Functional Programming

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Torsten Grust
University of Tübingen

Monads (“*The Programmable Semicolon*”)

Function composition via `(.)` is *the* FP idiom that constructs larger programs. In a sense, `(.)` serves the role of `;` (statement sequencing) in imperative PLs.

- **Sequencing functions:**

```
(.) :: (b -> c) -> (a -> b) -> (a -> c)
f . g = \x -> f (g x)

(>=>) :: (a -> b) -> (b -> c) -> (a -> c)
f >=> g = g . f
```

  - f_1 `>=>` f_2 `>=>` … `>=>` f_n composes the f_i in left-to-right order (think UNIX pipes).
  - (`>=>`,`id`) forms a monoid: `>=>` is associative with identity `id`.

Sequencing Partial Functions (a -> Maybe b)

- If the f_i in f_1 >=> f_2 >=> … >=> f_n are potentially failing (“partial”) functions, the composition should fail once the first function fails.
  - Return Nothing as soon as first function in sequence returns Nothing:

```
(>=>) :: (a -> Maybe b) -> (b -> Maybe c) -> (a -> Maybe c)
f >=> g = \x -> case f x of
                  Nothing -> Nothing
                  Just y  -> g y
```

Sequencing Exception-Generating Functions (`a -> Exc b`)

- Result of computations that may either throw an exception or return a regular value `b` (assumes suitable representation type for errors/exceptions, *e.g.*, `type Error = String`):

```
type Exc b = Either Error b
```

- In a composition, exceptions are propagated once occurred:

```
(>=>) :: (a -> Exc b) -> (b -> Exc c) -> (a -> Exc c)
f >=> g = \x ->
```

Sequencing Non-Deterministic Functions (a -> NonDet b)

- A non-deterministic function may respond with *any answer* in a list of possible of answers:

```
type NonDet b = [b]
```

- In a composition of non-deterministic functions, treat all possible outcomes alike:

```
(>=>) :: (a -> NonDet b) -> (b -> NonDet c) -> (a -> NonDet c)
f >=> g = \x -> concat $ map g $ f x
```

  or

```
f >=> g = \x -> concat [ g y | y <- f x ]
```

Sequencing Stateful Functions (a -> State -> (State, b))

- Perform regular computation (to yield a value of type b) **and** transform the current state of the system/program.
  - **State transformers** (assumes state representation State):

    ```
    type ST b = State -> (State, b)
    ```

  - Stateful functions thus have type a -> ST b.
- Composition (with state transformation s0 ⇢ s1 ⇢ s2):

```
(>=>) :: (a -> ST b) -> (b -> ST c) -> (a -> ST c)
f >=> g = \x s0 -> let (s1, y) = f x s0 in
                   let (s2, z) = g y s1 in      } simplifies to
                   (s2, z)                      } g y s1
```

Sequencing Side-Effecting Functions (`a -> World -> (World, b)`)

- Modeling **side effects:** functions consume current “world” state, return new world state along with result:

```
type World = ¯\_(ツ)_/¯ -- abstract (defined by GHC runtime)
type IO b = World -> (World, b)
```

- Values of type `IO b` are **actions** that
  1. *when performed* have a side-effect on the world, and then
  2. return a value of type `b`.

- Haskell built-ins with side-effects:
  - `print :: Show a => a -> IO ()`, `putStrLn :: String -> IO ()`
  - `getLine :: IO String`
  - `readFile :: FilePath -> IO String`
  - `(>=>) :: (a -> IO b) -> (b -> IO c) -> (a -> IO c)`

Type Class Monad

- These sequencing functions follow an obvious pattern:

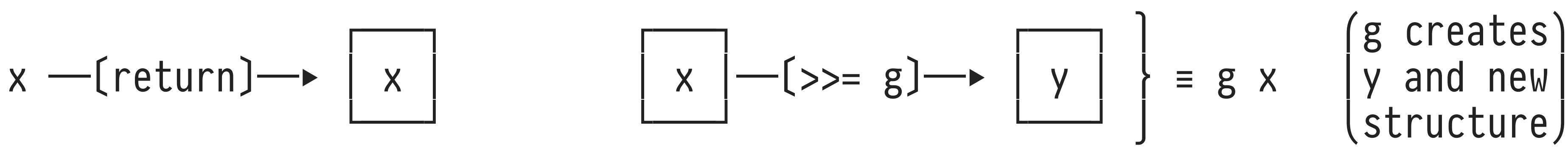
```
(>=>) :: (a ->        b) -> (b ->        c) -> (a ->        c)
(>=>) :: (a -> Maybe  b) -> (b -> Maybe  c) -> (a -> Maybe  c)
(>=>) :: (a -> Exc    b) -> (b -> Exc    c) -> (a -> Exc    c)
(>=>) :: (a -> NonDet b) -> (b -> NonDet c) -> (a -> NonDet c)
(>=>) :: (a -> ST     b) -> (b -> ST     c) -> (a -> ST     c)
(>=>) :: (a -> IO     b) -> (b -> IO     c) -> (a -> IO     c)

(>=>) :: (a ->      m b) -> (b ->      m c) -> (a ->      m c)
```

  - To instantiate (>=>) for different type constructors *m*, build a type class whose instances are **type constructors** (i.e. *m* is of kind * -> *).

Type Class Monad (Or: Why Haskell's Logo is ⋙=)

- Type class Monad (*m* has kind `* -> *`):

```
class Applicative m => Monad m where
  return :: a -> m a                      -- lifting
  (>>=)  :: m a -> (a -> m b) -> m b      -- sequencing ("bind")
```



- **NB:** Once a value is inside the monadic *m*-structure, class Monad does not let it “escape” again.

(>>=) vs. (>=>)

- **Bind** (>>=) is simpler/does less than (>=>):
  - f >=> g involves *two* applications (of f and g). Instead, s >>= g receives structure s, extracts its value x, and routes x to g (*one* application).
  - Nevertheless, both are closely related. (>=>) may be expressed in terms of (>>=):

```
(>=>) :: Monad m => (a -> m b) -> (b -> m c) -> (a -> m c)
f >=> g = \x -> f x >>= g
                └─┘
              :: m b
```

- (>=>) is also known as **Kleisli composition**.

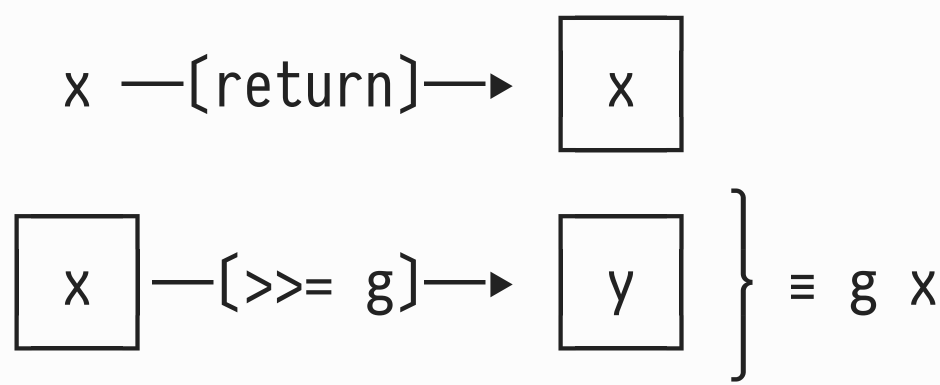
Lifting via **return**

- **Lifting** creates a trivial context/structure (recall pure):

Monad	lifting (return x)
Maybe	Just x
Exc	Right x
NonDet	[x]
ST	\s -> (s,x)

- **Example:** Make Maybe an instance of Monad:

```
instance Monad Maybe where
  return x = Just x

  Nothing >>= g = Nothing
  Just x  >>= g = g x
```



The Functor/Applicative/Monad Tower

- The type classes Functor, Applicative, and Monad are obviously closely related:

```
class Functor m     => Applicative m where …
class Applicative m => Monad m       where …
```

- Monad, Applicative, Functor form a "tower of abstractions":

expressiveness

Monad (return, (>>=)) — *high*

Applicative (pure, (<*>))

Functor (fmap) — *low*

A Unique Ability of Monad

- Monad is truly more expressive than the abstractions we have studied so far.

