ΘΕΜΑ 2: Σχεδιάσε έναν ταξινομητή LDA

$$\Sigma_w = P(w_1)\Sigma_1 + P(w_2)\Sigma_2$$

$$= \frac{1}{2}\begin{pmatrix} 11 & 9 \\ 9 & 11 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} \frac{13}{2} & \frac{9}{2} \\ \frac{9}{2} & \frac{13}{2} \end{pmatrix}$$

$$\det(\Sigma_w) = \left(\frac{13}{2}\right)^2 - \left(\frac{9}{2}\right)^2 = \frac{169}{4} - \frac{81}{4} = \frac{88}{4} = \underline{\underline{22}}$$

$$\Sigma_w^{-1} = \frac{1}{\det(\Sigma_w)}\begin{pmatrix} \frac{13}{2} & -\frac{9}{2} \\ -\frac{9}{2} & \frac{13}{2} \end{pmatrix} = \begin{pmatrix} \frac{13}{44} & -\frac{9}{44} \\ -\frac{9}{44} & \frac{13}{44} \end{pmatrix}$$

$$w = \Sigma_w^{-1}(\mu_1 - \mu_2) = \begin{pmatrix} \frac{13}{44} & -\frac{9}{44} \\ -\frac{9}{44} & \frac{13}{44} \end{pmatrix}\begin{pmatrix} -5-10 \\ 5-15 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{13}{44} & -\frac{9}{44} \\ -\frac{9}{44} & \frac{13}{44} \end{pmatrix}\begin{pmatrix} -15 \\ -10 \end{pmatrix} = \begin{pmatrix} -\frac{105}{44} \\ \frac{5}{44} \end{pmatrix}$$

Άρα $\boxed{w = \frac{1}{44}\begin{pmatrix} -105 \\ 5 \end{pmatrix}}$