Advanced SQL

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⑤
Window Functions

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1 ┊ Window Functions

With SQL:2003, the ISO SQL Standard introduced **window functions,** a new mode of row-based computation:
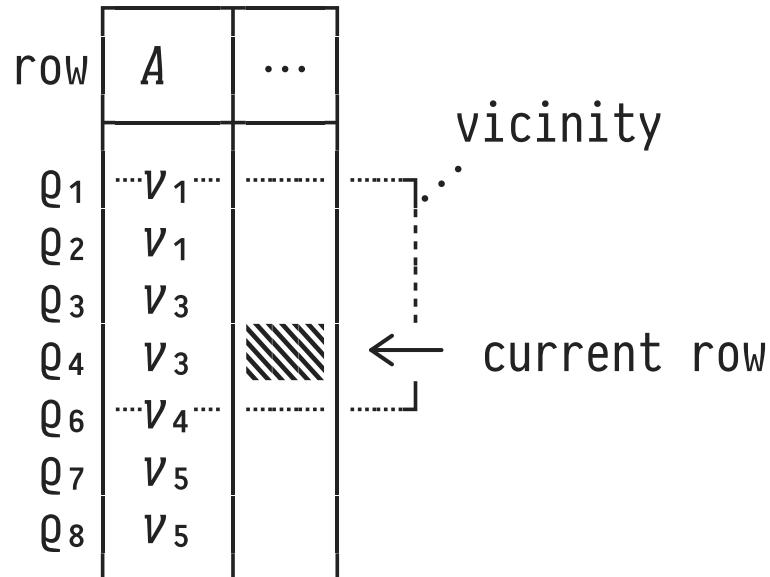
SQL Feature	Mode of Computation
function/operator	row → row
table-generating function	row → table of rows
aggregate	group of rows → row (one per group)
window function 👍	row vicinity → row (one per row)

SQL Modes of Computation

Window functions …

- … are **row-based:** each individual input row r is mapped to one result row,
- … use the **vicinity** around r to compute this result row.

Row Vicinity: Window Frames

row	A	$\cdots$
ϱ_1	v_1	
ϱ_2	v_1	
ϱ_3	v_3	
ϱ_4	v_3	
ϱ_6	v_4	
ϱ_7	v_5	
ϱ_8	v_5	

vicinity
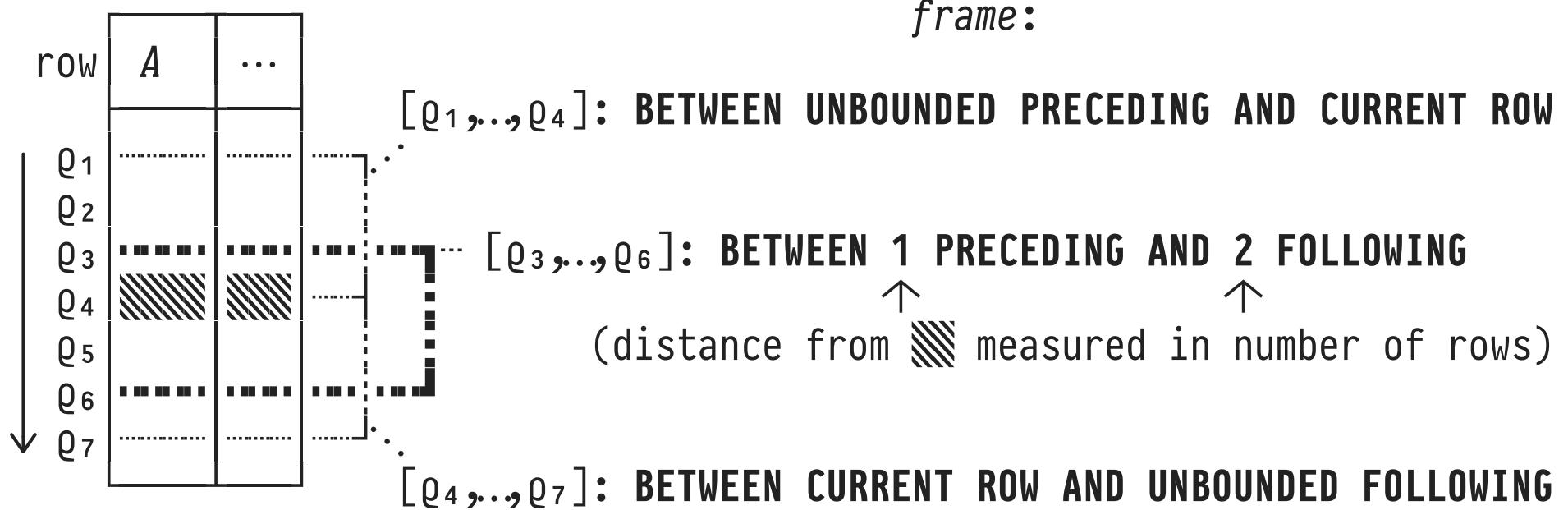
← current row

- Each row is the **current row** ▨ at one point in time.

- Row vicinity (**window, frame**) is based on either:

  ❶ row **position** (ROWS windows),
  ❷ row **values** v_i (RANGE windows),
  ❸ row **peers** (GROUPS windows).

- As the current row changes, the window *slides* with it.
- ⚠ Window semantics depend on a defined **row ordering**.

Window Frame Specifications (Variant: **ROWS**)

$$f \; \textbf{OVER} \; (\textbf{ORDER BY} \; e_1,\dots,e_n \; [\; \textbf{ROWS} \; \textit{frame} \;])$$

where the braces indicate: *window function* above f, *ordering criteria* above $e_1,\dots,e_n$, and *frame specification* above $\textbf{ROWS} \; \textit{frame}$.

frame:

$[\varrho_1,..,\varrho_4]$: **BETWEEN UNBOUNDED PRECEDING AND CURRENT ROW**

$[\varrho_3,..,\varrho_6]$: **BETWEEN 1 PRECEDING AND 2 FOLLOWING**

(distance from ▨ measured in number of rows)

$[\varrho_4,..,\varrho_7]$: **BETWEEN CURRENT ROW AND UNBOUNDED FOLLOWING**

row | A | $\dots$ with rows $\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6, \varrho_7$

Window Frame Specifications (Variant: RANGE)

$$f \ \text{OVER} \ (\text{ORDER BY} \ A \ [\ \text{RANGE} \ frame \ ])$$
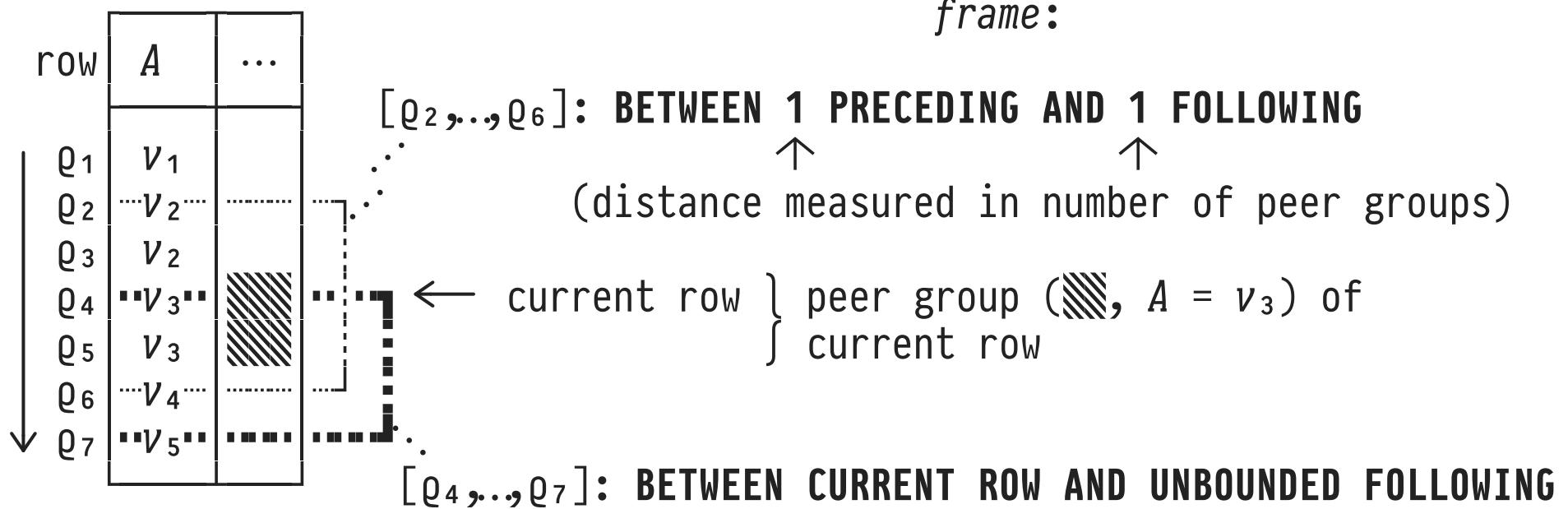
window function   one criterion   frame specification

frame:

$[\varrho_1,..,\varrho_4]$: **BETWEEN UNBOUNDED PRECEDING AND CURRENT ROW**

row	A	$\cdots$
ϱ_1	v_1	
ϱ_2	v_2	
ϱ_3	v_2	
ϱ_4	v_2	
ϱ_5	v_3	
ϱ_6	v_4	
ϱ_7	v_4	

current row $\}$ peer group (, $A = v_2$) of current row

$[\varrho_2,..,\varrho_5]$: **BETWEEN CURRENT ROW AND 2 FOLLOWING**

(distance measured in terms of $|v_2-v_i|$,
in this example: $v_3-v_2 \leqslant 2$ and $v_4-v_2 > 2$)
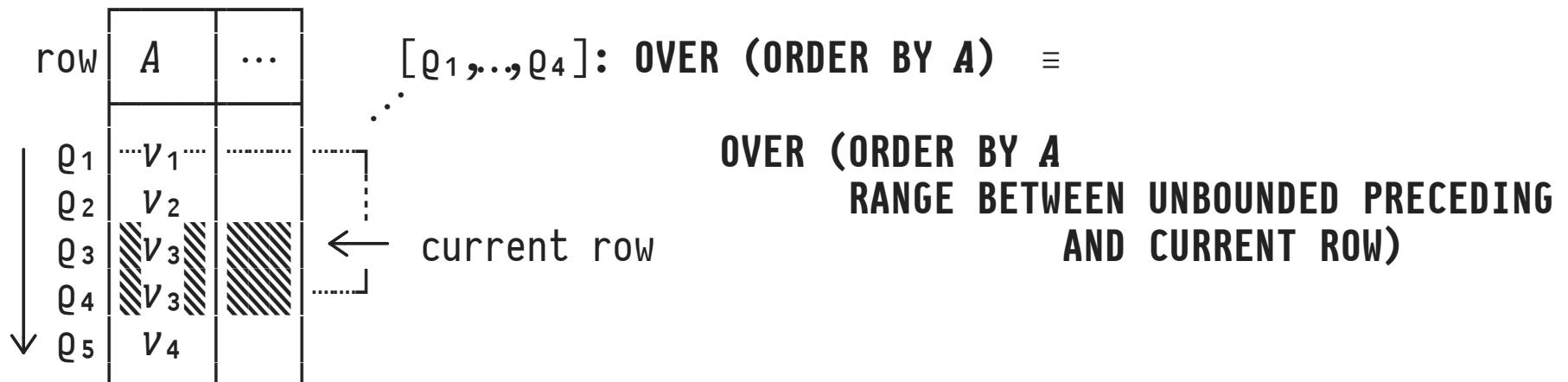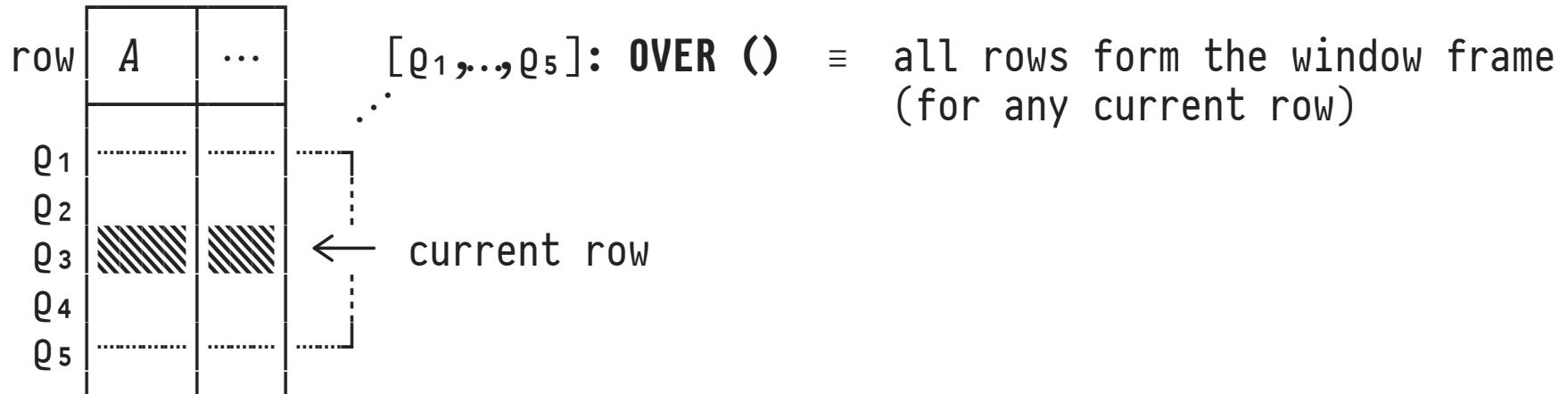
# Window Frame Specifications (Variant: GROUPS)

window function   ordering criteria   frame specification

$$f \ \textbf{ OVER (ORDER BY } e_1,\ldots,e_n \ [\ \textbf{GROUPS } \textit{frame} \ ])$$

frame:

row	A	$\cdots$
ϱ_1	v_1	
ϱ_2	v_2	
ϱ_3	v_2	
ϱ_4	v_3	
ϱ_5	v_3	
ϱ_6	v_4	
ϱ_7	v_5	

$[\varrho_2,..,\varrho_6]$: **BETWEEN 1 PRECEDING AND 1 FOLLOWING**

↑   ↑

(distance measured in number of peer groups)

← current row ⎱ peer group (▨, $A = v_3$) of
               ⎰ current row

$[\varrho_4,..,\varrho_7]$: **BETWEEN CURRENT ROW AND UNBOUNDED FOLLOWING**

Window Frame Specifications: Abbreviations

$[\varrho_1,..,\varrho_5]$: **OVER ()** $\equiv$ all rows form the window frame (for any current row)

row | A | $\cdots$

ϱ_1
ϱ_2
ϱ_3 $\leftarrow$ current row
ϱ_4
ϱ_5

$[\varrho_1,..,\varrho_4]$: **OVER (ORDER BY A)** $\equiv$

OVER (ORDER BY A
        RANGE BETWEEN UNBOUNDED PRECEDING
                AND CURRENT ROW)

row | A | $\cdots$

ϱ_1 v_1
ϱ_2 v_2
ϱ_3 v_3 $\leftarrow$ current row
ϱ_4 v_3
ϱ_5 v_4

WINDOW Frame Specifications: Exclusion[1]

window function   exclusion clause

$$f \;\; \textbf{OVER} \; (\textbf{ORDER BY} \; A \; \textit{frame} \; [\; \textit{exclusion} \;])$$

row	A	...
ϱ_1	v_1	
ϱ_2	v_2	
ϱ_3	v_3	
ϱ_4	v_3	
ϱ_5	v_3	
ϱ_6	v_4	

← current row

exclusion:

EXCLUDE NO OTHERS	$[\varrho_2, \varrho_3, \varrho_4, \varrho_5]$
EXCLUDE CURRENT ROW	$[\varrho_2, \varrho_3, \quad \varrho_5]$
EXCLUDE GROUP	$[\varrho_2 \qquad\qquad]$
EXCLUDE TIES	$[\varrho_2, \quad \varrho_4 \quad]$

[1] **Q:** Which *frame* would lead to a window as shown above?

WINDOW Clause: Name the Frame

Syntactic ⚲: If window frame specifications

1. become unwieldy because of verbose SQL syntax and/or
2. one frame is used multiple times in a query,

add a **WINDOW** clause to a SFW block to **name the frame**, *e.g.*:

```
SELECT … f OVER wᵢ … g OVER wⱼ …
FROM    …
WHERE   …
   ⋮
WINDOW w₁ AS (frame₁), …, wₙ AS (frameₙ)
ORDER BY …
```

Use SQL Itself to Explain Window Frame Semantics

Regular **aggregates** may act as window functions f. All **rows in the frame will be aggregated:**

```
SELECT w.row                 AS "current row",
       count(*)    OVER win AS "frame size",
       list(w.row) OVER win AS "rows in frame"
FROM   W AS w
WINDOW win AS (frame)
```

row	a	b
ϱ_1	1	●
ϱ_2	2	○
ϱ_3	3	○
ϱ_4	3	●
⋮	⋮	⋮

Table W

🔧 Q: What is the Chance of Fine Weather on Weekends?

