DB 2

11 – Sorting and Grouping

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1 ¦ A Family of Q_9: The Ubiquitous Sort

Recall table indexed (with B^+Tree index indexed_a only):

```
❶ SELECT i.*
  FROM   indexed AS i
  ORDER BY i.c

❷ SELECT DISTINCT i.c
  FROM   indexed AS i
```

```
❸ SELECT i.c, SUM(i.a) AS s
  FROM   indexed AS i
  GROUP BY i.c

❹ SELECT DISTINCT i1.a
  FROM   indexed AS i1,
         indexed AS i2
  WHERE  i1.a = i2.c :: int
```

All four queries are evaluated using the Sort plan operator.

Sorting Takes Time

- Operator Sort may be costly to evaluate and RDBMSs try to plan query execution without sorting if possible:
    - In queries ❶ to ❹ above, replace i.c (i2.c) by i.a and PostgreSQL will use Index (Only) Scans on the a-ordered B⁺Tree indexed_a instead of Sort.

- Sort is a **blocking operator** and introduces plan latency:

```
                     QUERY PLAN
Sort (cost=180530.84..183030.84 rows=1000000 width=41)
  Output: a, b, c
```

Sorting Needs Space

Sorting may need (lots of) **temporary working memory:**

❶ Try to stay RAM-resident if possible,
❷ otherwise, resort to a **disk-based sorting algorithm:**

```
QUERY PLAN
Sort  (cost=180530.84..183030.84 rows=1000000 …) (actual time=…)
1 Sort Method: quicksort  Memory: 102702kB
  Buffers: shared hit=9343
                                                    or
2 Sort Method: external sort  Disk: 50880kB
  Buffers: shared hit=9343, temp read=6360 written=6360
```

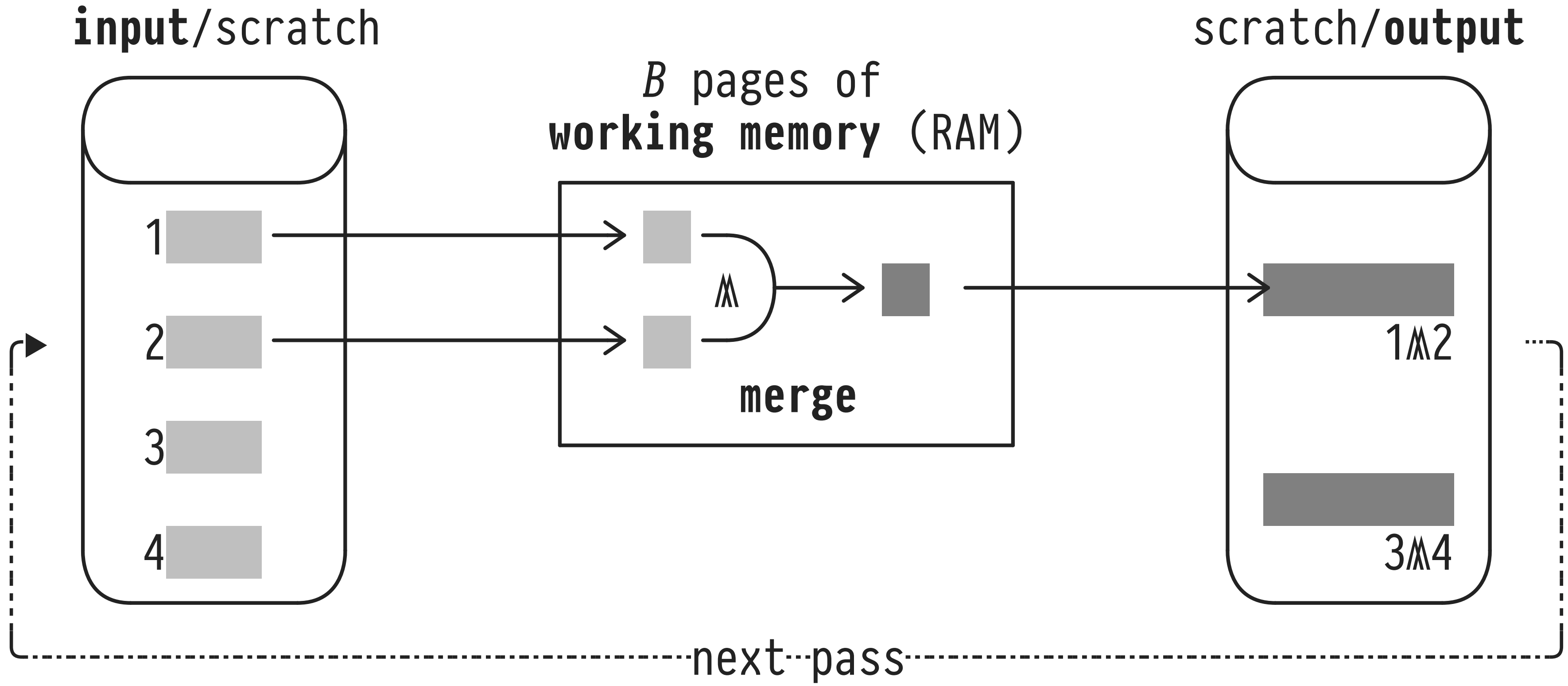
Now assume the following typical scenario:

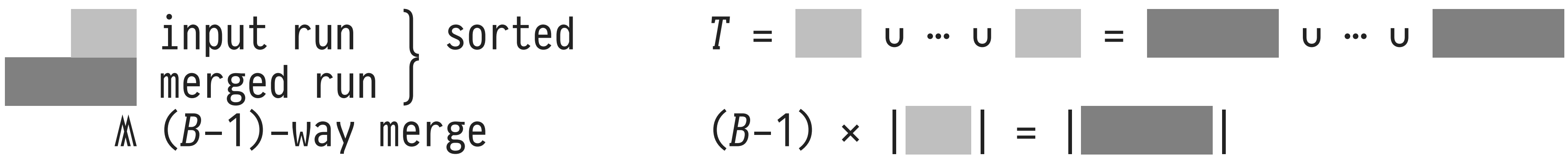
- input heap file T to be sorted: N pages,
- size of temporary working memory (RAM): $B \ll N$ pages,
- size of secondary scratch memory (disk): $\geqslant 2 \times N$ blocks.

