DB 2

09 – Ordered Indexes (B^+Trees)

Summer 2018

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1 ¦ Q_8 — Filtering an Indexed Table

Sequential scan (**Seq Scan**) and interpreted predicate evaluation go a long way. Large input tables call for significantly more **efficient support for value-based row access:**

```
SELECT i.b, i.c
FROM   indexed AS i
WHERE  i.a = 42 [i.c = 0.42] -- either filter on i.a or i.c
```

> Assume column a is **primary key** in table indexed: expect query workload that frequently identifies rows via predicates $a = k$. **Indexes** can support such queries.

Primary Key and Indexes

```
CREATE TABLE indexed (a int PRIMARY KEY, -- ⇒ NOT NULL
                      b text,
                      c numeric(3,2));   -- ± d.dd
```

DBMS expects predicates $a = k$ and creates an **index on column a**—a data structure associated with and maintained in addition to table indexed—to speed up evaluation:

```
CREATE INDEX indexed_a ON indexed USING btree (a);
```

1. Whenever possible/promising, index indexed_a[1] is (also) consulted when table indexed is queried.
2. When indexed is updated, indexed_a is maintained.

[1] PostgreSQL chooses index name indexed_pkey but let's follow a ‹*table*›_‹*column*› naming scheme here.

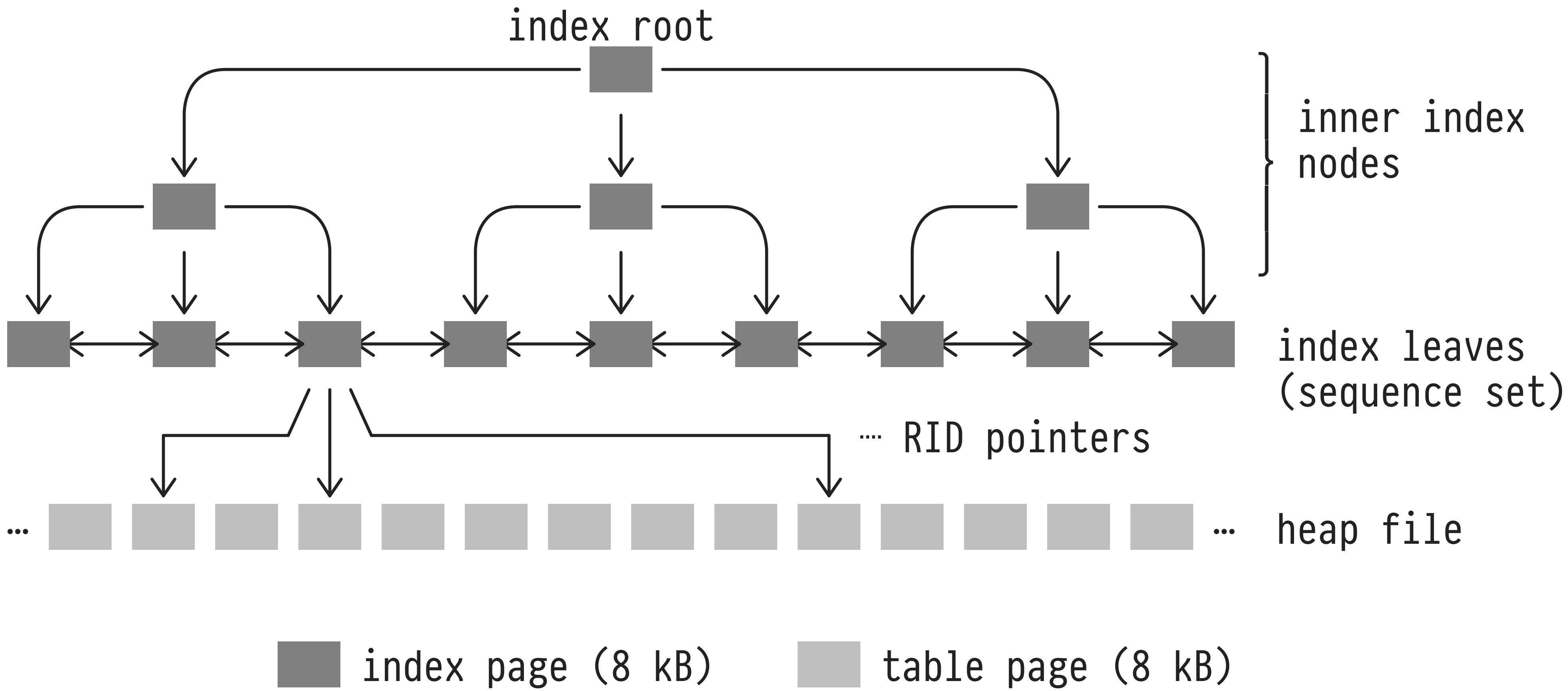
Using **EXPLAIN** on Q_8

```
EXPLAIN VERBOSE
  SELECT i.b, i.c
  FROM   indexed AS i    -- 10^6 rows
  WHERE  i.a = 42;       -- selection on key column a ⇒ ≤ 1 row will qualify
```

QUERY PLAN

Index Scan using indexed_a **on** indexed i (**cost**=0.42..8.44 **rows**=1 …)
  Output: b, c
  Index Cond: (i.a = 42)

- DBMS uses **Index Scan** (instead of **Seq Scan**), index scan will evaluate predicate i.a = k.
- System expects small result of a single row (rows=1), i.e., the predicate is assumed to be *very selective*.



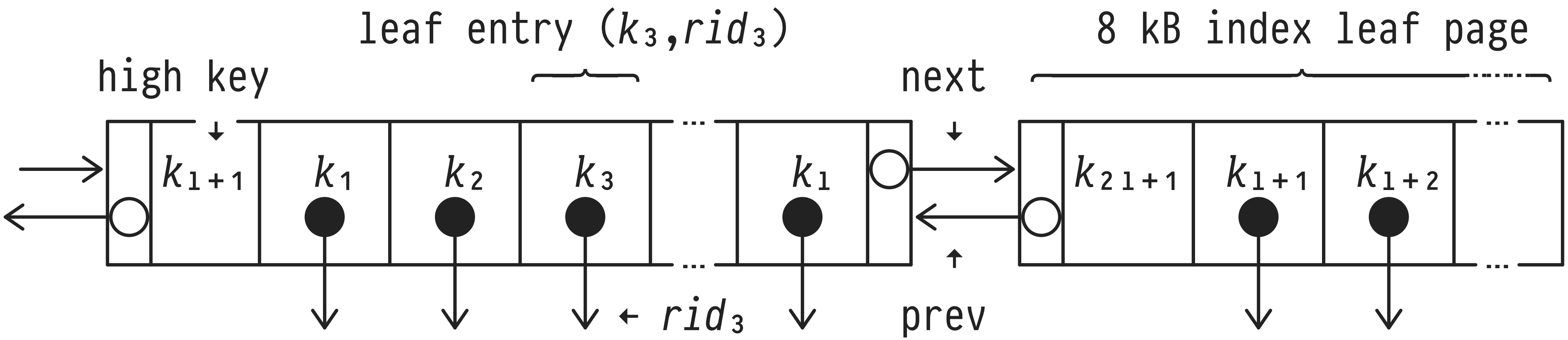
Anatomy of a B⁺Tree

Notes on B⁺Tree anatomy:

- A **B⁺Tree[2] index *I* on column *T*(*a*)** is an *ordered*, *n-ary* (*n* » 2), *balanced*, *block-oriented*, *dynamic* search tree.
- Inner nodes and leaves are formed by 8 kB **index pages.**
- Each inner node holds *n*−1 values of column *a* (**separators**) that allow to navigate the search tree structure.
- Leaves form a bidirectional chain, the **sequence set.**
- **Leaves use RIDs to point to rows** in the heap file of table *T*: besides *a* column values, *I* holds no data of *T*.

[2] Invented by Bayer and McCreight (1969) at Boeing Labs. The "B" in "B⁺Tree" does *not* stand for Bayer, binary, balanced, block, or Boeing. (We tried to find out, but Rudolf Bayer wouldn't say.)

B⁺Trees: Inside a Leaf Node



- Uses pointers prev/next to form the chained **sequence set.**
- **Leaf entries are ordered** by index keys k_i: $k_i \leqslant k_{i+1}$.
- RID rid_i points to a row t of T with $t.a = k_i$.
- The **high key** holds smallest key of *next* leaf (if any).

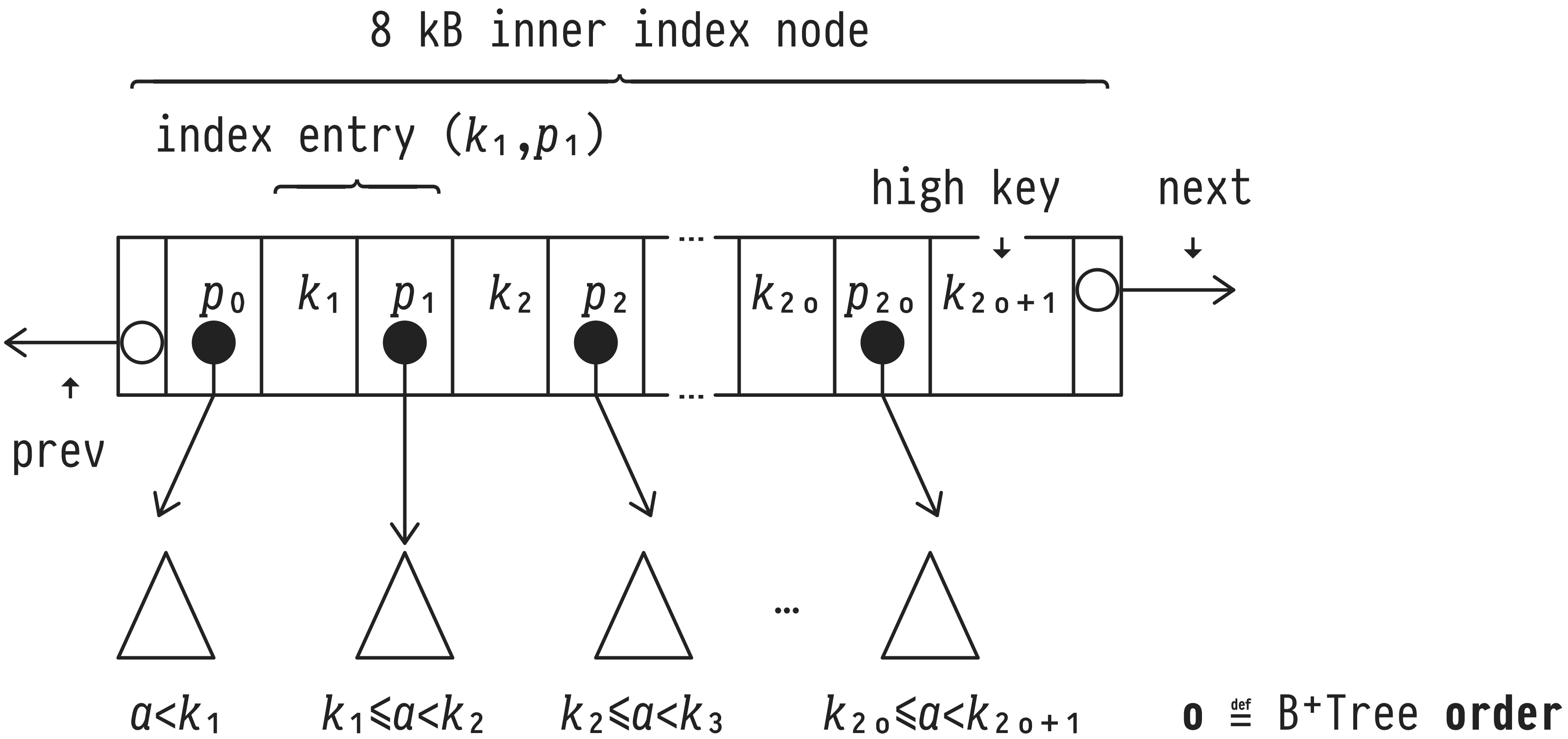
B⁺Trees: How to Find Rows t With $t.a = k$?

