INTRODUCTION TO RELATIONAL DATABASE SYSTEMS

DATENBANKSYSTEME 1 (INF 3131)

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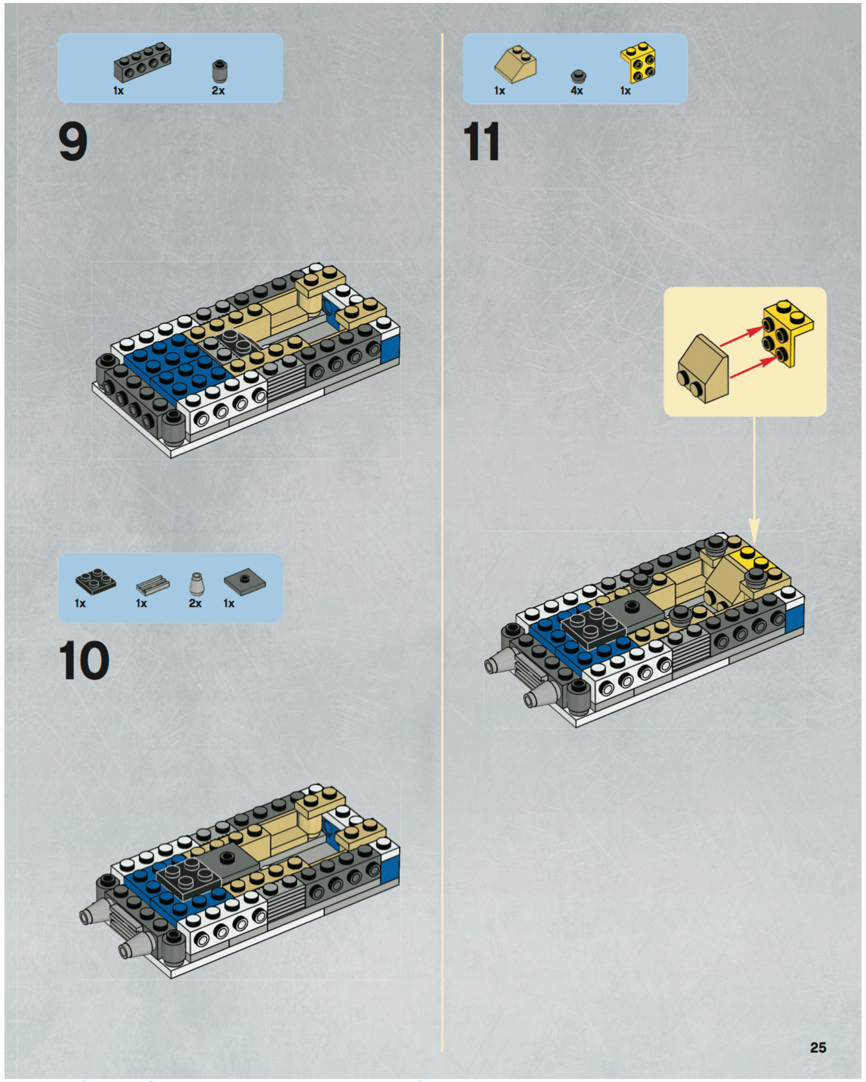
LEGO BUILDING INSTRUCTIONS

- Each LEGO *set* comes with *building instructions*, an illustrated booklet that details the individual steps of model construction.
  - One *page* in the booklet holds one or more instruction *steps* (steps are numbered 1, 2, ...).
  - Each step lists the *pieces* (with *color* and *quantity*) required to complete the step.
  - Each step comes with an *illustration* of where the listed pieces find their place in the model.
- What would be a reasonable **design for a building instructions table**? Clearly:

1. Do not include LEGO set details in instructions: instead, use a **foreign key** to refer to table sets.
2. Do not include LEGO piece details in instructions: instead, use a **foreign key** to refer to table bricks.
3. Represent page numbers, step numbers, image sizes as integers but formulate **constraints** that avoid data entry errors (e.g. negative page/step numbers).

