# DB 2

## 11 – Sorting and Grouping

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Torsten Grust
Universität Tübingen, Germany

# 1 ¦ A Family of $Q_9$: The Ubiquitous Sort

Recall table indexed (with B⁺Tree index indexed_a only):

❶
```
SELECT i.*
FROM   indexed AS i
ORDER BY i.c
```

❷
```
SELECT DISTINCT i.c
FROM   indexed AS i
```

❸
```
SELECT i.c, SUM(i.a) AS s
FROM   indexed AS i
GROUP BY i.c
```

❹
```
SELECT DISTINCT i1.a
FROM   indexed AS i1,
       indexed AS i2
WHERE  i1.a = i2.c :: int
```

All four queries are evaluated using the Sort plan operator.

## Sorting Takes Time

- Operator Sort may be costly to evaluate and RDBMSs try to plan query execution without sorting if possible:
  - In queries ❶ to ❹ above, replace i.c (i2.c) by i.a and PostgreSQL will use Index (Only) Scans on the a-ordered B⁺Tree indexed_a instead of Sort.

- Sort is a **blocking operator** and introduces plan latency:

```
QUERY PLAN
Sort (cost=180530.84..183030.84 rows=1000000 width=41)
 Output: a, b, c
```

# Sorting Needs Space

Sorting may need (lots of) **temporary working memory:**

❶ Try to stay RAM-resident if possible,
❷ otherwise, resort to a **disk-based sorting algorithm:**

```
                         QUERY PLAN
Sort  (cost=180530.84..183030.84 rows=1000000 …) (actual time=…)
1 Sort Method: quicksort  Memory: 102702kB
  Buffers: shared hit=9343
                                                            or
2 Sort Method: external sort  Disk: 50880kB
  Buffers: shared hit=9343, temp read=6360 written=6360
```

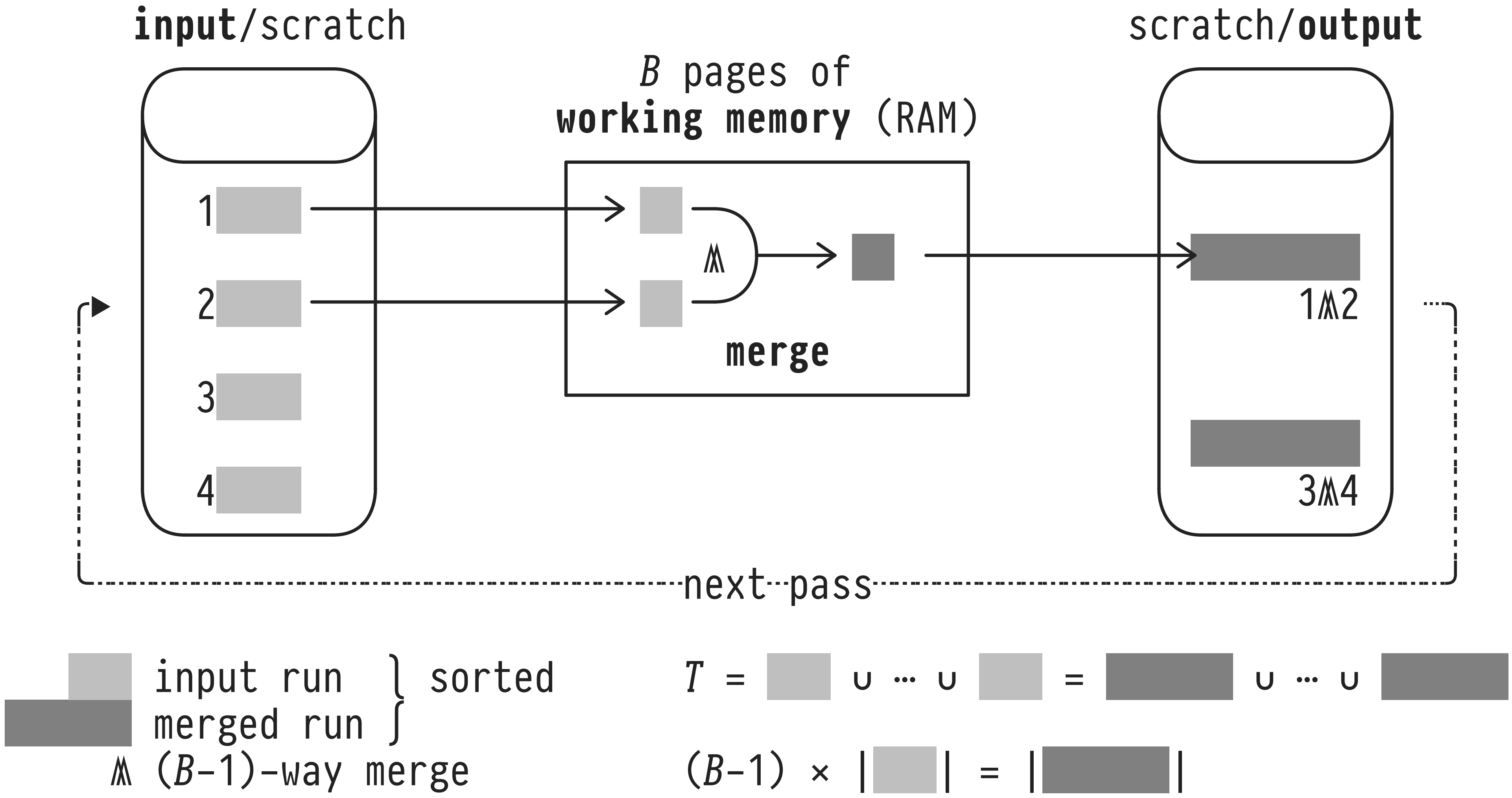
Now assume the following typical scenario:

- input heap file $T$ to be sorted: $N$ pages,
- size of temporary working memory (RAM): $B \ll N$ pages,
- size of secondary scratch memory (disk): $\geqslant 2 \times N$ blocks.

**External Merge Sort** can sort heap files of any size as long as $B \geqslant 3$ pages of working memory are available:

- reads unsorted input file, writes sorted output file,
- creates partially sorted sub-files (*runs*) on disk,
- multiple passes (the larger $B$, the fewer passes).