External Merge Sort can sort heap files of any size as long as $B \geqslant 3$ pages of working memory are available:

- reads unsorted input file, writes sorted output file,
- creates partially sorted sub-files (*runs*) on disk,
- multiple passes (the larger B, the fewer passes).

An External Merge Sort Pass (B = 3)





External Merge Sort

```
ExternalMergeSort(T,B):
  N ← #pages of T;
  R ← ⌈N/B⌉;                        } R: current number of runs

  split input T into R partitions pᵢ of B pages;  ⎫
  for each i ∈ 1…R                                ⎬ pass 0
   ⌊ run rᵢ ← in-memory sort of pᵢ;               ⎭

  while R > 1
   | R ← ⌈R / (B-1)⌉;                             ⎫
   | for each i ∈ 1…R                             ⎬ passes 1,2,…
   ⌊  ⌊ ⩚: merge next B-1 runs into one run;      ⎭

  return single sorted run;
```

External Merge Sort: Passes and I/O Operations

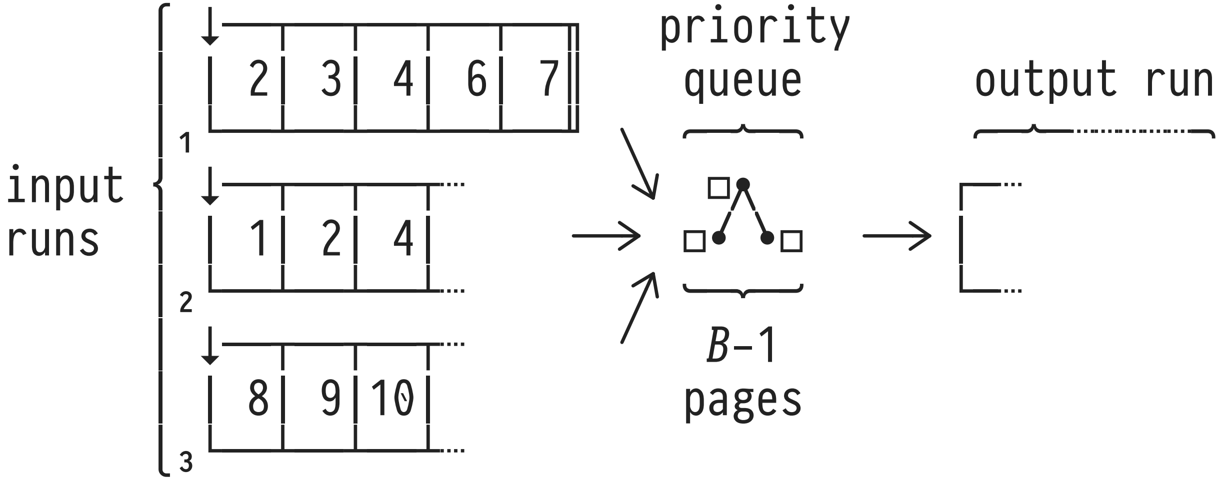
pass	input: #runs	input: run size	output: #runs	output: run size
1	$\lceil N/B \rceil$	B	$\lceil N/B \rceil$ / $(B-1)$	$B \times (B-1)$
2	$\lceil N/B \rceil$ / $(B-1)$	$B \times (B-1)$	$\lceil N/B \rceil$ / $(B-1)^2$	$B \times (B-1)^2$
3	$\lceil N/B \rceil$ / $(B-1)^2$	$B \times (B-1)^2$	$\lceil N/B \rceil$ / $(B-1)^3$	$B \times (B-1)^3$
$\vdots$				
n	$\lceil N/B \rceil$ / $(B-1)^{n-1}$	$B \times (B-1)^{n-1}$	$\lceil N/B \rceil$ / $(B-1)^n$	$B \times (B-1)^n$

- In each pass:
  N = input (#runs × run size) = output (#runs × run size).
  - Each pass performs $2 \times N$ I/O operations.

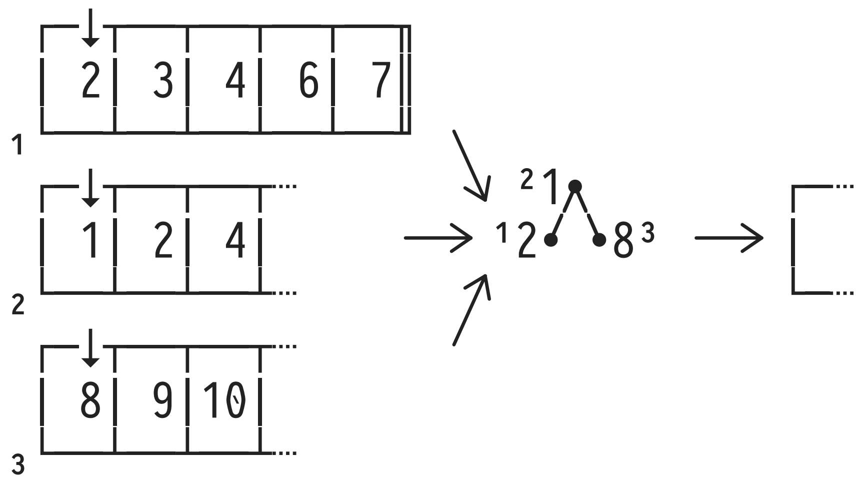
- # passes required by External Merge Sort with B buffers:

$$\underbrace{1}_{\text{pass 0}} + \underbrace{\lceil \log_{B-1} \lceil N/B \rceil \rceil}_{\text{merge passes}}$$

3 ¦ (B–1)-Way Merge (Passes 1,2,…)

