Advanced SQL

04 — Lists and Table-Generating Functions

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1 Lists: Aliens(?) Inside Table Cells

SQL tables adhere to the **First Normal Form** (1NF): values v inside table cells are *atomic* w.r.t. the tabular data model:

...	A	...
...	v	...

Let us now discuss the **list** data type:

- v may hold an ordered list of elements $[x_1,...,x_n]$.
- SQL treats v as an atomic unit, but ...
- ... list functions and operators also enable SQL to query the x_i individually (still, that's no ↯ with 1NF).

2 List Types

- For type τ, τ[] is the type of **homogeneous lists of elements of τ.**
  - τ may be built-in or user-defined (enums, row types).
  - List size is unspecified—the list is dynamic. (DuckDB also implements $\tau[n]$, the type of **arrays of fixed length** n.)

…	*A* :: int[]	…
…	[10,20,30]	…
…	[30,20,10]	…
…	[]	…
…	[NULL]	…

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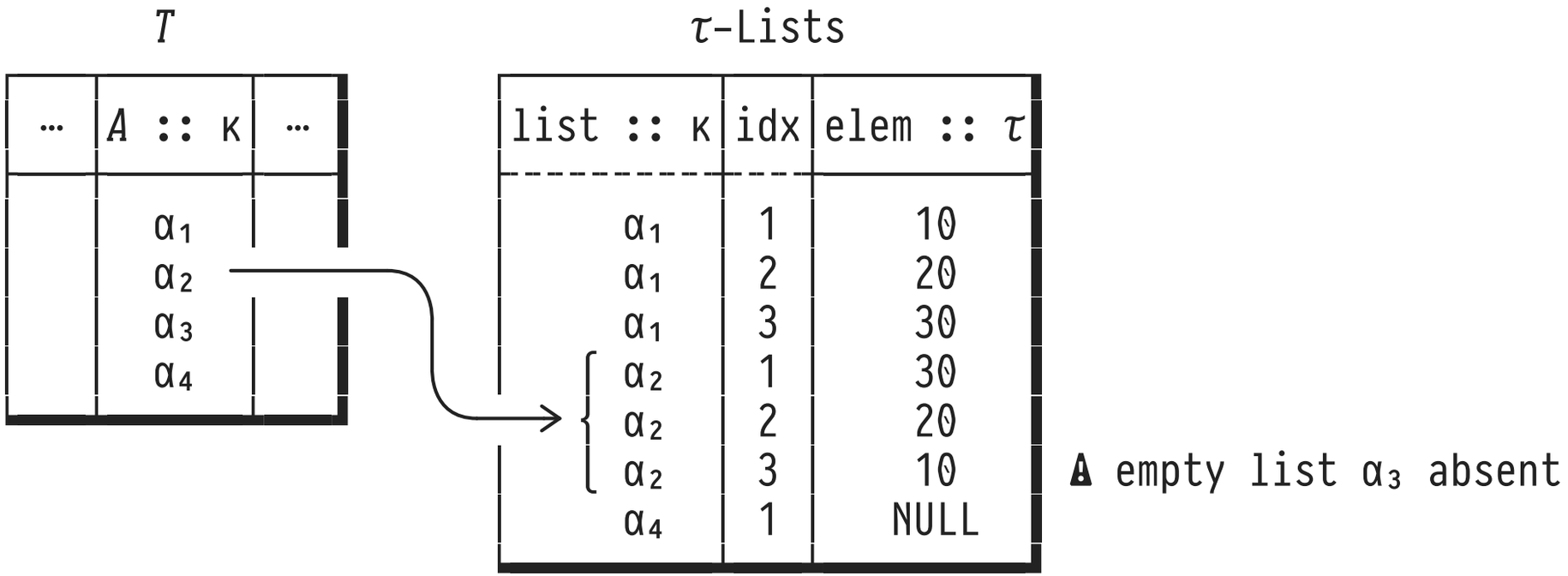
“Simulating” Lists (Tabular List Semantics)



- κ denotes a suitable key data type.
- List indexes are of type int and 1-based.

