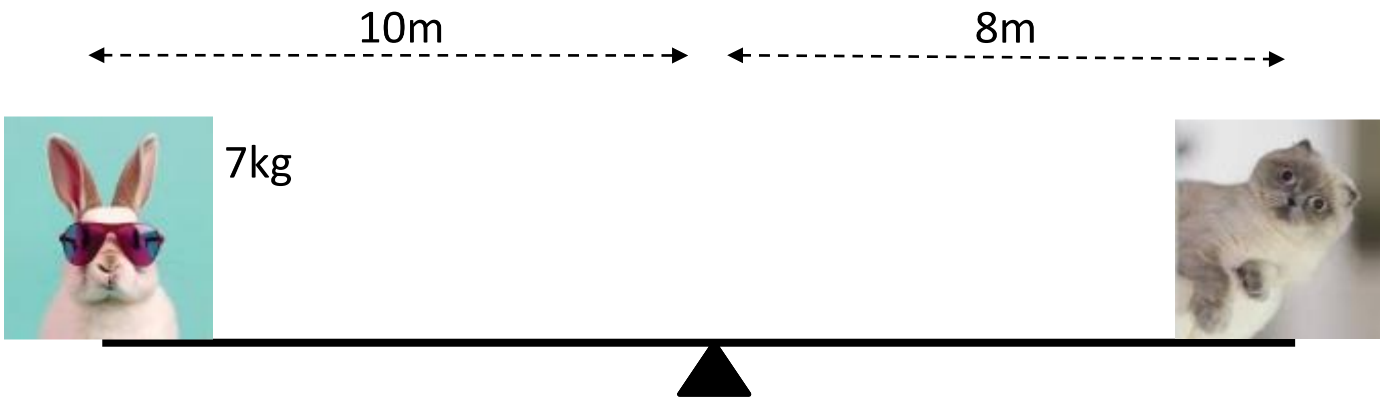
M2 Chapter 4: Moments

Equilibrium

This whole chapter in a nutshell...



✎ If a rigid body is in **equilibrium** then:

a. The resultant force in any direction is 0.
b. The resultant moment about any point is 0.

i.e. Forces up = forces down, as per Year 1

In other words, clockwise moments = anticlockwise moments

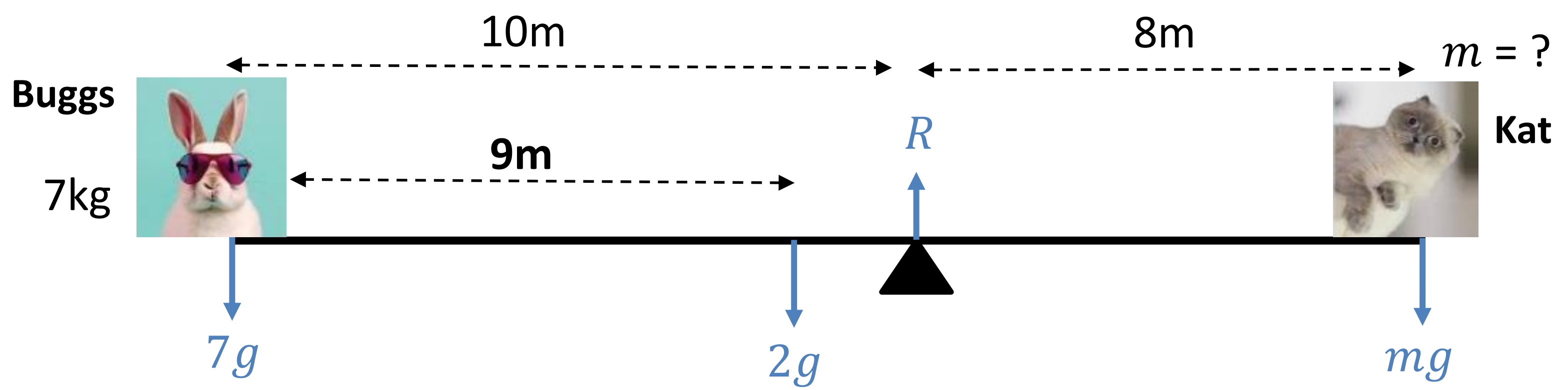
You will typically use **<u>both</u>** these properties to solve exam questions.

Example

Buggs and Kat are having fun on a **uniform** seesaw of mass 2kg. Buggs has a mass of 7kg and is 10m from the pivot. Tom is 8m from the pivot. The seesaw remains horizontal.

a) Determine the reaction force at the pivot of the seesaw.
b) Determine Tom's mass.

If a rod is uniform its centre of mass is at its centre, so we model its weight as acting at that point.



