Advanced SQL

05 — Window Functions

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Torsten Grust
Universität Tübingen, Germany

1 Window Functions

With SQL:2003, the ISO SQL Standard introduced **window functions**, a new mode of row-based computation:

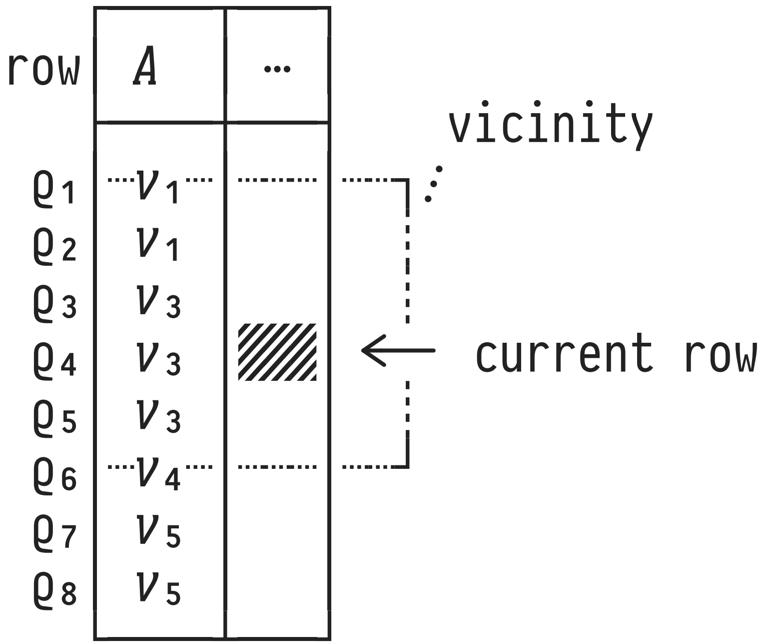
SQL Feature	Mode of Computation
function	row → row
table-generating function	row → table of rows
aggregate	group of rows → row (one per group)
window function ☜	row vicinity → row (one per row)

SQL Modes of Computation

Window functions …

- … are **row-based:** each individual input row r is mapped to one result row,
- … use the **vicinity** around r to compute this result row.

Row Vicinity: Window Frames



- Each row is the **current row** ▨ at one point in time.
- Row vicinity (**window**, **frame**) is based on either:

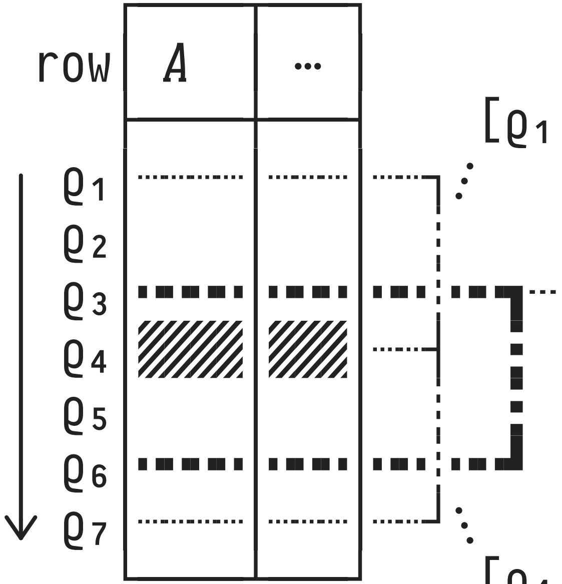
  ❶ row **position** (ROWS windows),
  ❷ row **values** v_i (RANGE windows),
  ❸ row **peers** (GROUPS windows).

- As the current row changes, the window *slides* with it.
- ⚠️ Window semantics depend on a defined **row ordering**.

Window Frame Specifications (Variant: ROWS)

window function ordering criteria frame specification

‹*f*› **OVER (ORDER BY** ‹e_1›,…,‹e_n› **[ROWS** ‹*frame*› **])**



‹*frame*›:

[ϱ1,…,ϱ4]: **BETWEEN UNBOUNDED PRECEDING AND CURRENT ROW**

[ϱ3,…,ϱ6]: **BETWEEN 1 PRECEDING AND 2 FOLLOWING**

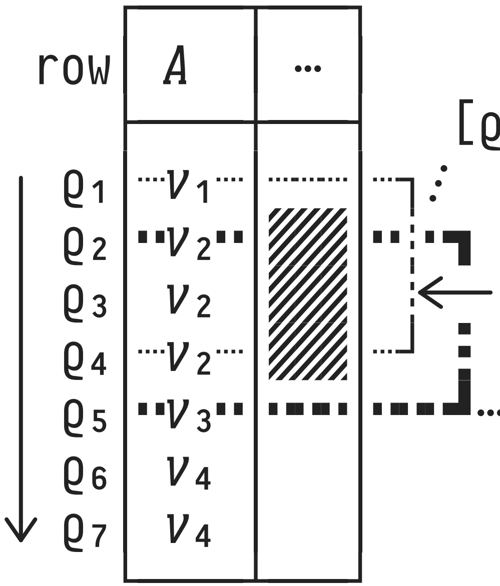
(distance from ▨ measured in number of rows)

[ϱ4,…,ϱ7]: **BETWEEN CURRENT ROW AND UNBOUNDED FOLLOWING**

Window Frame Specifications (Variant: RANGE)

window function — one column — frame specification

$$\underbrace{\langle f\rangle}_{\text{window function}}\ \textbf{OVER (ORDER BY}\ \underbrace{\langle A\rangle}_{\text{one column}}\ \textbf{[RANGE}\ \underbrace{\langle frame\rangle}_{\text{frame specification}}\ \textbf{])}$$



$\langle frame\rangle$:

$[\varrho_1,\dots,\varrho_4]$: **BETWEEN UNBOUNDED PRECEDING AND CURRENT ROW**

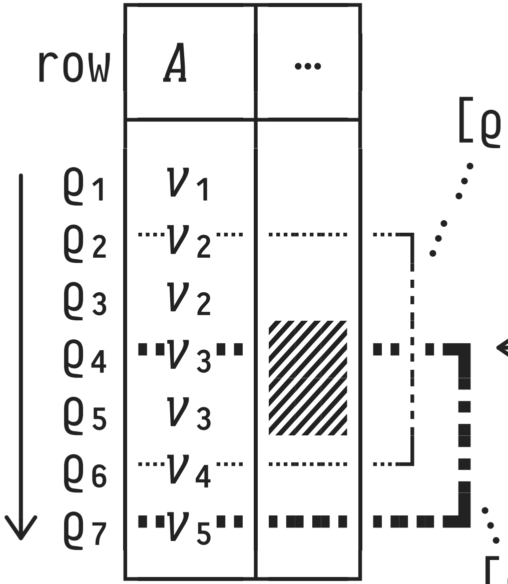
current row } peer group (▨, $A = v_2$) of current row

$[\varrho_2,\dots,\varrho_5]$: **BETWEEN CURRENT ROW AND 2 FOLLOWING**

(distance measured in terms of $|v_2 - v_i|$,
here: $v_3 - v_2 \leqslant 2$ and $v_4 - v_2 > 2$)

Window Frame Specifications (Variant: GROUPS)

$\overbrace{\langle f \rangle}^{\text{window function}}$ **OVER (ORDER BY** $\overbrace{\langle e_1 \rangle,\dots,\langle e_n \rangle}^{\text{ordering criteria}}$ **[GROUPS** $\overbrace{\langle \mathit{frame} \rangle}^{\text{frame specification}}$ **])**



$\langle \mathit{frame} \rangle$:

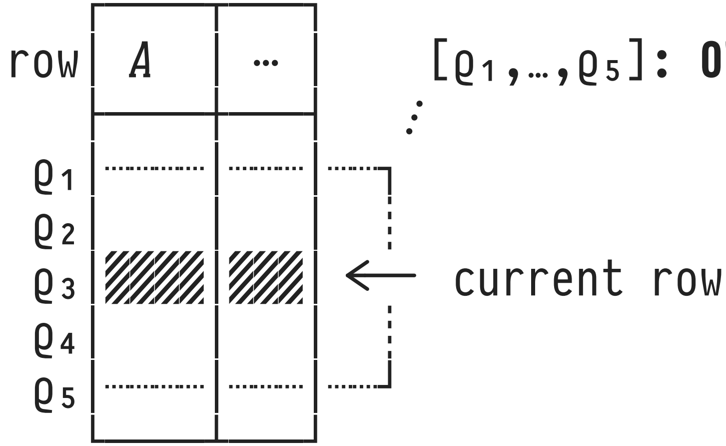
$[\varrho_2,\dots,\varrho_6]$: **BETWEEN 1 PRECEDING AND 1 FOLLOWING**

(distance measured in number of peer groups)

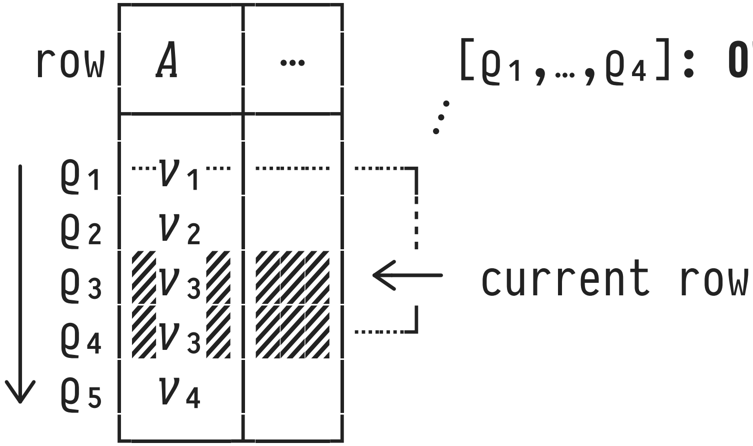
← current row } peer group (, $A = v_3$) of current row

$[\varrho_4,\dots,\varrho_7]$: **BETWEEN CURRENT ROW AND UNBOUNDED FOLLOWING**

Window Frame Specifications: Abbreviations



$[\varrho_1,\ldots,\varrho_5]$: **OVER ()** ≡ all rows form the window frame (for any current row)



$[\varrho_1,\ldots,\varrho_4]$: **OVER (ORDER BY ‹*A*›)** ≡

**OVER (ORDER BY ‹*A*›
RANGE BETWEEN UNBOUNDED PRECEDING
AND CURRENT ROW)**

WINDOW Clause: Name the Frame

Syntactic 🍭: If window frame specifications

1. become unwieldy because of verbose SQL syntax and/or
2. one frame is used multiple times in a query,

add a **WINDOW** clause to a SFW block to **name the frame**, *e.g.*:

```
SELECT … ‹f› OVER ‹w_i› … ‹g› OVER ‹w_j› …
FROM   …
WHERE  …
  ⋮
WINDOW ‹w_1› AS (‹frame_1›), …, ‹w_n› AS (‹frame_n›)
ORDER BY …
```

Use SQL Itself to Explain Window Frame Semantics

Regular **aggregates** may act as window functions *‹f›*. All **rows in the frame will be aggregated:**

```
SELECT w.row                          AS "current row",
       COUNT(*)           OVER win AS "frame size",
       array_agg(w.row) OVER win AS "rows in frame"
FROM   W AS w
WINDOW win AS (‹frame›)
```

row	a	b
ϱ_1	1	●
ϱ_2	2	○
ϱ_3	3	○
ϱ_4	3	●
⋮	⋮	⋮

Table W

Q: What is the Chance of Fine Weather on Weekends?

Input: Daily weather readings in sensors:

day	weekday	temp	rain
1	Fri	10	800
2	Sat	12	300
⋮	⋮	⋮	⋮

Table sensors

- The weather is fine on day ***d*** if—**on *d* and the two days prior**—the minimum temperature is above 15°C and the overall rainfall is less than 600ml/m².
- **Expected output:**

weekend?	% fine
f	29
t	43

2 ¦ **PARTITION BY**: Window Frames Inside Partitions

Optionally, we may **partition** the input table *before* rows are sorted and window frames are determined:

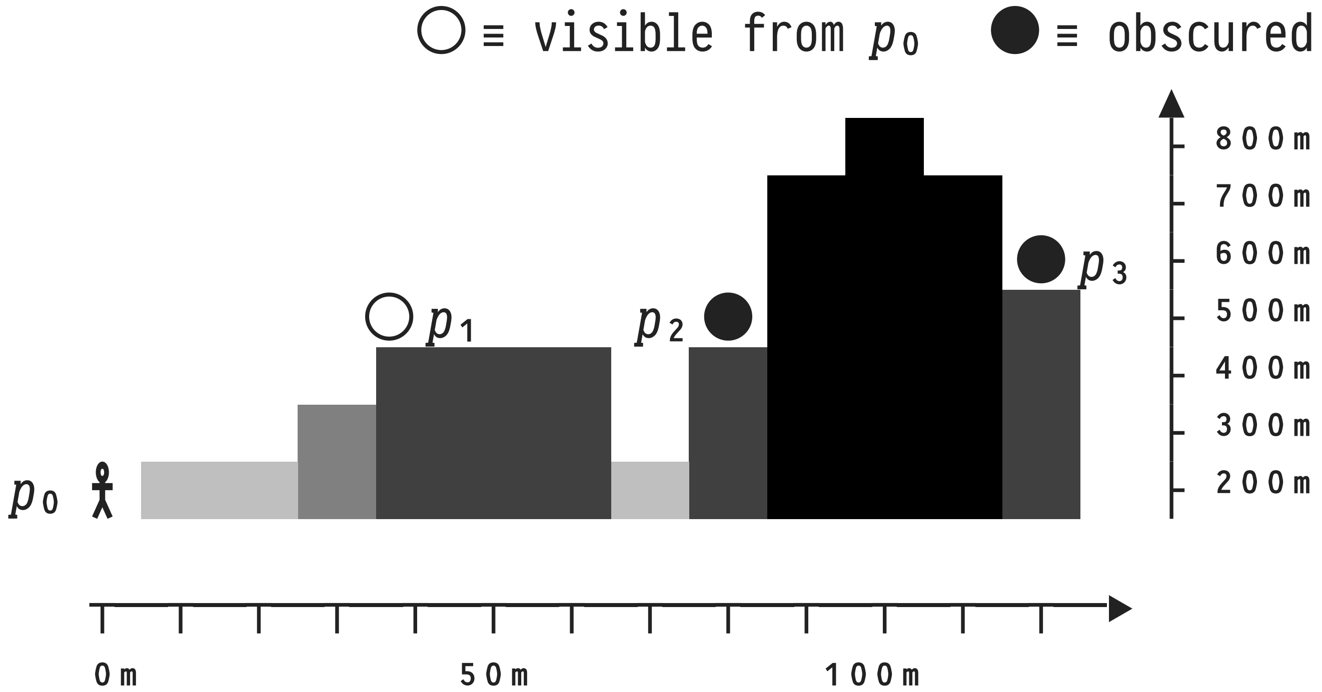
```
all input rows that agree on all ‹pi› form one partition

‹f› OVER ([ PARTITION BY ‹p1›,…,‹pm› ]
          [ ORDER BY ‹e1›,…,‹en› ]
          [ ‹frame› ])
```

- Note:

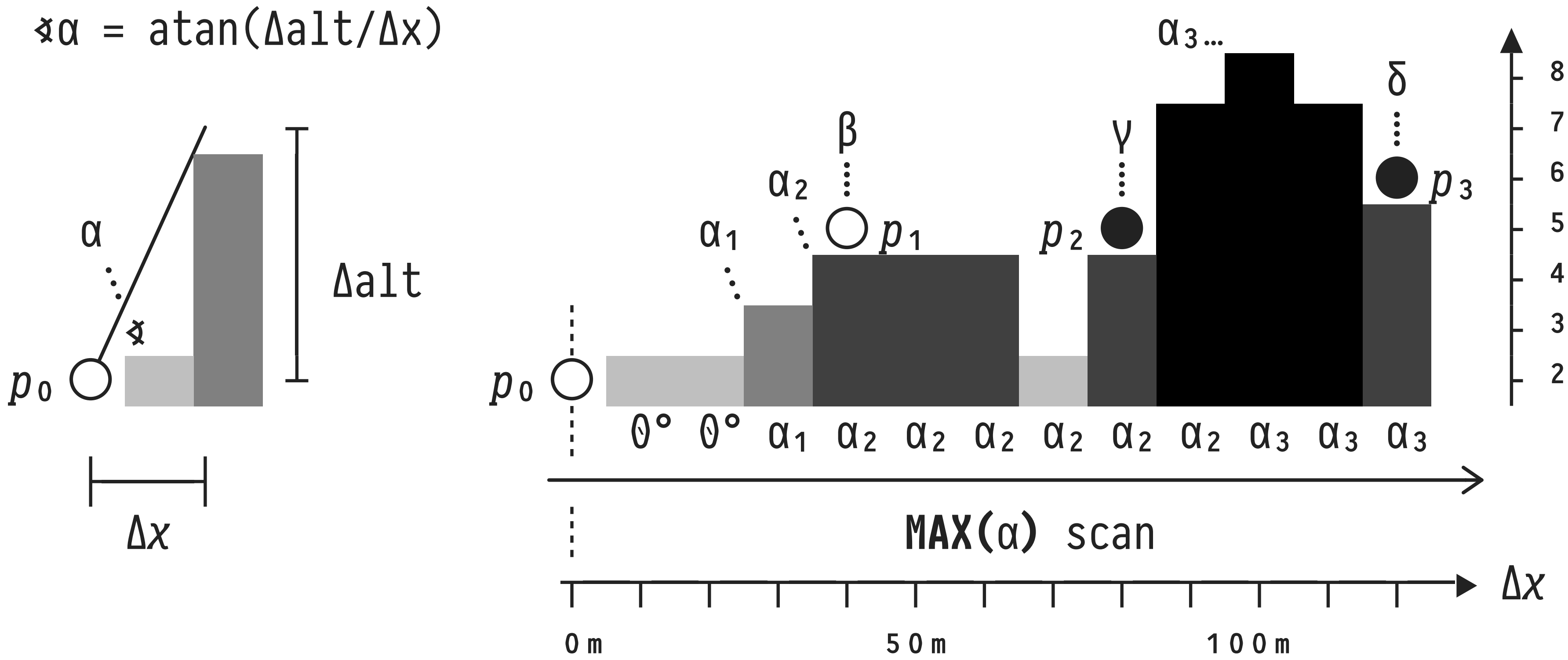
  1. Frames **never cross partitions.**
  2. BETWEEN ⋯ PRECEDING AND ⋯ FOLLOWING **respects partition boundaries.**

Q: Which Spots are Visible in a Hilly Landscape?



- From the viewpoint of p_0 (웃) we can see p_1, but…
  - … p_2 is **obscured** (no straight-line view from p_0),
  - … p_3 is **obscured** (lies behind the 800m peak).

Q: Visible Spots in a Hilly Landscape? — A: MAX Scan!



- We have $0° < \alpha_1 < \alpha_2 < \alpha_3$ and $\beta \geqslant \alpha_2$, $\gamma < \alpha_2$, $\delta < \alpha_3$.

  p_1 visible $p_{2,3}$ obscured

Q: Visible Spots in a Hilly Landscape? — A: MAX Scan!

- **Input:** Location of p_0 (here: x = 0) and 1D-map of hills:

x	alt
0	200
10	200
⋮	⋮
120	500

Table map

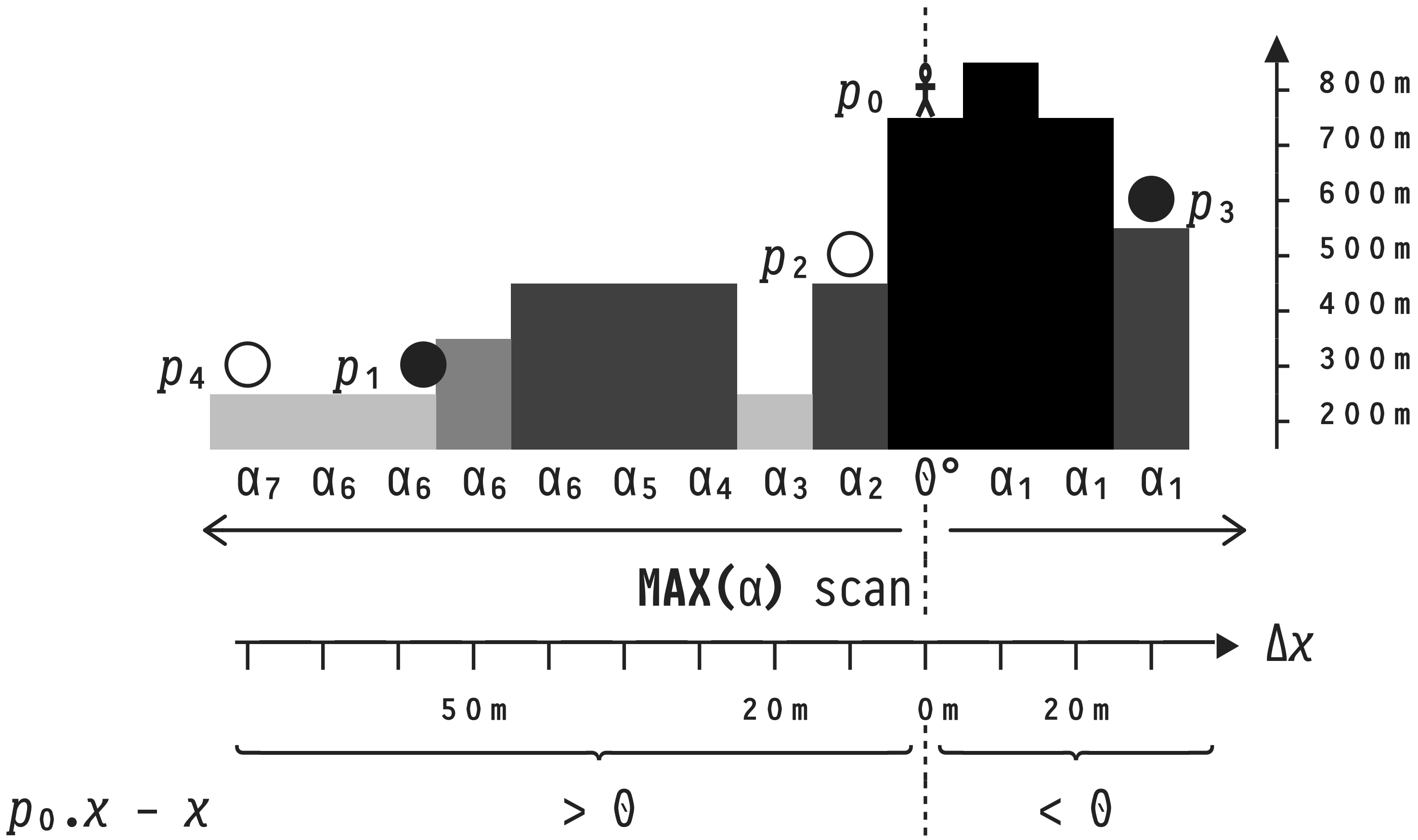
- **Output:** Can p_0 see the point on the hilltop at x?

x	visible?
0	true
10	true
⋮	⋮
120	false

Q: Visible Spots in a Hilly Landscape? — A: MAX Scan!

```
WITH
-- ❶ Angles α (in °) between p₀ and the hilltop at x
angles(x, angle) AS (
  SELECT m.x,
         degrees(atan((m.alt - p0.alt) /
                      abs(p0.x - m.x))) AS angle
  FROM   map AS m
  WHERE  m.x > p0.x),
-- ❷ MAX(α) scan (to the right of p₀)
max_scan(x, max_angle) AS (
  SELECT a.x,
         MAX(a.angle)
           OVER (ORDER BY abs(p0.x - a.x)) AS max_angle
  FROM   angles AS a),
⋮
```

Looking Left *and* Right: PARTITION BY



- Need MAX scans left *and* right of p_0 ⇒ use PARTITION BY.