LEGO BUILDING INSTRUCTIONS



Page 25 in Building Instruction for LEGO Set 9495 (Y-Wing)

LEGO BUILDING INSTRUCTIONS (TABLE DESIGN)

instructions

set	step	piece	color	quantity	page	img	width	height
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
9495-1	7	3010	2	2	24	<image07>	639	533
9495-1	7	3023	2	2	24	<image07>	639	533
9495-1	7	2877	86	1	24	<image07>	639	533
9495-1	8	3002	7	2	24	<image08>	650	522
9495-1	8	30414	1	2	24	<image08>	650	522
9495-1	9	30414	85	1	25	<image09>	541	638
9495-1	9	3062b	85	2	25	<image09>	541	638
9495-1	10	30033	11	1	25	<image10>	540	662
9495-1	10	2412b	86	1	25	<image10>	540	662
9495-1	10	4589b	86	2	25	<image10>	540	662
9495-1	10	87580	85	1	25	<image10>	540	662
9495-1	11	3039	2	1	25	<image11>	1042	558
9495-1	11	4073	85	4	25	<image11>	1042	558
9495-1	11	44728	3	1	25	<image11>	1042	558
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

REDUNDANCY

- The design of table instructions appears reasonable. We immediately spot a fair amount of **redundancy**, though. For example:

1. Step 10 of Set 9495 is printed on page 25. [represented 4 ×]
2. Step 7 of Set 9495 is illustrated by ‹image07›. [3 ×]
3. ‹image09› has dimensions 541 × 638 pixels. [2 ×]

- **Redundancy** comes with a number of serious problems, most importantly:
  - **Storage space is wasted.**
    Tables occupy more disk space than needed. Query processor has to touch/move more bytes. Archival storage (backup) requires more resources.
  - **Redundant copies will go out of sync.**
    Eventually, an update operation will miss a copy. The database instance now contains "multiple truths." Typically, this goes unnoticed by DBMS and user.

EMBEDDED FUNCTIONS AND REDUNDANCY

- In table instructions, the source of redundancy is the presence of **functions** that are **embedded in the table.**

Leibniz Principle
If f is a function defined on x and y, then

$$x = y \land f(x) = z \Rightarrow f(y) = z$$

- **Table instructions embeds the materialized functions**

1. printed_on(): maps set, step to the page it is printed on
2. illustrated_by(): maps set, step to the illustration stored in image img
3. image_size(): maps an image img to its width and height

FUNCTIONAL DEPENDENCIES

Functional Dependency (FD)

Let (R, α) denote a relational schema. Given $\beta \subseteq \alpha$ and $c \in \alpha$, the **functional dependency** $\beta \rightarrow c$ holds in R if

$$\forall\ t,u \in inst(R) : t.\beta = u.\beta \Rightarrow t.c = u.c$$

Read: "*If two rows agree on the columns in β, they also agree on column c.*" (β: function arguments, c: function result).

Notation: the FD $\beta \rightarrow \{c_1,\ldots,c_n\}$ abbreviates the set of FDs $\beta \rightarrow c_1$, …, $\beta \rightarrow c_n$.

- Note: If $c \in \beta$, then $\beta \rightarrow c$ is called a **trivial FD** that obviously holds for any instance of R. No interesting insight into R here.

FUNCTIONAL DEPENDENCIES

- FDs are **constraints** that document universally valid mini-world facts (e.g., "*a step is associated with one illustration*"). FDs thus need to hold in *all* database instances.

instructions

set	step	piece	color	quantity	page	img	width	height
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
9495-1	7	3010	2	2	24	<image07>	639	533
9495-1	7	3023	2	2	24	<image07>	639	533
9495-1	7	2877	86	1	24	<image07>	639	533
9495-1	8	3002	7	2	24	<image08>	650	522
9495-1	8	30414	1	2	24	<image08>	650	522
9495-1	9	30414	85	1	25	<image09>	541	638
9495-1	9	3062b	85	2	25	<image09>	541	638
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

- Which functional dependencies hold in table instructions?