# An External Merge Sort Pass ($B$ = 3)



# External Merge Sort

```
ExternalMergeSort(T,B):
  N ← #pages of T;
  R ← ⌈N/B⌉;                          } R: current number of runs

  split input T into R partitions pᵢ of B pages;  ⎫
  for each i ∈ 1…R                                ⎬ pass 0
  ⌊ run rᵢ ← in-memory sort of pᵢ;                ⎭

  while R > 1
  | R ← ⌈R / (B-1)⌉;                          ⎫
  | for each i ∈ 1…R                          ⎬ passes 1,2,…
  ⌊  ⌊ ⩚: merge next B-1 runs into one run;   ⎭

  return single sorted run;
```

# External Merge Sort: Passes and I/O Operations

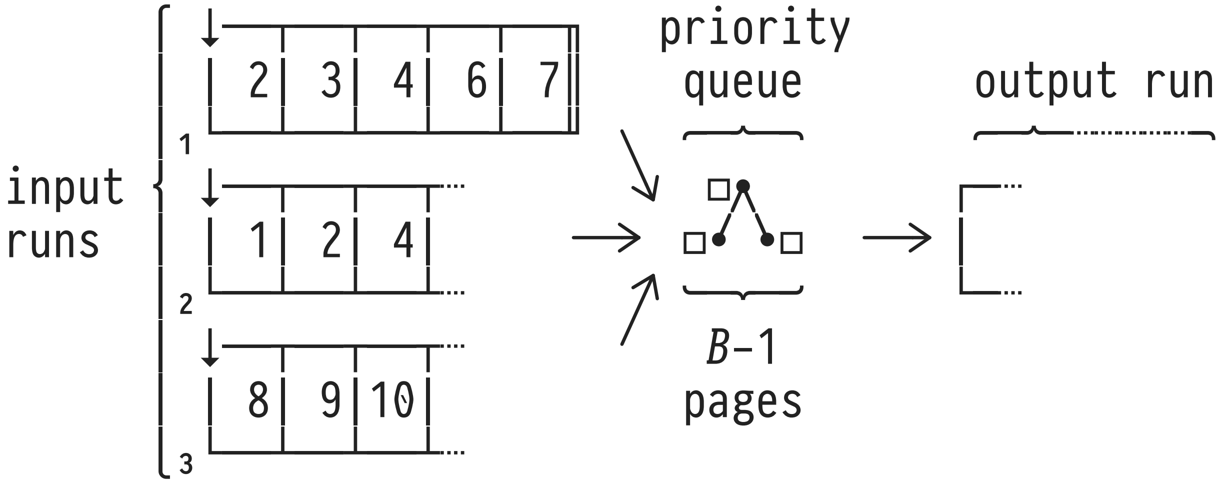
| pass | input: #runs | input: run size | output: #runs | output: run size |
|---|---|---|---|---|
| 1 | $\lceil N/B \rceil$ | $B$ | $\lceil N/B \rceil$ / $(B-1)$ | $B \times (B-1)$ |
| 2 | $\lceil N/B \rceil$ / $(B-1)$ | $B \times (B-1)$ | $\lceil N/B \rceil$ / $(B-1)^2$ | $B \times (B-1)^2$ |
| 3 | $\lceil N/B \rceil$ / $(B-1)^2$ | $B \times (B-1)^2$ | $\lceil N/B \rceil$ / $(B-1)^3$ | $B \times (B-1)^3$ |
| ⋮ | | | | |
| $n$ | $\lceil N/B \rceil$ / $(B-1)^{n-1}$ | $B \times (B-1)^{n-1}$ | $\lceil N/B \rceil$ / $(B-1)^n$ | $B \times (B-1)^n$ |

- In each pass:
  $N$ = input (#runs × run size) = output (#runs × run size).
  - Each pass performs $2 \times N$ I/O operations.

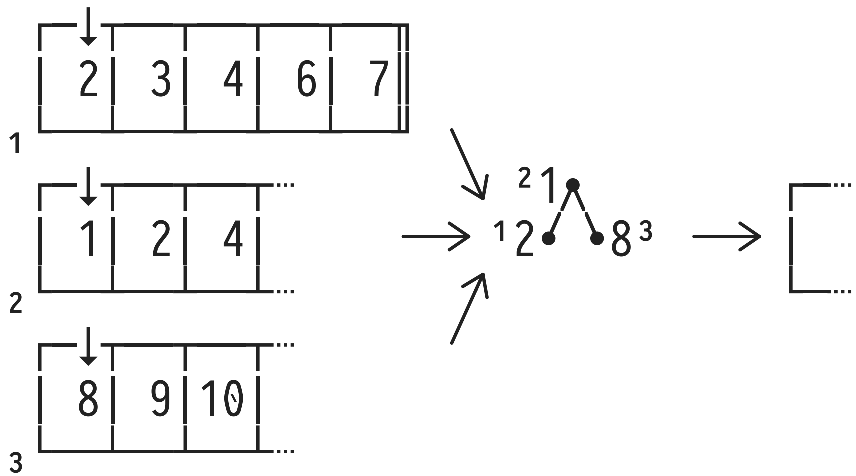
- # passes required by External Merge Sort with $B$ buffers:

$$\underbrace{1}_{\text{pass 0}} + \underbrace{\lceil \log_{B-1} \lceil N/B \rceil \rceil}_{\text{merge passes}}$$