Looking Left *and* Right: PARTITION BY

```
WITH
⋮
-- ❷ MAX(α) scan (left/right of p₀)
max_scan(x, max_angle) AS (
  SELECT a.x,                     --   ∊ {-1, 0 ,1}
         MAX(a.angle)             --  ⏞
           OVER (PARTITION BY sign(p0.x - a.x)
                 ORDER BY abs(p0.x - a.x)) AS max_angle
  FROM   angles AS a  --          ⏟
                      --       Δx > 0
),
⋮
```

- ∀a ∊ angles: a.x ≠ p0.x ⇒ We end up with **two** partitions.

3 ┆ Scans: Not Only in the Hills

Scans are a general and expressive computational pattern:

```
 ‹agg›(‹e›) OVER (ORDER BY ‹e₁›, …, ‹eₙ›
 ⏟               {ROWS,RANGE,GROUPS} BETWEEN
(ϕ,z,⊕)                UNBOUNDED PRECEDING AND CURRENT ROW)
```

- Available in a variety of forms in programming languages
  - Haskell: scanl z ⊕ xs, APL: ⊕\\xs, Python: accumulate:
    scanl ⊕ z $[x_1, x_2, \ldots] = [z,\ z \oplus x_1,\ (z \oplus x_1) \oplus x_2,\ \ldots]$
- In parallel programming: *prefix sums* (☞ Guy Blelloch)
  - Sorting, lexical analysis, tree operations, reg.exp. search, drawing operations, image processing, ...

4 ┆ Interlude: Quiz

Q: Assume *xs* ≡ '((b*2)-4×a×c)*0.5'. What is computed below?

```
SELECT inp.pos, inp.c,
       SUM((array[1,-1,0])[COALESCE(p.oc, 3)])
         OVER (ORDER BY inp.pos) AS d
FROM   unnest(string_to_array(xs, NULL))
         WITH ORDINALITY AS inp(c,pos),
       LATERAL (VALUES (array_position(array['(',')'],
                                       inp.c))) AS p(oc)
ORDER BY inp.pos;
```

Hint (this is the same query expressed in APL):

```
xs ← '((b*2)-4×a×c)*0.5'
+\ (1 ¯1 0)['()'⍳xs]
```

```
    window function
          ↓
        ‹f› OVER ([ PARTITION BY ‹p₁›,…,‹pₘ› ]
                  [ ORDER BY ‹e₁›,…,‹eₙ› ]
                  [ ‹frame› ])
```

Kinds of **window functions** ‹*f*›:

1. **Aggregates:** SUM(•), AVG(•), MAX(•), array_agg(•), … process all rows in the frame. ✓

2. **Row Access:** access row by *absolute/relative position* in ordered frame or partition: first/last/*n*th/*n* rows away.

3. **Row Ranking:** assign numeric *rank of row* in partition.

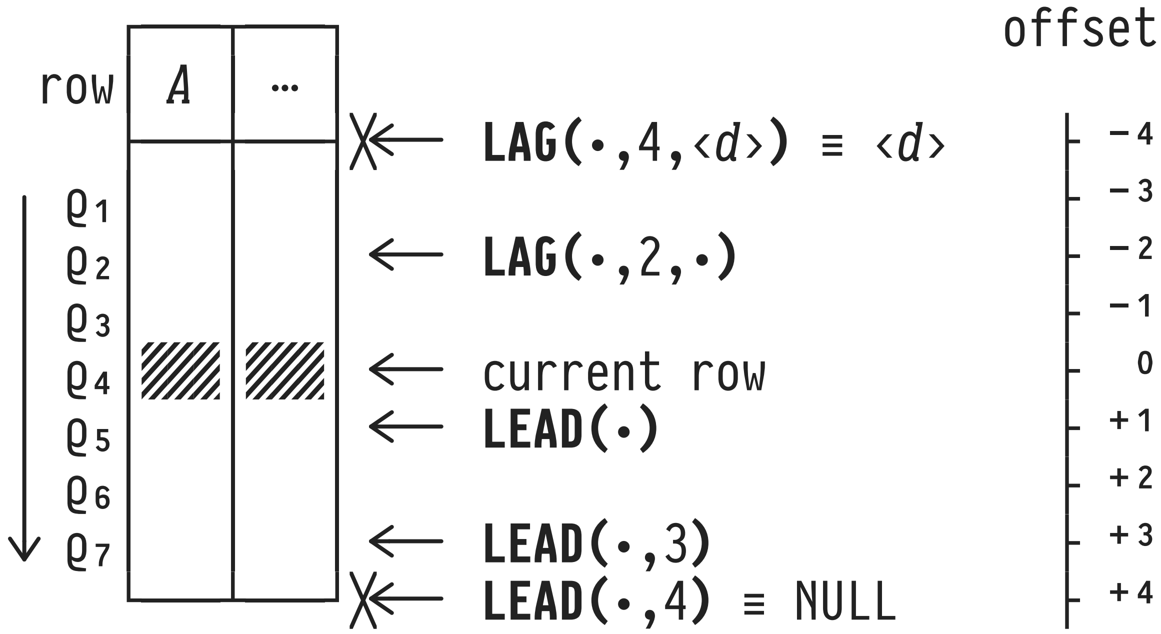
Row access at offset $\pm n$, relative to the current row:

```
-- evaluate e as if we were
-- n rows before the current row
--
     LAG(‹e›,‹n›,‹d›) OVER ([ PARTITION BY ‹p₁›,…,‹pₘ› ]
                               ORDER BY ‹e₁›,…,‹eₙ›)
```

Note:

- LEAD(‹e›,‹n›,‹d›): ⋯ n rows **after** the current row ⋯
- Scope is partition—no row access outside the partition.
- If there is no row at offset $\pm n$ ⇒ return value d.

LAG/LEAD: Row Offsets (Assume: No Partitioning, ORDER BY *A*)



- The frame of the current row is irrelevant for LAG/LEAD.
- Default parameters if absent: n ≡ 1, d ≡ NULL.

A March Through the Hills: Ascent or Descent?

```sql
SELECT m.x, m.alt,
       CASE sign(LEAD(m.alt, 1) OVER rightwards - m.alt)
            WHEN -1 THEN '↘' WHEN 1 THEN '↗'
            WHEN  0 THEN '→' ELSE '?'
       END AS climb,
       LEAD(m.alt, 1) OVER rightwards - m.alt AS "by [m]"
FROM   map AS m
WINDOW rightwards AS (ORDER BY m.x)  --  marching right
```

x	alt	climb	by [m]
0	200	→	0
⋮	⋮	⋮	⋮
90	700	↗	100
100	800	↘	-100
110	700	↘	-200
120	500	?	NULL

Crime Scene: Sessionization

A spy broke into the Police HQ computer system. A log records keyboard activity of user uid at time ts:

uid	ts
👮	05-25-2020 07:25:12
👮	05-25-2020 07:25:18
🕵	05-25-2020 08:01:55
🕵	05-25-2020 08:05:07
🕵	05-25-2020 08:05:30
👮	05-25-2020 08:05:39
🕵	05-25-2020 08:05:46

Table log

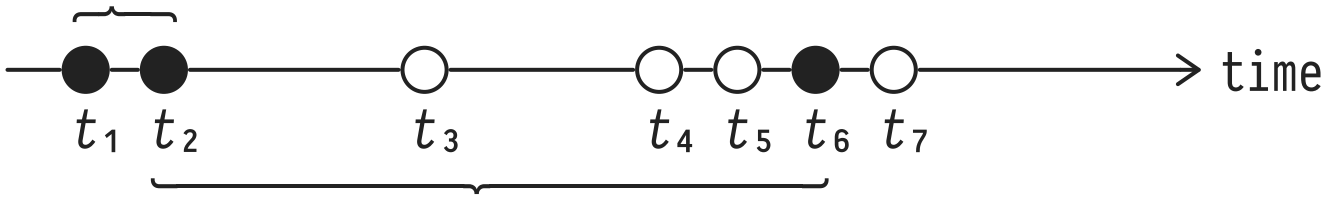
- **Q:** Can we **sessionize** the log so that investigators can identify *sessions* (≡ streaks of uninterrupted activity)?

