M1 Chapter 10: Forces and Motion

Vector Forces in 2D

Forces in 2 Dimensions

Vectors have both direction and magnitude, while scalars only have magnitude.

Vector quantity:	force, acceleration, velocity, displacement
Scalar only:	mass, time

$\sum \boldsymbol{F} = m\boldsymbol{a}$ is a vector equation.

Vector equations are true in any direction you chose. The trick is to choose the best "main" direction $\boldsymbol{i}$ as the direction of motion, then the perpendicular direction is $\boldsymbol{j}$.

[Textbook] Let $\boldsymbol{i}$ represent East and $\boldsymbol{j}$ North. A resultant force of $(3\boldsymbol{i} + 8\boldsymbol{j})$ N acts upon a particle of mass 0.5 kg.

(a) Find the acceleration of the particle in the form $(p\boldsymbol{i} + q\boldsymbol{j})$ ms^{-2}.

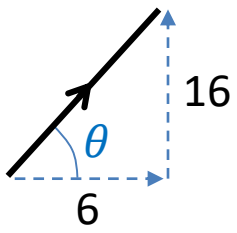
(b) Find the magnitude and bearing of the acceleration of the particle.

a

$$\begin{pmatrix}3\\8\end{pmatrix} = 0.5 \times a$$

$$\therefore a = \begin{pmatrix}6\\16\end{pmatrix} = (6\boldsymbol{i} + 16\boldsymbol{j}) \text{ ms}^{-2}$$

b



$$\left|\begin{pmatrix}6\\16\end{pmatrix}\right| = \sqrt{6^2 + 16^2} = 17.1 \text{ ms}^{-2} \text{ (3sf)}$$

Bearing: $90 - \tan^{-1}\left(\frac{16}{6}\right) = 020.6°$

Test Your Understanding

[Textbook] A boat is modelled as a particle of mass 60 kg being acted on by three forces.

$$F_1 = \begin{pmatrix} 80 \\ 50 \end{pmatrix} N, \qquad F_2 = \begin{pmatrix} 10p \\ 20q \end{pmatrix} N, \qquad F_3 = \begin{pmatrix} -75 \\ 100 \end{pmatrix} N$$

Given that the boat is accelerating at a rate of $\begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix}$ ms^{-2}, find the values of p and q.

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Resultant force:

$$\begin{pmatrix} 80 \\ 50 \end{pmatrix} + \begin{pmatrix} 10p \\ 20q \end{pmatrix} + \begin{pmatrix} -75 \\ 100 \end{pmatrix} = \begin{pmatrix} 5 + 10p \\ 150 + 20q \end{pmatrix} \text{N}$$

$$F = ma$$

$$\begin{pmatrix} 5 + 10p \\ 150 + 20q \end{pmatrix} = 60 \times \begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix} = \begin{pmatrix} 48 \\ -90 \end{pmatrix}$$

$\therefore 5 + 10p = 48 \quad \Rightarrow \quad p = 4.3$

and $150 + 20q = -90 \quad \Rightarrow \quad q = -12$

Classwork Exercise 10.4

Pearson Stats/Mechanics Year 1 Exercise Book
Pages 70-71
Questions 1 to 4.

Homework Exercise

In all the questions in this exercise **i** represents the unit vector due east, and **j** represents the unit vector due north.

1 A resultant force of $(\mathbf{i} + 4\mathbf{j})$ N acts upon a particle of mass 2 kg.

a Find the acceleration of the particle in the form $(p\mathbf{i} + q\mathbf{j})$ m s^{-2}.

b Find the magnitude and bearing of the acceleration of the particle.

2 A resultant force of $(4\mathbf{i} + 3\mathbf{j})$ N acts on a particle of mass m kg causing it to accelerate at $(20\mathbf{i} + 15\mathbf{j})$ m s^{-2}. Work out the mass of the particle.

3 A particle of mass 3 kg is acted on by a force **F**. Given that the particle accelerates at $(7\mathbf{i} - 3\mathbf{j})$ m s^{-2}:

a find an expression for **F** in the form $(p\mathbf{i} + q\mathbf{j})$ N

b find the magnitude and bearing of **F**.

4 Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on a particle of mass m. Find the acceleration of the particle, **a** m s^{-2}, given that:

a $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$ N, $\mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})$ N, $m = 0.25$ kg

b $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$ N, $\mathbf{F}_2 = (2\mathbf{i} + 3\mathbf{j})$ N, $m = 6$ kg

c $\mathbf{F}_1 = (-40\mathbf{i} - 20\mathbf{j})$ N, $\mathbf{F}_2 = (25\mathbf{i} + 10\mathbf{j})$ N, $m = 15$ kg

d $\mathbf{F}_1 = 4\mathbf{j}$ N, $\mathbf{F}_2 = (-2\mathbf{i} + 5\mathbf{j})$ N, $m = 1.5$ kg

Notation You are asked to find the acceleration as a vector, **a**. You can give your answer as a column vector or using **i–j** notation.

