# P1 Chapter 2: Quadratics

## Functions

# Function Machines

A function is something which **provides a rule on how to map inputs to outputs**.

We saw at GCSE that functions were a formal way of describing a ‘number machine’:

Input | | Output
--- | --- | ---
$x$ | $\rightarrow \boxed{\;f\;} \rightarrow$ | $2x$

Name of the function (often $f$ or $g$)

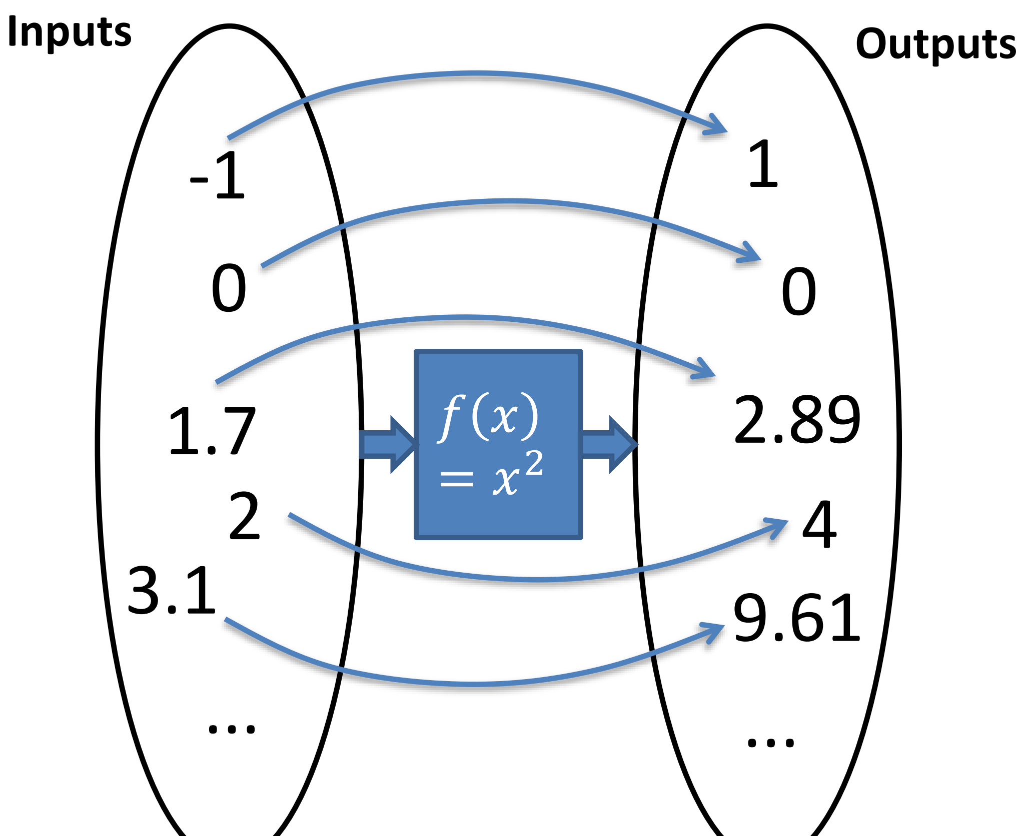
Input

Output

$$f(x) = 2x$$

# Function Maps

You'll cover functions extensively in future chapters, but for now, you need to understand the following concepts:




The domain of a function could potentially be **any** real number. If so, we'd write:

$$x \in \mathbb{R}$$

The input $x$... is a member of... the set of real numbers

We might be interested in what inputs $x$ give an output of 0. These are known as the **roots** of the function.

✏ The **domain** of a function is the set of possible inputs.

✏ The **range** of a function is the set of possible outputs.

✏ The **roots/zeroes** of a function are the values of $x$ for which $f(x) = 0$.

# Examples

If $f(x) = x^2 - 3x$ and $g(x) = x + 5, \quad x \in \mathbb{R}$

a) Find $f(-4)$

b) Find the values of $x$ for which $f(x) = g(x)$

c) Find the roots of $f(x)$.

d) Find the roots of $g(x)$.

**Fro Note**: The domain is usually stated for you.

a

?

b

?

c

?

d

?

