DB 2

09 – Ordered Indexes (B^+Trees)

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Torsten Grust
Universität Tübingen, Germany

1 ┆ Q_8 — Filtering an Indexed Table

Sequential scan (**Seq Scan**) and interpreted predicate evaluation go a long way. Large input tables call for significantly more **efficient support for value-based row access:**

```sql
SELECT i.b, i.c
FROM   indexed AS i
WHERE  i.a = 42 [i.c = 0.42] -- either filter on i.a or i.c
```

Assume column a is **primary key** in table indexed: expect query workload that frequently identifies rows via predicates $a = k$. **Indexes** can support such queries.

Primary Key and Indexes

```
CREATE TABLE indexed (a int PRIMARY KEY, -- ⇒ NOT NULL
                      b text,
                      c numeric(3,2));    -- ± d.dd
```

DBMS expects predicates a = *k* and creates an **index on column a**—a data structure associated with and maintained in addition to table indexed—to speed up evaluation:

```
CREATE INDEX indexed_a ON indexed USING btree (a);
```

1. Whenever possible/promising, index indexed_a[1] is (also) consulted when table indexed is queried. 🚀
2. When indexed is updated, indexed_a is maintained. ⚙

[1] PostgreSQL chooses index name indexed_pkey but let's follow a ‹*table*›_‹*column*› naming scheme here.

Using EXPLAIN on Q_8

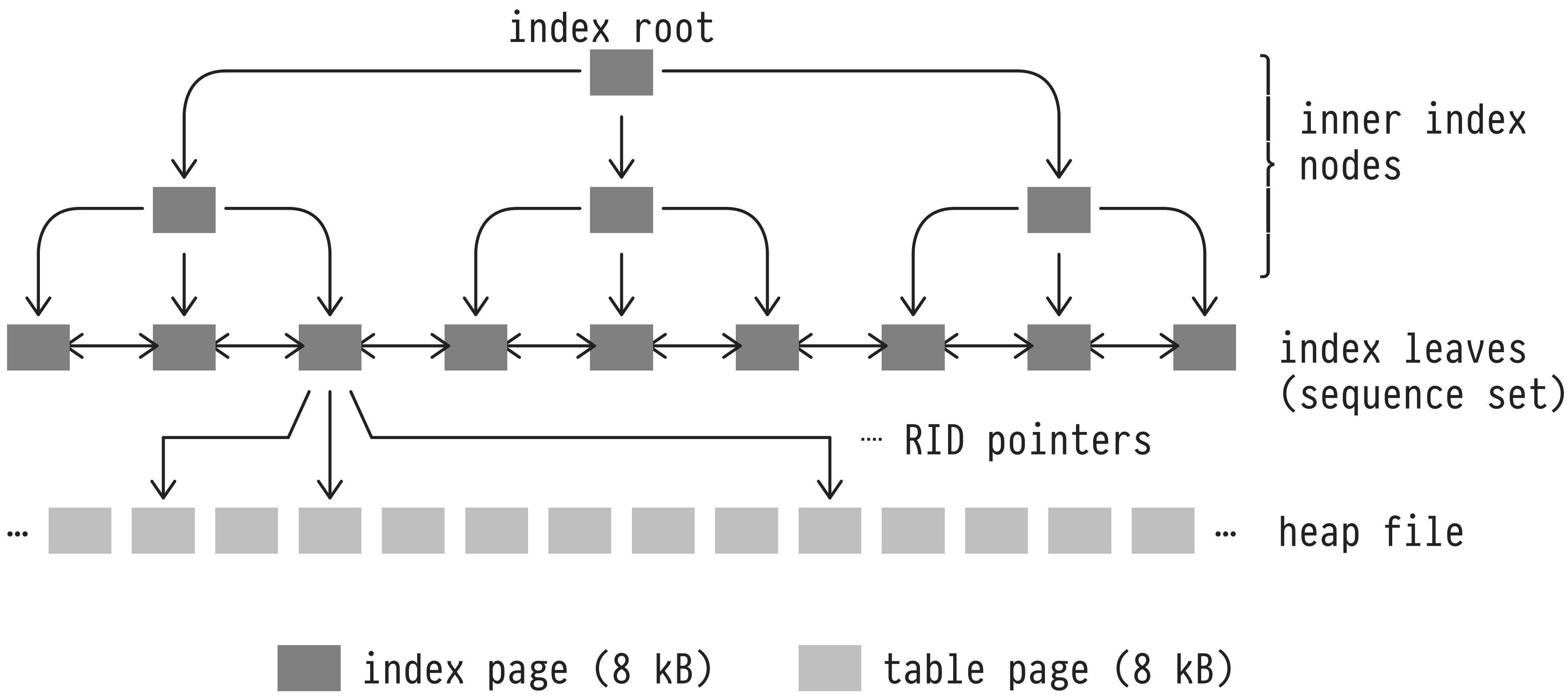
```
EXPLAIN VERBOSE
  SELECT i.b, i.c
  FROM   indexed AS i     -- 10⁶ rows
  WHERE  i.a = 42;        -- selection on key column a ⇒ ⩽ 1 row will qualify
```

QUERY PLAN

Index Scan using indexed_a on indexed i (cost=0.42..8.44 rows=1 …)
  Output: b, c
  Index Cond: (i.a = 42)

- DBMS uses **Index Scan** (instead of **Seq Scan**), index scan will evaluate predicate i.a = *k*.
- System expects small result of a single row (rows=1), i.e., the predicate is assumed to be *very selective*.

2 ┆ B⁺Trees: Ordered Indexes



Anatomy of a B⁺Tree

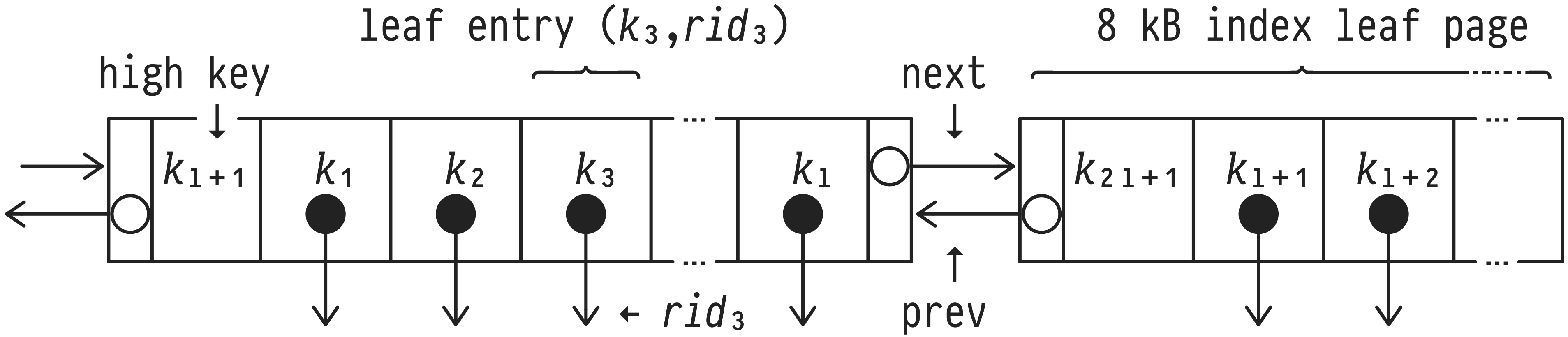
B⁺Trees: Ordered Indexes

Notes on B⁺Tree anatomy:

- A **B⁺Tree**[2] **index** I **on column** $T(a)$ is an *ordered*, *n-ary* ($n \gg 2$), *balanced*, *block-oriented*, *dynamic* search tree.
- Inner nodes and leaves are formed by 8 kB **index pages.**
- Each inner node holds $n-1$ values of column a (**separators**) that allow to navigate the search tree structure.
- Leaves form a bidirectional chain, the **sequence set.**
- **Leaves use RIDs to point to rows** in the heap file of table T: besides a column values, I holds no data of T.

[2] Invented by Bayer and McCreight (1969) at Boeing Labs. The "B" in "B⁺Tree" does *not* stand for Bayer, binary, balanced, block, or Boeing. (We tried to find out, but Rudolf Bayer wouldn't say.)

B⁺Trees: Inside a Leaf Node



- Uses pointers prev/next to form the chained **sequence set.**
- **Leaf entries are ordered** by index keys k_i: $k_i \leqslant k_{i+1}$.
- RID rid_i points to a row t of T with $t.a = k_i$.
- The **high key** holds smallest key of *next* leaf (if any).

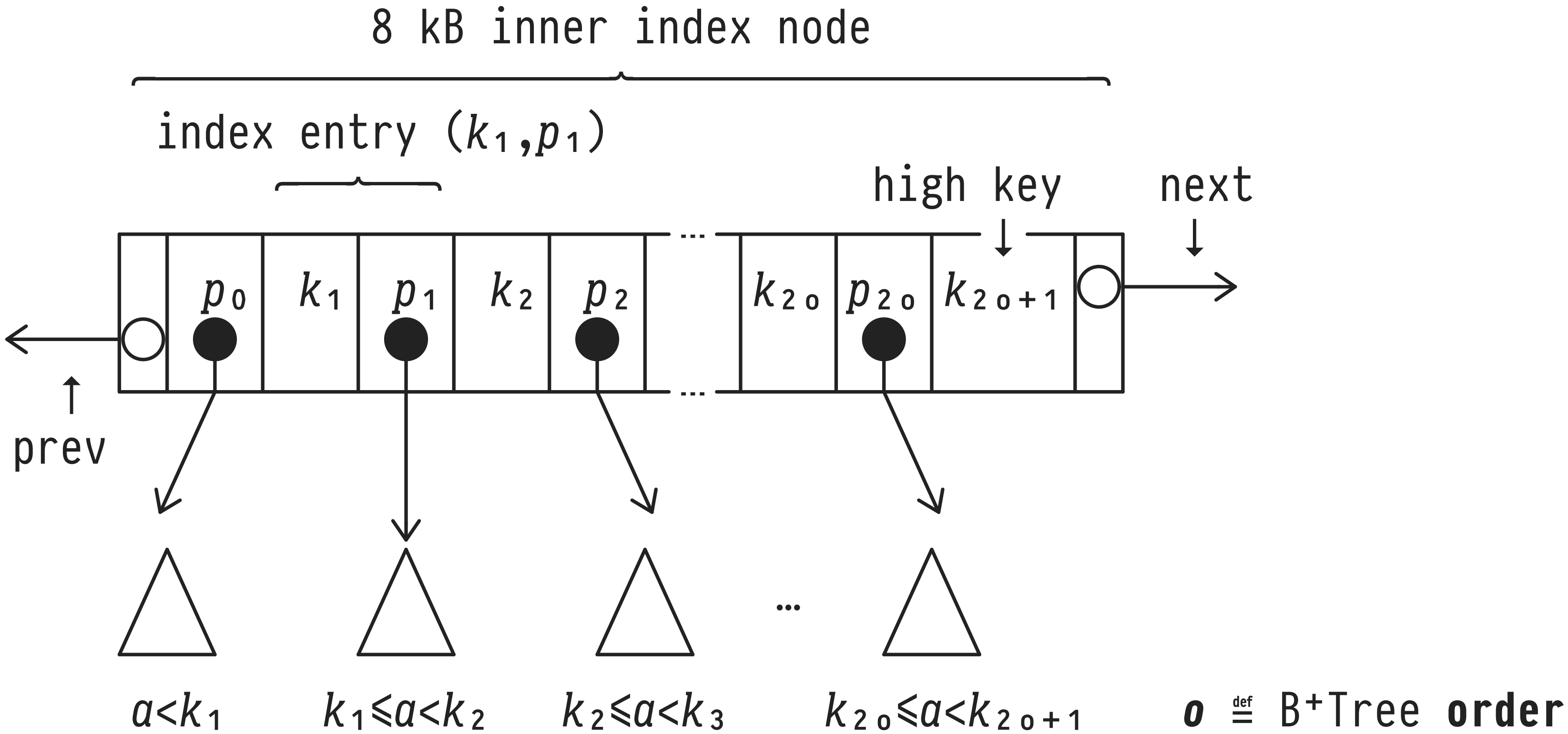
B⁺Trees: How to Find Rows t With $t.a = k$?