3 List Literals

One-dimensional list literals of type τ[]:

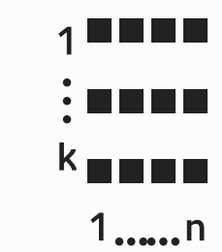
`[] :: τ[]`	empty list of elements of type τ
`[`x_1`,…,`x_n`]` `list_value(`x_1`,…,`x_n`)`	} all x_i of type τ

Multi-dimensional list literals of type τ[][]:

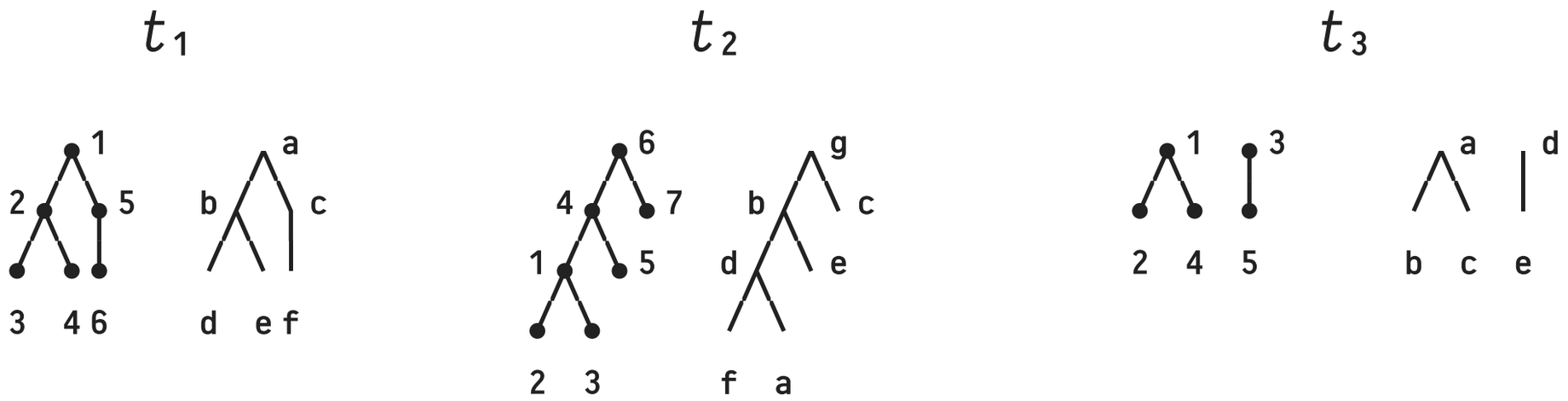
`[[`x_{11}`],[],[`x_{31}`,`x_{32}`]]` (ragged)

matrix: all sub-lists agree in size

`[[`x_{11}`,…,`x_{1n}`],…,[`x_{k1}`,…,`x_{kn}`]]`



Example: Tree Encoding (parents[i] ≡ parent of node i)



Tree shape and node labels held in separate in-sync lists:

tree	parents	labels
t_1	[NULL,1,2,2,1,5]	['a','b','d','e','c','f']
t_2	[4,1,1,6,4,NULL,6]	['d','f','a','b','e','g','c']
t_3	[NULL,1,NULL,1,3]	['a','b','d','c','e']
	1 2 3 4 5	1 2 3 4 5 ← index i

Constructing Lists

- **Append/prepend** element ※ to list or
- **concatenate** lists:

$$
\begin{aligned}
&\texttt{list\_append}\ ([x_1,\ldots,x_n],※) && \equiv && [x_1,\ldots,x_n,※]\\
&\texttt{list\_prepend}(※,[x_1,\ldots,x_n]) && \equiv && [※,x_1,\ldots,x_n]\\
&\\
&\texttt{list\_concat}([x_1,\ldots,x_n],\ [y_1,\ldots,y_m]) && \equiv && [x_1,\ldots,x_n,y_1,\ldots,y_m]\\
&\qquad\qquad [x_1,\ldots,x_n]\ \texttt{||}\ [y_1,\ldots,y_m] && \equiv && [x_1,\ldots,x_n,y_1,\ldots,y_m]
\end{aligned}
$$

- Academics: “*List type τ[] forms a monoid (τ[], ||, [])*
  with commutative operation || and neutral element [].”

