Stats1 Chapter 5: Probability

Tree Diagrams

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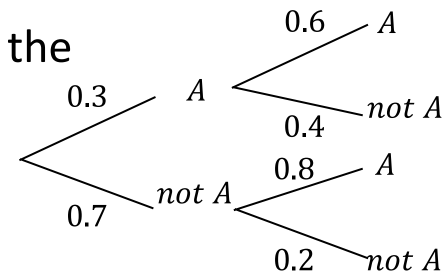
At GCSE we saw that tree diagrams were an effective way of showing the outcome of two events which happen **in succession**.

(Personal opinion however is that their use is easily avoidable)



There are 3 yellow and 2 green counters in a bag. I take two counters at random. Determine the probability that:

a) They are of the same colour.

b) They are of different colours.

a

?

Tip: List out the matching sequences of outcomes first, then find the probability for each.

b

?

The probability I hit a target on each shot is 0.3. I keep firing until I hit the target. Determine the probability I hit the target on the 5^{th} shot.

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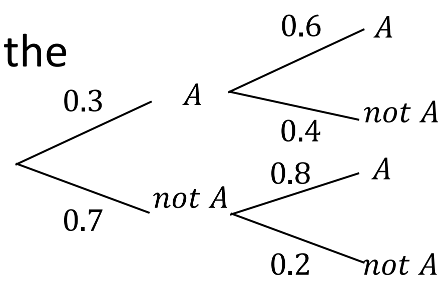

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a

$$YY: \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$$
$$GG: \frac{2}{5} \times \frac{1}{4} = \frac{2}{20}$$
$$P(same) = \frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5}$$

Tip: List out the matching sequences of outcomes first, then find the probability for each.

b

$$GY: \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$$
$$YG: \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$$
$$P(different) = \frac{6}{20} + \frac{6}{20} = \frac{3}{5}$$

(Or we could have used our answer to (a): $1 - \frac{2}{5} = \frac{3}{5}$)

Note that if the outcomes are the same but reordered, the probability will be the same. So we could have quickly done $\frac{2}{5} \times \frac{3}{4} \times 2$

The probability I hit a target on each shot is 0.3. I keep firing until I hit the target. Determine the probability I hit the target on the 5th shot.

$$0.7^4 \times 0.3 = 0.07203$$

Exercise 5D

Pearson Pure Mathematics Year 1/AS
Pages 35-36

Extension Questions

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1 [STEP I 2010 Q12] Prove that, for any real numbers x and y, $x^2 + y^2 \geq 2xy$.

(i) Carol has two bags of sweets. The first bag contains a red sweets and b blue sweets, whereas the second bag contains b red sweets and a blue sweets. Carol shakes the bags and picks one sweet from each bag without looking. Prove that the probability that the sweets are of the same colour cannot exceed the probability that they are of different colours.

(ii) Simon has three bags of sweets. The first bag contains a red sweet, b white sweets and c yellow sweets. The second bag contains b red sweets, c white sweets and a yellow sweets. The third bag contains c red sweets, a white sweets and b yellow sweets. Simon shakes the bags and picks one sweet from each bag without looking. Show that the probability that exactly two of the sweets are of the same colour is

$$\frac{3(a^2b + b^2c + c^2a + ab^2 + bc^2 + ca^2)}{(a+b+c)^3}$$

and find the probability that the sweets are all of the same colour. Deduce that the probability that exactly two of the sweets are of the same colour is at least 6 times the probability that the sweets are all of the same colour.

2 [STEP I 2011 Q12] I am selling raffle tickets for £1 per ticket. In the queue for tickets, there are m people each with a single £1 coin and n people each with a single £2 coin. Each person in the queue wants to buy a single raffle ticket and each arrangement of people in the queue is equally likely to occur. Initially, I have no coins and a large supply of tickets. I stop selling tickets if I cannot give the required change.

(i) In the case $n = 1$ and , $m \geq 1$, find the probability that I am able to sell one ticket each person in the queue.

(ii) By considering the first people in the queue, show that the probability that I am able to sell one ticket to each person in the queue in the case $n = 2$ and $m \geq 2$ is $\frac{m-1}{m+1}$

(iii) Show that the probability that I am able to sell one ticket to each person in the queue in the case $n = 3$ and $m \geq 3$ is $\frac{m-2}{m+1}$.