As described, a B⁺Tree is a **dense** index structure: every row t of T is represented by one leaf entry.

- The sequence set is ordered by keys k_i ⟹ a *binary search* for a key $k = k_i$ may sound viable, **BUT** the search would
  1. need to inspect $\log_2(|T|)$ keys in the sequence set and access just as many pages 👎, and
  2. “jump around” the sequence set in an unpredictable fashion, thus leading to random I/O. 👎

B⁺Trees exploit the sequence set ordering and erect an ***n*-ary search tree structure** (n large!) atop the leaf entries.

B⁺Trees: Inside an Inner Node



- The **separator** keys k_i are ordered: $k_i \leqslant k_{i+1}$.
- Page pointers p_j point to index (leaf or inner) nodes.

B⁺Trees: Notes on Inner Nodes

- Space in inner nodes is used economically: in a B⁺Tree of **order** o, any inner node—but the root node—is guaranteed to hold between o and $2 \times o$ ($\stackrel{\text{def}}{=}$ **fan-out** F) index entries.
- Given predicate $t.a = k$, perform **binary search inside node** to find B⁺Tree subtree with $k_i \leqslant k < k_{i+1}$.
- B⁺Tree is **balanced:** subtrees $\triangle$ are of identical height.
- **Path length** s from B⁺Tree root to leaf node **predictable:**

$$|T| \times \underbrace{1/F \times \cdots \times 1/F}_{s \text{ times}} = 1 \Leftrightarrow s = \log_F(|T|)$$

3 ¦ Index Scan

A B^+Tree is *the* index structure to support the evaluation of these kinds of conditions:[3]

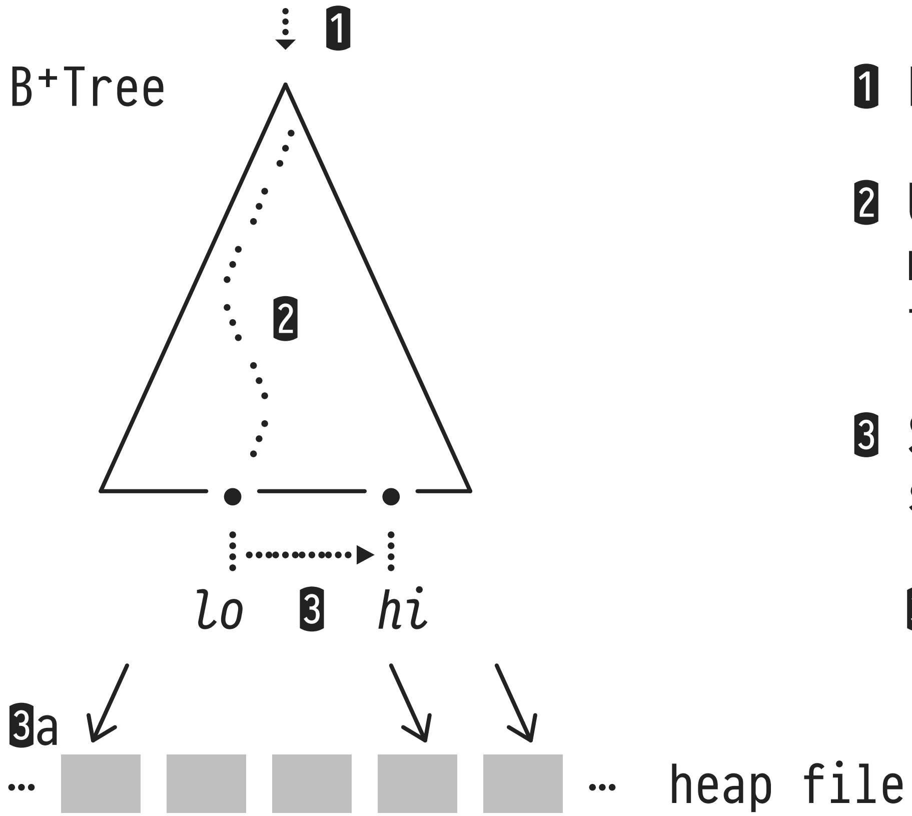
1. **Range predicates:** $lo \leqslant a \leqslant hi$
2. **Half-open ranges:** $lo \leqslant a$ or $a \leqslant hi$
3. **Equality predicates:** $a = lo$

- An **Index Scan** on index I for column $T(a)$ is parameterized by such a condition (PostgreSQL EXPLAIN: Index Cond).
- **Index Scan** uses lo to navigate the search tree structure and locate the start of relevant sequence set section.

[3] Half-open ranges are special range predicates where $hi = \infty$ ($lo = \infty$). Equality predicates are special range predicates where $lo = hi$.

Index Scan for Condition $lo \leqslant a \leqslant hi$

An **index scan** accesses the B+Tree index *and* the heap file:



1. Enter at B+Tree root page

2. Use key *lo* to **navigate the inner nodes** (search tree) until we reach the leaf level

3. Scan leaf entries in the sequence set section $lo \leqslant a \leqslant hi$, **extract RIDs**

   3a For each RID, **access heap file for table** T and return matching row

Navigating the Inner Nodes

Phase ❷ runs a vanilla traversal of a $2{\times}o$-way search tree:

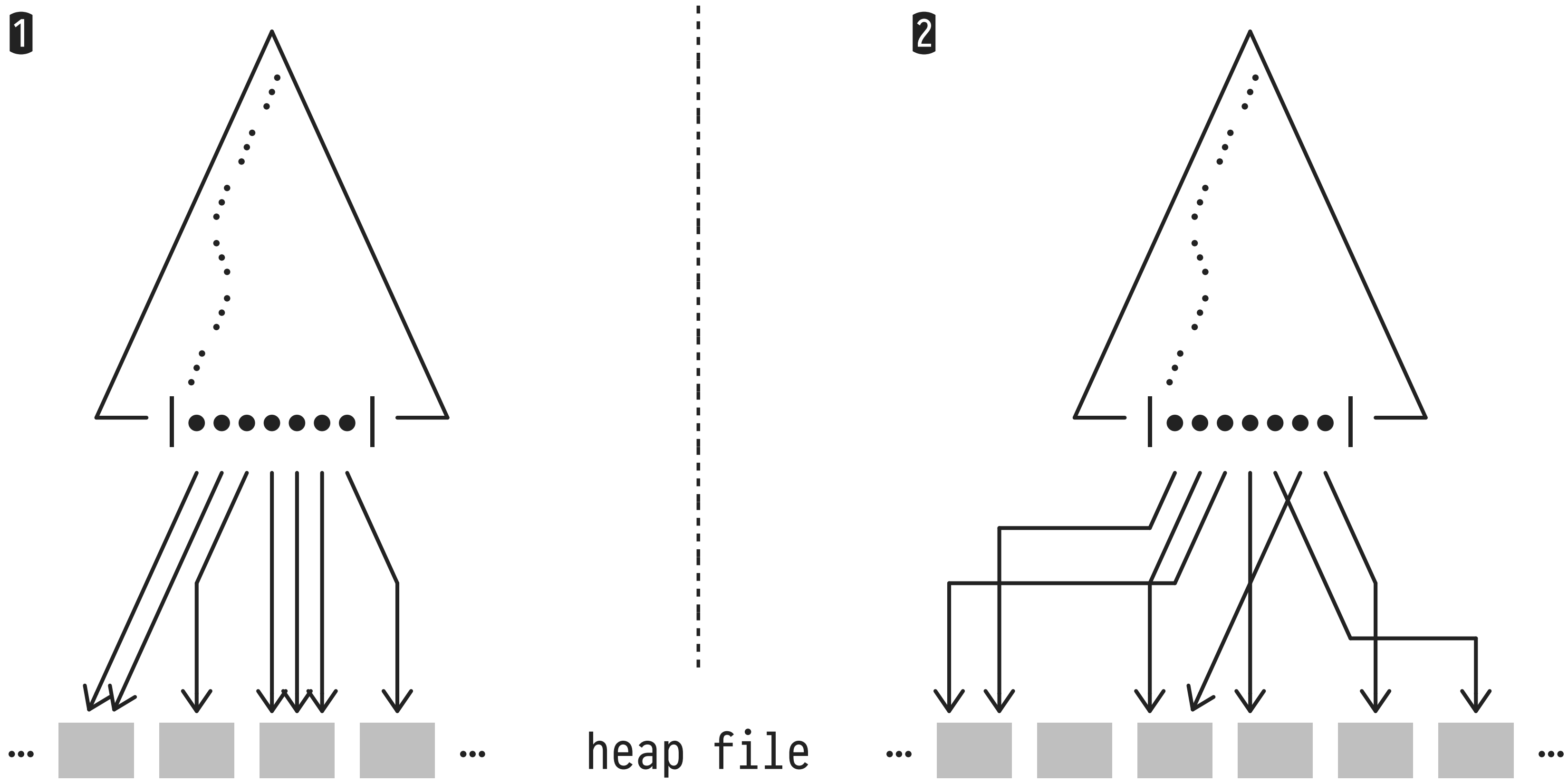
```
Search(lo):
  return TreeSearch(lo, root);

TreeSearch(lo, node):
  if (node is a leaf)
    return node;
  switch lo
    case lo < k₁
      return TreeSearch(lo, p₀);
    case kᵢ ⩽ lo < kᵢ₊₁
      return TreeSearch(lo, pᵢ);
    case k₂ₒ ⩽ lo
      return TreeSearch(lo, p₂ₒ);
```

Search: returns entry point for scan of sequence set

switch: use binary search to implement subtree choice

4 ┊ Order of Leaf Entries *vs* Order of Table Rows



❶ Order of leaf entry keys k_i ≡ row order in heap file. 👍

❷ Order of k_i in sequence set and row order do *not* match.

