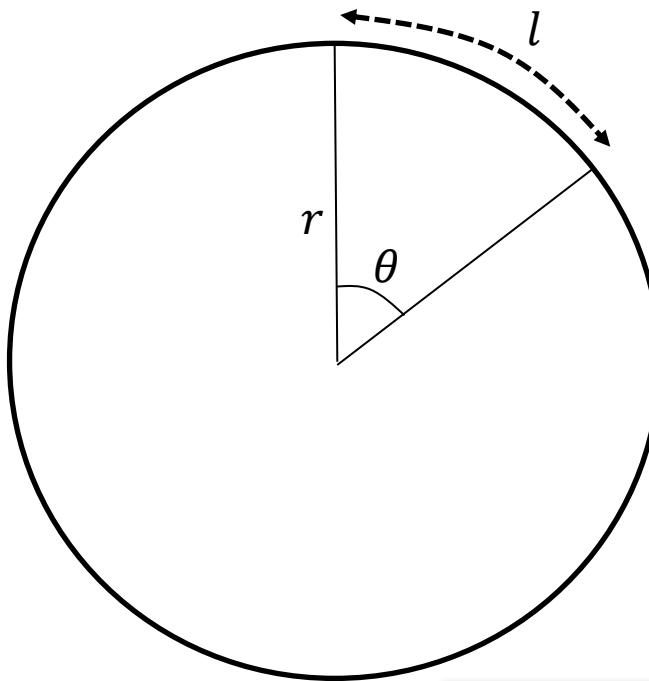

P2 Chapter 5: Radians

Arc Length

Arc length



Arc length in degrees:

$$l = \frac{\theta}{360} \times 2\pi r$$

Arc length in radians

From before, we know that 1 radian gives an arc of 1 radius in length, so θ radians must give a length of...

$$l = r\theta$$

Examples

[Textbook] Find the length of the arc of a circle of radius 5.2 cm, given that the arc subtends an angle of 0.8 radians at the centre of the circle.

?

[Textbook] An arc AB of a circle with radius 7 cm and centre O has a length of 2.45 cm. Find the angle $\angle AOB$ subtended by the arc at the centre of the circle

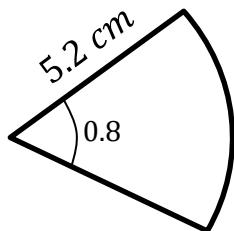
?

Fro Note: Whether your calculator is in degrees mode or radians mode is only relevant when using $\sin/\cos/\tan$ – it won't affect simple multiplication!

Terminology: 'Subtend' means **opposite** or extending beneath.

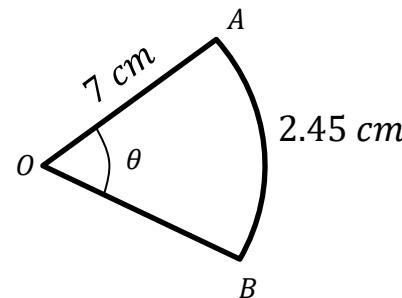
Examples

[Textbook] Find the length of the arc of a circle of radius 5.2 cm, given that the arc subtends an angle of 0.8 radians at the centre of the circle.



$$0.8 \times 5.2 = 4.16 \text{ cm}$$

[Textbook] An arc AB of a circle with radius 7 cm and centre O has a length of 2.45 cm. Find the angle $\angle AOB$ subtended by the arc at the centre of the circle



$$\theta \times 7 = 2.45$$

$$\theta = \frac{2.45}{7} = 0.35 \text{ rad}$$

From Note: Whether your calculator is in degrees mode or radians mode is only relevant when using $\sin/\cos/\tan$ – it won't affect simple multiplication!

Terminology: 'Subtend' means **opposite** or extending beneath.