  Monadic bind >>= can create entirely **new contexts/structure:**

```
                 These functions emit values (of type b) to be
                    embedded in existing/combined structures

($)   ::                             (a -> b) ->   a       ->   b
(<$>) :: Functor m =>                (a -> b) -> m a       -> m b
(<*>) :: Applicative m => m (a -> b) -> m a       -> m b

(>>=) :: Monad m =>                       m a -> (a -> m b) -> m b

                                         This function can generate
                                              a new context m!
```

Expressiveness: Monad > Applicative > Functor

- *A Monad m can do whatever an Applicative can.*
  Proof: Express Applicative's operations in terms of Monad's:

```
pure x    ≝  return x

              :: (a -> b)      :: a           :: b

u <*> v   ≝  u >>= \f -> v >>= \x -> return (f x)

        :: m (a -> b)   :: m a            :: m b
```

- Given this and fmap f v = pure f <*> v, *m* also is a Functor:

```
fmap f v  ≝  return f >>= \f -> v >>= \x -> return (f x)
```

A Dose of Syntactic Sugar: do-Notation

- Looong chains of `e₁ >>= \x -> e₂` are typical for monadic code, yet hard to read.
- *Imperative* programming languages have introduced established notation for programs that

  1. perform computation e_1, name the result `x`, and then
  2. perform computation e_2 (which may refer to variable `x`):

```
x := e₁;
e₂;          /* e₂ may use x */
```

- Haskell offers **syntactic sugar** that can mimic such sequential programs (but indeed is much more general): **do-notation**.

do-Notation

- Haskell's `do`-notation (e: expression of type m `a`, es: sequence of `;`-separated expressions e_1; e_2; … ; e_n):

```
do { e }                 ≡  e
do { x <- e; es }        ≡  e >>= \x -> do { es }
do { e; es }             ≡  e >>= \_ -> do { es }
do { let v = x; es }     ≡  let v = x in do { es }    -- a pure let
```

  - May use 2D layout instead of `{ … }` and `;`.
  - ⚠️ Syntactically mimics imperative statements, but still is an epxression that computes a monadic value: `do { … } ::` m `b`.

Example ❶: Interactively Copy a File (do-Notation)

- Using do-notation leads to almost Python-like syntax:

```
copyFile :: FilePath -> FilePath -> IO Int
copyFile from to = do
  content <- readFile from
  writeFile to content
  return (length content)

main :: IO ()
main = do
  putStrLn "Which file do you want to copy?"
  from <- getLine
  putStrLn "Where do you want to copy it to?"
  to <- getLine
  n <- copyFile from to
  putStrLn ("Copied " ++ show n ++ " bytes.")
```

Example ❷: Interactively Copy a File (>>= Explicit)

```
copyFile :: FilePath -> FilePath -> IO Int
copyFile from to =
  readFile from          >>= \content ->
  writeFile to content >>= \_        ->       -- � may use (>>)
  return (length content)

main :: IO ()
main =
  putStrLn "Which file do you want to copy?"  >>= \_ ->
  getLine                                     >>= \from ->
  putStrLn "Where do you want to copy it to?" >>= \_ ->
  getLine                                     >>= \to ->
  copyFile from to                            >>= \n ->
  putStrLn ("Copied " ++ show n ++ " bytes.")
```

⚠️ Do Not Let `do` Fool You

- This is **not** imperative programming in Haskell. Beware of actually thinking in , , JS, … terms.

- **Example:** Is this correct Haskell? What is the output?

```haskell
f :: IO Integer
f = do
  putStrLn "Running f ..."
  return 1893
  return 1904

main :: IO ()
main = do
  x <- f
  putStrLn (show x)
```

Input/Output

- But how can I/O possibly be pure? Reading a string with `getLine` may well yield different results on each execution.

