Stats Yr2 Chapter 3: Distribution-N

The Inverse Normal

Fx-CG50: Inverse Normal Distribution

We now know how to use a calculator to value of the variable to obtain a probability. But we might want to do the reverse: given a probability of being in a region, how do we find the value of the boundary?

[Textbook] $X \sim N(20, 3^2)$. Find, correct to two decimal places, the values of a such that:
a. $P(X < a) = 0.75$
b. $P(X > a) = 0.4$
c. $P(16 < X < a) = 0.3$

b

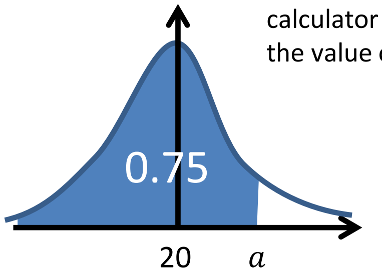
?

c

?

a

1. MENU → Statistics
2. Choose DIST
3. Choose NORM
4. Choose 'InvN'.
5. Put the area as 0.75 (this is the area <u>up to</u> the a value to determine). Put $\mu = 20$ and $\sigma = 3$.
6. You should get **22.0235**.



The 'area' requested by your calculator is the probability **<u>up to</u>** the value of interest (in this case a)

DRAW A SKETCH!

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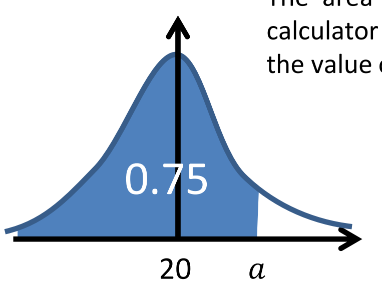

The 'area' requested by your calculator is the probability **<u>up to</u>** the value of interest (in this case a)

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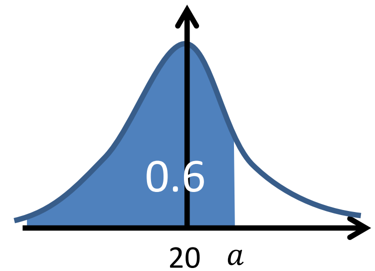
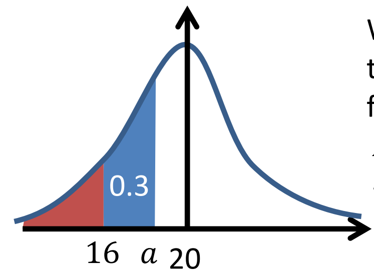
b



$P(X > a) = 0.4$
$\therefore P(X < a) = 0.6$
(i.e. if 40% is above a, then 60% must be below)

$a = 20.76$

c



We need to find the area up to a. From the sketch we can therefore see we first find the area up to 16 and add on the 0.3:
$P(X < 16) = 0.09121$
$\therefore P(X < a) = 0.3 + 0.09121 = 0.39121$
$\therefore a = 19.17$

REALLY, DRAW A SKETCH!

Further Example

If the IQ of a population is distributed using $X \sim N(100, 15^2)$.

a. Determine the IQ corresponding to the top 30% of the population.
b. Determine the interquartile range of IQs.

a

?

b

?

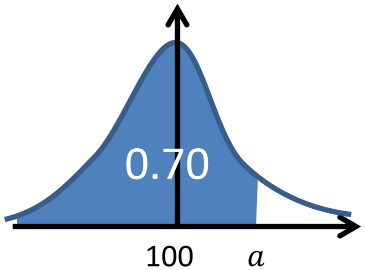
Further Example

If the IQ of a population is distributed using $X \sim N(100, 15^2)$.

a. Determine the IQ corresponding to the top 30% of the population.

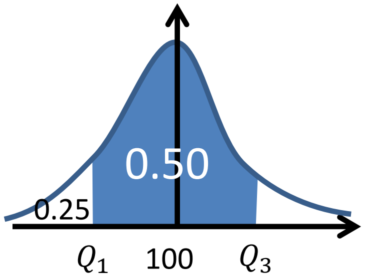
b. Determine the interquartile range of IQs.

a



$P(X > k) = 0.3$
$\therefore P(X < k) = 0.7$
$k = 107.87\ (2dp)$

b



$P(X < Q_1) = 0.25$
$\therefore Q_1 = 89.88$

$P(X < Q_3) = 0.75$
$\therefore Q_3 = 110.12$

$IQR = 110.12 - 89.88$
$= 20.24$

In general the quartiles of a normal distribution are approximately $\mu \pm \frac{2}{3}\sigma$

Test Your Understanding

$X \sim N(80, 7^2)$. Using your calculator,

a. determine the a such that $P(X > a) = 0.65$
b. determine the b such that $P(75 < X < b) = 0.4$
c. determine the c such that $P(c < X < 76) = 0.2$
d. determine the interquartile range of X.

a

?

c

?

b

?

d

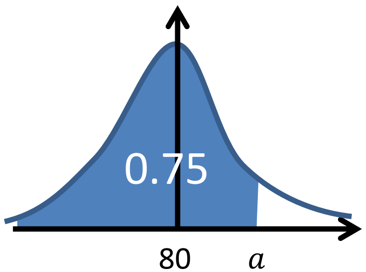
?