FUNCTIONAL DEPENDENCIES

- Given table R, check whether the FD $\{b_1,\ldots,b_n\} \rightarrow c$ holds in the current table instance:

```
SELECT DISTINCT 'The FD {b₁, …, bₙ} → c does not hold'
FROM   R AS r
GROUP BY r.b₁, …, r.bₙ
HAVING COUNT(DISTINCT r.c) > 1
```

Aggregate Functions

Optional modifier DISTINCT affects the computation of aggregate functions:

```
<aggregate>([ ALL ] expression)  -- aggregate all non-NULL values
<aggregate>(DISTINCT expression) -- aggregate all distinct non-NULL values
<aggregate>(*)                   -- aggregate all rows (count(*))
```

KEY → FD

- Note that a **key** implicitly defines a **particularly strong FD:** the key columns functionally determine *all* columns of the table.

Keys vs FDs (1)

Assume table $(R, \{a_1,\ldots,a_k, a_{k+1},\ldots,a_n\})$.

$$\{a_1,\ldots,a_k\} \text{ is a key of } R \Leftrightarrow$$
$$\{a_1,\ldots,a_k\} \rightarrow \{a_{k+1},\ldots,a_n\} \text{ holds.}$$

- So, keys are special FDs.
- Turning this around: FDs are a generalization of keys.

FD → (LOCAL, PARTIAL) KEY

Keys vs FDs (2)

Assume table R and FD $\beta \rightarrow c$. Then β is key in the sub-table of R defined by

SELECT DISTINCT β, c
FROM R

- **Example:** for table instructions and FD {set, step} → page the sub-table is

set	step	page
⋮	⋮	⋮
9495-1	7	24
9495-1	8	24
9495-1	9	25
9495-1	10	25
9495-1	11	25
⋮	⋮	⋮

(i.e., exactly the table materializing the function printed_on(), see above).

FUNCTIONAL DEPENDENCIES

- **Example:** recall table stores of the LEGO Data Warehouse scenario:

<u>store</u>	city	state	country
⋮	⋮	⋮	⋮
7	HAMBURG	Hamburg	Germany
8	LEIPZIG	Sachsen	Germany
9	MÜNCHEN	Bayern	Germany
10	MÜNCHEN PASING	Bayern	Germany
11	NÜRNBERG	Bayern	Germany
⋮	⋮	⋮	⋮
16	ARDEN FAIR MALL	CA	USA
17	DISNEYLAND RESORT	CA	USA
18	FASHION VALLEY	CA	USA
⋮	⋮	⋮	⋮

- List the FDs that hold in table stores.
- Does the mini-world suggest FDs not implied by the rows shown above?

FUNCTIONAL DEPENDENCIES

- An FD indicates the presence of a **materialized** function. Consider the following variant of the users and ratings table:

users

<u>user</u>	rating	stars
Alex	3	***
Bert	1	*
Cora	4	****
Drew	5	*****
Erik	1	*
Fred	3	***

- FD {rating} → stars materializes the **computable function** stars = f(rating) = repeat('*', rating) [see PostgreSQL's string function library].
- In such cases, good database design should consider to **trade materialization for computation**. Removes redundancy.

SQL: VIEWS

CREATE VIEW

Binds *query* to *name* which is **globally** visible. Whenever table *name* is referenced in subsequent queries, *query* is **re-evaluated** and its result returned (*no materialization* of the result of *query* is performed):

```
-- TEMPORARY: automatically drop view after current session
CREATE [ OR REPLACE ] [ TEMPORARY ] VIEW name
  AS query
```

- Compare with CTEs: local visibility in surrounding WITH statement only.
- A temporary view named *name* shadows a (regular, persistent) table of the same name.

SQL: VIEWS

- Views provide **data independence:** users and applications continue to refer to *name*, while the database designer may decide to replace a persistent table with a computed query or vice versa.
- **Example:** turn the materialized function stars = f(rating) into a computed function:

```
-- drop the materialized function from the table
ALTER TABLE users
  DROP COLUMN stars;

-- provide the three-column table that users/applications expect
CREATE TEMPORARY VIEW users(user, rating, stars)
  AS SELECT u.user, u.rating, repeat('*', u.rating) AS stars
     FROM   users AS u;
```

  - Since PostgreSQL's repeat() is a pure function, the FD rating → stars trivially holds in the view.

DERIVING FUNCTIONAL DEPENDENCIES

- Given a set F of FDs over table R, simple inference rules—the **Armstrong Axioms**—suffice to generate *all* FDs following from those in F.