Input: Daily weather readings in sensors:

day	weekday	temp	rain
1	Fri	10	800
2	Sat	12	300
⋮	⋮	⋮	⋮

Table sensors

- The weather is fine on day *d* if—**on *d* and the two days prior**—the minimum temperature is above 15°C and the overall rainfall is less than 600ml/m².
- **Expected output:**

weekend?	% fine
f	29
t	43

2 ⦙ PARTITION BY: Window Frames Inside Partitions
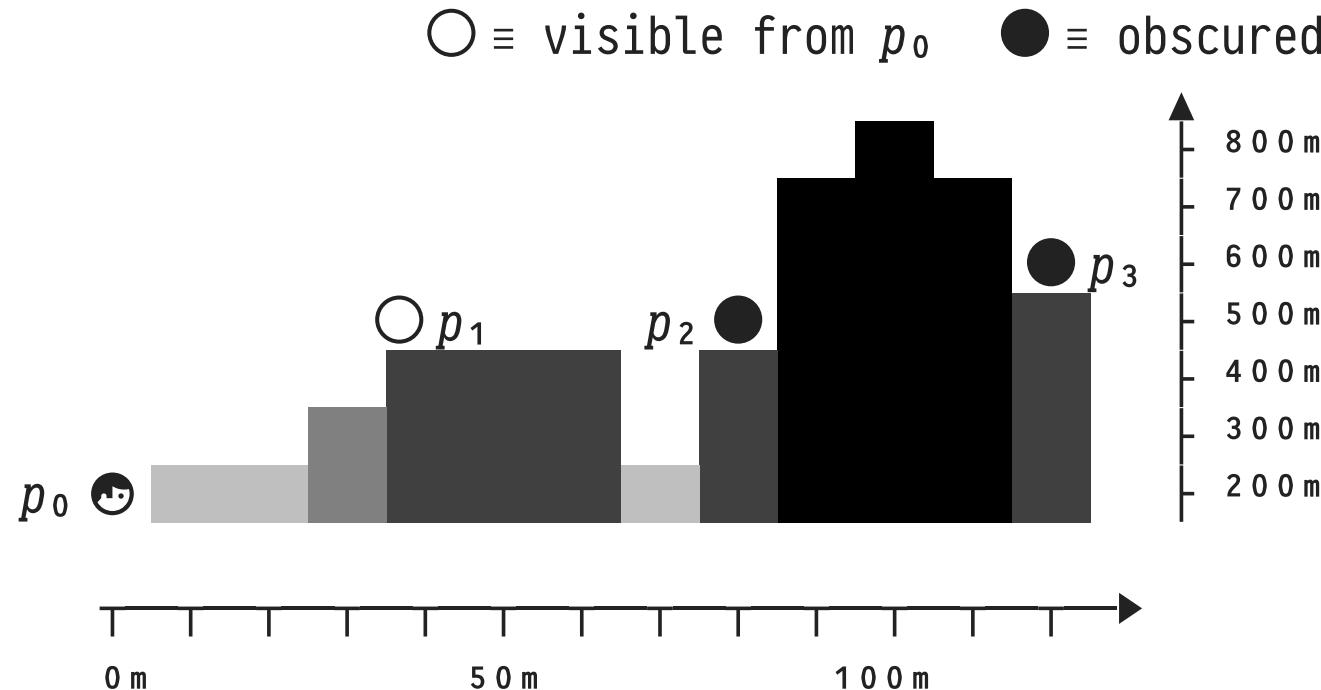
Optionally, we may **partition** the input table *before* rows are sorted and window frames are determined:

all input rows that agree on all p_i form one partition

f **OVER** ([**PARTITION BY** $p_1,\dots,p_m$]
      [**ORDER BY** $e_1,\dots,e_n$]
      [*frame*])

- Note:

1. Frames **never cross partitions.**
2. BETWEEN … PRECEDING AND … FOLLOWING **respects partition boundaries.**
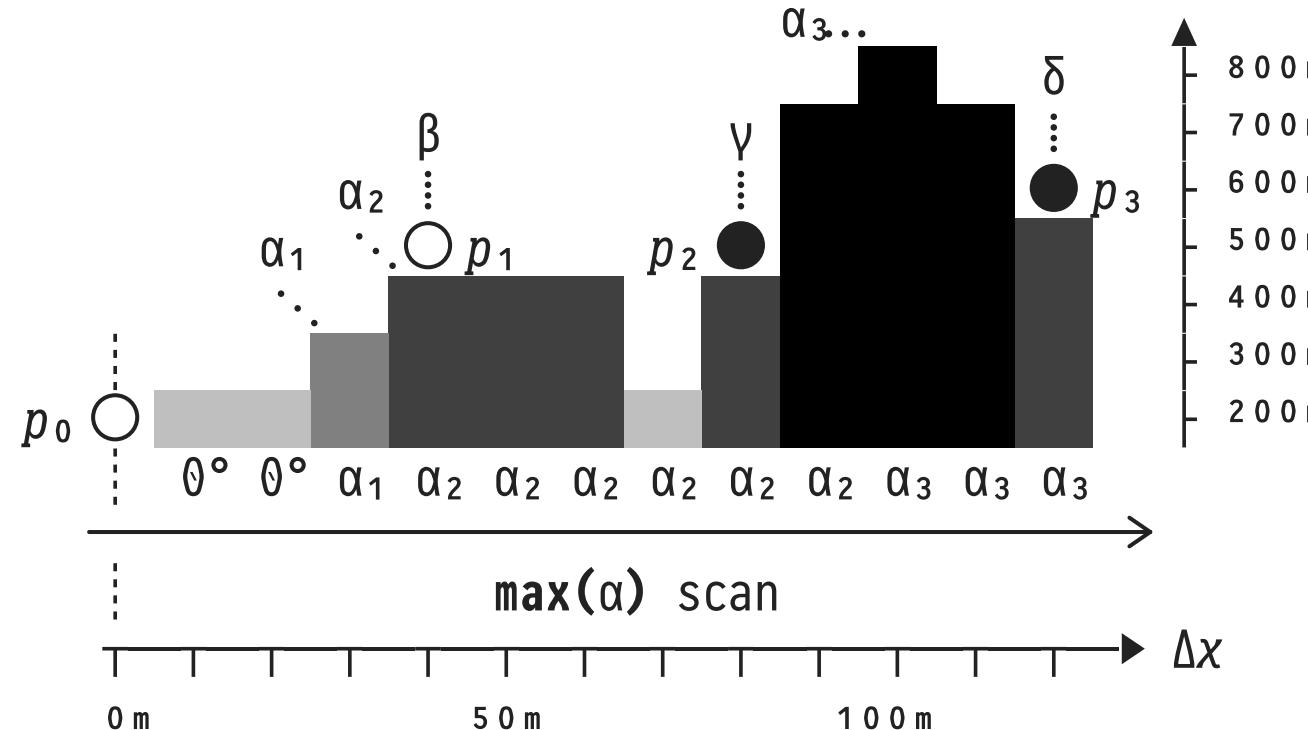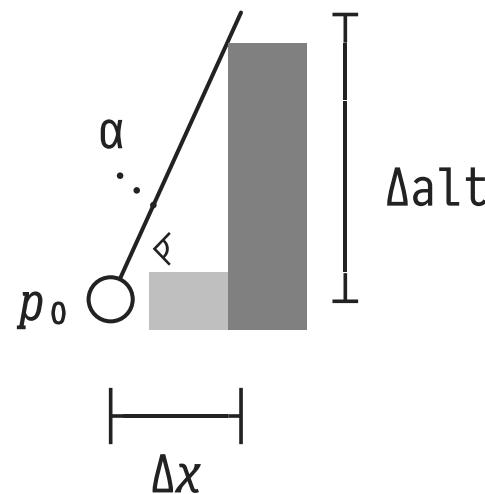
🔧 Q: Which Spots are Visible in a Hilly Landscape?

$\bigcirc \equiv$ visible from p_0 $\bullet \equiv$ obscured

- From the viewpoint of p_0 (😊) we can see p_1, but...
  - ... p_2 is **obscured** (no straight-line view from p_0),
  - ... p_3 is **obscured** (lies behind the 800m peak).

🔧 Q: Visible Spots in a Hilly Landscape? — A: MAX Scan!

$\angle\alpha = \mathrm{atan}(\Delta\mathrm{alt}/\Delta x)$

- We have $0° < \alpha_1 < \alpha_2 < \alpha_3$ and $\beta \geqslant \alpha_2$, $\gamma < \alpha_2$, $\delta < \alpha_3$.

