Mirage: Generating Enormous Databases for Complex Workloads

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***Abstract*—To optimize the query parallelism techniques, database developers require substantial workloads with specific query plans and customized output size for each operator (denoted as cardinality constraint). To this end, a rich body of query-aware database generators (QAG) are proposed. Given user specified database schema, column domain and query templates annotated with cardinality constraints, the QAG aims to generate a synthetic database and a set of synthetic queries such that all the cardinality constraints are guaranteed when executing synthetic queries on the database. However, the complex data dependencies hidden behind queries make previous QAGs suffer from deficiencies in supporting complex operators and controlling the generation errors. In this paper, we design a new generator *Mirage* with good properties of supporting complex operators and providing low error bounds for cardinality constraints. First, *Mirage* leverages the *Query Rewriting* and *Set Transforming Rules* to decouple dependencies between key and non-key columns, which could help generate each of them individually. Then, for the non-key columns, *Mirage* abstracts cardinality constraints of operators as placement requirements within each column's domain, and further models the generation problem as a classic bin packing problem. For the key columns, *Mirage* proposes a uniform representation of join cardinality constraints for all types of PK-FK joins and partition the data according to the matching status between PK and FK columns. Then, it formulates the key population as a *Constraint Programming* problem, which can be solved by an existing *CP Solver*. We have run extensive experiments and the results show that *Mirage* has a good scalability in data generation, and it greatly conquers all the previous work in either operator support or generation error.**

I. Introduction

Modern query execution engine supports high-performance query processing with the help of inter- and intra-query parallelisms. The inter-query parallelism [1] schedules different queries among multiple cluster nodes, while the intra-query parallelism [1] enables concurrent execution of multiple operators within a single query. Due to the complex orchestration of query and query operators during execution, the overall performance of the query engine is highly sensitive to the inter-query scheduling policies, intra-query parallelism optimization, dynamic memory management [2]–[7], etc. To conduct comprehensive evaluation and optimization of these performance critical algorithms, database developers would like to generate substantial workloads by customizing the query plans and the output size of each operator. We illustrate the scenarios desiring for these workloads as follows.

1) Inter-query Scheduling Policies. When scheduling queries among multiple nodes, the optimizer would place queries requiring different types of resources in the same node such that the resource contentions can be effectively avoided and the resource utilization is maximized. The state-of-the-art work leverages machine learning to learn a generalized schedule solution [4], [8] which works well under various workloads. However, it requires a number of diverse workloads for training. Specifically, the diversity of workloads is commonly represented by the diversity of query plans, query operators and the output size of each operator [4], [8].

2) Intra-query Parallelism Optimization. The optimization task divides a query plan into multiple parts, and then database can concurrently execute them with an appropriate level of parallelism to improve the execution efficiency. The state-of-the-practice work commonly uses heuristic rules to divide the query plan and decides the number of concurrent threads [9]–[12]. However, if the rules are designed with only a single type of workload, it is difficult to achieve optimal division or parallelism for the other workloads [2]. To design and validate better rules, the developers need to increase the diversity of query plans and the output size of each operator.

3) Dynamic Memory Management. The optimization task aims to achieve efficient memory usage of each single node by dynamic memory allocation and assignment for multiple queries [6], [7]. However, it is rather hard to estimate the required memory of a query before execution [6], [7]. This is because the memory usage varies dynamically during different stages of execution. For example, a substantial amount of memory is required for sorting data during the merge join, which is later released after the completion of join. Moreover, even for the same operator, the memory usage differs if their child operators have different output sizes. Therefore, when evaluating and optimizing the dynamic memory management algorithms, the developers need to construct workloads with various query plans and various output sizes of operators.

To meet the requirements described above, the Query-Aware Database Generator (QAG) [13]–[19] has been proposed. More specifically, given the user specified database metadata (including schema, table size and column domain), the query plans and the output size of each operator (denoted as cardinality constraint [13]), QAG generates a synthetic database and a set of synthetic queries. When executing synthetic queries on the database, QAG guarantees that the output size of each operator satisfies the cardinality constraints. Note that, the parameter variables in these query plans are anonymized, and database developers just need to design the query plan and annotate the cardinality constraint of each operator.

TABLE I
COMPARISON OF CURRENT QUERY AWARE DATA GENERATORS WITH *Mirage* IN OPERATOR SUPPORTABILITY

Related Work	Selection			Join				Projection	Theorical	Terabyte-
	Predicate	Arithmetic	Logical	Equi	Anti	Outer	Semi	Foreign Key	Relative Error	Generation
QAGen	Arbitrary	F	Arbitrary	T	F	F	F	T	Zero	F
MyBenchmark	Arbitrary	F	Arbitrary	T	F	F	F	T	No Guarantee	F
DCGen	$>, \geq, <, \leq, =$	F	DNF	T	F	F	F	F	Low	T
Hydra	$>, \geq, <, \leq, =$	F	DNF	T	F	F	F	F	Zero	T
Touchstone	Arbitrary	T	Simple	T	F	F	F	F	No Guarantee	T
Mirage	**Arbitrary**	**T**	**Arbitrary**	**T**	**T**	**T**	**T**	**T**	**Zero**	**T**

However, the QAG has been proven to be an *NP-complete* task [14]. This is because the cardinality constraints of various query operators in queries impose complexing joint data distribution requirements among different columns. To reduce the computational complexity of the problem, existing solutions usually choose to either support less operators or tolerate more relative errors (see Table I), among which *Mirage* is our method. Note, all operators in Table I exist in the common OLAP benchmark TPC-H. We observe that existing methods lack the support of: **1) Complex predicate connected by arithmetic and logical operators.** It introduces joint data distribution requirements among non-key columns [14]; **2) Outer/anti/semi join operators and projection on foreign keys in enormous database.** It introduces joint data distribution requirements between primary and foreign key columns. Moreover, existing work for this issue usually relies seriously on memory consumption [13], [16], [17] and are not scalable.

In this paper, we propose a new generator *Mirage*, which can not only successfully solve the complex joint data distribution requirements from different queries/operators, but also strictly guarantee the relative errors of the cardinality constraints on the AQTs. Specifically, it utilizes different techniques to deal with joint data distributions between different columns.

Between Key and Non-key Columns. It is caused by the uncertain execution order between selection and join operators in a query plan. If the join executes after the selection, it indicates that the join result relies on the selection result and the distribution of key columns relies on that of non-key columns, and vice versa. We propose to take a relational algebra-based *query rewriting* method to push down selection operators without breaking the cardinality constraints in query plans. Note that, the transformation is only used for designing a uniform generation method in *Mirage*, and users would finally execute the synthetic query with the original query plan. So, we reduce the bidirectional distribution dependencies to a unidirectional one, i.e., from key columns to non-key columns.

Between Non-key Columns. It is introduced by cardinality constraints of selections on multiple non-key columns which are connected by logical operators (e.g., $\wedge$) or arithmetic operators (e.g., +). Since the purpose of QAG is to guarantee the output size of each operator instead of that of each sub-predicate on the generated testing scenario, it inspires us to simplify the multi-column distribution dependencies by eliminating some sub-predicates to reduce complexity. Specifically, for a logical predicate, the *set transforming rules* can help to trim sub-predicates; for an arithmetic predicate, we can evade the problem of the joint data distribution requirement by transforming it into a parameter searching problem in the result space of arithmetic computations on its involved columns.

Between the Primary and Foreign Key Columns. It is caused by cardinality constraints on PK-FK join operators which impose some requirements on the match status between primary and foreign key columns. We define a uniform representation (join cardinality&distinct cardinality) of cardinality constraints for various join operators. To avoid the common conflicts of the population solutions from different joins [16] and reduce computational complexity, a multi-dimensional join visibility-oriented row representation and organization method is proposed for tables, which partitions rows by their visibility to the involved joins. Based on the uniform join constraint representations and partition-based data organizations, the problem of plotting the joint distributions of key columns is converted into a classic *Constraint Programming* (CP) problem [20] which can be simply solved by existing *CP Solver* [21].

Based on above solutions, we implement *Mirage*, a novel and generic QAG, which has the following contributions.

1) *Mirage* provides the most powerful and strong support to complex query operators or predicates as listed in Table I.

2) *Mirage* generates workloads with a theoretical zero error bound even having more supported operators.

3) *Mirage* is able to generate a database of any data size in a linear way with a controllable memory consumption.

4) We run extensive experiments. *Mirage* conquers the previous work by a wider operator support and lower errors.

II. PRELIMINARIES

A. Database and Annotated Query Template

A database D consists of multiple tables (i.e., relations). Each table R_i in D contains a single primary key column PK_i, a certain number of non-key columns $A_{i_1}, \cdots, A_{i_q}$, and zero or more foreign key columns $FK[R_{i_1}], \cdots, FK[R_{i_m}]$. Here, $FK[R_{i_1}]$ indicates that the foreign key of R_i refers to the primary key of R_{i_1}.

Cardinality Constraints of Table and Non-Key Column. Given a table R and a non-key column A in R, their cardinality constraints (denoted as $|R|$ and $|R|_A$) refer to the unique data values contained in R and A. $|R|$ can be viewed as the row count of R and $|R|_A$ can be viewed as the domain size of A.

Annotated Query Template (AQT). A query template can be created by parameterizing the input value of each predicate in a query plan [18]. An annotated query template AQT [14] labels the cardinality constraint on the output of each operator. For example, there are four AQTs in Fig. 1. Consider the selection operator $\sigma_{s_1<p_1}$ in AQT Q_1. The input value of its predicate is parameterized as p_1, and its cardinality constraint is labeled as 2, which indicates the output size of $\sigma_{s_1<p_1}$.

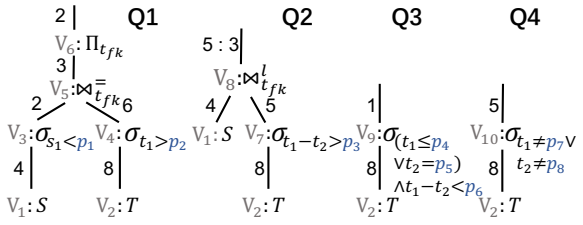


Fig. 1. Annotated Query Templates on Tables S and T

B. Cardinality Constraint of Query Operator

In this section, we first introduce the query operator view that represents the output of a query operator and then give the definition of cardinality constraints for three kinds of common query operators, which are selection, join, and projection.

Query Operator View (V). A query operator view represents the output rows of a query operator, and can be classified into the leaf view and internal view. A leaf view covers all rows of a specific table, and an internal view covers the execution results of a query operator which operates on its input views. Specifically, the selection and projection operators have only one input view, while the join operator has two input views.

Example 1. *The AQT Q_1 in Fig. 1 can be represented by query operator views from the bottom up as follows.*

$$\begin{aligned} leaf\ views: V_1 &= S & V_2 &= T \\ internal\ views: V_3 &= \sigma_{s_1<p_1}(V_1) & V_4 &= \sigma_{t_1>p_2}(V_2) \\ V_5 &= \bowtie^{=}_{t_{fk}}(V_3, V_4) & V_6 &= \Pi_{t_{fk}}(V_5) \end{aligned}$$

Selection View ($\sigma_P(V)$). A selection view is generated by performing a selection operator on an input view V with a logical predicate P. For simplicity, we mainly discuss the case in which the predicate P follows the conjunctive normal form (CNF). Note that a predicate with any other form can be transformed to CNF [22]. That is,

$$P{=}clause_i \wedge ... \wedge clause_n \quad s.t.\ clause_i{=}literal_i \vee ... \vee literal_n$$

Here, $literal_i$ can be a unary or an arithmetic predicate. More specifically, a unary predicate follows the form of $A_i \bullet p_k$ with $\bullet \in \{=, \neq, <, >, \leq, \geq, (not)\ in, (not)\ like\}$, e.g., $s_1 < p_1$ in V_3. An arithmetic predicate might operate on multiple non-key columns $A_i, ..., A_k$ through an arithmetic function $g()$, which usually follows the form of $g(A_i, ..., A_k) \circ p_k$, $\circ \in \{<, >, \leq, \geq\}$, e.g., $t_1\text{-}t_2 > p_3$ in V_7.

Selection Cardinality Constraint (SCC). A selection cardinality constraint $|\sigma_P(V)| = m$ requires that there should exactly exist m rows in the input view V which satisfy the predicate P. Moreover, the SCC can be further classified into three categories according to the predicate P, which are unary cardinality constraint (UCC), arithmetic cardinality constraint (ACC) and logical cardinality constraint (LCC). Specifically, the predicate in UCC/ACC is a simple unary/arithmetic predicate, while the predicate in LCC is a combination of multiple unary or arithmetic predicates by logical operators.

Join View ($\bowtie^{type}(V_l, V_r)$). A join view is generated by performing a join operator on two input views V_l and V_r with a specified join type. In general, the join type includes $equi\ join$ (=), $left/right/full\ outer\ join$ $(l/r/f)$, $left/right\ semi\ join(ls/rs)$, and $left/right\ anti\ join(la/ra)$. Following previous studies [13]–[19], we also focus on the PK-FK join in this paper.

TABLE II
OUTPUT SIZE OF JOINS ON V_l AND V_r.

Join Type	Equi	Outer			Semi		Anti	
		Left	Right	Full	Left	Right	Left	Right
JCC	T	T	F	F	F	T	F	T
JDC	F	T	F	T	T	F	T	F
Size	n_{jcc}	$\|V_l\|$-n_{jdc}+n_{jcc}	$\|V_r\|$	$\|V_l\|$-n_{jdc}+$\|V_r\|$	n_{jdc}	n_{jcc}	$\|V_l\|$-n_{jdc}	$\|V_r\|$-n_{jcc}

Join Cardinality/Distinct Constraint (JCC/JDC). To represent the output size of different join types in a general way, we abstract the join constraints by join cardinality constraint (JCC) and join distinct constraint (JDC). JCC requires that there should exactly exist n_{jcc} matched pairs of rows in the two input views V_l and V_r. For ease of presentation, hereinafter we assume that V_l contains the primary key (pk) of the referenced table, and V_r contains the foreign key (fk) of the referencing table. Then, if the pk value of a row in V_l equals the fk value of a row in V_r, the two rows can be considered as matched. JDC requires that there should exactly exist n_{jdc} distinct pk/fk values in all matched pairs of rows from V_l and V_r. JCC and JDC are used together to determine the output size of any type of join (see Table II). For example, suppose the view V_l left outer joins the view V_r, and the cardinality constraints are n_{jcc}:n_{jdc}. We can infer that there exist $|V_l| - n_{jdc}$ rows that are not matched with the rows in V_r and its output size is $|V_l| - n_{jdc} + n_{jcc}$.

Projection View ($\Pi_{PK/FK/A}(V)$). A projection view is generated by performing a projection operator on an input view V. It eliminates the duplicate rows in the result set.