Armstrong Axioms
Apply exhaustively to generate all FDs implied by FD set F.

Reflexivity:
If $\gamma \subseteq \beta$, then $\beta \rightarrow \gamma$.

Augmentation (with $c \in sch(R)$):
If $\beta \rightarrow \gamma$, then $\beta \cup \{c\} \rightarrow \gamma \cup \{c\}$

Transitivity:
If $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.

- Note: transitivity closely relates to **function composition:** if f, g are functions, so is $g \circ f$.

DERIVING FDS (COVER)

- **Problem:** Given a set $\alpha \subseteq sch(R)$ of columns and a set of FDs F over R, compute the **cover** α^+, i.e., the set of all columns functionally determined by α.

Cover

The **cover** α^+ of a set of columns α is the set of all columns c that are functionally determined by the columns in α (with respect to a given FD set F):

$$\alpha^+ := \{ \, c \mid F \text{ implies } \alpha \to c \, \}$$

- ⚠️ Should we find that $\alpha^+ = sch(R)$, then α is a super key for R.

DERIVING FDS (COVER)

- Compute the cover α^+ for a given set of FDs:

```
cover(α,F)  (input: column set α, FD set F, output: α⁺):

  ❶ X ← α

  ❷ repeat
    │ for each FD β → c ∈ F do
    │ │ if β ⊆ X then
    │ └ └ X ← X ∪ {c}
    until X did not change

  ❸ return X
```

DERIVING FDS (COVER)

- **Example:** In table instructions, compute $\{\text{set},\text{step}\}^+$ with $F = \{\ \{\text{set},\text{step}\} \rightarrow \text{page},\ \{\text{set},\text{step}\} \rightarrow \text{img},\ \{\text{img}\} \rightarrow \text{width},\ \{\text{img}\} \rightarrow \text{height}\ \}$.

instructions

set	step	piece	color	quantity	page	img	width	height

- Tracing column set X:

```
❶  X := {set,step}

⟲❷  FD {set,step} → page, {set,step} ⊆ X:  X := X ∪ {page}
    FD {set,step} → img,  {set,step} ⊆ X:  X := X ∪ {img}
    FD {img} → width,  {img} ⊆ X:          X := X ∪ {width}
    FD {img} → height, {img} ⊆ X:          X := X ∪ {height}

    All FDs considered.  X = {set, step, page, img, width, height}
    Repetition of ❷ does not add new columns to X.

❸  Return {set, step, page, img, width, height}.
```

DERIVING CANDIDATE KEYS

```
key(K,U,F)  (input: FD set F, output: set of all candidate keys for R)

  if U = ∅ then                                    } invariant: cover(K ∪ U, F) = sch(R)
  | return {K}
  else
  | X ← ∅
  | for each c ∈ U do
  | | if c ∉ cover(K ∪ (U \ {c}), F) then          } is c essential for the key?
  | | | X ← X ∪ key(K ∪ {c}, U \ {c}, F)           ⚑
  | | else
  | ⌊ ⌊ X ← X ∪ key(K, U \ {c}, F)
  |
  ⌊ return X
```

- Invoke via **key**($\emptyset$, $sch(R)$, F).
- Can optimize at ⚑: invoke **key**($K \cup \{c\}$, $U \setminus$ **cover**($K \cup \{c\}$, F), F) instead.

DATABASE DESIGN WITH FDS

- Typically it is a severe sign of **poor database design if tables embed functions**, i.e., if a table contains

FDs that are not implied by the primary key. ⚠️

- Consequences of table designs with non-key FDs / embedded functions:

1. **Redundancy** (see above ✓)
2. Update/Insertion/Deletion **Anomalies**
3. **RDBMS cannot protect the integrity** of non-key FDs, thus risk of inconsistency over time:
   - SQL DDL does *not* implement an `ALTER TABLE … ADD FUNCTIONAL DEPENDENCY …` statement.
   - Although FDs embody important mini-world facts they are easily violated without protection. (Can simulate this protection using SQL triggers or rewrite rules. Cumbersome. Inefficient.)