p_1 visible $p_{2,3}$ obscured

🔧 **Q: Visible Spots in a Hilly Landscape? — A: MAX Scan!**

- **Input:** Location of p_0 (here: $x = 0$) and 1D-map of hills:

x	alt
0	200
10	200
⋮	⋮
120	500

Table map

- **Output:** Can p_0 see the point on the hilltop at x?
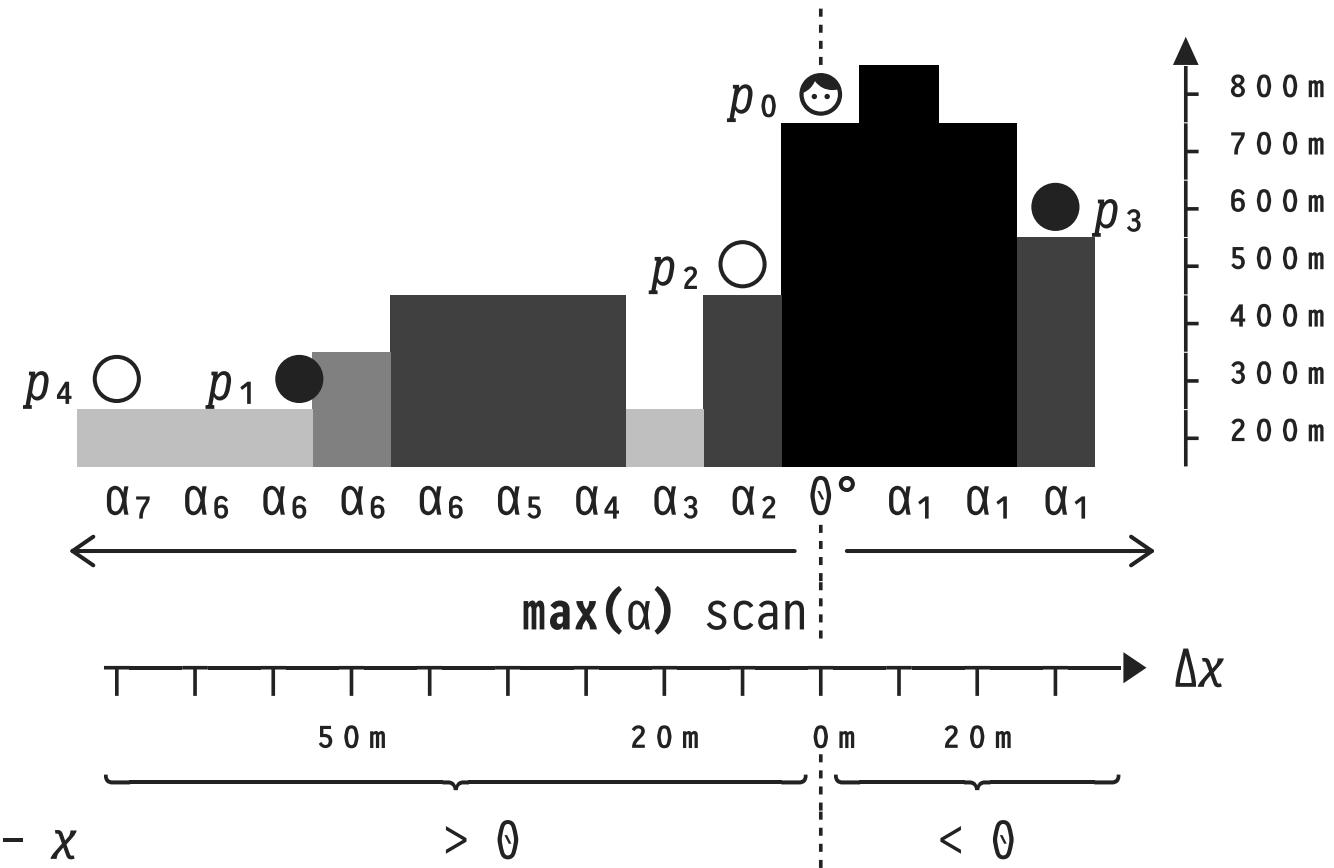
x	visible?
0	true
10	true
⋮	⋮
120	false

Q: Visible Spots in a Hilly Landscape? — A: MAX Scan!

```sql
WITH
-- ❶ Angles α (in °) between p₀ and the hilltop at x
angles(x, angle) AS (
  SELECT m.x,
         degrees(atan((m.alt - p0.alt) /
                      abs(p0.x - m.x))) AS angle
  FROM    map AS m
  WHERE   m.x > p0.x
),
-- ❷ max(α) scan (to the right of p₀)
max_scan(x, max_angle) AS (
  SELECT a.x,
         max(a.angle)
           OVER (ORDER BY abs(p0.x - a.x)) AS max_angle
  FROM    angles AS a
),
⋮
```

🔧 Looking Left *and* Right: **PARTITION BY**

- Need MAX scans left *and* right of p_0 ⟹ use PARTITION BY.

🔧 Looking Left *and* Right: PARTITION BY

```
WITH
  ⋮
-- ❷ max(α) scan (left/right of p₀)
max_scan(x, max_angle) AS (
  SELECT a.x,                    --    ∈ {-1, 0 ,1}
         max(a.angle)           --    ⌒‾‾‾‾‾‾‾⌒
           OVER (PARTITION BY sign(p0.x - a.x)
                 ORDER BY abs(p0.x - a.x)) AS max_angle
  FROM    angles AS a   --   ⌒‾‾‾‾‾‾⌒
                        --        Δx > 0
),
  ⋮
```

- $\forall a \in$ angles: $a.x \neq p0.x \Rightarrow$ We end up with **two** partitions.

3 ⦙ Scans: Not Only in the Hills

Scans are a general and expressive computational pattern:

$$\underbrace{agg(e)}_{(\phi,z,\oplus)} \textbf{ OVER } (\textbf{ORDER BY } e_1,\dots,e_n$$
$$\{\textbf{ROWS,RANGE}\} \textbf{ BETWEEN}$$
$$\textbf{UNBOUNDED PRECEDING AND CURRENT ROW})$$

- Available in a variety of forms in programming languages
  - Haskell: scanl $z \oplus xs$, APL: $\oplus \backslash xs$, Python: accumulate.
    scanl $\oplus$ z $[x_1,x_2,\dots]$ = $[z,\ z \oplus x_1,\ (z \oplus x_1) \oplus x_2,\ \dots]$

- In parallel programming: *prefix sums* (☞ Guy Blelloch)
  - Sorting, lexical analysis, tree operations, reg.exp.
    search, drawing operations, image processing, …

4 ┊ Interlude: Quiz

Q: Assume *xs* ≡ '((b*2)-4×a×c)*0.5'. What is computed below?

```sql
SELECT inp.pos, inp.c,
       sum([0,1,-1][p.oc]) OVER (ORDER BY inp.pos) AS d
FROM   unnest(string_split(xs(), ''))
         WITH ORDINALITY AS inp(c,pos),
       LATERAL (VALUES (list_position(['(',')'],
                                      inp.c) + 1)) AS p(oc)
ORDER BY inp.pos;
```

💡 **Hint** (this is the same query expressed in APL):

```
xs ← '((b*2)-4×a×c)*0.5'
+\ (1 ¯ 1 0)['()'⍳xs]
```

5 ┊ Beyond Aggregation: Window Functions

```
window function
      ↓
f OVER ([ PARTITION BY p_1,…,p_m ]
        [ ORDER BY e_1,…,e_n ]
        [ frame ])
```

Three kinds of **window functions** f:

1. **Aggregates:** sum(•), avg(•), max(•), list(•), … process all rows in the frame. ✓

2. **Row Access:** access row by *absolute/relative position* in ordered frame or partition: first/last/n^{th}/n rows away.

3. **Row Ranking:** assign numeric *rank of row* in partition.

6 ⦙ **LAG**/**LEAD**: Access Rows of the Past and Future

Row access at offset $\pm n$**,** relative to the current row:
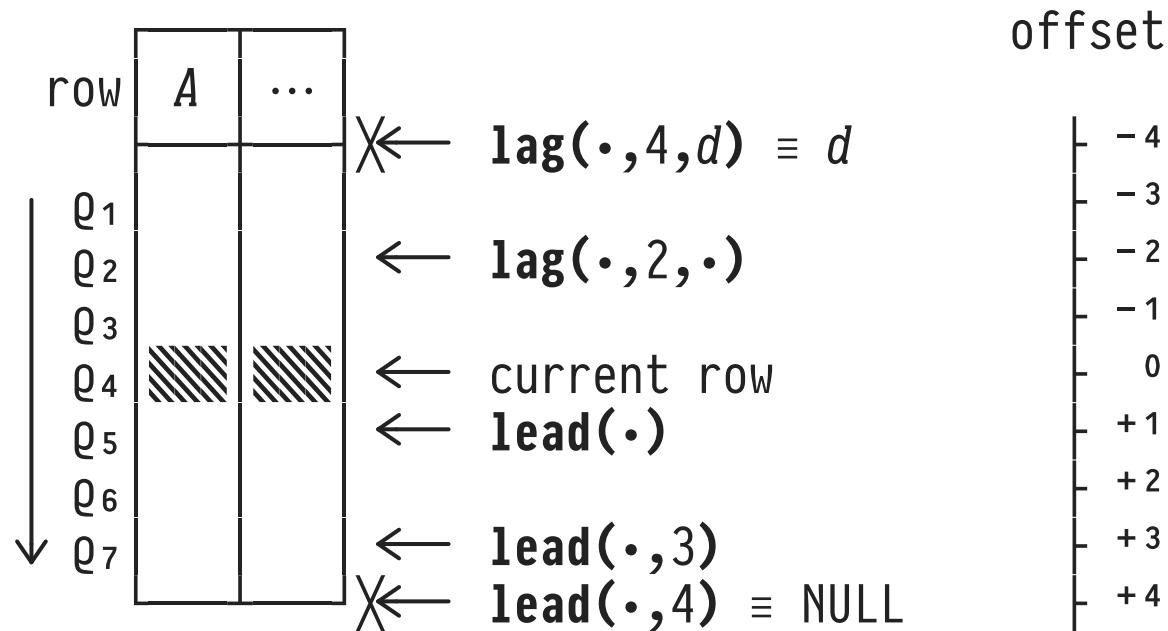
```
-- evaluate e as if we were
-- n rows before the current row
--
        lag(e,n,d) OVER ([ PARTITION BY p_1,…,p_m ]
                          ORDER BY e_1,…,e_n)
```

Note:

- lead(e,n,d): ⋯ n rows **after** the current row ⋯
- Scope is partition—no row access outside the partition.
- No row at offset $\pm n$? ⇒ return default value d.