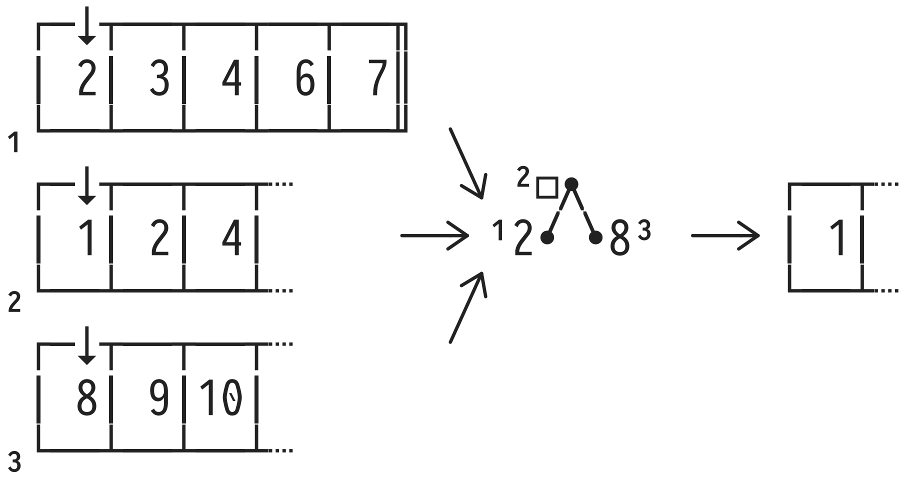


initial merge state

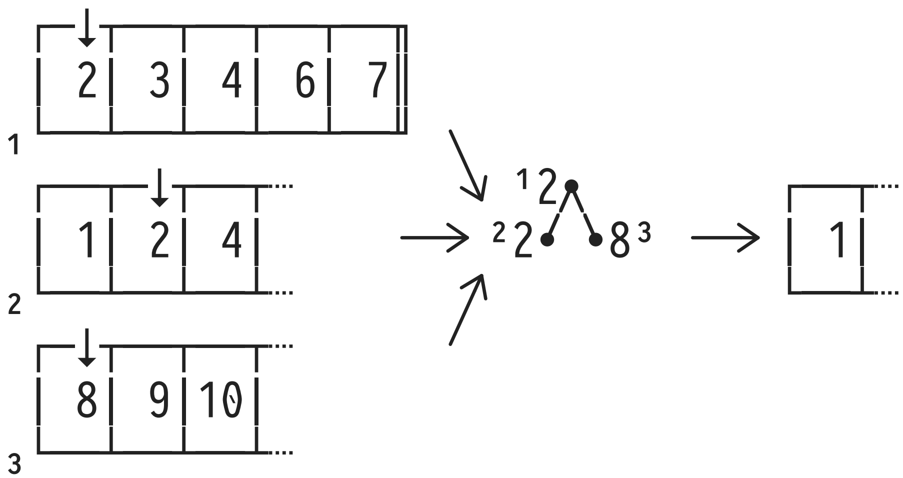


load queue,
entries from input run r_i
are tagged $\square^i$

(B-1)-Way Merge (Passes 1,2,…)

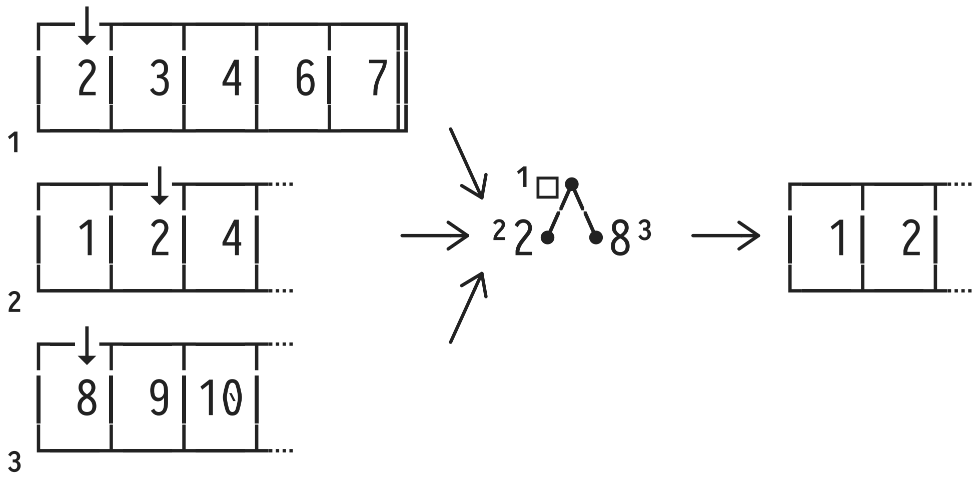


queue head → output run

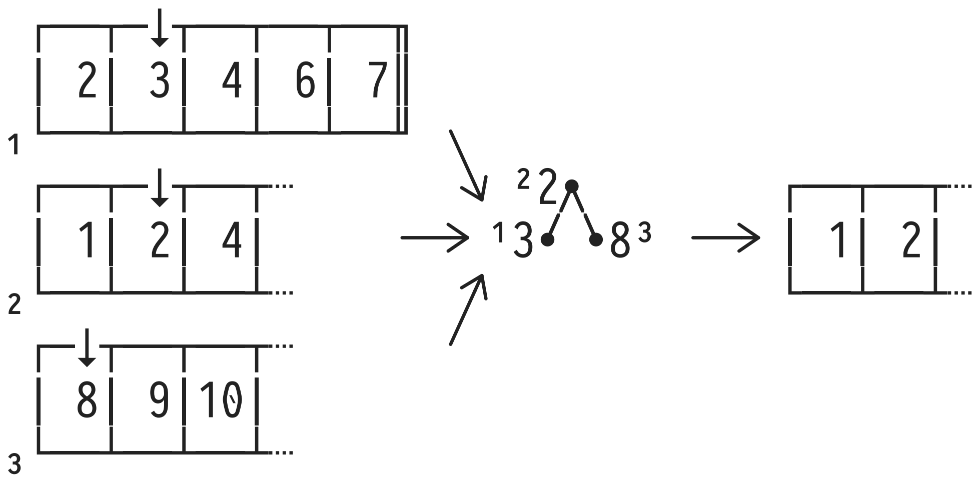


refill queue from input run

(B-1)-Way Merge (Passes 1,2,…)



queue head ➝ output run



refill queue from input run

⋮

External Merge Sort: Access Patterns and Blocked I/O

- I/O access patterns in
  - pass 0: sequential read/write chunks of B pages, 👍
  - merge passes 1,...: random reads from the $B-1$ runs. 👎

- 💡 Perform **blocked I/O** in merge passes 1,2,...:
  - Seek once to read $b > 1$ pages at a time from each run. Reduces per-page I/O cost by a factor of ≈ b.
  - Reduced fan-in: can only merge $\lfloor (B-1)/b \rfloor$ runs per pass.

