P1 Chapter 2: Quadratics

Solving Quadratics

Solving Quadratic Equations

$$x^2 + 5x = 6$$

There are three ways of solving a quadratic equation.
One is by *completing the square*, which we'll do later.
What are the other two ways?

By factorisation

$$x^2 + 5x - 6 = 0$$
$$(x + 6)(x - 1) = 0$$
$$x = -6, x = 1$$

Put in form. $ax^2 + bx + c = 0$ with 0 on one side

Factorise. If the product of two things is 0, at least one must be 0.

Using the quadratic formula.

If $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + 5x - 6 = 0$$
$$a = 1, b = 5, c = -6$$
$$x = \frac{-5 \pm \sqrt{25 + 24}}{2}$$
$$= \frac{-5 \pm 7}{2}$$
$$= 1 \text{ or } -6$$

1 :: Solving Quadratic Equations

Solving without factorising

If the subject only appears once however, it might be easier not to expand out/factorise:

$$(x-1)^2 = 5$$
$$x - 1 = \pm\sqrt{5}$$
$$x = 1 \pm \sqrt{5}$$

If you can't see why the $\pm$ is required, think about the solutions to: $x^2 = 4$.
$2^2 = 4$, but $(-2)^2 = 4$ as well!
So $x = \pm 2$.

Quadratics 'in disguise'

When we have an expression like say $x^2 + 3x - 2$, we say it is "quadratic in x".
In trigonometry you will have to solve equations like $(\sin x)^2 + 3\sin x + 2$. We say that the expression is "quadratic in $\sin x$".
Either use a suitable substitution so that you have a 'normal' quadratic, or go straight for the factorisation if you're feeling more confident (recommended!).

Solve $x - 6\sqrt{x} + 8 = 0$

Let $y = \sqrt{x}$, then:
$y^2 - 6y + 8 = 0$
$(y-2)(y-4) = 0$
$y = 2, y = 4 \quad \rightarrow \quad \sqrt{x} = 2, \sqrt{x} = 4$
$x = 4 \; or \; x = 16$

substitution

unchanged

$(\sqrt{x}-2)(\sqrt{x}-4) = 0$
$\sqrt{x} = 2 \; or \; \sqrt{x} = 4$
$x = 4 \; or \; x = 16$

Test Your Understanding

1 Solve $(x+3)^2 = x+5$ using factorisation.

?

2 Solve $(2x+1)^2 = 5$

?

3 Solve $\sqrt{x+3} = x-3$

?

4 Solve $2x + \sqrt{x} - 1 = 0$

?

Calculator method:
The 'equation' mode on your calculator will solve quadratics in the form $ax^2 + bx + c = 0$. When you're asked for the 'order', use 2 (we'll see why later).

Test Your Understanding

1 Solve $(x+3)^2 = x+5$ using factorisation.

$$x^2 + 6x + 9 = x + 5$$
$$x^2 + 5x + 4 = 0$$
$$(x+4)(x+1) = 0$$
$$x = -4 \; or \; x = -1$$

2 Solve $(2x+1)^2 = 5$

$$2x + 1 = \pm\sqrt{5}$$
$$2x = -1 \pm \sqrt{5}$$
$$x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{5}$$

3 Solve $\sqrt{x+3} = x - 3$

Squaring both sides:

$$x + 3 = (x-3)^2$$
$$x + 3 = x^2 - 6x + 9$$
$$x^2 - 7x + 6 = 0$$
$$(x-6)(x-1) = 0$$
$$x = 1 \; or \; x = 6$$

However, squaring both sides of an equation **can generate false solutions**. e.g. If $x = -2$, squaring gives $x^2 = 4$, which falsely creates the extra solution $x = 2$! We therefore need to check by substituting into the original equation, where we find only $x = 6$ works.

4 Solve $2x + \sqrt{x} - 1 = 0$

$$(2\sqrt{x} - 1)(\sqrt{x} + 1) = 0$$
$$\sqrt{x} = \frac{1}{2} \; or \; \sqrt{x} = -1$$
$$x = \frac{1}{4}$$

The latter gives no solution because the square root of a number can't give a negative number.

Calculator method:
The 'equation' mode on your calculator will solve quadratics in the form $ax^2 + bx + c = 0$. When you're asked for the 'order', use 2 (we'll see why later).