Projection Cardinality Constraint (PCC). A projection cardinality constraint $|\Pi_{PK/FK/A}(V)| = m$ requires that the size of projection result should exactly be m. As each primary key uniquely identifies a row in a table, the input and output cardinalities of a PK column are identical. Thus, the PK column is not of interest in the PCC. In addition, the performance of projection has been observed to be only influenced by its input size if its output cardinality is not very huge [18], [23]. Note that the projections on non-key columns in an OLAP database usually involve dimension columns. Although the dimension column can be of any data type, e.g., $time$ [24], $textual\ data$ [25], its cardinality is relatively small as it is filled with category data. For example, the max cardinality of all the dimension columns in these datasets [24]–[27] is 2112, which is far less than their table sizes. Thus, the output size of projections on non-key columns is relatively small and then has a marginal effect on the performance and memory management. However, the cardinality of an FK column is closely related to the cardinality of its referenced PK column, which may be huge and has a great impact on projection performance. Based on the observations, we only focus PCC on FK columns to simplify the complexity of QAG. Note, the PCC on an FK column declares the unique number of foreign keys in the output, and it can be converted to a JDC on its child join view. For example, the PCC of $|\Pi_{t_{fk}}(V_6)| = 2$ in Fig. 1 can be converted to a JDC of V_5 with $n_{jdc} = 2$. If a projection does not have a descendant join

view, *Mirage* automatically adds a virtual right semi join view as its child. Specifically, we set its left input view as the table referenced by the projected FK column, and set its right input view as the original input of the projection view. In this way, the output of the virtual join view is exactly consistent with the original input of the projection view. Then, we can convert the PCC to a JDC on its child virtual join view. Note, the virtual join views are only used to convert all the PCCs to JDCs such that the type of constraints can be reduced. They will finally be removed from query plans after our query aware database generation process is finished. For example, we add a child view V_{13} for the projection view V_{12} as shown in Fig. 2

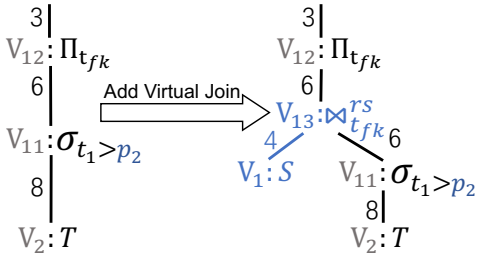


Fig. 2. Add a Virtual Join View for the Projection View

C. Problem Definition

We are now ready to formulate the problem of query aware database generation in Definition 1.

Definition 1. ***Query Aware Database Generation***. *Given the database schema, the cardinality constraints of tables and non-key columns, and the annotated query templates (*AQTs*), the query aware database generation (*QAG*) aims to (1) generate a synthetic database D, and (2) instantiate parameters in the* AQTs*, such that all the cardinality constraints are guaranteed if running the instantiated* AQTs *on the synthetic database D.*

Example 2. *Consider two tables S and T. S consists of a primary key s_{pk} and a non-key column s_1, T consists of a primary key t_{pk}, a foreign key t_{fk} which references s_{pk}, and two non-key columns t_1 and t_2. The cardinality constraints of D are $|S|$=4, $|T|$=8, $|S|_{s_1}$=4, $|T|_{t_1}$=5, $|T|_{t_2}$=4, and the cardinality constraints of* AQTs *are listed in Fig. 1. Then, the query aware database generation (*QAG*) aims to construct a synthetic database D and instantiate all parameters in* AQTs*. The results are shown as in Fig. 3.*

III. Design Overview

Figure 4 shows an overview of our *Mirage* framework. Its input configurations contain the database schema, the cardinality constraints of table and non-key columns, and the AQTs. For simplicity, we only discuss processing queries without subqueries. Note, $Mirage$ can also be extended to process subqueries, which is discussed in §VII-B.

Various cardinality constraints have imposed requirements of joint data distributions among different columns. In general, the joint data distributions can be classified into three types according to the involved columns, which are joint distributions between the primary key and foreign key, between key

$p_1 = 30$ $p_2 = 2$
$p_3 = 0$ $p_4 = 1$
$p_5 = 0$ $p_6 = 5$
$p_7 = 4$ $p_8 = 2$

(a) Instantiated Parameters

s_{pk}	s_1
1	10
2	20
3	30
4	40

t_{pk}	t_1	t_2	t_{fk}
1	3	2	1
2	3	2	1
3	3	2	3
4	5	3	3
5	5	1	4
6	4	4	2
7	2	3	4
8	1	4	4

(b) D: Table S and T

Fig. 3. Example of Query Aware Database Generation

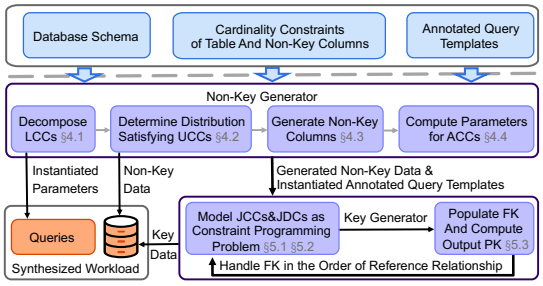


Fig. 4. Design Overview of $Mirage$

and non-key columns, and among different non-key columns. These requirements make the query aware database generation to be *NP-complete* [14]. To address this issue, we propose to first decouple the joint distributions (i.e., dependencies) between key and non-key columns, and then generate data and instantiate parameters for these two kinds of columns by non-key generator and key generator separately (see Fig. 4).

Decouple Dependencies Between Key and Non-key Columns. The requirement of the joint distributions between key and non-key columns is from the uncertain execution order between selection on non-key columns and join on key columns. If the selection executes after the join, it indicates that the filter result relies on the join result, i.e., the distribution of non-key columns relies on that of key columns. Otherwise, the distribution of key columns relies on that of non-key columns. Generally, current query optimizer usually tries to use its *rule based optimizer* module to push down select operators directly to each table, so as to reduce the volume of data transferred along the query tree. However, there exists some AQTs in which the selection executes after the join. For example, the residual predicate in TPC-H Q19 always executes after the join operator. To address this issue, we propose to rewrite the annotated query tree based on algebraic transformations so as to push down the selection without breaking the cardinality constraints (more details in §VII-A). With this mechanism, we avoid the bidirectional dependencies between key and non-key columns. It enables us to generate all non-key columns firstly, and then to generate key columns based on the distribution of non-key columns. Note, the rewritten query tree is only used in our database generation phase, the user would finally use the original query plan(i.e., selection executes after the join) and all the cardinality constraints in original plan are guaranteed when performing tests on the generated database.

Generate Non-Key Data. *Mirage* utilizes its *non-key generator* to populate non-key columns and instantiate the selection related parameters in AQTs, such that all the selection cardinality constraints (SCCs) can be satisfied. However, the selections with logical predicates and arithmetic predicates

usually operate on multiple non-key columns, then the corresponding logical cardinality constraints (LCC) and arithmetic cardinality constraints (ACC) would bring the requirements of joint distributions between these columns. As a result, it makes non-key data generation still *NP-complete* [14]. The main reason is the domain size of a joint distribution is the cumulative product of all involved columns' domain sizes, which leads to expensive domain space searching cost when instantiating parameters in AQTs. To address the issue, we propose to take the following four steps to eliminate requirements of joint distributions between non-key columns and thus reduce the computational complexity. Firstly, based on the observation that the QAG problem is required to guarantee the output size of each operator instead of that of each sub-predicate, we propose to apply *set transforming* rules to trim sub-predicates of LCCs without affecting the output size of each selection view. With this mechanism, we can decouple LCCs into individual unary cardinality constraints (UCCs) and ACCs (§IV-A). Considering that ACCs would also lead to the requirements of joint distributions between non-key columns (e.g., $t_1 - t_2 < p_6$ in Q_3 in Fig. 1), we propose to first populate all the non-key columns by only considering the UCCs on each column (§IV-B and §IV-C). After that, our aim is to find a valid parameter value for each ACC based on the populated non-key columns and its specified cardinality constraints (§IV-D). In details, §IV-B derives each column distribution according to all the UCCs on that column. For this purpose, we abstract it as a classic bin packing problem. Specifically, the column's domain space is considered as a container, and each UCC which specifies a specific constraint on the column's data distribution is considered as an item with a specific size. Then, solving the bin packing problem is equivalent to deriving a valid data distribution for the column such that all the associated UCCs are satisfied. Following the derived data distribution, §IV-C presents to instantiate parameters in UCCs and generate data for each non-key column. Lastly in §IV-D, to instantiate the parameter in each ACC, we first use its arithmetic function to compute the result view based on the involved non-key columns generated above (e.g., compute t_1-t_2 for all rows in columns t_1 and t_2). Then, we search the one dimensional result space and find a valid parameter value such that the ACC is satisfied.

Generate Key Data. The *key generator* of *Mirage* is used to populate key columns which satisfy all the join cardinality constraints. As the primary keys usually serve as an identifier of each row and there exists no distribution requirement on the PK column, we propose to follow previous studies [18] and generate them by an auto-incrementing integer generator. Then, our main aim is to populate FK columns according to the join constraints. For this purpose, we first formalize three FK populating rules for each JCC/JDC on two joined tables. Specifically, for any join view, the rules specify how to select primary keys in its left input view V_l and how to use them to populate foreign keys in its right input view V_r (§V-A). However, we observe that there might exist some conflicts if we populate foreign keys according to the JCC/JDC of each join view independently (e.g., use different primary keys to populate a foreign key). To address this issue, one naive method is to first enumerate all possible ways of populating foreign keys based on the JCC/JDC of each join view. Then, it selects one populating way from each join view and check whether there exists a conflict between them. However, it is highly inefficient. Thus, we propose to partition the table according to the overlaps between all the join input views V_l and V_r. Specifically, given a table whose primary keys are referenced by the foreign keys in another table, if its two rows are partitioned into the same partition, they must appear or disappear in any given left input view V_l at the same time. Similarly, the rows in the referencing table are partitioned according to their appearances in the right input views. In this way, each input view V_l/V_r can be represented by several partitions. Next, we integrate the partitions into the three FK populating rules for each join view described above, and then the problem can be modeled as a classic *Constraint Programming* (CP) problem [20], which can be easily solved by existing $CP\ Solver$ [21] (§V-B). Finally, to support the case of multi-table joins, we propose to build a directed graph according to the references between tables and perform a topological sorting on the graph. Then, we populate the FK column of each table in the topological order (§V-C).

$|V_4|=|\sigma_{t_1>p_2}(T)|=6$ $|V_9|=|\sigma_{(t_1\leq p_4\vee t_2=p_5)\wedge t_1-t_2<p_6}(T)|=1$
$|V_7|=|\sigma_{t_1-t_2>p_3}(T)|=5$ $|V_{10}|=|\sigma_{t_1\neq p_7\vee t_2\neq p_8}(T)|=5$

(a) Selection Cardinality Constraints of Table T

UCCs *on* t_1: $|V_4|=|\sigma_{t_1>p_2}(T)|=6$ $|V_9^{1(1)}|=|\sigma_{t_1\leq p_4}(T)|=1$
$|\overline{V_{10}^{1(1)}}|=|\sigma_{t_1=p_7}(T)|=3$
UCC *on* t_2: $|\overline{V_{10}^{1(2)}}|=|\sigma_{t_2=p_8}(T)|=3$
ACC *on* t_1, t_2: $|V_7|=|\sigma_{t_1-t_2>p_3}(T)|=5$

(b) Selection Cardinality Constraints after View Eliminations

$f_{t_1}(1)$=12.5% $f_{t_1}(2)$=12.5% $F_{t_1}(2,4)$=62.5% $f_{t_1}(4)$=37.5%
$f_{t_1}(5)$=12.5% $f_{t_2}(1)$=12.5% $f_{t_2}(2)$=37.5% $F_{t_2}(2,4)$=50%

(c) Distribution for Table T

Fig. 5. Selection Cardinality Constraints on Table T

IV. Non-Key Generator

In this section, we first introduce how to decouple LCCs into UCCs and ACCs (§IV-A). Then, we discuss how to use the bin packing method to deal with UCCs, including instantiating parameters in each UCC and generating non-key column data (§IV-B and §IV-C). Finally, we present how to instantiate parameters in each ACC (§IV-D).

A. Decouple Logical Dependencies

As the existence of LCCs would make it *NP-complete* to construct non-key columns, we first propose to decouple LCCs into UCCs and ACCs. Then, the joint data dependencies between non-key columns caused by LCCs can be decoupled. Recall that the logical predicate is assumed to be a CNF formula (see §II-B), then we can convert a selection view V containing a logical predicate $\wedge_{k=1}^{n} clause_k$ into the intersection of sub-selection views $\cap_{k=1}^{n} V^k$, where V^k is constructed by the sub-predicate $clause_k$:

TABLE III
ASSIGNED BOUNDARY VALUES FOR VIEW ELIMINATION

Comparator		$>, \geq$	$<, \leq$	$in,\ like,\ =$	$not\ in,\ not\ like,\ \neq$
Assigned Value	U	$-\infty$	$+\infty$	$/$	$NULL$
	$\emptyset$	$+\infty$	$-\infty$	$NULL$	$/$

$$V = \sigma_{\wedge_{k=1}^{n} clause_k}(R) = \cap_{k=1}^{n} \sigma_{clasue_k}(R) = \cap_{k=1}^{n} V^k$$

Each $clause_k$ can be further represented by a disjunction of $literal$s, and each sub-selection view V^k is decomposed as:

$$V^k = \sigma_{\vee_{i=1}^{m} literal_i}(R) = \cup_{i=1}^{m} \sigma_{literal_i}(R) = \cup_{i=1}^{m} V^{k(i)}$$

Here, we use $V^{k(i)}$ to represent a subsub-selection view constructed by $literal_i$ in $clause_k$. For example the selection view V_9 in Fig. 5a, it has a logical predicate P=$(t_1 \leq p_4 \vee t_2$=$p_5) \wedge t_1 - t_2 < p_6$. Then, we can decompose it as follows.

$$V_9 = \underline{\sigma_{t_1 \leq p_4 \vee t_2 = p_5}(T)} \cap \underline{\sigma_{t_1 - t_2 < p_6}(T)} \quad = V_9^1 \cap V_9^2$$
$$V_9^1 = \underline{\sigma_{t_1 \leq p_4}(T)} \cup \underline{\sigma_{t_2 = p_5}(T)} \quad = V_9^{1(1)} \cup V_9^{1(2)}$$

Since our goal is to guarantee the output size of a whole selection view instead of guaranteeing the concrete size of each sub-selection or subsub-selection view, we leverage the following two rules in the area of set theory to eliminate some subsub-selection views:

$$\mathbf{rule_1}: V_i \cap V_j = V_j \ \ if\ V_i \leftarrow U \quad \mathbf{rule_2}: V_i \cup V_j = V_j \ \ if\ V_i \leftarrow \emptyset.$$

The $\mathbf{rule_1}$ and $\mathbf{rule_2}$ indicate that we can eliminate any sub/subsub-selection view without affecting the output size if it meets certain criteria, such as the universal set U or empty set $\emptyset$. To this end, we try to assign boundary values to the parameters in related to the comparators in each sub/subsub-selection view's predicate. Table III lists the boundary values. More specifically, for any selection view V containing a logical predicate, our elimination procedure consists of two steps. 1) we try to set each sub-selection view V^k constructed by $clause_k$ as the universal set U. If all the V^k can be set as U, we only keep one sub-selection view and eliminate other sub-selection views; otherwise, we eliminate all the sub-selection views that can be set as U. 2) for each remaining V^k, we try to set each of its subsub-selection view $V^{k(i)}$ constructed by $literal_i$ in $clause_k$ as $\emptyset$. Then, we eliminate the subsub-selection views in a similar way to that of the first step.