External Merge Sort Parameters (Interactive)

I/O Characteristics and Performance of External Sorting

Database Characteristics

Database page size: 8 KiB
Available working space in database buffer (*B*): 16384 pages (that's 128.0 MiB)
I/O blocking factor (*b*): 64 pages

Disk Characteristics

Disk seek time: 3.4 ms
Disk read/write speed: 163 MiB/s
Resulting transfer time for a 8 KiB block: 0.049 ms

Size of Sort Problem

Size of input file to be sorted: 0.5 GiB (this makes for *N* = 65536 pages of input)

Resulting External Sort Behavior

Pass 0 will produce 4 runs, each of size 16384 pages .
We will need 1 merge passes, with a fan-in of 255.

Resulting I/O and Disk Seek Effort

The sort process will initiate 262144 I/O operations (reads and writes) and 2056 disk head seeks.

Resulting Overall Time for Sort Process

Disk seeking will need 0.1 minutes, while 0.2 minutes is spent on I/O itself.
Overall, we end up waiting **0.3 minutes** for the sort result.

Made with Tangle.js.

Interactive

- The *initial number of runs created in pass 0* influence overall sort performance:

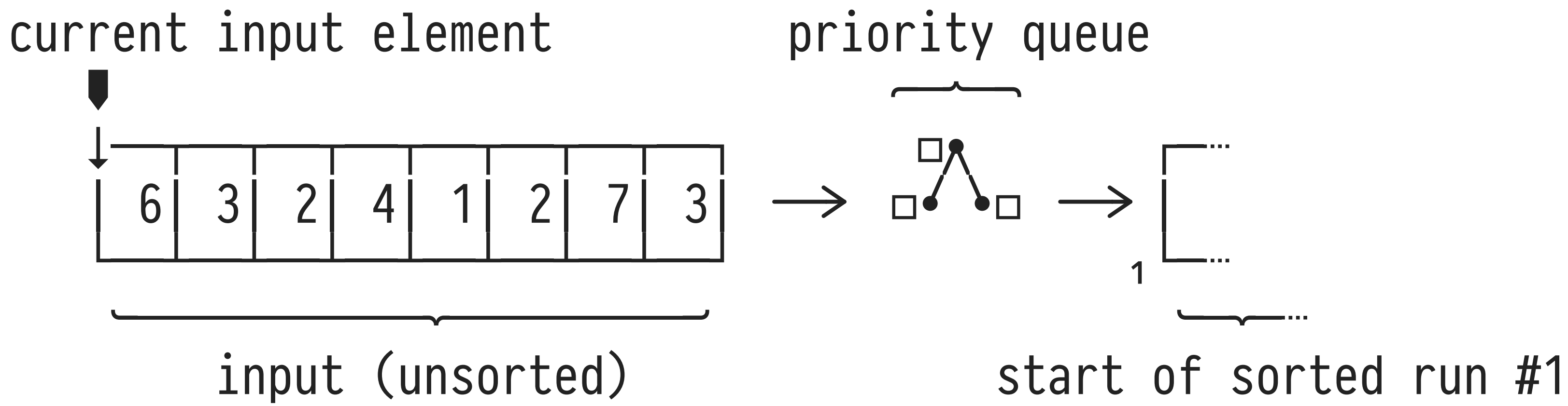
$$\text{\# I/O operations} = 2 \times N \times (1 + \lceil \log_{B-1} \underbrace{\lceil N/B \rceil}_{\text{\# runs created in pass 0}} \rceil)$$

- **Q:** Given only B buffers, can we create sorted runs *longer than* B pages?
  - **A:** Yes! In pass 0, use **Replacement Sort** (instead of QuickSort, for example).

