Stats1 Chapter 7: Hypothesis Testing

Chapter Practice

Key Points

1. The null hypothesis, H_0, is the hypothesis that you assume to be correct.
2. The alternative hypothesis, H_1, tells us about the parameter if your assumption is shown to be wrong.
3. Hypothesis tests with alternative hypotheses in the form $H_1: p < \dots$ and $H_1: p > \dots$ are called one-tailed tests.
4. Hypothesis tests with an alternative hypothesis in the form $H_1: p \neq \dots$ are called two-tailed tests.
5. A critical region is a region of the probability distribution which, if the test statistic falls within it, would cause you to reject the null hypothesis.
6. The critical value is the first value to fall inside of the critical region.
7. The actual significance level of a hypothesis test is the probability of incorrectly rejecting the null hypothesis.
8. For a two-tailed test the critical region is split at either end of the distribution.
9. For a two-tailed test, halve the significance level at each end you are testing.

Chapter Exercises

1 Mai commutes to work five days a week on a train. She does two journeys a day. Over a long period of time she finds that the train is late 20% of the time. A new company takes over the train service Mai uses. Mai thinks that the service will be late more often. In the first week of the new service the train is late 3 times. You may assume that the number of times the train is late in a week has a binomial distribution. Test, at the 5% level of significance, whether or not there is evidence that there is an increase in the number of times the train is late. State your hypothesis clearly. **(7 marks)**

2 A marketing company claims that Chestly cheddar cheese tastes better than Cumnauld cheddar cheese. Five people chosen at random as they entered a supermarket were asked to say which they preferred. Four people preferred Chestly cheddar cheese. Test, at the 5% level of significance, whether or not the manufacturer's claim is true. State your hypothesis clearly. **(7 marks)**

3 Historical information finds that nationally 30% of cars fail a brake test.

a Give a reason to support the use of a binomial distribution as a suitable model for the number of cars failing a brake test. **(1 mark)**

b Find the probability that, of 5 cars taking the test, all of them pass the brake test. **(2 marks)**

A garage decides to conduct a survey of their cars. A randomly selected sample of 10 of their cars is tested. Two of them fail the test.

c Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that cars in this garage fail less than the national average. **(7 marks)**

Chapter Exercises

4 The proportion of defective articles in a certain manufacturing process has been found from long experience to be 0.1.
A random sample of 50 articles was taken in order to monitor the production. The number of defective articles was recorded.

a Using a 5% level of significance, find the critical regions for a two-tailed test of the hypothesis that 1 in 10 articles has a defect. The probability in each tail should be as near 2.5% as possible. **(4 marks)**

b State the actual significance level of the above test. **(2 marks)**

Another sample of 20 articles was taken at a later date. Four articles were found to be defective.

c Test, at the 10% significance level, whether or not there is evidence that the proportion of defective articles has increased. State your hypothesis clearly. **(5 marks)**

5 It is claimed that 50% of women use Oriels powder. In a random survey of 20 women, 12 said they did not use Oriels powder.
Test, at the 5% significance level, whether or not there is evidence that the proportion of women using Oriels powder is 0.5. State your hypothesis carefully. **(6 marks)**

6 The manager of a superstore thinks that the probability of a person buying a certain make of computer is only 0.2.
To test whether this hypothesis is true the manager decides to record the make of computer bought by a random sample of 50 people who bought a computer.

a Find the critical region that would enable the manager to test whether or not there is evidence that the probability is different from 0.2. The probability of each tail should be as close to 2.5% as possible. **(4 marks)**

b Write down the significance level of this test. **(2 marks)**

15 people buy that certain make.

c Comment on this observation in light of your critical region. **(2 marks)**

7 **a** Explain what is meant by:

i a hypothesis test **ii** a critical value **iii** an acceptance region. **(3 marks)**

Johan believes the probability of him being late to school is 0.2. To test this claim he counts the number of times he is late in a random sample of 20 days.

b Find the critical region for a two-tailed test, at the 10% level of significance, of whether the probability he is late for school differs from 0.2. **(5 marks)**

c State the actual significance level of the test. **(1 mark)**

Johan discovers he is late for school in 7 out of the 20 days.

d Comment on whether Johan should accept or reject his claim that the probability he is late for school is 0.2. **(2 marks)**

8 From the large data set, the likelihood of a day with either zero or trace amounts of rain in Hurn in June 1987 was 0.5.

Poppy believes that the likelihood of a rain-free day in 2015 has increased.

In June 2015 in Hurn, 21 days were observed as having zero or trace amounts of rain.

