P2 Chapter 2: Graphing Functions

Solving Modulus Problems

Solving Modulus Problems

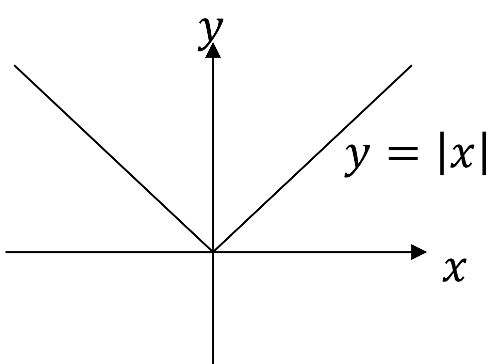
[Textbook] Given the function $f(x) = 3|x-1| - 2, x \in \mathbb{R}$,

(a) Sketch the graph of $y = f(x)$

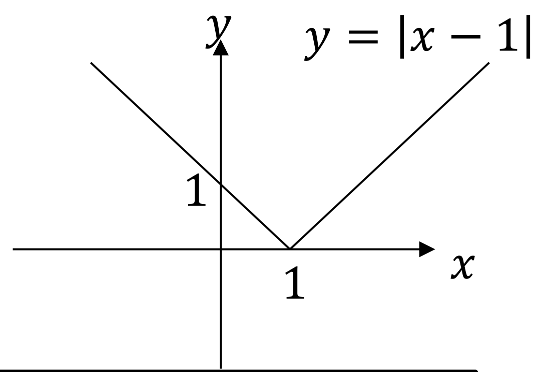
(b) State the range of f.

(c) Solve the equation $f(x) = \frac{1}{2}x + 3$

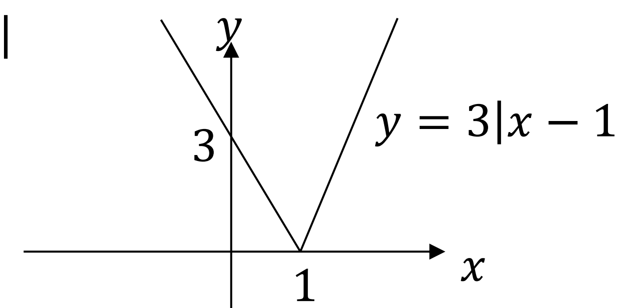
a It is often helpful to **sketch the graph in stages** as we apply more transformations:



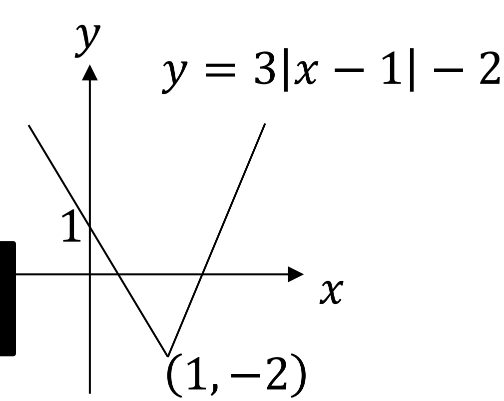
Start with the 'simplest' version of the graph, $y = |x|$



-1 is 'inside' function so translate 1 right.



3 is outside modulus function so affects y values.



-2 is outside modulus function so translate 2 down.

Solving Modulus Problems

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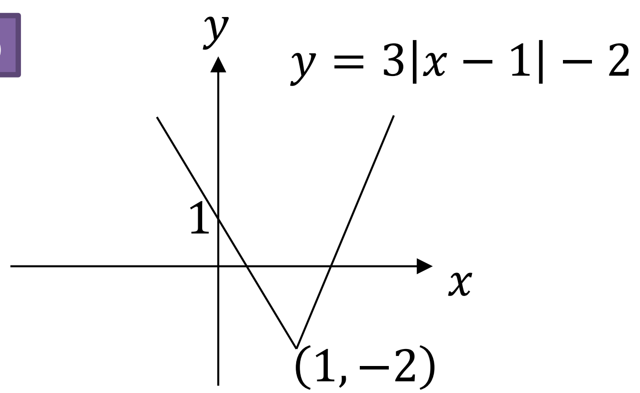
b