As described, a B⁺Tree is a **dense** index structure: every row t of T is represent by one leaf entry.

- The sequence set is ordered by keys k_i $\Rightarrow$ a *binary search* for a key $k = k_i$ may sound viable, **BUT** the search would
  1. need to inspect $\log_2(|T|)$ keys in the sequence set and access just as many pages 👎, and
  2. “jump around” the sequence set in an unpredictable fashion, thus leading to random I/O. 👎

B⁺Trees exploit the sequence set ordering and erect an ***n*-ary search tree structure** (n large!) atop the leaf entries.

B⁺Trees: Inside an Inner Node



- The **separator** keys k_i are ordered: $k_i \leqslant k_{i+1}$.
- Page pointers p_j point to index (leaf or inner) nodes.

- Space in inner nodes is used economically: in a B⁺Tree of **order** o, any inner node—but the root node—is guaranteed to hold between o and $2 \times o$ ($\stackrel{\text{def}}{=}$ **fan-out** F) index entries.
- Given predicate $t.a = k$, perform **binary search inside node** to find B⁺Tree subtree with $k_i \leqslant k < k_{i+1}$.
- B⁺Tree is **balanced:** subtrees △ are of identical height.
- **Path length** s from B⁺Tree root to leaf node **predictable:**

$$|T| \times \underbrace{1/F \times \cdots \times 1/F}_{s \text{ times}} = 1 \Leftrightarrow s = \log_F(|T|)$$

A B^+Tree is *the* index structure to support the evaluation of these kinds of conditions:[3]

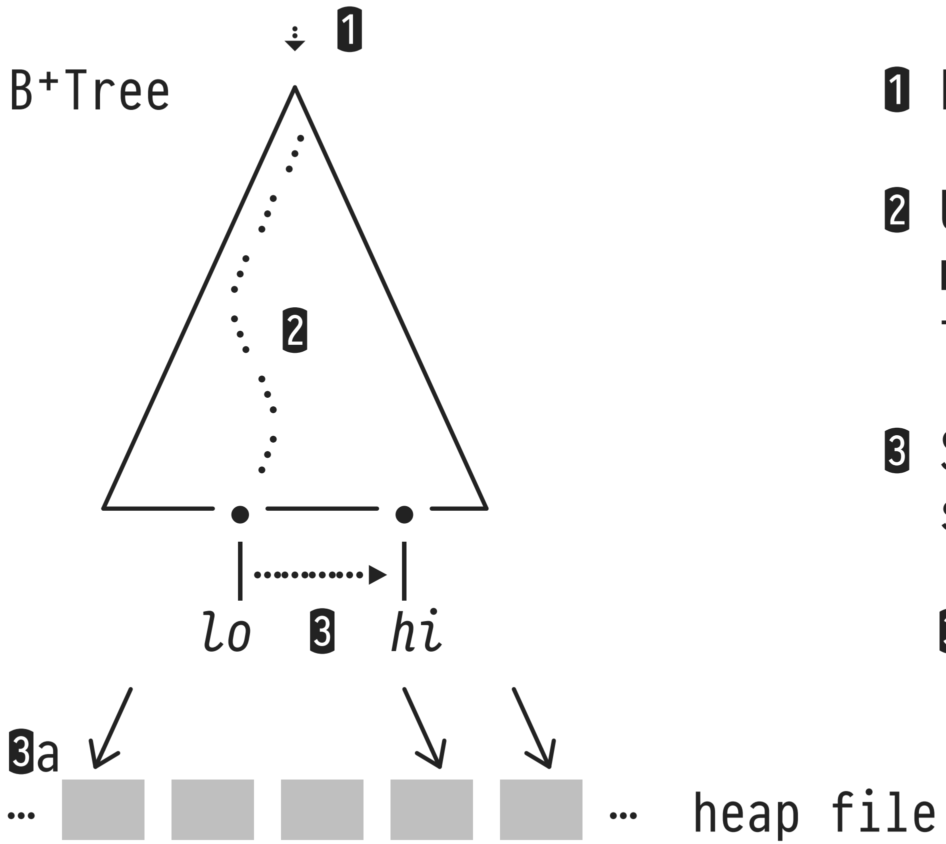
1. **Range predicates:** $lo \leqslant a \leqslant hi$
2. **Half-open ranges:** $lo \leqslant a$ or $a \leqslant hi$
3. **Equality predicates:** $a = hi$

- An **Index Scan** on index I for column $T(a)$ is parameterized by such a condition (PostgreSQL EXPLAIN: Index Cond).
- **Index Scan** uses lo to navigate the search tree structure and locate the start of relevant sequence set section.

[3] Half-open ranges are special range predicates where $hi = \infty$ ($lo = \infty$). Equality predicates are special range predicates where $lo = hi$.

Index Scan for Condition $lo \leqslant a \leqslant hi$

An **index scan** access the B⁺Tree index *and* the heap file:



1 Enter at B⁺Tree root page

2 Use key *lo* to **navigate the inner nodes** (search tree) until we reach the leaf level

3 Scan leaf entries in the sequence set section $lo \leqslant a \leqslant hi$, **extract RIDs**

3a For each RID, **access heap file for table** *T* and return matching row

Navigating the Inner Nodes

Phase ❷ runs a vanilla traversal of a 2×o-way search tree:

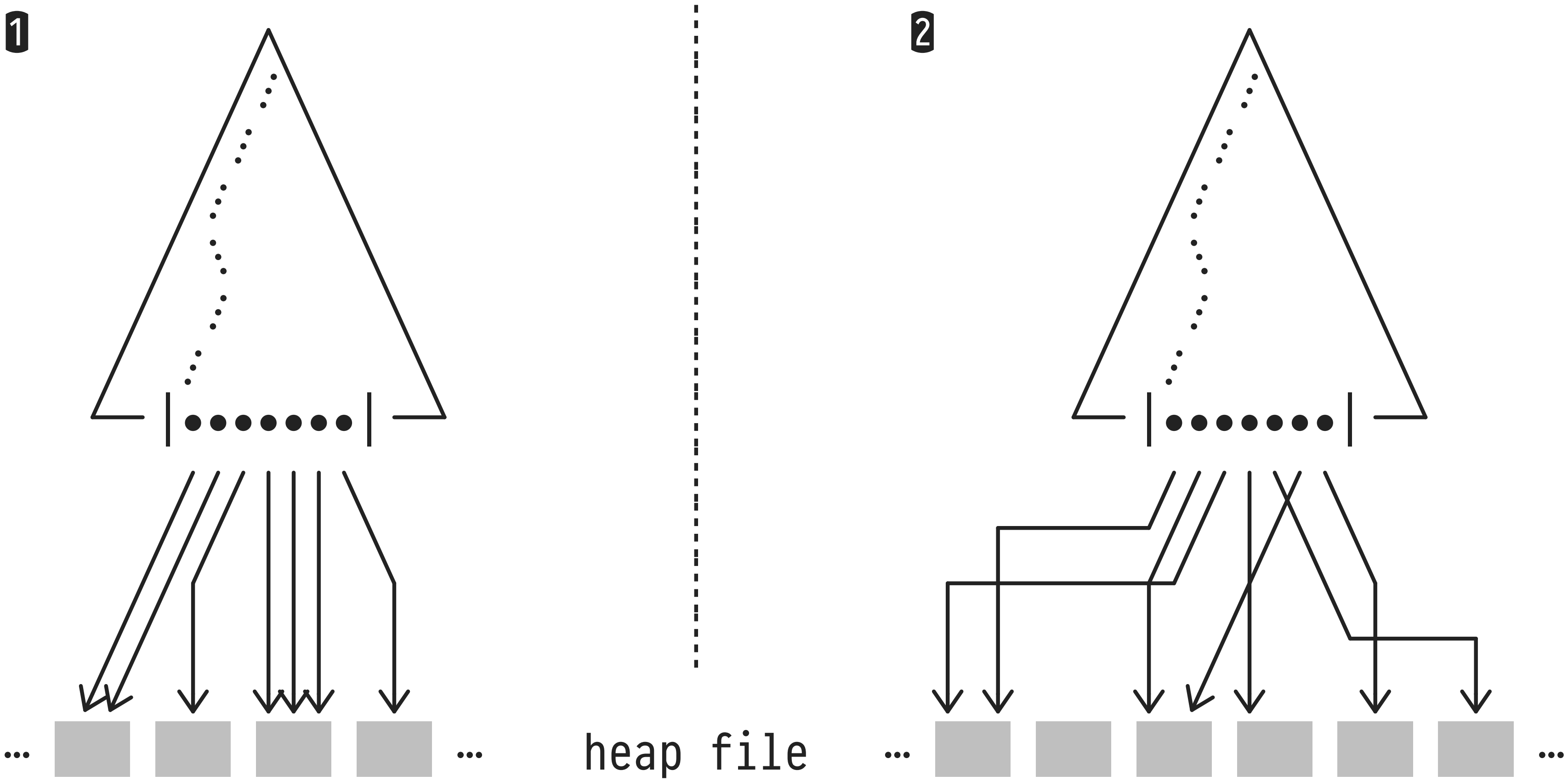
```
Search(lo):
  return TreeSearch(lo, root);
```

returns entry point for scan of sequence set

```
TreeSearch(lo, node):
  if (node is a leaf)
   ⌊ return node;
  switch lo
   | case lo < k1
   |  ⌊ return TreeSearch(lo, p0);
   | case ki ⩽ lo < ki+1
   |  ⌊ return TreeSearch(lo, pi);
   | case k2o ⩽ lo
   ⌊  ⌊ return TreeSearch(lo, p2o);
```

use binary search to implement subtree choice

Order of Leaf Entries vs. Order of Table Rows



❶ Order of leaf entry keys k_i ≡ row order in heap file. 👍

❷ Order of k_i in sequence set and row order do *not* match.

Clustered Indexes

An index I for column $T(a)$ is **clustered** if the order of leaf entries coincides with T's row order (i.e., both I's sequence set and T's heap file are ordered by a):

Given entries $\langle k_i, p_i \rangle$ and $\langle k_j, p_j \rangle$, $k_i \leqslant k_j \Rightarrow p_i \leqslant p_j$.

- An **Index Scan** over a *clustered* index
  1. collects matching rows from adjacent heap file pages ($\Rightarrow$ sequential I/O 👍),
  2. will find many matching rows on each loaded heap file page ($\Rightarrow$ less page I/O 👍).