Key Idea:

Decouple the **pure construction** of I/O actions from their **effectful execution.**

1. Construct (complex) I/O action of type `IO a` using `getLine`, `putStrLn`, `>>=`, … **No I/O or side effects happened yet.**

2. Perform constructed `IO a` action: **side effects will occur** and value of type `a` is returned. *Only* the `IO a` action returned by function `main` is performed by Haskell's runtime system.

Roll Your Own I/O DSL

- GHC's internal representation of IO actions is opaque. Likewise, the internals of the Haskell runtime—the interpreter for these IO actions—are hidden.

- To better see what is going on, build our own versions of both in terms of a simple **IO action DSL:**

  1. The *constructors* build IO actions (*e.g.*, printing/reading strings). Supply a `Monad` instance for the constructors to use `do`-notation to conveniently sequence complex IO actions.

  2. The *observer* `runIO` translates the resulting IO action into a regular Haskell `IO a` action that can then be performed with side effects.

Monad Laws

- Haskell presently cannot enforce (or even check) this, but `return` and `>>=` (or `>=>`) are expected to behave "sanely":

```
return >=> g         ≡   g                       (left identity)
f      >=> return    ≡   f                       (right identity)
(f >=> g) >=> h      ≡   f >=> (g >=> h)         (associativity)
```

  - ⇒ (`>=>`, `return`) forms a monoid over the carrier of functions `a -> m b`.

"A monad is just a monoid in the category of endofunctors. What's the problem?" — James Iry

- Category is *Hask* (objects: Haskell types, arrows: functions)
- `m` is the endofunctor: `m b` is a Haskell type again

Merging Monads: Monad Transformers

The monads we have seen so far are specialists in representing computations with *a single* type of side effect:

Monad	represents computation that…
Maybe a	may yield an a or return no result at all
Either e a	throws an exception e or succeeds with a
NonDet a ([a])	yields one of many possible a
ST a	reads/modifies current state, then returns a
IO a	performs input/output, then returns a
Reader e a	reads environment e, then returns a

- Can we build a monad to represent computations that perform I/O **and** may potentially throw an exception?
  - Do we need to start from scratch?
  - Or can we **combine the Either and IO monads** systematically?

Either + IO: ❶ Verifying E-Mail Addresses

- Start with a computation that "verifies" a given e-mail address (does it contain exactly one '@')?

```
data LoginError = InvalidEmail
  deriving Show

checkEmail :: String -> Either LoginError String
checkEmail email = case splitOn "@" email of
                     [_name, _domain] -> Right email
                     _                -> Left InvalidEmail
```

  - Case Left *e* represents an exception that subsequent computations need to handle (monad-trans-1.hs).

Either + IO: ❷ Adding I/O

- Go interactive: ask for e-mail address, then verify it:

```
askForEmail :: IO (Either LoginError String)
askForEmail = do
  putStrLn "Enter e-mail address:"
  email <- getLine
  return (checkEmail email)
```

- As long as exception-generating computation and I/O do not interact, we are doing fine.
  - Here: perform all I/O, *then* verify e-mail (and we're done).

- If, instead, exception-generating computation and I/O mix, the code quickly gets nested and messier (monad-trans-2.hs).

Either + IO: ❸ Build a Custom Combined Monad

We are operating in the **combined** (or: **stacked**) **monad** `IO (Either LoginError String)`. Generalized:

```
-- A computation that performs I/O and then either
-- 1 fails with an exception (of type e) or
-- 2 succeeds and returns with a result (of type a):

newtype EitherIO e a = EitherIO { runEitherIO :: IO (Either e a) }
```

- Recall that this `newtype` declaration defines two type conversions (at no runtime cost):

```
EitherIO    :: IO (Either e a) -> EitherIO e a
runEitherIO :: EitherIO e a    -> IO (Either e a)
```