UPDATE/INSERTION/DELETION ANOMALIES

- Recall table instructions and embedded FD {img} → {width, height}:

instructions

set	step	piece	color	quantity	page	img	width	height

- **Update anomaly:**
  Changing a **single mini-world fact** requires the modification of **multiple rows.**
  [Modifying image size requires to search/update entire instruction table.]
- **Insertion anomaly:**
  A new mini-world **fact cannot be stored** unless it is put in larger context.
  [No place to record width/height dimension of a new image yet unused in an instruction manual.]
- **Deletion anomaly:**
  A formerly stored mini-world fact vanishes once its (last) context is deleted.
  [Information about image width/height is lost once last instruction manual including that image is deleted from instructions.]

BOYCE–CODD NORMAL FORM

Boyce–Codd Normal Form (BCNF)

Table R is in **Boyce–Codd Normal Form (BCNF)** if and only if all its FDs are already implied by its key constraints.

For table R in BCNF and any FD $\beta \rightarrow c$ of R one of the following holds:

1. The FD is trivial, i.e., $c \in \beta$.
2. The FD follows from a key because β (or a subset of it) already is a key of R.

- A table in BCNF does not exhibit the three anomalies (no embedded functions).
- All FDs in table in BCNF are protected by the RDBMS through PRIMARY KEY (or UNIQUE) constraints.

BOYCE–CODD NORMAL FORM

- **Examples:**
  - Table instructions is not in BCNF: key FD {set, step, piece, color} → {quantity, page, img, width, height} does not imply {set, step} → {page, img} or {img} → {width, height}:

    instructions

set	step	piece	color	quantity	page	img	width	height

  - Table users not in BCNF: {rating} → {stars} not implied by key FD:

    users

name	rating	stars

  - Table stores not in BCNF: {state} → {country} not implied by key FD:

    stores

store	city	state	country

BCNF SCHEMA DECOMPOSITION

split(R,F) (**input:** table R with FD set F **output:** splitted relation schemata)

  if $\beta \rightarrow c \in F$ with $c \notin \beta$ and β does not contain a key of R **then**
  split and replace R:

$$\overbrace{R_1((sch(R) \setminus \mathbf{cover}(\beta, F)) \cup \beta) \qquad R_2(\mathbf{cover}(\beta, F))}^{R}$$

  split(R_1, $F|sch(R_1)$)
  split(R_2, $F|sch(R_2)$)

- Notes:
  - $F|C$ denotes FD set F restricted to those $\beta \rightarrow c$ for which $\beta \cup \{c\} \subseteq C$.
  - For each split: $sch(R_1) \cup sch(R_2) = sch(R)$ and $sch(R_1) \cap sch(R_2) = \beta$.

BCNF: AFTER DECOMPOSITION (1)

- Resultant BCNF tables after **split**(instructions, *F*) has been completed:

parts (1/3)

set	step	piece	color	quantity
⋮	⋮	⋮	⋮	⋮
9495-1	7	3010	2	2
9495-1	7	3023	2	2
9495-1	7	2877	86	1
9495-1	8	3002	7	2
9495-1	8	30414	1	2
9495-1	9	30414	85	1
9495-1	9	3062b	85	2
⋮	⋮	⋮	⋮	⋮

- Note: It is rather straightforward to name the newly generated tables: these tables represent a *single* real-world concept.

BCNF: AFTER DECOMPOSITION (2)

- Resultant BCNF tables after **split**(instructions, *F*) has been completed:

layouts (2/3)

set	step	page	img
⋮	⋮	⋮	⋮
9495-1	7	24	<image07>
9495-1	8	24	<image08>
9495-1	9	24	<image09>
⋮	⋮	⋮	⋮

illustrations (3/3)

img	width	height
⋮	⋮	⋮
<image07>	639	533
<image08>	650	522
<image09>	541	638
⋮	⋮	⋮

- To tie the BNCF tables together, establish foreign keys pointing from parts to layouts and from layouts to illustrations.

BCNF: RECONSTRUCTION

- Use an **equi-join to reconstruct** the original wide table instructions from its constituent tables:

Reconstruction after BCNF decomposition

Perform an equi-join over the (non-empty) schema intersections of the BCNF tables:

```
SELECT p.set, p.step, p.piece, p.color, p.quantity,
       l.page, l.img,
       i.width, i.height
FROM   parts AS p, layouts AS l, illustrations AS i
WHERE  p.set = l.set AND p.step = l.step
AND    l.img = i.img
```

- It may make sense to use CREATE VIEW to reestablish the wide table for users and applications.