# 3 ¦ ($B$–1)-Way Merge (Passes 1,2,…)

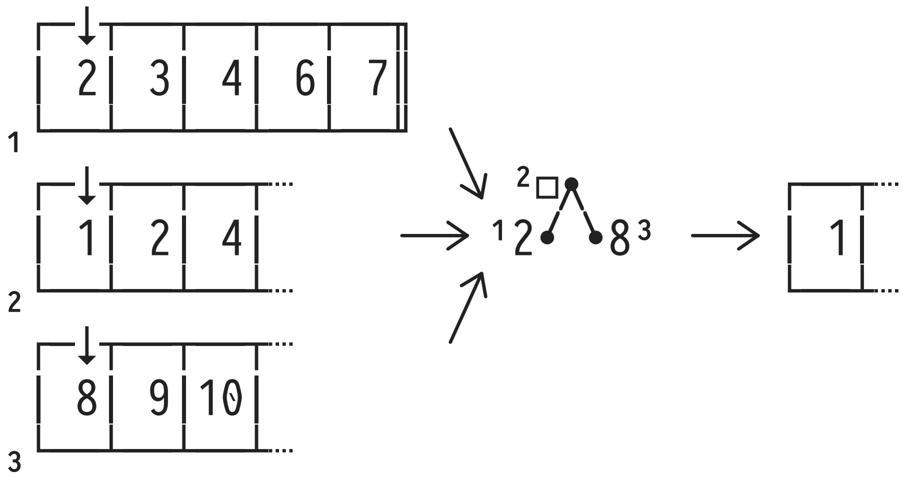



initial merge state

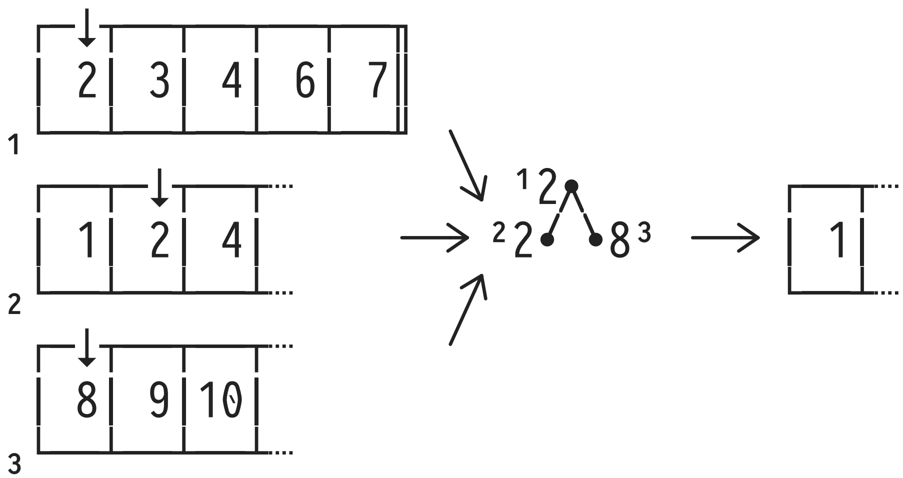



load queue,
entries from input run $r_i$
are tagged $\square^i$

# ($B$-1)-Way Merge (Passes 1,2,…)

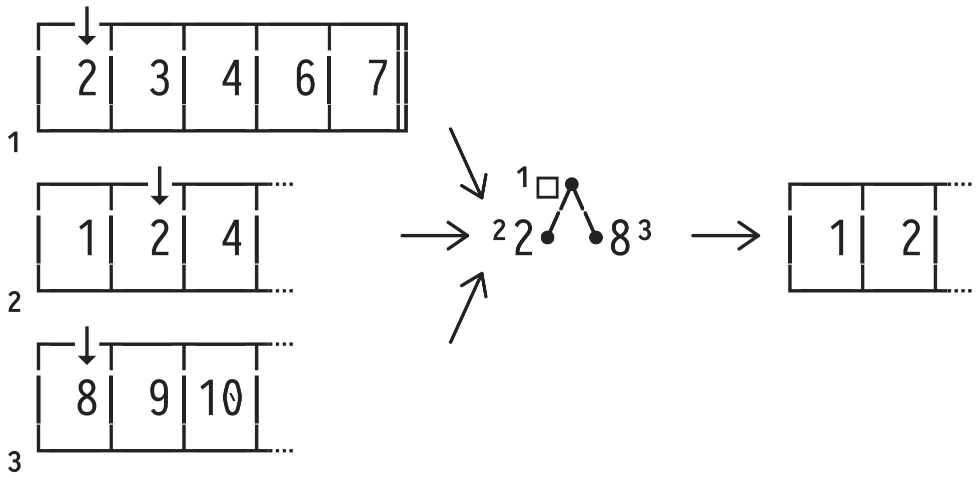



queue head → output run

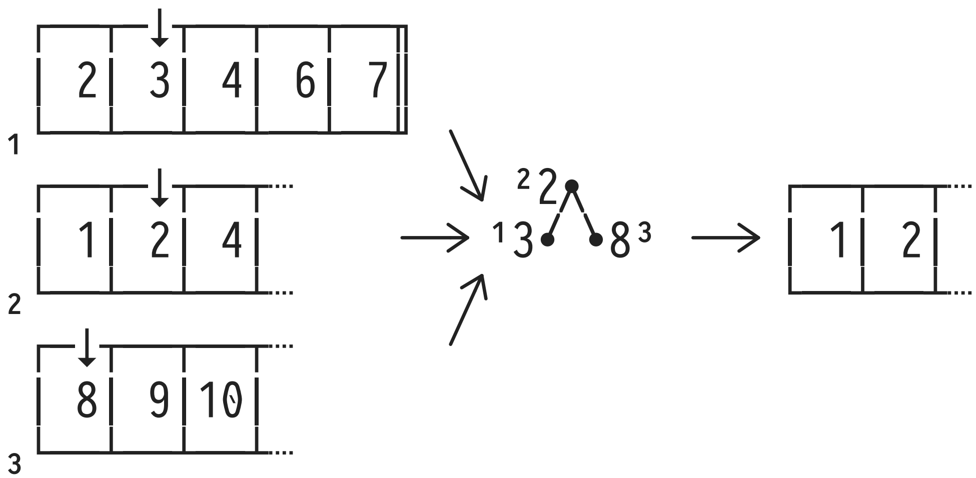



refill queue from input run

# $(B-1)$-Way Merge (Passes 1,2,…)



queue head → output run



refill queue from input run

⋮

## External Merge Sort: Access Patterns and Blocked I/O

- I/O access patterns in
  - pass 0: sequential read/write chunks of $B$ pages, 👍
  - merge passes 1,…: random reads from the $B-1$ runs. 👎

- 💡 Perform **blocked I/O** in merge passes 1,2,…:
  - Seek once to read $b > 1$ pages at a time from each run. Reduces per-page I/O cost by a factor of $\approx b$.
  - Reduced fan-in: can only merge $\lfloor (B-1)/b \rfloor$ runs per pass.

