P1 Chapter 11: Vectors

Vector Modelling

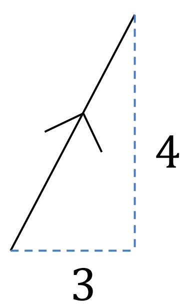
Modelling

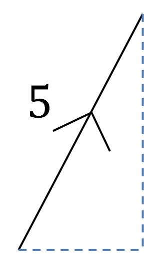
In Mechanics, you will see certain things can be represented as a simple number (without direction), or as a vector (with direction):

Remember a 'scalar' just means a number (in the context of vectors). It can be obtained using the **magnitude** of the vector.

Vector Quantity	**Equivalent Scalar Quantity**
Velocity e.g. $\begin{pmatrix}3\\4\end{pmatrix}\ km/h$	Speed $= 5\ km/h$
Displacement e.g. $\begin{pmatrix}-5\\12\end{pmatrix}\ km$	Distance $= 13\ km$

This means the position vector of the object changes by $\begin{pmatrix}3\\4\end{pmatrix}$ each hour.



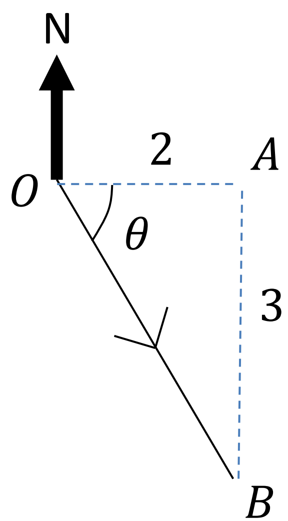


...which is equivalent to moving 5km each hour.

Example

[Textbook] A girl walks 2 km due east from a fixed point O to A, and then 3 km due south from A to B. Find

a) the total distance travelled
b) the position vector of B relative to O
c) $\left|\overrightarrow{OB}\right|$
d) The bearing of B from O.



a $2 + 3 = 5km$

b $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} km$

c $\left|\overrightarrow{OB}\right| = \sqrt{2^2 + 3^2} = \sqrt{13} = 3.61km$ (3sf)

d $90° + \tan^{-1}\left(\frac{3}{2}\right) = 56.3°$

Further Example

[Textbook] In an orienteering exercise, a cadet leaves the starting point O and walks 15 km on a bearing of $120°$ to reach A, the first checkpoint. From A he walks 9 km on a bearing of $240°$ to the second checkpoint, at B. From B he returns directly to O.

Find:

a) the position vector of A relative to O

b) $|\overrightarrow{OB}|$

c) the bearing of B from O

d) the position vector of B relative O.

I have no specific advice to offer except:

1. Draw a BIG diagram.
2. Remember bearings are measured clockwise from North.
3. Don't forget units (on vectors!)

a

?

b

?

c

?

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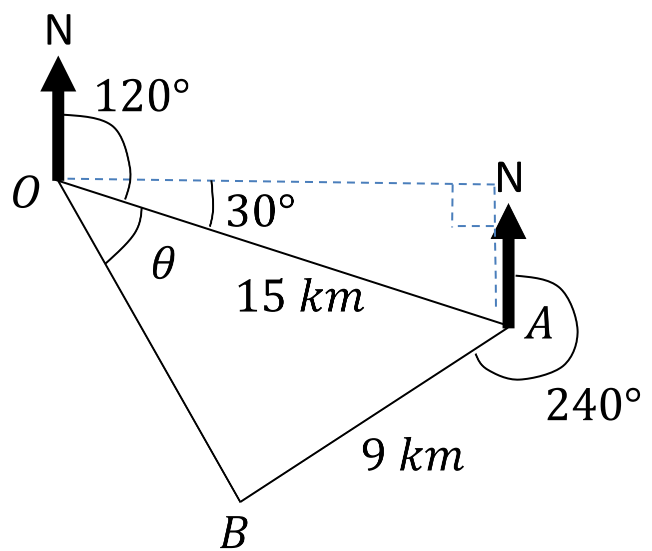
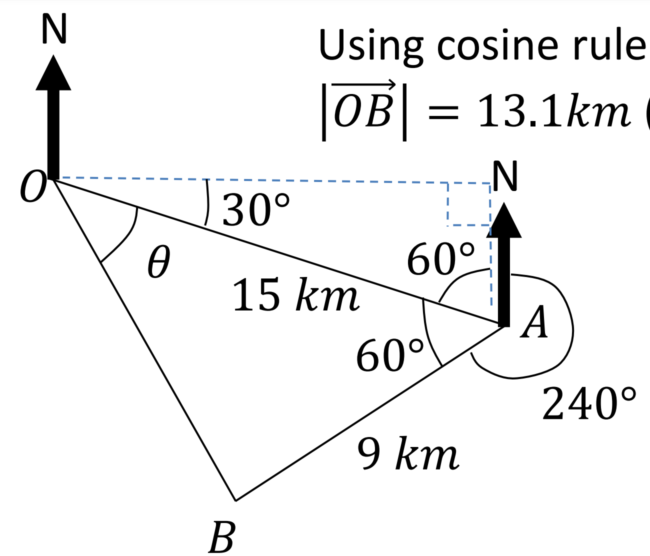
a