LAG/LEAD: Row Offsets (Assume: No Partitioning, ORDER BY A)

- The frame of the current row is irrelevant for lag/lead.
- Default parameters if absent: $n \equiv 1$, $d \equiv$ NULL.

🔧 A March Through the Hills: Ascent or Descent?

```
SELECT  m.x, m.alt,
        CASE sign(lead(m.alt, 1) OVER rightwards – m.alt)
             WHEN –1 THEN '↘' WHEN 1 THEN '↗'
             WHEN  0 THEN '→' ELSE '?'
        END AS climb,
        lead(m.alt, 1) OVER rightwards – m.alt AS "by [m]"
FROM    map AS m
WINDOW  rightwards AS (ORDER BY m.x)  --  marching right
```

x	alt	climb	by [m]
0	200	→	0
⋮	⋮	⋮	⋮
90	700	↗	100
100	800	↘	–100
110	700	↘	–200
120	500	?	NULL

🔧 Crime Scene: Sessionization

A spy broke into the Police HQ computer system. A log records keyboard activity of user uid at time ts:

uid	ts
	2025-12-11 07:25:12
	2025-12-11 07:25:18
	2025-12-11 08:01:55
	2025-12-11 08:05:07
	2025-12-11 08:05:30
	2025-12-11 08:05:39
	2025-12-11 08:05:46
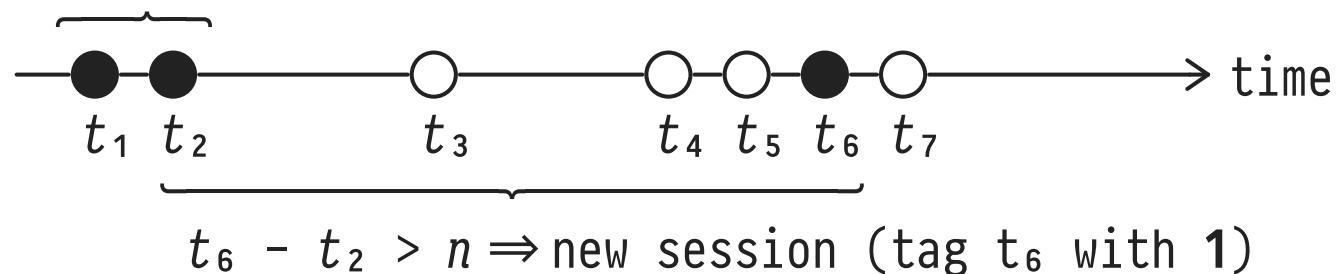
Table log

- **Q:** Can we **sessionize** the log so that investigators can identify *sessions* (≡ streaks of uninterrupted activity)?

🔧 Sessionization (Query Plan)
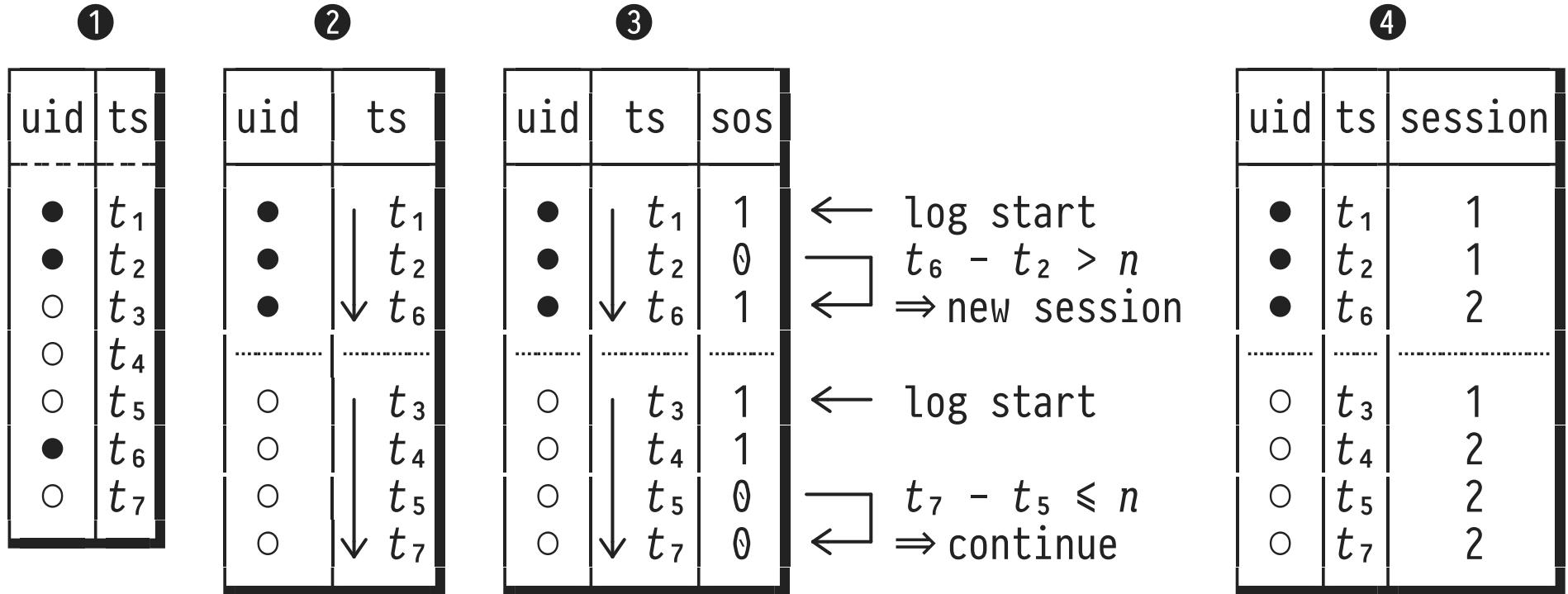
1. Cop and spy sessions happen independently (even if interleaved): partition `log` into 👮/● and 🕵/○ rows.

2. **Tag** keyboard activities (below: tagging for ●):

$t_2 - t_1 \leqslant$ threshold $n \Rightarrow$ continue session (tag t_2 with **0**)

$t_6 - t_2 > n \Rightarrow$ new session (tag t_6 with **1**)

3. **Scan** the tagged table and derive session IDs by maintaining a **runnning sum** of *start of session* tags.

🔧 Sessionization (Query Plan)

❶

uid	ts
●	t_1
●	t_2
○	t_3
○	t_4
○	t_5
●	t_6
○	t_7

❷

uid	ts
●	t_1
●	t_2
●	t_6
○	t_3
○	t_4
○	t_5
○	t_7

❸

uid	ts	sos
●	t_1	1
●	t_2	0
●	t_6	1
○	t_3	1
○	t_4	1
○	t_5	0
○	t_7	0

⟵ log start

$t_6 - t_2 > n$
⟹ new session

⟵ log start

$t_7 - t_5 \leqslant n$
⟹ continue

❹

uid	ts	session
●	t_1	1
●	t_2	1
●	t_6	2
○	t_3	1
○	t_4	2
○	t_5	2
○	t_7	2
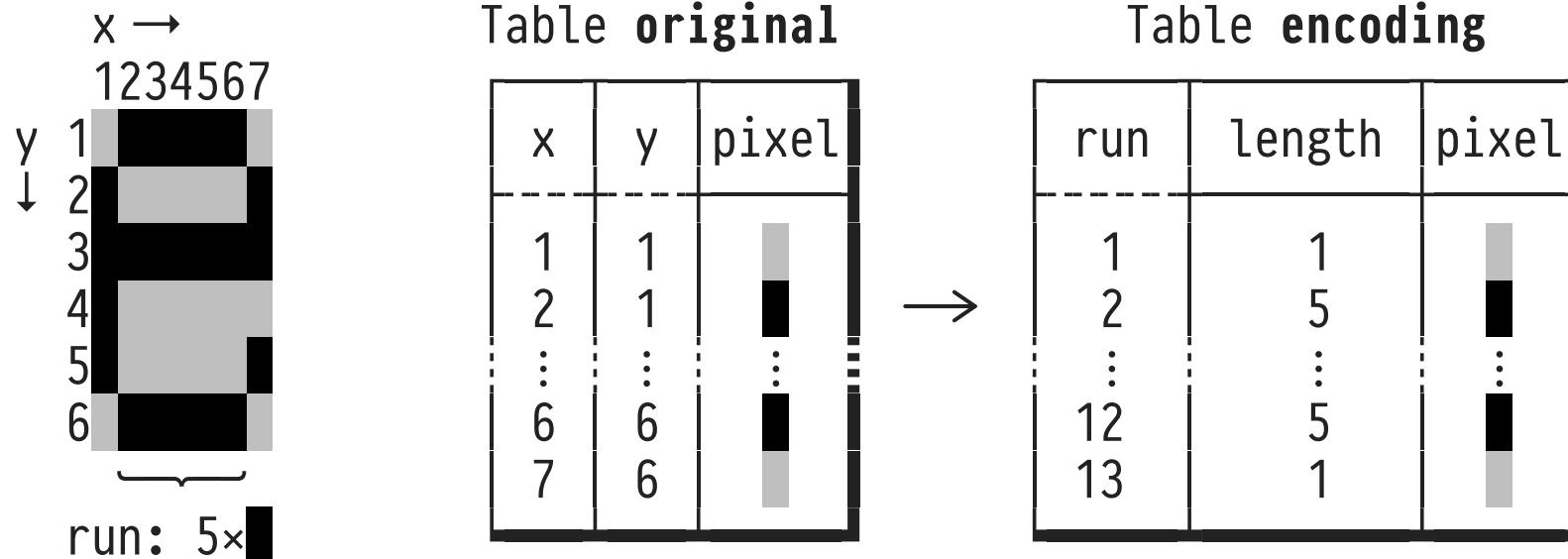
- At log start, always begin a new session (sos = 1).
- How to assign *global session IDs* (○'s sessions: 3, 4)?