# Examples

If $f(x) = x^2 - 3x$ and $g(x) = x + 5$,   $x \in \mathbb{R}$

a) Find $f(-4)$

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c) Find the roots of $f(x)$.

d) Find the roots of $g(x)$.

**Fro Note**: The domain is usually stated for you.

a

$$f(-4) = (-4)^2 - 3(-4)$$
$$= 28$$

b

Conceptually, we're looking for the inputs of the functions which give the same outputs. We can just equate the output expressions.

$$x^2 - 3x = x + 5$$
$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$
$$x = 5 \; or \; x = -1$$

c

The roots are the inputs which give an output of 0. So set output expression to 0.

$$x^2 - 3x = 0$$
$$x(x-3) = 0$$
$$x = 0 \; or \; x = 3$$

d

$$x + 5 = 0$$
$$x = -5$$

# Examples

Determine the minimum value of the function $f(x) = x^2 - 6x + 2$, and state the value of $x$ for which this minimum occurs.

This means we want to minimise the **output** of the function.

You might try a (clumsy) approach of trying a few values of $x$ and try to see what makes the output as small as possible…

?

But the best way to find the minimum/maximum value of a quadratic is to **complete the square**:

?

# Examples

Determine the minimum value of the function $f(x) = x^2 - 6x + 2$, and state the value of $x$ for which this minimum occurs.

This means we want to minimise the **output** of the function.

You might try a (clumsy) approach of trying a few values of $x$ and try to see what makes the output as small as possible…

$$f(1) = 1 - 6 + 2 = -3$$
$$f(2) = 4 - 12 + 2 = -6$$
$$f(3) = 9 - 18 + 2 = -7$$
$$f(4) = 16 - 24 + 2 = -6$$

This looks like the minimum as the value starts going up after.

But the best way to find the minimum/maximum value of a quadratic is to **complete the square**:

$$f(x) = (x-3)^2 - 7$$

Since anything squared is at least 0, the smallest we can make the bracket is 0, which occurs when $x = 3$.

$$f(1) = (-2)^2 - 7 = -3$$
$$f(2) = (-1)^2 - 7 = -6$$
$$f(3) = 0^2 - 7 = -7$$
$$f(4) = 1^2 - 7 = -6$$

✎ If $f(x) = (x+a)^2 + b$, the minimum value of $f(x)$ is $b$, which occurs when $x = -a$.

# Quickfire Questions

| $f(x)$ | Completed square | Min/max value of $f(x)$ | $x$ for which this min/max occurs |
|---|---|---|---|
| $x^2 + 4x + 9$ | ? | ? | ? |
| $x^2 - 10x + 21$ | ? | ? | ? |
| $10 - x^2$ | ? | ? | ? |
| $8 - x^2 + 6x$ | ? | ? | ? |

# Quickfire Questions

| $f(x)$ | Completed square | Min/max value of $f(x)$ | $x$ for which this min/max occurs |
|---|---|---|---|
| $x^2 + 4x + 9$ | $(x+2)^2 + 5$ | $5$ | $-2$ |
| $x^2 - 10x + 21$ | $(x-5)^2 - 4$ | $-4$ | $5$ |
| $10 - x^2$ | Already completed | $10$ | $0$ |
| $8 - x^2 + 6x$ | $17 - (x-3)^2$ | $17$ | $3$ |

# Test Your Understanding

1 Find the minimum value of $f(x) = 2x^2 + 12x - 5$ and state the value of $x$ for which this occurs.

?

2 Find the roots of the function $f(x) = 2x^2 + 3x + 1$

?

3 Find the roots of the function $f(x) = x^4 - x^2 - 6$

?

# Test Your Understanding

**1** Find the minimum value of $f(x) = 2x^2 + 12x - 5$ and state the value of $x$ for which this occurs.

$$f(x) = 2(x^2 + 6x) - 5$$
$$= 2\left((x+3)^2 - 9\right) - 5$$
$$= 2(x+3)^2 - 18 - 5$$
$$= 2(x+3)^2 - 23$$

Minimum value is -23.
$x$ at which this occurs is -3.