Exercise 2.1

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1 **(i)** Use the substitution $\sqrt{x} = y$ (where $y \geqslant 0$) to find the real root of the equation

$$x + 3\sqrt{x} - \tfrac{1}{2} = 0\,.$$

(ii) Find all real roots of the following equations:

(a) $x + 10\sqrt{x+2} - 22 = 0\,;$

(b) $x^2 - 4x + \sqrt{2x^2 - 8x - 3} - 9 = 0\,.$

? ii(a)

? ii(b)

? i

Exercise 2.1

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$$y^2 + 3y - \frac{1}{2} = 0$$
$$2y^2 + 6y - 1 = 0$$
$$y = \frac{-6 \pm \sqrt{36+8}}{4}$$
$$\sqrt{x} = -\frac{3}{2} \pm \frac{\sqrt{11}}{2}$$

Since $\sqrt{x} > 0$:

$$\sqrt{x} = -\frac{3}{2} + \frac{\sqrt{11}}{2}$$
$$x = 5 - \frac{3}{2}\sqrt{11}$$

(ii)(a)
Use $y = \sqrt{x+2}$, thus $x = y^2 - 2$
$\rightarrow y^2 - 2 + 10y - 22 = 0$
... $\rightarrow x = 2$

(ii)(b): $x = 2 \pm \sqrt{10}$

Exercise 1.1

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Homework Exercise

1 Solve the following equations using factorisation:

a $x^2 + 3x + 2 = 0$ **b** $x^2 + 5x + 4 = 0$ **c** $x^2 + 7x + 10 = 0$ **d** $x^2 - x - 6 = 0$

e $x^2 - 8x + 15 = 0$ **f** $x^2 - 9x + 20 = 0$ **g** $x^2 - 5x - 6 = 0$ **h** $x^2 - 4x - 12 = 0$

2 Solve the following equations using factorisation:

a $x^2 = 4x$ **b** $x^2 = 25x$ **c** $3x^2 = 6x$ **d** $5x^2 = 30x$

e $2x^2 + 7x + 3 = 0$ **f** $6x^2 - 7x - 3 = 0$ **g** $6x^2 - 5x - 6 = 0$ **h** $4x^2 - 16x + 15 = 0$

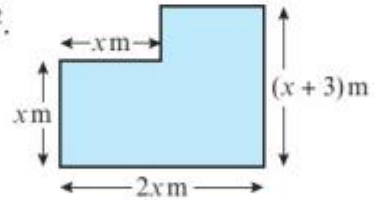
3 Solve the following equations:

a $3x^2 + 5x = 2$ **b** $(2x - 3)^2 = 9$ **c** $(x - 7)^2 = 36$ **d** $2x^2 = 8$ **e** $3x^2 = 5$

f $(x - 3)^2 = 13$ **g** $(3x - 1)^2 = 11$ **h** $5x^2 - 10x^2 = -7 + x + x^2$

i $6x^2 - 7 = 11x$ **j** $4x^2 + 17x = 6x - 2x^2$

4 This shape has an area of $44\,\text{m}^2$.
Find the value of x.



Problem-solving

Divide the shape into two sections:

5 Solve the equation $5x + 3 = \sqrt{3x + 7}$.

6 Solve the following equations using the quadratic formula.
Give your answers exactly, leaving them in surd form where necessary.

a $x^2 + 3x + 1 = 0$ **b** $x^2 - 3x - 2 = 0$ **c** $x^2 + 6x + 6 = 0$ **d** $x^2 - 5x - 2 = 0$

e $3x^2 + 10x - 2 = 0$ **f** $4x^2 - 4x - 1 = 0$ **g** $4x^2 - 7x = 2$ **h** $11x^2 + 2x - 7 = 0$

Homework Exercise

7 Solve the following equations using the quadratic formula.
Give your answers to three significant figures.

a $x^2 + 4x + 2 = 0$ **b** $x^2 - 8x + 1 = 0$ **c** $x^2 + 11x - 9 = 0$ **d** $x^2 - 7x - 17 = 0$

e $5x^2 + 9x - 1 = 0$ **f** $2x^2 - 3x - 18 = 0$ **g** $3x^2 + 8 = 16x$ **h** $2x^2 + 11x = 5x^2 - 18$

8 For each of the equations below, choose a suitable method and find all of the solutions.
Where necessary, give your answers to three significant figures.

a $x^2 + 8x + 12 = 0$ **b** $x^2 + 9x - 11 = 0$

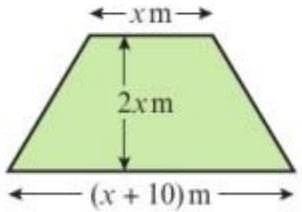
c $x^2 - 9x - 1 = 0$ **d** $2x^2 + 5x + 2 = 0$

e $(2x + 8)^2 = 100$ **f** $6x^2 + 6 = 12x$

g $2x^2 - 11 = 7x$ **h** $x = \sqrt{8x - 15}$

Hint You can use any method you are confident with to solve these equations.