Accessing List Elements: Indexing / Slicing

- List **indexes** i are 1-based (let $xs \equiv [x_1, x_2, \ldots, x_n]$):

$$
\begin{array}{rll}
xs[i] & \equiv x_i & (1 \leqslant i \leqslant n) \\
(\text{NULL})[i] & \equiv \text{NULL} & \\
xs[\text{NULL}] & \equiv \text{NULL} & \\
xs[i{:}j] & \equiv [x_i, \ldots, x_j] & (i > j\text{: } []) \\
xs[i{:}\ ] & \equiv [x_i, \ldots, x_n] & \\
xs[\ {:}j] & \equiv [x_1, \ldots, x_j] &
\end{array}
$$

- Access the last element / from the list back:

$$
\begin{array}{lll}
xs[\text{len}(xs)] & \equiv x_n & \\
xs[-i] & \equiv x_{n-(i-1)} & (1 \leqslant i \leqslant n)
\end{array}
$$

Searching for Elements in Lists

Indexing accesses lists by position. **Searching** accesses list by **contents**, instead.

- Let $xs \equiv [x_1,\ldots,x_{i-1},\divideontimes,x_{i+1},\ldots,x_{j-1},\divideontimes,x_{j+1},\ldots,x_n]$ and comparison operator $\theta \in \{=,<,>,<>,<=,>=\}$:

$x\ \theta\ \text{ANY}(xs)$	$\equiv$	$\exists\ i \in \{1,\ldots,n\}: x\ \theta\ xs[i]$
$x\ \theta\ \text{ALL}(xs)$	$\equiv$	$\forall\ i \in \{1,\ldots,n\}: x\ \theta\ xs[i]$
list_has(xs,x)	$\equiv$	$x = \text{ANY}(xs)$
list_has_any$(xs,[y_1,\ldots,y_m])$	$\equiv$	$\exists\ i \in \{1,\ldots,m\}: y_i = \text{ANY}(xs)$
list_position$(xs,\divideontimes)$	$\equiv$	i (if $\divideontimes$ not found: NULL)

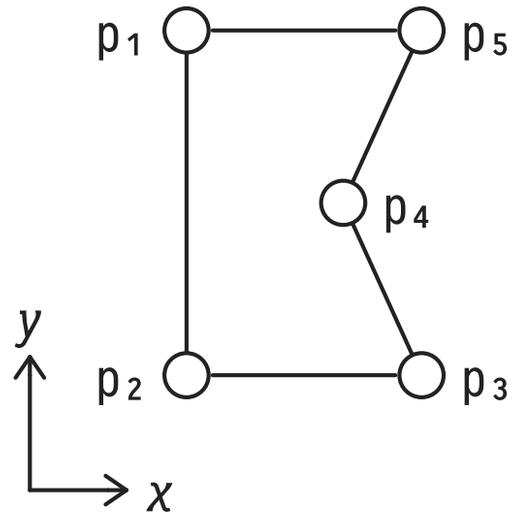
Advanced List Processing (think Haskell, APL)

```
-- map and fold(l1)
list_transform(xs, x -> e(x))     ≡ [e(x₁),…,e(xₙ)]
list_reduce(xs, (a,x) -> a ⊗ x)) ≡ (…((x₁ ⊗ x₂) ⊗ x₃)…) ⊗ xₙ
-- apply Boolean mask (bᵢ :: boolean)
list_where(xs, [b₁,…,bₙ])  ≡ [xᵢ | i ∈ [1,…,n], bᵢ = true]
-- filter and flatten
list_filter(xs, x -> p(x)) ≡ list_where(xs,
                               list_transform(xs, x -> p(x))
flatten(xss)               ≡ list_reduce(xss,
                               (xs₁,xs₂) -> xs₁ || xs₂)
```

Also: position-aware map (access list index i of element x):

```
list_transform(xs, (x,i) -> e(x,i))  ≡ [e(x₁,1),…,e(xₙ,n)]
```

Farewell, Tables? Use SQL as a List Programming Language?



	x	y
p1	1	6
p2	1	1
p3	7	1
p4	5	4
p5	7	6

- Area of the 2D polygon $p_1 \cdots p_5$ (“shoe lace” formula):