🔧 Sessionization (Query Plan)

1. Cop and spy sessions happen independently (even if interleaved): partition `log` into 👮/● and 🕵/○ rows.

2. **Tag** keyboard activities (below: tagging for ●):

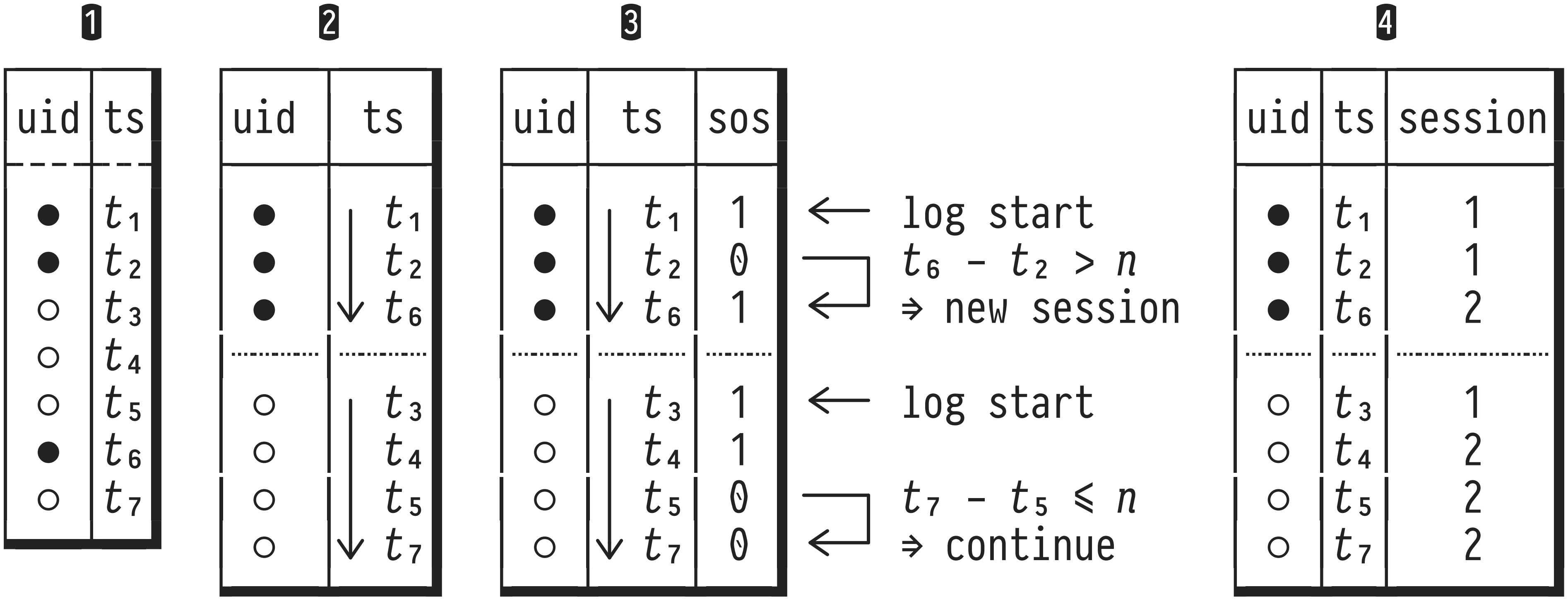
$t_2 - t_1 \leqslant n \Rightarrow$ continue session (tag t_2 with **0**)



$t_6 - t_2 > n \Rightarrow$ new session (tag t_6 with **1**)

3. **Scan** the tagged table and derive session IDs by maintaining a **runnning sum** of *start of session* tags.

Sessionization (Query Plan)



- At log start, always begin a new session (sos = 1).
- How to assign *global session IDs* (○'s sessions: 3, 4)?

Image Compression by Run-Length Encoding

Compress image by identifying pixel **runs** of the same color:

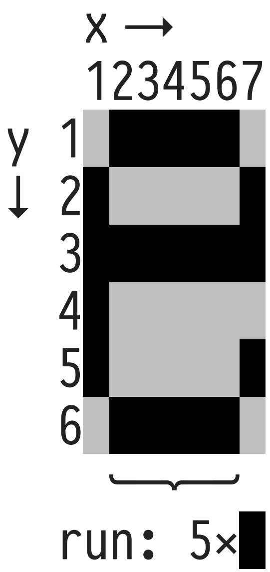


Table **original**

x	y	pixel
1	1	
2	1	
⋮	⋮	⋮
6	6	
7	6	

→

Table **encoding**

run	length	pixel
1	1	
2	5	
⋮	⋮	⋮
12	5	
13	1	

- Here: assumes a row-wise linearization of the pixel map.
- In b/w images we may omit column pixel in table encoding.

Run-Length Encoding (Query Plan)

❶

x	y	pixel	change?	
1	1		t	1
2	1		t	1
3	1		f	0
4	1		f	0
5	1		f	0
6	1		f	0
7	1		t	1
⋮	⋮	⋮	⋮	⋮

❷

x	y	pixel	change?	Σ change?
1	1		1	1
2	1		1	2
3	1		0	2
4	1		0	2
5	1		0	2
6	1		0	2
7	1		1	3
⋮	⋮	⋮	⋮	⋮

run #2 of length 5

❶: LAG(pixel,1,*undefined*): pixel @ (1,1) always “changes.”

❷: A SUM() scan of change? yields run identifiers.

7 ¦ FIRST_VALUE, LAST_VALUE, NTH_VALUE: In-Frame Row Access

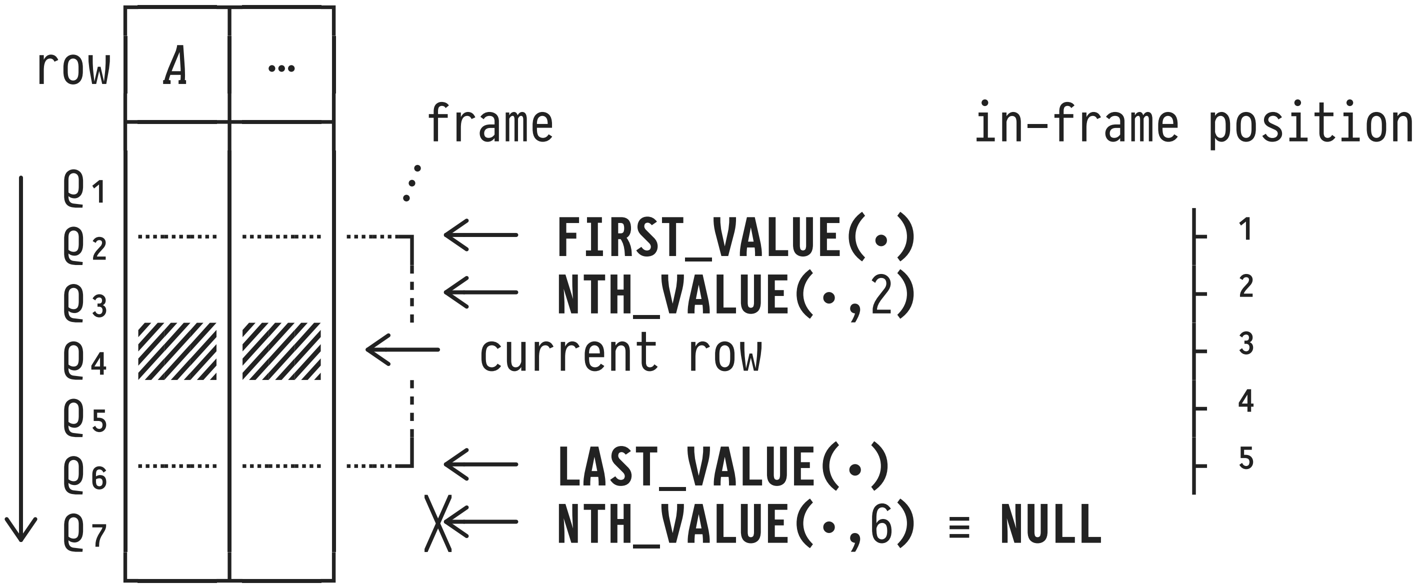
Aggregates reduce *all rows* inside a frame to a single value. Now for something different:

- **Positional access to individual rows** inside a frame is provided by three window functions:

```
-- evaluate expression e as if we were at
-- the first/last/nth row in the frame
--
          FIRST_VALUE(‹e›)
          LAST_VALUE(‹e›)          OVER (…)
          NTH_VALUE(‹e›,‹n›)
```

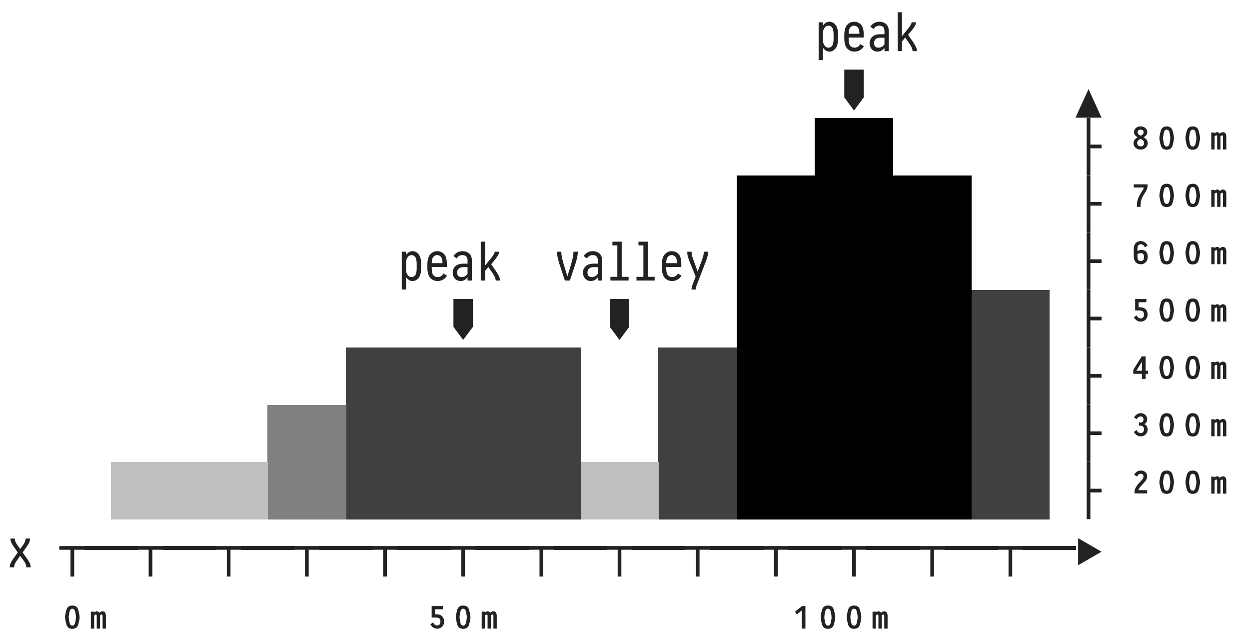
- NTH_VALUE(‹e›,‹n›): No n^{th} row in frame ⇒ return NULL.

In-Frame Row Access



- We have FIRST_VALUE(‹*e*›) ≡ NTH_VALUE(‹*e*›,1).

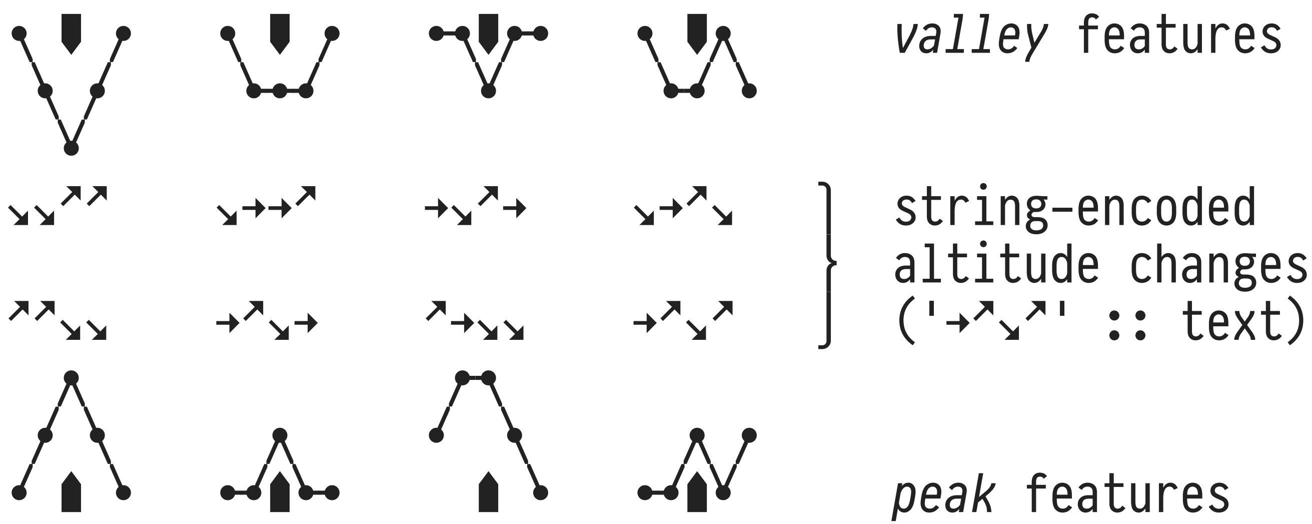
Detecting Landscape Features



- Detect features in hilly landscape. Attach label ∈ {peak,valley,-} to every location x.
- Feature defined by relative altitude change **in vicinity.**

Detecting Landscape Features (Query Plan)