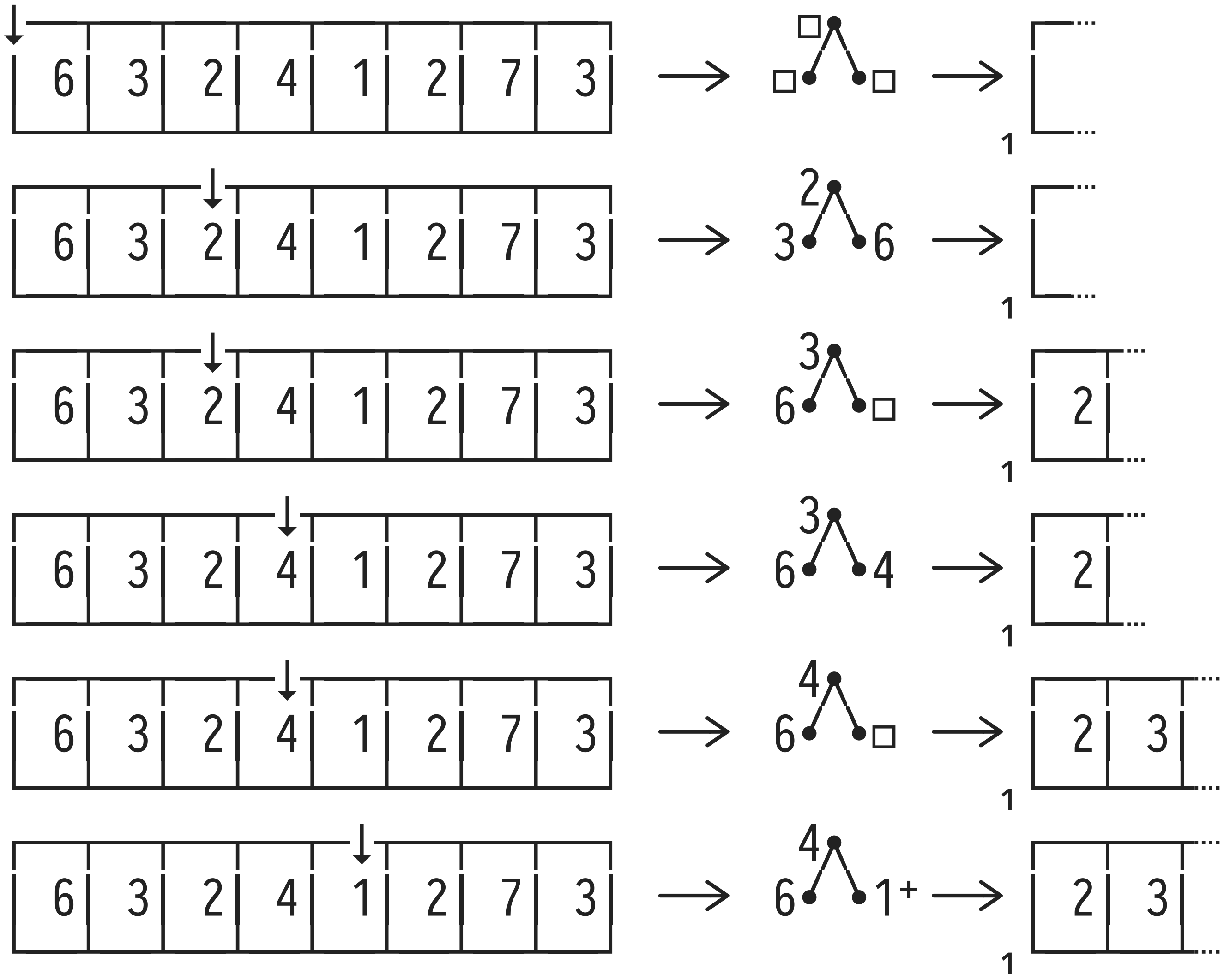
Replacement Sort

Again, use $B-1$ buffer pages to set up a **priority queue:**

1. Elements arriving too late for inclusion in current run are marked (□⁺) and receive lower priority.
2. When all elements in queue are marked, close the current run, unmark all elements, open a new run.



Replacement Sort (B = 4)



initial state

load queue

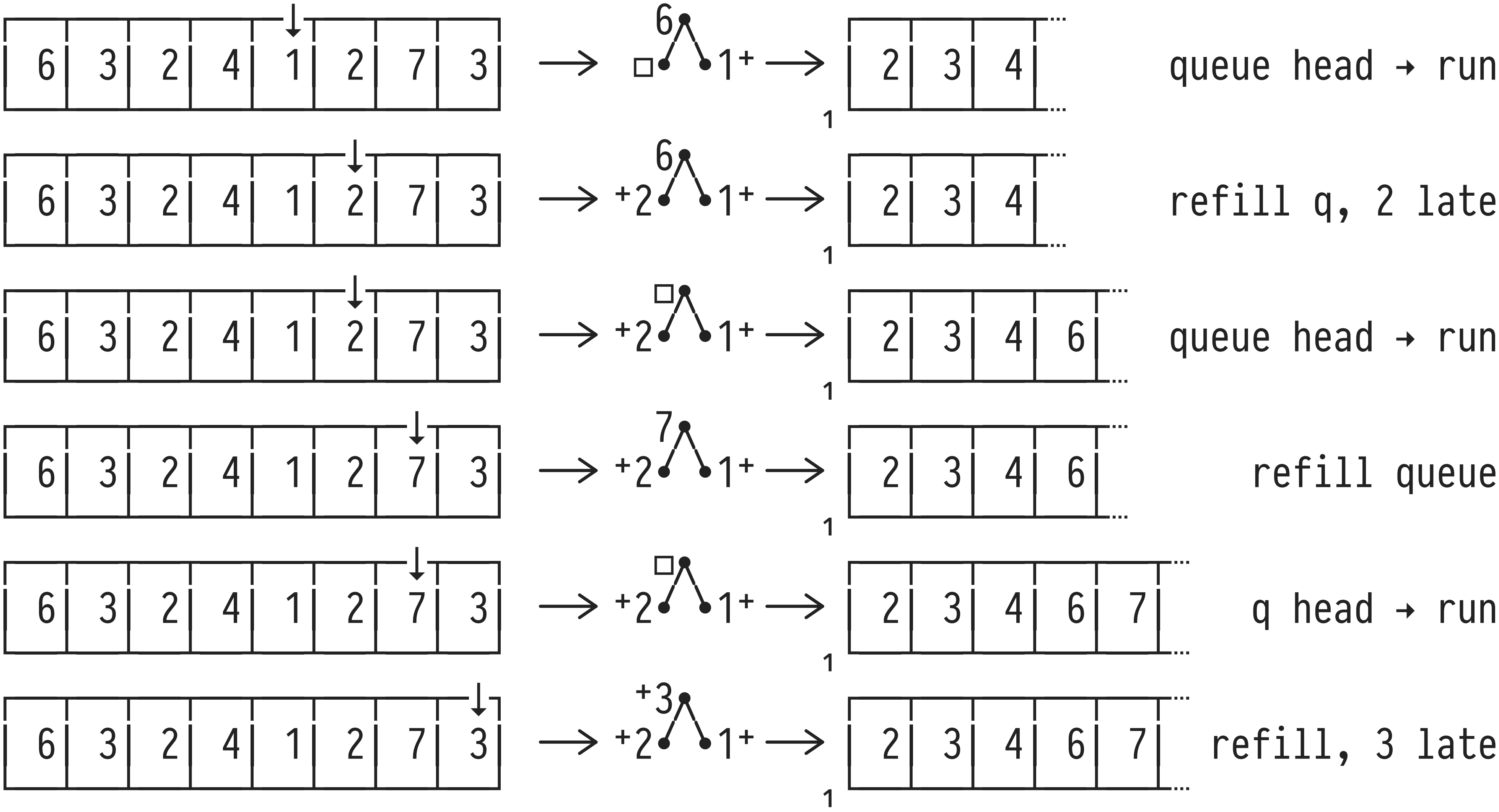
queue head → run #1

refill queue from input

queue head → run

refill q, 1 is late (□⁺)

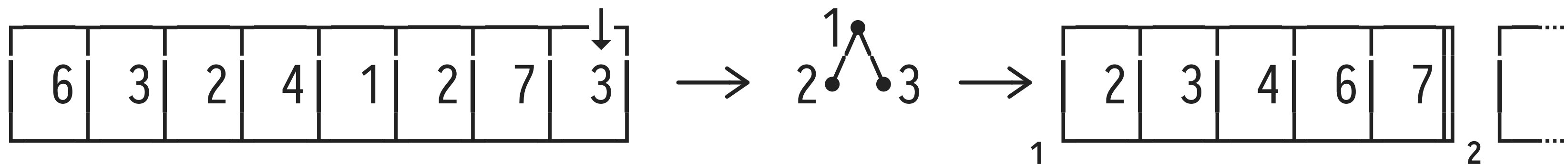
Replacement Sort ($B = 4$)



Replacement Sort (B = 4)

All entries in queue are late (□⁺):

- Close current run #1, open new run #2.
- Reorder entries in queue, continue processing.



- Replacement Sort produces runs of length $\approx 2 \times (B-1) > B$ (see Knuth, TAoCP, volume 3, p. 254).
- Replacement Sort generates longer runs if input file is almost sorted (*e.g.*, consider a heap file that was once clustered but has received a few updates since then).