$x \rightarrow$	1	1	7	5	7	1
	X	X	X	X	X	
$y \rightarrow$	6	1	1	4	6	6

$$\frac{1}{2} \times \left(\begin{array}{l} \quad p_1.x \times p_2.y - p_2.x \times p_1.y \\ + \; p_2.x \times p_3.y - p_3.x \times p_2.y \\ + \; \cdots \end{array} \right)$$

4 ¦ Bridging Lists and Tables: unnest & aggregate list

```
SELECT t.elem
FROM   unnest([x1,…,xn]) AS t(elem)
                ≡ xs
```

≡

elem
x_1
⋮
x_n

Table t

```
SELECT list(t.elem) AS xs
FROM   (VALUES (x1),
               ⋮
              (xn)) AS t(elem)
```

≡

xs
$[x_1,\ldots,x_n]$

- unnest(•): a *table-returning function*. More on that soon.
- ⚠️ Preservation of order of the x_i is *not* guaranteed...

Representing Order (Indices) As First-Class Values

```
SELECT t.*
FROM   unnest([x1,…,xn])
       WITH ORDINALITY AS t(elem,idx)
                                ↑
```

≡

elem	idx
x_1	1
⋮	⋮
x_n	n

recall ordered aggregates

```
SELECT list(t.elem ORDER BY t.idx) AS xs
FROM   (VALUES (x1,1),
                  ⋮
               (xn,n)) AS t(elem,idx)
```

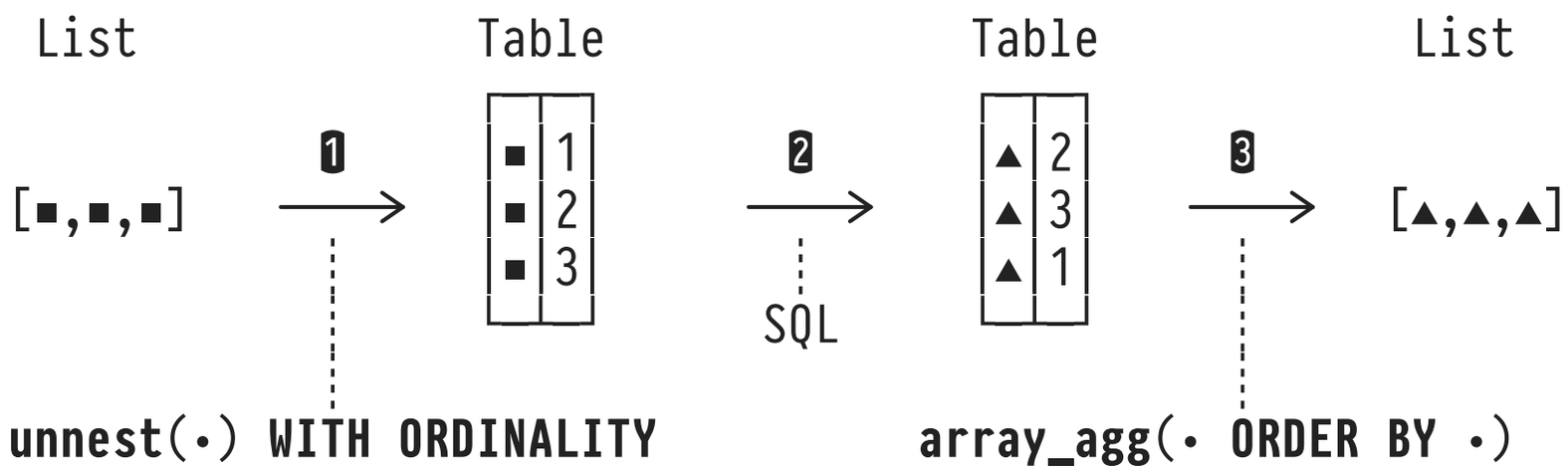
≡

xs
$[x_1,\ldots,x_n]$

- $f(\cdots)$ WITH ORDINALITY adds a trailing column (see ↑) of ascending indices 1,2,... to the output of function f.

A Relational List Programming Pattern

Availability of unnest(•) and ordered aggregate list(•) suggests a pattern for **relational list programming:**



- At ❷ use the full force of SQL, read/transform/generate elements and their positions at will.
- ❶+❸ constitute **overhead:** an RDBMS is *not* a list PL.