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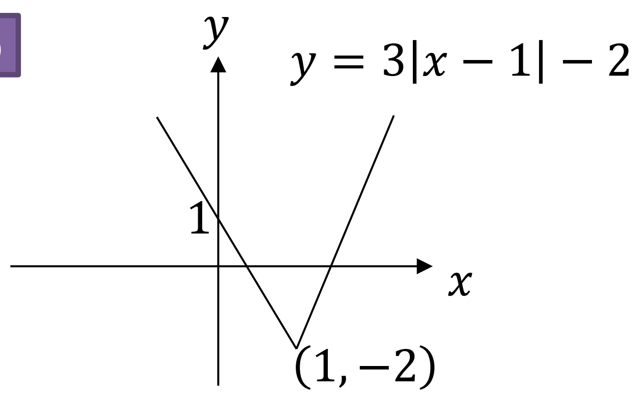

From the graph we observe the possible outputs (i.e. y values):

$$f(x) \geq -2$$

Solving Modulus Problems

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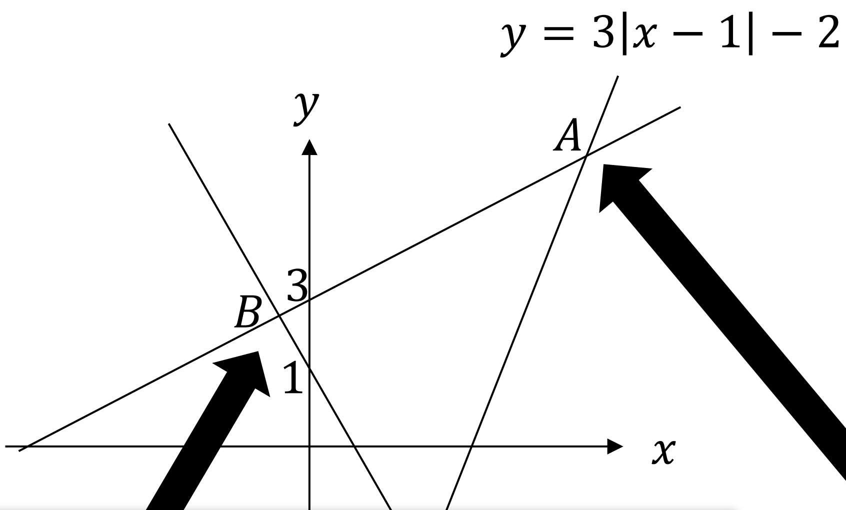
C



In Year 1 we saw how we could sketch the line representing each side of the equation, then find the point of intersection, i.e.

$$y = 3|x-1| - 2$$
$$y = \frac{1}{2}x + 3$$

At B, this is the intersection of the negated reflected line (i.e. $y = 3(-x+1) - 2$) with the line $y = \frac{1}{2}x + 3$

$$3(-x+1) - 2 = \frac{1}{2}x + 3$$
$$x = -\frac{4}{7}$$

Fro Note: Only the modulus bit is negated, not the whole equation.

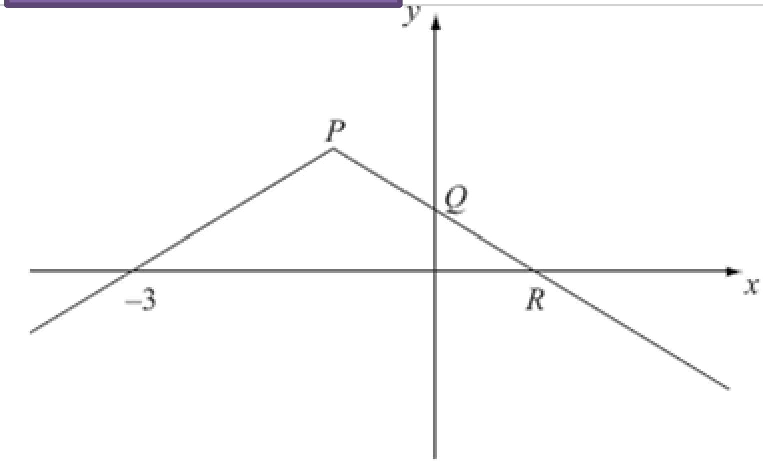
At A, this is the intersection of the original unreflected line (i.e. $y = 3(x-1) - 2$) with the line $y = \frac{1}{2}x + 3$

$$3(x-1) - 2 = \frac{1}{2}x + 3$$
$$x = \frac{16}{5}$$

Test Your Understanding

C4 June 2008 Q3

You can sketch this function by starting with $y = |x|$ and gradually transform it as per the previous example.