Example 3. *Consider the cardinality constraint $|V_9| = 1$ in Fig. 5a. Firstly, we observe that both V_9^1 and V_9^2 can be U if we set p_4=$+\infty$ and p_6=$+\infty$. Then, we keep one sub-selection view and eliminate the other one. Suppose V_9^2 is eliminated by setting p_6=$+\infty$. Secondly, we try to set subsub-selection views of V_9^1 as $\emptyset$. Specifically, both $V_9^{1(1)}$ and $V_9^{1(2)}$ can be $\emptyset$ if we set $p_4 = -\infty$ and $p_5 = null$. Then, we eliminate one subsub-selection view. Suppose $V_9^{1(2)}$ is eliminated, then we simplify the cardinality constraint from $|V_9| = 1$ to $|V_9^{1(1)}| = 1$.*

Note, if all the predicates in a $clause$ only contain comparators of $not\ in$, $not\ like$ and $\neq$, then none of the subsub-selection view can be set as $\emptyset$ (see Table III). To address this issue, we make use of the *De Morgan's law* [28] shown in $\mathbf{rule_3}$ to convert it to an equivalent constraint which is easier to deal with. Here, $|U_V|$ denotes the universal set size of the selection view V. In our case, $|U_V|$ equals the number of rows in the table that is operated by V. For example, consider the selection view V_{10} operating on table T in Fig. 5a. Its universal set size is $|T| = 8$. Then, we can convert the cardinality constraint of $|V_{10}| = 5$ as $|\sigma_{t_1=p_7}(T) \cap \sigma_{t_2=p_8}(T)| = 3$.

$$\mathbf{rule_3}: |V^{k(1)} \cup ... \cup V^{k(m)}| = n \Leftrightarrow |\overline{V^{k(1)}} \cap ... \cap \overline{V^{k(m)}}| = |U_V| - n$$

Theorem 1 shows that our elimination procedure can reduce a selection view V to its subsub-selection view $V^{k(i)}$ (Case1) or the conjunction of ω unary views $\cap_{j=1}^{\omega} V_e^j$ (Case2) whose comparators are in, $like$ or $=$. Note that Case2 requires that some values must coexist in the same row. For example, the conversion of V_{10} requires that there must exactly exist three rows whose $t_1 = p_7$ and $t_2 = p_8$. Nevertheless, we can first derive valid distribution for each column based on its cardinality constraints (§IV-B). Then, after all the non-key columns are generated, we add a post-processing step to bound these values into same rows (§IV-C). Fig. 5b shows the elimination result of selection views V_4, V_7, V_9 and V_{10}.

Theorem 1. *Given a selection view V, our elimination procedure can reduce it to one of the following two cases: $V^{k(i)}$ or $\cap_{j=1}^{\omega} V_e^j$, where $V^{k(i)}$ is a subsub-selection view of V, V_e^j is a unary view whose comparator is in, $like$ or $=$, and ω is the number of unary views.*

Proof. In our elimination procedure, the first step is to set each sub-selection view as U. Given a sub-selection view V^k, from table III, we can see that it cannot be set as U if each literal in $clause_k$ only contains comparators of in, $like$ and $=$. Suppose there exists q sub-selection views which cannot be set as U.

If $q > 0$, the first step would reduce V to $\cap_{j=1}^{q} V^{i_j}$, where all the literals in the i_j^{th} clause (i.e., $clause_{i_j}$) only contain comparators of in, $like$ and $=$. As a result, in the second step, each subsub-selection view of V^{i_j} can be as $\emptyset$. Next, *Mirage* only keeps one subsub-selection view for each V^{i_j}, and finally reduces V to $\cap_{j=1}^{q} V_e^j$.

If $q = 0$, the first step would reduce V to a single sub-selection view V^k. Suppose there exist s subsub-selection views of V^k which cannot be set as $\emptyset$. If $s = 0$, the second step of *Mirage* would reduce it to $V^{k(i)}$. Otherwise, it reduces V^k to $\cup_{j=1}^{s} V^{k(i_j)}$, where the i_j^{th} literal (i.e., $literal_{i_j}$) in $clause_k$ only contains the comparator of $not\ in$, $not\ like$ and $\neq$. From $rule_3$, we can see that $\cup_{j=1}^{s} V^{k(i_j)}$ can be converted to $\cap_{j=1}^{s} \overline{V^{k(i_j)}}$, where $\overline{V^{k(i_j)}}$ is a unary view whose comparator is in, $like$ or $=$.

The theorem is proved. □

B. Solve Unary Selection Operators

As each UCC involves a single column, we propose to use UCCs to derive the data distribution of each non-key column independently. Specifically, for each non-key column A, we use the cumulative distribution function CDF_A to represent its data distribution. Since different columns usually have various data types and domain spaces, we first normalize the original

domain space of each non-key column to its cardinality space of an integer type such that all the UCCs can be resolved in a general way. For example, consider a non-key column A whose cardinality constraint is $|R|_A$ (i.e., the domain size of A). We assume that all the parameters in $UCCs$ are integers in $(0, |R|_A]$. In this way, deriving CDF in the original domain space is converted to deriving CDF in the cardinality space. Note that, $Mirage$ transfers the generated data into the expected data type by a data transformer as defined in [18].

Definition 2. ***Cumulative Distribution Function of Non-Key Column.*** *Given a non-key column A, the cumulative distribution function $F_A(p)$ is defined as the probability of a row whose attribute A takes a value less than or equal to p. Moreover, $F_A(p_i, p_j)$ is defined as the probability of a row whose attribute A lies in the interval $(p_i, p_j]$, where $p_i < p_j$, and $f_A(p)$ is defined as the probability of a row whose attribute A takes a value equaling to p.*

To further simplify the process of deriving CDF_A, we also propose to convert all the comparators in UCCs (see Table III) into two kinds of comparators, which are $=$ and $\leq$. Specifically, the UCC with a comparator in can be transformed into the union of multiple UCCs with the = comparator. Similarly, a UCC with a comparator $like$ can be converted into an in comparator with a pre-defined number of distinct matching values. In addition, based on commutativity property from *De Morgan's Law* [28], the comparators $>, \geq$ and $\neq$ can be easily converted to $\leq$ and $=$. For example, $|\sigma_{A>p}(R)|$=k is equal to $|\sigma_{A\leq p}(R)|$=$|R|-k$. Then, we use the two kinds of UCCs to derive $F_A(p)$ and $f_A(p)$ respectively. If $|\sigma_{A\leq p}(R)|$=k, we have $F_A(p)$=$k/|R|$, and if $|\sigma_{A=p}(R)| = k$, we have $f_A(p)$=$k/|R|$. For example, consider the three UCCs on column t_1 in Fig. 5b. From $|\sigma_{t_1>p_2}(T)|$=6, $|\sigma_{t_1\leq p_4}(T)|$ =1, $|\sigma_{t_1=p_7}(T)|$=3 and $|T|$=8, we can infer that $F_{t_1}(p_2)$=(8-6)/8=25%, $F_{t_1}(p_4)$=1/8=12.5% and $f_{t_1}(p_7)$=3/8=37.5%. Next, given a non-key column A and its associated CDFs $F_A(p)$ and $f_A(p)$, we take three steps to instantiate all the parameters of UCCs on column A.

(1) Determine the Partial Order for Each Parameter p in $F_A(p)$. As $F_A(p)$ monotonically increases with p, p_j must be greater than p_i if $F_A(p_i)$<$F_A(p_j)$. Then, for each parameter p and its corresponding CDF $F_A(p)$, we propose to sort the pair $(F_A(p), p)$ in ascending order of $F_A(p)$. Suppose we have n parameters in column A's UCCs whose comparators can be converted to $\leq$. After ordering, the partial order of these n parameters is represented by $p_{h_1}, \cdots, p_{h_n}$, and each p_{h_i} corresponds to one of the parameters. Then we can divide the cardinality space $(0, |R|_A]$ into $n+1$ ranges which satisfy

$$F_A(0, p_{h_1}) + \sum_{j=1}^{n-1} F_A(p_{h_j}, p_{h_{j+1}}) + F_A(p_{h_n}, |R|_A) = 1$$

(2) Determine the Partial Order for Each Parameter p in $f_A(p)$. In the step, we put each parameter p in $f_A(p)$ into one of the $n+1$ ranges properly. Specifically, suppose we would put k parameters $p_{e_1}, \cdots, p_{e_k}$ into the range $(p_i, p_j]$, we should guarantee that the sum of data existence probabilities of $p_{e_1}, \cdots, p_{e_k}$ does not exceed the data existence probability of the range $(p_i, p_j]$, i.e., $\sum_{j=1}^{k} f_A(p_{e_j}) \leq F_A(p_i, p_j)$. Then, the problem can be regarded as a bin packing problem, which is *NP-complete* [29]. We propose a greedy method to address it. For each parameter p in $f_A(p)$, we always find the range $(p_i, p_j]$ which can accommodate p validly and has the smallest value of $F_A(p_i, p_j) - \sum_{j=1}^{k} f_A(p_{e_j})$. Here, $p_{e_1}, \cdots, p_{e_k}$ are parameters already put into the range $(p_i, p_j]$. After putting p into $(p_i, p_j]$, we repeat the above process until all parameters are in the n+1 ranges. However, our greedy method might fail to find valid ranges for a parameter p if it cannot fit within any range $(p_i, p_j]$. To ensure the cardinality size of each UCC, we can first check whether there exists a parameter p' which has been in a range with its $f_A(p')$=$f_A(p)$. If so, we specify p=p'. Otherwise, we transform $f_A(p)$=x into $f_A(p_1)$=$x_1, \cdots, f_A(p_q)$=x_q, with $x_1 + \cdots + x_q = x$. This can be implemented by converting the UCC from $|\sigma_{A=p}(R)|$=$x \cdot |R|$ to $|\sigma_{A\ in\ (p_1,\cdots,p_q)}(R)|$=$x \cdot |R|$. Obviously, the *UCC* with a smaller granularity of cardinality requirement is more likely to be satisfied. After putting k parameters $p_{e_1}, \cdots, p_{e_k}$ into a range $(p_i, p_j]$, we should specify the data existence probability of each newly split range. As we only need to guarantee $F_A(p_{e_i}, p_{e_{i+1}}) \geq f_A(p_{e_{i+1}})$, there are various ways to allocate the rest probability $F_A(p_i, p_j) - \sum_{j=1}^{k} f_A(p_{e_j})$ to each range, e.g., random allocation.

(3) Instantiate Parameters According to Partial Orders. Suppose that there are n extra ranges divided by $F_A(p)$ and m extra ranges divided by $f_A(p)$, respectively. With the two steps above, we have divided the cardinality space $(0, |R|_A]$ into n+m+1 ranges. As the data existence probability of each range $(p_i, p_j]$ (i.e., $F_A(p_i, p_j)$) is greater than zero, we first assign one unique value to each range. For the rest of $|R|_A$-m-n-1 unique values, we can assign them to each range arbitrarily. In our case, we assign these values to each range in a uniform way. Note that, when assigning unique values to a range $(p_i, p_j]$, the data existence probability $F_A(p_i, p_j)$ should not be violated. For example, consider a range $(p_i, p_j]$, if its parameter p_j has a constraint $f_A(p_j)$, then the number of assigned unique values to that range is at most $|R| \cdot (F_A(p_i, p_j)$-$f(p_j))$. Then, we instantiate parameters according to the number of assigned unique values to each range. Specifically, each parameter p_i is instantiated as the number of unique values which locates at or before p_i.

Example 4. *Consider the column t_1 of table T in Example 2, and its domain size is 5, then we have $F_{t_1}(0) = 0\%$ and $F_{t_1}(5) = 100\%$. As discussed before, we can derive that $F_{t_1}(p_2) = 25\%$, $F_{t_1}(p_4) = 12.5\%$ and $f_{t_1}(p_7) = 37.5\%$ based on the three UCCs and $|T| = 8$.*

An example is shown in Fig. 6. The first step is to sort the pair $(F_{t_1}(p), p)$ for each parameter p in $F_{t_1}(p)$. As $F_{t_1}(p_4) < F_{t_1}(p_2)$, then we have the partial order of parameters as $p_4 < p_2$ and the cardinality space can be divided into three ranges $(0, p_4]$, $(p_4, p_2]$ and $(p_2, 5]$, where $F_{t_1}(0, p_4) = 12.5\%$, $F_{t_1}(p_4, p_2) = 12.5\%$ and $F_{t_1}(p_2, 5) = 75\%$. Next, our second step is to find a proper range for each parameter p in $f_{t_1}(p)$. In our case, there exists only one $f_{t_1}(p_7)$ and only the third range has a data existence probability that is greater than

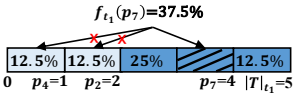


Fig. 6. CDFs of Column t_1

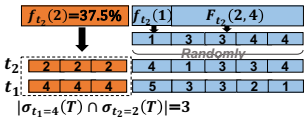


Fig. 7. Generate Table T

$f_{t_1}(p_7)$, then we put p_7 in the range $(p_2, 5]$. This further splits the range into $(p_2, p_7]$ and $(p_7, 5]$. Note, we only need to guarantee that $F_{t_1}(p_2, p_7) \geq f_{t_1}(p_7)$, so we can allocate 25% and 12.5% to the two ranges, then we have $F_{t_1}(p_2, p_7)$ $=62.5\%$ and $F_{t_1}(p_7, 5) = 12.5\%$. Finally, our last step is to instantiate parameters. As the first two steps have divided the cardinality space into 4 ranges, we first assign one unique value to each range. Then, only one unique value left in the domain space. Moreover, from $|T|*F(0, p_4) = |T|*F(p_4, p_2)$ $= |T|*F(p_7, 5) = 1$, we observe that only the data existence probability $F_{t_1}(p_2, p_7)$ is not violated if we assign an extra value to that range. Thus, we can infer that only the range $(p_2, p_7]$ contains more than one (i.e., 2) unique value. At last, we have $p_4 = 1$, $p_2 = 2$ and $p_7 = 4$. Fig. 5c shows the CDFs for non-key columns t_1 and t_2 after processing t_2.

C. Generate Data Based on CDF

In this section, we first discuss how to generate data for each non-key column, and then introduce how to arrange values from different non-key columns in each relational table.

Generate Data for Each Non-Key Column. Given a non-key column A, we take two steps to generate its data. Firstly, we deal with each parameter p in $f_A(p)$. Suppose p is instantiated as a_p (see §IV-B). Then we can infer that the existence probability of the data a_p is $f_A(p)$. To this end, we generate $|R| \cdot f_A(p)$ data items with the value of a_p. For example in Fig. 7, column t_2 is populated with 3=$|T|*f_{t_2}(2)$ rows valued 2 based on the CDFs in Fig. 5c. Secondly, for the other data items, we first use $F_A(p_i, p_j)$ to derive the volume of data items in the range $(p_i, p_j]$. Then, we generate data according to the number of unique values in that range. Suppose there exist μ unique values in $(p_i, p_j]$. Since we are only concerned about the number of data items regarding $F_A(p_i, p_j)$, we can generate totally $|R| \cdot F_A(p_i, p_j)$ data items with μ unique values based on any given distribution, e.g., uniform distribution. For example in Fig. 7, we assign 4=$|T|*F_{t_2}(2, 4)$ rows with two unique values (i.e., 3,4) uniformly into column t_2.

Arrange Values from Different Non-Key Columns. After generating data for non-key columns, our next step is to arrange them in each table. Recall that our decoupling procedure might reduce a selection view V into $\cap_{j=1}^{\omega} V_e^j$, where V_e^j is a unary view whose comparator is in, $like$ or $=$. Thus, the $\cap$ operator and $=$ comparator in the cardinality constraint $|\cap_{j=1}^{\omega} V_e^j| = n$ requires that the column values associated with each view V_e^j must be bound to n same rows. To this end, we propose to first populate rows with values meeting the cardinality constraints of selection views that follow the format of $\cap_{j=1}^{\omega} V_e^j$. For example in Fig. 7, based on the requirement $|\sigma_{t_1=p_7=4}(T) \cap \sigma_{t_2=p_8=2}(T)| = 3$ from V_{10}, we bind 3 rows valued (4,2). Then, we populate the rest generated data into rows randomly to satisfy the other selection constraints on the non-key columns. Further, the primary key for each row can be generated alongside populating its non-key columns. As primary keys usually have no semantic meaning, we propose to generate them by an auto-incremental integer generator.