Test Your Understanding

$X \sim N(80, 7^2)$. Using your calculator,

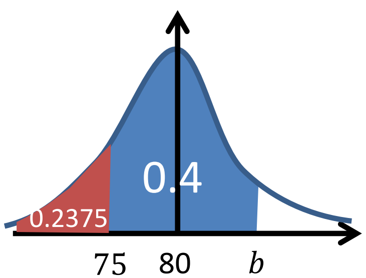
a. determine the a such that $P(X > a) = 0.65$
b. determine the b such that $P(75 < X < b) = 0.4$
c. determine the c such that $P(c < X < 76) = 0.2$
d. determine the interquartile range of X.

a



$P(X > a) = 0.65$
$\therefore P(X < a) = 0.35$
$a = 77.303\ (3dp)$

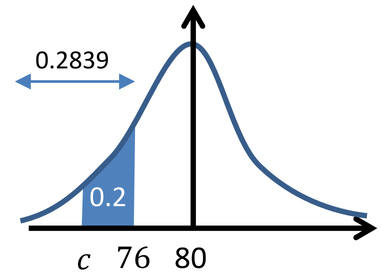
b



$P(X < 75) = 0.2375$
$\therefore P(X < b)$
$= 0.2375 + 0.4$
$= 0.6375$

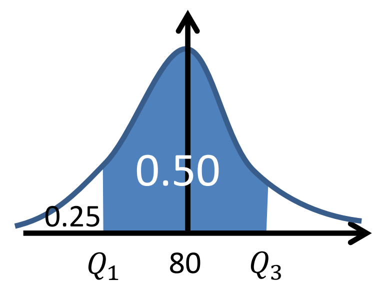
$b = 82.463$

c



$P(X < 76) = 0.2839$
$P(X < c) = 0.2839 - 0.2 = 0.0839$
$c = 70.34\ (2dp)$

d



$P(X < Q_1) = 0.25 \quad \therefore Q_1 = 75.28$
$P(X < Q_3) = 0.75 \quad \therefore Q_3 = 84.72$
$IQR = 84.72 - 75.28 = 9.44$

Exercise 3.3

Pearson Stats/Mechanics Year 2

Pages 24-25

Homework Exercise

1 The random variable $X \sim N(30, 5^2)$. Find the value of a, to 2 decimal places, such that:

a $P(X < a) = 0.3$ **b** $P(X < a) = 0.75$ **c** $P(X > a) = 0.4$ **d** $P(32 < X < a) = 0.2$

2 The random variable $X \sim N(12, 3^2)$. Find the value of a, to 2 decimal places, such that:

a $P(X < a) = 0.1$ **b** $P(X > a) = 0.65$

c $P(10 \leqslant X \leqslant a) = 0.25$ **d** $P(a < X < 14) = 0.32$

3 The random variable $X \sim N(20, 12)$.

a Find the value of a and the value of b such that:

i $P(X < a) = 0.40$ **ii** $P(X > b) = 0.6915$

b Find $P(b < X < a)$.

4 The random variable $Y \sim N(100, 15^2)$.

a Find the value of a and the value of b such that:

i $P(Y > a) = 0.975$ **ii** $P(Y < b) = 0.10$

b Find $P(a < Y < b)$.

5 The random variable $X \sim N(80, 16)$.

a Find the value of a and the value of b such that:

i $P(X > a) = 0.40$ **ii** $P(X < b) = 0.5636$

b Find $P(b < X < a)$.

6 The masses, M kg, of a population of badgers are modelled as $M \sim N(4.5, 0.6^2)$.

For this population, find:

a the lower quartile

b the 80th percentile

c Explain without calculation why $Q_2 = 4.5$ kg.

Homework Exercise

7 The percentage scores, X, of a group of learner drivers in a theory test is modelled as a normal distribution with $X \sim N(72, 6^2)$.

a Find the value of a such that $P(X < a) = 0.6$. **(1 mark)**

b Find the interquartile range of the scores. **(2 marks)**

8 The masses, Y grams, of a brand of chocolate bar are modelled as $Y \sim N(60, 2^2)$.

a Find the value of y such that $P(Y > y) = 0.2$. **(1 mark)**

b Find the 10% to 90% interpercentile range of masses. **(2 marks)**

c Tom says that the median is equal to the mean. State, with a reason, whether Tom is correct. **(1 mark)**

9 The distribution of heights, H cm, of a large group of men is modelled using $H \sim N(170, 10^2)$. A frock coat is a coat that goes from the neck of a person to near the floor. A clothing manufacturer uses the information to make three different lengths of frock coats. The table below shows the proportion of each size they will make.

Short	Regular	Long
30%	50%	20%

a The company wants to advertise a range of heights for which the regular frock coat is suitable. Use the model to suggest suitable heights for the advertisement. **(4 marks)**

b State one assumption you have made in deciding these values. **(1 mark)**

Homework Answers

For Chapter 3, student answers may differ slightly from those shown here when calculators are used rather than table values.

1	**a**	27.38	**b**	33.37	**c**	31.27	**d**	35.30
2	**a**	8.16	**b**	10.85	**c**	12.02	**d**	11.45

3 **a** **i** 19.1 **ii** 18.3
b 0.0915

4 **a** **i** 70.6 **ii** 80.8 **b** 0.075

5 **a** **i** 81.0 **ii** 80.6 **b** 0.0364

6 **a** 4.095 (3 d.p.) **b** 5.005 (3 d.p.)
c 4.5 is the mean, so 50% of badgers will have a mass less than 4.5.

7 **a** 73.52 (2 d.p.) **b** 8.09 (2 d.p.)

8 **a** 61.68 (2 d.p.) **b** 5.13 (2 d.p.)
c Tom is correct in this case; the normal distribution is symmetric about the mean, so 50% of bars will have mass less than the mean.

9 **a** Short: Up to 165 cm,
Regular: Between 165 cm and 178 cm
Long: Over 178 cm
b That the population follows the normal distribution over the whole range of values i.e. that there are no extreme outliers.