Tip: First draw on forces. Click to sketch

Hint: Choose a suitable point to take moments about.

Method 1:

Equating moments about pivot:

$$(2g \times 1) + (7g \times 10) = (mg \times 8)$$

$$72g = 8mg \quad \rightarrow \quad m = \frac{72g}{8g} = 9 \text{ kg}$$

Equating forces in vertical direction,

$$R = 7g + 2g + mg = 176N$$

Method 2:

Equating moments about Kat:

$$8R = (9 \times 2g) + (18 \times 7g)$$

$$R = 18g = 176N$$

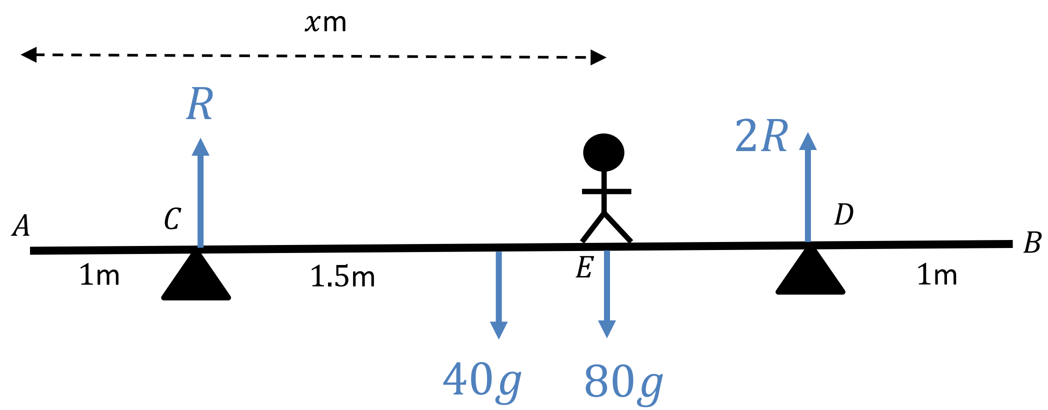
Equating forces in vertical direction:

$$7g + 2g + mg = 176$$

$$m = 9 \text{ kg}$$

Two Pivots

[Textbook] A uniform beam AB, of mass 40 kg and length 5m, rests horizontally on support at C and D, where $AC = DB = 1$ m. When a person of mass 80kg stands on the beam at E the magnitude of the reaction at D is twice the magnitude of the reaction at C.
By modelling the beam as a rod and the man as a particle, find the distance AE.



Note: A rod has no mass, so there is no reaction force of the rod on the man, only reaction forces from the pivots.

?

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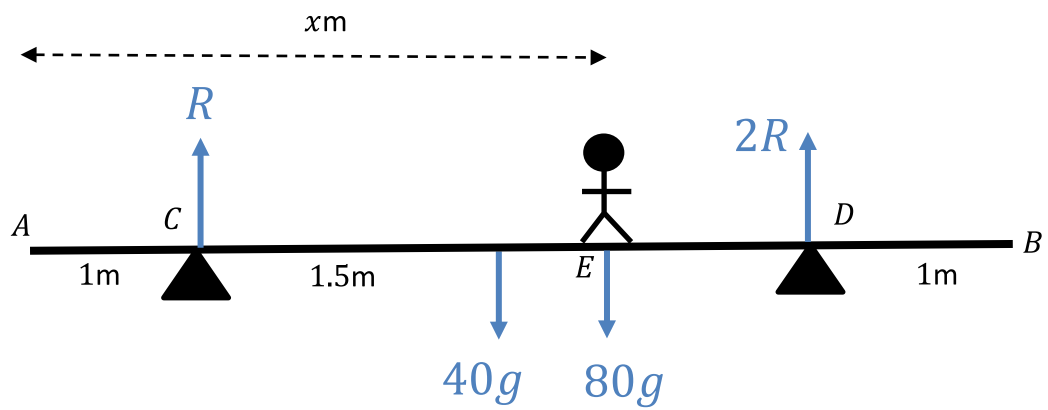


Note: A rod has no mass, so there is no reaction force of the rod on the man, only reaction forces from the pivots.

Resolving vertically:

$$R + 2R = 40g + 80g$$
$$3R = 120g$$
$$R = 40g$$

Let distance AE be x m.

Taking moments about A:

$$(40g \times 2.5) + (80g \times x) = (40g \times 1) + (80g \times 4)$$

...