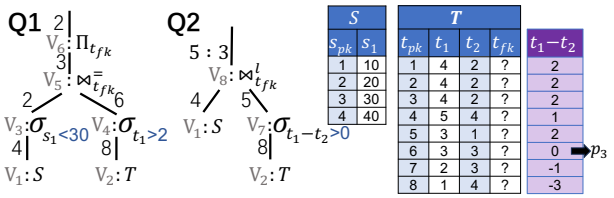


Fig. 8. Non-key Column Data and Instantiated Queries

D. Solver of Arithmetic Selection Operator

The arithmetic cardinality constraint (ACC) can be satisfied by taking advantage of the column data populated above. Specifically, given an ACC $|\sigma_{g(A_i...A_j)\bullet p}(R)| = n$, where $g()$ is an arithmetic function, $A_i, ..., A_k$ are the non-key columns operated by $g()$, and $\bullet$ is a comparator. We first calculate the result view $g(A_i...A_j)(R)$ based on the generated non-key columns. Then, our aim is to find a parameter p such that there exactly exist n items in the result view which satisfies the predicate $\bullet p$. Take the comparator $\leq$ as an example, we can set p as the n^{th} largest value in the result view. For example, consider the ACC $|V_7| = |\sigma_{t_1-t_2>p_3}(T)| = 5$ in Fig. 5b. We first calculate the result view $V_{rs} = g(t_1, t_2) = t_1 - t_2$ based on the populated data in t_1 and t_2. As the comparator is $>$, we compute the result as shown in Fig. 8 and then set p_3 as the $3rd$ largest value in the result (i.e., $p_3 = 0$). However, as the real-world table referred by our generator is usually rather large, this makes it expensive to calculate the result view. Moreover, the large table might also lead to large volumes of data in the result view that could not be fully accommodated in memory. As a result, it further increases the complexity of finding parameters. To address this issue, we propose to sample a small batch of rows from the table to approximate its data distributions. Then, we perform the above parameter instantiation process based on the sampled rows. In addition, based on Hoeffding's Inequality [30], the number of sampled rows can be calculated as $\frac{ln2-ln(1-\alpha)}{2\delta^2}$ according to the given error bound δ and confidence level α.

After solving all the ACCs, we have finished generating non-key column data and instantiating all their parameters. Next, we transfer the generated data and the instantiated query templates containing join operators to *key generator* for populating foreign keys. For example, the parameters in $Q_1 \sim Q_4$ are instantiated as following; then we transfer the generated table S and T, and the instantiated Q_1 and Q_2 in Fig. 8 to *key generator* for populating t_{fk}.

p_1=30 p_2=2 p_3=0 p_4=1 p_5=0 p_6=5 p_7=4 p_8=2

V. Key Generator

In this section, we first present how to deal with join cardinality and distinct constraints (JCC&JDC) on a single join in §V-A. Then, we discuss how to fuse multiple JCCs&JDCs on the same two tables without conflicts in §V-B. Finally, we extend the solution to multiple tables in §V-C.

A. Single Join Constraint on Two Tables

As defined in §II-B, a join view $\bowtie^{type}(V_l, V_r)$ has two input child views V_l and V_r. For a single join view, its JCC requires that there exist n_{jcc} matched pairs of rows in two input views (i.e. the primary key of a row in V_l equals the foreign key of another row in V_r). Its JDC requires that there exactly exist n_{jdc} distinct pk/fk values in all matched pairs of rows. From the definition of JCC, we can infer that we should select some primary keys from V_l and then populate them to the foreign key column of n_{jcc} rows in V_r (denoted as $PF_{V_l \to V_r}$=n_{jcc}). Meanwhile, we can also infer that the rest of $|V_r|-n_{jcc}$ foreign keys in V_r should not match any primary key in V_l. To this end, we need to populate these foreign keys with primary keys which do not exist in V_l (denoted as $PF_{\overline{V_l} \to V_r}$=$|V_r|-n_{jcc}$). In addition, JDC imposes an additional constraint which requires that there exist n_{jdc} distinct foreign keys in all matched pairs of rows from V_l and V_r. Thus, we must exactly select n_{jdc} primary keys in V_l when populating foreign keys in V_r (denoted as $PF^d_{V_l \to V_r}$=n_{jdc}). To summarize, we derive three foreign key populating rules from join cardinality and distinct constraints in Equation 1.

$$PF_{V_l \to V_r}=n_{jcc} \quad PF_{\overline{V_l} \to V_r}=|V_r|-n_{jcc} \quad PF^d_{V_l \to V_r}=n_{jdc} \tag{1}$$

Recall that we always firstly push down selection operators (see §III) and then both V_l and V_r can be immediately constructed after generating non-key column data and instantiating the corresponding parameters. Next, we can populate the foreign keys in V_r by applying the populating rules in Equation 1. For the foreign keys which are not in V_r, they can be populated by any primary keys in the referenced table. This is because they would not join with the primary keys in V_l and do not affect the output of $\bowtie^{type}(V_l, V_r)$.

Example 5. *Consider the join view* V_5= $\bowtie^{=}_{t_{fk}}(V_3, V_4)$ *in Fig. 8. From its join cardinality constraints* n_{jcc}=3*, there exist* 3 *matched pairs of rows from* V_3 *and* V_4*. Additionally, from* $|V_6|$=$|\Pi_{t_{fk}}(V_5)|$=2*, we can infer that* 2 *distinct* s_{pk}/t_{fk} *in matched rows, i.e.,* n_{jdc}=2*. Based on Eqn. 1, the three foreign key populating rules for* V_4 *are listed as follows.*

$$PF_{V_3 \to V_4}=3 \quad PF_{\overline{V_3} \to V_4}=|V_4|-n_{jcc}=3 \quad PF^d_{V_3 \to V_4}=2$$

Fig. 9a shows an example of populating foreign keys in V_4*. For ease of presentation, we only list the primary and foreign keys. According to the rules* $PF_{V_3 \to V_4}=3$ *and* $PF^d_{V_3 \to V_4}=2$*, we select* 2 *primary keys in* V_3 *to populate* 3 *foreign keys in* V_4*. As shown in Fig. 9a, we can populate the first three foreign keys in* V_4 *as* 1*,* 2 *and* 2*. According to the rule* $PF_{\overline{V_3} \to V_4}=3$*, we should populate the rest foreign keys in* V_4 *by any primary key in* $\overline{V_3}$*, e.g.* 3 *in* $\overline{V_3}$*. Finally, since* $\overline{V_4}$ *does not affect the output of* $\bowtie^{=}_{t_{fk}}(V_3, V_4)$*, the foreign keys in it can be populated with any primary key in table* S*, e.g., the primary key* 1*.*

B. Multiple Join Constraints on Two Tables

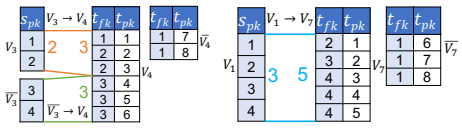
Suppose there exist m join views on two tables S and T, where S is a referenced table and T is a referencing table. If we populate the foreign keys according to the constraints of each join view individually, we might use different primary keys to populate the foreign key at the same row of table T. For example, consider the row in T with primary key 1, its foreign key is populated as 1 in Fig. 9a and 2 Fig. 9b. To address this issue, one naive method is to firstly enumerate all possible foreign key population results, and then find a feasible result which does not lead to any conflict. However, it's computationally prohibitive to enumerate population results for two joined tables. Specifically, for each foreign key in the referencing table T, it can be populated by any one of the primary keys in the referenced table S. Thus, there exist $|S|$ population results for each foreign key. Considering that the referencing table has $|T|$ foreign keys, then the total number of population results is $|S|^{|T|}$.

Note that, there exist some overlaps of primary keys between the input left child views. For example, consider the left child views V_1 and V_3 in Fig. 9, both of them contain the primary keys 1 and 2. Similarly, there also exist some overlaps of foreign keys between the input right child views. This motivates us to design a table partitioning based method to reduce the computational complexity. The basic idea is that we can first partition each table into disjoint partitions according to the overlaps of primary/foreign keys between input views. Then, for each partition in the referenced table, we derive its populating rules regarding partitions in the referencing table. By combining all the populating rules, we can formalize it as the *Constraint Programming* (CP) problem. Finally, we take advantage of the existing solver of CP problem to get the result of fusing multiple join constraints.

(1) Partition Tables. As a row can be uniquely identified by its primary key, for easier presentation, we use the primary key to represent a row. In addition, we denote the left and right child view of the k^{th} join view as V^k_l and V^k_r, respectively. If a primary key in the referenced table S is contained in V^k_l, it has the chance to (be input to) participate in the join, then it is an input join candidate for the k^{th} join view. Specifically, we use a status value $0/1$ to indicate whether a row in S is a join candidate of a given join view. Suppose there exist m join views, then each row in S is associated with a m dimensional status vector. Similarly, each row in the referencing table T also has a m dimensional status vector, which indicates whether it is an input join candidate in each of the m right child views, i.e., whether V^k_r contains the row's foreign key.

Example 6. *Fig. 10 shows the* 2 *dimensional status vectors of tables* S *and* T *regarding two join views* V_5 =$\bowtie^{=}_{t_{fk}}(V_3, V_4)$ *and* V_8 =$\bowtie^{l}_{t_{fk}}(V_1, V_7)$ *in Fig. 8. Specifically, the status vectors of table* S *are constructed based on left child views* V_1 *and* V_3*, while the status vectors of table* T *are constructed based on right child views* V_4 *and* V_7*. Consider the referenced table* S*, as its primary keys* 1 *and* 2 *are contained in* V_1 *and* V_3*, we set both the status vectors of the two rows as* $(1, 1)$*.*

Next, we partition the two joined tables according to their status vectors. Specifically, if the status vectors of two rows have the same value, we put them into the same partition. For example, consider table S in Fig. 10. The status vectors of its 4 rows are $(1, 1)$, $(1, 1)$, $(1, 0)$ and $(1, 0)$, respectively.



(a) Populate t_{fk} for V_5 (b) Populate t_{fk} for V_8

Fig. 9. Populate t_{fk} Based on Populating Rules

Status Vector		S	
V_1	V_3	s_{pk}	Partition
1	1	1, 2	S_1
1	0	3, 4	S_2
0	1	/	/
0	0	/	/

Status Vector		T	
V_4	V_7	t_{pk}	Partition
1	1	1,2,3,4,5	T_1
1	0	6	T_2
0	1	/	/
0	0	7, 8	T_3

Fig. 10. Partitions of S and T According to Status Vectors

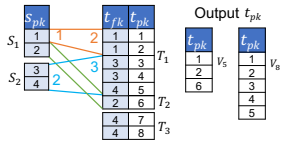


Fig. 11. Unified Population for t_{fk}

Then, we partition S into two partitions S_1 and S_2, where S_1 consists of rows 1∼2 and S_2 consists of rows 3∼4. Similarly, table T can be partitioned into T_1, T_2 and T_3. Since the dimension of status vectors is m, the number of partitions is at most 2^m. However, it is much smaller than 2^m in practice because dependencies among statuses can greatly reduce the enumeration space of status vectors. Specifically, we observe that the number of partitions is much smaller than 2^m in real-world applications. More specifically, recall that we always firstly push down selection operators (see §III), then each child view V_l^k (resp. V_r^k) is the output of the selection view in S (resp. T). Thus, we can infer that the status of a row in table S/T is determined by the selection result on that table. For example, given a row in S, if the k^{th} bit in its status vector is set as 1, then it must be in the output of the selection view under V_l^k. Further, as described in §IV-A, our selection view elimination procedure can reduce any selection view with a logical predicate into the view with a unary or an arithmetic predicate. Suppose there exist j unary and $m-j$ arithmetic selection views after reduction. We assume that the j unary selection views cover α columns in the partitioned table, and each of these columns is associated with $x_1, \cdots, x_\alpha$ unary selection views, $s.t. \sum_{i=1}^{\alpha} x_i = j$. Then, according to our cumulative distribution function based analysis described in §IV-B, the parameters in x_i unary selections would divide the domain space of a column into at most $x_i + 1$ ranges. Since all the rows in a range must be selected/filtered together by a given unary selection, we can infer that the j bits that are associated with unary selection views in the status vector would at most have $\Pi_{i=1}^{\alpha}(x_i+1)$ distinct values. For the $m-j$ bits associated with arithmetic selection views, they have at most 2^{m-j} distinct values even if they are independent of each other. Taken together, the number of partitions is at most $\Pi_{i=1}^{\alpha}(x_i+1)\cdot 2^{m-j}$. In real-world applications, we observe that there usually exist a few arithmetic selections on each table. For example, the number of arithmetic selections on a table is only 3 in the TPC-H workload. In addition, we also observe that the unary selection views usually cover a small number of columns in each table (i.e., α is small). For example, the unary selection views cover at most 4 columns in any given table in the TPC-H workload. Thus, the actual number of partitions is much less than 2^m.

(2) Derive Populating Rules Based on Partitions. Consider the status vector of a referenced partition S_i, if its k^{th} bit is set as 1, then we can infer that all the primary keys in S_i are contained in V_l^k. For a referencing partition T_i, setting its k^{th} bit as 1 indicates that all the foreign keys in T_i are contained in V_r^k. For ease of presentation, hereinafter we use S_i/T_i to represent the PK/FK partitions. Then, for any join view V^k, key population from V_l^k to V_r^k is

$$V_l^k \to V_r^k = \bigcup_{S_i \subseteq V_l^k} S_i \to \bigcup_{T_j \subseteq V_r^k} T_j = \bigcup_{S_i \subseteq V_l^k, T_j \subseteq V_r^k} S_i \to T_j \quad (2)$$

Based on Equation 2, we formalize two populating rules for each pair of partitions S_i and T_j. The two rules require to use z primary keys in S_i for populating y foreign keys in T_j.

$$PF_{S_i \to T_j} = y \;\; s.t.\; y \in \{0, 1, \ldots, |T_j|\} \quad (3)$$
$$PF^d_{S_i \to T_j} = z \;\; s.t.\; z \in \{0, 1, \ldots, |S_i|\}$$

In addition, from Equation 2, we can convert three populating rules on a join view in Equation 1 into the ones on their associated partitions. Here, n^k_{jcc} and n^k_{jdc} denote the join cardinality and distinct constraints on the k^{th} join view.

$$PF_{V_l^k \to V_r^k} = \sum_{S_i \subseteq V_l^k,\, T_j \subseteq V_r^k} PF_{S_i \to T_j} = n^k_{jcc}$$
$$PF^d_{V_l^k \to V_r^k} = \sum_{S_i \subseteq V_l^k,\, T_j \subseteq V_r^k} PF^d_{S_i \to T_j} = n^k_{jdc} \quad (4)$$
$$PF_{\overline{V_l^k} \to V_r^k} = \sum_{S_i \subseteq \overline{V_l^k},\, T_j \subseteq V_r^k} PF_{S_i \to T_j} = |V_r^k| - n^k_{jcc}$$

Note, for any partition S_i in V_l^k, if its primary keys are selected to populate foreign keys in more than one partition in V_r^k, *Mirage* avoids selecting the primary key repeatedly to populate different partitions in V_r^k. If not, we would have $\sum_{S_i \subseteq V_l^k,\, T_j \subseteq V_r^k} PF^d_{S_i \to T_j} < n^k_{jdc}$.

Further, our populating rules should also make sure that the foreign keys in each partition should are exactly covered. That is, for any partition T_j, its number of foreign keys populated by partitions in table S must equal to its number of rows.

$$\sum_{S_i \subseteq S} PF_{S_i \to T_j} = |T_j| \quad (5)$$

Example 7. *Consider the join view $V_5 = \bowtie^{=}_{t_{fk}} (V_3, V_4)$ in Example 5 again. From Fig. 10, we can see that $V_3 = S_1$, $\overline{V_3} = S_2$ and $V_4 = T_1 \cup T_2$. Based on Equation 4, the three populating rules in Example 5 can be converted as*

$$PF_{V_3 \to V_4} = PF_{S_1 \to T_1} + PF_{S_1 \to T_2} = 3$$
$$PF_{\overline{V_3} \to V_4} = PF_{S_2 \to T_1} + PF_{S_2 \to T_2} = 3$$
$$PF^d_{V_3 \to V_4} = PF^d_{S_1 \to T_1} + PF^d_{S_1 \to T_2} = 2$$

Based on Equation 5, the two following equations makes sure that the foreign keys in T_1 and T_2 are exactly covered.

$$PF_{S_1 \to T_1} + PF_{S_2 \to T_1} = |T_1| = 5 \quad PF_{S_1 \to T_2} + PF_{S_2 \to T_2} = |T_2| = 1$$

(3) Construct Constraint Programming Problem. By combining all of these populating rules in Equations 3 ∼ 5, we can formalize the problem as a *Constraint Programming* (CP) problem [20]. Then, after finding a feasible solution for a set of variables stated in the constraint equations, we can follow it to populate foreign keys in each partition T_j. However, decomposing populating rules of join views into partitions would enlarge the solution space and lead to contradictory solutions. To address this issue, we further introduce three constraints to ensure the validity of populating results.