Homework Exercise

5 A particle of mass 8 kg is at rest. It is acted on by three forces, $\mathbf{F}_1 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ N, $\mathbf{F}_2 = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ N and $\mathbf{F}_3 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ N.

a Find the magnitude and direction of the acceleration of the particle, **a** m s^{-2}.

b Find the time taken for the particle to travel a distance of 20 m.

Hint Use $s = ut + \frac{1}{2}at^2$ with $s = 20$ and $u = 0$.

6 Two forces, $(2\mathbf{i} + 3\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N, act on a particle P. The resultant of the two forces is **R**. Given that **R** acts in a direction which is parallel to the vector $(-\mathbf{i} + 4\mathbf{j})$, show that $4p + q + 11 = 0$. **(4 marks)**

Problem-solving

You can write **R** in the form $(-k\mathbf{i} + 4k\mathbf{j})$ N for some constant k.

7 A particle of mass 4 kg starts from rest and is acted upon by a force **R** of $(6\mathbf{i} + b\mathbf{j})$ N. **R** acts on a bearing of 045°.

a Find the value of b. **(1 mark)**

b Calculate the magnitude of **R**. **(2 marks)**

c Work out the magnitude of the acceleration of the particle. **(2 marks)**

d Find the total distance travelled by the particle during the first 5 seconds of its motion. **(3 marks)**

Homework Exercise

8 Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a particle. $\mathbf{F}_1 = (-3\mathbf{i} + 7\mathbf{j})$ N, $\mathbf{F}_2 = (\mathbf{i} - \mathbf{j})$ N and $\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})$ N.

a Given that this particle is in equilibrium, determine the value of p and the value of q.

Force $\mathbf{F}_2$ is removed.

b Given that in the first 10 seconds of its motion the particle travels a distance of 12 m, find the exact mass of the particle in kg.

9 A particle of mass m kg is acted upon by forces of $(5\mathbf{i} + 6\mathbf{j})$ N, $(2\mathbf{i} - 2\mathbf{j})$ N and $(-\mathbf{i} - 4\mathbf{j})$ N causing it to accelerate at $7\,\text{m s}^{-2}$. Work out the mass of the particle. Give your answer correct to 2 d.p.

10 Two forces, $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ N and $\begin{pmatrix} p \\ q \end{pmatrix}$ N, act on a particle P of mass m kg. The resultant of the two forces is $\mathbf{R}$.

a Given that $\mathbf{R}$ acts in a direction which is parallel to the vector $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, show that $2p + q + 9 = 0$. **(4 marks)**

b Given also that $p = 1$ and that P moves with an acceleration of magnitude $15\sqrt{5}\,\text{m s}^{-2}$, find the value of m. **(7 marks)**

Challenge

A particle of mass 0.5 kg is acted on by two forces:

$\mathbf{F}_1 = -4\mathbf{i}$ N $\qquad$ $\mathbf{F}_2 = (k\mathbf{i} + 2k\mathbf{j})$ N

where k is a positive constant.

Given that the particle is accelerating at a rate of $8\sqrt{17}\,\text{m s}^{-2}$, find the value of k.

Homework Answers

1 **a** $(0.5\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-2}$

b $2.06\ \text{m s}^{-2}$ (3 s.f.) on a bearing of 014° (to the nearest degree).

2 0.2 kg

3 **a** $(21\mathbf{i} - 9\mathbf{j})$ N

b 22.8 N (3 s.f.) on a bearing of 113° (to the nearest degree).

4 **a** $(-4\mathbf{i} + 32\mathbf{j})\,\text{m s}^{-2}$ **b** $(\frac{5}{6}\mathbf{i} - \frac{1}{6}\mathbf{j})\,\text{m s}^{-2}$

c $(-\mathbf{i} - \frac{2}{3}\mathbf{j})\,\text{m s}^{-2}$ **d** $(-\frac{4}{3}\mathbf{i} + 6\mathbf{j})\,\text{m s}^{-2}$

5 **a** $\sqrt{0.8125}\ \text{ms}^{-2}$ on a bearing of 146° (to the nearest degree).

b 6.66 s

6 $\mathbf{R} = (-k\mathbf{i} + 4k\mathbf{j})$ N

So $4k = 3 + q$ (1), $-k = 2 + p$ (2) and $-4k = 8 + 4p$ (3)

Adding equations (1) and (3) gives $4p + q + 11 = 0$

7 **a** $b = 6$ **b** $6\sqrt{2}$ N

c $\frac{3\sqrt{2}}{2}\,\text{m s}^{-2}$ **d** $\frac{75\sqrt{2}}{4}$ m

8 **a** $p = 2, q = -6$ **b** $\frac{25\sqrt{2}}{6}$ kg

9 0.86 kg

10 **a** $5 + q = -2k$ (1), $2 + p = k$ (2) and $4 + 2p = 2k$ (3)

Adding equations (1) and (3) gives $2p + q + 9 = 0$

b 0.2 kg

Challenge

$k = 8$