Index I for column $T(a)$ is **clustered** if the order of leaf entries coincides with T's row order (i.e., both I's sequence set and T's heap file are ordered by a):

Given entries $\langle k_i, p_i \rangle$ and $\langle k_j, p_j \rangle$, $k_i \leqslant k_j \Rightarrow p_i \leqslant p_j$.

- An **Index Scan** over a *clustered* index
  1. collects matching rows from adjacent heap file pages ($\Rightarrow$ sequential I/O 👍),
  2. will find many matching rows on each accessed heap file page ($\Rightarrow$ less page I/O 👍).

Non-Clustered Indexes

Sad fact: only *one*—among the many possible—indexes for a table may be clustered. Most indexes are non-clustered.

- An **Index Scan** over a *non-clustered* index
  1. will find matching rows potentially scattered across all heap file pages (⇒ random I/O 👎),
  2. will find few matching rows on each accessed heap file page and may access the same page more than once (⇒ as many page I/Os as matching rows 👎).

PostgreSQL addresses this challenge through **RID sorting**, implemented via **Bitmap Index Scan** & **Bitmap Heap Scan**.

Bitmap Index Scan & Bitmap Heap Scan

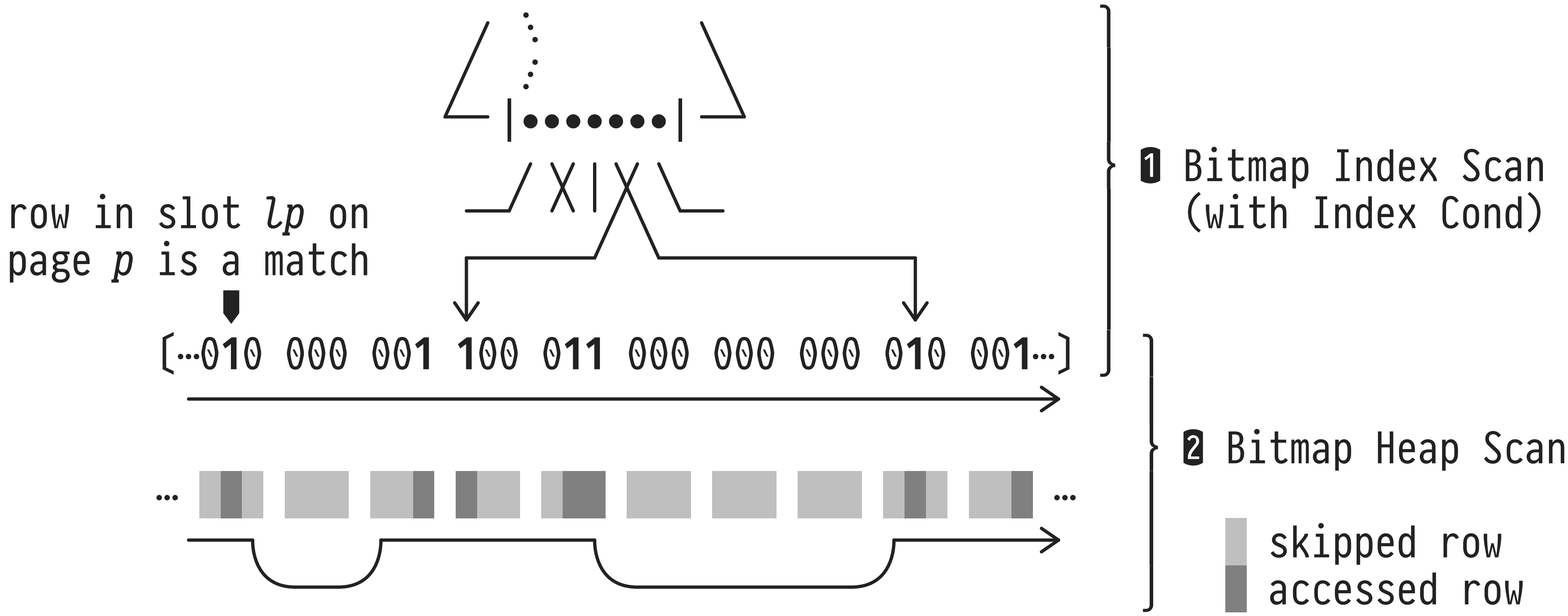
```
                 QUERY PLAN
-------------------------------------------------
❷ Bitmap Heap Scan on indexed i
    Recheck Cond: (c = 0.42)
    Heap Blocks: exact=2964
    -> ❶ Bitmap Index Scan on indexed_c
          Index Cond: (c = 0.42)
```

bitmap of
matched rows

❶ **Bitmap Index Scan:** perform Index Scan and create **bitmap** that encodes *heap file locations* of rows matching the Index Cond. Do *not* access rows in heap file yet.

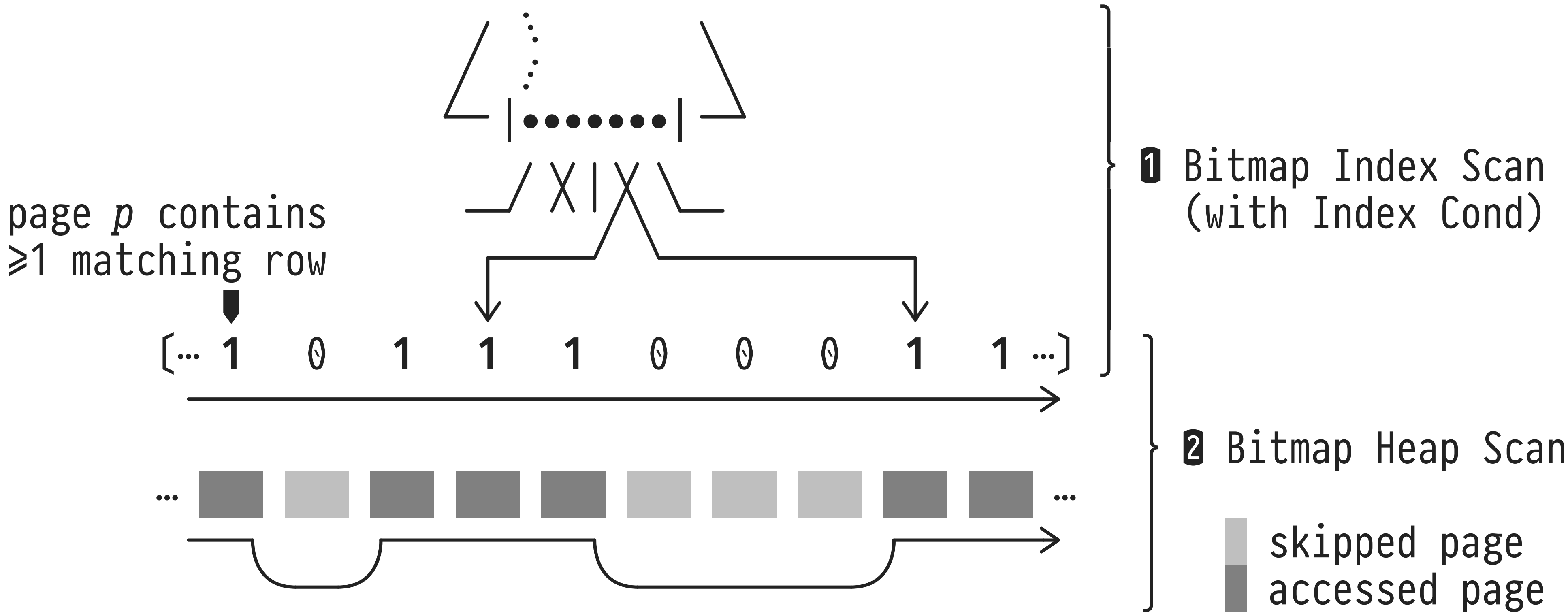
❷ **Bitmap Heap Scan:** scan heap file once, only access those rows (pages) that have been marked **1** in the bitmap.

Bitmap Index Scan & Bitmap Heap Scan: Row-Level Bitmap



Bitmap Heap Scan performs one sequential scan (with skips) of the heap file, regardless of RID order in sequence set.

Bitmap Index Scan & Bitmap Heap Scan: Page-Level Bitmap



Working memory tight ⇒ build **page-level** bitmap. ⚠️ In ❷, need to **recheck condition** for all rows on accessed pages.

5 ┊ CLUSTERing Based on an Index

If the workload depends on top performance of particular predicates supported by *non-clustered* index *I*, we may

> **physically reorder the rows of underlying table's *T* heap file** to coincide with the key order in *I*'s sequence set (i.e., *I* will become a *clustered* index[4]):

```
CLUSTER [VERBOSE] ‹T› USING ‹I›;
CLUSTER ‹T›;   -- re-cluster once T's rows get out of order
```

- ⚠️ Subsequent updates on *T* can destroy the perfect clustering. (May need to re-cluster *T* in intervals.)

[4] At a price, of course: formerly *clustered* indexes on *T* will turn into *non-clustered* indexes.

B^+Trees…

1. **economically utilize space** in inner/leaf nodes (minimum node occupancy 50%, typical fill factor 67%),
2. are **balanced** trees and thus require a **predictable number of page I/Os** to traverse from root to sequence set—enables query optimizer to forecast B^+Tree access cost.

DBMSs maintain properties 1. and 2. when rows are **inserted** into/**deleted** from an B^+Tree-indexed table.[5]

[5] Some real B^+Tree implementations of row deletion deviate from the textbook to keep things simpler.