- Composability. The number of populated foreign keys must not be less than the number of populated distinct foreign keys, i.e., $PF_{S_i \to T_j} \geq PF^d_{S_i \to T_j}$
- Expressibility. If a partition S_i uses its primary keys to populate foreign keys in another partition T_j, then at least one primary key is used, i.e., $PF_{S_i \to T_j} > 0 \Rightarrow PF^d_{S_i \to T_j} > 0$
- Coverability. The total number of selected primary keys must not exceed the size of primary key candidate set (recall that we cannot select a primary key repeatedly to populate foreign keys of different partitions in V^k_r), i.e., $|S_i| \geq \sum_{T_j \subseteq V^k_r} PF^d_{S_i \to T_j}$

After integrating all the three constraints into the CP problem, we propose to use the *Or-Tools* [21] as our *CP solver*. It improves its efficiency by utilizing the constraint propagation method to prune the search space. Then we can use the solution to populate foreign keys.

Example 8. *Consider the join constraints from two join views V_5 and V_8. After we include the above three constraints, we can find a feasible solution without contradictions as follows.*

$PF_{S_1 \to T_1}$=2	$PF_{S_1 \to T_2}$=1	$PF_{S_2 \to T_1}$=3	$PF_{S_2 \to T_2}$=0
$PF^d_{S_1 \to T_1}$=1	$PF^d_{S_1 \to T_2}$=1	$PF^d_{S_2 \to T_1}$=2	$PF^d_{S_2 \to T_2}$=0

Fig. 11 presents the result of populating foreign keys based on this solution. Specifically, from $PF_{S_1 \to T_1} = 2$ and $PF^d_{S_1 \to T_1} = 1$, we should select one primary key from S_1 to populate two foreign keys in T_1, e.g., $\{1, 1\}$. In addition, recall that the foreign keys in T_3 can be populated by arbitrary primary keys, we populate T_3 with 4.

Note that CP Problem may be unsolvable due to the constraints are exactly conflict with each other. If so, we keep multiple foreign key columns in the table, the number of which is the number of conflict distributions. And each AQT uses the corresponding column based on its distribution requirement.

C. Join Constraints on Multiple Tables

For any referenced table S and referencing table T, after fusing all their join constraints and populating foreign keys in T, we can directly get the join result of each join view on S and T. Moreover, we can also derive the primary keys of table T output by each join view, and use them to populate the foreign keys in another table that references T. Based on the observation, we propose to build a topological order among tables based on their PK-FK references [31], and populate foreign keys in each table according to the topological order. Specifically, each table is represented as a vertex in the graph, and if the primary keys in a given table S are referenced by the foreign keys in another table T, we add a link between the corresponding vertices. Then, we find a topological order of the graph by using the graph sorting algorithm [31]. Finally, we populate foreign keys in each table according to their topological order. More specifically, after populating foreign keys in a specific table T, we compute the intermediate join result regrading T and the table referenced by T for all the multi-table join views containing T. Then, we collect T's primary keys in the join result. Next, by applying the method described in §V-B, we use these primary keys to populate the foreign keys in another table referencing T.

VI. Discussion

A. Discussion of Error Bound

In this section, we discuss the bound of errors introduced by our non-key column generator and key column generator. For ease of presentation, hereinafter we use $|V_i|$ to represent the required output size imposed by the cardinality constraint on the i^{th} query operator view, and use $|\hat{V}_i|$ to represent its actual output size on our simulated database.

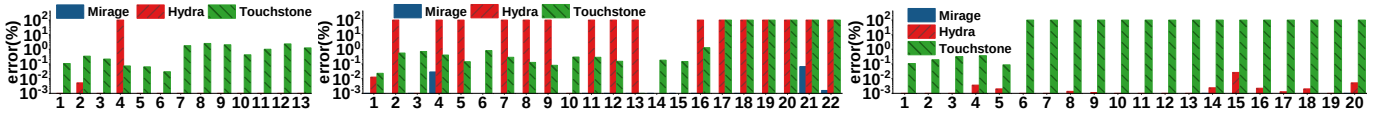
Error Bound of the Non-key Column Generator. There exist three kinds of cardinality constraints involved in non-key columns, which are LCC, UCC and ACC. First, based on the decoupling rules $rule_1 \sim rule_3$ in §IV-A, all LCCs can be decomposed to UCCs and ACCs with the help of equivalent transformations. Thus, for the V_i from a LCC, we can guarantee that $|V_i| = |\hat{V}_i|$ as long as we ensure that the decomposed UCCs and ACCs are satisfied. Second, for the case of V_i associated with a UCC, as the data distribution of each non-key column is derived according to all the UCCs on that column. Theorem 2 proves that our data generation and parameter instantiation methods based on CDFs (see §IV-B and §IV-C) could guarantee $|V_i| = |\hat{V}_i|$. Finally, for the case of V_i associated with an ACC, if we calculate the parameters of an ACC based on the whole of generated data (in §IV-D), we have $|V_i| = |\hat{V}_i|$. However, to avoid memory overflows caused by large volumes of data, small sampling data is used to calculate the parameters. Even so, we still have the error bound as δ in a confidence level α if the sampling data size is no less than $\frac{ln2 - ln(1-\alpha)}{2\delta^2}$.

Theorem 2. *For any query operator view V_i whose cardinality constraint is a UCC, our non-key generator guarantees that $|\hat{V}_i| = |V_i|$.*

Proof. Revisit our algorithm in §IV-B. First, we transfer UCC into distribution requirements $F_A(p)$ or $f_A(p)$ in §IV-B, where p can a parameter. Secondly, we divide whole domain space of the non-key column into multiple sub-ranges by ordering the distribution requirements from $F_A(p)$. Therefore, $F(p)$ is exactly equal to the accumulation of the probablities from preceding sub-ranges, i.e., the following equation. Then, the step dose not introduce any error for $F(p)$.

$$\sum [F_A(p_i) - F_A(p_j)] = [F_A(p) - F_A(0)] = F_A(p)$$
$$s.t.,\ 0 \leq p_i < p_j \leq p\ \&\ \nexists p \in (p_i, p_j)$$

Then, we put the distribution requirements from $f_A(p)$ in these sub-ranges by solving bin-packing problem. Because putting $f_A(p_k)$ into range $F(p_i, p_j)$ dose not change the probability of $f_A(p_k)$, the step dose not lead any error of $f_A(p)$. After putting $f_A(p_k)$ into $F(p_i, p_j)$, the sub-range $(p_i, p_j]$ can be divided into $(p_i, p_k - 1]$, $(p_k - 1, p_k]$ and $(p_k, p_j]$. We update the probabilities of the new ranges with the remaining probability, i.e., $F(p_i, p_k - 1) + F(p_k, p_j) =$



(a) Query of SSB (b) Query of TPC-H (c) Query Group of TPC-DS

Fig. 12. Relative Errors for Various Workloads including SSB, TPC-H, and TPC-DS (100% error means no support)

$F(p_i, p_j) - f_A(p_k)$. Therefore, the total probability of sub-range $(p_i, p_j]$ dose not change, i.e., the step dose not still introduce any error for $F(p)$.

In summary, our probability assignment does not introduce any error for distribution requirements $F(p)$ and $f(p)$. In §IV-C, we exactly generate $|R| * F(p_i, p_j)$ values for each range $(p_i, p_j]$. Therefore, the non-key generator can satisfy UCC, i.e., $|UCC|$=$|\hat{UCC}|$. □

Error Bound of the Key Column Generator. As all the equations in §V-B are strictly constructed to satisfy the JCCs and JDCs of all join views, the procedure of fusing populating results from multiple join views would not introduce any error. Thus, we focus on the error bound of dealing join constraints for a single join view. Suppose the join view V_i has the constrains of n_{jcc_i} and n_{jdc_i}, and its left and right child input views are V_l^i and V_r^i, respectively. According to the Equation 1 in §V-A, the population of a join view can be achieved when and only when the number of input primary keys in V_l^i is no less than n_{jdc_i} and the number of input rows in V_r^i is no less than n_{jcc_i}, i.e.,

$$|V_l^i| \geq n_{jdc_i} \quad |V_r^i| \geq n_{jcc_i} \quad s.t. \quad n_{jcc_i} \geq n_{jdc_i} \tag{6}$$

If the data sampling method is not applied when solving the ACCs, we can guarantee that the output size of each selection view in the simulated database is exactly the same as that of the original database. Then, as the join executes after selection, the input size of each join view also equals to that of the original database. Thus, we must have a solution satisfying the constraints of n_{jcc_i} and n_{jdc_i}. Similarly, there also exists no error for the subsequent join views in the same query. However, the sampling method would lead to the case in which the input size of a join view (i.e., the output size of a selection view) in the simulated database is smaller than that of the original database.

More specifically, if there exists that $|\hat{V_r^i}| < |V_r^i|$, it might lead to the case of $|\hat{V_r^i}| < n_{jcc_i}$, breaking the requirement in Equation 6. In this case, we propose to resize the constraint n_{jcc_i} to $|\hat{V_r^i}|$, which is the largest valid constraint that can be satisfied by the current input size $|\hat{V_r^i}|$. In this way, we can reduce the errors to the utmost extent. Then, from $|V_r^i| \geq n_{jcc_i}$ in Equation 6, we can derive that relative error introduced by such a resizing is also bounded by δ:

$$\frac{|(n_{jcc_i} - |\hat{V_r^i}|)|}{n_{jcc_i}} = |1 - \frac{|\hat{V_r^i}|}{n_{jcc_i}}| \leq |1 - \frac{|\hat{V_r^i}|}{|V_r^i|}| \leq \delta \tag{7}$$

Similarly, we have the same conclusion for the relative error of the constraint n_{jdc_i}.

Note that the relative error for any join view can be calculated in the same way as Eqn. 7. Therefore, the relative error of each join view is no bigger than the relative error introduced by its input views. So the relative error of any subsequent join view is no bigger than the error bound δ of the initial select views.

In summary, when *Mirage* does not sample data for deciding parameters of ACC, it can generate data satisfying all CCs without any error, or else the relative error of each view cannot exceed δ, which can be adjusted considering the memory limitation (in §IV-D). In our experiment, it is verified to be an especially small number.

B. Discussion of Time Complexity

In the section, we discuss the time complexity of *Mirage*. Suppose there exist $K + J$ literals in all the annotated query templates, among them K literals are unary predicates and J literals are arithmetic predicates. Moreover, the generated database is supposed to consist of M columns and L rows in total. For the non-key generator, the logical dependency decoupling process needs to assign boundary values to the parameters in literals, thus its time complexity is $\mathcal{O}(K + J)$. For solving UCCs, let N be the total number of UCCs on a given non-key column A. Firstly, we sort each parameter p in $F_A(p)$, whose time complexity of is $\mathcal{O}(NlogN)$. Secondly, for each parameter p in $f_A(p)$, we use a binary search to find the smallest range that can accommodate it and then the time complexity is $\mathcal{O}(NlogN)$. Thirdly, the time complexity of assigning unique values and instantiating parameters is $\mathcal{O}(N)$. Thus, the total time complexity of UCCs on a non-key column is $\mathcal{O}(NlogN)$. Since there are K UCCs on all the non-key columns, the total time complexity is $\mathcal{O}(KlogK)$. For solving ACCs, the time complexity of computing the result view of the i^{th} ACC is $\mathcal{O}(g_i * L)$, where g_i represents the computation cost of the i^{th} ACC's arithmetic function. The time complexity of selecting a certain number of items in the result view that satisfy the predicate is $\mathcal{O}(L)$. Since there are J ACCs on all the non-key columns, the total time complexity of solving ACCs is $\mathcal{O}(\sum_1^J g_i L)$. Lastly, the time complexity of generating L rows for at most M non-key columns is $\mathcal{O}(L * M)$. Taken together, the total time complexity of generating non-key columns is $\mathcal{O}(J + KlogK + L(\sum_{i=1}^J g_i + M))$.

For the key generator, the time complexity of computing the status vectors according to the outputs of selection operators is $\mathcal{O}(L(K + \sum_1^J g_i))$. This is because the time complexity of computing the result of K UCCs over L rows is $\mathcal{O}(K * L)$, and the time complexity of computing the result of J ACCs over L rows is $\mathcal{O}(\sum_1^J g_i L)$. Therefore, the time complexity of computing the status vectors is $\mathcal{O}(L(K + \sum_1^J g_i))$. As the *Constraint Programming (CP)* problem is NP-hard, its time complexity is unavailable. Finally, the time complexity of

generating L rows for at most M foreign key columns based on the result of the CP solver is $\mathcal{O}(L * M)$.

C. Discussion of Assumption And Conflict

We implicitly include two assumptions of data correlation in our paper.

1) The result of a logical predicate is equivalent to the result of its kept sub-predicates after decoupling. The assumption is used to decouple logical cardinality constraints (LCC). Therefore, we can simplify the complex joint distribution requirements among columns from LCC and just deal with unary cardinality constraints (UCC) for each column independently.
2) The parameter of UCC with comparator $=$ ($UCC_{=}$) can be put into any appropriate range satisfying its data existence probability. The assumption is used to put the parameter of $UCC_{=}$. Since parameters in the annotated query template (AQT) are anonymized, we can not determine the original range that locates the parameter of the $UCC_{=}$. Based on the assumption, we just put the parameter into any appropriate range instead of the exact original range.

Next, we discuss the conflicts introduced by the two assumptions and discuss how to resolve them.

The first assumption may introduce conflicts when the non-key generator solves UCC. Specifically, if a cardinality constraint of $UCC_{=}$ is obtained after decoupling, it may cause a totally different data space requirement from the original one to the involved column. Therefore, we may not be able to find a feasible space for placing the parameter of the $UCC_{=}$ in the divided ranges of the column. To solve the conflict, we propose to divide the $UCC_{=}$ into multiple small-sized $UCC'_{=}$. Then it is easy to find ranges to locate the parameters in the small sized $UCC'_{=}$, which aggregates to satisfy the cardinality requirement in $UCC_{=}$. We revise the content in § **4.2** to highlight the solution. An example is provided in Example 9. After dividing, the comparator $=$ of $UCC_{=}$ is converted to comparator in.