Non-Clustered Indexes

Sad fact: only *one*—among the many possible—indexes for a table may be clustered. Most indexes are non-clustered.

- An **Index Scan** over a *non-clustered* index
  1. will find matching rows potentially scattered across all heap file pages (⇒ random I/O 👎),
  2. will find few matching rows on each loaded heap file page and may access the same page more than once (⇒ as many page I/Os as matching rows 👎).

PostgreSQL addresses this challenge through **RID sorting**, implemented via **Bitmap Index Scan** & **Bitmap Heap Scan**.

Bitmap Index Scan & Bitmap Heap Scan

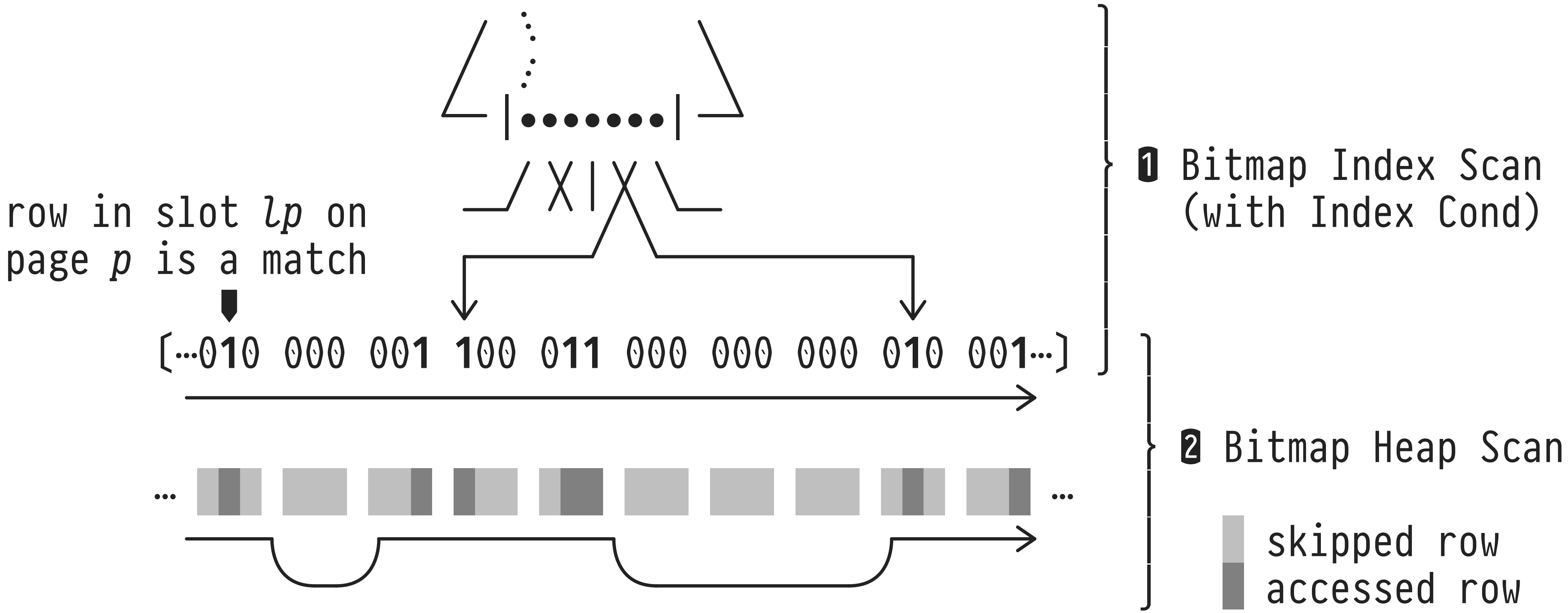
```
          QUERY PLAN
❷ Bitmap Heap Scan on indexed i
    Recheck Cond: (c = 0.42)
    Heap Blocks: exact=2964
    -> ❶ Bitmap Index Scan on indexed_c
            Index Cond: (c = 0.42)
```

bitmap of
matched rows

❶ **Bitmap Index Scan:** perform Index Scan and create **bitmap** that encodes *heap file locations* of rows matching the Index Cond. Do *not* access rows in heap file yet.

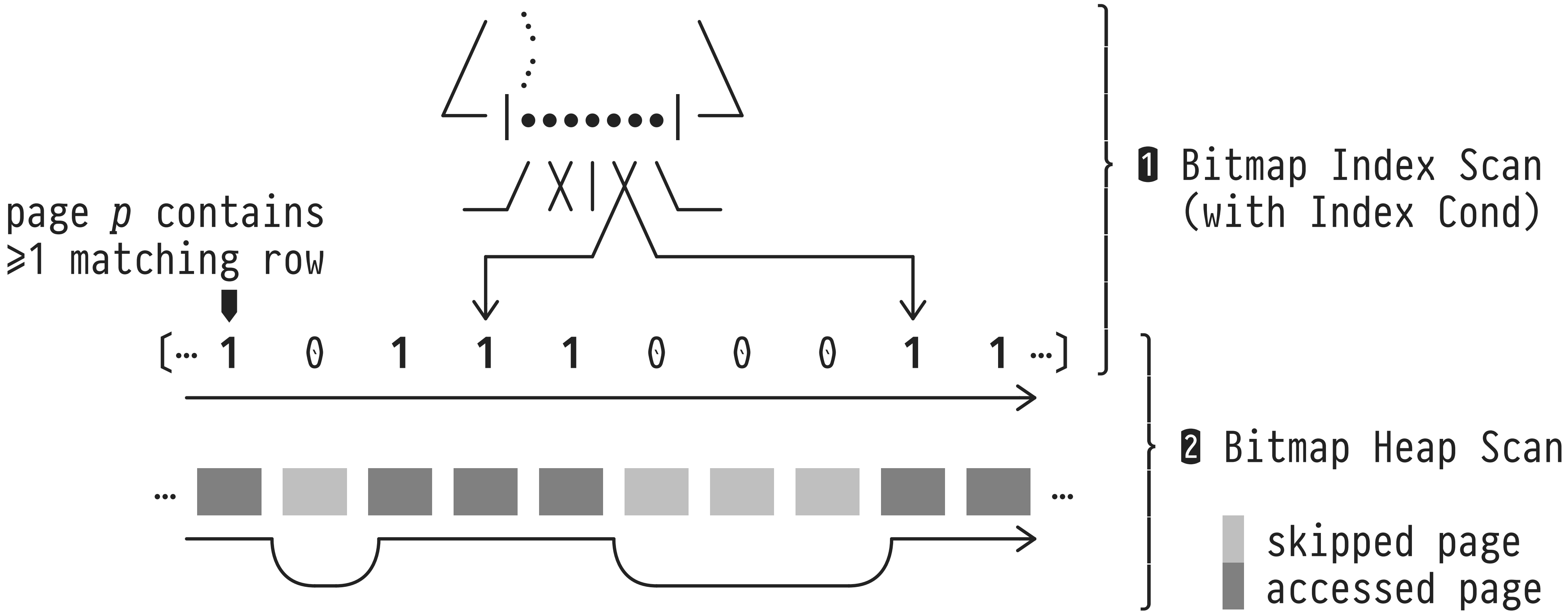
❷ **Bitmap Index Scan:** scan heap file once, only access those rows (pages) that have been marked **1** in the bitmap.

Bitmap Index Scan & Bitmap Heap Scan: Row-Level Bitmap



Bitmap Heap Scan performs one sequential scan (with skips) of the heap file, regardless of RID order in sequence set.

Bitmap Index Scan & Bitmap Heap Scan: Page-Level Bitmap



Working memory tight ⇒ build **page-level** bitmap. ⚠️ In ❷, need to **recheck condition** for all rows on accessed pages.

5 ¦ CLUSTERing Based on an Index

If the workload depends on top performance of particular predicates supported by *non-clustered* index *I*, we may

> **physically reorder the rows of underlying table's *T* heap file** to coincide with the key order in *I*'s sequence set (i.e., *I* will become a *clustered* index[4]):

```
CLUSTER [VERBOSE] ‹T› USING ‹I›;
CLUSTER ‹T›;    -- re-cluster once T's rows get out of order
```

- ⚠️ Subsequent updates on *T* can destroy the perfect clustering. (May need to re-cluster *T* in intervals.)

[4] At a price, of course: formerly *clustered* indexes on *T* will turn into *non-clustered* indexes.

B+Trees…

1. **economically utilize space** in inner/leaf nodes (minimum node occupancy 50%, typical fill factor 67%),
2. are **balanced** trees and thus require a **predictable number of page I/Os** to traverse from root to sequence set—enables query optimizer to forecast B+Tree access cost.

DBMSs maintain properties 1. and 2. when rows are **inserted** into/**deleted** from an B+Tree-indexed table.[5]

[5] Some real B+Tree implementations of row deletion deviate from the textbook to keep things simpler.

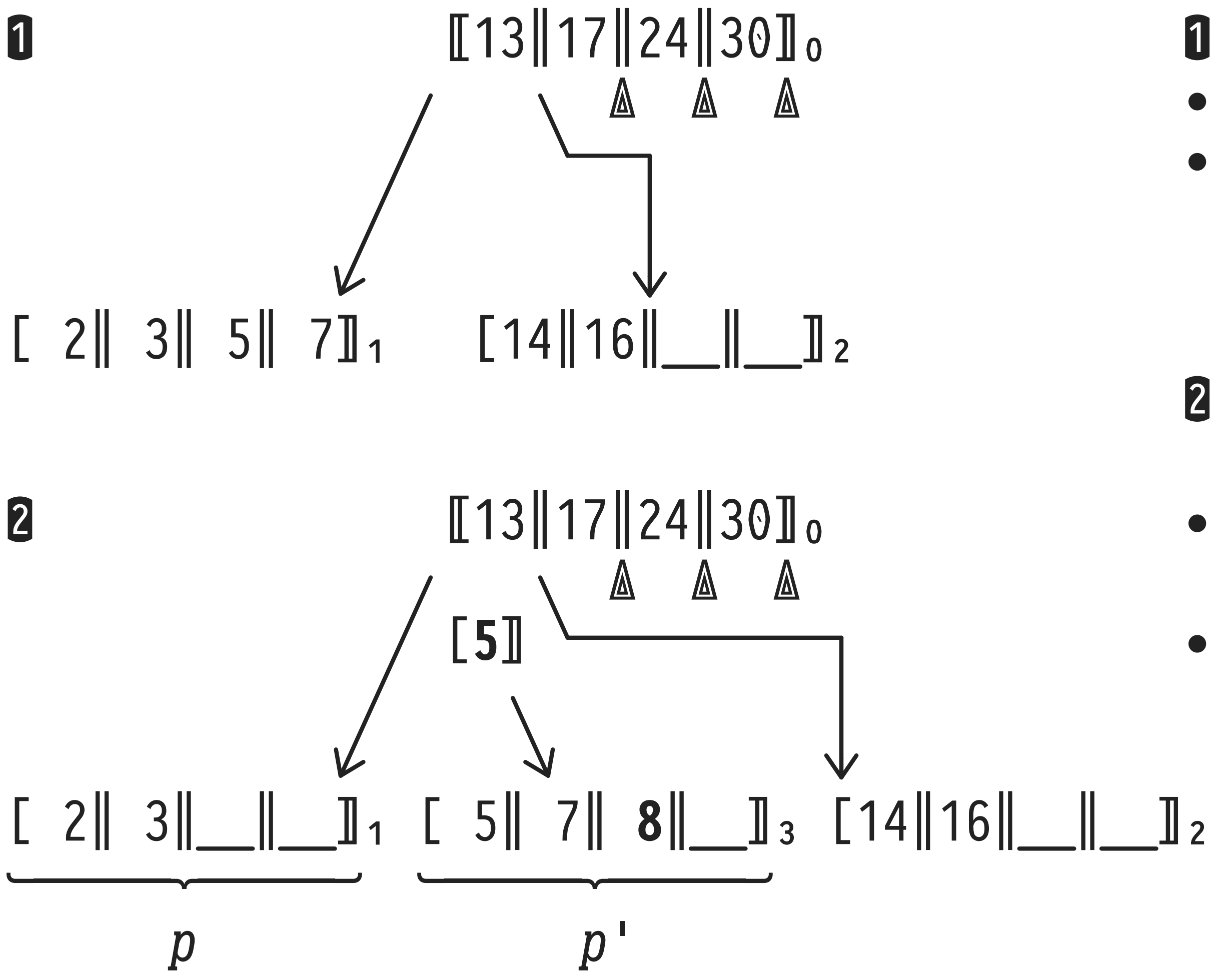
B⁺Tree Insertion for New Entry $\langle k, rid \rangle$ (Sketch)