$$\overrightarrow{OA} = \begin{pmatrix} 15\cos 30° \\ 15\sin 30° \end{pmatrix} = \begin{pmatrix} 13.0 \\ 7.5 \end{pmatrix} \ km$$

b



Using cosine rule on ΔOAB,

$|\overrightarrow{OB}| = 13.1km$ (3sf)

c Using sine rule on ΔOAB:

$$\theta = 36.6 \ldots °$$

$\therefore$ Bearing $= 120 + 36.6° = 157°$ (3sf)

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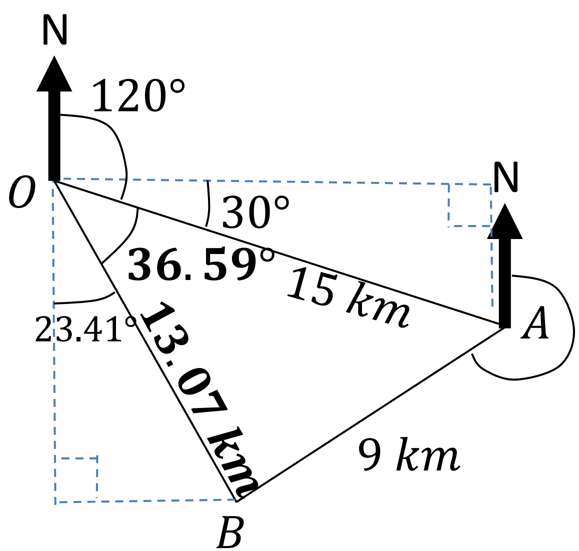

$$\overrightarrow{OB} = \begin{pmatrix} 13.07 \sin 23.41° \\ -13.07 \cos 23.41° \end{pmatrix} = \begin{pmatrix} 5.19 \\ -12.0 \end{pmatrix} km$$

(I think the textbook made a rounding error due to use of rounded values. They put $\begin{pmatrix} 5.1 \\ -12.1 \end{pmatrix}$)

Exercise 11.6

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Pages 90-91

Homework Exercise

1 Find the speed of a particle moving with these velocities:

a $(3\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ b $(24\mathbf{i} - 7\mathbf{j})\,\text{km h}^{-1}$

c $(5\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ d $(-7\mathbf{i} + 4\mathbf{j})\,\text{cm s}^{-1}$

Hint Speed is the magnitude of the velocity vector.

2 Find the distance moved by a particle which travels for:

a 5 hours at velocity $(8\mathbf{i} + 6\mathbf{j})\,\text{km h}^{-1}$

b 10 seconds at velocity $(5\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

c 45 minutes at velocity $(6\mathbf{i} + 2\mathbf{j})\,\text{km h}^{-1}$

d 2 minutes at velocity $(-4\mathbf{i} - 7\mathbf{j})\,\text{cm s}^{-1}$.

Hint Find the speed in each case then use:
Distance travelled = speed × time

3 Find the speed and the distance travelled by a particle moving in a straight line with:

a velocity $(-3\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ for 15 seconds b velocity $(2\mathbf{i} + 5\mathbf{j})\,\text{m s}^{-1}$ for 3 seconds

c velocity $(5\mathbf{i} - 2\mathbf{j})\,\text{km h}^{-1}$ for 3 hours d velocity $(12\mathbf{i} - 5\mathbf{j})\,\text{km h}^{-1}$ for 30 minutes.

4 A particle P is accelerating at a constant speed.
When $t = 0$, P has velocity $\mathbf{u} = (2\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$
and at time $t = 5$ s, P has velocity $\mathbf{v} = (16\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-1}$.