**2** Find the roots of the function $f(x) = 2x^2 + 3x + 1$

$$2x^2 + 3x + 1 = 0$$
$$(2x+1)(x+1) = 0$$
$$x = -\frac{1}{2} \ or\ x = -1$$

**3** Find the roots of the function $f(x) = x^4 - x^2 - 6$

$$x^4 - x^2 - 6 = 0$$
$$(x^2+2)(x^2-3) = 0$$
$$x^2 = -2 \ or\ x^2 = 3$$
$$x = \pm\sqrt{3}$$

# Exercise 2.3

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# Homework Exercise

**1** Using the functions $f(x) = 5x + 3$, $g(x) = x^2 - 2$ and $h(x) = \sqrt{x+1}$, find the values of:

**a** $f(1)$ **b** $g(3)$ **c** $h(8)$ **d** $f(1.5)$ **e** $g(\sqrt{2})$

**f** $h(-1)$ **g** $f(4) + g(2)$ **h** $f(0) + g(0) + h(0)$ **i** $\dfrac{g(4)}{h(3)}$

**2** The function $f(x)$ is defined by $f(x) = x^2 - 2x$, $x \in \mathbb{R}$.
Given that $f(a) = 8$, find two possible values for $a$.

**Problem-solving**

Substitute $x = a$ into the function and set the resulting expression equal to 8.

**3** Find all of the roots of the following functions:

**a** $f(x) = 10 - 15x$ **b** $g(x) = (x + 9)(x - 2)$ **c** $h(x) = x^2 + 6x - 40$

**d** $j(x) = 144 - x^2$ **e** $k(x) = x(x + 5)(x + 7)$ **f** $m(x) = x^3 + 5x^2 - 24x$

**4** The functions p and q are given by $p(x) = x^2 - 3x$ and $q(x) = 2x - 6$, $x \in \mathbb{R}$.
Find the two values of $x$ for which $p(x) = q(x)$.

**5** The functions f and g are given by $f(x) = 2x^3 + 30x$ and $g(x) = 17x^2$, $x \in \mathbb{R}$.
Find the three values of $x$ for which $f(x) = g(x)$.

**6** The function f is defined as $f(x) = x^2 - 2x + 2$, $x \in \mathbb{R}$.

**a** Write $f(x)$ in the form $(x + p)^2 + q$, where $p$ and $q$ are constants to be found. **(2 marks)**

**b** Hence, or otherwise, explain why $f(x) > 0$ for all values of $x$, and find the minimum value of $f(x)$. **(1 mark)**

# Homework Exercise

**7** Find all roots of the following functions:

**a** $f(x) = x^6 + 9x^3 + 8$

**b** $g(x) = x^4 - 12x^2 + 32$

**c** $h(x) = 27x^6 + 26x^3 - 1$

**d** $j(x) = 32x^{10} - 33x^5 + 1$

**e** $k(x) = x - 7\sqrt{x} + 10$

**f** $m(x) = 2x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 12$

**Hint** The function in part **b** has four roots.

**8** The function f is defined as $f(x) = 3^{2x} - 28(3^x) + 27$, $x \in \mathbb{R}$.

**a** Write $f(x)$ in the form $(3^x - a)(3^x - b)$, where $a$ and $b$ are real constants. **(2 marks)**

**b** Hence find the two roots of $f(x)$. **(2 marks)**

**Problem-solving**

Consider $f(x)$ as a function of a function.

# Homework Answers

**1** **a** 8 **b** 7 **c** 3 **d** 10.5 **e** 0
**f** 0 **g** 25 **h** 2 **i** 7

**2** $a = 4$ or $a = -2$

**3** **a** $\frac{2}{3}$ **b** 2 and $-9$ **c** $-10$ and 4
**d** 12 and $-12$ **e** 0, $-5$ and $-7$ **f** 0, 3 and $-8$

**4** $x = 3$ and $x = 2$

**5** $x = 0$, $x = 2.5$ and 6

**6** **a** $(x - 1)^2 + 1$
$p = -1$, $q = 1$
**b** Squared terms are always $\geqslant 0$, so the minimum value is $0 + 1 = 1$

**7** **a** $-2$ and $-1$ **b** 2, $-2$, $2\sqrt{2}$ and $-2\sqrt{2}$
**c** $-1$ and $\frac{1}{3}$ **d** $\frac{1}{2}$ and 1
**e** 4 and 25 **f** 8 and $-27$

**8** **a** $(3^x - 27)(3^x - 1)$ **b** 0 and 3