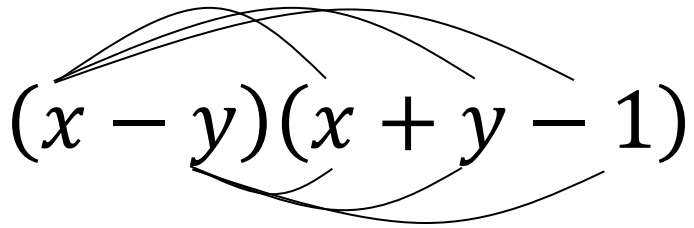
P1 Chapter 1: Algebra

Expanding Brackets

Expanding Brackets

If you have ever been taught 'FOIL' to multiply brackets please purge it from your mind now – instead:

Multiply each term in the first bracket by each term in the second.



$$(x-y)(x+y-1)$$

$$= x^2 + xy - x - xy - y^2 + y$$

$$= x^2 - y^2 - x + y$$

Tip: My order is "first term in first brackets times each in second, then second term in first bracket times each in second, etc."

Tip: For more than 2 brackets, multiply two out each time to reduce the number of brackets by one.

$$(x+1)(x+2)(x+3)$$

$$= (x+1)(x^2+5x+6)$$

$$= x^3 + 5x^2 + 6x + x^2 + 5x + 6$$

$$= x^3 + 6x^2 + 11x + 6$$

Test Your Understanding

1 Expand and simplify

$$(x+5)(x-2)(x+1)$$

$$= (x+5)(x^2 - x - 2)$$
$$= x^3 - x^2 - 2x + 5x^2 - 5x - 10$$
$$= x^3 + 4x^2 - 7x - 10$$

2 Expand and simplify:

$$2(x-3)(x-4)$$

$$= 2(x^2 - 7x + 12)$$
$$= 2x^2 - 14x + 24$$

3 Expand and simplify:

$$(2x-1)^3$$

$$= (2x-1)(2x-1)(2x-1)$$
$$= (2x-1)(4x^2 - 4x + 1)$$
$$= 8x^3 - 8x^2 + 2x - 4x^2 + 4x - 1$$
$$= 8x^3 - 12x^2 + 6x - 1$$

Test Your Understanding

1 Expand and simplify

$$(x+5)(x-2)(x+1)$$

?

2 Expand and simplify:

$$2(x-3)(x-4)$$

?

3 Expand and simplify:

$$(2x-1)^3$$

?

Exercise 1.2

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1 *[MAT 2002 1B]*

Of the following three alleged algebraic identities, at least one is wrong.

(i) $yz(z-y)+zx(x-z)+xy(y-x)$
$=(z-y)(x-z)(y-x)$

(ii) $yz(z-y)+zx(x-z)+xy(y-x)$
$=(z-y)(z-x)(y-x)$

(iii) $yz(x+y)+zx(z+x)+xy(y+x)$
$=(z+y)(z+x)(y+x)$

Which of the following statements are correct? Tick all that apply.

- [ ] (i)
- [ ] (ii)
- [ ] (iii)

?

2 *[MAT 2007 1E]*

If x and n are integers then

$$(1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$$

is:

- negative when $n>5$ and $x<5$
- negative when n is odd and $x>5$
- negative when n is a multiple of 3 and $x>5$
- negative when n is even and $x<5$

?

Exercise 1.2

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1 *[MAT 2002 1B]*

Of the following three alleged algebraic identities, at least one is wrong.

(i) $yz(z-y)+zx(x-z)+xy(y-x)$
$=(z-y)(x-z)(y-x)$
(ii) $yz(z-y)+zx(x-z)+xy(y-x)$
$=(z-y)(z-x)(y-x)$
(iii) $yz(x+y)+zx(z+x)+xy(y+x)$
$=(z+y)(z+x)(y+x)$

Which of the following statements are correct? Tick all that apply.

- ☐ (i)
- ☐ (ii)
- ☐ (iii)

Solution: (ii) only

2 *[MAT 2007 1E]*

If x and n are integers then

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is:

- ○ negative when $n>5$ and $x<5$
- ○ negative when n is odd and $x>5$
- ○ negative when n is a multiple of 3 and $x>5$
- ○ negative when n is even and $x<5$

Solution: n is odd and $x>5$

Homework Exercise

1 Expand and simplify if possible:

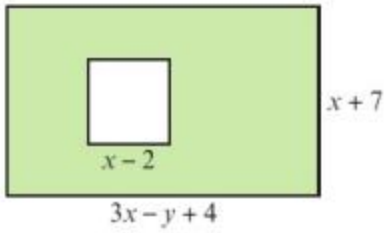
a $(x+4)(x+7)$	**b** $(x-3)(x+2)$	**c** $(x-2)^2$
d $(x-y)(2x+3)$	**e** $(x+3y)(4x-y)$	**f** $(2x-4y)(3x+y)$
g $(2x-3)(x-4)$	**h** $(3x+2y)^2$	**i** $(2x+8y)(2x+3)$
j $(x+5)(2x+3y-5)$	**k** $(x-1)(3x-4y-5)$	**l** $(x-4y)(2x+y+5)$
m $(x+2y-1)(x+3)$	**n** $(2x+2y+3)(x+6)$	**o** $(4-y)(4y-x+3)$
p $(4y+5)(3x-y+2)$	**q** $(5y-2x+3)(x-4)$	**r** $(4y-x-2)(5-y)$

