ΘΕΜΑ 3:

$D = \{x_1, x_2, \dots, x_n\}$ ανεξάρτητα από ~~...~~ $\quad p(x|\lambda) = \dfrac{\lambda^x e^{-\lambda}}{x!}$

Θέλω να εκτιμήσω το λ με MLE.

$$L_D(\lambda) = P(D|\lambda) = \prod_{k=1}^{n} p(x_k|\lambda)$$

~~...~~ $\quad \boxed{\lambda_{MLE} = \underset{\lambda}{\arg\max}\, P(D|\lambda)}$

$$\ell_D(\lambda) = \ln L_D(\lambda)$$
$$= \sum_{k=1}^{n} \ln p(x_k|\lambda)$$

Πρέπει $\dfrac{\partial}{\partial\lambda} \ell_D(\lambda) = 0$

$$\Rightarrow \frac{\partial}{\partial\lambda}\left(\sum_{k=1}^{n} \ln p(x_k|\lambda)\right) = 0$$

$$\Rightarrow \sum_{k=1}^{n} \frac{\partial}{\partial\lambda}\left(\ln \frac{\lambda^{x_k}\cdot e^{-\lambda}}{x!}\right) = 0$$

$$\Rightarrow \sum_{k=1}^{n} \frac{x!}{\lambda^{x_k} e^{-\lambda}} \cdot \frac{\partial}{\partial\lambda}\left(\frac{\lambda^{x_k}\cdot e^{-\lambda}}{x!}\right) = 0 \quad \Rightarrow$$

$$\Rightarrow \sum_{k=1}^{n} \frac{\cancel{x!}}{\lambda^{x_k} e^{-\lambda}} \cdot \frac{x_k \lambda^{x_k - 1} e^{-\lambda} - \lambda^{x_k} e^{-\lambda}}{\cancel{x!}} = 0$$

$$\Rightarrow \sum_{k=1}^{n} \frac{\cancel{\lambda^{x_k} \cdot e^{-\lambda}} \left(\frac{x_k}{\lambda} - 1 \right)}{\cancel{\lambda^{x_k} \cdot e^{-\lambda}}} = 0$$

$$\Rightarrow \sum_{k=1}^{n} \frac{x_k}{\lambda} - n = 0$$

$$\Rightarrow \frac{1}{\lambda} \sum_{k=1}^{n} x_k = n$$

$$\Rightarrow \boxed{\lambda_{MLE} = \frac{1}{n} \cdot \sum_{k=1}^{n} x_k}$$