🔧 Image Compression by Run-Length Encoding

Compress image by identifying pixel **runs** of the same color:

x	y	pixel
1	1	
2	1	
⋮	⋮	⋮
6	6	
7	6	

Table **original** → Table **encoding**

run	length	pixel
1	1	
2	5	
⋮	⋮	⋮
12	5	
13	1	

run: 5×

- Here: assumes a row-wise linearization of the pixel map.
- In b/w images we may omit column pixel in table encoding.

🔧 Run-Length Encoding (Query Plan)

❶

x	y	pixel	change?	
1	1		t	1
2	1		t	1
3	1		f	0
4	1		f	0
5	1		f	0
6	1		f	0
7	1		t	1
⋮	⋮	⋮	⋮	⋮

❷

x	y	pixel	change?	Σ change?
1	1		1	1
2	1		1	2
3	1		0	2
4	1		0	2
5	1		0	2
6	1		0	2
7	1		1	3
⋮	⋮	⋮	⋮	⋮

run #2 of length 5

❶: lag(pixel,1,*undefined*): pixel @ (1,1) always "changes."

❷: A sum() scan of change? yields run identifiers.

7 ┊ FIRST_VALUE, LAST_VALUE, NTH_VALUE: In-Frame Row Access

Aggregates reduce *all rows* inside a frame to a single value.
Now for something different:
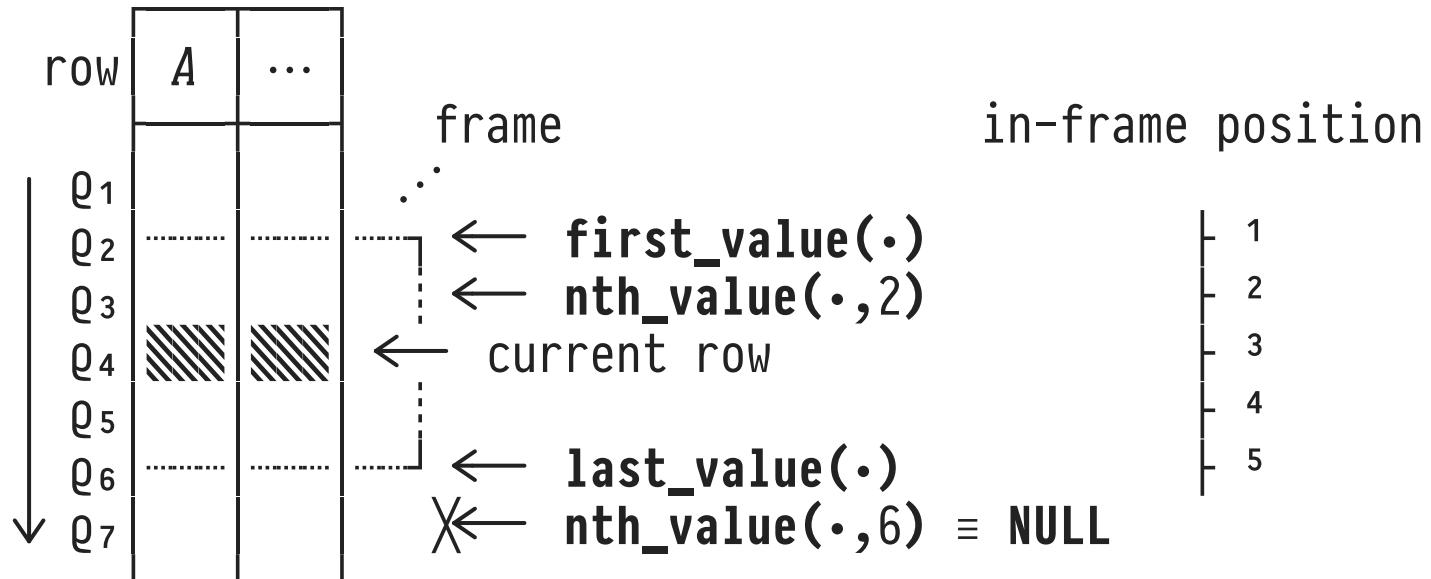
- **Positional access to individual rows** inside a frame is
  provided by three window functions:

```
-- evaluate expression e as if we were at
-- the first/last/nᵗʰ row in the frame
--
            first_value(e)
            last_value(e)    }  OVER (…)
            nth_value(e,n)
```
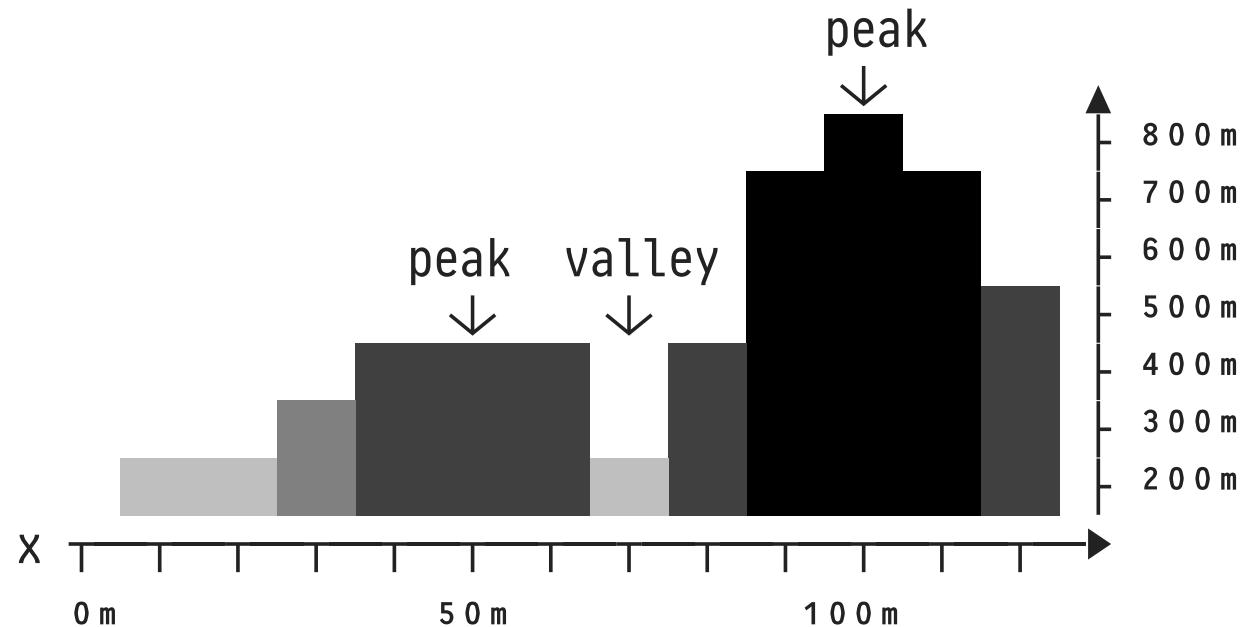
- nth_value(e,n): No n^{th} row in frame $\Rightarrow$ return NULL.