Grouping coarsens the granularity of data processing (individual rows ↘ groups of rows):

```
❷ SELECT g.g, SUM(g.a) AS s -- out: 10⁴ groups (aggregates)
  FROM   grouped AS g       -- in:  10⁶ rows
❶ GROUP BY g.g
```

❶ **Partition** table grouped by criterion g.g (all rows agreeing on g.g form one group),

❷ output group criterion and **aggregates** of the group's member rows (the group member rows themselves are never output).

Grouping: Sorting vs. Hashing

input

a	g
1	42
2	11
3	42
4	42
5	38
6	11

GROUP BY g

a	g
2	11
6	11
5	38
3	42
1	42
4	42

scan/Σa

11: Σa=8
38: Σa=5
42: Σa=8

- scan sorted table for group boundaries
- aggregate while scanning

Sorting

Hashing

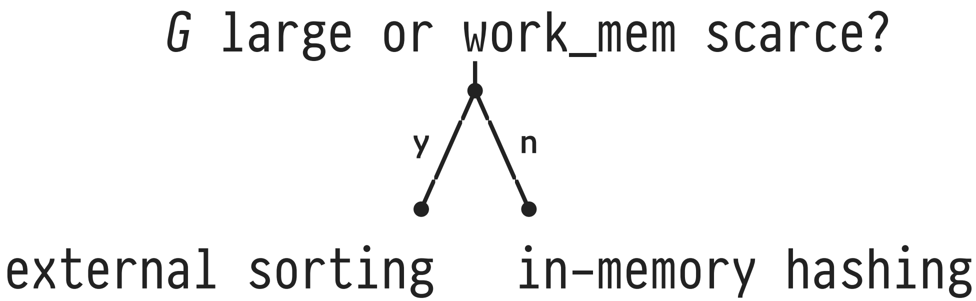
key	buckets—Σa
k_1	$^{g=42}$|8, $^{g=38}$|5
k_2	
k_3	$^{g=11}$|8

- hash buckets hold grouping criterion and aggregate value

Grouping: Sorting vs. Hashing

PostgreSQL plans for sorting vs. hashing based on

1. the available working memory (work_mem) and
2. the estimated number G of resulting groups:



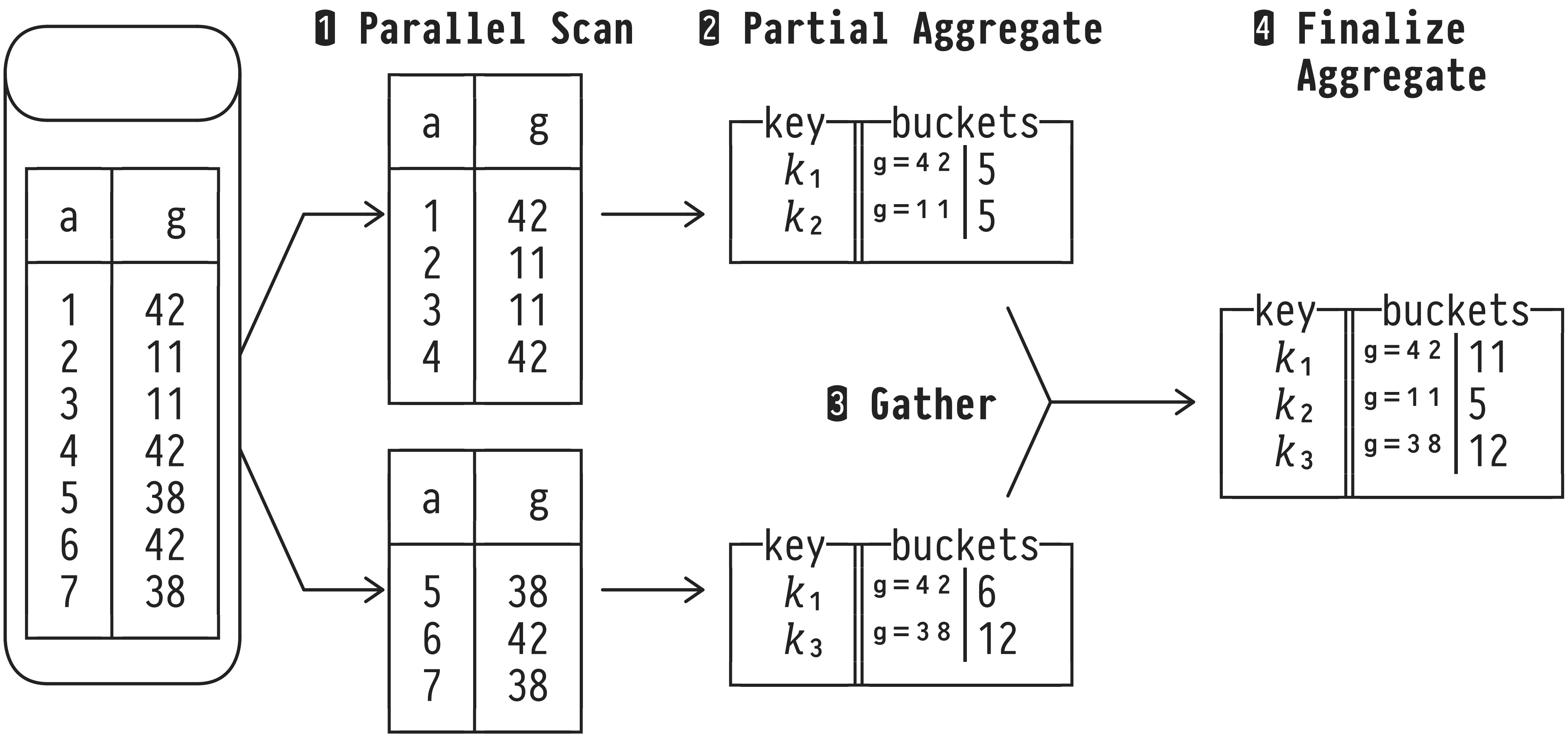
- Often, G is unknown or cannot be derived (*e.g.*, GROUP BY g.g % 2 $\Rightarrow G \leqslant 3$ not understood by PostgreSQL).
  - $\Rightarrow$ Overestimate G conservatively, use sorting.