1. Use Search(k) to **find leaf page** p which should hold the entry for k.
2. If p has **enough space** to hold new entry (i.e., at most $2 \times o - 1$ entries in p), **simply insert** $\langle k, rid \rangle$ into p.
3. Otherwise, node p must be **split** into p and p' and a new **separator** has to be inserted↻ into the parent of p.

   Splitting happens recursively↻ and may eventually lead to a split of the root node (increasing B⁺Tree height).

4. **Distribute** the entries of p and new entry $\langle k, rid \rangle$ onto pages p and p'.

B+Tree Insertion and Leaf Node Split



1 Insert new entry ‹**8**,*rid*›

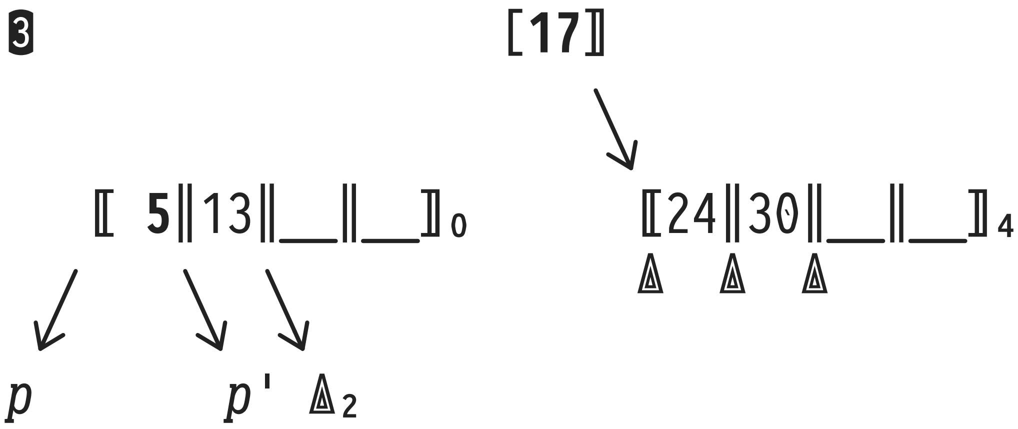
- Search(**8**) returns leaf *p* = 1
- Leaf 1 is full ⇒ split

2 Leaf 1 split into leaves *p* = 1 and *p*' = 3

- Distribute {2,3,5,7,**8**} between leaves 1 and 3
- **Copy** new separator [**5**]] into parent node 0

p'

B⁺Tree Insertion and Inner Node Split

3


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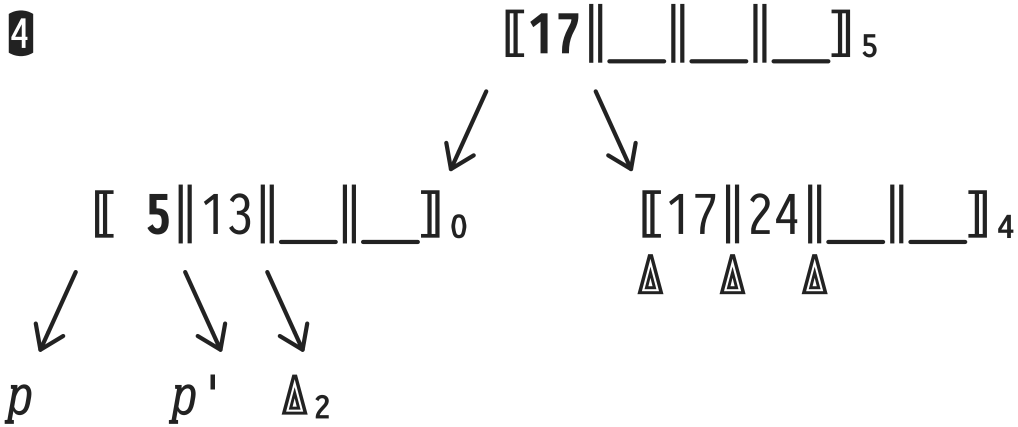


3 Inner node 0 (here: root) is full ⇒ split

- Inner node 0 splits into nodes *p* = 0 and *p*' = 4
- Distribute {**5**,13,24,30} ▲ between nodes 0 and 4
- **Move** new separator [**17**]] into parent of node 0

p'

4 Split node 0 has been the old root

- Create new root node 5, has ⟦**17**‖ as only entry
- B⁺Tree height has increased

- Splitting starts at the leaf level and continues upward as long as inner index nodes are fully occupied (holding $2 \times o$ entries).

- ⚠️ Unlike during a *leaf* split, an *inner* node split **moves**[6] the new separator ⟦***sep***⟧ discriminating between p and p' upwards and recursively inserts it into the parent. **Q:** Why?

- **Q:** How often do you expect a root node split to happen?

[6] A leaf node split **copies** the new separator upwards, i.e., the entry ⟦***sep***⟧ also remains at the leaf level.

B+Tree Insertion Example: Insert ‹42,*rid*›

1

- Search(**42**) returns leaf 3

```
                        [84|__|__|__]0

          [50|82|__|__]1                    [87|90|__|__]2

[41|44|45|__]3 [50|64|80|81]4 [82|83|__|__]5   [84|85|86|__]6 [87|88|89|__]7 [90|92|__|__]8
```

2

- Leaf 3 has capacity
- ⇒ Sorted insert of **42** in leaf 3

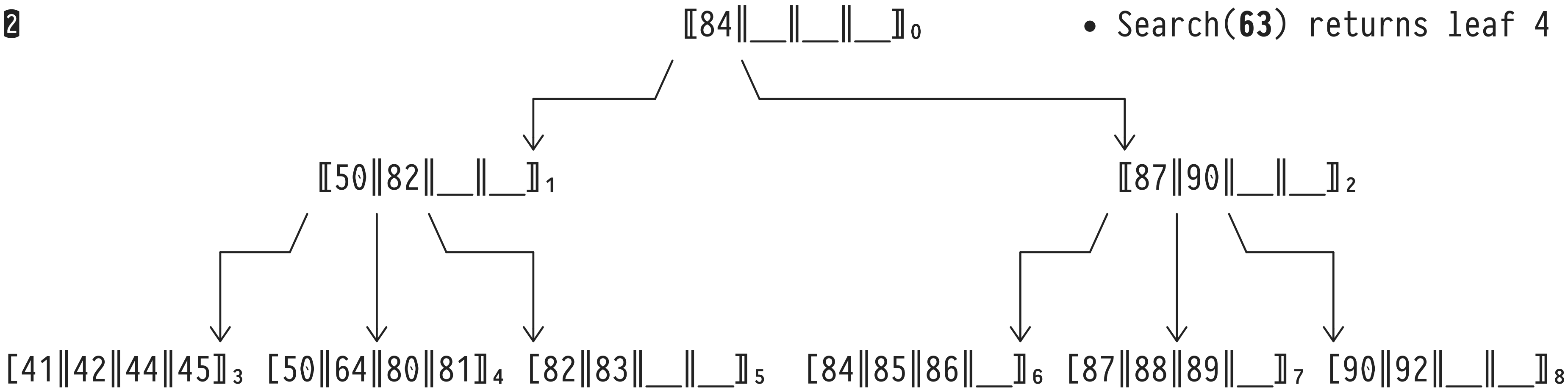
```
                        [84|__|__|__]0

          [50|82|__|__]1                    [87|90|__|__]2

[41|42|44|45]3 [50|64|80|81]4 [82|83|__|__]5   [84|85|86|__]6 [87|88|89|__]7 [90|92|__|__]8
```

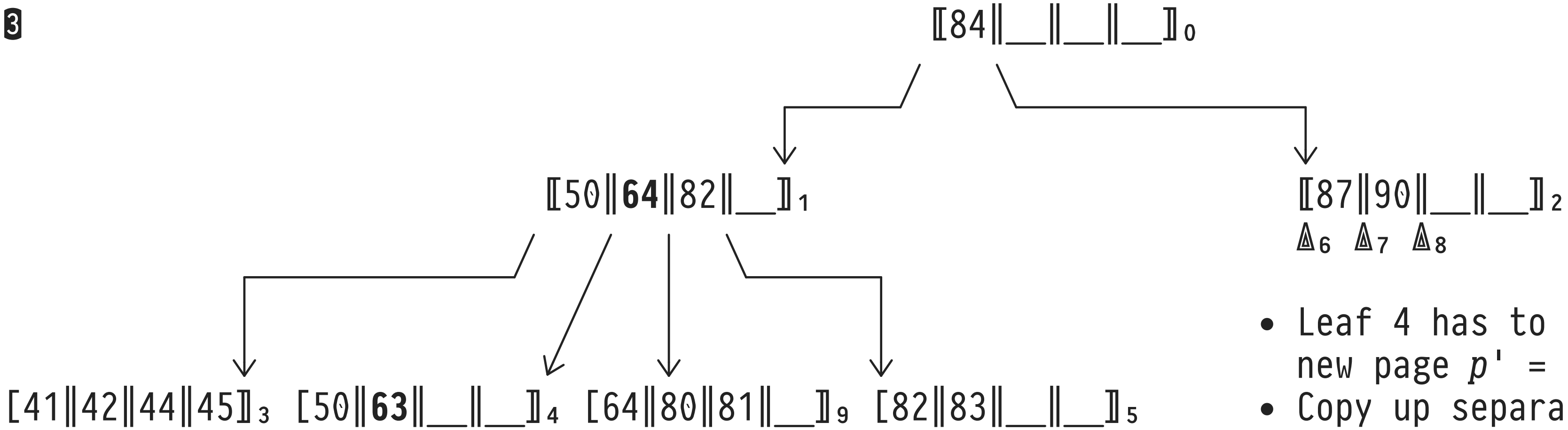
B$^+$Tree Insertion Example: Insert ‹63,*rid*›