Using a 5% significance level, test whether or not there is evidence to support Poppy's claim. **(6 marks)**

9 A single observation x is to be taken from a binomial distribution $B(30, p)$. This observation is used to test H_0: $p = 0.35$ against H_1: $p \neq 0.35$.

a Using a 5% level of significance, find the critical region for this test. The probability of rejecting either tail should be as close as possible to 2.5%. **(3 marks)**

b State the actual significance level of this test. **(2 marks)**

The actual value of X obtained is 4.

c State a conclusion that can be drawn based on this value giving a reason for your answer. **(2 marks)**

10 A pharmaceutical company claims that 85% of patients suffering from a chronic rash recover when treated with a new ointment.

A random sample of 20 patients with this rash is taken from hospital records.

a Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new ointment. **(2 marks)**

b Given that the claim is correct, find the probability that the ointment will be successful for exactly 16 patients. **(2 marks)**

The hospital believes that the claim is incorrect and the percentage who will recover is lower. From the records an administrator took a random sample of 30 patients who had been prescribed the ointment. She found that 20 had recovered.

c Stating your hypotheses clearly, test, at the 5% level of significance, the hospital's belief. **(6 marks)**

Chapter Exercises

Large data set

You will need access to the large data set and spreadsheet software to answer these questions.

1 The proportion of days with a recorded daily mean temperature greater than 15 °C in Leuchars between May 1987 and October 1987 was found to be 0.163 (3 s.f.).

A meteorologist wants to use a randomly chosen sample of 10 days to determine whether the probability of observing a daily mean temperature greater than 15 °C has increased significantly between 1987 and 2015.

- **a** Using a significance level of 5%, determine the critical region for this test.
- **b** Select a sample of 10 days from the 2015 data for Leuchars, and count the number of days with a mean temperature of greater than 15 °C.
- **c** Use your observation and your critical region to make a conclusion.

2 From the large data set, in Beijing in 1987, 23% of the days from May to October had a daily mean air temperature greater than 25 °C. Using a sample of size 10 from the data for daily mean air temperature in Beijing in 2015, test, at the 5% significance level, whether the proportion of days with a mean air temperature greater than 25 °C increased between 1987 and 2015.

Chapter Answers

1 H_0: $p = 0.2$, H_1: $p > 0.2$, $P(X \geqslant 3) = 0.3222 > 0.05$
There is insufficient evidence to reject H_0.
There is no evidence that the trains are late more often.

2 H_0: $p = 0.5$, H_1: $p > 0.5$, $P(X \geqslant 4) = 0.1875 > 0.05$
There is insufficient evidence to reject H_0.
There is insufficient evidence that the manufacturer's claim is true.

3 **a** Fixed number; independent trials; two outcomes (pass or fail); p constant for each car.
b 0.16807
c $0.3828 > 0.05$
There is insufficient evidence to reject H_0.
There is no evidence that the garage fails fewer than the national average.

4 **a** Critical region $X \leqslant 1$ and $X \geqslant 10$.
b 0.0583
c H_0: $p = 0.1$, H_1: $p > 0.1$, $P(X \geqslant 4) = 0.133 > 0.1$.
Accept H_0. There is no evidence that the proportion of defective articles has increased.

5 H_0: $p = 0.5$, H_1: $p \neq 0.5$, $P(X \leqslant 8) = 0.252 > 0.025$ (two-tailed)
There is insufficient evidence to reject H_0.
There is no evidence that the claim is wrong.

6 **a** Critical region is $X \leqslant 4$ and $X \geqslant 16$
b 0.0493
c There is insufficient evidence to reject H_0.
There is no evidence to suggest that the proportion of people buying that certain make of computer differs from 0.2.

7 **a** **i** The theory, methods, and practice of testing a hypothesis by comparing it with the null hypothesis.
ii The critical value is the first value to fall inside of the critical region.
iii The acceptance region is the region where we accept the null hypothesis.
b Critical region $X = 0$ and $X \geqslant 8$
c 4.36%
d As 7 does not lie in the critical region, H_0 is not rejected. Therefore, the proportion of times that Johan is late for school has not changed.

8 $P(X \geqslant 21) = 0.021 < 0.05$. Therefore there is sufficient evidence to support Poppy's claim that the likelihood of a rain-free day has increased.

9 **a** Critical region $X \leqslant 5$ and $X \geqslant 16$
b 5.34%
c $X = 4$ is in the critical region so there is enough evidence to reject H_0.

10 **a** $X \sim B(20, 0.85)$
b 0.1821
c Test statistic is proportion of patients who recover.
H_0: $p = 0.85$, H_1: $p < 0.85$
$P(X \leqslant 20) = 0.00966$
$0.00966 < 0.05$ so there is enough evidence to reject H_0. The percentage of patients who recover after treatment with the new ointment is lower than 85%.

Large data set

1 **a** The critical region is $X \geqslant 5$
b Students' answers
c Students' answers
2 Students' answers