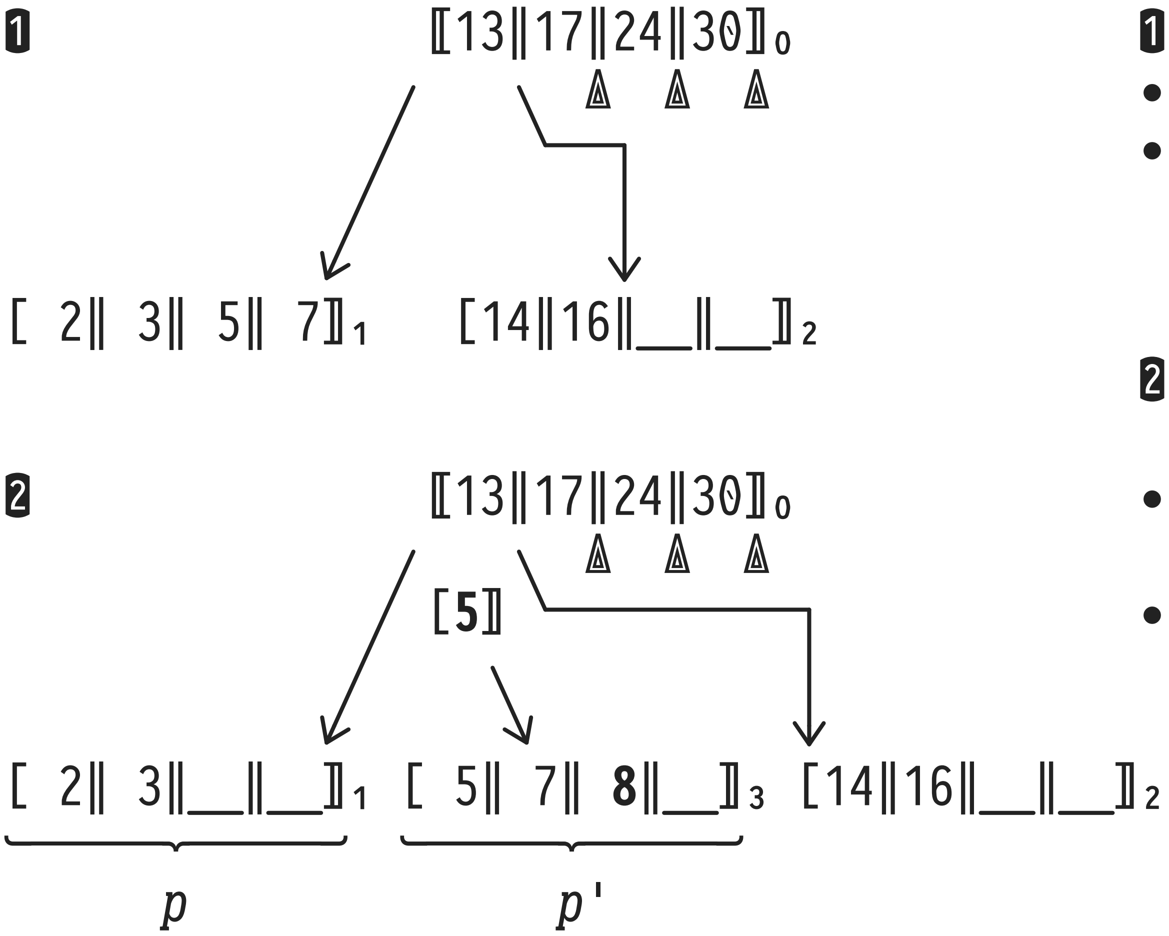
B^+Tree Insertion for New Entry $\langle k, rid\rangle$ (Sketch)

1. Use Search(k) to **find leaf page** p which should hold the entry for k.
2. If p has **enough space** to hold new entry (i.e., at most $2 \times o - 1$ entries in p), **simply insert** $\langle k, rid\rangle$ into p.
3. Otherwise, node p must be **split** into p and p' and a new **separator** has to be inserted↻ into the parent of p.

   Splitting happens recursively↻ and may eventually lead to a split of the root node (increasing B^+Tree height).

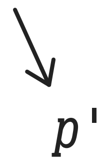
   - **Distribute** the entries of p and new entry $\langle k, rid\rangle$ onto pages p and p'.

B⁺Tree Insertion and Leaf Node Split

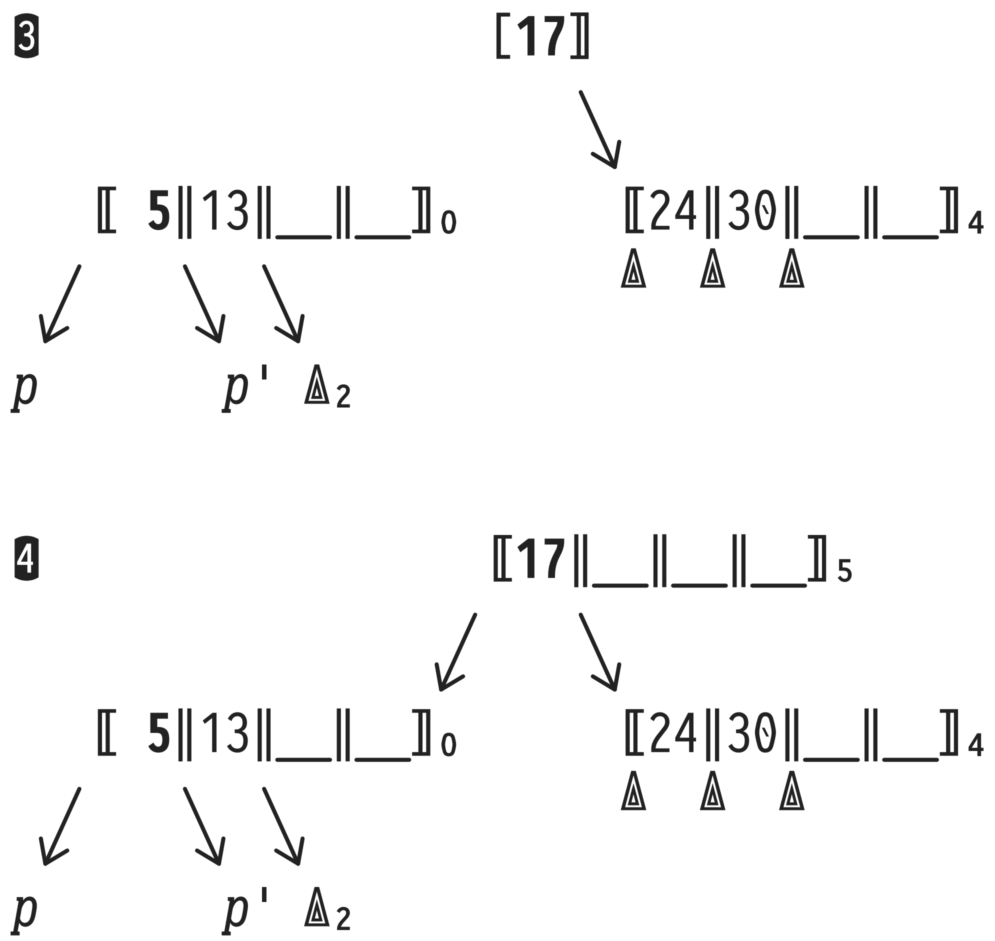


❶ Insert new entry ‹**8**,*rid*›
- Search(**8**) returns leaf *p* = 1
- Leaf 1 is full ⇒ split

❷ Leaf 1 split into leaves *p* = 1 and *p*' = 3
- Distribute {2,3,5,7,**8**} between leaves 1 and 3
- **Copy** new separator [**5**⟧ into parent node 0



B⁺Tree Insertion and Inner Node Split



3 Inner node 0 (here: root) is full ⇒ split

- Inner node 0 splits into old node 0 and new *p*'' = 4
- Distribute {**5**,13,24,30} between nodes 0 and 4
- **Move** new separator [**17**⟧ into parent of node 0

p''

4 Split node 0 has been the old root

- Create new root node 5, has ⟦**17**‖ as only entry
- B⁺Tree height has increased

B^+Tree Insertion Notes

- Splitting starts at the leaf level and continues upward as long as inner index nodes are fully occupied (holding $2 \times o$ entries).

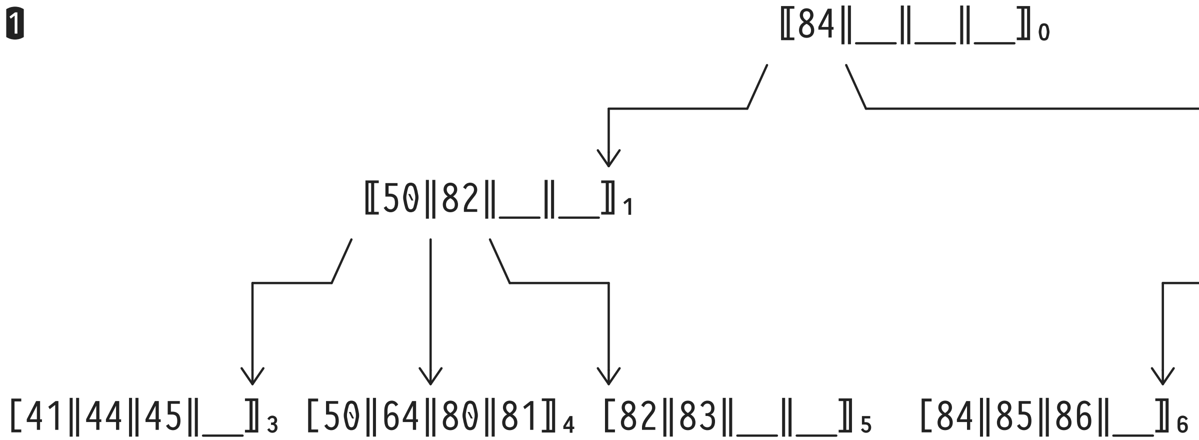
- ⚠️ Unlike during a *leaf* split, an *inner* node split **moves**[6] the new separator [***sep***] discriminating between p and p' upwards and recursively inserts it into the parent. **Q:** Why?

- **Q:** How often do you expect a root node split to happen?

[6] A leaf node split **copies** the new separator upwards, i.e., the entry [***sep***] also remains at the leaf level.