# External Merge Sort Parameters (Interactive)

## I/O Characteristics and Performance of External Sorting

**Database Characteristics**

Database page size: 8 KiB
Available working space in database buffer (*B*): 16384 pages (that's 128.0 MiB)
I/O blocking factor (*b*): 64 pages

**Disk Characteristics**

Disk seek time: 3.4 ms
Disk read/write speed: 163 MiB/s
Resulting transfer time for a 8 KiB block: 0.049 ms

**Size of Sort Problem**

Size of input file to be sorted: 0.5 GiB (this makes for *N* = 65536 pages of input)

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**Resulting External Sort Behavior**

Pass 0 will produce 4 runs, each of size 16384 pages .
We will need 1 merge passes, with a fan-in of 255.

**Resulting I/O and Disk Seek Effort**

The sort process will initiate 262144 I/O operations (reads and writes) and 2056 disk head seeks.

**Resulting Overall Time for Sort Process**

Disk seeking will need 0.1 minutes, while 0.2 minutes is spent on I/O itself.
Overall, we end up waiting **0.3 minutes** for the sort result.

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Made with Tangle.js.

Interactive

- The *initial number of runs created in pass 0* influence overall sort performance:

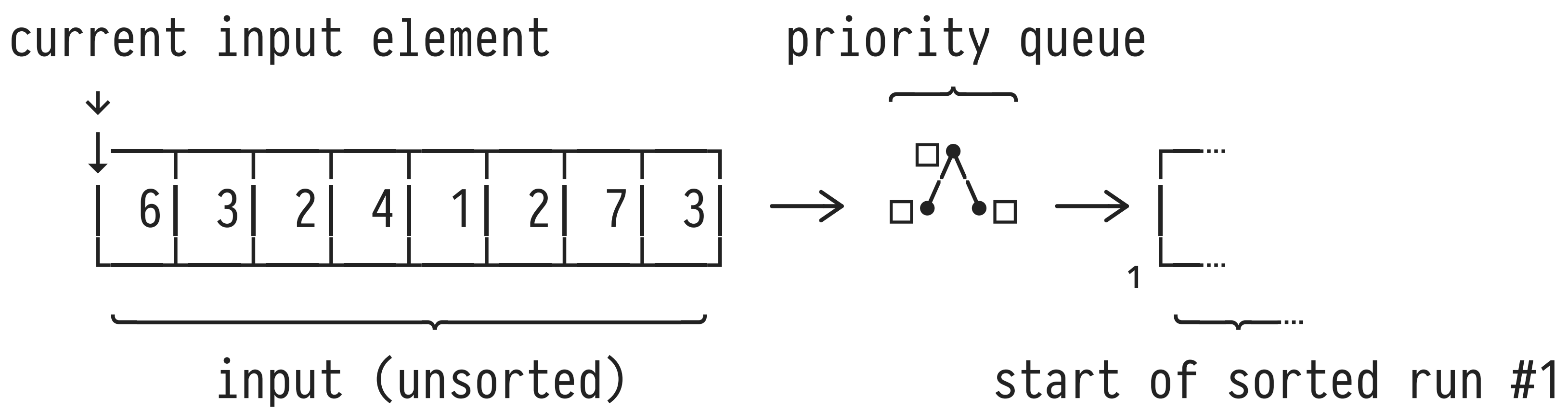
$$\#\ \text{I/O operations} = 2 \times N \times \left(1 + \left\lceil \log_{B-1} \underbrace{\lceil N/B \rceil}_{\#\ \text{runs created in pass 0}} \right\rceil\right)$$

- **Q:** Given only $B$ buffers, can we create sorted runs *longer than* $B$ pages?
  - **A:** Yes! In pass 0, use **Replacement Sort** (instead of QuickSort, for example).