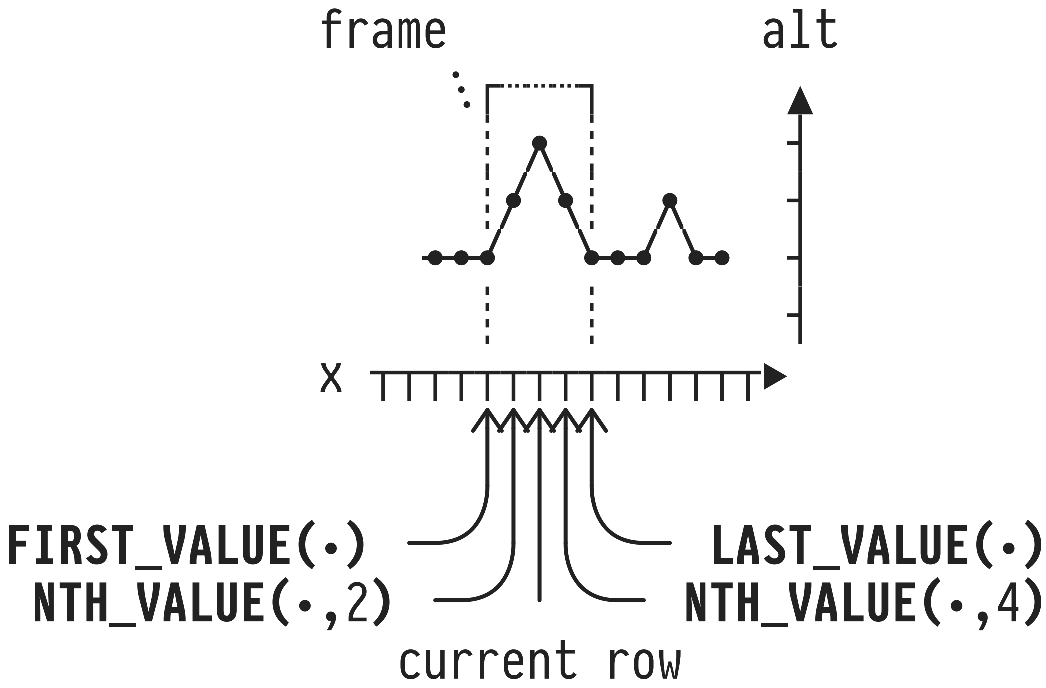
1. Track relative altitude changes in a sliding **x-window of size 5:**



2. **Pattern match** on change strings to detect features.

Altitude Changes in a Sliding Window

- Frame: ROWS BETWEEN 2 PRECEDING AND 2 FOLLOWING (5 rows):



- FIRST_VALUE(alt) < NTH_VALUE(alt,2) ⇒ ascent ('↗').

Altitude Changes in a Sliding Window

```
-- Find slopes in ±2 vicinity around point x
SELECT m.x, slope(
 sign(FIRST_VALUE(m.alt) OVER w-NTH_VALUE(m.alt,2) OVER w),
 sign(NTH_VALUE(m.alt,2) OVER w-m.alt                    ),
 sign(m.alt                   -NTH_VALUE(m.alt,4) OVER w),
 sign(NTH_VALUE(m.alt,4) OVER w-LAST_VALUE(m.alt)  OVER w)
)
FROM   map AS m
WINDOW w AS (ORDER BY m.x ROWS BETWEEN 2 PRECEDING
                                   AND 2 FOLLOWING)
```

- Recall: 1D landscape represented as table `map(x,alt)`.
- UDF encodes altitude changes: `slope(-1,-1,0,1) ≡ '↘↘→↗'`.

Row Pattern Matching (SQL:2016)

SQL:2016 introduced an entirely new SQL construct, **row pattern matching** (MATCH_RECOGNIZE):

1. ORDER BY: Order the rows of a table.
2. DEFINE: Tag rows that satisfy given predicates.
3. PATTERN: Specify a **regular expression over row tags**, find matches in the ordered sequence of rows.
4. MEASURES: For each match, evaluate expressions that measure its features (matched rows, length, …).

- ⚠️ As of May 2020, not supported by . Currently implemented in Oracle® only.

Row Pattern Matching (SQL:2016)

```
SELECT *
FROM   map
MATCH_RECOGNIZE (
 ORDER BY x
 MEASURES FIRST(x,1)     AS x,
          MATCH_NUMBER() AS feature,
          CLASSIFIER()   AS slope
 ONE ROW PER MATCH
 AFTER MATCH SKIP TO NEXT ROW
 PATTERN (((DOWN DOWN|DOWN EVEN|UP DOWN|EVEN DOWN)…)
 DEFINE UP   AS UP.alt   > PREV(UP.alt),     --
        DOWN AS DOWN.alt < PREV(DOWN.alt),   --  row tags
        EVEN AS EVEN.alt = PREV(EVEN.alt)    --
)
```

Output

x	feature	slope
50	1	DOWN
70	2	UP
100	3	DOWN

Countless problem scenarios involve the **number** (position) or **rank** of the current row in an *ordered sequence* of rows.

- Family of **window functions to number/rank rows:**

```
ROW_NUMBER()      ⎫
                  ⎪        --  intra-partition ranking ✓
DENSE_RANK()      ⎪        --
RANK()            ⎬  OVER ([ PARTITION BY ‹p₁›,…,‹pₘ› ]
                  ⎪          [ ORDER BY ‹e₁›,…,‹eₙ› ] )
PERCENT_RANK()    ⎪        --
CUME_DIST()       ⎪        -- ranking w/o ORDER BY ↯
NTILE(‹n›)        ⎭
```

- Scope is partition (if present)—*‹frame›* is irrelevant.

Numbering and Ranking Rows — ‹*f*›() OVER (ORDER BY *A*)

Table W | *f*

row	*A*	ROW_NUMBER	DENSE_RANK	RANK
ϱ1	1	1	1	1
ϱ2	2	2	2	2
ϱ3	**3**	3	3	3
ϱ4	**3**	4	3	3
ϱ5	**3**	5	3	3
ϱ6	4	6	4	6
ϱ7	**6**	7	5	7
ϱ8	**6**	8	5	7
ϱ9	7	9	6	9

┄ Rows that agree on
┄ the sort criterion
┄ (here: *A*) …

- … number randomly
- … rank equally

⌶ Mind the ranking gap (think Olympics)

- In general: DENSE_RANK() ⩽ RANK() ⩽ ROW_NUMBER()

Once More: Find the Top n Rows in a Group

species	length	height	legs
⋮	⋮	⋮	∊ {2,4,NULL}

Table dinosaurs

```
SELECT tallest.legs, tallest.species, tallest.height
FROM (SELECT d.legs, d.species, d.height,
ROW_NUMBER()·····RANK() OVER (PARTITION BY d.legs
                              ORDER BY d.height DESC) AS n
      FROM  dinosaurs AS d
      WHERE d.legs IS NOT NULL) AS tallest
WHERE n <= 3
```

- RANK() *vs* ROW_NUMBER(): both OK, but different semantics!
- Need a subquery: window functions *not* allowed in WHERE.

Identify Consecutive Ranges

- What you often encounter in scientific papers :
  “... *as Knuth has shown in [5,2,14,3,1,42,6,10,7,13]* ...”

- What you want to see :
  “... *as Knuth has shown in [1–3,5–7,10,13&14,42]* ...”

Table **citations**

ref
5
2
⋮
13

Output

ref	range
1	r_0
2	r_0
⋮	⋮
42	r_4

references belong to the same range

Identify Consecutive Ranges (Query Plan)

❶

ref
5
2
14
3
1
42
6
10
7
13

❷

ref		ROW_NUMBER()			
1	−	1	=	0	range 0 $\equiv r_0$
2	−	2	=	0	
3	−	3	=	0	
5	−	4	=	1	range 1 $\equiv r_1$
6	−	5	=	1	
7	−	6	=	1	
10	−	7	=	3	range 3 $\equiv r_2$
13	−	8	=	5	range 5 $\equiv r_3$
14	−	9	=	5	
42	−	10	=	32	range 32 $\equiv r_4$

subtract

Numbering and Ranking Rows — ‹*f*› OVER (ORDER BY *A*)

row	*A*	PERCENT_RANK	CUME_DIST	NTILE(3)
ϱ1	1	0	$^1/_9$	1
ϱ2	2	$^1/_8$	$^2/_9$	1
ϱ3	**3**	$^2/_8$	$^5/_9$	1
ϱ4	**3**	$^2/_8$	$^5/_9$	2
ϱ5	**3**	$^2/_8$	$^5/_9$	2
ϱ6	4	$^5/_8$	$^6/_9$	2
ϱ7	**6**	$^6/_8$	$^8/_9$	3
ϱ8	**6**	$^6/_8$	$^8/_9$	3
ϱ9	7	$^8/_8$	$^9/_9$	3