Example 9. *Consider a table T containing two columns t_1 and t_2, its cardinality constraints are $|T| = 4$, $|T|_{t_1} = 3$ and $|T|_{t_2} = 3$. The following three cardinality constraints are from two queries over table T.*

$$|V_1| = |T| = 4 \quad |V_2| = |\sigma_{t_1=p_1 \vee t_2>p_2}(T)| = 3$$
$$|V_3| = |\sigma_{t_1=p_3}(T)| = 2$$

According to our decomposing algorithm in § 4.1, the second view V_2 can be decomposed into two unary views, i.e., V_2^1=$\sigma_{t_1=p_1}(T)$ and V_2^2=$\sigma_{t_2>p_2}(T) = V_2^1 \cup V_2^2$.

$$V_2 = \sigma_{t_1=p_1}(T) \cup \sigma_{t_2>p_2}(T) = V_2^1 \cup V_2^2$$

After decoupling, we can set $p_2 = \infty$ and then $V_2^2 = \emptyset$. Finally, we eliminate view V_2^2 and keep the following two unary cardinality constraints on column t_1.

$$|V_2^1| = |\sigma_{t_1=p_1}(T)| = 3 \qquad |V_3| = |\sigma_{t_1=p_3}(T)| = 2$$

We represent their corresponding CDF requests as $f(p_1) = \frac{3}{4} = 75\%$ and $f(p_3) = \frac{2}{4} = 50\%$. Since $f(p_1) + f(p_3) \geq 100\%$, we cannot locate the parameters in the two $UCC_{=}$ in domain range (0, 3] simultaneously. Therefore, we divide $UCC_{=}$ of V_2^1 into two small sized $UCC'_{=}$, which introduces two parameters, and then we have $\sigma_{t_1 \ in \ (p'_1,p_4)}(T)$, and its cardinality constraint is $|V_2^1| = |\sigma_{t_1 \ in \ (p'_1,p_4)}(T)| = 3$. Therefore, we can obtain the following two CDF requests.

$$f(p_1) = f(p'_1) + f(p_4) = \frac{3}{4} = 75\% \quad f(p_3) = \frac{2}{4} = 50\%$$

If the generated data is 1,2,3 for column t_1, a feasible instantiation is p'_1=p_3 = 1 and p_4 = 2, which divide the space into three ranges, i.e., $(0, p'_1 = p_3], (p'_1 = p_3, p_4], (p_4, 3]$. The corresponding CDF is $f(1) = 50\%$, $f(2) = 25\%$ and $f(3) = 25\%$.

The second assumption may introduce conflicts when Key Generator merges multiple distribution requirements to a single foreign key column. Specifically, the Constraint Programming Problem described in § **5.2** may be unsolvable due to the conflict, that is we may not be able to merge the distribution requirements into one column. An example is provided in Example 10. In such a case, we will split one foreign key column into multiple foreign key columns in a table, the number of which is the number of conflict distributions (can not be merged together), and each AQT uses the corresponding column generated based on its distribution requirement. We add the discussion in § **5.2** to highlight the solution.

Example 10. *Suppose the database D in production contains two tables S and T. S consists of a primary key s_{pk} and a non-key column s_1, T consists of a primary key t_{pk}, a foreign key t_{fk} which references s_{pk}. The original data in D is presented in Fig. 14a with the cardinality constraints $|S| = 2$, $|T| = 1$, $|S|_{s_1} = 2$. The example AQTs are illustrated in Fig. 13. Noted that if we set p_1 =3 and p_2=3 based on the original data in Fig. 14a, all cardinality constraints in AQTs hold, so there is a solution for this QAGen problem.*

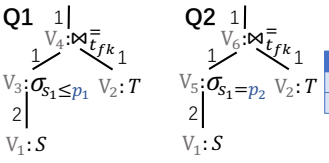


Fig. 13. Annotated Query Templates for Table S and T

s_{pk}	s_1
1	3
2	7

t_{pk}	t_{fk}
1	1

(a) Original Data

s_{pk}	s_1
1	1
2	2

t_{pk}	t_{fk}
1	?

(b) Generated Data Without t_{fk}

Fig. 14. Data of Table S and T

However, considering the way that we deal with the two UCCs of the non-key column s_1, i.e., $|\sigma_{s_1<p_1}(S)| = 1$ (for V_3) and $|\sigma_{s_1=p_2}(S)| = 1$ (for V_5), we infer their CDF requests are $F(p_1) = 50\%$ and $f(p_2) = 50\%$. Suppose the generated data for s_1 is 1 and 2 as in Fig. 14b. First, since the cardinality of s_1 is 2, we initialize the domain range of s_1 as (0,2], and divide the range into $(0, p_1]$ and $(p_1, 2]$ by $F(p_1) = 50\%$. Then we have two CDFs for s_1 divided by p_1 as $F(0, p_1) = 50\%$ and $F(p_1, 2) = 50\%$. Note that p_2 can be put into either of the

two ranges. However, if we put p_2 into range $(p_1, 2]$, we have $p_1 = 1$ and $p_2 = 2$, which cause a conflict when populating t_{fk}. In the case when p_1=1 and p_2=2, V_3 and V_5 output the primary keys valued 1 and 2 respectively. Correspondingly, to satisfy the two $\bowtie_{t_{fk}=}$ for V_4 and V_6, we must have two foreign keys valued 1 and 2 respectively. But we can only populate one t_{fk} in table T. Then we can not find a solution to satisfy the cardinality constraints in the two AQTs.

In such a case, the generated table T splits the column t_{fk} into two foreign key columns, i.e., t_{fk1} and t_{fk2} as in Fig. 16. We instantiate each AQT based on the data from the individual foreign key column as in Fig 15.

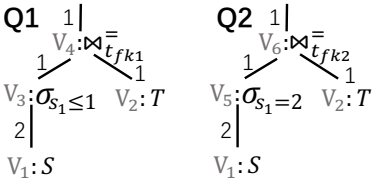


Fig. 15. Populated Query with Different Foreign Key Columns

s_{pk}	s_1
1	1
2	2

t_{pk}	t_{fk1}	t_{fk2}
1	1	2

Fig. 16. Generated Data

Notice that these conflicts are highly related to the associations between multiple cardinality constraints. For the test workloads, our experiments show that 1) all the cardinality requests from $UCC_=$ are well satisfied without dividing any $UCC_=$ into smaller size $UCC'_=$; 2) all the populations of foreign key columns accomplish the merge, i.e., no foreign key column splitting introduced. Though we have solutions for the conflicts introduced by the two assumptions, we have not met the conflicts in our experimental benchmark scenarios.

VII. Extension

A. Extension to Residual Predicate

There are three general cases where the query optimizer would not push down the residual predicate filter entirely to the base table:

- The residual predicate contains a logical predicate that operates on multiple tables, e.g., the residual predicate in TPC-H q19.
- The residual predicate contains an arithmetic predicate that operates on multiple tables.
- The residual predicate operates on one table and the query optimizer can push down it entirely. However, it is more cost-effective to execute the join before selection.

Logical predicate crosses tables. For simplicity, we mainly discuss the case in which the residual predicate follows the disjunctive normal form (DNF). Note that, a predicate with any other form can be transformed to DNF [22]. That is,

$$(LeftTableClause_1 \wedge RightTableClause_1) \vee ... \vee (LeftTableClause_m \wedge RightTableClause_m).$$

More specifically, the query optimizer would first push down the union of predicate filters $LeftTableClause_1 \vee ... \vee LeftTableClause_m$ and $RightTableClause_1 \vee ... \vee RightTableClause_m$ to the left and right tables, respectively. Next, our proposed decoupling dependency procedure would try to eliminate $m-1$ sub-predicates and keep only one sub-predicate in both of the pushed-down predicate filters. As described above, if we guarantee that the two kept sub-predicates belong to the same sub-predicate of the residual predicate (i.e., $LeftTableClause_i \wedge RightTableClause_i$), then the sub-predicate $LeftTableClause_i \wedge RightTableClause_i$ would always hold. Moreover, as the residual predicate is a union of multiple sub-predicates, then it would also always hold.

Below, we will specifically outline the process. Consider the union of predicates pushed down to the left table, i.e., $LeftTableClause_1 \vee ... \vee LeftTableClause_m$. Its corresponding view is as follows.

$$\sigma_{LeftTableClause_1 \vee ... \vee LeftTableClause_m}(LeftTable) = \sigma_{LeftTableClause_1}(LeftTable) \cup ... \cup \sigma_{LeftTableClause_m}(LeftTable)$$

In the first decoupling phase, we find all sub-selection views that can be instantiated to be $\emptyset$. If all the selection views can be set as $\emptyset$, we keep all the sub-selection views; otherwise, we only keep the sub-selection views that can not be set as $\emptyset$. Suppose we finally keep $n^l (\leq m)$ sub-selection views based on the predicates on the left table, i.e., V^l. We process the predicates on the right table similarly and keep $n^r (\leq m)$ sub-selection views, i.e., V^r.

If there are two sub-selection views in V^l and V^r generated from the same i-th pair of clauses in the residual predicate, we keep only these two clauses in the decoupling clauses and then the $LeftTableClause_i \wedge RightTableClause_i$ always holds, then reset the other sub-selection views to $\emptyset$. Finally, the problem can be simplified to be processed by Mirage.

If there are no sub-selection views in both V^l and V^r generated from the same i-th pair of predicates, the residual predicate does not always hold. Then Mirage cannot deal with this situation. However, this situation may only happen when $n^l + n^r \leq m$ according to the pigeonhole principle, then we further infer that $n^l < m$ and $n^r < m$. In other words, each kept clause cannot be set to $\emptyset$. Based on the Table. 3 in § 4.1, the comparators of all literals in all n^l (resp. n^r) clauses must strictly belong to $\neq, not\ like, not\ in$. As far as we know, the situation does not exist in current OLAP benchmarks, e.g., TPC-H, and public workloads, e.g., Public BI benchmark.

In summary, Mirage always supports residual predicates as long as either of the decoupling result does not contain $\neq, not\ like, not\ in$.

Example 11. *As shown in Fig. 17, the annotated query template for TPC-H q19 contains a residual predicate listing 3 which operates on both tables part and lineitem. Thus, listing 3 could not be directly pushed down to the two base tables. Note that the residual predicate listing 3 is usually applied alongside the join operator during the execution. To reduce the join cost, the query optimizer would apply the union of predicate filters in listing 3 to each table, which are listing 1 and 2. For this case, as listing 3 imposes dependencies between multiple non-key columns from two tables, our query*

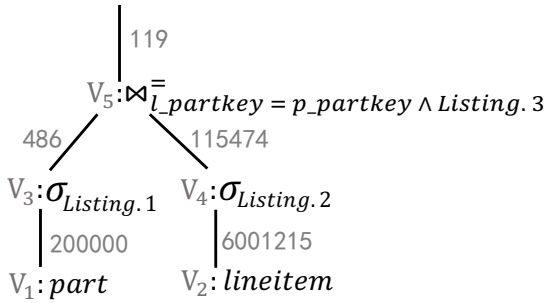


Fig. 17. The annotated query template of TPC-H q19 in PostgreSQL, whose residual predicate *listing* 3 is usually applied alongside the join operator during the execution.

Listing 1. the union of predicates on the table *part*

```
( p_brand = p1 AND p_size >= p2 AND p_size <=
    p3 AND
  p_container IN (p4, p5, p6, p7) )
      --PartClause1
OR
( p_brand = p8 AND p_size >= p9 AND p_size <=
    p10 AND
  p_container IN (p11, p12, p13, p14) )
      --PartClause2
OR
( p_brand = p15 AND p_size >= p16 AND p_size <=
    p17 AND
  p_container IN (p18, p19, p20, p21) )
      --PartClause3
```

Listing 2. the union of predicates on the table *lineitem*

```
( l_shipmode IN (p22, p23) AND l_shipinstruct =
    p24 AND
  l_quantity >= p25 AND l_quantity <= p26 )
          --LineitemClause1
OR
( l_shipmode IN (p27, p28) AND l_shipinstruct =
    p29
  AND l_quantity >= p30 AND l_quantity <= p31 )
      --LineitemClause2
OR
( l_shipmode IN (p32,p33) AND l_shipinstruct =
    p34
  AND l_quantity >= p35 AND l_quantity <= p36 )
      --LineitemClause3
```

Listing 3. the residual logical predicate on *part* and *lineitem*

```
(PartClause1 AND LineitemClause1) OR
(PartClause2 AND LineitemClause2) OR
(PartClause3 AND LineitemClause3)
```

rewriting method also could not push down the selection without breaking the cardinality constraints in the AQT. Fortunately, our proposed sub-selection view elimination method (see Section 4.1) could effectively decouple the dependencies between non-key columns. This further helps us simplify the complexity of dealing with the residual predicate filter.

More specifically, the logical predicate in Fig. 1 follows the disjunctive normal form (DNF), which is $PartClause1 \vee PartClause2 \vee PartClause3$. As shown in Table 3 in our paper, we can assign proper boundary values to the parameters of some sub-predicates such that we can keep only one sub-predicate and eliminate the others. For example, by setting $p_{10} = -\infty$ and $p_{17} = -\infty$, we can make $\sigma_{PartClause2}(part) = \sigma_{PartClause3}(part) = \emptyset$. Then, we can eliminate $PartClause2$ and $PartClause3$ and reduce listing 1 to $PartClause1$. Similarly, we can also reduce listing 2 to $LineitemClause1$ by setting $p_{31} = -\infty$ and $p_{36} = -\infty$. That is, after finishing the procedure of decoupling dependencies, all the left (resp. right) child input rows of V_5 in Fig. 17 would satisfy $PartClause1$ in the table part (resp. $LineitemClause1$ in the table lineitem). Then, the sub-predicate $PartClause1$ AND $LineitemClause1$ of the residual predicate listing 3 (see Figs. 17 and 3) would always hold. With this mechanism, we do not need to take into account the residual predicate when generating the foreign keys of table lineitem to guarantee the output size of the join operator (see V_5). Note that, the foreign key index could not be leveraged to improve the performance of join as its inputs are intermediate execution results, and the join condition computing cost is determined by the number of input primary keys and foreign keys. Fortunately, our Mirage could guarantee the input size of the join operator.

Arithmetic predicate crosses tables. For the arithmetic predicate, Mirage cannot push down to any single table. Therefore, Mirage tries to eliminate the predicate like dealing with logical predicate across multiple tables. Based on Table. 3 in § **4.1**, Mirage can eliminate the arithmetic predicate. For example, the residual predicate is an arithmetic predicate in the following AQT. We can set $p_2 = -\infty$ to eliminate $s_1 + t_1 > p_2$.

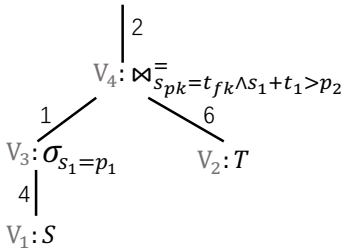


Fig. 18. The AQT with a residual arithmetic predicate

Select After Join Because of Lower Cost. Consider the logical query plan $\sigma_{P_T}(\sigma_{P_S}(S) \bowtie^{=}_{t_{fk}} T)$, where predicate P_S (resp. P_T) only involves table S (resp. T) and P_T is executed after join. It commonly occurs in nested loop join using foreign key index. Specifically, if P_S filters a larger number of rows of S, it is cheaper to execute the nested loop join using the foreign key index over t_{fk} firstly and then apply P_T to the join result. In such a query plan, the size of foreign key index scan is equal to the size of matched rows, so we need to deal

with three cardinality constraints as follows.

$$|\sigma_{P_S}(S)| = n_1 \quad |\sigma_{P_S}(S) \bowtie^{=}_{t_{fk}} T| = n_2$$
$$|\sigma_{P_T}(\sigma_{P_S}(S) \bowtie^{=}_{t_{fk}} T)| = n_3$$

$Mirage$ supports the first two cardinality constraints. For supporting the last one, $Mirage$ replaces it with an equivalent transformation, i.e., $|\sigma_{P_S}(S) \bowtie^{=}_{t_{fk}} \sigma_{P_T}(T)| = n_3$. However, the transformation introduces an extra cardinality constraint for $\sigma_{P_T}(T)$, i.e., $|\sigma_{P_T}(T)| = n_4$. If customers allow, $Mirage$ obtains n_4 by executing $\sigma_P(T)$ in the original database. Otherwise, we are inspired by the join predicate independence assumption from traditional query optimizer [32] and assume that P_T filters the same proportion of rows on the table T as on the join result. Then $Mirage$ calculates n_4 by the following equation.

$$\frac{|\sigma_{P_T}(T)|}{|T|} = \frac{|\sigma_{P_T}(\sigma_{P_S}(S) \bowtie^{=}_{t_{fk}} T)|}{|\sigma_{P_S}(S) \bowtie^{=}_{t_{fk}} T|} = \frac{n_3}{n_2} \Rightarrow n_4 = |\sigma_{P_T}(T)| = \frac{n_3}{n_2}|T|$$

B. Extension to Subquery

$Mirage$ can be extended to support the nested subqueries. Usually, DBMS provides ways to eliminate subqueries for efficiency [33] and then in most of the AQTs, we see no subqueries. Here we illustrate the solution for the subqueries that cannot be eliminated by running TPC-H on PostgreSQL in Example [33].