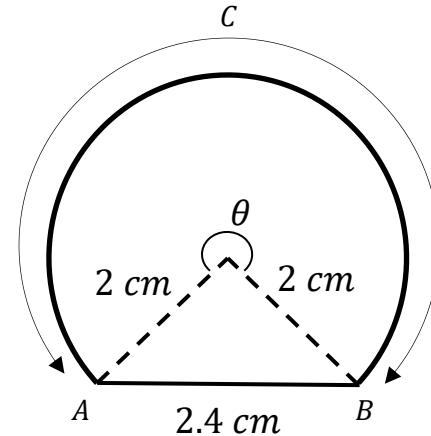
Further Examples

[Textbook] An arc AB of a circle, with centre O and radius r cm, subtends an angle of θ radians at O . The perimeter of the sector AOB is P cm. Express r in terms of P and θ .

?

[Textbook] The border of a garden pond consists of a straight edge AB of length 2.4m, and a curved part C , as shown in the diagram. The curve part is an arc of a circle, centre O and radius 2m.

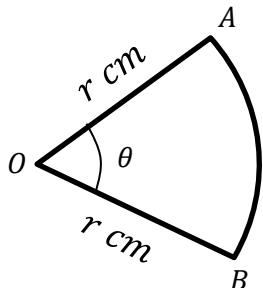
Find the length of C .



?

Further Examples

[Textbook] An arc AB of a circle, with centre O and radius r cm, subtends an angle of θ radians at O . The perimeter of the sector AOB is P cm. Express r in terms of P and θ .



$$P = 2r + \theta r$$

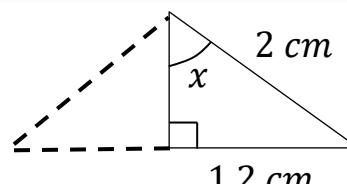
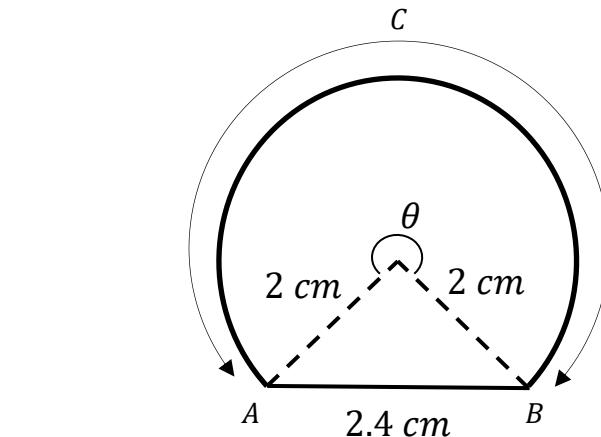
$$P = r(2 + \theta)$$

$$P$$

$$r = \frac{P}{2 + \theta}$$

[Textbook] The border of a garden pond consists of a straight edge AB of length 2.4m, and a curved part C , as shown in the diagram. The curve part is an arc of a circle, centre O and radius 2m.

Find the length of C .



Fro Tip: Trigonometry on right-angled triangles is always simpler than using sine/cosine rule.

$$x = \sin^{-1}\left(\frac{1.2}{2}\right) = 0.6435 \dots \text{rad}$$

Angles round a point add to 2π .