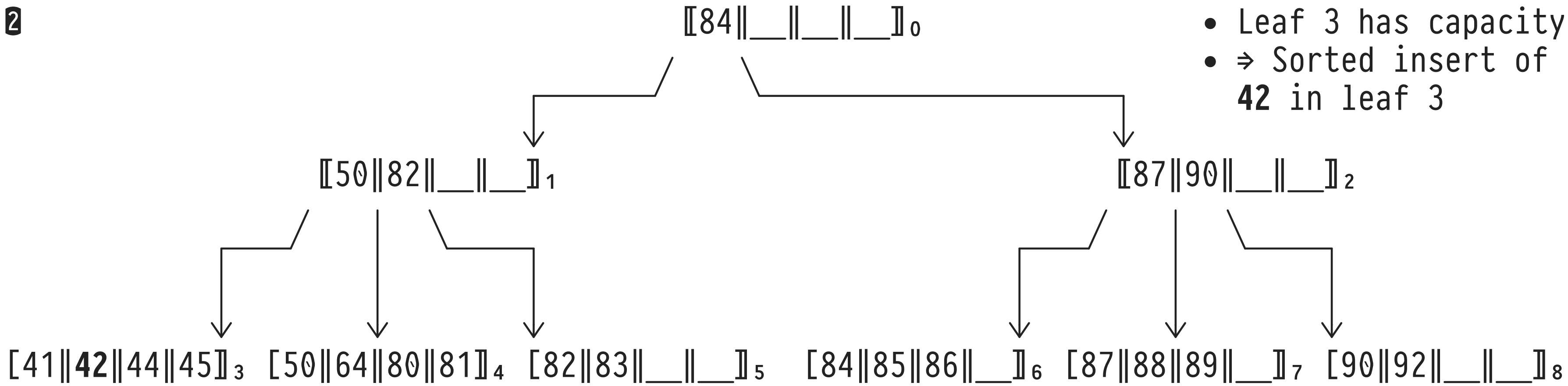
B⁺Tree Insertion Example: Insert ‹42,*rid*›

1



- Search(**42**) returns leaf 3

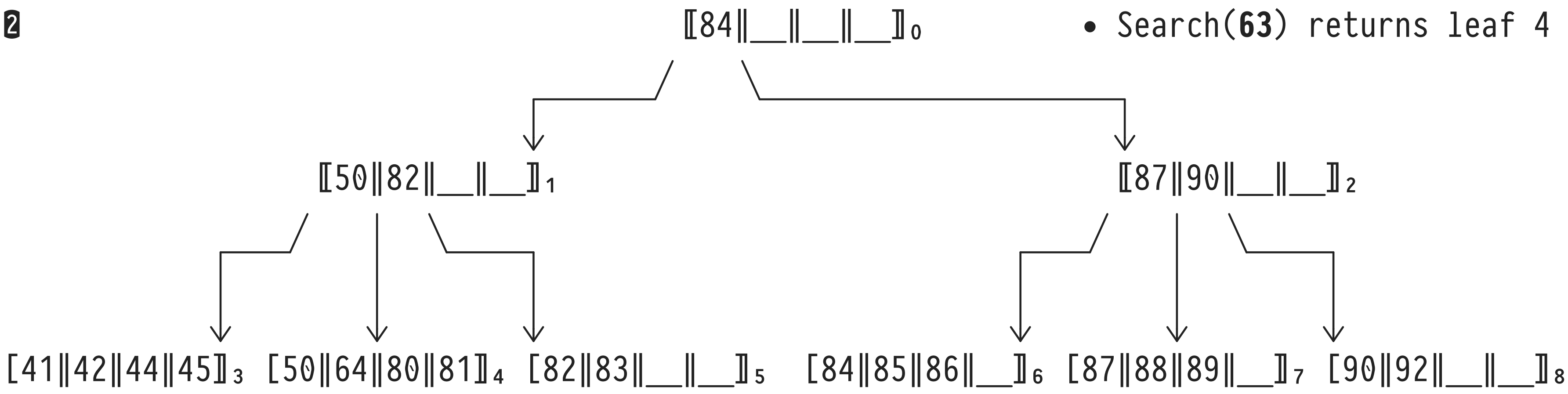
2



- Leaf 3 has capacity
- ⇒ Sorted insert of **42** in leaf 3

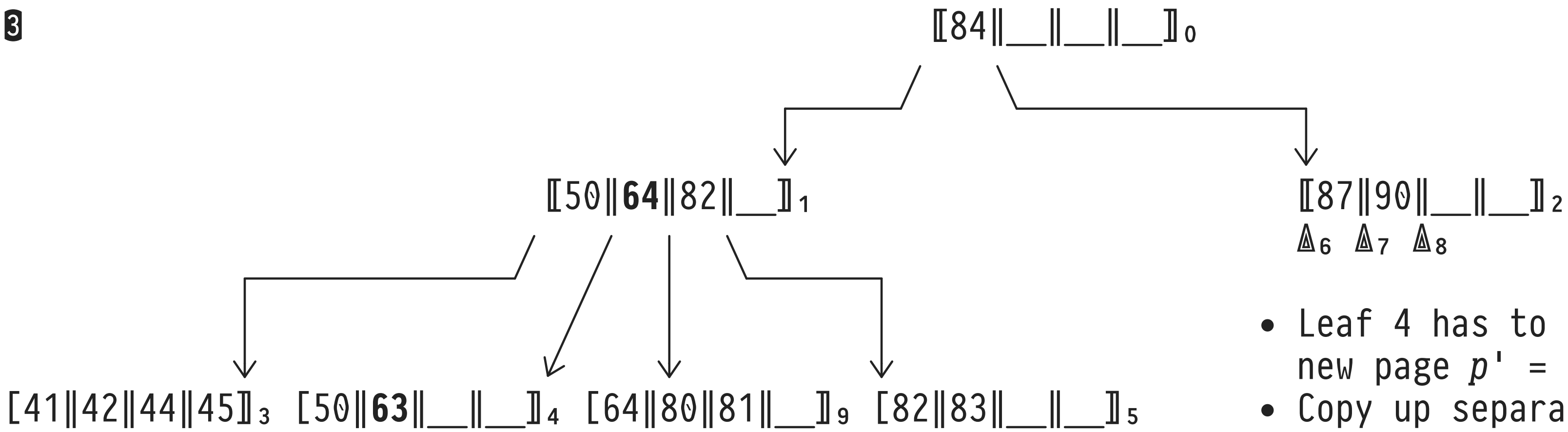
B⁺Tree Insertion Example: Insert ‹**63**,*rid*›

2



- Search(**63**) returns leaf 4

3



- Leaf 4 has to split, new page p' = 9
- Copy up separator [**64**⟧

B⁺Tree Insertion Example: Insert ‹75,*rid*›

3

⟦84‖__‖__‖__⟧$_0$

⟦50‖64‖82‖__⟧$_1$ ⟦87‖90‖__‖__⟧$_2$

⍋$_6$ ⍋$_7$ ⍋$_8$

[41‖42‖44‖45⟧$_3$ [50‖63‖__‖__⟧$_4$ [64‖80‖81‖__⟧$_9$ [82‖83‖__‖__⟧$_5$

- Search(**75**) returns leaf 9

4

⟦84‖__‖__‖__⟧$_0$

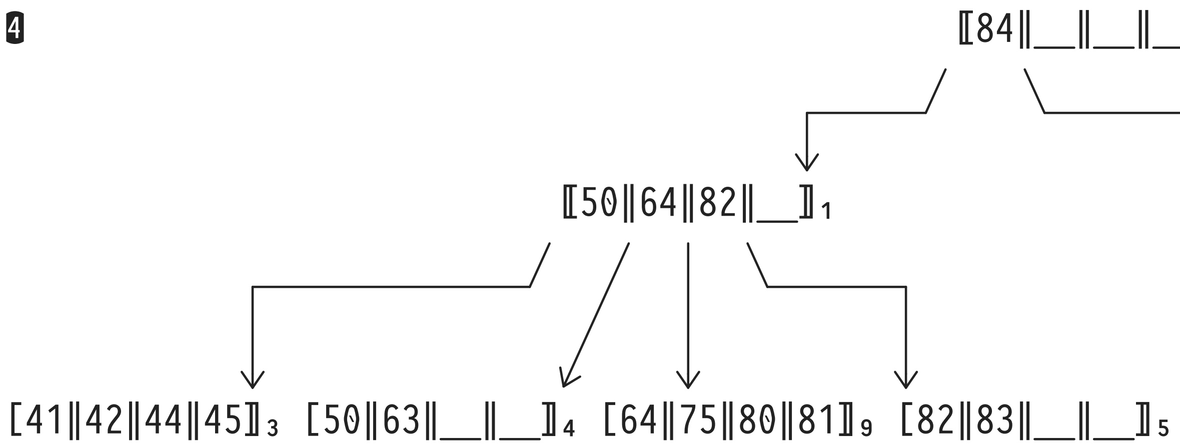
⟦50‖64‖82‖__⟧$_1$ ⍋$_2$

[41‖42‖44‖45⟧$_3$ [50‖63‖__‖__⟧$_4$ [64‖**75**‖80‖81⟧$_9$ [82‖83‖__‖__⟧$_5$

- Leaf 9 has capacity
- ⇒ Sorted insert of **75** in leaf 9

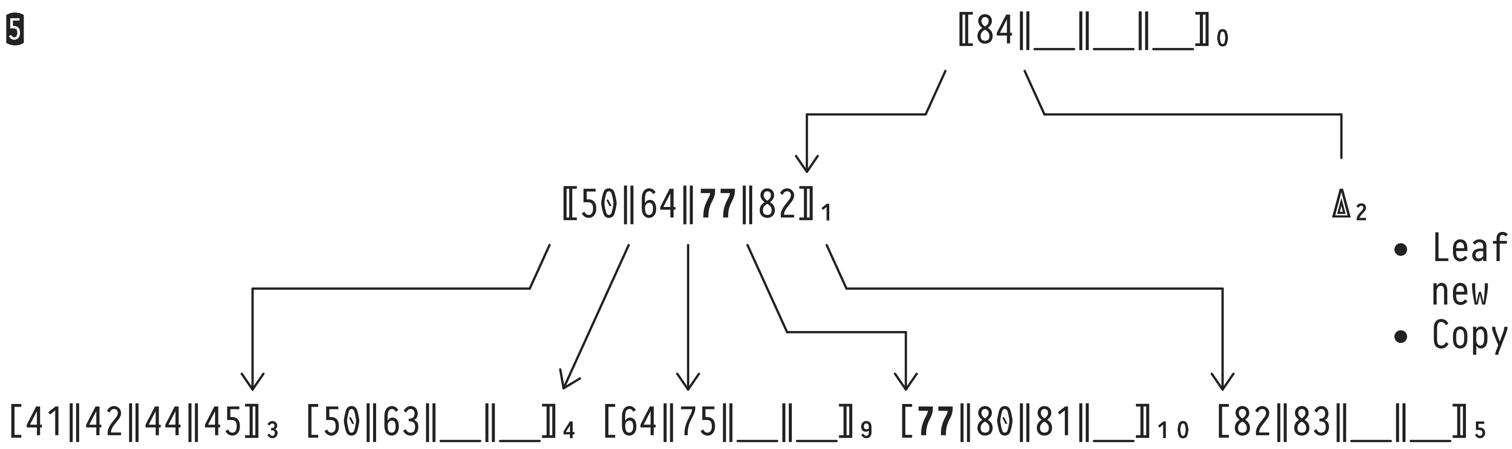
B$^+$Tree Insertion Example: Insert ‹77,*rid*›

4



- Search(**77**) returns leaf 9
- Leaf 9 is full (already holds 2×*o* = 4 entries)

5



- Leaf 9 has to split, new page *p*' = 10
- Copy up separator [**77**⟧

B⁺Tree Insertion Example: Insert ‹**35**,*rid*›

5

- Search(**35**) returns leaf 3
- Leaf 3 is full ⇒ split,
  new page *p*' = 11

⟦84‖__‖__‖__⟧$_0$

⟦50‖64‖77‖82⟧$_1$ ⍋$_2$

[41‖42‖44‖45⟧$_3$ [50‖63‖__‖__⟧$_4$ [64‖75‖__‖__⟧$_9$ [77‖80‖81‖__⟧$_{10}$ [82‖83‖__‖__⟧$_5$