6 ┆ Parallel Grouping and Aggregation

Grouping and aggregation are query operations that are straightforward to **parallelize:**

- Spawn **workers,** each of which execute in // (on dedicated CPU core). Constrain max number of workers to fit host.
- Try to **evenly distribute work** (*e.g.*, data volume) among workers.
- Assign a **leader** thread/process that coordinates workers and **gathers partial query results.**
- After gathering, **merge/finalize partial results** to produce a single complete query result.

Parallel Grouping (GROUP BY g — SUM(a))



Parallel Grouping for Q_{10}

```
EXPLAIN
  SELECT g.g, SUM(g.a) AS s
  FROM   grouped AS g
  GROUP BY g.g;
```

QUERY PLAN
Finalize HashAggregate (cost=13869.28..13969.02 …)
Group Key: g
-> **Gather** (cost=11675.00..13769.54 …)
Workers Planned: 2 ⬅ *‖ism degree*: 3 (2 *worker* + 1 *leader*)
-> **Partial HashAggregate** (cost=10675.00..10774.74 …)
Group Key: g
-> **Parallel Seq Scan** on grouped g (cost=0.00..8591.67 …)

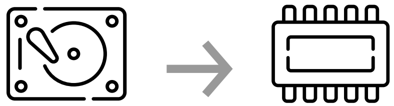
Partial Aggregation and Finalization

- Parallel evaluation of aggregate *AGG* depends on its **distributivity** over $\uplus$ (bag union):

$$AGG(X \uplus Y) = AGG(\{AGG(X)\} \uplus \{AGG(Y)\}).$$

- Many SQL aggregates (COUNT, SUM, MAX, MIN, AVG, bool_and, bool_or, …) exhibit this property:

$$\mathrm{SUM}(\underbrace{X \uplus Y}_{\text{distribute work}}) = \mathrm{SUM}(\{\underbrace{\mathrm{SUM}(X)}_{\text{partial aggregates}}\} \uplus \{\underbrace{\mathrm{SUM}(Y)}_{\text{partial aggregates}}\}) = \mathrm{SUM}(X) \underset{\substack{\uparrow \\ \text{finalize}}}{+} \mathrm{SUM}(Y)$$



```
CREATE TABLE sorted (a text, s int);
  ⋮
SELECT s.a, s.s
FROM   sorted AS s
ORDER BY s.s [, s.a]   -- single- or multi-column criteria
```

MonetDB's BATs already provide **ordered row storage.** Some ORDER BY queries will thus be no-ops (recall tail properties sorted, revsorted).

Otherwise, use **order indexes**—either persistent or computed on the fly—to apply column re-ordering.

Recall: Order Indexes (ORDER BY s.s)

a

head	tail
0@0	a
1@0	b
2@0	c
3@0	d
4@0	e
5@0	f
6@0	g
7@0	h
8@0	i
9@0	j

s

tail
40
0
50
30
50
10
50
10
10
20

oidx^{s}

head	tail
0@0	1@0
1@0	7@0
2@0	8@0
3@0	5@0
4@0	9@0
5@0	3@0
6@0	0@0
7@0	2@0
8@0	6@0
9@0	4@0

$a^{ord(s)}$

head	tail
0@0	b
1@0	h
2@0	i
3@0	f
4@0	j
5@0	d
6@0	a
7@0	c
8@0	g
9@0	e

$s^{ord(s)}$

tail
0
10
10
10
20
30
40
50
50
50

…**algebra.projection**(oidx^{s},·)

Order Indexes on the Fly: algebra.sort

a b c d e f g h i j k l m n o p r s t u v w x y z

```
EXPLAIN
  SELECT s.a, s.s
  FROM   sorted AS s
  ORDER BY s.s;

sorted :bat[:oid] := sql.tid(sql, "sys", "sorted");
s0     :bat[:int] := sql.bind(sql, "sys", "sorted", "s", …);
s      :bat[:int] := algebra.projection(sorted, s0);
(s^ord(s), oidx^s, gidx^s)            desc      nil last   stable
                  := algebra.sort(s, false, false, false);
a0     :bat[:str] := sql.bind(sql, "sys", "sorted", "a", …);
a      :bat[:str] := algebra.projection(sorted, a0);
a^ord(s):bat[:str] := algebra.projectionpath(oidx^s, sorted, a0);

io.print(a^ord(s), s^ord(s));
```

Persistent Order Indexes

If sorting is central to the query workload, create a **persistent order index** that is immediately applicable:

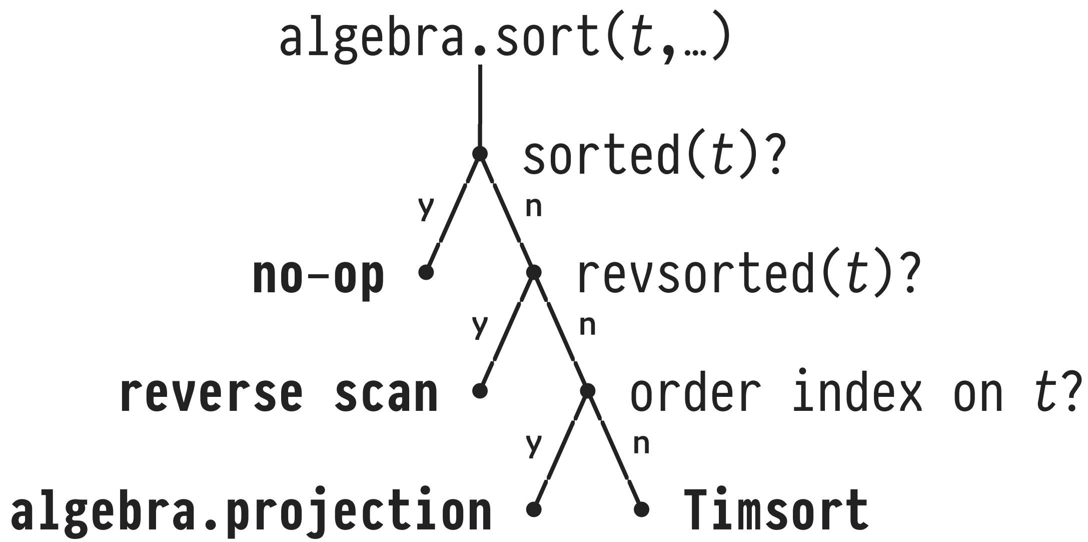
```
ALTER TABLE sorted SET READ ONLY;     -- ⚠️

CREATE ORDERED INDEX oidxˢ ON sorted(s);
```

- Order indexes are **static** structures that are *not* dynamically maintained (as opposed to B^+Trees).
  If order index has been created...
  1. on the fly: throw away on table update,
  2. persistent: read-only table, no updates at all.