In-Frame Row Access

- We have first_value(e) ≡ nth_value(e,1).
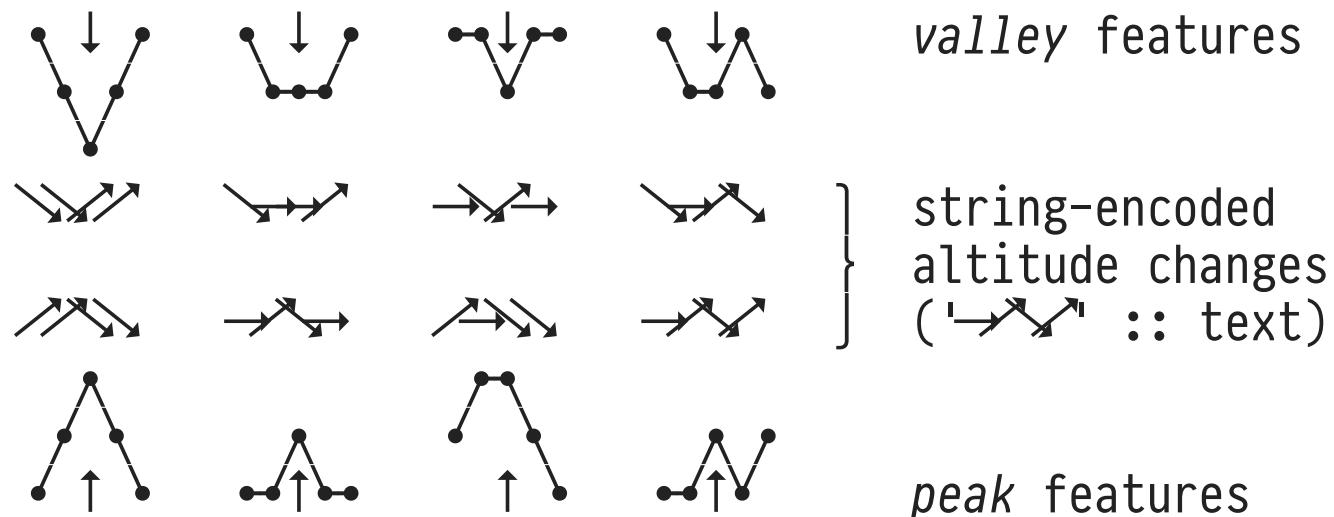
# 🔧 Detecting Landscape Features

- Detect features in a hilly landscape. Attach label $\in$ {peak,valley,-} to every location x.

- Feature defined by relative altitude change **in vicinity.**

🔧 **Detecting Landscape Features (Query Plan)**

- -
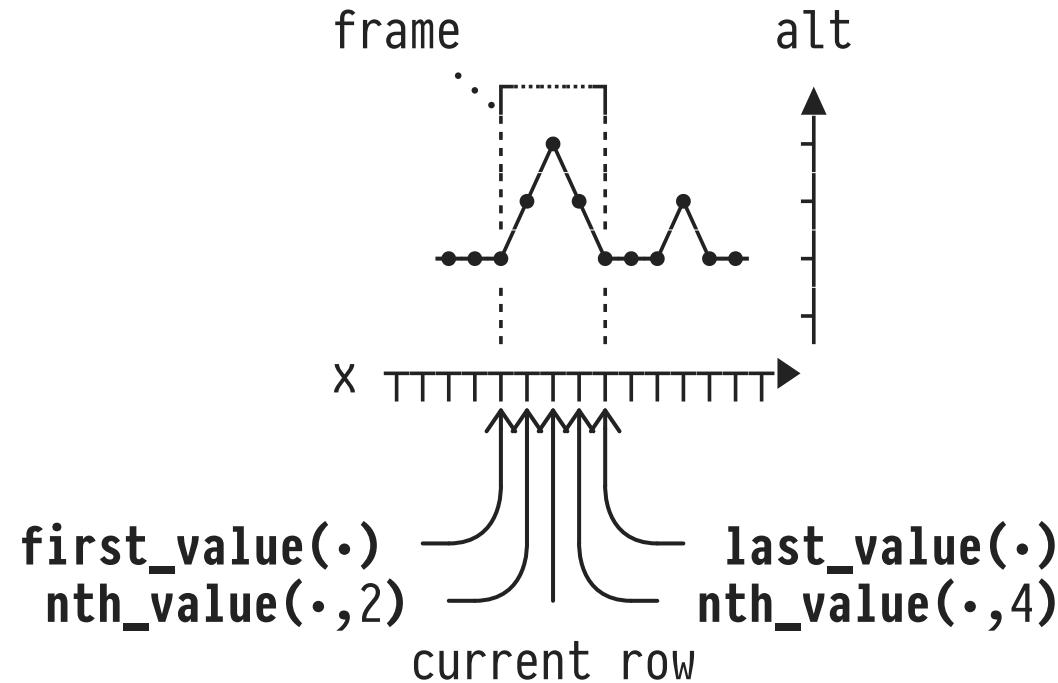
1. Track relative altitude changes in a sliding **x-window of size 5:**

valley features

string-encoded altitude changes (`'➚➘➚'` :: text)

peak features

2. **Pattern match** on change strings to detect features.

🔧 Altitude Changes in a Sliding Window

- Frame: ROWS BETWEEN 2 PRECEDING AND 2 FOLLOWING (5 rows):

- first_value(alt) < nth_value(alt,2) ⇒ ascent ('↗').

🔧 Altitude Changes in a Sliding Window

```sql
-- Find slopes in ±2 vicinity around point x

SELECT m.x,
  slope(sign(first_value(m.alt) OVER w-nth_value(m.alt,2) OVER w)) ||
  slope(sign(nth_value(m.alt,2) OVER w-m.alt                   )) ||
  slope(sign(m.alt                     -nth_value(m.alt,4) OVER w)) ||
  slope(sign(nth_value(m.alt,4) OVER w-last_value(m.alt)  OVER w))
FROM   map AS m
WINDOW w AS (ORDER BY m.x ROWS BETWEEN 2 PRECEDING AND 2 FOLLOWING)
```

- Recall: 1D landscape represented as table map(x,alt).
- Encode alt changes: slope(-1) ≡ '↗', slope(0) ≡ '→'
  (here: slope(•) implemented as a DuckDB macro).

Row Pattern Matching (SQL:2016)

- -

SQL:2016 introduced an entirely new SQL construct, **row pattern matching** (MATCH_RECOGNIZE):

1. ORDER BY: Order the rows of a table.
2. DEFINE: Tag rows that satisfy given predicates.
3. PATTERN: Specify a **regular expression over row tags**, find matches in the ordered sequence of rows.
4. MEASURES: For each match, evaluate expressions that measure its features (matched rows, length, …).

⚠️ As of December 2025, not supported by 🔘.
(Work is underway here in Tübingen to rectify this.)

Row Pattern Matching (SQL:2016)

```
SELECT *                            Output
FROM    map
MATCH_RECOGNIZE (
 ORDER BY x
 MEASURES first(x,1)     AS x,
          match_number() AS feature,
          classifier()   AS slope
 ONE ROW PER MATCH
 AFTER MATCH SKIP TO NEXT ROW
 PATTERN (((DOWN DOWN|DOWN EVEN|UP DOWN|EVEN DOWN)…)
 DEFINE UP    AS UP.alt   > PREV(UP.alt),   --
        DOWN  AS DOWN.alt < PREV(DOWN.alt), --  } row tags
        EVEN  AS EVEN.alt = PREV(EVEN.alt)  --
)
```

x	feature	slope
50	1	DOWN
70	2	UP
100	3	DOWN

8 ┊ Numbering and Ranking Rows

Countless problem scenarios involve the **number** (position) or **rank** of the current row in an *ordered sequence* of rows.

- Family of **window functions to number/rank rows:**

```
row_number()
                        -- intra-partition ranking 👍
dense_rank()            --
rank()          OVER ([ PARTITION BY p₁,…,pₘ ]
                      [ ORDER BY e₁,…,eₙ ] )
percent_rank()          --
cume_dist()             -- ranking w/o ORDER BY 👎
ntile(n)
```

- Scope is partition (if present)—*frame* is irrelevant.

Numbering and Ranking Rows — *f*() OVER (ORDER BY *A*)

Table W   *f*

row	A	row_number	dense_rank	rank
ϱ1	1	1	1	1
ϱ2	2	2	2	2
ϱ3	**3**	3	3	3
ϱ4	**3**	4	3	3
ϱ5	**3**	5	3	3
ϱ6	4	6	4	6
ϱ7	**6**	7	5	7
ϱ8	**6**	8	5	7
ϱ9	7	9	6	9

⋯ Rows that agree on
⋯ the sort criterion
⋯ (here: *A*) …
- … number randomly
- … rank equally

⌶ Mind the ranking gap
 (think Olympics)

- In general: dense_rank() ⩽ rank() ⩽ row_number()

🔧 Once More: Find the Top *n* Rows in a Group

species	length	height	legs
⋮	⋮	⋮	∈ {2,4,NULL}

Table dinosaurs

```
SELECT tallest.legs, tallest.species, tallest.height
FROM (SELECT d.legs, d.species, d.height,

row_number()⋯ rank() OVER (PARTITION BY d.legs
                           ORDER BY d.height DESC) AS n
        FROM  dinosaurs AS d
        WHERE d.legs IS NOT NULL) AS tallest
WHERE tallest.n <= 3
```

- rank() *vs* row_number(): both OK, but different semantics!
- Need a subquery: window functions *not* allowed in WHERE (but see SQL filtering clause QUALIFY).

🔧 Identify Consecutive Ranges

- What you often encounter in scientific papers ☎:
  "... *as Knuth has shown in [5,2,14,3,1,42,6,10,7,13]* ..."

