P1 Chapter 11: 3D Vectors

Applications to Mechanics

Application to Mechanics

Out of displacement, speed, acceleration, force, mass and time, all but mass and time are vectors. Clearly these can act in 3D space.

	Vector		Scalar	
Force	$\begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} N$	→	$\sqrt{3^2 + 4^2 + (-1)^2}$ $= 5.10\ N$	
Acceleration	$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} ms^{-2}$	→	$1.41\ ms^{-2}$	
Displacement	$\begin{pmatrix} 12 \\ 3 \\ 4 \end{pmatrix} m$	→	$13\ m$	**Distance**
Velocity	$\begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} ms^{-1}$	→	$5\ m$	**Speed**

Example

[Textbook] A particle of mass 0.5 kg is acted on by three forces.

$$F_1 = (2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k})\ N$$
$$F_2 = (-\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k})\ N$$
$$F_3 = (4\boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k})\ N$$

a. Find the resultant force R acting on the particle.
b. Find the acceleration of the particle, giving your answer in the form $(p\boldsymbol{i} + q\boldsymbol{j} + r\boldsymbol{k})$ ms^{-2}.
c. Find the magnitude of the acceleration.

Given that the particle starts at rest,
d. Find the velocity of the particle at t=5 seconds.

a. ?

b. ?

c. ?

d. ?

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a. $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \\ -3 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} N$

b. $\boldsymbol{F} = m\boldsymbol{a}$

$$\begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} = 0.5a \quad \therefore \boldsymbol{a} = \begin{pmatrix} 10 \\ -2 \\ -6 \end{pmatrix} ms^{-2}$$

c. $|\boldsymbol{a}| = \sqrt{10^2 + (-2)^2 + (-6)^2} = \sqrt{140}$ ms^{-2}

d. $\boldsymbol{a} = \begin{pmatrix} 50 \\ -10 \\ -30 \end{pmatrix} ms^{-2}$

Exercise 12.4

Pearson Pure Mathematics Year 2/AS
Pages 108-109

Homework Exercise

1 A particle is acted upon by forces of $(3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ N, $(7\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$ N and $(-5\mathbf{i} - 3\mathbf{j})$ N.

a Work out the resultant force **R**.

b Find the exact magnitude of the resultant force.

2 A particle, initially at rest, is acted upon by a force that causes the particle to accelerate at $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})\,\text{m s}^{-2}$ for 2 seconds. Work out the distance travelled by the particle.

3 A body of mass 4 kg is moving with a constant velocity when it is acted upon by a force of $(2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ N.

a Find the acceleration of the body while the force acts.

b Find the magnitude of this acceleration to 3 s.f.

4 A particle of mass 6 kg is acted on by two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Given that $\mathbf{F}_1 = (7\mathbf{i} + 3\mathbf{j} + \mathbf{k})$ N, and that the particle is accelerating at $(2\mathbf{i} - \mathbf{k})\,\text{m s}^{-2}$, find $\mathbf{F}_2$, giving your answer in the form $(p\mathbf{i} + q\mathbf{j} + r\mathbf{k})$ N.

5 A particle of mass 2 kg is in static equilibrium and is acted upon by three forces:

$\mathbf{F}_1 = (\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ N
$\mathbf{F}_2 = (-\mathbf{i} + 3\mathbf{j} + b\mathbf{k})$ N
$\mathbf{F}_3 = (a\mathbf{j} - 2\mathbf{k})$ N

a Find the values of the constants a and b.

$\mathbf{F}_2$ is removed. Work out:

b the resultant force **R**

c the acceleration of the particle, giving your answer in the form $(p\mathbf{i} + q\mathbf{j} + r\mathbf{k})\,\text{m s}^{-2}$

d the magnitude of this acceleration

e the angle the acceleration vector makes with the unit vector **j**.

Homework Exercise

6 In this question **i** and **j** are the unit vectors due east and north, and **k** is the unit vector vertically upwards. An aeroplane of mass 1200 kg is initially in level flight. The forces of thrust **T**, lift **L**, and the combined forces of wind and air resistance **F**, acting on the aeroplane are modelled as:

$$\mathbf{T} = 2800\mathbf{i} - 1800\mathbf{j} + 300\mathbf{k}$$
$$\mathbf{L} = 11\,000\mathbf{k}$$
$$\mathbf{F} = -900\mathbf{i} + 500\mathbf{j}$$

a Taking $g = 9.8\ \text{m s}^{-2}$, find the magnitude of the acceleration of the aeroplane.

b Determine whether the aeroplane is ascending or descending, and find the size of the obtuse angle its acceleration makes with the vector **k**.

Homework Answers

1 **a** $(5\mathbf{i} - \mathbf{j} + 4\mathbf{k})$ N **b** $\sqrt{42}$ N

2 $2\sqrt{29}$ m

3 **a** $(\frac{1}{2}\mathbf{i} - \frac{5}{4}\mathbf{j} + \frac{3}{4}\mathbf{k})\,\text{m s}^{-2}$ **b** $1.54\,\text{m s}^{-2}$

4 $(5\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$ N

5 **a** $a = -2, b = 4$ **b** $(\mathbf{i} - 3\mathbf{j} - 4\mathbf{k})$ N

c $(\frac{1}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} - 2\mathbf{k})\,\text{m s}^{-2}$ **d** $\frac{1}{2}\sqrt{26}\,\text{m s}^{-2}$

e $126°$

6 **a** $1.96\,\text{m s}^{-2}$ **b** Descending, $101.3°$