Nested Structs + Lists vs. JSON

- SQL type constructors `struct(…)` and `τ[]` nest arbitrarily: may build complex tree-shaped structures much like JSON.
- DuckDB supports bidirectional casting between SQL and JSON values:
  - SQL structs ↔ JSON objects, SQL lists ↔ JSON arrays.
  - SQL → JSON ✓ ⚠️ Prerequisites for SQL ← JSON casting:
    - JSON object keys needs to be known *a priori*.
    - JSON arrays need to be homogeneous.
- SQL's `unnest(•)`/`list(•)` processing idiom applies.

5 DuckDB: Key/Value Maps (Type Constructor `map`)

- Recall the container types:
  - `struct`: map fixed fields to values of varied types.
  - τ`[]`: map dynamic `int` index set to values of type τ.
- Additional container type in DuckDB:
  - `map(`τ_1`,`τ_2`)`: map dynamic set of keys (of type τ_1) to values (of type τ_2).
- **Example:** two equivalent `map(int,boolean)` literals:

```
map {1:true, 3:true, 4:false}
                ≡
map([1,3,4], [true,true,false])
```

DuckDB: Accessing/Constructing Maps

Let $m \equiv \mathtt{map}([k_1,\ldots,k_n], [v_1,\ldots,v_n])$, $k_i \neq k_j$ for $i \neq j$.

```
m[k_i]              ≡ [v_i]  -- if k_i ∈ [k_1,...,k_n]
m[k_i]              ≡ []     -- otherwise (NB. m[•] cannot fail)

cardinality(m) ≡ n
map_keys(m)    ≡ [k_1,…,k_n]
map_values(m)  ≡ [v_1,…,v_n]

map_entries(m) ≡ {k_1:v_1,…,k_n:v_n}
map_from_entries([{□:k_1,□:v_1},…,{□:k_n,□:v_n}]) ≡ m -- any □

map_concat(m_1,m_2)  -- merges maps m_1 and m_2
                     -- (keys in m_2 overwrite those in m_1)
```

6 Table-Generating Functions

What is the **type** of unnest(•)?

- unnest(•) establishes a bridge between lists and SQL's tabular data model:

$$\text{unnest} :: \tau[] \to \text{TABLEOF } \tau$$

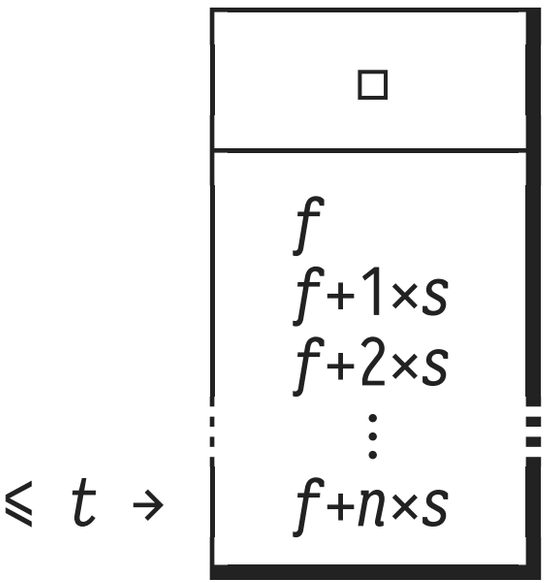
- In SQL, functions of type $\tau_1 \to \text{TABLEOF } \tau_2$ are known as **table-generating or set-returning functions.** May be invoked wherever a query expects a table (FROM clause): compositionality.

- Several of these functions are built into DuckDB.

Series Generators

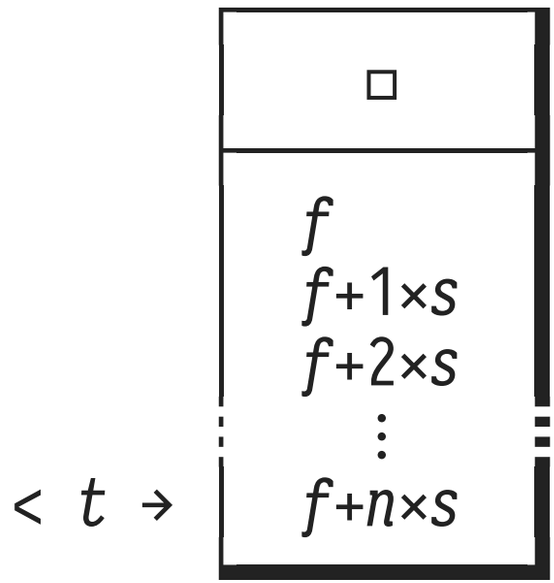
Built-in table-generating functions that generate **tables of consecutive numbers/values:**

generate_series(f,t,s)