❷



- Search(**63**) returns leaf 4

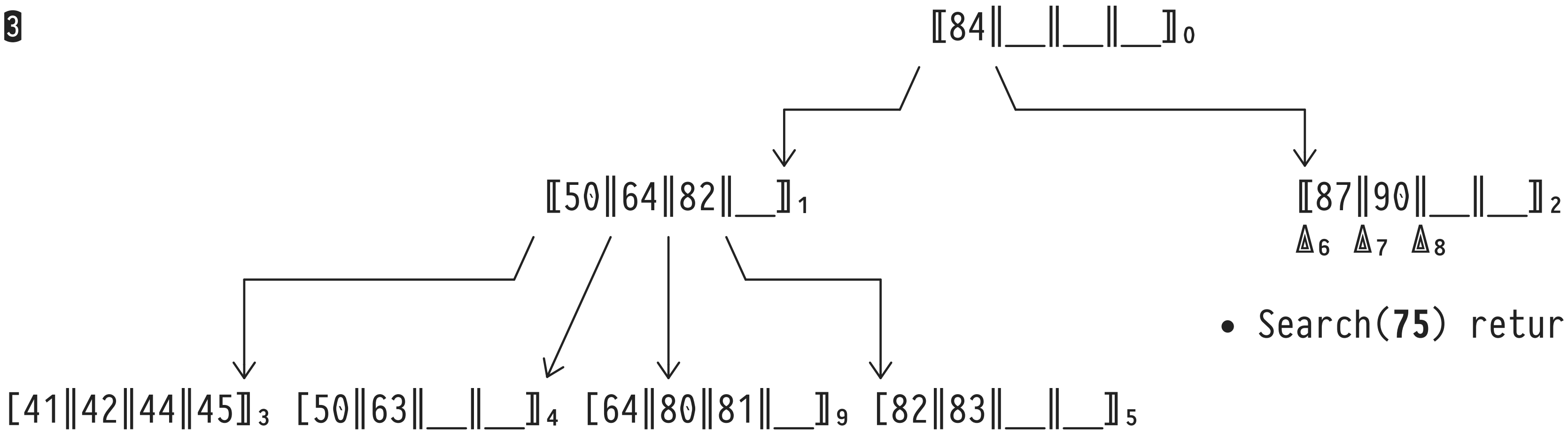
❸



- Leaf 4 has to split, new page p' = 9
- Copy up separator [**64**⟧

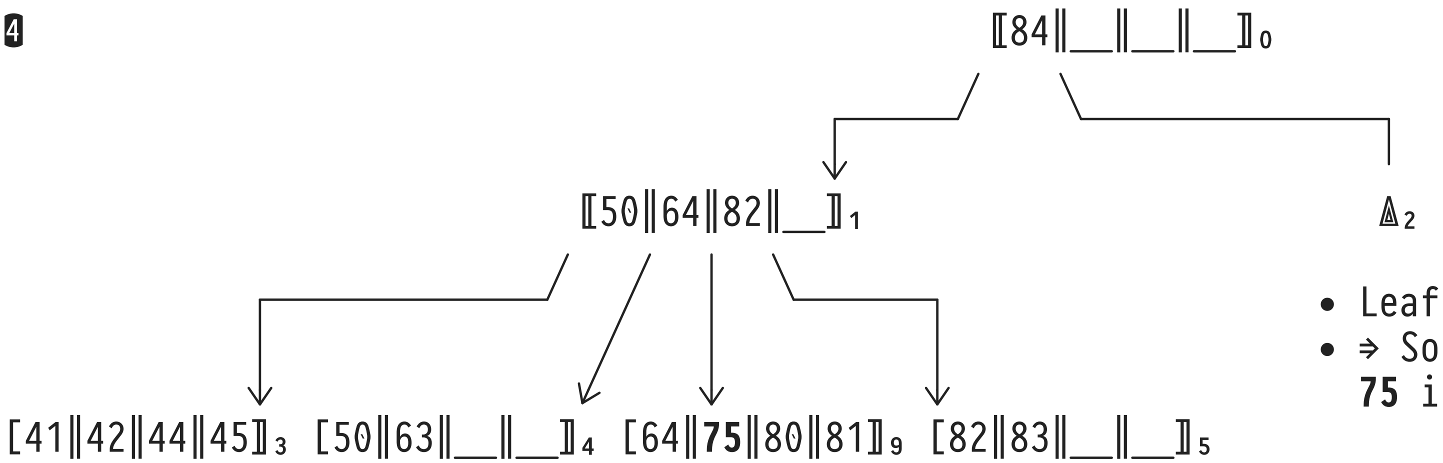
B+Tree Insertion Example: Insert ‹75,*rid*›

❸



- Search(**75**) returns leaf 9

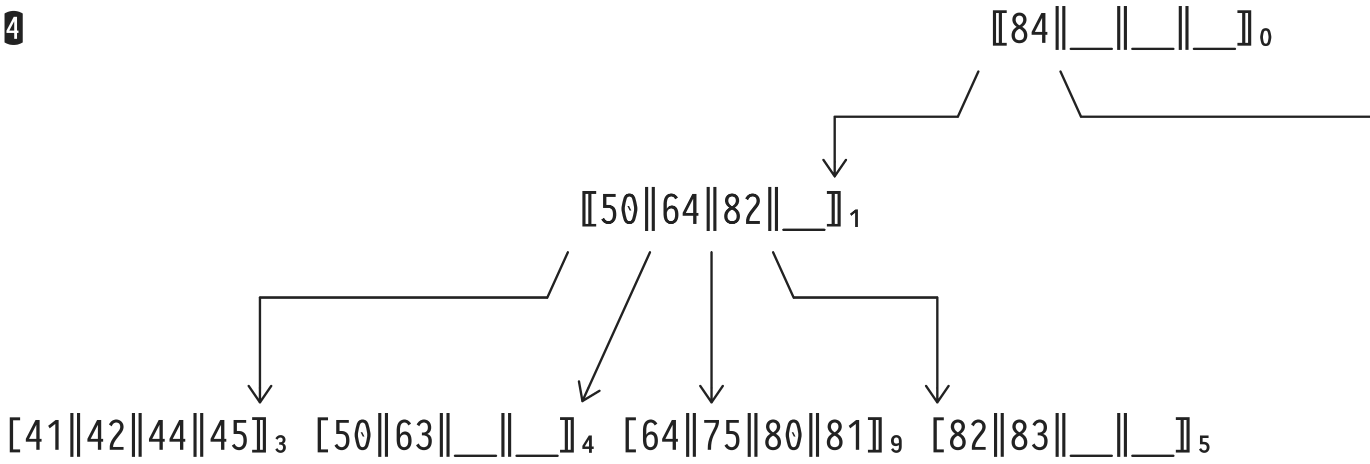
❹



- Leaf 9 has capacity
- ⇒ Sorted insert of **75** in leaf 9

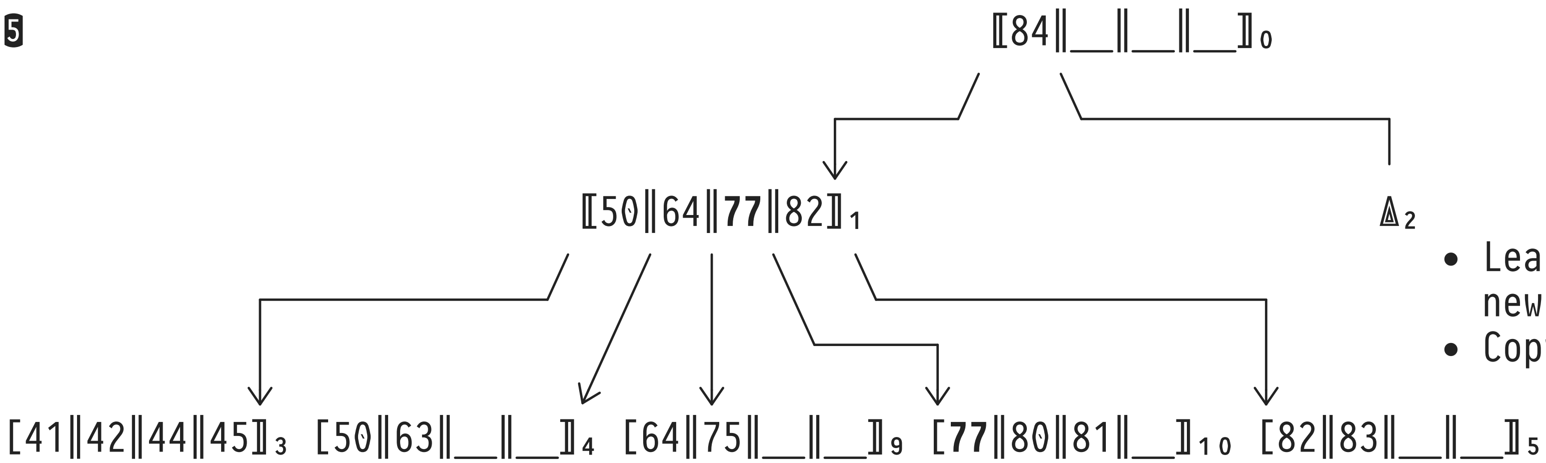
B$^+$Tree Insertion Example: Insert ‹77,*rid*›

4



- Search(**77**) returns leaf 9
- Leaf 9 is full (already holds 2×o = 4 entries)