6

- Copy up [**42**⟧
  ⇒ split inner node 1,
  new page *p*" = 12
- Move up [**64**⟧

⟦**64**‖84‖__‖__⟧$_0$

⟦**42**‖50‖__‖__⟧$_1$ ⟦77‖82‖__‖__⟧$_{12}$ ⍋$_2$

[**35**‖41‖__‖__⟧$_3$ [42‖44‖45‖__⟧$_{11}$ [50‖63‖__‖__⟧$_4$ [64‖75‖__‖__⟧$_9$ [77‖80‖81‖__⟧$_{10}$ [82‖83‖__‖__⟧$_5$

B^+Tree Insertion Algorithm[7] (1)

```
TreeInsert(‹k,rid›,node):
  if (node is a leaf)
   | return LeafInsert(‹k,rid›,node);
  else
   | switch k                                              ⎫
   |  | case k < k_1                                       ⎪
   |  |  ⌊ ‹sep,ptr› ← TreeInsert(‹k,rid›,p_0);            ⎪
   |  | case k_i ⩽ k < k_{i+1}                             ⎬ see Search()
   |  |  ⌊ ‹sep,ptr› ← TreeInsert(‹k,rid›,p_i);            ⎪
   |  | case k_{2o} ⩽ k                                    ⎪
   |  ⌊  ⌊ ‹sep,ptr› ← TreeInsert(‹k,rid›,p_{2o});         ⎭
   | if (sep = ⊥)                                          ⎫ upwards split?
   |  | return ‹⊥,⊥›;                                      ⎬ ⇒ no
   | else                                                  ⎪
   ⌊  ⌊ return InnerInsert(‹sep,ptr›,node);                ⎭ ⇒ yes
```

[7] Note: ‹*sep*,*ptr*› ≡ ⟦*sep*⟧ in our discussion above.

B⁺Tree Insertion Algorithm (2)

```
LeafInsert(‹k,rid›,node):
  if (node has < 2×o entries)
    | insert ‹k,rid› into node;
    | return ‹⊥,⊥›;                 } ‹⊥,_› ≡ no upwards split required
  else
    | p' ← allocate leaf page;
    | [‹k₁,rid₁›,…,‹k₂ₒ₊₁,rid₂ₒ₊₁›] ← entries of node ∪ ‹k,rid›;
    | node ← [k₁|rid₁|…|kₒ|ridₒ|__‖__];
    | p'   ← [kₒ₊₁|ridₒ₊₁|…|k₂ₒ₊₁|rid₂ₒ₊₁|__‖__];
    ⌊ return ‹kₒ₊₁,p'›;             } new separator to be copied upwards
```

- **Copy** upwards: entry $\langle k_{o+1}, rid_{o+1}\rangle$ remains in leaf p'.

B⁺Tree Insertion Algorithm (3)

```
InnerInsert(⟨sep,ptr⟩,node):
  if (node has < 2×o entries)
  │ insert ⟨sep,ptr⟩ into node;
  │ return ⟨⊥,⊥⟩;                } ⟨⊥,_⟩ ≡ no upwards split required
  else
  │ p' ← allocate inner node page;
  │ [p_0,⟨k_1,p_1⟩,…,⟨k_2o+1,p_2o+1⟩] ← entries of node ∪ ⟨sep,ptr⟩;
  │ node ← [p_0|k_1|p_1|…|k_o|p_o|__‖__];
  │ p'   ← [p_o+1|k_o+2|p_o+2|…|k_2o+1|p_2o+1|__‖__];
  └ return ⟨k_o+1,p'⟩;           } new separator to be moved upwards
```

- **Move** upwards: new entry $\langle k_{o+1}, p' \rangle$ returned for insertion at parent. No entry $\langle k_{o+1}, \_ \rangle$ remains at level of *node*/p'.

B⁺Tree Insertion Algorithm (Top Level)

Insert(‹*k*,*rid*›) is the top-level B⁺Tree insertion routine:

```
Insert(‹k,rid›):
  ‹k',ptr› ← TreeInsert(‹k,rid›,root);  } root ≡ old root page
  if (k' ≠ ⊥)
  │ r ← [root|k'|ptr|__‖__‖__];          } r ≡ new root page
  └ root ← r
```

```
        ⟦k'‖__‖__‖__⟧ ◀ new root page r
         ↙   ↘
      root     ptr
```

- Note: Insert() may leave us with a new root node that violates the minimum occupancy rule. ¯\(ツ)/¯

B⁺Tree Insertion: Redistribution (1)

Can improve average occupancy and delay height increase on B⁺Tree insertion through **redistribution:**

❶ ⟦13‖17‖24‖30⟧$_0$

⍋ ⍋ ⍋

[2‖ 3‖ 5‖ 7⟧$_1$ ⇆ [14‖16‖__‖__⟧$_2$ ⇆ …

❶ Insert new entry ‹**6**,*rid*›
- Search(**6**) returns leaf 1
- Leaf 1 is full, but its right **sibling** 2 has capacity

- Use sequence set chain pointers (⇆) to inspect **sibling** nodes for spare capacity.
- **Push** entry from overflowing node to sibling and ⚠️ **update separator in parent node** to reflect this redistribution.

B+Tree Insertion: Redistribution (2)

```
❷                  ⟦7‖17‖24‖30⟧_0
                        ⍋   ⍋   ⍋

[ 2‖ 3‖ 5‖ 6⟧_1 ⇆ [ 7‖14‖16‖__⟧_2 ⇆ …
```

❷ Push entry ‹7,*rid*'› to leaf 2
- Place ‹**6**,*rid*› in leaf 1
- Update separator (13 → **7**) in parent node 0
- B+Tree remains at height 2

- Inspecting node sibling involves additional page I/O. 👎
- Actual implementations use redistribution on the index leaf level only (if at all).

7 ¦ B^+Tree Deletion of Entry With Key k (Sketch)

1. Use Search(k) to **find the leaf** p holding entry ‹k,rid›.
2. **Simply delete** ‹k,rid› from p.[8]
3. If p now holds < o entries, leaf p **underflows.** Any sibling of p with spare entries?
   - Yes, use **redistribution** to move an entry into p.
   - No, **merge** p and a sibling leaf p' of o entries. Delete↻ the now obsolete separator of p and p' in their parent node.

     Deletion propagates upwards and may eventually leave the root node empty (decreases B^+Tree height).

[8] **Q:** If ‹k,rid› is the leftmost entry in p, do we need to update the associated separator entry in p's parent node? Why not?

B⁺Tree Deletion (No Underflow)

❶ ⟦24‖30‖__‖__⟧$_0$

◬$_3$

[**19**‖20‖22‖__⟧$_1$ [24‖27‖29‖__⟧$_2$

❶ Delete entry with key k = **19**

- Search(**19**) returns leaf 1
- Leaf 1 has > o entries, node will not underflow

❷ ⟦24‖30‖__‖__⟧$_0$

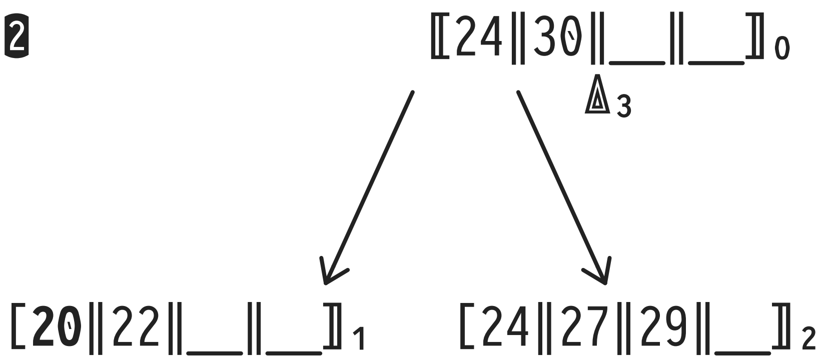
◬$_3$

[20‖22‖__‖__⟧$_1$ [24‖27‖29‖__⟧$_2$

❷ Simply delete entry ‹**19**,*rid*› from leaf 1

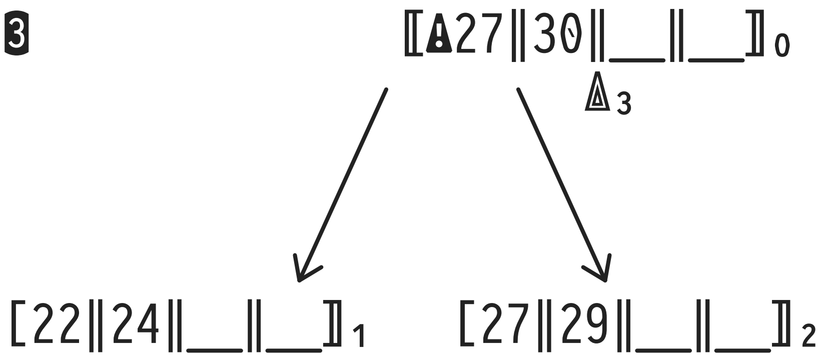
B+Tree Deletion and Redistribution

❷



❷ Delete entry with key k = **20**
- Search(**20**) returns leaf 1
- Leaf 1 has minimum occupancy of o entries ⇛ will underflow

❸



❸ Sibling p' = 2 has one entry to spare ⇛ redistribution
- Move entry ‹24,*rid*'› from leaf 2 to leaf 1
- Update separator (24 → 27) in parent node 0

B⁺Tree Deletion and Leaf Node Merging

❸ ⟦27‖30‖__‖__⟧$_0$ △$_3$

[22‖**24**‖__‖__⟧$_1$ [27‖29‖__‖__⟧$_2$

❸ Delete entry with key k = **24**
- Search(**24**) returns leaf 1
- Leaf 1 has minimum occupancy, no sibling with spare entries

