P1 Chapter 10: Trigonometry Equations

Harder Trig Equations

Harder Equations

Harder questions replace the angle θ with a linear expression.

Solve $\cos 3x = -\frac{1}{2}$ in the interval $0 \leq x \leq 360°$.

$$0 \leq 3x < 1080°$$

$$3x = \cos^{-1}\left(-\frac{1}{2}\right) = 120°$$

$$3x = 120°, 240°, 480°, 600°, 840°, 960°$$

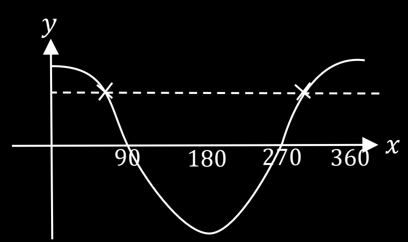
$$x = 40°, 80°, 160°, 200°, 280°, 320°$$

STEP 1: Adjust the range of values for θ to match the expression inside the cos.

STEP 2: Immediately after applying an inverse trig function (and BEFORE dividing by 3!), find all solutions up to the end of the interval.

STEP 3: Then do final manipulation to each value.

Froflections: As mentioned before, in general you tend to get a pair of values per 360° (for any of sin/cos/tan), except for $\cos\theta = \pm 1$ or $\sin\theta = \pm 1$:



Thus once getting your first pair of values (e.g. using $\sin(180 - \theta)$ or $\cos(360 - \theta)$ to get the second value), keep adding 360° to generate new pairs.

Further Examples

Solve $\sin(2x + 30°) = \frac{1}{\sqrt{2}}$ in the interval $0 \leq x \leq 360°$.

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Solve $\sin x = 2\cos x$ in the interval $0 \leq x < 300°$

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Further Examples

Solve $\sin(2x+30°)=\frac{1}{\sqrt{2}}$ in the interval $0 \leq x \leq 360°$.

$$30° \leq 2x+30° \leq 750°$$
$$2x+30° = 45°, 135°, 405°, 495°$$
$$2x = 15°, 105°, 375°, 465°$$
$$x = 7.5°, 52.5°, 187.5°, 232.5°$$

To get from x to $2x+30°$ we double and add 30°. So do the same to the upper and lower bound!

Solve $\sin x = 2\cos x$ in the interval $0 \leq x < 300°$

$\tan x = 2$
$x = \tan^{-1} 2 = 63.43°, 243.43°$

By dividing both sides by $\cos x$, the $\sin x$ becomes $\tan x$ and the $\cos x$ disappears, leaving a trig equation helpfully only in terms of one trig function.

Test Your Understanding

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Solve, for $0 \leq x < 180°$,

$$\cos(3x - 10°) = -0.4,$$

giving your answers to 1 decimal place. You should show each step in your working.

(7)

?

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$\cos^{-1}(-0.4) = 113.58 \ (\alpha)$	Awrt 114	B1
$3x - 10 = \alpha \Rightarrow x = \dfrac{\alpha + 10}{3}$	Uses their α to find x. Allow $x = \dfrac{\alpha \pm 10}{3}$ **not** $\dfrac{\alpha}{3} \pm 10$	M1
Note: If $x = \dfrac{\alpha \pm 10}{3}$ is not clearly applied from their first angle it may be recovered if applied to their second or third angle.		
$x = 41.2$	Awrt	A1
$(3x - 10 =) 360 - \alpha \ (246.4....)$	$360 - \alpha$ (can be implied by 246.4...)	M1
$x = 85.5$	Awrt	A1
$(3x - 10 =) 360 + \alpha \ (= 473.57....)$	$360 + \alpha$ (Can be implied by 473.57...)	M1
$x = 161.2$	Awrt	A1
Note 1: Do not penalise incorrect accuracy more than once and penalise it the first time it occurs. E.g if answers are only given to the nearest integer (41, 85, 161) only the first A mark that would otherwise be scored is lost.		
Note 2: Ignore any answers outside the range. For extra answers in range in an otherwise **fully** correct solution lose final A1		
Note 3: Lack of working means that it is sometimes not clear where their intermediate angles are coming from. In these cases, if the final answers are incorrect score M0.		
Note 4: Candidates are unlikely to be working in radians deliberately but may have their calculator in radian mode (gives α = 1.98). In such cases the main scheme should be applied and the method marks are available. If you suspect that the candidate is working in radians correctly then please use the review mechanism and/or consult your team leader.		