3 I have an unfair coin with a fixed probability p of heads. Determine how the unfair coin could be used to simulate a fair coin, i.e. you declare “Heads” or “Tails” each with probability 0.5.

?

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3 I have an unfair coin with a fixed probability p of heads. Determine how the unfair coin could be used to simulate a fair coin, i.e. you declare "Heads" or "Tails" each with probability 0.5.

Throw the coin twice. Then the probability of each sequence:

$$\boldsymbol{P(HH) = p^2 \quad P(TT) = (1-p)^2}$$
$$\boldsymbol{P(HT) = p(1-p) \quad P(TH) = p(1-p)}$$

Note that two of these have the same probability. So if the first throw is Heads and the second Tails, declare "Heads", or if Tails then Heads, declare "Tails". If the two throws are the same, repeat the process until the two throws are different.

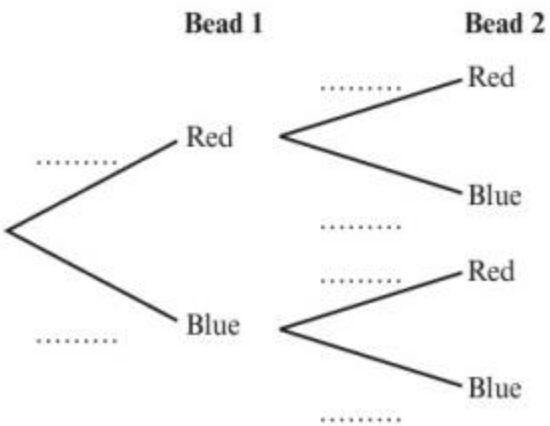
Homework Exercise

1 A bag contains three red beads and five blue beads. A bead is chosen at random from the bag, the colour is recorded and the bead is replaced. A second bead is chosen and the colour recorded.

a Copy and complete this tree diagram to show the outcomes of the experiment.

b Find the probability that both beads are blue.

c Find the probability that the second bead is blue.



2 A box contains nine cards numbered 1 to 9. A card is drawn at random and not replaced. It is noted whether the number is odd or even. A second card is drawn and it is also noted whether this number is odd or even.

a Draw a tree diagram to represent this experiment.

b Find the probability that both cards are even.

c Find the probability that one card is odd and the other card is even.

Hint The first card is not replaced.

3 The probability that Charlie takes the bus to school is 0.4. If he doesn't take the bus, he walks. The probability that Charlie is late to school if he takes the bus is 0.2. The probability he is late to school if he walks is 0.3.

a Draw a tree diagram to represent this information.

b Find the probability that Charlie is late to school.

Homework Exercise

4 Mr Dixon plays golf. The probability that he scores par or under on the first hole is 0.7. If he scores par or under on the first hole, the probability he scores par or under on the second hole is 0.8. If he doesn't score par or under on the first hole, the probability that he scores par or under on the second hole is 0.4.

a Draw a tree diagram to represent this information. **(3 marks)**

b State whether the events 'scores par or under on the first hole' and 'scores par or under par on the second hole' are independent. **(1 mark)**

c Find the probability that Mr Dixon scores par or under on only one hole. **(3 marks)**

5 A biased coin is tossed three times and it is recorded whether it falls heads or tails. P(heads) = $\frac{1}{3}$

a Draw a tree diagram to represent this experiment. **(3 marks)**

b Find the probability that the coin lands on heads all three times. **(1 mark)**

c Find the probability that the coin lands on heads only once. **(2 marks)**

The whole experiment is repeated for a second trial.

d Find the probability of obtaining either 3 heads or 3 tails in both trials. **(3 marks)**

6 A bag contains 13 tokens, 4 coloured blue, 3 coloured red and 6 coloured yellow. Two tokens are drawn from the bag without replacement.

a Find the probability that both tokens are yellow. **(2 marks)**

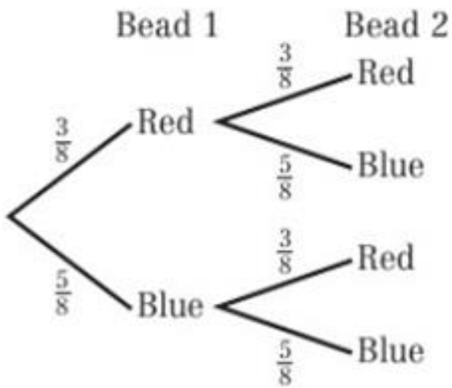
A third token is drawn from the bag.

