P1 Chapter 11: Vectors

Representing Vectors

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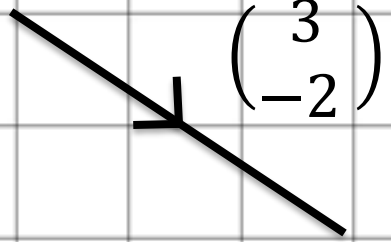
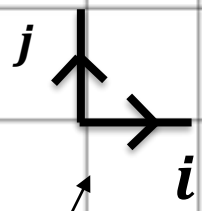


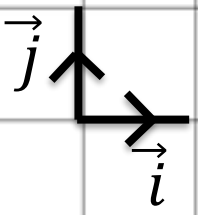
You should already be familiar that the value of a vector is the **displacement** in the x and y direction (if in 2D).

$$\vec{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \qquad \vec{b} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$\vec{a} + \vec{b} =$?

$2\vec{a} =$?





Bold notation is used in textbooks and exam papers but is awkward for handwriting.

A **unit vector** is a vector of magnitude 1. $\vec{i}$ and $\vec{j}$ are unit vectors in the x-axis and y-axis respectively.

$$\vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

e.g. $\begin{pmatrix} 4 \\ 3 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 4\vec{i} + 3\vec{j}$

Examples

If $\boldsymbol{a} = 3\boldsymbol{i}, \quad \boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j}, \quad \boldsymbol{c} = \boldsymbol{i} - 2\boldsymbol{j}$ then:

1) Write $\boldsymbol{a}$ in vector form.
2) Find $\boldsymbol{b} + 2\boldsymbol{c}$ in $\boldsymbol{i}, \boldsymbol{j}$ form.

1 ?

2 ?

Exercise 11.2

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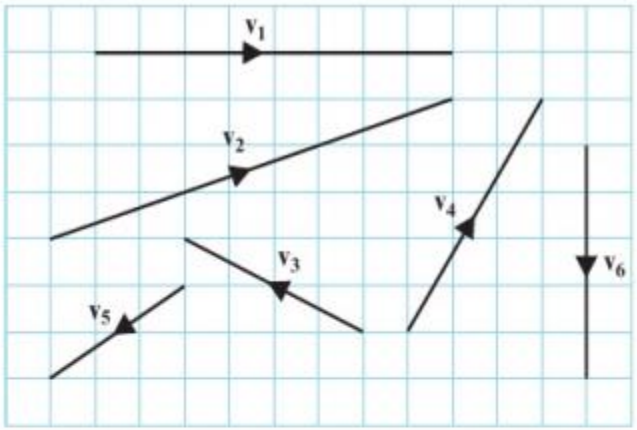
Homework Exercise

1 These vectors are drawn on a grid of unit squares.
Express the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, $\mathbf{v}_5$ and $\mathbf{v}_6$ in:
(i) **i, j** notation (ii) column vector form



2 Given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} - \mathbf{j}$, find these vectors in terms of **i** and **j**.

a $4\mathbf{a}$ **b** $\frac{1}{2}\mathbf{a}$ **c** $-\mathbf{b}$ **d** $2\mathbf{b} + \mathbf{a}$

e $3\mathbf{a} - 2\mathbf{b}$ **f** $\mathbf{b} - 3\mathbf{a}$ **g** $4\mathbf{b} - \mathbf{a}$ **h** $2\mathbf{a} - 3\mathbf{b}$

3 Given that $\mathbf{a} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 11 \\ -3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -8 \\ -1 \end{pmatrix}$ find:

a $5\mathbf{a}$ **b** $-\frac{1}{2}\mathbf{c}$ **c** $\mathbf{a} + \mathbf{b} + \mathbf{c}$ **d** $2\mathbf{a} - \mathbf{b} + \mathbf{c}$

e $2\mathbf{b} + 2\mathbf{c} - 3\mathbf{a}$ **f** $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

4 Given that $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{j}$, find:

a λ if $\mathbf{a} + \lambda\mathbf{b}$ is parallel to the vector **i** **b** μ if $\mu\mathbf{a} + \mathbf{b}$ is parallel to the vector **j**