$x = AE = 3.25$ m

Test Your Understanding

Edexcel M1(Old) May 2013(R) Q8

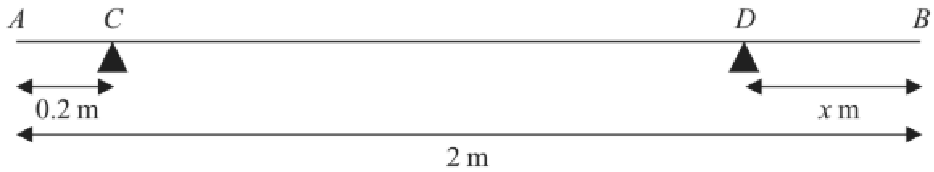


Figure 5

A uniform rod AB has length 2 m and mass 50 kg. The rod is in equilibrium in a horizontal position, resting on two smooth supports at C and D, where $AC = 0.2$ metres and $DB = x$ metres, as shown in Figure 5. Given that the magnitude of the reaction on the rod at D is twice the magnitude of the reaction on the rod at C,

(a) find the value of x.

(6)

The support at D is now moved to the point E on the rod, where $EB = 0.4$ metres. A particle of mass m kg is placed on the rod at B, and the rod remains in equilibrium in a horizontal position. Given that the magnitude of the reaction on the rod at E is four times the magnitude of the reaction on the rod at C,

(b) find the value of m.

(7)

a

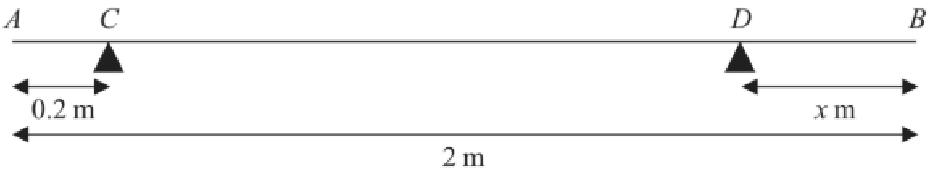
?

b

?

Test Your Understanding

Edexcel M1(Old) May 2013(R) Q8



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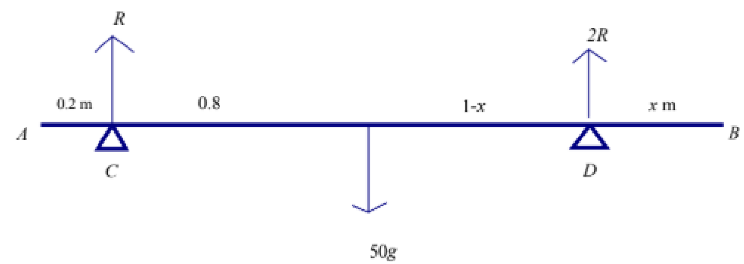
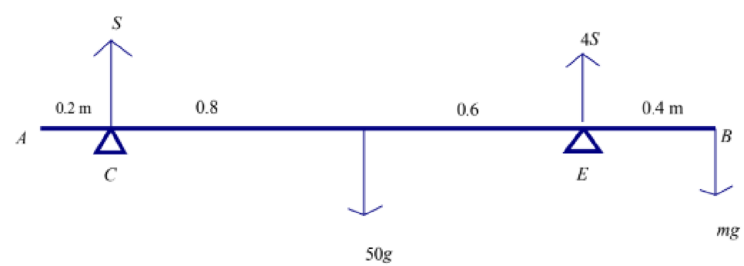
a



Vertical equilibrium: $R + 2R = 50g$,	M1A1
Moments about C: $50g \times 0.8 = (1.8 - x) \times 2 \times R$	M1A1
$3 \times 0.8 = 3.6 - 2x$, $x = 0.6$	**DM1**A1

b



S, $4S$	B1
Vertical equilibrium: $S + 4S = (50 + m)g = 5S$	M1A1
Moments about B: $50g \times 1 = 4S \times 0.4 + S \times 1.8 = 3.4S$	M1A1
$50 \times \dfrac{5}{3.4} = (50 + m)$	**DM1**
$m = 400/17$, 24, 23.5 or better	A1

Exercise 4.3

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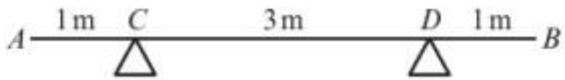
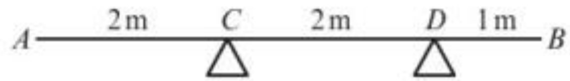
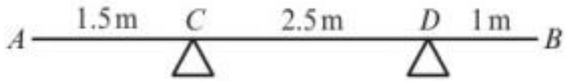
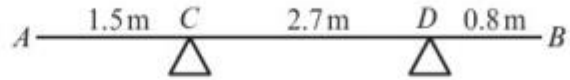
Homework Exercise

1 AB is a uniform rod of length 5 m and weight 20 N. In these diagrams AB is resting in a horizontal position on supports at C and D. In each case, find the magnitudes of the reactions at C and D.