Given that $f(x) = 2 - |x + 1|$,

(c) find the coordinates of the points P, Q and R, **(3)**

(d) solve $f(x) = \frac{1}{2}x$. **(5)**

a

?

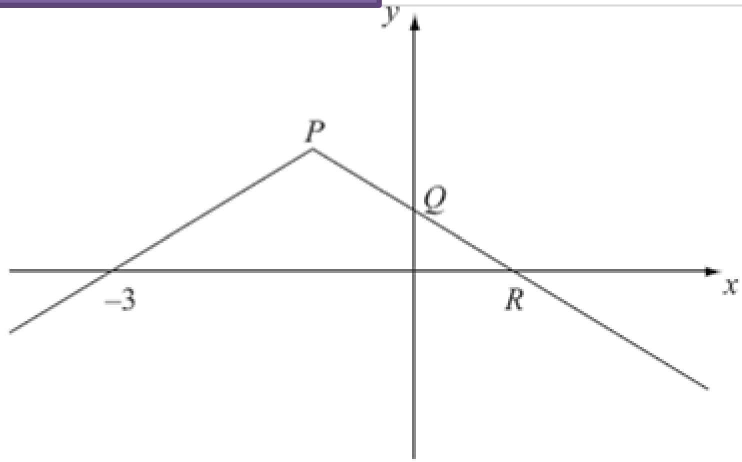
b

?

Test Your Understanding

C4 June 2008 Q3

You can sketch this function by starting with $y = |x|$ and gradually transform it as per the previous example.



Given that $f(x) = 2 - |x + 1|$,

(*c*) find the coordinates of the points P, Q and R, **(3)**

(*d*) solve $f(x) = \frac{1}{2}x$. **(5)**

a

Q: When $x = 0$, $f(x) = 2 - |0 + 1| = 1$

$$Q(0,1)$$

R: When $y = 0$, $2 - |x + 1| = 0$

$$|x + 1| = 2$$

Either $x + 1 = 2 \quad \rightarrow \quad x = 1$

Or $-x - 1 = 2 \quad \rightarrow x = -3$

$$R(1,0)$$

P: Graph is at its maximum when $|x + 1| = 0$

Thus $x = -1$ (alternative by symmetry, -1 is halfway between -3 and 1)

$$P(-1,2)$$

b

$$2 - |x + 1| = \frac{1}{2}x$$

When $x + 1$ is not reflected:

$$2 - x - 1 = \frac{1}{2}x$$

$$x = \frac{2}{3}$$

When $x + 1$ is reflected:

$$2 + x + 1 = \frac{1}{2}x$$

$$x = -6$$

Check:

$2 - \left|\frac{2}{3} + 1\right| = \frac{1}{2}\left(\frac{2}{3}\right)$ works

$2 - |-6 + 1| = \frac{1}{2}(-6)$ works

Exercise 2.7

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Extension

[MAT 2006 1I]
The equation $|x| + |x - 1| = 0$ has how many solutions?

?

Exercise 2.7

Pearson Pure Mathematics Year 2/AS
Page 14

Extension

[MAT 2006 1I]
The equation $|x| + |x - 1| = 0$ has how many solutions?

0 solutions. $|\boldsymbol{x}|$ and $|\boldsymbol{x} - \mathbf{1}|$ are each at least 0, and thus must both be 0 to add to 0. But if $|\boldsymbol{x}| = \mathbf{0}$ then $\boldsymbol{x} = \mathbf{0}$, but then $|\boldsymbol{x} - \mathbf{1}| = \mathbf{1}$.
Alternatively, if we sketch $\boldsymbol{y} = |\boldsymbol{x}| + |\boldsymbol{x} - \mathbf{1}|$ we can see it never touches the x-axis, and therefore has no roots.