b Write down the probability that the third token is yellow, given that the first two are yellow. **(1 mark)**

c Find the probability that all three tokens are different colours. **(4 marks)**

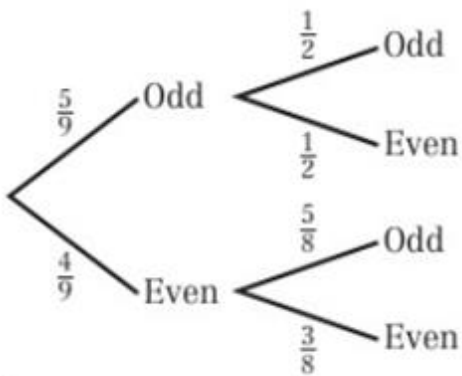
Homework Answers

1 a



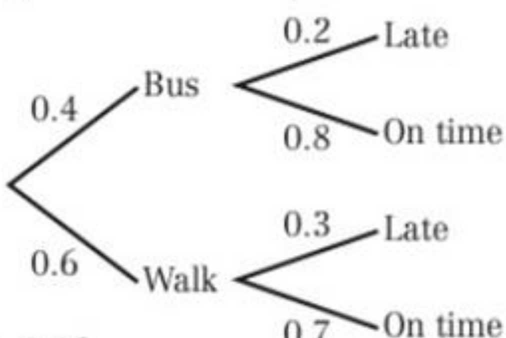
b $\frac{25}{64}$ **c** $\frac{5}{8}$

2 **a**



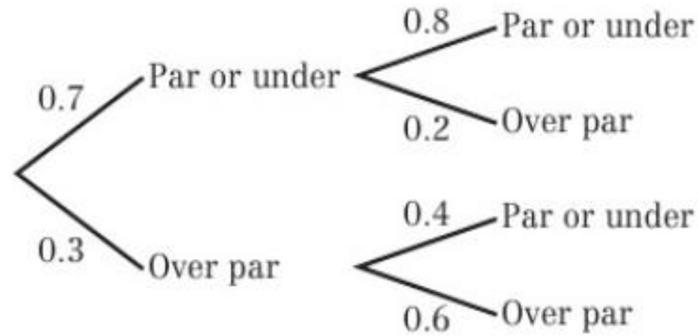
b $\frac{1}{6}$ **c** $\frac{5}{9}$

3 **a**



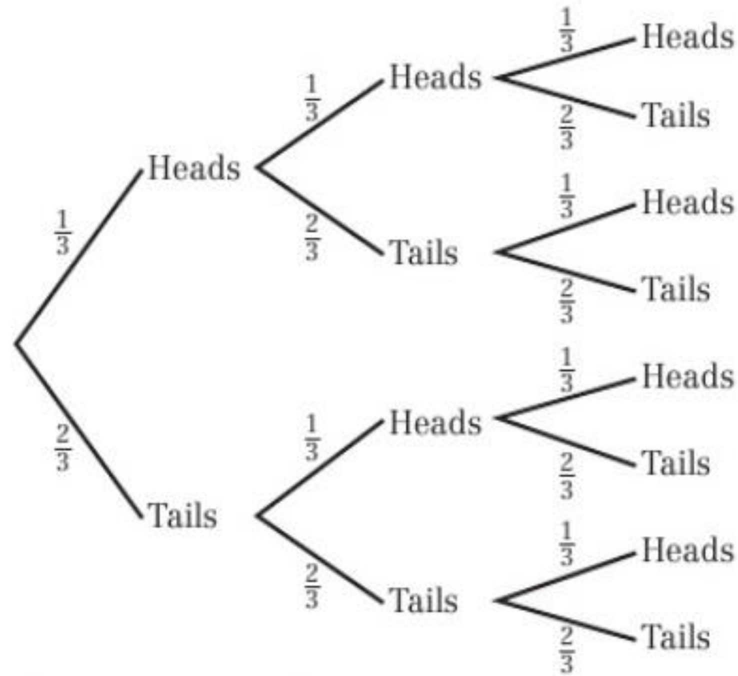
b 0.26

4 **a**



b Not independent **c** 0.26

5 **a**



b $\frac{1}{27}$ **c** $\frac{4}{9}$ **d** $\frac{1}{9}$

6 **a** $\frac{5}{26}$ **b** $\frac{4}{11}$ **c** $\frac{36}{143}$