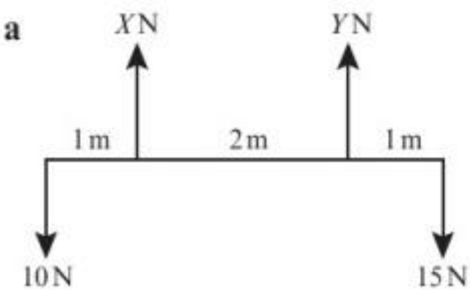
a


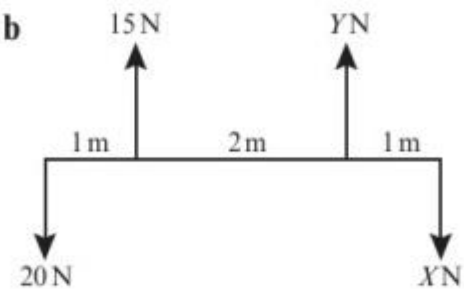
b


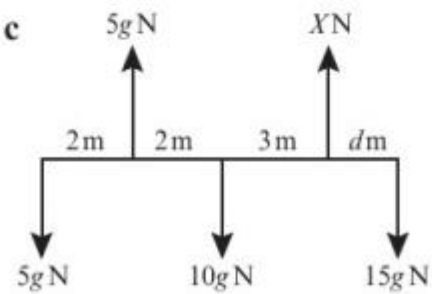
c


d


2 Each of these diagrams shows a light rod in equilibrium in a horizontal position under the action of a set of forces. Find the values of the unknown forces and distances.

a


b


c


3 Jack and Jill are playing on a seesaw made from a uniform plank AB, of length 5 m pivoted at M, the midpoint of AB. Jack has mass 35 kg and Jill has mass 28 kg. Jill sits at A and Jack sits at a distance x m from B. The plank is in equilibrium. Find the value of x.

4 A uniform rod AB, of length 3 m and mass 12 kg, is pivoted at C, where $AC = 1$ m. A vertical force **F** applied at A maintains the rod in horizontal equilibrium. Calculate the magnitude of **F**.

Homework Exercise

5 A broom consists of a broomstick of length 130 cm and mass 5 kg and a broomhead of mass 5.5 kg attached at one end. By modelling the broomstick as a uniform rod and the broomhead as a particle, find where a support should be placed so that the broom will balance horizontally.

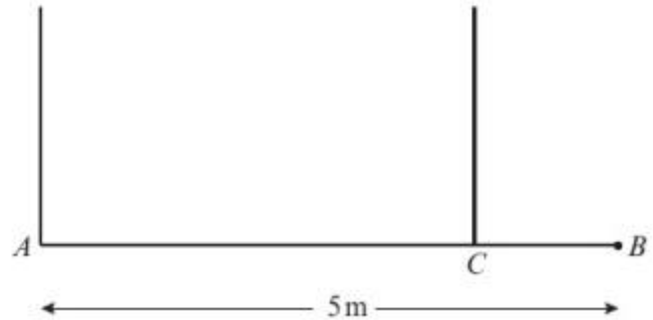
6 A uniform rod AB, of length 4 m and weight 20 N, is suspended horizontally by two vertical strings attached at A and at B. A particle of weight 10 N is attached to the rod at point C, where $AC = 1.5$ m.

a Find the magnitudes of the tensions in the two strings.

The particle is moved so that the rod remains in horizontal equilibrium with the tension in the string at B 1.5 times the tension in the string at A.

b Find the new distance of the particle from A.

7 A uniform beam AB, of length 5 m and mass 60 kg, has a load of 40 kg attached at B. It is then held horizontally in equilibrium by two vertical wires attached at A and C. The tension in the wire at C is four times the tension in the wire at A. By modelling the beam as a uniform rod and the load as a particle, calculate:

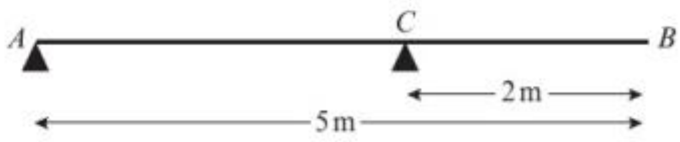


a the tension in the wire at C **(2 marks)**

b the distance CB. **(5 marks)**

Homework Exercise

8 A uniform plank AB has length 5 m and mass 15 kg. The plank is held in equilibrium horizontally by two smooth supports A and C as shown in the diagram, where $BC = 2$ m.