Homework Exercise

1 For each function

i sketch the graph of $y = \mathrm{f}(x)$

ii state the range of the function.

a $\mathrm{f}: x \mapsto 4|x| - 3, x \in \mathbb{R}$

b $\mathrm{f}(x) = \frac{1}{3}|x + 2| - 1, x \in \mathbb{R}$

c $\mathrm{f}(x) = -2|x - 1| + 6, x \in \mathbb{R}$

d $\mathrm{f}: x \mapsto -\frac{5}{2}|x| + 4, x \in \mathbb{R}$

Hint For part **b** transform the graph of $y = |x|$ by:

- A translation by vector $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$
- A vertical sketch with scale factor $\frac{1}{3}$
- A translation by vector $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

2 Given that $\mathrm{p}(x) = 2|x + 4| - 5, x \in \mathbb{R}$,

a sketch the graph of $y = \mathrm{p}(x)$

b shade the region of the graph that satisfies $y \geqslant \mathrm{p}(x)$.

3 Given that $\mathrm{q}(x) = -3|x| + 6, x \in \mathbb{R}$,

a sketch the graph of $y = \mathrm{q}(x)$

b shade the region of the graph that satisfies $y < \mathrm{q}(x)$.

4 The function f is defined as

$\mathrm{f}: x \mapsto 4|x + 6| + 1, x \in \mathbb{R}$.

a Sketch the graph of $y = \mathrm{f}(x)$.

b State the range of the function.

c Solve the equation $\mathrm{f}(x) = -\frac{1}{2}x + 1$.

Homework Exercise

5 Given that $g(x) = -\frac{5}{2}|x - 2| + 7, x \in \mathbb{R}$,

a sketch the graph of $y = g(x)$

b state the range of the function

c solve the equation $g(x) = x + 1$.

6 The functions m and n are defined as

$m(x) = -2x + k, x \in \mathbb{R}$

$n(x) = 3|x - 4| + 6, x \in \mathbb{R}$

where k is a constant.

The equation $m(x) = n(x)$ has no real roots.
Find the range of possible values for the constant k. **(4 marks)**

Problem-solving

$m(x) = n(x)$ has no real roots means that $y = m(x)$ and $y = n(x)$ do not intersect.

7 The functions s and t are defined as

$s(x) = -10 - x, x \in \mathbb{R}$

$t(x) = 2|x + b| - 8, x \in \mathbb{R}$

where b is a constant.

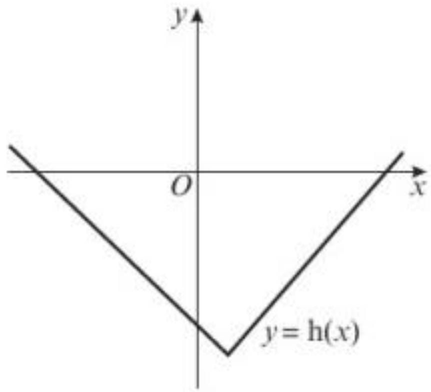
The equation $s(x) = t(x)$ has exactly one real root. Find the value of b. **(4 marks)**

Homework Exercise

8 The function h is defined by

$h(x) = \frac{2}{3}|x - 1| - 7, x \in \mathbb{R}$

The diagram shows a sketch of the graph $y = h(x)$.



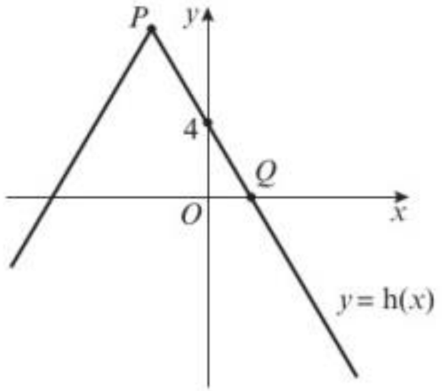
a State the range of h. **(1 mark)**

b Give a reason why h^{-1} does not exist. **(1 mark)**

c Solve the inequality $h(x) < -6$. **(4 marks)**

d State the range of values of k for which the equation $h(x) = \frac{2}{3}x + k$ has no solutions. **(4 marks)**