Tactical Optimization for `algebra.sort`

- `algebra.sort` aims to avoid actual sorting effort based on properties of BAT *t* and the presence of order indexes:



- If all else fails, apply in-memory sort algorithm **Timsort** (1993; hybrid of merge/insertion sort, run-aware).

8 ┆ Multi-Criteria ORDER BY

Multi-column ordering criteria require special treatment: `algebra.sort(s)` only receives single criterion `s`.

```
SELECT s.a, s.s
FROM   sorted AS s
ORDER BY s.s, s.a -- s₁ < s₂ ⇔ s₁.s < s₂.s ∨
                  --           (s₁.s = s₂.s ∧ s₁.a < s₂.a)
```

- 💡 Let `algebra.sort(s)` return *three* result BATs:
  1. $s^{ord(s)}$ (the ordered input `s`) ✓
  2. $oidx^{s}$ (order index) ✓
  3. $gidx^{s}$ (groups rows that agree on criterion `s`).

Multi-Criteria ORDER BY: Group Index gidx

$s^{ord(s)}$ ✓

head	tail
0@0	0
1@0	10
2@0	10
3@0	10
4@0	20
5@0	30
6@0	40
7@0	50
8@0	50
9@0	50

,

$oidx^s$ ✓

head	tail
0@0	1@0
1@0	7@0
2@0	8@0
3@0	5@0
4@0	9@0
5@0	3@0
6@0	0@0
7@0	2@0
8@0	6@0
9@0	4@0

,

$gidx^s$

head	tail	s =
0@0	0@0	0
1@0	1@0	10
2@0	1@0	10
3@0	1@0	10
4@0	2@0	20
5@0	3@0	30
6@0	4@0	40
7@0	5@0	50
8@0	5@0	50
9@0	5@0	50

:= algebra.sort

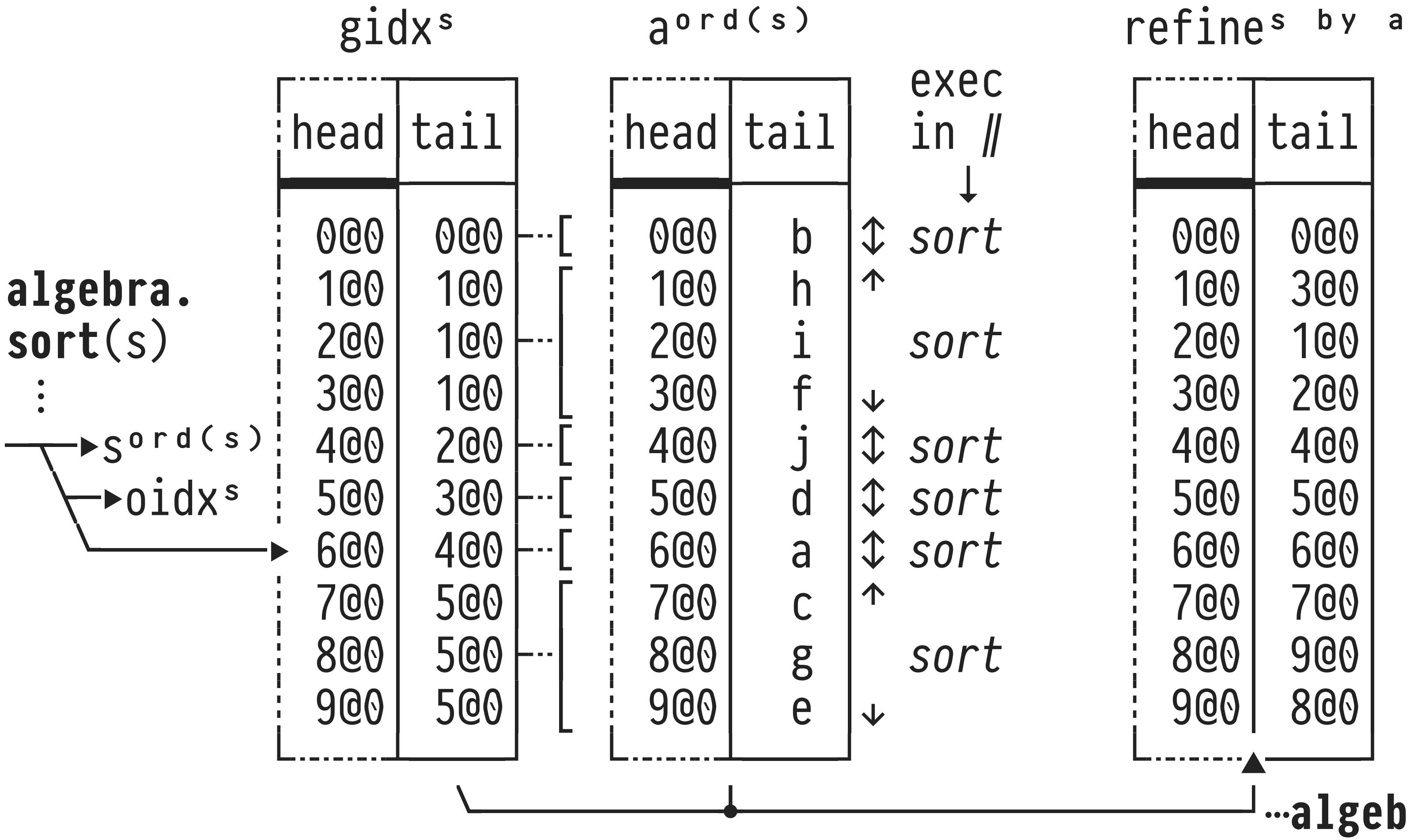
s

head	tail
0@0	40
1@0	0
2@0	50
3@0	30
4@0	50
5@0	10
6@0	50
7@0	10
8@0	10
9@0	20

3 output BATs

input BAT

Multi-Criteria **ORDER BY s,a**: Refine **ORDER BY s** by **a**



- Apply this to $c^{ord(s)}$ to obtain $c^{ord(sa)}$.
- Apply this to $oidx^{s}$ to obtain $oidx^{sa}$.