5

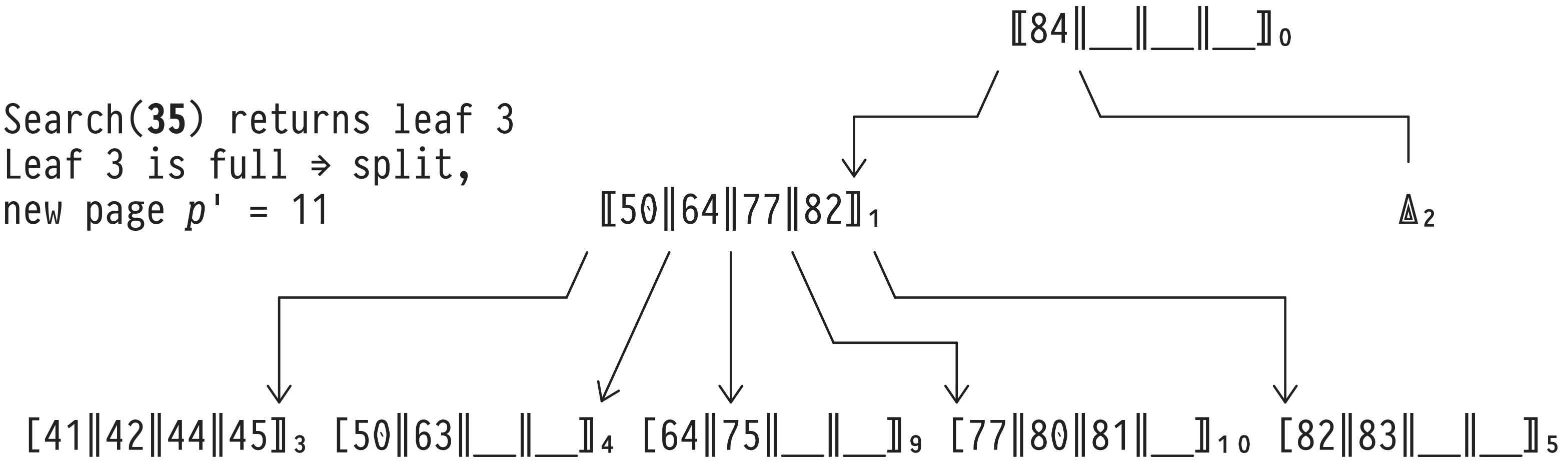


- Leaf 9 has to split, new page *p*' = 10
- Copy up separator [**77**⟧

B^+Tree Insertion Example: Insert ‹35,*rid*›

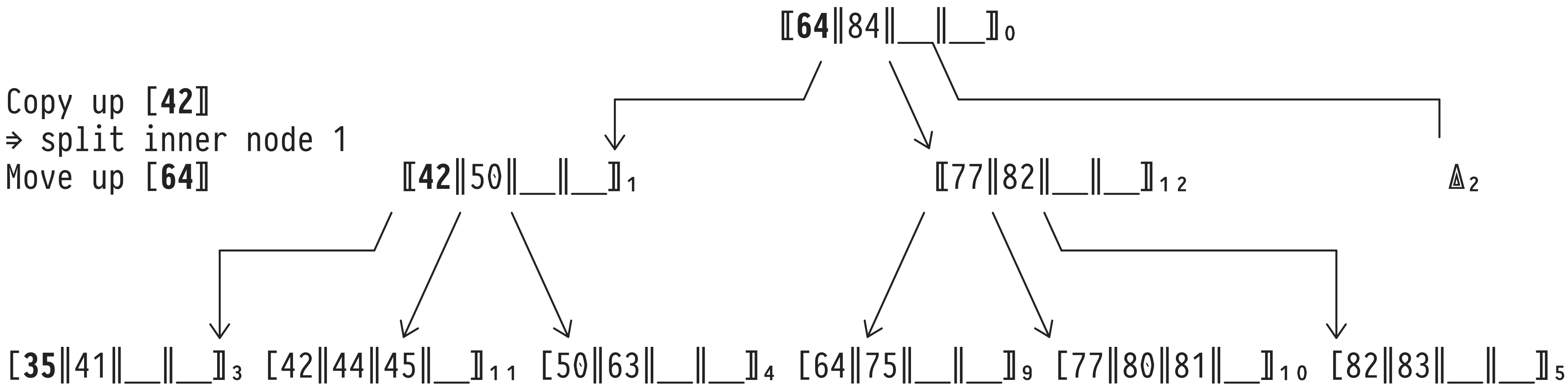
5

- Search(**35**) returns leaf 3
- Leaf 3 is full ⇛ split,
  new page *p*' = 11



6

- Copy up [**42**⟧
  ⇛ split inner node 1
- Move up [**64**⟧



B+Tree Insertion Algorithm[7] (1)

```
TreeInsert(‹k,rid›,node):
  if (node is a leaf)
   | return LeafInsert(‹k,rid›,node);
  else
   | switch k                                         ⎫
   |  | case k < k₁                                   ⎪
   |  |  ⌊ ‹sep,ptr› ← TreeInsert(‹k,rid›,p₀);        ⎪
   |  | case kᵢ ⩽ k < kᵢ₊₁                            ⎬ see Search()
   |  |  ⌊ ‹sep,ptr› ← TreeInsert(‹k,rid›,pᵢ);        ⎪
   |  | case k₂ₒ ⩽ k                                  ⎪
   |  ⌊  ⌊ ‹sep,ptr› ← TreeInsert(‹k,rid›,p₂ₒ);       ⎭
   | if (sep = ⊥)
   |  | return ‹⊥,⊥›;
   | else
   ⌊  ⌊ return InnerInsert(‹sep,ptr›,node);
```

[7] Note: ‹*sep*,*ptr*› ≡ [*sep*]] in our discussion above.

B$^+$Tree Insertion Algorithm (2)

```
LeafInsert(‹k,rid›,node):
  if (node has < 2×o entries)
   │ insert ‹k,rid› into node;
   │ return ‹⊥,⊥›;                } ‹⊥,_› ≡ no upwards split required
  else
   │ p' ← allocate leaf page;
   │ [‹k_1,rid_1›,…,‹k_{2o+1},rid_{2o+1}›] ← entries of node ∪ ‹k,rid›;
   │ node ← [k_1|rid_1|…|k_o|rid_o|__‖__⟧;
   │ p'   ← [k_{o+1}|rid_{o+1}|…|k_{2o+1}|rid_{2o+1}|__‖__⟧;
   └ return ‹k_{o+1},p'›;
```

- **Copy** upwards: entry $\langle k_{o+1}, rid_{o+1} \rangle$ remains in leaf p'.

B⁺Tree Insertion Algorithm (3)

```
InnerInsert(‹sep,ptr›,node):
  if (node has < 2×o entries)
   │ insert ‹sep,ptr› into node;
   │ return ‹⊥,⊥›;               } ‹⊥,_› ≡ no upwards split required
  else
   │ p' ← allocate inner node page;
   │ [p₀,‹k₁,p₁›,…,‹k₂ₒ₊₁,p₂ₒ₊₁›] ← entries of node ∪ ‹sep,ptr›;
   │ node ← ⟦p₀|k₁|p₁|…|kₒ|pₒ|__‖__⟧;
   │ p'   ← ⟦pₒ₊₁|kₒ₊₂|pₒ₊₂|…|k₂ₒ₊₁|p₂ₒ₊₁|__‖__⟧;
   └ return ‹kₒ₊₁,p'›;
```

- **Move** upwards: new entry $\langle k_{o+1}, p' \rangle$ returned for insertion at parent. No entry $\langle k_{o+1}, \_ \rangle$ remains at level of *node*/*p'*.

B⁺Tree Insertion Algorithm (Top Level)

Insert(‹*k*,*rid*›) is the top-level B⁺Tree insertion routine:

```
Insert(‹k,rid›):
  ‹k',ptr› ← TreeInsert(‹k,rid›,root);  } root ≡ old root page
  if (k' ≠ ⊥)
   │ r ← [root|k'|ptr|__‖__‖__];         } r ≡ new root page
   └ root ← r
```

```
      [k'‖__‖__‖__]  ◀ new root page r
      ↙    ↘
   root     ptr
```

- Note: Insert() may leave us with a new root node that violates the minimum occupancy rule. ¯\(ツ)/¯

B⁺Tree Insertion: Redistribution (1)

Can improve average occupancy and delay height increase on B⁺Tree insertion through **redistribution:**

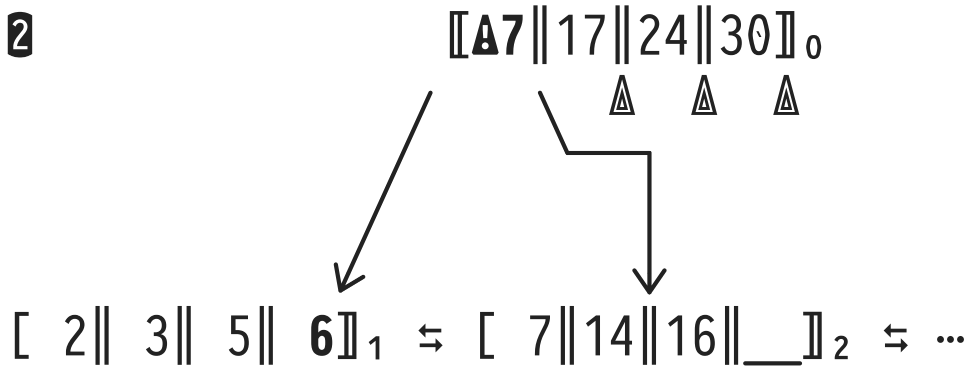
❶ ⟦13‖17‖24‖30⟧$_0$

[2‖ 3‖ 5‖ 7⟧$_1$ ⇆ [14‖16‖__‖__⟧$_2$ ⇆ …

❶ Insert new entry ‹**6**,*rid*›
- Search(**6**) returns leaf 1
- Leaf 1 is full, but its right **sibling** 2 has capacity

- Use sequence set chain pointers (⇆) to inspect **sibling** nodes for spare capacity.
- **Push** entry from overflowing node to sibling and ⚠️ **update separator in parent node** to reflect this redistribution.

B$^+$Tree Insertion: Redistribution (2)



2 Push entry ‹7,*rid*'› to leaf 2

- Place ‹**6**,*rid*› in leaf 1
- Update separator (13 → **7**) in parent node 0
- B$^+$Tree remains at height 2

- Inspecting node sibling involves additional page I/O. 👎
- Actual implementations use redistribution on the index leaf level only (if at all).

B^+Tree Deletion of Entry With Key k (Sketch)

1. Use Search(k) to **find the leaf** p holding entry ‹k,rid›.
2. **Simply delete** ‹k,rid› from p.[8]
3. If p now holds < o entries, leaf p **underflows.** Any sibling of p with spare entries?
   - Yes, use **redistribution** to move an entry into p.
   - No, **merge** p and a sibling leaf p' of o entries. Delete the now obsolete separator of p and p' in their parent node.

     Deletion propagates upwards and may eventually leave the root node empty (decreases B^+Tree height).

[8] **Q:** If ‹k,rid› is the leftmost entry in p, do we need to update the associated separator entry in p's parent node? Why not?

B⁺Tree Deletion (No Underflow)

1

⟦24‖30‖__‖__⟧$_0$

𝔸$_3$

[**19**‖20‖22‖__⟧$_1$ [24‖27‖29‖__⟧$_2$

1 Delete entry with key k = **19**

- Search(**19**) returns leaf 1
- Leaf 1 has > o entries, node will not underflow

2

⟦24‖30‖__‖__⟧$_0$

𝔸$_3$

[20‖22‖__‖__⟧$_1$ [24‖27‖29‖__⟧$_2$

2 Simply delete entry ‹**19**,*rid*› from leaf 1

B$^+$Tree Deletion and Redistribution

2

$[\![24 \| 30 \| \_\_ \| \_\_]\!]_0$ $\mathbb{A}_3$

$[\mathbf{20} \| 22 \| \_\_ \| \_\_]\!]_1$ $[24 \| 27 \| 29 \| \_\_]\!]_2$

2 Delete entry with key k = **20**

- Search(**20**) returns leaf 1
- Leaf 1 has minimum occupancy of o entries ⇛ will underflow

3

$[\![\mathbf{A}27 \| 30 \| \_\_ \| \_\_]\!]_0$ $\mathbb{A}_3$

$[22 \| 24 \| \_\_ \| \_\_]\!]_1$ $[27 \| 29 \| \_\_ \| \_\_]\!]_2$

3 Sibling p' = 2 has one entry to spare ⇛ redistribution

- Move entry ‹24,*rid*'› from leaf 2 to leaf 1
- Update separator (24 → 27) in parent node 0