$$\theta = 2\pi - 2x = 4.9961 \dots \text{rad}$$

$$\therefore C = 2 \times 4.9961 = 9.99 \text{ m (3sf)}$$

Test Your Understanding

Edexcel C2 Jan 2005 Q7

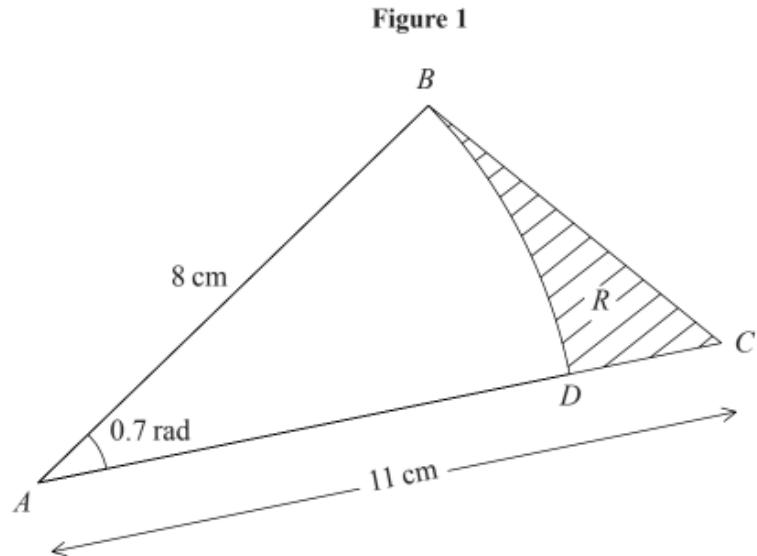


Figure 1 shows the triangle ABC , with $AB = 8 \text{ cm}$, $AC = 11 \text{ cm}$ and $\angle BAC = 0.7$ radians. The arc BD , where D lies on AC , is an arc of a circle with centre A and radius 8 cm . The region R , shown shaded in Figure 1, is bounded by the straight lines BC and CD and the arc BD .

Find

- The length of the arc BD .
- The perimeter of R , giving your answer to 3 significant figures.

a

?

b

?

Test Your Understanding

Edexcel C2 Jan 2005 Q7

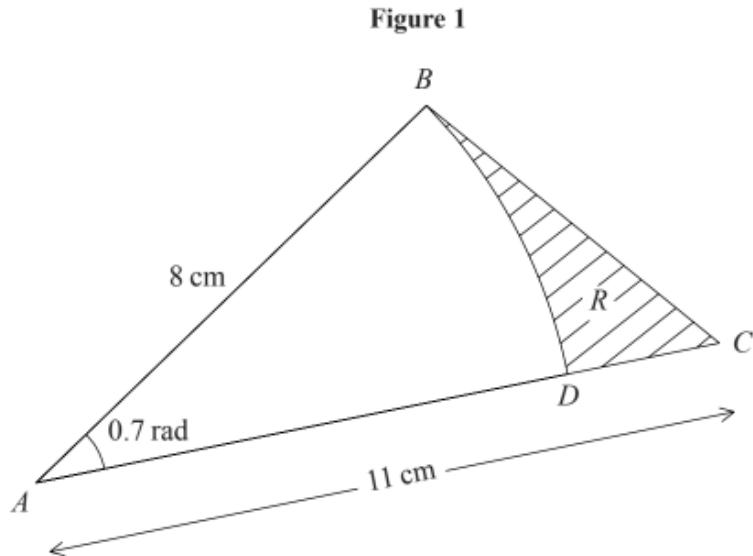


Figure 1 shows the triangle ABC , with $AB = 8 \text{ cm}$, $AC = 11 \text{ cm}$ and $\angle BAC = 0.7$ radians. The arc BD , where D lies on AC , is an arc of a circle with centre A and radius 8 cm. The region R , shown shaded in Figure 1, is bounded by the straight lines BC and CD and the arc BD .

Find

- The length of the arc BD .
- The perimeter of R , giving your answer to 3 significant figures.

a Length of arc $BD = 0.7 \times 8 = 5.6 \text{ cm}$

b Perimeter $= BD + CD + BC$

$$CD = 11 - 8 = 3$$

$$BC = \sqrt{8^2 + 11^2 - 2 \times 8 \times 11 \times \cos(0.7)} = 7.09 \text{ cm}$$

$$\therefore P = 5.6 + 3 + 7.09 = 15.7 \text{ cm (to 3sf)}$$

Exercise 5.2

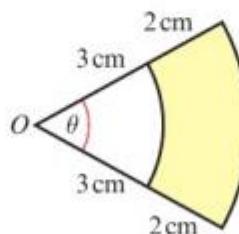
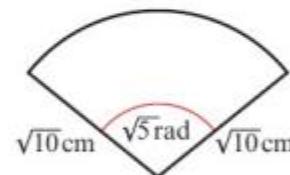
Pearson Pure Year 2