BCNF: AFTER DECOMPOSITION

- Decomposition for table users:

users (R_1)

user	rating
Alex	3
Bert	1
Cora	4
Drew	5
Erik	1
Fred	3

render (R_2)

rating	stars
1	*
3	***
4	****
5	******

- The RDBMS protects the FDs (keys): translation from rating to stars in table render is always consistent. No redundancy in table render.

BCNF DECOMPOSITION: LOSSLESS SPLITS

- BNCF decomposition builds on the assumption that **no information is lost** during the splits: original table R can be reconstructed by an equi-join of R_1 and R_2.
- Not all decompositions are **lossless**, however. Consider:

R

A	B	C
a_1	b_1	c_1
a_1	b_1	c_2
a_1	b_2	c_1

and its decomposition into R_1(A, B), R_2(A, C). This is a somewhat arbitrary split, *not* suggested by an FD. The equi-join of R_1 and R_2 (on A) is:

A	B	C
a_1	b_1	c_1
a_1	b_1	c_2
a_1	b_2	c_1
a_1	b_2	c_2

$\Rightarrow$ An extra (bogus!) row has been reconstructed by the join. **Information has been lost.**

BCNF: LOSSLESS SPLITS

Decomposition Theorem

Consider the decomposition of table R into R_1 and R_2. The reconstruction of R from R_1, R_2 via an equi-join on $sch(R_1) \cap sch(R_2)$ is **lossless** if

1. $sch(R_1) \cup sch(R_2) = sch(R)$ and
2. $sch(R_1) \cap sch(R_2)$ is a key of R_1 or R_2 (or both).

- The splits "along the FD $\beta \rightarrow c$" performed by **split**(R,F) will always be lossless:

  1. $sch(R_1) \cup sch(R_2) = sch(R)$ and $sch(R_1) \cap sch(R_2) = \beta$ ✔
  2. Since $sch(R_2) =$ **cover**(β,F), β is a key for R_2 ✔

- We will never lose information through BCNF decomposition.

BCNF: NON-DETERMINISM, LOSS OF FDS

- ⚠️ BCNF is **not deterministic:** arbitrary choice of the “split FD” in algorithm **split**(R,F) leads to different decompositions, in general:

 1. For table instructions: splitting along FD {set,step} → {page, img} or {img} → {width,height} first makes no difference. (Try it.)
 2. But consider R(<u>A,B</u>,C,D,E) with FDs {C,D} → E and {B} → E.

- ⚠️ BCNF decomposition **may fail to preserve dependencies:** given FD $\beta \rightarrow c$, the column set $\beta \cup \{c\}$ may be distributed across multiple tables. The FD is “lost” (cannot be enforced by the system).

  - Consider FDs {zip} → {city, state} and {street, city, state} → zip in table zipcodes.

    1. What are the candidate keys for zipcodes?
    2. What is a BCNF decomposition for zipcodes?

zipcodes

zip	street	city	state

DENORMALIZATION VS. DECOMPOSITION

- BCNF and decomposition come with significant benefits but are no panacea. There are valid reasons to leave database tables in **denormalized form:**

1. **Performance:**
   Decomposition requires table reconstruction via equi-joins which incur query evaluation costs. Denormalized table save this effort at the cost of storing information redundantly.

2. **Preservation of FDs:**
   In specific applications, preservation of mission-critical FDs may be a higher priority than the removal of redundancy.

- **Columnar database systems** perform **full decomposition** (beyond the splits required by BCNF normalization): $R(\underline{id}, A, B, C, \ldots)$ decomposed into $R_1(\underline{id}, A)$, $R_2(\underline{id}, B)$, $R_3(\underline{id}, C)$, … (binary tables).
  - Queries other than `SELECT r.* FROM R AS r` can selectively access the R_i, reading less bytes from persistent storage.
  - DBMS internals simplified: every row is guaranteed to have exactly two fields.