B+Tree Deletion and Leaf Node Merging

❸ ⟦27‖30‖__‖__⟧$_0$
⍋$_3$

[22‖**24**‖__‖__⟧$_1$ [27‖29‖__‖__⟧$_2$

❸ Delete entry with key k = **24**
- Search(**24**) returns leaf 1
- Leaf 1 has minimum occupancy, no sibling with spare entries

❹ ⟦30‖__‖__‖__⟧$_0$
⍋$_3$

[22‖27‖29‖__⟧$_1$ [__‖__‖__‖__⟧$_2$♲

❹ Merge leaf nodes 1 and 2, mark empty page 2 as garbage
- In parent 0, delete0 obsolete separator [27⟧

B$^+$Tree Deletion and Leaf Node Merging (Empty Root)

❺ ⟦30‖__‖__‖__⟧$_0$

[22‖29‖__‖__⟧$_1$ [33‖**35**‖__‖__⟧$_3$

❹ Delete entry with key *k* = **35**

- Search(**35**) returns leaf 3
- Leaf 3 has minimum occupancy, no sibling with spare entries

❻ ⟦__‖__‖__‖__⟧$_0$♲

root

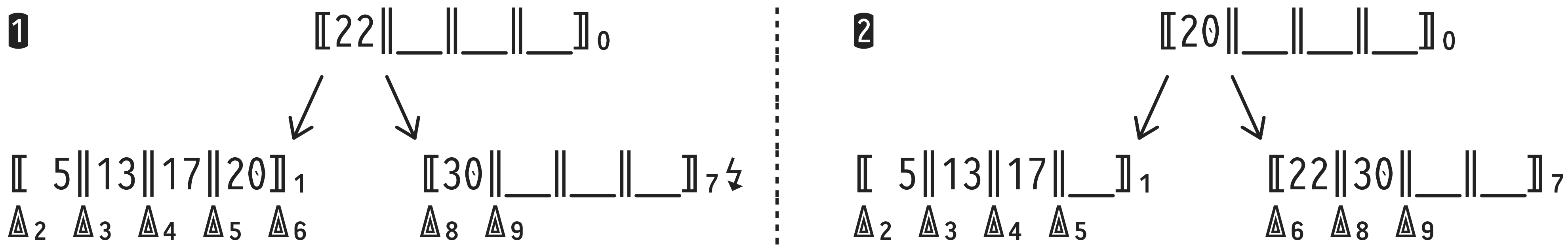
[22‖29‖33‖__⟧$_1$ [__‖__‖__‖__⟧$_3$♲

❹ Merge leaf nodes 1 and 3, mark empty page 3 as garbage

- In parent 0, delete obsolete separator [30⟧
- Old root empty (⇛ garbage), mark page 1 as the new root
- B$^+$Tree height decreases

B⁺Tree Deletion and Inner Node Redistribution

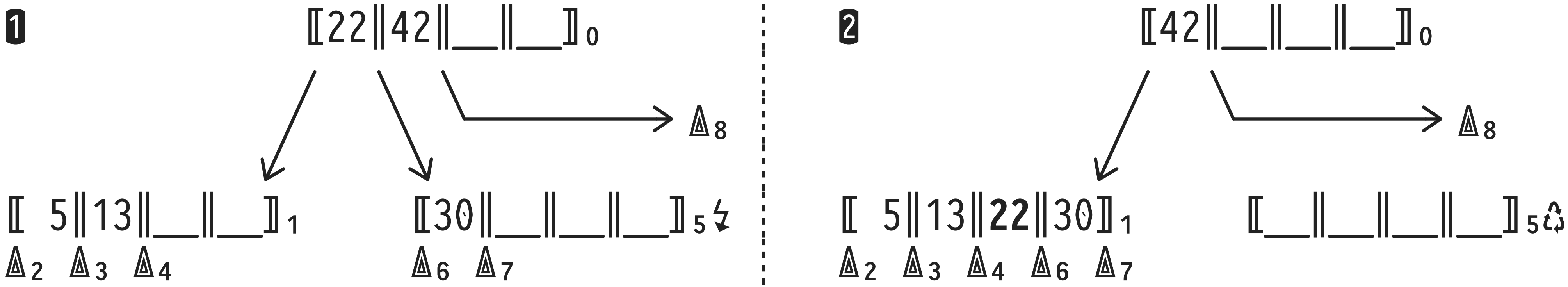
- **Redistribution** is also defined for **inner nodes.** Suppose we encounter underflow ❶ during ↻ deletion propagation:



- Inner node 1 has two spare entries. "Rotate entry ⟦20⟧ through parent" to underflowed inner node 7.

  N.B.: Semantics of subtree Δ_6 (holds index entries with $k \geqslant 20 \wedge k < 22$) are preserved.

- Likewise, **inner nodes** may also be **merged.** The underflow in ❶ cannot be handled by redistribution:



- Note how the separator **22** has been **pulled down**↻ from the parent to discriminate between subtrees Δ_4 and Δ_6:
  - Δ_4: $k \geqslant 13 \wedge k < 22$
  - Δ_6: $k \geqslant 22 \wedge k < 30$

The higher the **fan-out** F, the more index entries fit in a B+Tree of fixed height. How to maximize F?

- For entries $\langle k,p \rangle$ in indexes over text/char columns, we may have $|k| \gg |p|$.[9] Can we reduce the size of k?

- 💡 Search() and TreeInsert() do *not* inspect the actual key values but only use $<$/$\leqslant$ to direct tree traversals.
  - ⇒ May **shorten (truncate) string keys** as long as the ordering relation is preserved.
  - This applies to index entries inner nodes only. Leaf level keys remain as is.

[9] The implementation (thus size) of page pointers p is prescribed by the DBMS. Nothing to win here.

B+Trees: Key Suffix Truncation

❶ Original B+Tree

```
                                  ↙
                 ⟦irish‖ironical‖irregular‖__⟧0
                 ⍋1                       ⍋4
                      ↙         ↘
  [irksome‖iron‖ironage‖__⟧2     [irrational‖irreducible‖__‖__]3
```

❷ B+Tree after **key suffix truncation**

```
                                   ↙
                 ⟦␣␣␣␣␣‖␣␣␣␣␣␣␣␣‖␣␣␣␣␣␣␣␣␣‖__⟧0
                 ⍋1                       ⍋4
                      ↙         ↘
  [irksome‖iron‖ironage‖__⟧2     [irrational‖irreducible‖__‖__]3
```

⚠️ While truncating, preserve the **separator** semantics.

B+Trees: Key Prefix Compression

Observation: string keys within a B+Tree inner node often **share a common prefix.**

- 💡 Store common prefix only once (e.g., as "k_0").
- Violating the 50% occupancy rule can help compression.

❶ Original B+Tree

⟦irish‖ironical‖irregular‖__⟧$_0$

$\mathbb{A}_1$ $\mathbb{A}_2$ $\mathbb{A}_3$ $\mathbb{A}_4$

❷ B+Tree after **key prefix compression**

[ir+‖ish‖onical‖regular‖__⟧$_0$

k_0 $\mathbb{A}_1$ $\mathbb{A}_2$ $\mathbb{A}_3$ $\mathbb{A}_4$

Grab a hot cup of ☕ and start a war on Stack Overflow:[10]

Q: *Which order of operations is better?*

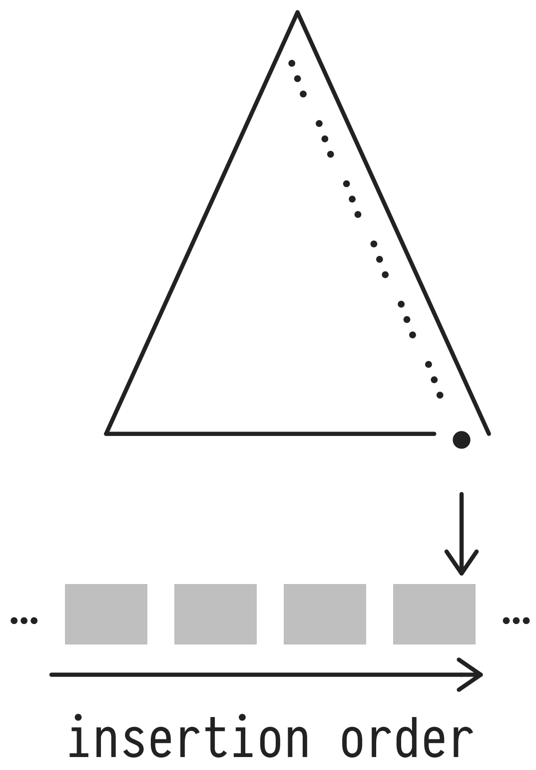
```
❶ CREATE TABLE T (…);
❷ INSERT INTO T VALUES (‹5 × 10⁶ rows›);
❸ CREATE INDEX I ON T USING btree (…);
```

or

```
❶ CREATE TABLE T (…);
❸ CREATE INDEX I ON T USING btree (…);
❷ INSERT INTO T VALUES (‹5 × 10⁶ rows›);
```

[10] See, for example, https://stackoverflow.com/questions/5910486/indexes-on-a-table-database

If insertions happen in index key order (i.e., ascending values of k), we observe a particular B^+Tree access pattern:



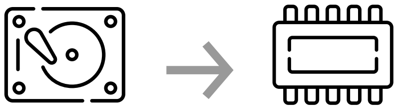
heap file (sorted on keys k)

- TreeInsert() will always traverse path ⋱, will always hit the righmost leaf.

⇛ Fix rightmost leaf in buffer, insert next entry right there (*no* traversal from root). Node splits only occur along path ⋱.

- We effectively create a clustered index.

11 ¦ Q_8 — Filtering a Table



```sql
SELECT i.b, i.c
FROM   indexed AS i
WHERE  i.a = 42 [i.c = 0.42] -- either filter on i.a or i.c
```

Indexes in MonetDB play a secondary role and are *not* organized in tree shapes.

MMDBMSs try to exploit that data resides in directly adressable memory and primarily aim to avoid access to separate index data structures (to avoid pointer chasing and potential cache misses).

Using **EXPLAIN** on Q_8: Filter on Column **a**

```
sql> EXPLAIN SELECT i.b, t.c
             FROM   indexed AS i
             WHERE  i.a = 42;
  ⋮
  indexed :bat[:oid] := sql.tid(sql, "sys", "indexed");
  a0      :bat[:int] := sql.bind(sql, "sys", "indexed", "a", 0:int);
  p1      :bat[:oid] := algebra.thetaselect(a0, indexed, 42:int, "=="); ◀ ≡ a = 42
  c0      :bat[:sht] := sql.bind(sql, "sys", "indexed", "c", 0:int);
  c       :bat[:sht] := algebra.projection(p1, c0);
  b0      :bat[:str] := sql.bind(sql, "sys", "indexed", "b", 0:int);
  b       :bat[:str] := algebra.projection(p1, b0);
  ⋮
```

- MonetDB uses `algebra.thetaselect(…, 42:int, "==")` to implement the predicate filter.