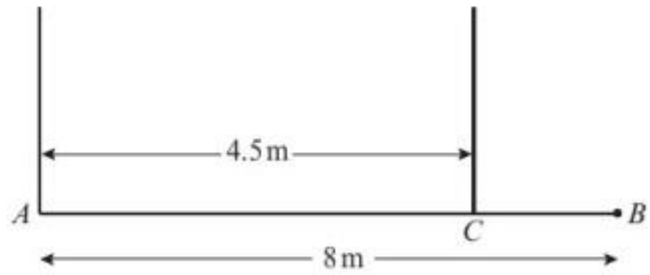


a Find the reaction on the plank at C. **(3 marks)**

A person of mass 45 kg stands on the plank at the point D and it remains in equilibrium. The reactions on the plank at A and C are now equal.

b Find the distance AD. **(7 marks)**

9 A uniform beam AB has weight W N and length 8 m. The beam is held in a horizontal position in equilibrium by two vertical light inextensible wires attached to the beam at the points A and C where $AC = 4.5$ m, as shown in the diagram. A particle of weight 30 N is attached to the beam at B.



a Show that the tension in the wire attached to the beam at C is $\left(\frac{8}{9}W + \frac{160}{3}\right)$ N. **(4 marks)**

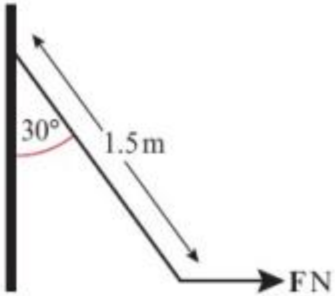
b Find, in terms of W, the tension in the wire attached to the beam at A. **(3 marks)**

Given that the tension in the wire attached to the beam at C is twelve times the tension in the wire attached to the beam at A,

c find the value of W. **(3 marks)**

Homework Exercise

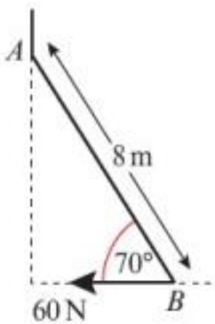
10 A metal lever of mass 5 kg and length 1.5 m is attached by a smooth hinge to a vertical wall. The lever is held at an angle of 30° to the vertical by a horizontal force of magnitude **F** N applied at the other end of the lever. By modelling the lever as a uniform rod, find the value of **F**. **(4 marks)**



11 A uniform ladder, AB, is leaning against a smooth vertical wall on rough horizontal ground at an angle of 70° to the horizontal. The ladder has length 8 m, and is held in equilibrium by a frictional force of magnitude 60 N acting horizontally at B, as shown in the diagram.

a Write down the magnitude of the normal reaction of the wall on the ladder at A. **(1 mark)**

b Find the mass of the ladder. **(4 marks)**

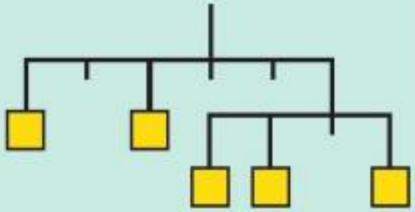


Problem-solving

In part **b** you can ignore the normal reaction at B by taking moments about that point.

Challenge

The diagram shows a kinetic sculpture made from hanging rods. The distances between the points marked on each rod are equal. Arrange 1 kg, 2 kg, 3 kg, 4 kg and 5 kg weights onto the marked squares, using each weight once, so that the sculpture hangs in equilibrium with the rods horizontal.



Homework Answers

1 **a** 10 N, 10 N **b** 15 N, 5 N
c 12 N, 8 N **d** 12.6 N, 7.4 N
2 **a** 7.5, 17.5 **b** 30, 35 **c** 245, $2\frac{2}{3}$
3 0.5 m
4 59 N
5 31 cm from the broomhead
6 **a** 16.25 N, 13.75 N **b** 3.2 m
7 **a** 784 N **b** 0.625 m
8 **a** 122.5 N **b** 1.17 m
9 **a** $\frac{9}{2}T_C = 4W + 8 \times 30$

$\frac{9}{2}T_C = 4W + 240$

$9T_C = 8W + 480$

$T_C = \frac{8}{9}W + \frac{160}{3}$

b $T_A = \frac{W}{9} - \frac{70}{3}$ **c** 750 N
10 14.1 N
11 **a** $R_A = 60$ N **b** 33.6 kg

Challenge
3 kg, 5 kg, 1 kg, 2 kg, 4 kg (from left to right)