Hint The units of acceleration will be m/s² or m s⁻².

The acceleration vector of the particle is given by the formula: $\mathbf{a} = \dfrac{\mathbf{v} - \mathbf{u}}{t}$

Find the acceleration of P in terms of $\mathbf{i}$ and $\mathbf{j}$.

5 A particle P of mass $m = 0.3$ kg moves under the action of a single constant force $\mathbf{F}$ newtons.
The acceleration of P is $\mathbf{a} = (5\mathbf{i} + 7\mathbf{j})\,\text{m s}^{-2}$.

a Find the angle between the acceleration and $\mathbf{i}$. **(2 marks)**

Force, mass and acceleration are related by the formula $\mathbf{F} = m\mathbf{a}$.

b Find the magnitude of $\mathbf{F}$. **(3 marks)**

Homework Exercise

6 Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, are given by the vectors $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N.
The resultant force, $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$ acts in a direction which is parallel to the vector $(2\mathbf{i} - \mathbf{j})$.

a Find the angle between $\mathbf{R}$ and the vector $\mathbf{i}$. **(2 marks)**

b Show that $p + 2q = 5$. **(3 marks)**

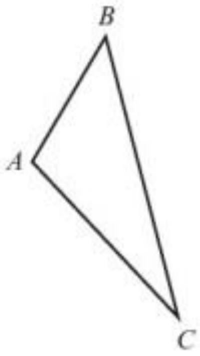
c Given that $p = 1$, find the magnitude of $\mathbf{R}$. **(3 marks)**

7 The diagram shows a sketch of a field in the shape of a triangle ABC.
Given $\overrightarrow{AB} = 30\mathbf{i} + 40\mathbf{j}$ metres and $\overrightarrow{AC} = 40\mathbf{i} - 60\mathbf{j}$ metres,

a find $\overrightarrow{BC}$ **(2 marks)**

b find the size of $\angle BAC$, in degrees, to one decimal place **(4 marks)**

c find the area of the field in square metres. **(3 marks)**



8 A boat has a position vector of $(2\mathbf{i} + \mathbf{j})$ km and a buoy has a position vector of $(6\mathbf{i} - 4\mathbf{j})$ km, relative to a fixed origin O.

a Find the distance of the boat from the buoy.

b Find the bearing of the boat from the buoy.

The boat travels with constant velocity $(8\mathbf{i} - 10\mathbf{j})$ km/h.

c Verify that the boat is travelling directly towards the buoy

d Find the speed of the boat.

e Work out how long it will take the boat to reach the buoy.

Problem-solving

Draw a sketch showing the initial positions of the boat, the buoy and the origin.

Homework Answers

1 **a** $5\,\mathrm{m\,s^{-1}}$ **b** $25\,\mathrm{km\,h^{-1}}$
c $5.39\,\mathrm{m\,s^{-1}}$ **d** $8.06\,\mathrm{cm\,s^{-1}}$

2 **a** 50 km **b** 51.0 m
c 4.74 km **d** 967 cm

3 **a** $5\,\mathrm{m\,s^{-1}}$, 75 m **b** $5.39\,\mathrm{m\,s^{-1}}$, 16.2 m
c $5.39\,\mathrm{km\,h^{-1}}$, 16.2 km **d** $13\,\mathrm{km\,h^{-1}}$, 6.5 km

4 $(2.8\mathbf{i} - 1.6\mathbf{j})\,\mathrm{m\,s^{-2}}$

5 **a** 54.5° **b** $0.3\sqrt{74}$ Newtons

6 **a** 26.6° below **i**
b $\mathbf{R} = (3 + p)\mathbf{i} + (q - 4)\mathbf{j}$, $3 + p = 2\lambda$ and
$q - 4 = -\lambda \Rightarrow \lambda = 4 - q$
$3 + p = 2(4 - q) \Rightarrow 3 + p = 8 - 2q$ so $p + 2q = 5$
c $|\mathbf{R}| = 2\sqrt{5}$ newtons

7 **a** $10\mathbf{i} - 100\mathbf{j}$ **b** 109.4° **c** $1700\,\mathrm{m^2}$

8 **a** $\sqrt{41}$ **b** 303.7°
c $\overrightarrow{AB} = 4\mathbf{i} - 5\mathbf{j}$, $\mathbf{v} = 2(4\mathbf{i} - 5\mathbf{j})$ so the boat is travelling directly towards the buoy.
d $2\sqrt{41}$ **e** 30 minutes