Either + IO: ❸ Build a Custom Combined Monad

- The essence (behavior) of the combined monad:

```
instance Monad (EitherIO e) where

                trivial (= no) I/O, computation succeeds

 return x = EitherIO $ (returnio . Right) x

                                               ❶ computation failed,
                                                    trivial I/O

 x >>= f = EitherIO $ runEitherIO x >>=io either (returnio . Left)
                                                 (runEitherIO . f)

                          perform I/O,
                     then yield Either e a       ❷ computation succeeded,
                                                    continue with f
```

❹ Lifting Values Into the Combined EitherIO Monad

- ⚠️ To run existing code in the new EitherIO monad, we need to convert/**lift** IO *a* (and Either *e a*) values into EitherIO *e a* values. Otherwise, type errors (⚡) occur:

```
askForEmail :: EitherIO LoginError String
askForEmail = do
 putStrLn "Enter e-mail address:" ⚡1 :: IO ()
 email <- getLine                 ⚡1 :: IO String
 return (checkEmail email)        ⚡2 :: IO (Either Lo…r String)
```

  - ⚡[1]: Has type IO *a* but we need EitherIO *e a*.
  - ⚡[2]: Has type IO (Either *e a*) but we need EitherIO *e a*.

❹ Lifting Values Into the Combined EitherIO Monad

- Both **liftings** (Either *e a* ⇢ EitherIO *e a* ⇠ IO *a*) are readily defined and help to write EitherIO computations concisely:

```
liftEither :: Either e a -> EitherIO e a
liftEither x = EitherIO (return x)

                   :: IO (Either e a)

liftIO :: IO a -> EitherIO e a
liftIO x = EitherIO (fmap Right x)

                 :: IO (Either e a) [recall: IO is a functor]
```

- That is all we need to rewrite our login/password application code (🖹 monad-trans-3.hs).

❺ Throwing and Catching Exceptions

- In `userLogin`, `liftEither (Left` *`exc`*`)` is used to raise (or “throw”) exception *exc*. Capturing that idiom:

```
throw :: e -> EitherIO e a
throw exc = liftEither (Left exc)
```

- As a counterpart to `throw`, “catch” exceptions thrown by a computation and invoke a **handler** to cope with the exception:

```
    computation that may
     throw an exception

catch :: EitherIO e a -> (e -> EitherIO e a) -> EitherIO e a

                                  handler (receives
                                 exception as input)
```

❺ Throwing and Catching Exceptions

- Exception handlers allow to model complex application behavior.
  Example:

  ❶ Attempt user login.

  ❷ Success?
  - Yes, logged in.
  - First failure?
    - Yes, offer 2nd attempt. Goto ❶.
    - No, abort. Print error message.

  - Implementation (monad-trans-4.hs):

```haskell
loginDialogue :: EitherIO LoginError String
loginDialogue = do
  email <- userLogin `catch` wrongPassword `catch` abortLogin
  liftIO $ putStrLn ("Logged in as " ++ email)
  return email
```

❻ Generalizing From IO to Any Monad *m*

- Abstract from IO and combine Either with any monad *m*. We arrive at the **monad transformer** EitherT *e m a*:

```
-- Run a computation in monad m and then either
-- ❶ fail with an exception (of type e) or
-- ❷ succeed and return with a result (of type a):

newtype EitherT e m a = EitherT { runEitherT :: m (Either e a) }
```

- See the Haskell transformers library for a collection of such monad transformers.
  - Example: An implementation of EitherT is found in Control.Monad.Trans.ExceptT.

❻ Generalizing From IO to Any Monad *m*

```
instance Monad m => Monad (EitherT e m) where

        trivial m-computation, computation succeeds

 return x = EitherT $ (returnₘ . Right) x
```

```
                                              ❶ computation failed,
                                                trivial m-computation

 x >>= f = EitherT $ runEitherT x >>=ₘ either (returnₘ . Left)
                                              (runEitherT . f)

                    perform m-computation x,
                     then yield Either e a    ❷ computation succeeded,
                                                continue with f
```

- See monad-trans-5.hs.