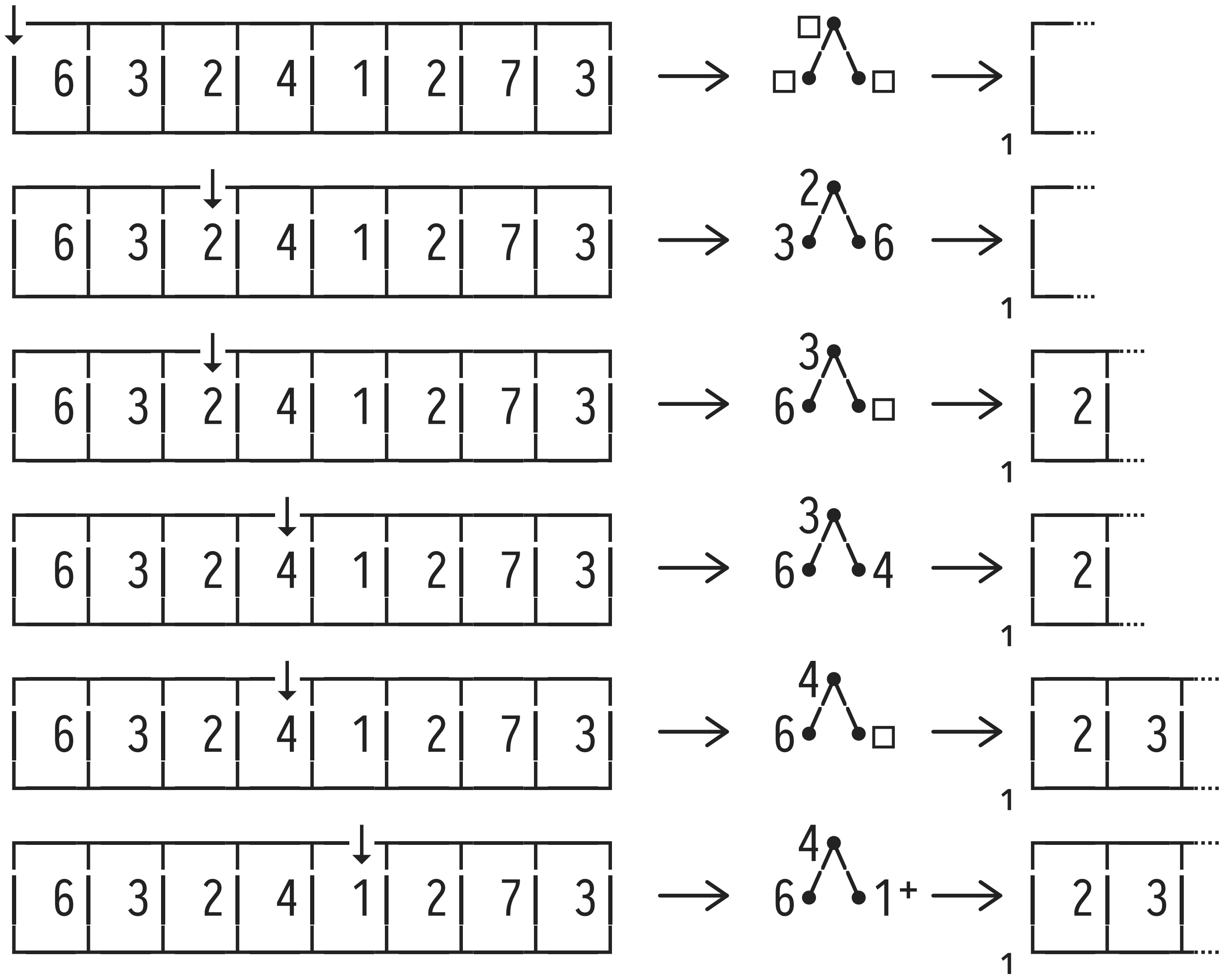
# Replacement Sort

Again, use $B-1$ buffer pages to set up a **priority queue:**

1. Elements arriving too late for inclusion in current run are marked ($\square^+$) and receive lower priority.
2. When all elements in queue are marked, close the current run, unmark all elements, open a new run.



# Replacement Sort ($B$ = 4)



initial state

load queue

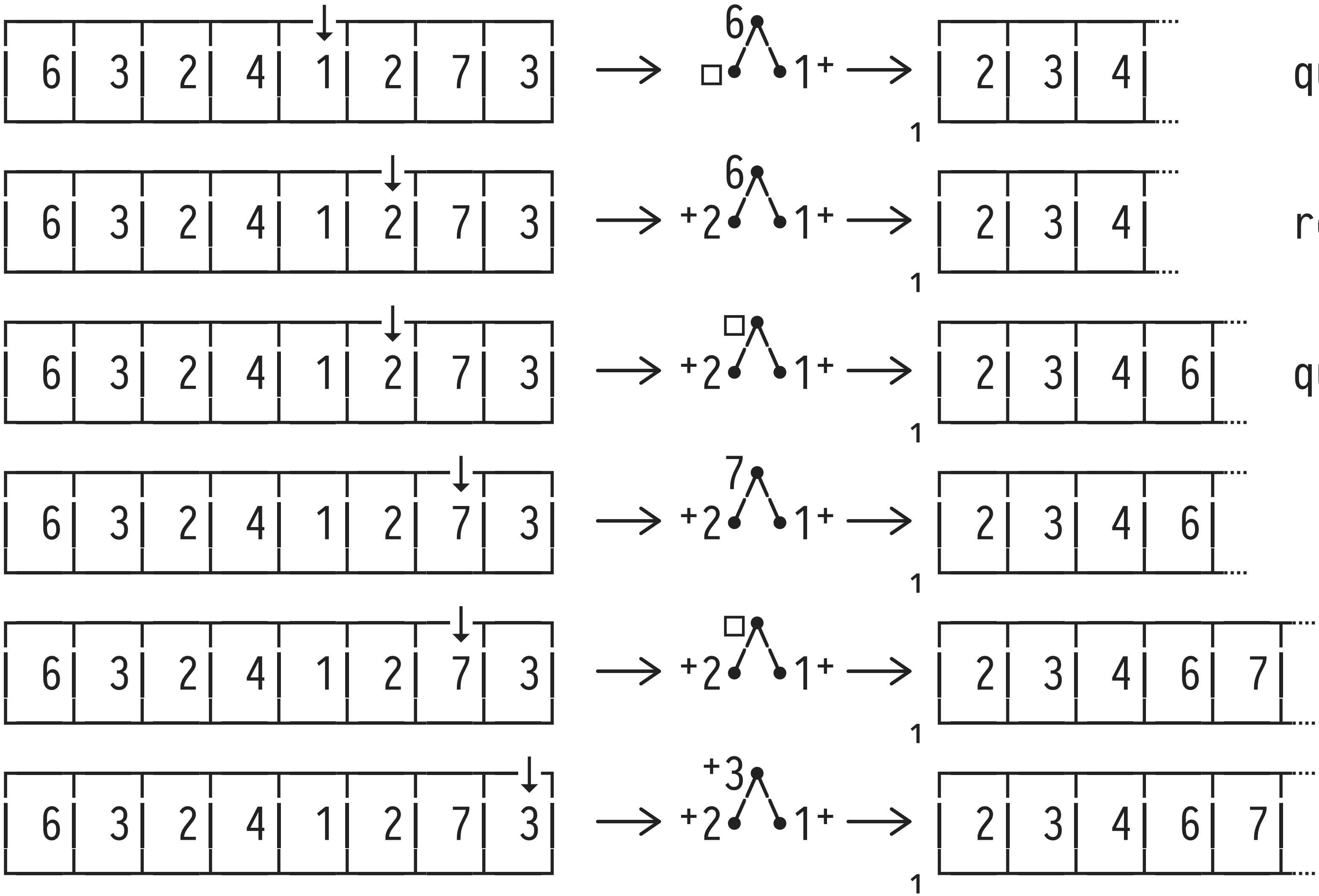
queue head → run #1

refill queue from input

queue head → run

refill q, 1 is late (□⁺)

# Replacement Sort ($B = 4$)




queue head → run

refill q, 2 late

queue head → run

refill queue

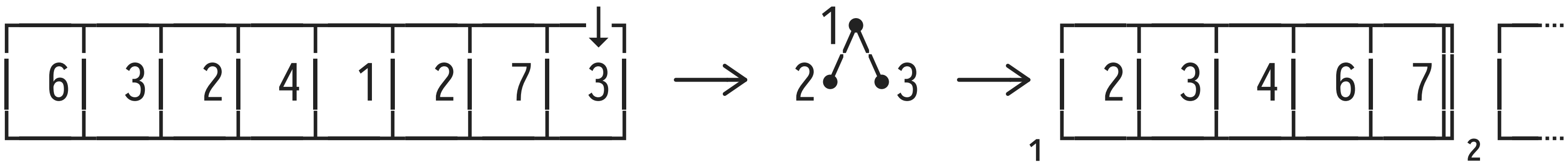
q head → run

refill, 3 late

# Replacement Sort ($B$ = 4)

All entries in queue are late (□⁺):

- Close current run #1, open new run #2.
- Reorder entries in queue, continue processing.




