ΘΕΜΑ 2: Λογιστική παλινδρόμηση με Οφαλοποίηση.

α) Αν θ_j και $x_j^{(i)}$ είναι η j-οστή συνιστώσα των διανυσμάτων $\theta = [\theta_1, \theta_2, \dots, \theta_n]^T$ και $x^{(i)} = [x_1^{(i)}, x_2^{(i)}, \dots, x_n^{(i)}]^T$.

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{k} -y^{(i)} \ln(h_\theta(x^{(i)})) - (1-y^{(i)}) \ln(1-h_\theta(x^{(i)})) + \frac{\lambda}{2m} \sum_{j=1}^{n} \theta_j^2$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m} \frac{\partial}{\partial \theta_j} \Big(\overbrace{\sum -y^{(i)} \ln(h_\theta(x^{(i)})) - (1-y^{(i)}) \ln(1-h_\theta(x^{(i)}))}^{\text{Από } 1^{\text{ο}} \text{ θέμα}} \Big) + \frac{\partial}{\partial \theta_j} \left(\frac{\lambda}{2m} \sum_{j=1}^{n} \theta_j^2 \right)$$

$$= \frac{1}{m} \sum_{k=1}^{m} x_j^{(i)} \left(h_\theta(x^{(i)}) - y^{(i)} \right) + \frac{\lambda}{2m} \sum_{j=1}^{n} \frac{\partial}{\partial \theta_j} \theta_j^2$$

$$= \boxed{\frac{1}{m} \sum_{k=1}^{m} x_j^{(i)} \left(h_\theta(x^{(i)}) - y^{(i)} \right) + \frac{\lambda}{m} \theta_j}$$