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Homework Exercise

- 1 An arc AB of a circle, centre O and radius r cm, subtends an angle θ radians at O .
The length of AB is l cm.
- a Find l when: i $r = 6, \theta = 0.45$ ii $r = 4.5, \theta = 0.45$ iii $r = 20, \theta = \frac{3}{8}\pi$
b Find r when: i $l = 10, \theta = 0.6$ ii $l = 1.26, \theta = 0.7$ iii $l = 1.5\pi, \theta = \frac{5}{12}\pi$
c Find θ when: i $l = 10, r = 7.5$ ii $l = 4.5, r = 5.625$ iii $l = \sqrt{12}, r = \sqrt{3}$
- 2 A minor arc AB of a circle, centre O and radius 10 cm, subtends an angle x at O . The major arc AB subtends an angle $5x$ at O . Find, in terms of π , the length of the minor arc AB .
- 3 An arc AB of a circle, centre O and radius 6 cm, has length l cm. Given that the chord AB has length 6 cm, find the value of l , giving your answer in terms of π .
- 4 The sector of a circle of radius $\sqrt{10}$ cm contains an angle of $\sqrt{5}$ radians, as shown in the diagram. Find the length of the arc, giving your answer in the form $p\sqrt{q}$ cm, where p and q are integers.
- 5 Referring to the diagram, find:
- a the perimeter of the shaded region when $\theta = 0.8$ radians.
b the value of θ when the perimeter of the shaded region is 14 cm.
- 6 A sector of a circle of radius r cm contains an angle of 1.2 radians. Given that the sector has the same perimeter as a square of area 36 cm^2 , find the value of r .
- 7 A sector of a circle of radius 15 cm contains an angle of θ radians. Given that the perimeter of the sector is 42 cm, find the value of θ .

Notation The **minor arc** AB is the shorter arc between points A and B on a circle.

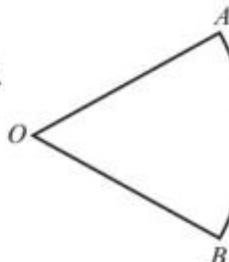
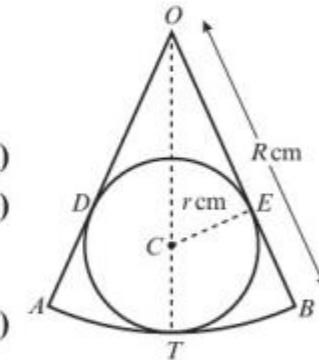
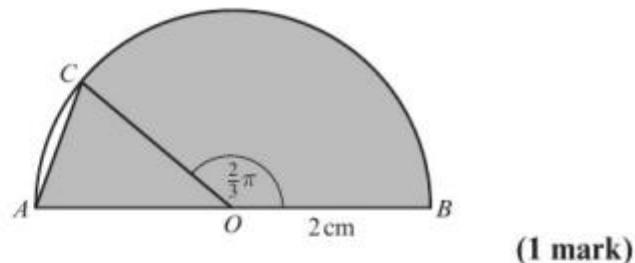


Problem-solving

The radius of the larger arc is $3 + 2 = 5$ cm.