Using **EXPLAIN** on Q_8: Filter on Column c[11]

```
sql> EXPLAIN SELECT i.b, t.c
             FROM   indexed AS i
             WHERE  i.c = 0.42;
   ⋮
  indexed :bat[:oid] := sql.tid(sql, "sys", "indexed");
  c0      :bat[:sht] := sql.bind(sql, "sys", "indexed", "c", 0:int);
  p1      :bat[:oid] := algebra.thetaselect(c0, indexed, 42:int, "=="); ◀ ≡ c = 0.42
  c       :bat[:sht] := algebra.projection(p1, c0);
  b0      :bat[:str] := sql.bind(sql, "sys", "indexed", "b", 0:int);
  b       :bat[:str] := algebra.projection(p1, b0);
   ⋮
```

- Plan is nearly identical (modulo access to the a BAT).
- MonetDB *appears* to use the same `algebra.thetaselect(…, 42:int, "==")` MAL operation.

[11] Note how MonetDB maps the domain of type numeric(3,2) of column c, i.e., the set $N_{3,2} \equiv \{-9.99,\ldots,9.99\}$ with $|N_{3,2}| = 1999$, to a 16-bit value of type :sht. Nifty.

BAT Tail Properties

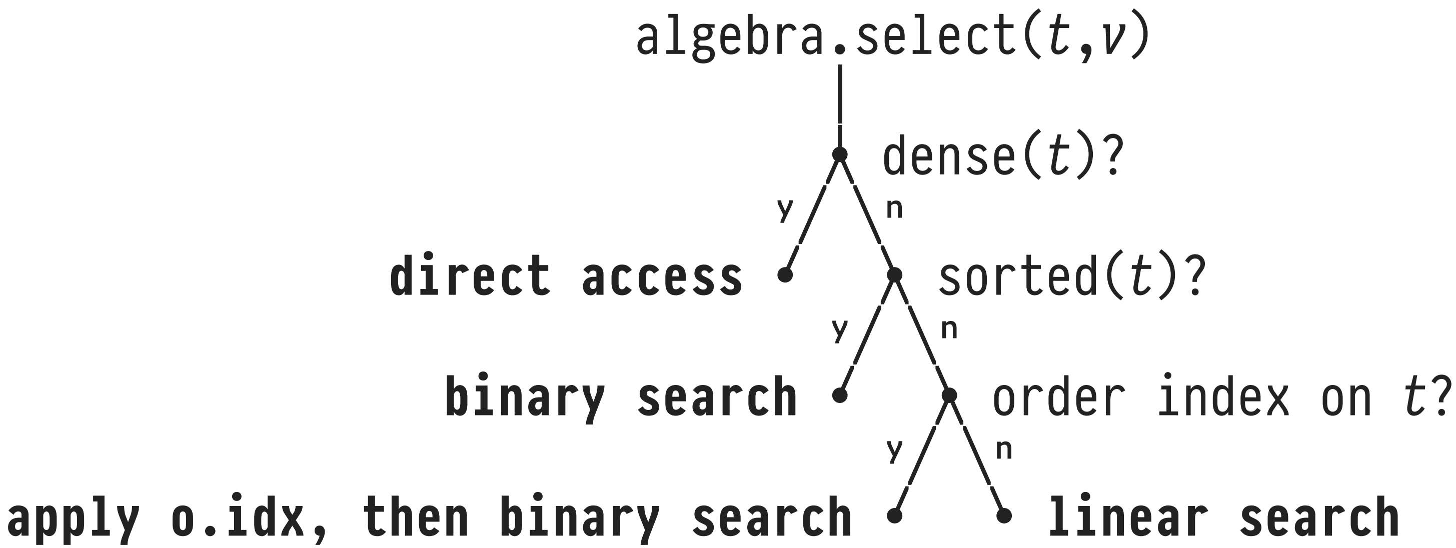
When MonetDB constructs a BAT *t*, a family of tail column **properties** *prop*(*t*) is derived/maintained:[12]

BAT Property *prop*(*t*)	**Description**
dense (tails of type :oid only)	ascending values, no gaps
key	unique values
sorted	ascending values
revsorted	descending values
nil/nonil	at least one/no nil value

- Use bat.info(*t*) to inspect current properties of *t*.
- ⚠️ Incomplete: *t*'s tail may be sorted although sorted(*t*) = false (⇒ but not ⇔).

[12] Additional properties nokey, nosorted, norevsorted give “proofs” (tail positions) why property does not hold. Example: nosorted = 3 ≡ tail value for row 3@0 < tail value for row 2@0.

MAL operations inspect BAT properties at *query runtime*, select one of several efficient implementations:



- This is coined **tactical optimization** (as opposed to strategical query optimization at *query compile time*).

The Tactics of **algebra.select: dense(*t*)**

If input BAT *t* is **dense**, use **positional access** and **slicing** to evaluate equality and range selections:

algebra.select(*t*,42@0)

head	tail
0@0	39@0
1@0	40@0
2@0	41@0
3@0	42@0
4@0	43@0
5@0	44@0

offset 3 = 42@0-39@0

hseqbase(*t*)

algebra.select(*t*,40@0,42@0,*t*,*t*,*f*)

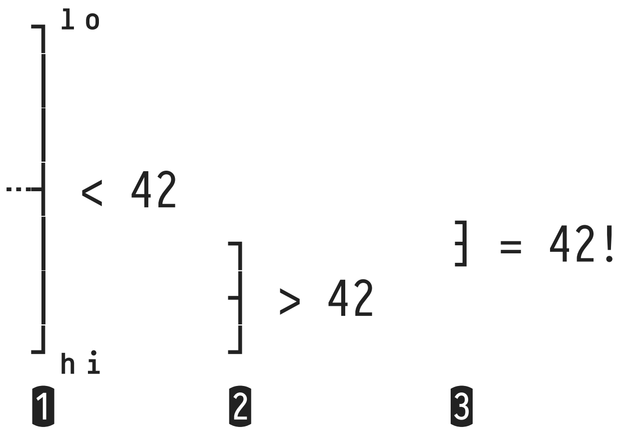
head	tail
0@0	39@0
1@0	40@0
2@0	41@0
3@0	42@0
4@0	43@0
5@0	44@0

↕ ≡ algebra.slice(*t*,1,3)

The Tactics of algebra.select: sorted(t)

algebra.select(t,42)

head	tail
0@0	38
1@0	39
2@0	40
3@0	41
4@0	42
5@0	43
6@0	44



Binary Search:

- Test middle value (pivot) between limits *lo* and *hi*
- Recurse into upper or lower partition based on test
- Finishes in $\log_2(|t|)$ steps

- **N.B.:** Unpredictable branches ($\lesseqgtr$ 42?) and jumps of pivot position less than ideal for CPU.

The Tactics of algebra.select: Order Indexes

algebra.select(*t*,*v*)

algebra.projection(*oidx*,*t*)

head	tail
0@0	38
1@0	39
2@0	40
3@0	41
4@0	42
5@0	43
6@0	44

❷

Order Index *oidx*

head	tail
0@0	5@0
1@0	1@0
2@0	0@0
3@0	6@0
4@0	2@0
5@0	4@0
6@0	3@0

❶

BAT *t*

head	tail
0@0	40
1@0	39
2@0	42
3@0	44
4@0	43
5@0	38
6@0	41

- Row [*i*@0,*j*@0] ∈ *oidx*: value at offset *j* is *i*th largest in tail. Tactic: ❶ Apply *oidx*, ❷ then use binary search.

Creating Order Indexes (On the Fly)

MonetDB may *automatically* create a temporary order index to support predicates $lo \leqslant a \leqslant hi$ or other order-sensitive queries (e.g., ORDER BY, GROUP BY).

- Check current properties of column BATs and presence of indexes in MonetDB system table sys.storage:

```
sql> SELECT column, sorted, revsorted, "unique", orderidx
     FROM   sys.storage('sys', 'indexed');
+-----------------+--------+-----------+--------+----------+
| column          | sorted | revsorted | unique | orderidx |
+=================+========+===========+========+==========+
| a               | true   | null      | true   |        0 |
| b               | null   | null      | null   |        0 |
| c               | false  | false     | null   |        0 |
+-----------------+--------+-----------+--------+----------+
```

Creating Order Indexes (Manually)

If this seems beneficial for the **query workload**, clients may *manually* create an order index.

- ⚠️ Order indexes are **static** (i.e., not maintained under updates—costly) ⇒ underlying table must be *read-only*:

  ```
  <create and populate table T>
  sql> ALTER TABLE T SET READ ONLY;
  sql> CREATE ORDERED INDEX I ON T(a);
  ```

  - Order index *I* is made persistent (in a `*.torderidx` disk file) and will be used by future `algebra.select()`s on column *a*.

With **column cracking**,[13] MonetDB introduced a **self-organizing** (partially) ordered index structure.

- A **cracker index** for column a is created/updated as a **by-product of processing range predicates** $lo \leqslant a \leqslant hi$.
  - In the cracker index, the a values $\in [lo, hi]$ are stored physically contiguous.
- If the **query workload** focuses only on a subset of column a, that part is indexed with fine granularity (while the other parts remain largely non-indexed).

[13] *"Database Cracking"*, S. Idreos, M. Kersten, S. Manegold. Proc. CIDR, Asilomar (CA, USA), 2007.

Column Cracking As a By-Product of Query Processing

❶ BAT a

head	tail
0@0	17
1@0	3
2@0	8
3@0	6
4@0	2
5@0	15
6@0	13
7@0	4
8@0	12

$\xrightarrow{Q_i}$

❷ Cracker Index

head	tail	
0@0	4	
1@0	3	$\leqslant 5 \quad s_1$
2@0	2	
3@0	6	$> 5 \quad s_2$
4@0	8	
5@0	15	
6@0	13	$\geqslant 10 \quad s_3$
7@0	17	
8@0	12	

$\xrightarrow{Q_j}$

❸ Cracker Index

head	tail	
0@0	2	$\leqslant 3 \quad s_4$
1@0	3	
2@0	4	$> 3 \quad s_5$
3@0	6	$> 5 \quad s_6$
4@0	8	
5@0	12	$\geqslant 10 \quad s_7$
6@0	13	
7@0	17	$\geqslant 14 \quad s_8$
8@0	15	

- Q_i: … **WHERE** $a > 5$ **AND** $a < 10$ Result: slice s_2
- Q_j: … **WHERE** $a > 3$ **AND** $a < 14$ Result: slices $s_5 + s_6 + s_7$

Column Cracking Notes

- MonetDB implements slicing in terms of *views*[14] of the source BAT, no data copying involved. Cost free.

- $\forall x \in s_i,\ y \in s_{i+1}: x < y$: a fully cracked column ($\forall_i |s_i| = 1$) is completely ordered. This is uncommon (workload skew).

- First cracking step (❶→❷) copies source BAT. All further steps physically reorganize the cracker index.

- Physical cracker index reorganization ("tail shuffling") can be efficiently performed *in-situ*.

[14] A possible BAT view: (*source BAT*, *first row*, *last row*).

Cracker Index Reorganization For Predicate $a < hi$

Reorganize column vector *a*[] between row offsets *start* and *end*, relocate its elements *in-situ*:

```
CrackInTwo(a,start,end,hi):
 while (start < end)
   if (a[start] < hi)
     start ← start + 1;
   else
     while (a[end] ⩾ hi ∧ end > start)
       end ← end - 1;
     swap(a[start], a[end]);  ⁂
     start ← start + 1;
     end ← end - 1;
```

- ⁂ Either $a[start] \geqslant hi \wedge a[end] < hi$ or $start = end$.