- What you want to see ☺:
  "... *as Knuth has shown in [1–3,5–7,10,13&14,42]* ..."

Table **citations**

ref
5
2
⋮
13

Output

ref	range
1	r_0
2	r_0
⋮	⋮
42	r_4

references belong to the same range

🔧 Identify Consecutive Ranges (Query Plan)

❶	❷					
ref	ref	**row_number()**				
5	1	−	1	=	0	⎤
2	2	−	2	=	0	⎬ range 0 ≡ r_0
14	3	−	3	=	0	⎦
3	5	−	4	=	1	⎤
1	6	−	5	=	1	⎬ range 1 ≡ r_1
42	7	−	6	=	1	⎦
6	10	−	7	=	3	⎬ range 3 ≡ r_2
10	13	−	8	=	5	⎤ range 5 ≡ r_3
7	14	−	9	=	5	⎦
13	42	−	10	=	32	⎬ range 32 ≡ r_4

subtract

Numbering and Ranking Rows — f OVER (ORDER BY A)
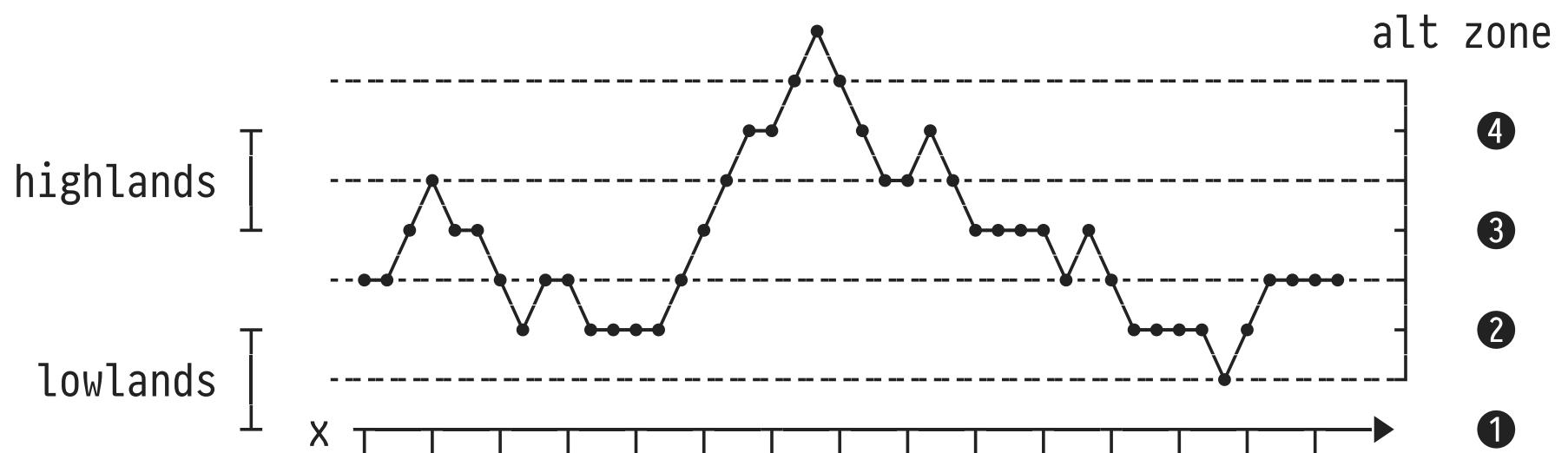
row	A	percent_rank	cume_dist	ntile(3)
ρ1	1	0	$1/9$	1
ρ2	2	$1/8$	$2/9$	1
ρ3	**3**	$2/8$	$5/9$	1
ρ4	**3**	$2/8$	$5/9$	2
ρ5	**3**	$2/8$	$5/9$	2
ρ6	4	$5/8$	$6/9$	2
ρ7	**6**	$6/8$	$8/9$	3
ρ8	**6**	$6/8$	$8/9$	3
ρ9	7	$8/8$	$9/9$	3

⋯ Rows that agree on
⋯ the sort criterion
⋯ (here: A) rank equally

← current row is in the nth of 3 chunks of rows

n% of the other rows rank lower than the current row

the current row and lower ranked rows make up n% of all rows

🔧 Altitudinal Mountain Zones

- Classify the altitudes of a mountain range into
  1. **equal-sized** vegetation **zones** and
  2. lowlands (altitude in the lowest **20%**) and highlands (between **60%–80%** of maximum altitude).

🔧 Altitudinal Mountain Zones
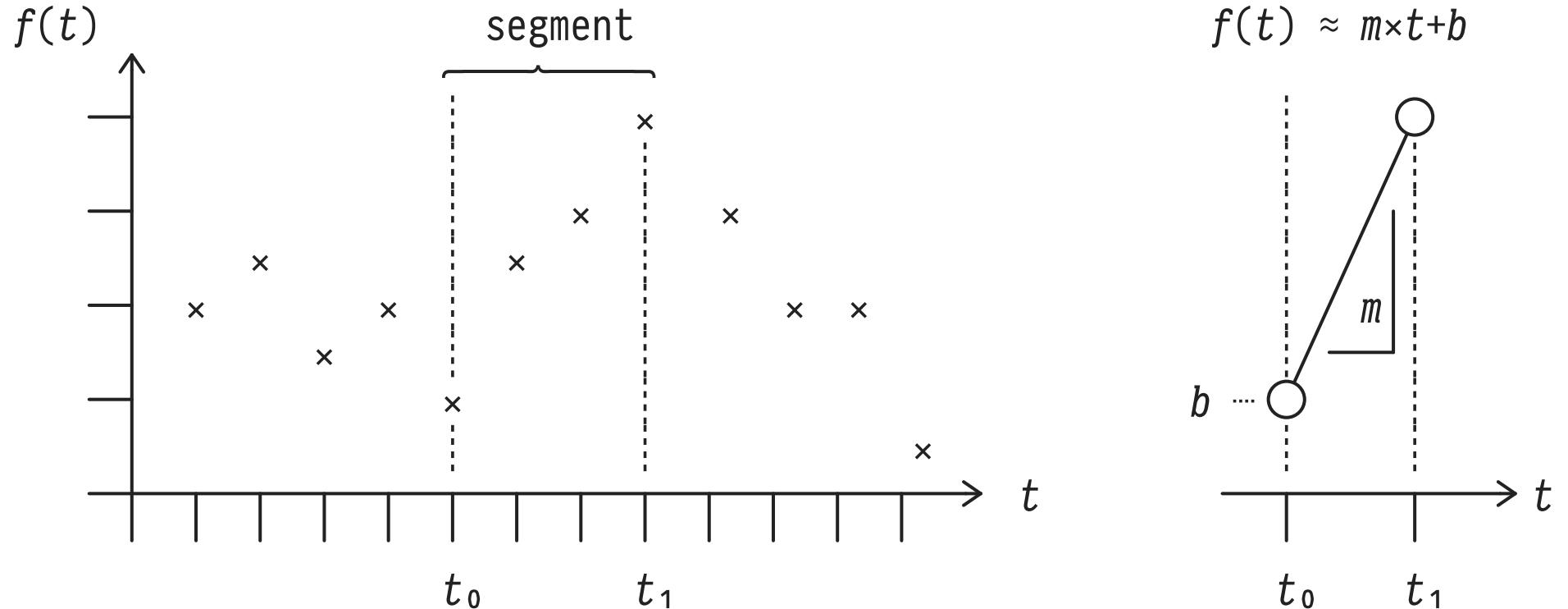
```sql
-- Classify altitudinal zones in table mountains(x, alt)
--
SELECT
  m.x, m.alt,
  ntil(4) OVER altitude AS zone,
  CASE
    WHEN percent_rank() OVER altitude BETWEEN 0.6 AND 0.8
        THEN 'highlands'
    WHEN percent_rank() OVER altitude < 0.2
        THEN 'lowlands'
        ELSE '-'
  END AS region
FROM    mountains AS m
WINDOW altitude AS (ORDER BY m.alt)
ORDER BY m.x;
```

🔧 Linear Approximation of a Time Series

1. `ntile(n)` **segments** time series at desired granularity.
2. Compute m, b in each **segment** ≡ **window frame**.

9 ⋮ Summary: Window Function Semantics[2]

Scope	Computation	Function	Description
frame	aggregation	(aggregates)	sum, avg, max, list, …
	row access	first_value(e)	e at first row in frame
		last_value(e)	e at last row in frame
		nth_value(e,n)	e at n^{th} row in frame
partition	row access	lag(e,n,d)	e at n rows *before* current row
		lead(e,n,d)	e at n rows *after* current row
	ranking	row_number()	number of current row
		rank()	rank with gaps ("Olympics")
		dense_rank()	rank without gaps
		percent_rank()	relative rank of current row
		cume_dist()	ratio of rows up to —"—
		ntile(n)	rank on a scale $\{1,2,…,n\}$

[2] first_value(e): expression e will be evaluated *as if* we are at the first row in the frame.
lag(e,n,d): default expression d is returned if there is no row at offset n before the current row.