2 Expand and simplify if possible:

a $5(x+1)(x-4)$	**b** $7(x-2)(2x+5)$	**c** $3(x-3)(x-3)$
d $x(x-y)(x+y)$	**e** $x(2x+y)(3x+4)$	**f** $y(x-5)(x+1)$
g $y(3x-2y)(4x+2)$	**h** $y(7-x)(2x-5)$	**i** $x(2x+y)(5x-2)$
j $x(x+2)(x+3y-4)$	**k** $y(2x+y-1)(x+5)$	**l** $y(3x+2y-3)(2x+1)$
m $x(2x+3)(x+y-5)$	**n** $2x(3x-1)(4x-y-3)$	**o** $3x(x-2y)(2x+3y+5)$
p $(x+3)(x+2)(x+1)$	**q** $(x+2)(x-4)(x+3)$	**r** $(x+3)(x-1)(x-5)$
s $(x-5)(x-4)(x-3)$	**t** $(2x+1)(x-2)(x+1)$	**u** $(2x+3)(3x-1)(x+2)$
v $(3x-2)(2x+1)(3x-2)$	**w** $(x+y)(x-y)(x-1)$	**x** $(2x-3y)^3$

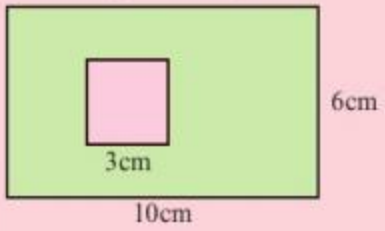
Homework Exercise

3 The diagram shows a rectangle with a square cut out.
The rectangle has length $3x - y + 4$ and width $x + 7$.
The square has length $x - 2$.
Find an expanded and simplified expression for the shaded area.



Problem-solving

Use the same strategy as you would use if the lengths were given as numbers:



4 A cuboid has dimensions $x + 2$ cm, $2x - 1$ cm and $2x + 3$ cm.
Show that the volume of the cuboid is $4x^3 + 12x^2 + 5x - 6\,\text{cm}^3$.

5 Given that $(2x + 5y)(3x - y)(2x + y) = ax^3 + bx^2y + cxy^2 + dy^3$, where a, b, c and d are constants, find the values of a, b, c and d. **(2 marks)**

Challenge

Expand and simplify $(x + y)^4$.

Links You can use the binomial expansion to expand expressions like $(x + y)^4$ quickly. → Section 8.3

Homework Answers

1

a $x^2 + 11x + 28$

b $x^2 - x - 6$

c $x^2 - 4x + 4$

d $2x^2 + 3x - 2xy - 3y$

e $4x^2 + 11xy - 3y^2$

f $6x^2 - 10xy - 4y^2$

g $2x^2 - 11x + 12$

h $9x^2 + 12xy + 4y^2$

i $4x^2 + 6x + 16xy + 24y$

j $2x^2 + 3xy + 5x + 15y - 25$

k $3x^2 - 4xy - 8x + 4y + 5$

l $2x^2 + 5x - 7xy - 4y^2 - 20y$

m $x^2 + 2x + 2xy + 6y - 3$

n $2x^2 + 15x + 2xy + 12y + 18$

o $13y - 4x + 12 - 4y^2 + xy$

p $12xy - 4y^2 + 3y + 15x + 10$

q $5xy - 20y - 2x^2 + 11x - 12$

r $22y - 4y^2 - 5x + xy - 10$

2

a $5x^2 - 15x - 20$

b $14x^2 + 7x - 70$

c $3x^2 - 18x + 27$

d $x^3 - xy^2$

e $6x^3 + 8x^2 + 3x^2y + 4xy$

f $x^2y - 4xy - 5y$

g $12x^2y + 6xy - 8xy^2 - 4y^2$

h $19xy - 35y - 2x^2y$

i $10x^3 - 4x^2 + 5x^2y - 2xy$

j $x^3 + 3x^2y - 2x^2 + 6xy - 8x$

k $2x^2y + 9xy + xy^2 + 5y^2 - 5y$

l $6x^2y + 4xy^2 + 2y^2 - 3xy - 3y$

m $2x^3 + 2x^2y - 7x^2 + 3xy - 15x$

n $24x^3 - 6x^2y - 26x^2 + 2xy + 6x$

o $6x^3 + 15x^2 - 3x^2y - 18xy^2 - 30xy$

p $x^3 + 6x^2 + 11x + 6$

q $x^3 + x^2 - 14x - 24$

r $x^3 - 3x^2 - 13x + 15$

s $x^3 - 12x^2 + 47x - 60$

t $2x^3 - x^2 - 5x - 2$

u $6x^3 + 19x^2 + 11x - 6$

v $18x^3 - 15x^2 - 4x + 4$

w $x^3 - xy^2 - x^2 + y^2$

x $8x^3 - 36x^2y + 54xy^2 - 27y^3$

3 $2x^2 - xy + 29x - 7y + 24$

4 $4x^3 + 12x^2 + 5x - 6\,\text{cm}^3$

5 $a = 12, b = 32, c = 3, d = -5$