$s \equiv 1$, if absent
f,t: numbers/timestamps

range(f,t,s)

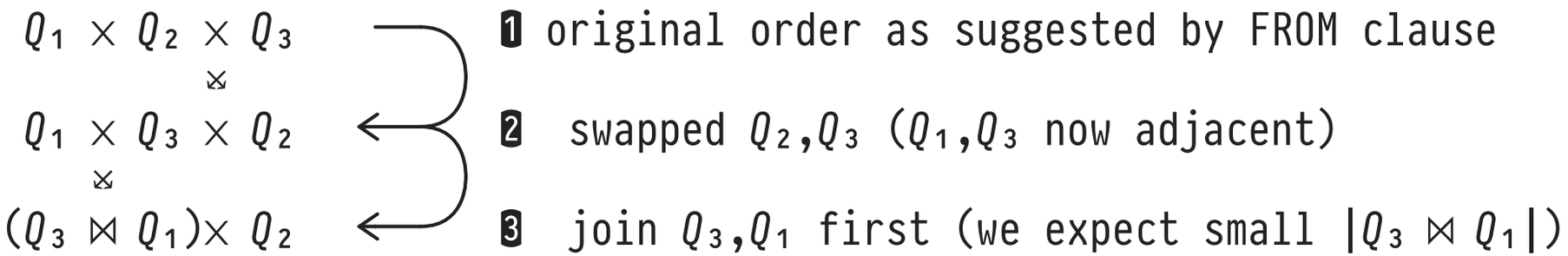


$s \equiv 1$, if absent
stop value t excluded

7 ',' in the FROM Clause and Row Variable References

```
SELECT …
FROM   Q1 AS t1, Q2 AS t2, Q3 AS t3 -- ti<j not free in Qj
```

- Q: Why is $t_{i<j}$ *not* usable in Q_j?
- A: "… *the ',' in FROM is commutative and associative*…". Query optimization might rearrange the Q_j:



But Dependent Iteration in FROM is Useful...

Recall (find largest label in each tree t_1):

```
SELECT t1.tree, MAX(t2.label) AS "largest label"
--          Q1                 Q2
--
FROM   Trees AS t1, unnest(t1.labels) AS t2(label)
GROUP BY t1.tree;
```

- **Dependent iteration** (here: Q_2 depends on t_1 defined in Q_1) has its uses and admits intuitve query formulation.

- **NB.** DuckDB's “friendly SQL” analyzes dependencies between FROM clause entries, introduces LATERAL automatically.

LATERAL:[1] Dependent Iteration for Everyone

Prefix Q_j with LATERAL in the FROM clause to announce dependent iteration:

SELECT …
FROM Q_1 **AS** t_1, …, **LATERAL** Q_j **AS** t_j, …

may refer to $t_1, \ldots, t_{j-1}$

- Works for *any* table-valued SQL expression Q_j, subqueries in (…) in particular.
  - Good style: be explicit and use LATERAL even on DuckDB.

[1] Lateral /ˈlæt(ə)rəl/ a. [Latin lateralis]: *sideways*

LATERAL: SQL's for each-Loop

LATERAL admits the formulation of **nested-loops** computation:

```
SELECT e
FROM   Q1 AS t1, LATERAL Q2 AS t2, LATERAL Q3 AS t3
```

is evaluated just like this nested loop:

```
for t1 in Q1
  for t2 in Q2(t1)
    for t3 in Q3(t1,t2)
      return e(t1,t2,t3)
```

- Convenient, intuitive, and perfectly OK.
  But much like hand-cuffs for the query optimizer. ⚠️