Example 12. *There are 13 queries with subqueries in TPC-H, i.e., Q_2, Q_4, Q_7, Q_8, Q_9, Q_{11}, Q_{13}, Q_{16}, Q_{17}, Q_{18}, Q_{20}, Q_{21}, Q_{22}. PostgreSQL eliminates subqueries for 7 of them, i.e., Q_4, Q_7, Q_8, Q_9, Q_{13}, Q_{18}, Q_{21}. But there are still 6 queries having subqueries in their query plans, i.e., Q_2, Q_{11}, Q_{16}, Q_{17}, Q_{20} and Q_{22}.*

We classify the subqueries that cannot be eliminated into the following two cases.

1) An uncorrelated subquery returning a single row, e.g., Listing 4 simplified from Q_{11}. For this case, database first executes the sub-query independently and saves its result as a temporary variable. Then the database replaces the sub-query in its parent query with the temporary variable and executes its parent query, e.g., Q_{11} and Q_{22}.

Listing 4. An uncorrelated subquery returning a single row

```
1 select * from T where t1 >
2         (select avg(t1) as tavg from T)
```

2) There is no unnesting policy that can be applied to eliminate the subquery in current database, e.g., *line 2* of Listing [34] simplified from Q_{16}. Note that in this case, MySQL can eliminate the subquery with antijoin, and the result is then an unnested query plan, e.g., *line 4* of Listing [34]. However, PostgreSQL cannot eliminate it due to the lack of left anti join operator. Given the lack of unnesting policies in PostgreSQL, Q_2, Q_{16}, Q_{17} and Q_{20} have subqueries in their AQTs.

Listing 5. An uncorrelated subquery can be unnested

```
1 -- An uncorrelated subquery
2 select * from T where tfk not in
3         (select spk from S where s1 > 1)
4 -- The unnested query by antijoin
5 select * from T antijoin S on tfk=spk where s1
      > 1
```

For the first case, we parameterize the result of the subquery as a parameter p in the parameter query, and an example is provided in Example 13. Then we can deal with the subquery and the parent query as two separate AQTs. But after dealing with the two AQTs, the instantiated value p of the parent query may be different from the execution result r of the subquery on the generated data. Then we take an adjustment value $\delta(=p-r)$ in the subquery to make the result of the subquery consistent with the instantiated parameter value p of the parent query. The policy is suitable to Q_{11} and Q_{22} in TPC-H.

Example 13. *Consider a table T with only a non-key column t_1. There are three rows in the table, i.e., $1, 5, 6$. The query in Listing 4 contains an uncorrelated subquery returning a single row (an aggregated value). Usually, the database system executes $select\ avg(t_1)\ as\ t_{avg}\ from\ t$ firstly to get t_{avg}, i.e., 4, and then executes $select * from\ t\ where\ t_1 > 4$ (=t_{avg}) to get the results, i.e., $5, 6$.*

*After parameterizing the returned result of the subquery as p_1 in its parent query, the original query can be divided into two independent queries, which are 'select avg(t_1) as t_{avg} from T' and 'select * from T where $t_1 > p_1$'. There are only one cardinality constraint in the AQT of the parent query, i.e, $|\sigma_{t_1>p_1}(T)| = 2$. After dealing with the constraint, if $Mirage$ populates data $1, 2, 3$ for table T, it instantiates parameter p_1=1 to guarantee $|\sigma_{t_1>p_1}(T)| = 2$ on the generated data. However, the execution result of the subquery on the generated data is t_{avg}=2. We now have $t_{avg} \neq p_1$. Then, we initiate an adjustment value δ=$p_1 - t_{avg}$ = 1-2 = -1 for the subquery by revising it as 'select $avg(t_1) + \delta\ as\ t_{avg}\ from\ t$'. Finally, we can combine the two instantiated AQTs to generate an equivalent new revised query as following.*

```
1 select * from T where t1 >
2         (select avg(t1) −1 as tavg from T)
```

For the second case, the subquery can not be eliminated by its unnesting policy, which may be uncorrelated or correlated with the parent query. We present the solutions for them separately as follows.

Uncorrelated subquery. The example is shown in *line 2* of Listing 5 which is simplified from Q_{16} in TPC-H. We can unnest its subquery with an antijoin operator (query rewriting in *line 4* of Listing 5), which is taken by MySQL [34]. In $Mirage$, before processing the AQT containing uncorrelated subquery, it merges the plan of the uncorrelated subquery into its parent query plan and then deals with the merged AQT without a subquery. $Mirage$ can use the existed query rewriting rules [35] to launch the merge. For example, any subquery of the form '*IN (select * from * where *)*' or '*EXISTS (select * from * where *)*' is transformed into a left semi join; any negation subquery of the form '*NOT IN (select **

*from * where *)*' or '*NOT EXISTS (select * from * where *)*' is transformed into a left anti join, e.g., the *antijoin* in MySQL [34]. The policy can be applied to Q_{16} in TPC-H.

Correlated subquery. An example is shown in Listing 6, which is simplified from Q_2 in TPC-H. It is related to three tables, i.e., *part*, *lineitem*, *supplier*.

Its physical plan from PostgreSQL is visualized in Fig. 19. In the plan, It is divided into three execution stages. Firstly, PostgreSQL executes *Hash Join* between *part* and *lineitem*, which is the *plan before subquery*. Secondly, PostgreSQL invokes the correlated subquery as an extra filter for each output *p_partkey* from the first step hash join, which is the *plan of subquery*. Finally, PostgreSQL executes another Hash Join between the result of the subquery and *supplier*, which is *plan after subquery*. Obviously, *Mirage* can guarantee the cardinality constraints in the first stage. The difficulty is to ensure that the execution cost of the second stage is similar to the original query, and the cardinality from the second stage is the same with the original query so as to further guarantee the cost of the third stage.

Listing 6. A simplified query from Q_2 in TPC-H

```
select *
from part join lineitem on p_partkey =
    l_partkey
         join supplier on s_suppkey =
             l_suppkey
where p_size = 2
  and l_quantity <= (select min(l_quantity)
      as a
                     from lineitem
                     where l_partkey =
                         p_partkey)
  and s_acctbal < 2000;
```

To guarantee similar execution cost in the second stage of the subquery. Note that the scan in the correlated subquery is always the index-scan because of that its input from the first stage usually involves only a few rows. Therefore, each invocation to process the subquery is similar in cost. Since Mirage can guarantee the cardinality constraints in the first stage, the number of invocations to process the correlated subquery on the generated database can always be the same with the original one. Finally, the total execution cost of the second stage is similar to the original execution on the real database, which is only related to the cardinality from the first stage processing.

To guarantee the same output cardinality from the second stage. Since the framework of *Mirage* cannot support the subquery, *Mirage* then cannot control the cardinality from the second stage (i.e., from the subquery). Therefore, before dealing with the *AQT* with the correlated subquery, *Mirage* takes the way to replace the plan of the subquery with a semantically equivalent plan (as well as the equivalent cardinality constraint) without subquery. Note that the parameters in the re-written equivalent plan (in Fig. 20) for the subquery in the original real plan (in Fig. 19) always have the one-to-one correspondence. Notice that, *Mirage* can support the data generation and parameter instantiation on the re-

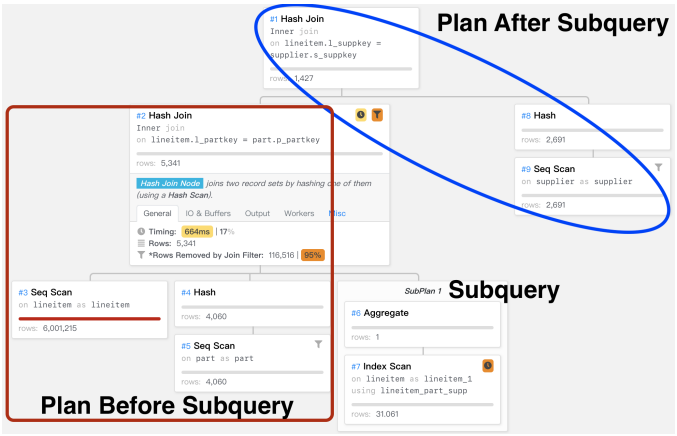


Fig. 19. Query plan of Listing 6

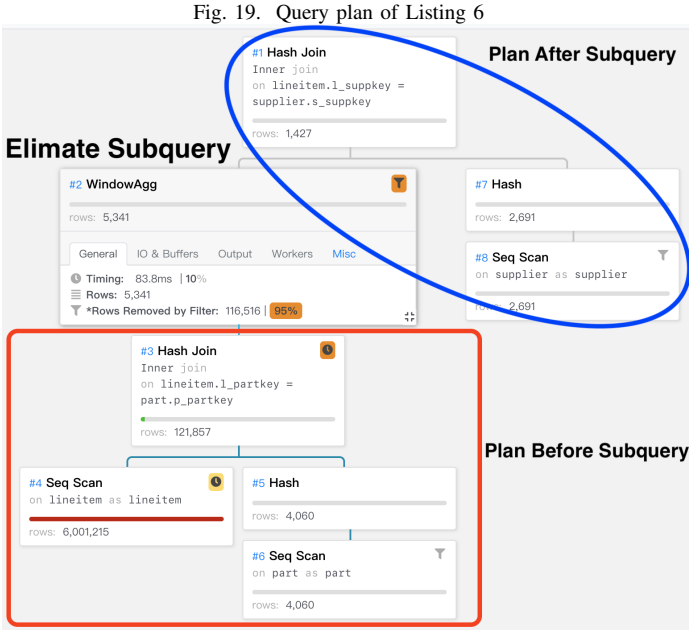


Fig. 20. Query plan without subquery of Listing 3

written query plan. By taking the semantically equivalent plan replacement, *Mirage* guarantees the same cardinality from the second stage. This kind of subquery rewriting has been widely studied [36], [37]. *Mirage* achieves the replacement with the help of the well-studied query rewriting technique. For example, we replace the plan of the subquery in Fig. 19 with a WindowAgg operator [36], which is presented in Fig. 20. *Mirage* supports WindowAgg because it is a physical operator of projection. Note that all the cardinality constraints of the *plan after subquery* can be well supported by *Mirage*. Since its input cardinality from the subquery is the same with the original one, we then achieve an accurate simulation for the query with correlated subquery. In TPC-H, we take this way to support Q_2, Q_{17} (by rule in work [36]) and Q_{20} (by rule in work [37]).

In summary, *Mirage* can support the generation with subqueries based on some well-studied rewriting policies. We can then transform and eliminate the subquery into an equivalent query plan without subquery, which can be successfully processed by *Mirage* but still guarantees the cardinality

constraints in the original query plan.

VIII. Related Work

Data-aware generators and query-aware generators are two types of synthetic database generation methods.

Data-aware generators [38]–[42] aim to generate a database closely related to the given data characteristics and the generation of a database instance is independent of workload. For example, *Alexander* [40] designs pseudo-random number generators and *Torlak* [38] uses kinds of multi-dimensional models. Although the generated database satisfies the given data distribution, the cardinality of each operator cannot be guaranteed [18]. **Query-aware generators** [13]–[19], [43] aim to generate a database that satisfies the cardinality constraints of required query plans. *QAGen* [13] is the first work for query aware database generation. It has powerful support in operators, but can only generate one database for one query at a time. Subsequent work focuses on query aware database generation of multiple queries, which has been proved *NP-complete* [14]. Then these works make a great effort to reduce the computational complexity rather than improve the support capability to operators. Following $QAGen$, $MyBenchmark$ [16] adopts a Query-Oriented Divide and Conquer solution. It first uses $QAGen$ to generate an individual database for each query and then tries its best to merge them. However, $MyBenchmark$ can not guarantee to generate one single database. *Touchstone* [19] then proposes to generate one database by launching a k-round of random distribution samples satisfying some specific rules and then choosing the one with the minimum error. However, Touchstone cannot sample for the cardinality constraints from arbitrary logical predicates or support semi-/anti-join. *DCGen* [14] and *Hydra* [19] transfer the generation task into multiple *Linear Programming* (LP) problems. However, the LP model cannot be adapted for the arithmetic filter operator, and outer-/semi-join.$Loki$ [43] works well only for solving the selection cardinality constraints. Compared to these works, *Mirage* can give the most powerful support to complex operators and guarantee error to be *zero* theoretically.

IX. Experiment

We conduct extensive experiments on *Mirage*. 1) To tell whether *Mirage* can conquer the state-of-the-art work in query aware database generation; 2) To expose the effectiveness of the technology designed for *Mirage*.

Database: Since all generators aim to guarantee the cardinality constraints of operators, we can take any database to present the results. Here, we select PostgreSQL (v.14.2) as the test database since it provides a convenient interface to check the cardinality constraints of tables and annotated query templates. The machine is equipped with 2 $\times$ Intel(R) Xeon(R) Gold 6240R CPU, 390 GB memory, and 2.5 TB disk.

Workloads: We compare *Mirage* with the two most recent work, i.e., *Touchstone* [18] and *Hydra* [19] with their provided codes. We select three classic benchmark scenarios, i.e., SSB [44], TPC-H [45] and TPC-DS [46], to provide the query plans and the cardinality constraints. SSB is a benchmark with 13 simple queries. TPC-H is the most popular OLAP benchmark with 22 queries having complex operators. In order to test the generation scalability, *Hydra* produces a scenario with large-scale workloads from TPC-DS, but removes its unsupported complex operators, e.g., *arithmetic* selections. In this workload, we take all the 100 distinct queries. Their official generation tools are used to compose the inputs of $Mirage$ [44]–[46]. Due to the slow data import speed of database, when comparing performance on the database, we take a small scale factor SF=1; for demonstrating the generation efficiency, a large default SF=200 is used.

Metrics: We adopt relative error, i.e., $\frac{\sum ||V_i|-|\hat{V}_i||}{\sum |V_i|}$ in [18] to measure the accuracy for query Q. $|V_i|$ represents the required cardinality of the i^{th} view V_i in query Q. $|\hat{V}_i|$ represents the cardinality of $\hat{V}_i$ in instantiated query $\hat{Q}$ on the generated database. $Relative\ error$ represents the cardinality deviation between Q and $\hat{Q}$, and the smaller is better.

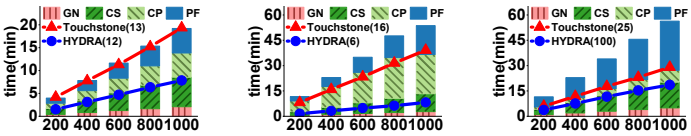
Setting: We take the sampling-based parameter instantiation for ACC. Its default sampling size is 4 million ($4M$) rows with theoretical error bound 0.1% in a confidence level 99.9%. In order to control memory usage, we use a batch generation strategy. Specifically, for each batch, we first generate non-key columns according to their CDF, and then populate foreign keys based on the join constraints whose values are scaled down according to the ratio between batch size and table size. The default batch size is set as $7M$ rows. Note that we use GN, CS, CP, and PF to represent the methods of generating non-key columns, computing status vectors, constraint programming solving, and populating foreign keys.