Exercise 10.5

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Homework Exercise

1 Find the values of θ, in the interval $0 \leqslant \theta \leqslant 360°$, for which:

a $\sin 4\theta = 0$ **b** $\cos 3\theta = -1$ **c** $\tan 2\theta = 1$

d $\cos 2\theta = \frac{1}{2}$ **e** $\tan \frac{1}{2}\theta = -\frac{1}{\sqrt{3}}$ **f** $\sin(-\theta) = \frac{1}{\sqrt{2}}$

2 Solve the following equations in the interval given:

a $\tan(45° - \theta) = -1, 0 \leqslant \theta \leqslant 360°$ **b** $2\sin(\theta - 20°) = 1, 0 \leqslant \theta \leqslant 360°$

c $\tan(\theta + 75°) = \sqrt{3}, 0 \leqslant \theta \leqslant 360°$ **d** $\sin(\theta - 10°) = -\frac{\sqrt{3}}{2}, 0 \leqslant \theta \leqslant 360°$

e $\cos(70° - x) = 0.6, 0 \leqslant \theta \leqslant 180°$

3 Solve the following equations in the interval given:

a $3\sin 3\theta = 2\cos 3\theta, 0 \leqslant \theta \leqslant 180°$

b $4\sin(\theta + 45°) = 5\cos(\theta + 45°), 0 \leqslant \theta \leqslant 450°$

c $2\sin 2x - 7\cos 2x = 0, 0 \leqslant x \leqslant 180°$

d $\sqrt{3}\sin(x - 60°) + \cos(x - 60°) = 0, -180° \leqslant x \leqslant 180°$

4 Solve for $0 \leqslant x \leqslant 180°$ the equations:

a $\sin(x + 20°) = \frac{1}{2}$ **(4 marks)**

b $\cos 2x = -0.8$, giving your answers to 1 decimal place. **(4 marks)**

Homework Exercise

5 **a** Sketch for $0 \leqslant x \leqslant 360°$ the graph of $y = \sin(x + 60°)$ **(2 marks)**

b Write down the exact coordinates of the points where the graph meets the coordinate axes. **(3 marks)**

c Solve, for $0 \leqslant x \leqslant 360°$, the equation $\sin(x + 60°) = 0.55$, giving your answers to 1 decimal place. **(5 marks)**

6 **a** Given that $4\sin x = 3\cos x$, write down the value of $\tan x$. **(1 mark)**

b Solve, for $0 \leqslant \theta \leqslant 360°$, $4\sin 2\theta = 3\cos 2\theta$ giving your answers to 1 decimal place. **(5 marks)**

7 The equation $\tan kx = -\frac{1}{\sqrt{3}}$, where k is a constant and $k > 0$, has a solution at $x = 60°$

a Find a possible value of k. **(3 marks)**

b State, with justification, whether this is the only such possible value of k. **(1 mark)**

Challenge

Solve the equation $\sin(3x - 45°) = \frac{1}{2}$ in the interval $0 \leqslant x \leqslant 180°$.

Homework Answers

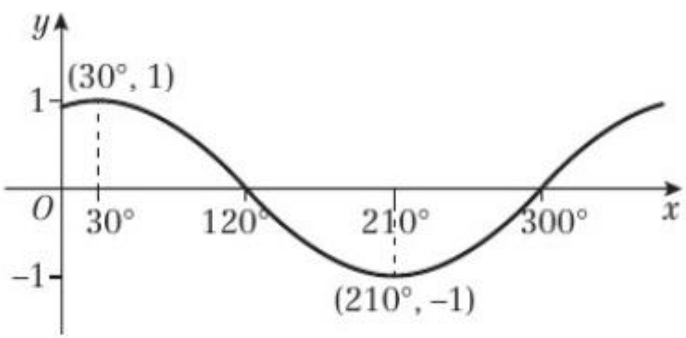
1 **a** 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°
b 60°, 180°, 300°
c $22\frac{1}{2}°$, $112\frac{1}{2}°$, $202\frac{1}{2}°$, $292\frac{1}{2}°$
d 30°, 150°, 210°, 330°
e 300°
f 225°, 315°

2 **a** 90°, 270° **b** 50°, 170° **c** 165°, 345°
d 250°, 310° **e** 16.9°, 123°

3 **a** 11.2°, 71.2°, 131.2° **b** 6.3°, 186.3°, 366.3°
c 37.0°, 127.0° **d** –150°, 30°

4 **a** 10°, 130° **b** 71.6°, 108.4°

5 **a**



b $\left(0°, \frac{\sqrt{3}}{2}\right)$, (120°, 0), (300°, 0)

c 86.6°, 333.4°

6 **a** 0.75
b 18.4°, 108.4°, 198.4°, 288.4°

7 **a** 2.5
b No: increasing k will bring another 'branch' of the tan graph into place.

Challenge
25°, 65°, 145°