LATERAL Example: Find the Top *n* Rows Among a Peer Group

Which are **the three tallest** two- and four-legged dinosaurs?

```
SELECT locomotion.legs, tallest.species, tallest.height
FROM   (VALUES (2), (4)) AS locomotion(legs),
       LATERAL (SELECT d.*
                FROM   dinosaurs AS d
                WHERE  d.legs = locomotion.legs ←
                ORDER BY d.height DESC
                LIMIT 3) AS tallest
```

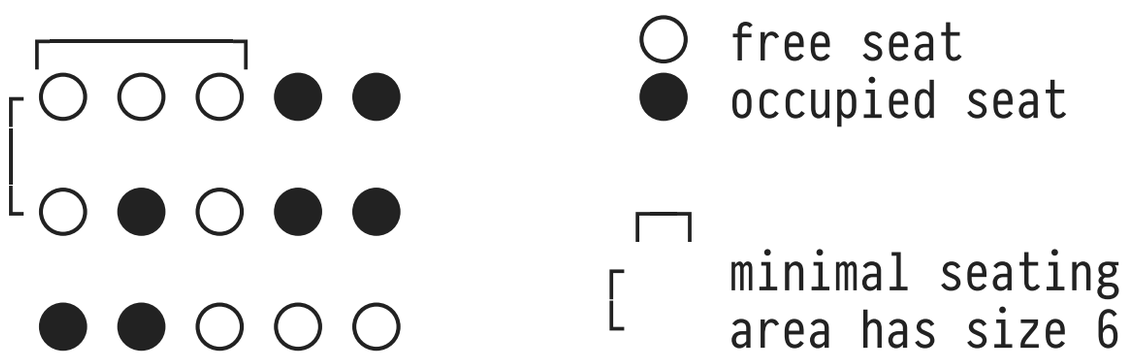
legs	species	height
2	Tyrannosaurus	7
2	Ceratosaurus	4
2	Spinosaurus	2.4
4	Supersaurus	10
4	Brachiosaurus	7.6
4	Diplodocus	3.6

8 ACM ICPC: Finding Seats

ACM ICPC Task **Finding Seats** (South American Regionals, 2007)

"K friends go to the movies but they are late for tickets. To sit close to each other, they look for K free seats such that the rectangle containing these seats has minimal area."

- Assume $K = 5$:



Finding Seats: Parse the ICPC Input Format

- Typical ICPC character-based input format:

...XX␍ .X.XX␍ XX...	. free seat X occupied seat ␍ new line

- **Parse into table** making seat position/status explicit:

row	col	taken?
1	1	false
1	2	false
1	3	false
1	4	true
⋮	⋮	⋮
3	5	false

Table seats

Finding Seats: Parse the ICPC Input Format (Table seats)

```
-- Assume cinema ≡ E'...XX\n.X.XX\nXX...'

SELECT  row.pos, col.pos, col.x = 'X' AS "taken?"
FROM    -- rows
        unnest(string_split(cinema, E'\n'))
        WITH ORDINALITY AS row(xs, pos),
        -- columns
        LATERAL unnest(string_split(row.xs, ''))
                WITH ORDINALITY AS col(x, pos)
```

- string_split(cinema, E'\n') yields a list of three row strings: '...XX', '.X.XX', 'XX...'.
- string_split(row.xs, '') splits string row.xs into a list of individual characters (= seats).

Finding Seats: A Problem Solution (Generate and Test)

- **Query Plan:**

  1. Parse the input into a seating plan table `seats`.
  2. **Generate all** possible north-west (*nw*) and south-east (*se*) corners of rectangular seating areas:
     - For each such ⌜*nw*,*se*⌟ rectangle, scan its seats and **test** whether the number of free seats is ⩾ *K*.
     - If so, record *nw* together with the rectangle's `width`/`height`.
  3. Among these rectangles with sufficient seating space, select one with minimal area.

🔧 Finding Seats: Generating All Possible Rectangles

Generate all ⌜*nw*,*se*⌟ corners for rectangles in the seating plan (table seats):

```sql
SELECT nw, se
FROM   seats AS nw,          -- self-join
       seats AS se
WHERE  nw.col <= se.col      -- ⌜nw is to the top left of se⌟
AND    nw.row <= se.row
```

- This generates $\left(\sum_{r=1}^{\mathrm{rows}} r\right) \times \left(\sum_{c=1}^{\mathrm{cols}} c\right)$ rectangles.
- Generally: If possible, test/filter early.