Rows that agree on the sort criterion (here: ***A***) rank equally

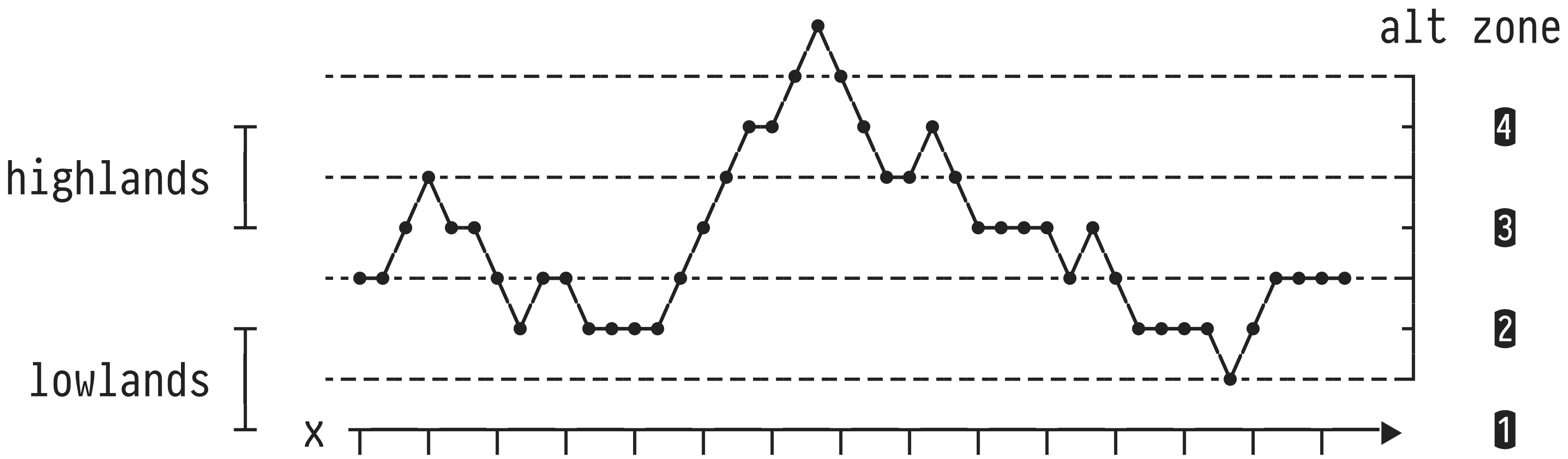
current row is in the *n*th of 3 chunks of rows

n% of the other rows rank lower than the current row

the current row and lower ranked rows make up *n*% of all rows

Altitudinal Mountain Zones

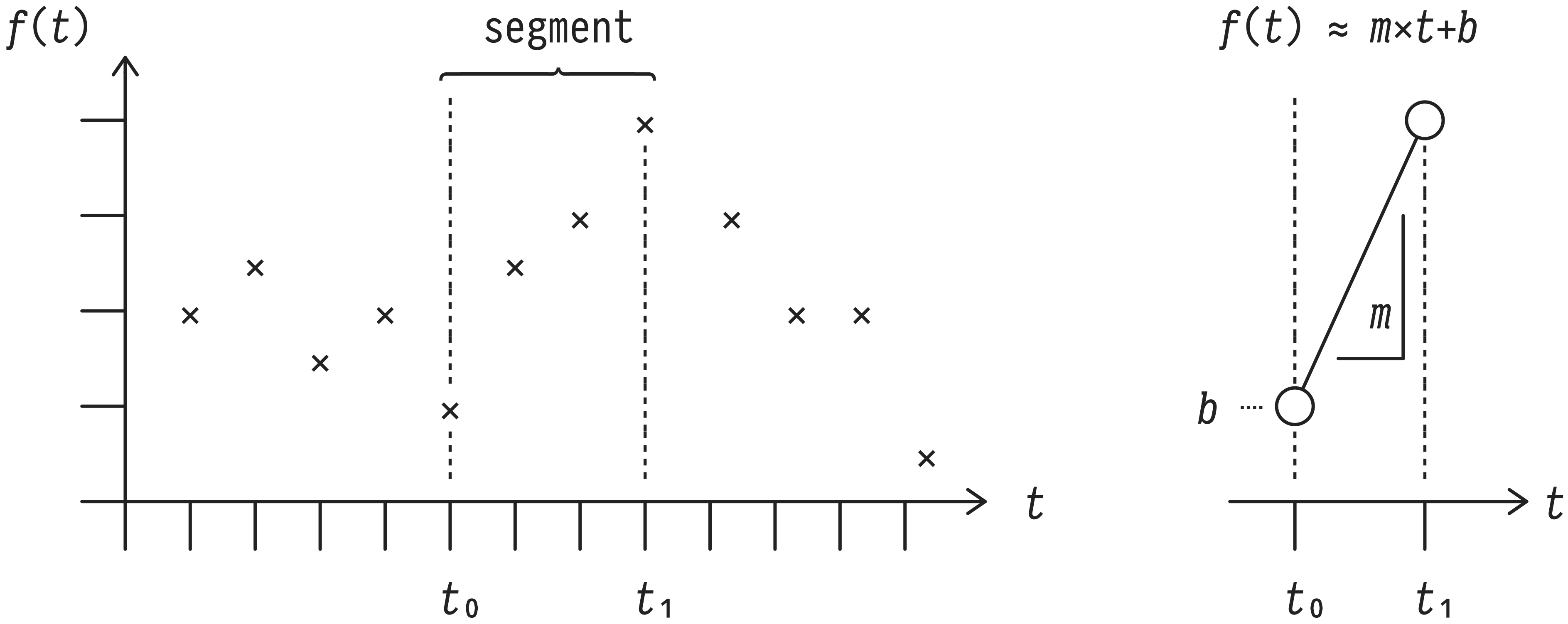
- Classify the altitudes of a mountain range into
  1. **equal-sized** vegetation **zones** and
  2. lowlands (altitude in the lowest **20%**) and highlands (between **60%–80%** of maximum altitude).



🔧 Altitudinal Mountain Zones

```sql
-- Classify altitudinal zones in table mountains(x, alt)
--
SELECT
  m.x, m.alt,
  NTILE(4) OVER altitude AS zone,
  CASE
    WHEN PERCENT_RANK() OVER altitude BETWEEN 0.6 AND 0.8
         THEN 'highlands'
    WHEN PERCENT_RANK() OVER altitude < 0.2
         THEN 'lowlands'
         ELSE '-'
  END AS region
FROM   mountains AS m
WINDOW altitude AS (ORDER BY m.alt)
ORDER BY m.x;
```

Linear Approximation of a Time Series



1. NTILE(‹*n*›) **segments** time series at desired granularity.
2. Compute *m*, *b* in each **segment ≡ window frame.**

9 Summary: Window Function Semantics[1]

Scope	Computation	Function	Description
frame	aggregation	(aggregates)	SUM, AVG, MAX, array_agg, …
	row access	FIRST_VALUE(*e*)	*e* at first row in frame
		LAST_VALUE(*e*)	*e* at last row in frame
		NTH_VALUE(*e*,*n*)	*e* at n^{th} row in frame
partition	row access	LAG(*e*,*n*,*d*)	*e* at *n* rows *before* current row
		LEAD(*e*,*n*,*d*)	*e* at *n* rows *after* current row
	ranking	ROW_NUMBER()	number of current row
		RANK()	rank with gaps (“Olympics”)
		DENSE_RANK()	rank without gaps
		PERCENT_RANK()	relative rank of current row
		CUME_DIST()	ratio of rows up to –"–
		NTILE(*n*)	rank on a scale {1,2,…,*n*}

[1] FIRST_VALUE(*e*): expression *e* will be evaluated *as if* we are at the first row in the frame.
LAG(*e*,*n*,*d*): default expression *d* is returned if there is no row at offset *n* before the current row.