9 This trapezium has an area of $50\,\text{m}^2$.
Show that the height of the trapezium is equal to $5(\sqrt{5} - 1)\,\text{m}$.



Problem-solving

Height must be positive. You will have to discard the negative solution of your quadratic equation.

Challenge

Given that x is positive, solve the equation

$$\frac{1}{x} + \frac{1}{x+2} = \frac{28}{195}$$

Hint Write the equation in the form $ax^2 + bx + c = 0$ before using the quadratic formula or factorising.

Homework Answers

1 **a** $x = -1$ or $x = -2$ **b** $x = -1$ or $x = -4$

c $x = -5$ or $x = -2$ **d** $x = 3$ or $x = -2$

e $x = 3$ or $x = 5$ **f** $x = 4$ or $x = 5$

g $x = 6$ or $x = -1$ **h** $x = 6$ or $x = -2$

2 **a** $x = 0$ or $x = 4$ **b** $x = 0$ or $x = 25$

c $x = 0$ or $x = 2$ **d** $x = 0$ or $x = 6$

e $x = -\frac{1}{2}$ or $x = -3$ **f** $x = -\frac{1}{3}$ or $x = \frac{3}{2}$

g $x = -\frac{2}{3}$ or $x = \frac{3}{2}$ **h** $x = \frac{3}{2}$ or $x = \frac{5}{2}$

3 **a** $x = \frac{1}{3}$ or $x = -2$ **b** $x = 3$ or $x = 0$

c $x = 13$ or $x = 1$ **d** $x = 2$ or $x = -2$

e $x = \pm\sqrt{\frac{5}{3}}$ **f** $x = 3 \pm \sqrt{13}$

g $x = \dfrac{1 \pm \sqrt{11}}{3}$ **h** $x = 1$ or $x = -\frac{7}{6}$

i $x = -\frac{1}{2}$ or $x = \frac{7}{3}$ **j** $x = 0$ or $x = -\frac{11}{6}$

4 $x = 4$

5 $x = -1$ or $x = -\frac{2}{25}$

6 **a** $x = \frac{1}{2}(-3 \pm \sqrt{5})$ **b** $x = \frac{1}{2}(3 \pm \sqrt{17})$

c $x = -3 \pm \sqrt{3}$ **d** $x = \frac{1}{2}(5 \pm \sqrt{33})$

e $x = \frac{1}{3}(-5 \pm \sqrt{31})$ **f** $x = \frac{1}{2}(1 \pm \sqrt{2})$

g $x = 2$ or $x = -\frac{1}{4}$ **h** $x = \frac{1}{11}(-1 \pm \sqrt{78})$

7 **a** $x = -0.586$ or $x = -3.41$ **b** $x = 7.87$ or $x = 0.127$

c $x = 0.765$ or $x = -11.8$ **d** $x = 8.91$ or $x = -1.91$

e $x = 0.105$ or $x = -1.90$ **f** $x = 3.84$ or $x = -2.34$

g $x = 4.77$ or $x = 0.558$ **h** $x = 4.89$ or $x = -1.23$

8 **a** $x = -6$ or $x = -2$ **b** $x = 1.09$ or $x = -10.1$

c $x = 9.11$ or $x = -0.110$ **d** $x = -\frac{1}{2}$ or $x = -2$

e $x = 1$ or $x = -9$ **f** $x = 1$

g $x = 4.68$ or $x = -1.18$ **h** $x = 3$ or $x = 5$

9 Area $= \frac{1}{2}(2x)\,(x + (x+10)) = 50\,\text{m}^2$

So $x^2 + 5x - 25 = 0$

Using the quadratic formula:

$x = \frac{1}{2}(-5 \pm 5\sqrt{5})$

Height $= 2x = 5\,(\sqrt{5} - 1)\,\text{m}$

Challenge

$x = 13$