9 The diagram shows a sketch of part of the graph $y = h(x)$, where $h(x) = a - 2|x + 3|, x \in \mathbb{R}$.
The graph intercepts the y-axis at $(0, 4)$.



a Find the value of a. **(2 marks)**

b Find the coordinates of P and Q. **(3 marks)**

c Solve $h(x) = \frac{1}{3}x + 6$. **(5 marks)**

Homework Exercise

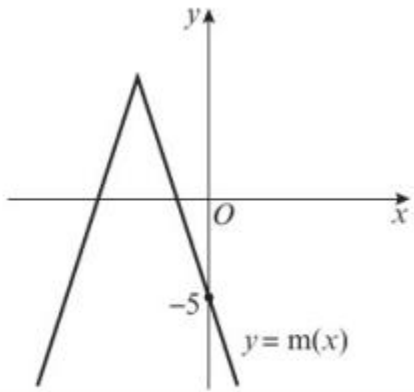
10 The diagram shows a sketch of part of the graph $y = \mathrm{m}(x)$, where $\mathrm{m}(x) = -4|x + 3| + 7$, $x \in \mathbb{R}$.

a State the range of m. **(1 mark)**

b Solve the equation $\mathrm{m}(x) = \frac{3}{5}x + 2$. **(4 marks)**

Given that $\mathrm{m}(x) = k$, where k is a constant, has two distinct roots

c state the set of possible values for k. **(4 marks)**



Challenge

1 The functions f and g are defined by

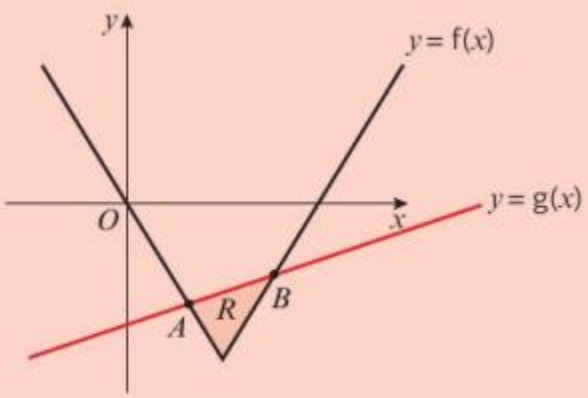
$\mathrm{f}(x) = 2|x - 4| - 8$, $x \in \mathbb{R}$

$\mathrm{g}(x) = x - 9$, $x \in \mathbb{R}$

The diagram shows a sketch of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$.

a Find the coordinates of the points A and B.

b Find the area of the region R.