- Replacement Sort produces runs of length $\approx 2 \times (B-1) > B$ (see Knuth, TAoCP, volume 3, p. 254).
- Replacement Sort generates longer runs if input file is almost sorted (*e.g.*, consider a heap file that was once clustered but has received a few updates since then).

## 5 ┊ $Q_{10}$: Grouping

**Grouping** coarsens the granularity of data processing (individual rows ↘ groups of rows):

```
❷ SELECT g.g, SUM(g.a) AS s -- out: 10⁴ groups (aggregates)
  FROM   grouped AS g        -- in:  10⁶ rows
❶ GROUP BY g.g
```

❶ **Partition** table grouped by criterion g.g (all rows agreeing on g.g form one group),

❷ output group criterion and **aggregates** of the group's member rows (the group member rows themselves are never output).

# Grouping: Sorting vs. Hashing

input

| a | g |
|---|---|
| 1 | 42 |
| 2 | 11 |
| 3 | 42 |
| 4 | 42 |
| 5 | 38 |
| 6 | 11 |

**GROUP BY** g

| a | g |
|---|---|
| 2 | 11 |
| 6 | 11 |
| 5 | 38 |
| 3 | 42 |
| 1 | 42 |
| 4 | 42 |

scan/Σa

11: Σa=8

38: Σa=5

42: Σa=8

- scan sorted table for group boundaries
- aggregate while scanning

**Sorting**

**Hashing**

| key | buckets—Σa |
|---|---|
| $k_1$ | $^{g=42}$\|8, $^{g=38}$\|5 |
| $k_2$ | |
| $k_3$ | $^{g=11}$\|8 |

- hash buckets hold grouping criterion and aggregate value

# Grouping: Sorting vs. Hashing

PostgreSQL plans for sorting vs. hashing based on

1. the available working memory (work_mem) and
2. the estimated number *G* of resulting groups:

*G* large or work_mem scarce?

y n

external sorting in-memory hashing

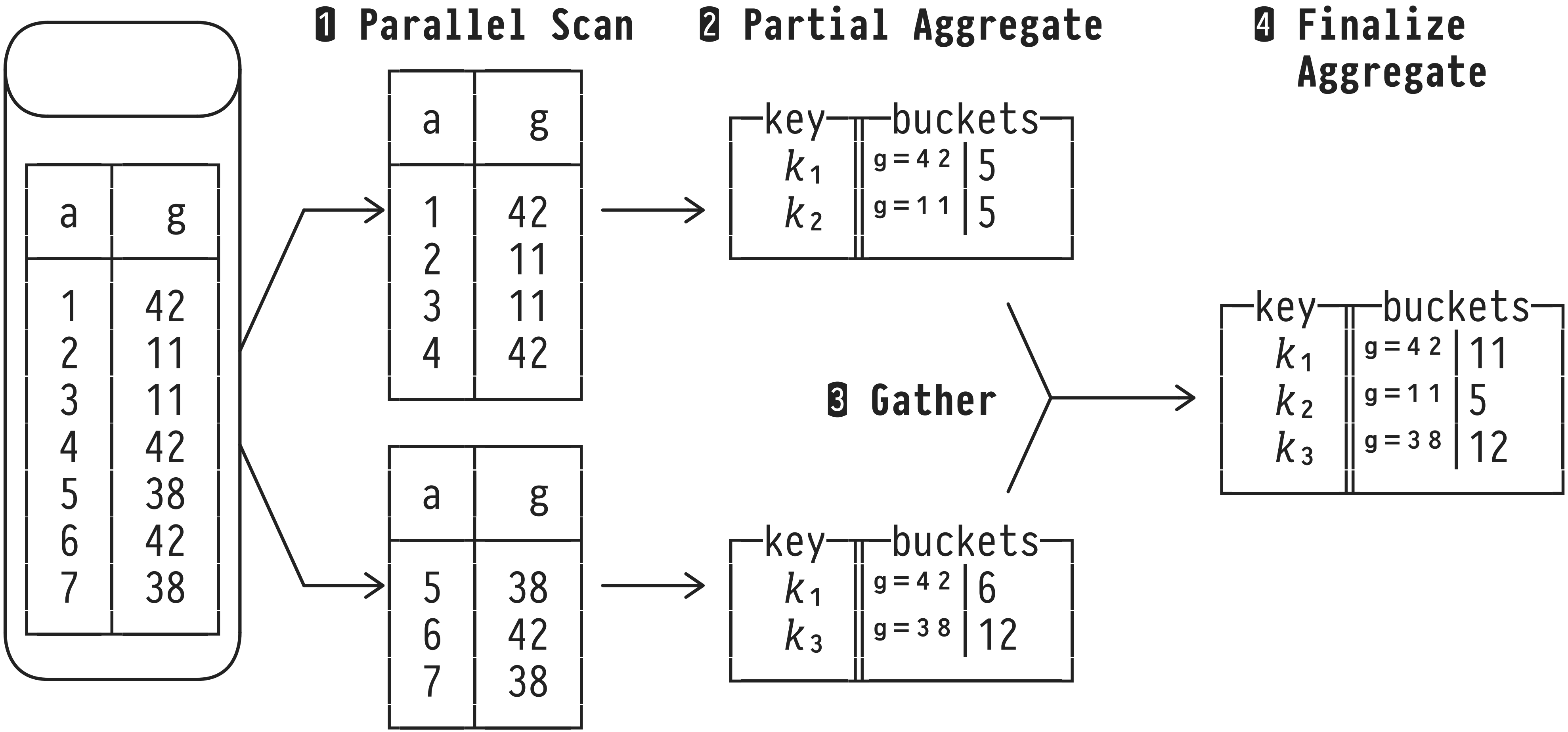
- Often, *G* is unknown or cannot be derived (*e.g.*, GROUP BY g.g % 2 ⇒ $G \leqslant 3$ not understood by PostgreSQL).
  - ⇒ Overestimate *G* conservatively, use sorting.