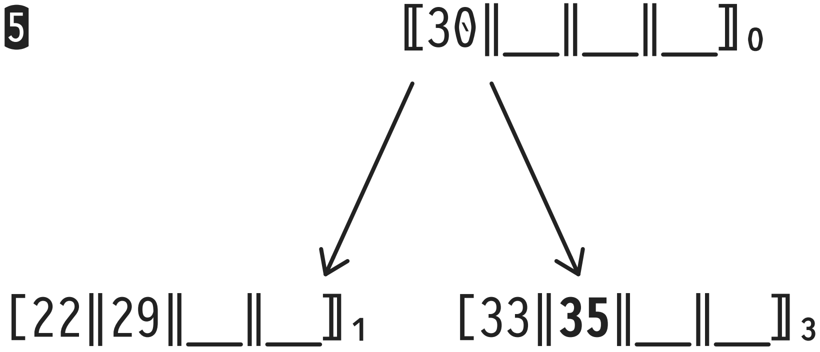
❹ ⟦30‖__‖__‖__⟧$_0$ △$_3$

[22‖27‖29‖__⟧$_1$ [__‖__‖__‖__⟧$_2$♻

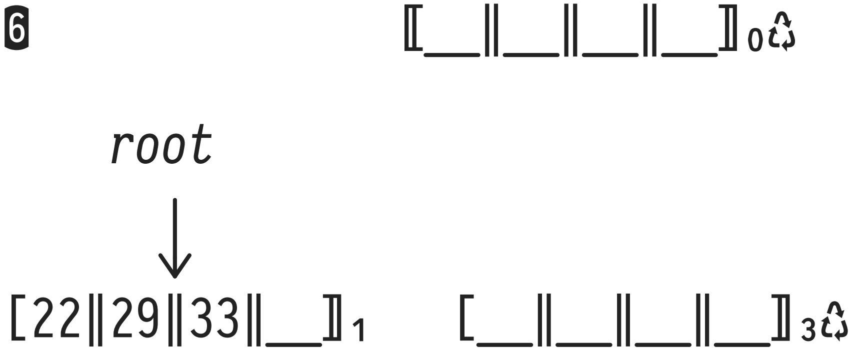
❹ Merge leaf nodes 1 and 2, mark empty page 2 as garbage
- In parent 0, delete0 obsolete separator [27⟧

B⁺Tree Deletion and Leaf Node Merging (Empty Root)

❺



❻



❺ Delete entry with key k = **35**
- Search(**35**) returns leaf 3
- Leaf 3 has minimum occupancy, no sibling with spare entries

❻ Merge leaf nodes 1 and 3, mark empty page 3 as garbage
- In parent 0, delete obsolete separator [30⟧
- Old root empty (⇒ garbage), mark page 1 as the new root
- B⁺Tree height decreases

B⁺Tree Deletion and Inner Node Redistribution

- **Redistribution** is also defined for **inner nodes.** Suppose we encounter underflow ❶ during ↻ deletion propagation:

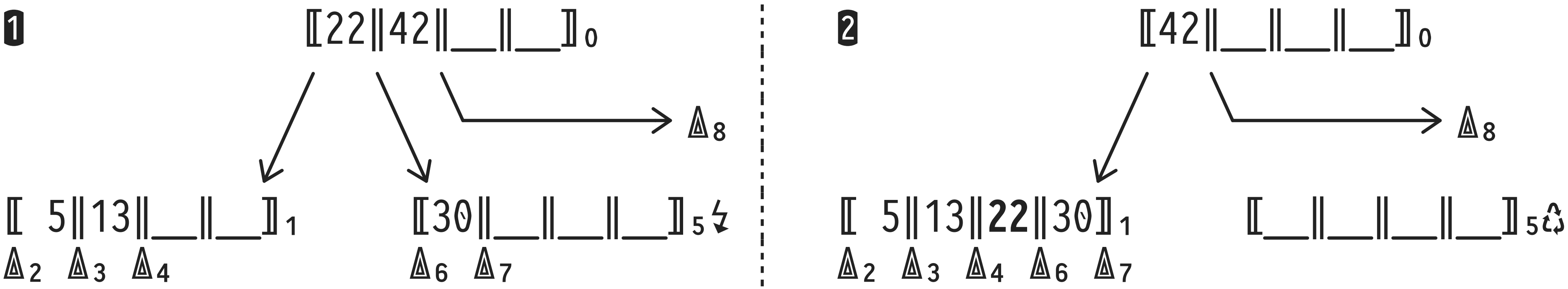
```
1              ⟦22‖__‖__‖__⟧0           |  2              ⟦20‖__‖__‖__⟧0
                 ↙    ↘                 |                   ↙    ↘
⟦ 5‖13‖17‖20⟧1        ⟦30‖__‖__‖__⟧7↯   |  ⟦ 5‖13‖17‖__⟧1       ⟦22‖30‖__‖__⟧7
𝔸2 𝔸3 𝔸4 𝔸5 𝔸6         𝔸8 𝔸9              |  𝔸2 𝔸3 𝔸4 𝔸5          𝔸6 𝔸8 𝔸9
```

- Inner node 1 has two spare entries. “Rotate entry [20] through parent” to underflowed inner node 7.

  NB: Semantics of subtree $\mathbb{A}_6$ (holds index entries with $k \geqslant 20 \wedge k < 22$) are preserved.

B⁺Tree Deletion and Inner Node Merging

- Likewise, **inner nodes** may also be **merged.** The underflow in ❶ cannot be handled by redistribution:



- Note how the separator **22** has been **pulled down** from the parent to discriminate between subtrees $\triangle_4$ and $\triangle_6$:
  - $\triangle_4$: $k \geqslant 13 \wedge k < 22$
  - $\triangle_6$: $k \geqslant 22 \wedge k < 30$

The higher the **fan-out** F, the more index entries fit in a B⁺Tree of fixed height. How to maximize F?

- For entries $\langle k,p \rangle$ in indexes over text/char columns, we may have $|k| \gg |p|$.[9] Can we reduce the size of k?
- 💡 Search() and TreeInsert() do *not* inspect the actual key values but only use </⩽ to direct tree traversals.
  - ⇒ May **shorten (truncate) string keys** as long as the ordering relation is preserved.
  - This applies to index entries in inner nodes only. Leaf level keys remain as is.

[9] The implementation (thus size) of page pointers p is prescribed by the DBMS. Nothing to win here.

B⁺Trees: Key Suffix Truncation

❶ Original B⁺Tree

```
                    ↙
        ⟦irish‖ironical‖irregular‖__⟧0
        ⍋1    ↙         ↘         ⍋4
[irksome‖iron‖ironage‖__⟧2    [irrational‖irreducible‖__‖__]3
```

❷ B⁺Tree after **key suffix truncation**

```
                    ↙
        ⟦␣␣␣␣␣␣‖␣␣␣␣␣␣␣␣␣␣‖␣␣␣␣␣␣␣␣␣␣␣‖__⟧0
        ⍋1    ↙         ↘         ⍋4
[irksome‖iron‖ironage‖__⟧2    [irrational‖irreducible‖__‖__]3
```

⚠️ While truncating, preserve the **separator** semantics.

B+Trees: Key Prefix Compression

Observation: string keys within a B+Tree inner node often **share a common prefix.**

- 💡 Store common prefix only once (e.g., as "k_0").
- Violating the 50% occupancy rule can help compression.

❶ Original B+Tree

⟦irish‖ironical‖irregular‖__⟧$_0$

$\mathbb{A}_1$ $\mathbb{A}_2$ $\mathbb{A}_3$ $\mathbb{A}_4$

❷ B+Tree after **key prefix compression**

[ir+‖ish‖onical‖regular‖__⟧$_0$

k_0 $\mathbb{A}_1$ $\mathbb{A}_2$ $\mathbb{A}_3$ $\mathbb{A}_4$

Grab a hot cup of ☕ and start a war on Stack Overflow:[10]

Q: *Which order of operations is better?*

```
❶ CREATE TABLE T (…);
❷ INSERT INTO T VALUES (‹5 × 10^6 rows›);
❸ CREATE INDEX I ON T USING btree (…);
```

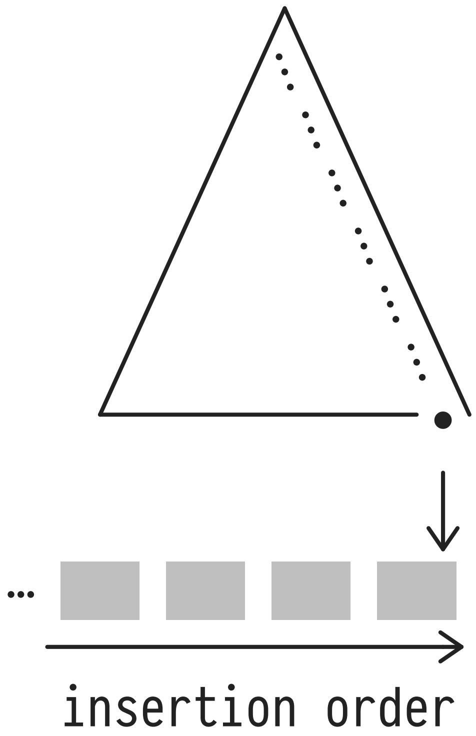
or

```
❶ CREATE TABLE T (…);
❸ CREATE INDEX I ON T USING btree (…);
❷ INSERT INTO T VALUES (‹5 × 10^6 rows›);
```

[10] See, for example, https://stackoverflow.com/questions/5910486/indexes-on-a-table-database

B+Tree Bulk Loading

If insertions happen in index key order (i.e., ascending values of *k*), we observe a particular B+Tree access pattern:



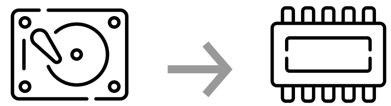
heap file (sorted on keys *k*)

- TreeInsert() will always traverse path ⋱, will always hit the righmost leaf.

⇒ Fix rightmost leaf in buffer, insert next entry right there (*no* traversal from root). Node splits only occur along path ⋱.

- We effectively create a clustered index.

10 ¦ Q_8 — Filtering a Table



```
SELECT i.b, i.c
FROM   indexed AS i
WHERE  i.a = 42 [i.c = 0.42] -- either filter on i.a or i.c
```

Indexes in MonetDB play a secondary role and are *not* organized in tree shapes.

MMDBMSs try to exploit that data resides in directly-adressable memory and primarily aim to avoid access to separate index data structures (to avoid pointer chasing and potential cache misses).

Using **EXPLAIN** on Q_8: Filter on Column **a**

```
sql> EXPLAIN SELECT i.b, i.c
             FROM   indexed AS i
             WHERE  i.a = 42;
  ⋮
 indexed :bat[:oid] := sql.tid(sql, "sys", "indexed");
 a0      :bat[:int] := sql.bind(sql, "sys", "indexed", "a", 0:int);
 p1      :bat[:oid] := algebra.thetaselect(a0, indexed, 42:int, "=="); ◀ ≡ a = 42
 c0      :bat[:sht] := sql.bind(sql, "sys", "indexed", "c", 0:int);
 c       :bat[:sht] := algebra.projection(p1, c0);
 b0      :bat[:str] := sql.bind(sql, "sys", "indexed", "b", 0:int);
 b       :bat[:str] := algebra.projection(p1, b0);
  ⋮
```

- MonetDB uses `algebra.thetaselect(…, 42:int, "==")` to implement the predicate filter.

