ΘΕΜΑ 6°: Minimum risk

Έχουμε $l_1 = \lambda_{11} P(\omega_1) P(x|\omega_1) + \lambda_{21} P(\omega_2) P(x|\omega_2)$

$$\Rightarrow \boxed{l_1 = P(\omega_2) P(x|\omega_2)}$$

$$l_2 = \lambda_{12} P(\omega_1) P(x|\omega_1) + \lambda_{22} P(\omega_2) P(x|\omega_2)$$

$$\Rightarrow \boxed{l_2 = \frac{1}{2} P(\omega_1) P(x|\omega_1)}$$

όπου $L = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix} = \begin{pmatrix} 0 & 0,5 \\ 1 & 0 \end{pmatrix}$

- Αν $l_1 < l_2$ αποφασίζω ω_1
- Αν $l_2 < l_1$ αποφασίζω ω_2

} Το όριο x_0 βρίσκεται στο $l_1 = l_2$

$$l_1 = l_2$$

$$\Rightarrow P(\cancel{\omega_2}) P(x|\omega_2) = \frac{1}{2} P(\cancel{\omega_1}) P(x|\omega_1)$$

$$\Rightarrow \frac{\cancel{x_0}}{2^{\cancel{2}}} e^{-\frac{x_0^2}{2\cdot 4}} = \frac{1}{2} \cancel{x_0} e^{-\frac{x_0^2}{2}}$$

$x_0 \neq 0$

$$\Rightarrow \ln\left(\frac{1}{2} e^{-\frac{x_0^2}{8}}\right) = \ln\left(e^{-\frac{x_0^2}{2}}\right)$$

$$\Rightarrow -\frac{x_0^2}{8} - \ln 2 = -\frac{x_0^2}{2}$$

$$\Rightarrow -x_0^2 - 8\ln 2 = -4x_0^2$$

$$\Rightarrow 3x_0^2 = 8\ln 2$$

$$\Rightarrow \boxed{x_0 = \sqrt{\frac{8\ln 2}{3}}}$$