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# Functional Programming

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## Assignment #4

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### Exercise 1: Minesweeper (10 Points)

The objective of Minesweeper is to clear a rectangular field which contains hidden *mines*. For each field cell the player steps on, he is given a hint about the number of mines in the direct neighborhood.

We want to implement a function to compute these numbers for a given field with visible mines:

```
minesweep :: [[Char]] -> [[Int]]
```

**Example:**

```
field = [ "␣␣␣␣␣␣␣␣"
        , "***␣␣␣␣␣"
        , "*␣*␣␣␣␣␣"
        , "***␣␣␣␣␣"
        ]

Main> minesweep field
[ [ 2, 3, 2, 1, 0, 0, 0, 0 ]
, [ 3, 5, 3, 2, 0, 0, 0, 0 ]
, [ 5, 8, 5, 3, 0, 0, 0, 0 ]
, [ 3, 5, 3, 2, 0, 0, 0, 0 ]
]
```

Follow these steps to solve the problem:

1. The actual algorithm shall be formulated as a combination of helper functions, you have to implement in advance:
   (a) Write a function `num :: Char -> Int` which takes a field cell and and returns `1`, if it is mined (`'*'`), and `0`, if it is safe (`' '`).
   (b) Write a function `shiftL :: Num a =>[a] -> [a]` which shifts a given list of numbers to the left, i. e. drops the first element and appends a new element `0` to the end of the list.
   (c) Write a function `shiftR :: Num a =>[a] -> [a]` which shifts a given list of numbers to the right.

(d) Write a function

```
zipWith3' :: (a -> b -> c -> d) -> [a] -> [b] -> [c] -> [d]
```

It takes a function `f :: (a -> b -> c -> d)` that combines three elements, as well as three lists of equal length and returns a list where each element is the combination (using `f`) of the input lists' elements at the same position.[1]

**Example:**

```
zipWith3' (\a b c -> a * b + c) [1,2] [3,4] [5,6] ≡ [8,14]
```

(e) Use functions `shiftL`, `shiftR` and `zipWith3'` to implement a function

```
addNeighbours :: [Int] -> [Int]
```

For each element of a given list of integers, the function returns the sum of the element itself, together with its left and right neighbour.

**Example:**

```
addNeighbours [1,1,1,0,1] ≡ [2,3,2,2,1]
```

(f) Write a function which transposes the rows and columns of a given matrix (list of lists of equal length):[2]

```
transpose :: [[a]] -> [[a]]
```

**Example:**

```
transpose [[1,2,3],[4,5,6]] ≡ [[1,4],[2,5],[3,6]]
```

2. Implement `minesweep` as a combination of `Prelude` function `map` and the helper functions `num`, `addNeighbours` and `transpose` defined in step 1.

**Exercise 2: Regular Expressions (10 Points)**

Finite state machines aren't the only method to implement regular expression matching. Here, we will build a regular expression matcher using the *derivatives of a regular expression*.

To implement this approach, we first need a representation for regular expressions on a given alphabet of symbols.

1. Define a data type `RegExp a` for regular expressions, which are one of:
   - the empty string $\varepsilon$,
   - a symbol of the alphabet (symbols have type $a$ – typically $\mathbf{a} \equiv \mathbf{Char}$),
   - a concatenation $r_1 r_2$ of two regular expressions $r_1$ and $r_2$ ($r_1$ followed by $r_2$),
   - the *Kleene star* $r^*$ of a regular expression $r$ ($r$ repeated zero or more times),
   - an alternation $r_1|r_2$ of two regular expressions $r_1$ and $r_2$ ($r_1$ or $r_2$),
   - a special regular expression $\varnothing$ which accepts no input at all.
2. Write an instance of type class `Show` for `RegExp a` to print a regular expression as string of characters $\varepsilon$, $a$, $^*$, $^+$, $|$, $\varnothing$. Use parentheses () to group parts of the regular expression, if necessary.

The *derivative of a regular expression* $r$ with respect to symbol $a$ is another regular expression $r'$. If input $s$ is accepted by $r$, then $r'$ accepts $s$ with the starting symbol $a$ removed. For example, consider the regular expression $r = \mathtt{ab}^*$. The derivative of $r$ with respect to `a` is $\mathtt{b}^*$ and the derivative of $\mathtt{b}^*$ with respect to `b` is again $\mathtt{b}^*$. However, the derivative of $r$ with respect to `b` is the regular expression $\varnothing$.

These derivatives can be used to implement regular expression matching.

[1] Obviously you shall implement this function on your own and must not use the `Prelude` function `zipWith3`.
[2] You must not use the `Data.List` function `transpose`, but implement the function on your own.

First we need a function $\nu(r)$ to test whether a regular expression $r$ is nullable. We say that $r$ is nullable, if $r$ accepts the empty string $\varepsilon$:

$$
\begin{aligned}
\nu(\varepsilon) &= \texttt{True} \\
\nu(a) &= \texttt{False} \\
\nu(r^*) &= \texttt{True} \\
\nu(r_1 r_2) &= \nu(r_1) \land \nu(r_2) \\
\nu(r_1 | r_2) &= \nu(r_1) \lor \nu(r_2) \\
\nu(\varnothing) &= \texttt{False}
\end{aligned}
$$

3. Write a function `nullable :: RegExp a -> Bool` implementing $\nu$.

Now we can define a function $\partial_a(r)$ to compute the derivative of a regular expression $r$ with respect to a symbol $a$:

$$
\begin{aligned}
\partial_a(\varepsilon) &= \varnothing \\
\partial_a(b) &= \begin{cases} \varepsilon & , \textit{ if } a = b \\ \varnothing & , \textit{ if } a \neq b \end{cases} \\
\partial_a(r^*) &= \partial_a(r r^*) \\
\partial_a(r_1 r_2) &= \begin{cases} \partial_a(r_1) r_2 \,|\, \partial_a(r_2) & , \textit{ if } \nu(r_1) \\ \partial_a(r_1) r_2 & , \textit{ if } \neg\nu(r_1) \end{cases} \\
\partial_a(r_1 | r_2) &= \partial_a(r_1) \,|\, \partial_a(r_2) \\
\partial_a(\varnothing) &= \varnothing
\end{aligned}
$$

4. Write a function `derive :: Eq a =>RegExp a -> a -> RegExp a` implementing $\partial$.

Supposed we have a regular expression $r$ and a string of symbols $s = a_1 \ldots a_n$. To test whether $r$ accepts $s$, we can make use of a successive application of $\partial$ to $r$ with respect to the symbols of $s$. If and only if the final derivative with respect to $a_n$ is nullable, i.e., matches the empty string $\varepsilon$, the regular expression $r$ matches the whole string $s$:

$$
r \text{ matches } s \quad \Leftrightarrow \quad \nu(\partial_{a_n}(\cdots \partial_{a_1}(r)))
$$

5. Write a function `match :: Eq a =>RegExp a -> [a] -> Bool` implementing the regular expression matcher. Remember to provide some tests.

**[Optional:]** Extend your definitions of `RegExp a`, `nullable` and `derive` to also support the *Kleene plus*: A regular expression can also be $r^+$ (the regular expression $r$ repeated one or more times).