Homework Exercise

5 Given that $\mathbf{c} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{d} = \mathbf{i} - 2\mathbf{j}$, find:

a λ if $\mathbf{c} + \lambda\mathbf{d}$ is parallel to $\mathbf{i} + \mathbf{j}$

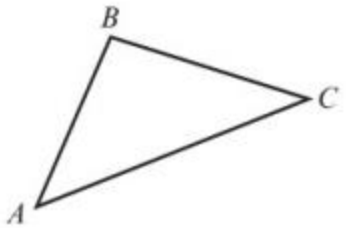
b μ if $\mu\mathbf{c} + \mathbf{d}$ is parallel to $\mathbf{i} + 3\mathbf{j}$

c s if $\mathbf{c} - s\mathbf{d}$ is parallel to $2\mathbf{i} + \mathbf{j}$

d t if $\mathbf{d} - t\mathbf{c}$ is parallel to $-2\mathbf{i} + 3\mathbf{j}$

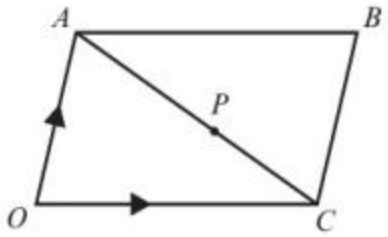
6 In triangle ABC, $\overrightarrow{AB} = 4\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{AC} = 5\mathbf{i} + 2\mathbf{j}$.
Find BC.

(2 marks)



7 $OABC$ is a parallelogram.
P divides AC in the ratio 3 : 2. $\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OC} = 7\mathbf{i}$.
Find in **i**, **j** format and column vector format:

a $\overrightarrow{AC}$ **b** $\overrightarrow{OP}$ **c** $\overrightarrow{AP}$



8 $\mathbf{a} = \begin{pmatrix} j \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 10 \\ k \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

Given that $\mathbf{b} - 2\mathbf{a} = \mathbf{c}$, find the values of j and k.

(2 marks)

Problem-solving

You can consider $\mathbf{b} - 2\mathbf{a} = \mathbf{c}$ as two linear equations. One for the x-components and one for the y-components.

Homework Exercise

9 $\mathbf{a} = \begin{pmatrix} p \\ -q \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} q \\ p \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$

Given that $\mathbf{a} + 2\mathbf{b} = \mathbf{c}$, find the values of p and q. **(2 marks)**

10 The resultant of the vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = p\mathbf{i} - 2p\mathbf{j}$ is parallel to the vector $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$.
Find:

a the value of p **(4 marks)**

b the resultant of vectors **a** and **b**. **(1 mark)**

Homework Answers

1 $\mathbf{v}_1$: $8\mathbf{i}$, $\begin{pmatrix}8\\0\end{pmatrix}$ $\mathbf{v}_2$: $9\mathbf{i}+3\mathbf{j}$, $\begin{pmatrix}9\\3\end{pmatrix}$ $\mathbf{v}_3$: $-4\mathbf{i}+2\mathbf{j}$, $\begin{pmatrix}-4\\2\end{pmatrix}$

$\mathbf{v}_4$: $3\mathbf{i}+5\mathbf{j}$, $\begin{pmatrix}3\\5\end{pmatrix}$ $\mathbf{v}_5$: $-3\mathbf{i}-2\mathbf{j}$, $\begin{pmatrix}-3\\-2\end{pmatrix}$ $\mathbf{v}_6$: $-5\mathbf{j}$, $\begin{pmatrix}0\\-5\end{pmatrix}$

2 **a** $8\mathbf{i}+12\mathbf{j}$ **b** $\mathbf{i}+1.5\mathbf{j}$ **c** $-4\mathbf{i}+\mathbf{j}$

d $10\mathbf{i}+\mathbf{j}$ **e** $-2\mathbf{i}+11\mathbf{j}$ **f** $-2\mathbf{i}-10\mathbf{j}$

g $14\mathbf{i}-7\mathbf{j}$ **h** $-8\mathbf{i}+9\mathbf{j}$

3 **a** $\begin{pmatrix}45\\35\end{pmatrix}$ **b** $\begin{pmatrix}4\\0.5\end{pmatrix}$ **c** $\begin{pmatrix}12\\3\end{pmatrix}$

d $\begin{pmatrix}-1\\16\end{pmatrix}$ **e** $\begin{pmatrix}-21\\-29\end{pmatrix}$ **f** $\begin{pmatrix}10\\2\end{pmatrix}$

4 **a** $\lambda = 5$ **b** $\mu = -\frac{3}{2}$

5 **a** $\lambda = \frac{1}{3}$ **b** $\mu = -1$

c $s = -1$ **d** $t = -\frac{1}{17}$

6 $\mathbf{i}-\mathbf{j}$

7 **a** $\overrightarrow{AC} = 5\mathbf{i}-4\mathbf{j} = \begin{pmatrix}5\\-4\end{pmatrix}$ **b** $\overrightarrow{OP} = 5\mathbf{i}+\frac{8}{5}\mathbf{j} = \begin{pmatrix}5\\\frac{8}{5}\end{pmatrix}$

c $\overrightarrow{AP} = 3\mathbf{i}-\frac{12}{5}\mathbf{j} = \begin{pmatrix}3\\-\frac{12}{5}\end{pmatrix}$

8 $j = 4, k = 11$

9 $p = 3, q = 2$

10 **a** $p = 5$ **b** $8\mathbf{i}-12\mathbf{j}$