## 6 ¦ Parallel Grouping and Aggregation

Grouping and aggregation are query operations that are straightforward to **parallelize:**

- Spawn **workers,** each of which execute in // (on dedicated CPU core). Constrain max number of workers to fit host.
- Try to **evenly distribute work** (*e.g.*, data volume) among workers.
- Assign a **leader** thread/process that coordinates workers and **gathers partial query results.**
- After gathering, **merge/finalize partial results** to produce a single complete query result.

# Parallel Grouping (GROUP BY g — SUM(a))



# Parallel Grouping for $Q_{10}$

```
EXPLAIN
  SELECT g.g, SUM(g.a) AS s
  FROM   grouped AS g
  GROUP BY g.g;
```

| QUERY PLAN |
| --- |
| **Finalize HashAggregate** (cost=13869.28..13969.02 …)<br>Group Key: g<br>-> **Gather** (cost=11675.00..13769.54 …)<br>Workers Planned: 2 ← ‖ism *degree*: 3 (2 *worker* + 1 *leader*)<br>-> **Partial HashAggregate** (cost=10675.00..10774.74 …)<br>Group Key: g<br>-> **Parallel Seq Scan** on grouped g (cost=0.00..8591.67 …) |

## Partial Aggregation and Finalization

- Parallel evaluation of aggregate *AGG* depends on its **distributivity** over $\uplus$ (bag union):

$$AGG(X \uplus Y) = AGG(\{AGG(X)\} \uplus \{AGG(Y)\}).$$

- Many SQL aggregates (COUNT, SUM, MAX, MIN, AVG, bool_and, bool_or, …) exhibit this property:

$$\mathrm{SUM}(\underbrace{X \uplus Y}_{\text{distribute work}}) = \mathrm{SUM}(\{\underbrace{\mathrm{SUM}(X)}_{\text{partial}}\} \uplus \{\underbrace{\mathrm{SUM}(Y)}_{\text{aggregates}}\}) = \mathrm{SUM}(X) \underset{\substack{\uparrow \\ \text{finalize}}}{+} \mathrm{SUM}(Y)$$

# 7 ┆ $Q_9$: Sorting in MonetDB

```
CREATE TABLE sorted (a text, s int);
 ⋮
SELECT s.a, s.s
FROM   sorted AS s
ORDER BY s.s [, s.a]   -- single- or multi-column criteria
```

MonetDB's BATs already provide **ordered row storage.** Some ORDER BY queries will thus be no-ops (recall tail properties sorted, revsorted).

Otherwise, use **order indexes**—either persistent or computed on the fly—to apply column re-ordering.

# Recall: Order Indexes (ORDER BY s.s)

| head | a: tail | s: tail |
|---|---|---|
| 0@0 | a | 40 |
| 1@0 | b | 0 |
| 2@0 | c | 50 |
| 3@0 | d | 30 |
| 4@0 | e | 50 |
| 5@0 | f | 10 |
| 6@0 | g | 50 |
| 7@0 | h | 10 |
| 8@0 | i | 10 |
| 9@0 | j | 20 |

| head | $oidx^s$: tail |
|---|---|
| 0@0 | 1@0 |
| 1@0 | 7@0 |
| 2@0 | 8@0 |
| 3@0 | 5@0 |
| 4@0 | 9@0 |
| 5@0 | 3@0 |
| 6@0 | 0@0 |
| 7@0 | 2@0 |
| 8@0 | 6@0 |
| 9@0 | 4@0 |

| head | $a^{ord(s)}$: tail | $s^{ord(s)}$: tail |
|---|---|---|
| 0@0 | b | 0 |
| 1@0 | h | 10 |
| 2@0 | i | 10 |
| 3@0 | f | 10 |
| 4@0 | j | 20 |
| 5@0 | d | 30 |
| 6@0 | a | 40 |
| 7@0 | c | 50 |
| 8@0 | g | 50 |
| 9@0 | e | 50 |

…**algebra.projection**($oidx^s$,·)

# Order Indexes on the Fly: algebra.sort

```
EXPLAIN
  SELECT s.a, s.s
  FROM   sorted AS s
  ORDER BY s.s;

sorted :bat[:oid] := sql.tid(sql, "sys", "sorted");
s0     :bat[:int] := sql.bind(sql, "sys", "sorted", "s", …);
s      :bat[:int] := algebra.projection(sorted, s0);
(sord(s), oidxs, gidxs)                desc↓    ↓nil last  ↓stable
                  := algebra.sort(s, false, false, false);
a0     :bat[:str] := sql.bind(sql, "sys", "sorted", "a", …);
a      :bat[:str] := algebra.projection(sorted, a0);
aord(s):bat[:str] := algebra.projectionpath(oidxs, sorted, a0);

io.print(aord(s), sord(s));
```

## Persistent Order Indexes

If sorting is central to the query workload, create a **persistent order index** that is immediately applicable:

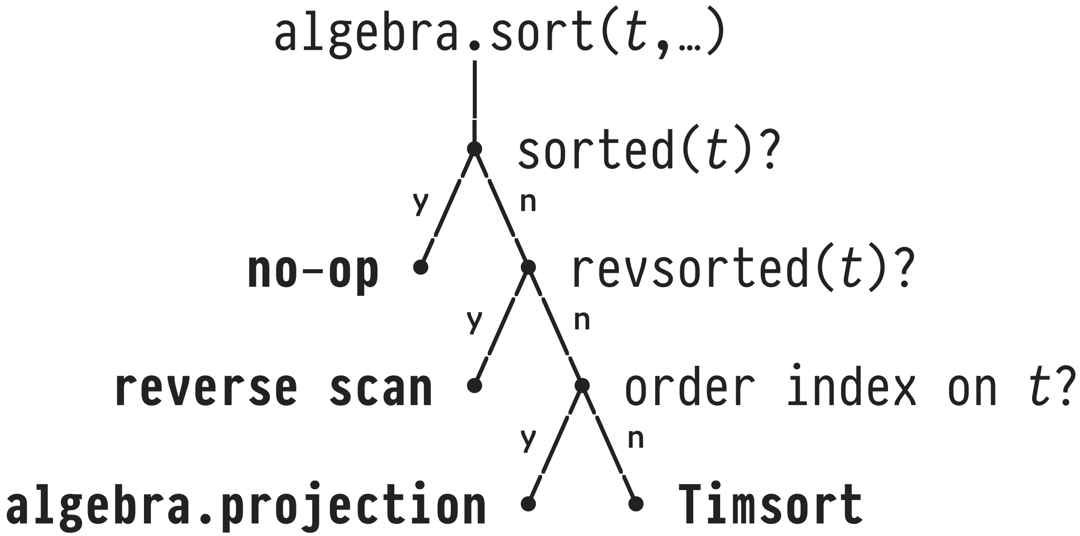
```
ALTER TABLE sorted SET READ ONLY;      -- ⚠

CREATE ORDERED INDEX oidxˢ ON sorted(s);
```

- Order indexes are **static** structures that are *not* dynamically maintained (as opposed to $B^+$Trees).
  If order index has been created...
  1. on the fly: throw away on table update,
  2. persistent: read-only table, no updates at all.

# Tactical Optimization for algebra.sort

- algebra.sort aims to avoid actual sorting effort based on properties of BAT *t* and the presence of order indexes:




- If all else fails, apply in-memory sort algorithm **Timsort** (1993; hybrid of merge/insertion sort, run-aware).

## 8 ¦ Multi-Criteria ORDER BY

**Multi-column** ordering criteria require special treatment: `algebra.sort(s)` only receives single criterion s.

```
SELECT s.a, s.s
FROM   sorted AS s
ORDER BY s.s, s.a -- s₁ < s₂ ⇔ s₁.s < s₂.s ∨
                  --          (s₁.s = s₂.s ∧ s₁.a < s₂.a)
```

- 💡 Let `algebra.sort(s)` return *three* result BATs:
  1. $s^{ord(s)}$ (the ordered input s) ✓
  2. $oidx^{s}$ (order index) ✓
  3. $gidx^{s}$ (groups rows that agree on criterion s).

# Multi-Criteria **ORDER BY**: Group Index **gidx**

$s^{ord(s)}$ ✓

| head | tail |
|---|---|
| 0@0 | 0 |
| 1@0 | 10 |
| 2@0 | 10 |
| 3@0 | 10 |
| 4@0 | 20 |
| 5@0 | 30 |
| 6@0 | 40 |
| 7@0 | 50 |
| 8@0 | 50 |
| 9@0 | 50 |

,

$oidx^s$ ✓

| head | tail |
|---|---|
| 0@0 | 1@0 |
| 1@0 | 7@0 |
| 2@0 | 8@0 |
| 3@0 | 5@0 |
| 4@0 | 9@0 |
| 5@0 | 3@0 |
| 6@0 | 0@0 |
| 7@0 | 2@0 |
| 8@0 | 6@0 |
| 9@0 | 4@0 |

,

**$gidx^s$**

| head | tail | s = |
|---|---|---|
| 0@0 | 0@0 | 0 |
| 1@0 | 1@0 | 10 |
| 2@0 | 1@0 | 10 |
| 3@0 | 1@0 | 10 |
| 4@0 | 2@0 | 20 |
| 5@0 | 3@0 | 30 |
| 6@0 | 4@0 | 40 |
| 7@0 | 5@0 | 50 |
| 8@0 | 5@0 | 50 |
| 9@0 | 5@0 | 50 |

:= algebra.sort

s

| head | tail |
|---|---|
| 0@0 | 40 |
| 1@0 | 0 |
| 2@0 | 50 |
| 3@0 | 30 |
| 4@0 | 50 |
| 5@0 | 10 |
| 6@0 | 50 |
| 7@0 | 10 |
| 8@0 | 10 |
| 9@0 | 20 |

3 output BATs

input BAT

# Multi-Criteria **ORDER BY s,a**: Refine **ORDER BY s** by **a**

**algebra.sort**(s)

⋮

→ $s^{ord(s)}$

→ $oidx^{s}$

$gidx^{s}$

| head | tail |
|---|---|
| 0@0 | 0@0 |
| 1@0 | 1@0 |
| 2@0 | 1@0 |
| 3@0 | 1@0 |
| 4@0 | 2@0 |
| 5@0 | 3@0 |
| 6@0 | 4@0 |
| 7@0 | 5@0 |
| 8@0 | 5@0 |
| 9@0 | 5@0 |

$a^{ord(s)}$

| head | tail | exec in ∥ |
|---|---|---|
| 0@0 | b | *sort* |
| 1@0 | h | |
| 2@0 | i | *sort* |
| 3@0 | f | |
| 4@0 | j | *sort* |
| 5@0 | d | *sort* |
| 6@0 | a | *sort* |
| 7@0 | c | |
| 8@0 | g | *sort* |
| 9@0 | e | |

$refine^{s\ by\ a}$

| head | tail |
|---|---|
| 0@0 | 0@0 |
| 1@0 | 3@0 |
| 2@0 | 1@0 |
| 3@0 | 2@0 |
| 4@0 | 4@0 |
| 5@0 | 5@0 |
| 6@0 | 6@0 |
| 7@0 | 7@0 |
| 8@0 | 9@0 |
| 9@0 | 8@0 |

…**algebra.sort***($a^{ord(s)}$, $gidx^{s}$)

- Apply this to $c^{ord(s)}$ to obtain $c^{ord(sa)}$.
- Apply this to $oidx^{s}$ to obtain $oidx^{sa}$.