Homework Exercise

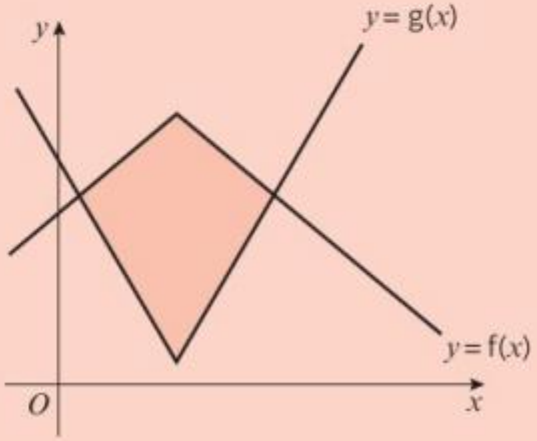
Challenge

2 The functions f and g are defined as:

$f(x) = -|x - 3| + 10, x \in \mathbb{R}$

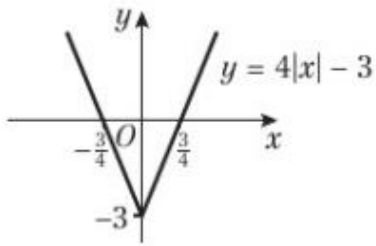
$g(x) = 2|x - 3| + 2, x \in \mathbb{R}$

Show that the area of the shaded region is $\frac{64}{3}$

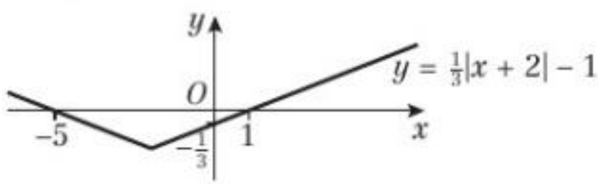


Homework Answers

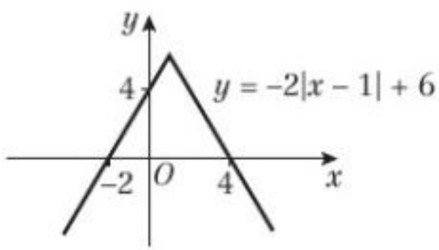
1 **a** Range $f(x) \geqslant -3$



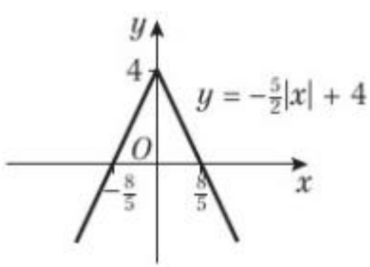
b Range $f(x) \geqslant -1$



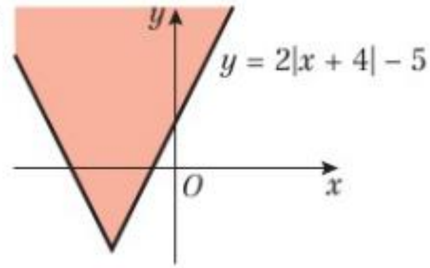
c Range $f(x) \leqslant 6$



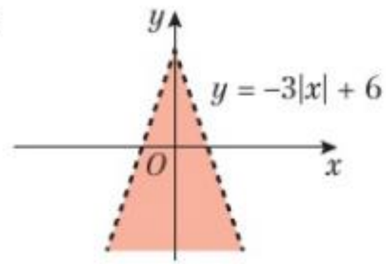
d Range $f(x) \leqslant 4$



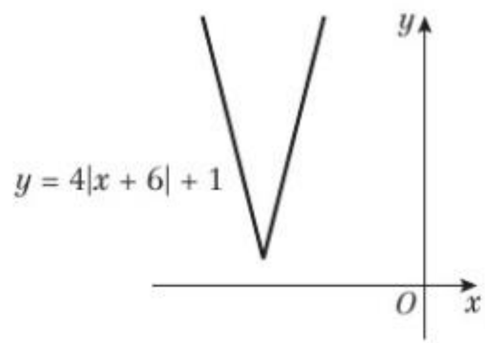
2 **a, b**



3 **a, b**



4 **a**

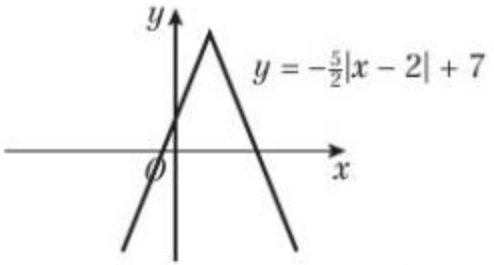


b $f(x) \geqslant 1$

c $x = -\frac{16}{3}$ and $x = -\frac{48}{7}$

Homework Answers

5 **a**



b $g(x) \leqslant 7$ **c** $x = -\frac{2}{3}$ and $x = \frac{22}{7}$

6 $k < 14$

7 $b = 2$

8 **a** $h(x) \geqslant -7$

b Original function is many-to-one, therefore the inverse is one-to-many, which is not a function.

c $-\frac{1}{2} < x < \frac{5}{2}$ **d** $k < -\frac{23}{3}$

9 **a** $a = 10$ **b** $P(-3, 10)$ and $Q(2, 0)$

c $x = -\frac{6}{7}$ and $x = -6$

10 **a** $m(x) \leqslant 7$ **b** $x = -\frac{35}{23}$ and $x = -5$

c $k < 7$

Challenge

1 **a** $A(3, -6)$ and $B(7, -2)$

b 6 units^2.

2 Graphs intersect at $x = \frac{1}{3}$ and $x = \frac{17}{3}$.
Maximum point of $f(x)$ is (3, 10). Minimum point of $g(x)$ is (3, 2). Using area of a kite, area $= \frac{64}{3}$