Homework Exercise

- 8 In the diagram AB is the diameter of a circle, centre O and radius 2cm. The point C is on the circumference such that $\angle COB = \frac{2}{3}\pi$ radians.
- a State the value, in radians, of $\angle COA$. (1 mark)
- The shaded region enclosed by the chord AC , arc CB and AB is the template for a brooch.
- b Find the exact value of the perimeter of the brooch. (5 marks)
- 9 The points A and B lie on the circumference of a circle with centre O and radius 8.5cm. The point C lies on the major arc AB . Given that $\angle ACB = 0.4$ radians, calculate the length of the minor arc AB .
- 10 In the diagram OAB is a sector of a circle, centre O and radius R cm, and $\angle AOB = 2\theta$ radians. A circle, centre C and radius r cm, touches the arc AB at T , and touches OA and OB at D and E respectively, as shown.
- a Write down, in terms of R and r , the length of OC . (1 mark)
- b Using $\triangle OCE$, show that $R \sin \theta = r(1 + \sin \theta)$. (3 marks)
- c Given that $\sin \theta = \frac{3}{4}$ and that the perimeter of the sector OAB is 21cm, find r , giving your answer to 3 significant figures. (7 marks)
- 11 The diagram shows a sector AOB . The perimeter of the sector is twice the length of the arc AB . Find the size of angle AOB .



Homework Exercise

- 12 A circular Ferris wheel has 24 pods equally spaced on its circumference.

Given the arc length between each pod is $\frac{3\pi}{2}$ m, and modelling each pod as a particle,

- a calculate the diameter of the Ferris wheel.

Given that it takes approximately 30 seconds for a pod to complete one revolution,

- b estimate the speed of the pod in km/h.

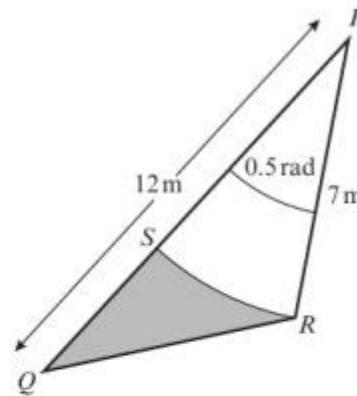
- 13 The diagram above shows a triangular garden, PQR , with

$PQ = 12$ m, $PR = 7$ m and $\angle QPR = 0.5$ radians. The curve SR is a small path separating the shaded patio area and the lawn, and is an arc of a circle with centre at P and radius 7 m.

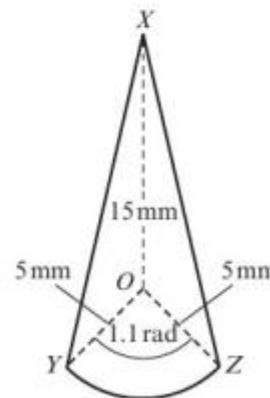
Find:

- a the length of the path SR (2 marks)

- b the perimeter of the shaded patio, giving your answer to 3 significant figures. (4 marks)



- 14 The shape XYZ shown is a design for an earring.



The straight lines XY and XZ are equal in length. The curve YZ is an arc of a circle with centre O and radius 5 mm. The size of $\angle YOZ$ is 1.1 radians and $XO = 15$ mm.

- a Find the size of $\angle XOZ$, in radians, to 3 significant figures. (2 marks)
- b Find the total perimeter of the earring, to the nearest mm. (6 marks)

Homework Answers

- 1** a i 2.7 ii 2.025 iii 7.5π
 b i $\frac{50}{3}$ ii 1.8 iii 3.6
 c i $\frac{4}{3}$ ii 0.8 iii 2

2 $\frac{10}{3}\pi$ cm **3** 2π **4** $5\sqrt{2}$ cm
5 a 10.4 cm b 1.25 rad
6 7.5 **7** 0.8
8 a $\frac{1}{3}\pi$ b $6 + \frac{4}{3}\pi$ cm
9 6.8 cm
10 a $R - r$
 b $\sin \theta = \frac{r}{R - r} \Rightarrow (R - r) \sin \theta = r \Rightarrow (R \sin \theta - r \sin \theta) = r$
 $\Rightarrow R \sin \theta = r + r \sin \theta \Rightarrow R \sin \theta = r(1 + \sin \theta)$.
 c 2.43 cm

11 2 rad
12 a 36 m b 13.6 km/h
13 a 3.5 m b 15.3 m
14 a 2.59 rad b 44 mm