqrt{2}$ **d** $\sqrt{30}$

4 $k = 5$ or $k = 9$ **5** $k = 10$ or $k = -4$

Challenge

a $(1, -3, 4), (1, -3, -2), (7, 3, 4), (7, 3, -2), (7, -3, -2)$

b $6\sqrt{5}$