Using EXPLAIN on Q_8: Filter on Column c[11]

```
sql> EXPLAIN SELECT i.b, i.c
             FROM   indexed AS i
             WHERE  i.c = 0.42;
   ⋮
  indexed :bat[:oid] := sql.tid(sql, "sys", "indexed");
  c0      :bat[:sht] := sql.bind(sql, "sys", "indexed", "c", 0:int);
  p1      :bat[:oid] := algebra.thetaselect(c0, indexed, 42:sht, "=="); ◀ ≡ c = 0.42
  c       :bat[:sht] := algebra.projection(p1, c0);
  b0      :bat[:str] := sql.bind(sql, "sys", "indexed", "b", 0:int);
  b       :bat[:str] := algebra.projection(p1, b0);
   ⋮
```

- Plan is nearly identical (modulo access to the a BAT).
- MonetDB *appears* to use the same `algebra.thetaselect(…, 42:sht, "==")` MAL operation.

[11] Note how MonetDB maps the domain of type numeric(3,2) of column c, i.e., the set $\mathbb{N}_{3,2} \equiv \{-9.99,\ldots,9.99\}$ with $|\mathbb{N}_{3,2}| = 1999$, to a 16-bit value of type :sht. Nifty.

BAT Tail Properties

When MonetDB constructs a BAT t, a family of tail column **properties** $prop(t)$ is derived/maintained:[12]

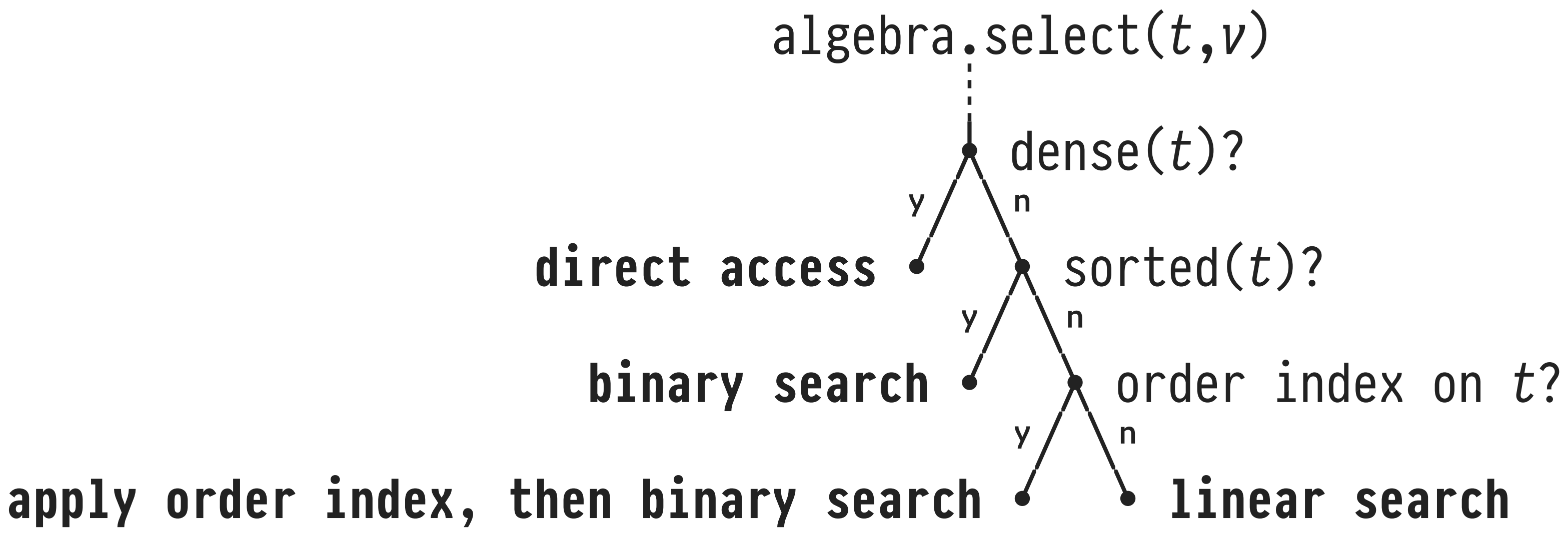
BAT Property $prop(t)$	Description
dense (tails of type :oid only)	ascending values, no gaps
key	unique values
sorted	strictly ascending values
revsorted	strictly descending values
nil/nonil	at least one/no nil value

- Use bat.info(t) to inspect current properties of t.
- ⚠️ Incomplete: t's tail may be sorted although sorted(t) = false ($\Rightarrow$ but not $\Leftrightarrow$).

[12] Additional properties *nokey*, *nosorted*, *norevsorted* give “proofs” (tail positions) why property does not hold. Example: nosorted = 3 ≡ tail value for row 3@0 < tail value for row 2@0.

MonetDB: Tactical Optimization

MAL operations inspect BAT properties at *query runtime*, select one of several efficient implementations:



- This is coined **tactical optimization** (as opposed to strategical query optimization at *query compile time*).

The Tactics of algebra.select: dense(t)

If input BAT t is **dense**, use **positional access** and **slicing** to evaluate equality and range selections:

algebra.select(t,42@0)

head	tail
0@0	39@0
1@0	40@0
2@0	41@0
3@0	42@0
4@0	43@0
5@0	44@0

offset 3 = 42@0-39@0

hseqbase(t)

algebra.select(t,40@0,42@0,t,t,f)

head	tail
0@0	39@0
1@0	40@0
2@0	41@0
3@0	42@0
4@0	43@0
5@0	44@0

↕ ≡ algebra.slice(t,1,3)

The Tactics of algebra.select: sorted(t)

algebra.select(t,42)

head	tail
0@0	38
1@0	39
2@0	40
3@0	41
4@0	42
5@0	43
6@0	44

lo

< 42

hi

> 42

= 42!

1 2 3

Binary Search:

- Test middle value (pivot) between limits *lo* and *hi*
- Recurse into upper or lower partition based on test
- Finishes in $\log_2(|t|)$ steps

- **NB:** Unpredictable branches (⩽ 42?) and jumps of pivot position less than ideal for CPU.

The Tactics of algebra.select: Order Indexes

algebra.select(t,v)

algebra.projection(*oidx*,*t*)

head	tail
0@0	38
1@0	39
2@0	40
3@0	41
4@0	42
5@0	43
6@0	44

❷

Order Index *oidx*

head	tail
0@0	5@0
1@0	1@0
2@0	0@0
3@0	6@0
4@0	2@0
5@0	4@0
6@0	3@0

❶

BAT *t*

head	tail
0@0	40
1@0	39
2@0	42
3@0	44
4@0	43
5@0	38
6@0	41

- Row [*i*@0,*j*@0] ∈ *oidx*: value at offset *j* is *i*th largest in tail. Tactic: ❶ Apply *oidx*, ❷ then use binary search.

Creating Order Indexes (On the Fly)

MonetDB may *automatically* create a temporary order index to support predicates $lo \leqslant a \leqslant hi$ or other order-sensitive queries (e.g., ORDER BY, GROUP BY).

- Check current properties of column BATs and presence of indexes in MonetDB system table sys.storage:

```
sql> SELECT column, sorted, revsorted, "unique", orderidx
     FROM   sys.storage('sys', 'indexed');
+---------------+--------+-----------+--------+----------+
| column        | sorted | revsorted | unique | orderidx |
+===============+========+===========+========+==========+
| a             | true   | null      | true   |        0 |
| b             | null   | null      | null   |        0 |
| c             | false  | false     | null   |        0 |
+---------------+--------+-----------+--------+----------+
```

Creating Order Indexes (Manually)

If this seems beneficial for the **query workload**, clients may *manually* create an order index.

- ⚠️ Order indexes are **static** (i.e., not maintained under updates—costly) ⇒ underlying table must be *read-only*:

```
<create and populate table T>
sql> ALTER TABLE T SET READ ONLY;
sql> CREATE ORDERED INDEX I ON T(a);
```

  - Order index *I* is made persistent (in a `*.torderidx` disk file) and will be used by future `algebra.select()`s on column *a*.

With **column cracking**,[13] MonetDB introduced a **self-organizing** (partially) ordered index structure.

- A **cracker index** for column a is created/updated as a **by-product of processing range predicates** $lo \leqslant a \leqslant hi$.
    - In the cracker index, the a values $\in [lo, hi]$ are stored physically contiguous.
- If the **query workload** focuses only on a subset of column a, that part is indexed with fine granularity (while the other parts remain largely non-indexed).

[13] *"Database Cracking"*, S. Idreos, M. Kersten, S. Manegold. Proc. CIDR, Asilomar (CA, USA), 2007.

Column Cracking As a By-Product of Query Processing

❶ BAT a

head	tail
0@0	17
1@0	3
2@0	8
3@0	6
4@0	2
5@0	15
6@0	13
7@0	4
8@0	12

$\xrightarrow{Q_i}$

❷ Cracker Index

head	tail	
0@0	4	$\leqslant 5$ s_1
1@0	3	
2@0	2	
3@0	6	> 5 s_2
4@0	8	
5@0	15	$\geqslant 10$ s_3
6@0	13	
7@0	17	
8@0	12	

$\xrightarrow{Q_j}$

❸ Cracker Index

head	tail	
0@0	2	$\leqslant 3$ s_4
1@0	3	
2@0	4	> 3 s_5
3@0	6	> 5 s_6
4@0	8	
5@0	12	$\geqslant 10$ s_7
6@0	13	
7@0	17	$\geqslant 14$ s_8
8@0	15	

- Q_i: … **WHERE** $a > 5$ **AND** $a < 10$ | Result: slice s_2
- Q_j: … **WHERE** $a > 3$ **AND** $a < 14$ | Result: slices $s_5 + s_6 + s_7$

Column Cracking Notes

- MonetDB implements slicing in terms of *views*[14] of the source BAT, no data copying involved. Cost free.

- $\forall x \in s_i, y \in s_{i+1}: x < y$: a fully cracked column ($\forall_i |s_i| = 1$) is completely ordered. This is uncommon (workload skew).

- First cracking step (❶→❷) copies source BAT. All further steps physically reorganize the cracker index.

- Physical cracker index reorganization ("tail shuffling") can be efficiently performed *in-situ*.

[14] A possible BAT view: (*source BAT*, *first row*, *last row*).

Cracker Index Reorganization For Predicate $a < hi$

Reorganize column vector $a[]$ between row offsets $start$ and end, relocate its elements *in-situ*:

```
CrackInTwo(a,start,end,hi):
 while (start < end)
   if (a[start] < hi)
     start ← start + 1;
   else
     while (a[end] ⩾ hi ∧ end > start)
       end ← end - 1;
     swap(a[start], a[end]);  ⁂
     start ← start + 1;
     end ← end - 1;
```

- ⁂ Either $a[start] \geqslant hi \land a[end] < hi$ or $start = end$.