A. Comparison of Workload Support Ability and Fidelity

We first compare the complex workload support ability as well as the generation fidelity with the state-of-the-art work, i.e., *Touchstone* and *Hydra*. We plot the relative errors for the benchmark queries in Fig. 12. Note, if a query is not supported by a specific method, we mark its relative error as 100%. To plot the errors clearly for TPC-DS, we divide every 5 queries of TPC-DS into a group and show the result of each group.

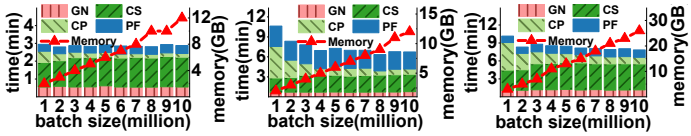
Touchstone can give full support to generate the simple SSB application as in Fig. 12a and a TPC-H application with its first 16 queries as in Fig. 12b. But it can not scale well with the number of queries, which can generate a TPC-DS application with only 25 queries as in Fig. 12c. For the lack of operator support, even for SSB, *Hydra* can not process Q_4 for generation and for TPC-H application, it can only generate an application with 6 out of all 22 queries. Even though *Hydra* can support all queries in its TPC-DS workload, it has a higher relative error than *Mirage*. *Mirage* gives a strong support to almost all queries in these three applications. For the generation fidelity, it performs the best. Only for TPC-H, sampling-based parameter instantiation of ACC introduces errors, which are around 1‰.

In summary, *Mirage* has the **widest** support of operators with **lower** errors and totally conquers its related work.



(a) SSB (b) TPC-H (c) TPC-DS

Fig. 21. Data Size vs. Generation Efficiency



(a) SSB (b) TPC-H (c) TPC-DS

Fig. 22. Batch Size vs. Generation Efficiency

B. Comparison of Generation Efficiency

The OLAP database usually has big volume of data, which requires the generation tool to have a high generation speed. We compare generation time of *Mirage* with *Touchstone* and *Hydra* in Fig. 21 by changing SF from 200 to 1000. Note that each tool only generates the application scenarios with its supported queries (size labeled on figures).

For fair comparisons, we only analyze the results with the same workload size. Specifically, for SSB, $Mirage$ is as fast as $Touchstone$; for TPC-DS, $Mirage$ is $2\times$ slower than *Hydra*. For TPC-H, though $Mirage$ is slower, it supports more queries in generation. So let us study the reason why $Mirage$ is slower than $Hydra$ on TPC-DS (in Fig. 21c). *Hydra* resolves all cardinality constraints as a linear programming problem (LP) and generates a small-sized initial dataset, which is simply duplicated for a large database. Therefore, it does not introduce new computation cost in duplicating, but its generated dataset has an extremely high duplication ratio. However, in order to ensure the diversity of dataset, $Mirage$ calculates CS, CP, and PF for each round of data population, which causes the slower generation speed. To reduce the time of population, $Mirage$ can take a similar way to generate a small dataset as $Hydra$. Take TPC-DS for example. $Mirage$ is about $2\times$ faster than $Hydra$, because $Mirage$ only puts the join constraints into a CP problem, but $Hydra$ models all the constraints into an LP problem.

In summary, *Mirage* **guarantees to generate data instead of duplicating** from a small-sized data. It is more practical.

C. Memory Consumption

Maintaining too much data dependency in memory, e.g., PK-FK dependency, has always bottlenecked the data generation efficiency [18], [19]. To balance generation performance and memory usage, *Mirage* can generate data in batch. Given a table, a large batch usually occupies more memory, but it decreases the total rounds of generations. Here, we illustrate the impact of batch size for generation by changing batch size from $1M$ to $10M$ rows in Fig. 22.

For a given workload, the generation time for GN, CS, and PF is only relevant to table size, which is stable. However, a larger batch size reduces the number of rounds to solve CP, which decreases generation time for CP. But the gain from CP decreases as the batch size increases. For example, the turning point is $4M$ rows for TPC-H, after which we obtain only a slender performance improvement from CP. Fig. 22 also shows the memory required for the generation is linearly related to the batch size. Since TPC-DS has wider tables, it consumes memory for data generation about twice as much as that of TPC-H or SSB. To trade off the overall generation efficiency against memory usage, taking the batch sized $7M$ rows (the default setting) almost uniformly reaches the peak performance for all benchmarks. In such a setting, the maximum memory used is no more than 28GB.

In summary, *Mirage* can reach its peak performance with **conservative memory** usages decided by the batch size.

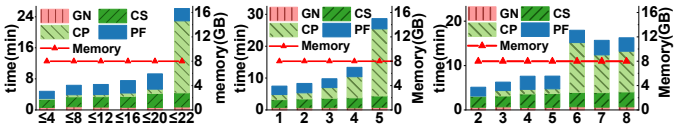
1) Generation Scalability: We demonstrate the generation scalability of $Mirage$ by varying the numbers of query instances, query templates, and involved join tables. We use TPC-H as the test workload and generate data with SF=200. To evaluate the efficiency of $Mirage$ under different numbers of query templates, we add query templates in TPC-H from 4 to 22 gradually and use 5 query instances for each query template (in Fig. 23a). To evaluate the efficiency of $Mirage$ under different numbers of query instances per template, we vary the number of query instances per template in TPC-H from 1 to 5 (in Fig. 23b). To evaluate the efficiency of $Mirage$ with different numbers of join tables, we divide the 22 Queries in TPC-H into 7 groups based on the join table accessing relationship. Specifically, queries in i^{th} group can only join with the tables involved in $1^{st} \sim (i-1)^{th}$ groups. We use TPC-H as the test workload and generate data with SF=200. We present the tables involved in query templates in Table IV. The query templates from left column to right column in Table IV have a partial order relationship based on the involved tables. That is any query template of the current column cannot involve any table right to it. For example, Q_{14} involves table $Part$ and $Lineitem$.

TABLE IV
THE QUERIES AND INVOLVED TABLES

Tables	Orders, Lineitem	Customer	Part	Nation	Supplier	Partsupp	Region
Queries	$Q_{1,4,6,12}$	$Q_{3,13,18,22}$	$Q_{14,17,19}$	Q_{10}	$Q_{7,15,21}$	$Q_{9,11,16,20}$	$Q_{2,5,8}$

Based on Table IV, we construct seven groups of workloads. In the i-th group, we randomly sample 100 query templates in the first i columns and then instantiate them, parameters of which are populated by the official tool [1] and different with each other. Therefore, the queries in the (i+1)-th group involve more join tables than those in the i-th group. In the i^{th} group, we randomly sample 100 query templates, which involve joins with at most i tables and the result is shown in Fig. 23c. Note, as we observed that the total time of generating non-key columns is relatively small, we do not show the running time of each step in the non-key generator. We also observe that GN, CS, and PF scale well with the increasing number of query templates and query instances per template. As the

[1]TPC-H Benchmark: http://www.tpc.org/tpch/



(a) Query Templates (b) Query Instances (c) Join Tables

Fig. 23. Generation Scalability of $Mirage$ Based on TPC-H

CP problem is NP-Hard, we observed that the time of CP increases super-linearly. In Fig. 23c, when the join involves more than 5 tables, the time of CP increases suddenly. Because more query templates will introduce exponential growth of the status vectors of $l\_orderkey$, i.e., the sharp increase of parameters in the CP solver. Furthermore, we can see that the memory consumption is quite stable, because of the batch-based data generation strategy to control the memory usage.

In summary, *Mirage* is **linearly scalable** with GN, CS and PF; CP is NP-Hard, which is **super-linearly scalable**.

X. Conclusion and Future Work

To optimize query parallelism techniques, we propose *Mirage*, a query-aware data generator that provides the most powerful support to complex OLAP workloads. It achieves data generation in a linear way with $zero$ error bound theoretically. Though *Mirage* performs the best among all the related work, it still could not cover all operators, e.g., non-PK-FK join, which will be left as our future work.

REFERENCES

[1] C. Chekuri, W. Hasan, and R. Motwani, "Scheduling problems in parallel query optimization," in *PODS*, 1995, pp. 255–265.

[2] K. Goda, Y. Hayamizu, H. Yamada, and M. Kitsuregawa, "Out-of-order execution of database queries," in *VLDB*, vol. 13, no. 12, 2020, pp. 3489–3501.

[3] L. Landgraf, F. Wolf, A. Boehm, and W. Lehner, "Memory efficient scheduling of query pipeline execution," in *CIDR*, 2022, pp. 9–12.

[4] I. Sabek, T. S. Ukyab, and T. Kraska, "Lsched: A workload-aware learned query scheduler for analytical database systems," in *SIGMOD*, 2022, pp. 1228–1242.

[5] B. Wagner, A. Kohn, and T. Neumann, "Self-tuning query scheduling for analytical workloads," in *SIGMOD*, 2021, pp. 1879–1891.

[6] J. Wang and M. Balazinska, "Toward elastic memory management for cloud data analytics," in *SIGMOD*, 2016, pp. 1–4.

[7] Z. Yu, Z. Bei, and X. Qian, "Datasize-aware high dimensional configurations auto-tuning of in-memory cluster computing," in *ASPLOS*, 2018, pp. 564–577.

[8] H. Mao, M. Schwarzkopf, S. B. Venkatakrishnan, Z. Meng, and M. Alizadeh, "Learning scheduling algorithms for data processing clusters," in *SIGCOMM*, 2019, pp. 270–288.

[9] J. Cieslewicz, J. Berry, B. Hendrickson, and K. A. Ross, "Realizing parallelism in database operations: insights from a massively multithreaded architecture," in *DaMoN*, 2006, pp. 4–es.

[10] S. Manegold, M. L. Kersten, and P. Boncz, "Database architecture evolution: Mammals flourished long before dinosaurs became extinct," in *VLDB*, vol. 2, no. 2, 2009, pp. 1648–1653.

[11] D. J. DeWitt, R. Gerber, G. Graefe, M. Heytens, K. Kumar, and M. Muralikrishna, "Gamma-a high performance dataflow database machine," University of Wisconsin-Madison Department of Computer Sciences, Tech. Rep., 1986.

[12] G. Graefe, "Volcano an extensible and parallel query evaluation system," in *TKDE*, vol. 6, no. 1, 1994, p. 120–135.

[13] C. Binnig, D. Kossmann, E. Lo, and M. T. Özsu, "QAGen: generating query-aware test databases," in *SIGMOD*, 2007, pp. 341–352.

[14] A. Arasu, R. Kaushik, and J. Li, "Data generation using declarative constraints," in *SIGMOD*, 2011, p. 685–696.

[15] A. Gilad, S. Patwa, and A. Machanavajjhala, "Synthesizing Linked Data Under Cardinality and Integrity Constraints," in *SIGMOD*, 2021, pp. 619–631.

[16] E. Lo, N. Cheng, W. W. Lin, W. K. Hon, and B. Choi, "MyBenchmark: generating databases for query workloads," in *VLDBJ*, vol. 23, no. 6, 2014, pp. 895–913.

[17] E. Lo, N. Cheng, and W.-K. Hon, "Generating databases for query workloads," in *VLDB*, vol. 3, no. 1–2, 2010, p. 848–859.

[18] Y. Li, R. Zhang, X. Yang, Z. Zhang, and A. Zhou, "Touchstone: Generating enormous query-aware test databases," in *USENIX ATC*, 2018, pp. 575–586.

[19] A. Sanghi, R. Sood, J. R. Haritsa, and S. Tirthapura, "Scalable and dynamic regeneration of big data volumes," in *EDBT*, 2018, pp. 301–312.

[20] F. Rossi, P. Van Beek, and T. Walsh, *Handbook of constraint programming*. Elsevier, 2006.

[21] L. Perron and V. Furnon, "Or-tools," Google, 2021. [Online]. Available: https://developers.google.com/optimization/

[22] S. J. Russell and P. Norvig, *Artificial intelligence a modern approach*. London, 2010.

[23] I. Müller, P. Sanders, A. Lacurie, W. Lehner, and F. Färber, "Cache-efficient aggregation: Hashing is sorting," in *SIGMOD*, 2015, p. 1123–1136.

[24] S. Macke, M. Aliakbarpour, I. Diakonikolas, A. Parameswaran, and R. Rubinfeld, "Rapid approximate aggregation with distribution-sensitive interval guarantees," in *ICDE*, 2021, pp. 1703–1714.

[25] Z. Miao, A. Lee, and S. Roy, "Lensxplain: Visualizing and explaining contributing subsets for aggregate query answers," in *VLDB*, vol. 12, no. 12, 2019, pp. 1898–1901.

[26] Y. Diao, P. Guzewicz, I. Manolescu, and M. Mazuran, "Efficient exploration of interesting aggregates in rdf graphs," in *SIGMOD*, 2021, pp. 392–404.

[27] X. Liang, S. Sintos, Z. Shang, and S. Krishnan, "Combining aggregation and sampling (nearly) optimally for approximate query processing," in *SIGMOD*, 2021, pp. 1129–1141.

[28] I. M. Copi, C. Cohen, and K. McMahon, *Introduction to logic*. Routledge, 2016.

[29] M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman & Co., 1990.

[30] W. Hoeffding, "Probability inequalities for sums of bounded random variables," in *The collected works of Wassily Hoeffding*, 1994, pp. 409–426.

[31] A. B. Kahn, "Topological sorting of large networks," in *Commun. ACM*, vol. 5, no. 11, 1962, p. 558–562.

[32] P. G. Selinger, M. M. Astrahan, D. D. Chamberlin, R. A. Lorie, and T. G. Price, "Access path selection in a relational database management system," in *SIGMOD*, 1979, pp. 23–34.

[33] S. Bellamkonda, R. Ahmed, A. Witkowski, A. Amor, M. Zait, and C.-C. Lin, "Enhanced subquery optimizations in oracle," *VLDB*, vol. 2, no. 2, pp. 1366–1377, 2009.

[34] M. Documentation. (2023) Mysql 8.0 anti-join. [Online]. Available: https://dev.mysql.com/blog-archive/antijoin-in-mysql-8/

[35] ——. (2023) Mysql 8.0 reference manual: Semi-joins. [Online]. Available: https://dev.mysql.com/doc/refman/8.0/en/semijoins.html

[36] C. Zuzarte, H. Pirahesh, W. Ma, Q. Cheng, L. Liu, and K. Wong, "Winmagic: Subquery elimination using window aggregation," in *SIGMOD*, 2003, pp. 652–656.

[37] P. Fent, A. Birler, and T. Neumann, "Practical planning and execution of groupjoin and nested aggregates," *The VLDB Journal*, pp. 1–26, 2022.

[38] E. Torlak, "Scalable test data generation from multidimensional models," in *SIGSOFT*, 2012, pp. 1–11.

[39] E. Shen and L. Antova, "Reversing statistics for scalable test databases generation," in *Proceedings of the Sixth International Workshop on Testing Database Systems*, 2013, pp. 1–6.

[40] A. Alexandrov, K. Tzoumas, and V. Markl, "Myriad: scalable and expressive data generation," in *VLDB*, vol. 5, no. 12, 2012, pp. 1890–1893.

[41] N. Bruno and S. Chaudhuri, "Flexible database generators," in *VLDB*, 2005, pp. 1097–1107.

[42] K. Houkjær, K. Torp, and R. Wind, "Simple and realistic data generation," in *VLDB*, 2006, pp. 1243–1246.

[43] P. Negi, L. Bindschaedler, M. Alizadeh, T. Kraska, J. Leeka, A. Gruenheid, and M. Interlandi, "Unshackling database benchmarking from synthetic workloads," in *ICDE*, 2023, pp. 3659–3662.

[44] P. O'Neil, E. O'Neil, X. Chen, and S. Revilak, "The star schema benchmark and augmented fact table indexing," in *TPCTC*, 2009, pp. 237–252.

[45] TPC-H, "TPC-H benchmark," http://www.tpc.org/tpch/, 1999.

[46] TPC-DS, "TPC-DS benchmark," http://www.tpc.org/tpcds/, 1999.