

# Functional Programming

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## Monads (“*The Programmable Semicolon*”)

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Function composition via `(.)` is *the* FP idiom that constructs larger programs. In a sense, `(.)` serves the role of `;` (statement sequencing) in imperative PLs.

- **Sequencing functions:**

```
(.) :: (b -> c) -> (a -> b) -> (a -> c)
f . g = \x -> f (g x)
```

```
(>=>) :: (a -> b) -> (b -> c) -> (a -> c)
f >=> g = g . f
```

- $f_1 \Rightarrow f_2 \Rightarrow \dots \Rightarrow f_n$  composes the  $f_i$  in left-to-right order (think UNIX pipes).
- $(\Rightarrow, id)$  forms a monoid:  $\Rightarrow$  is associative with identity  $id$ .

## Sequencing Partial Functions ( $a \rightarrow \text{Maybe } b$ )

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- If the  $f_i$  in  $f_1 \Rightarrow f_2 \Rightarrow \dots \Rightarrow f_n$  are potentially failing (“partial”) functions, the composition should fail once the first function fails.
  - Return **Nothing** as soon as first function in sequence returns **Nothing**:

```
(=>) :: (a -> Maybe b) -> (b -> Maybe c) -> (a -> Maybe c)
f ==> g = \x -> case f x of
    Nothing -> Nothing
    Just y   -> g y
```

## Sequencing Exception-Generating Functions (**a** -> **Exc** **b**)

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- Result of computations that may either throw an exception or return a regular value **b** (assumes suitable representation type for errors/exceptions, *e.g.*, `type Error = String`):

```
type Exc b = Either Error b
```

- In a composition, exceptions are propagated once occurred:

```
(>=>) :: (a -> Exc b) -> (b -> Exc c) -> (a -> Exc c)  
f >=> g = \x ->
```

## Sequencing Non-Deterministic Functions (**a** -> **NonDet** **b**)

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- A non-deterministic function may respond with *any answer* in a list of possible of answers:

```
type NonDet b = [b]
```

- In a composition of non-deterministic functions, treat all possible outcomes alike:

```
(>=>) :: (a -> NonDet b) -> (b -> NonDet c) -> (a -> NonDet c)
f >=> g = \x -> concat $ map g $ f x
```

or

```
f >=> g = \x -> concat [ g y | y <- f x ]
```

## Sequencing Stateful Functions ( $a \rightarrow \text{State} \rightarrow (\text{State}, b)$ )

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- Perform regular computation (to yield a value of type  $b$ ) **and** transform the current state of the system/program.
  - **State transformers** (assumes state representation  $\text{State}$ ):

```
type ST b = State -> (State, b)
```

- Stateful functions thus have type  $a \rightarrow \text{ST } b$ .
- Composition (with state transformation  $s_0 \mapsto s_1 \mapsto s_2$ ):

```
(>=>) :: (a -> ST b) -> (b -> ST c) -> (a -> ST c)
f >=> g = \x s0 -> let (s1, y) = f x s0 in
                  let (s2, z) = g y s1 in      } simplifies to
                  (s2, z)                       } g y s1
```

## Sequencing Side-Effecting Functions ( $a \rightarrow \text{World} \rightarrow (\text{World}, b)$ )

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- Modeling **side effects**: functions consume current “world” state, return new world state along with result:

```
type World = "\_(ツ)_/" -- abstract (defined by GHC runtime)
type IO b = World -> (World, b)
```

- Values of type  $\text{IO } b$  are **actions** that
  1. *when performed* have a side-effect on the world, and then
  2. return a value of type  $b$ .
- Haskell built-ins with side-effects:
  - `print :: Show a => a -> IO ()`, `putStrLn :: String -> IO ()`
  - `getLine :: IO String`
  - `readFile :: FilePath -> IO String`
  - `(>=>) :: (a -> IO b) -> (b -> IO c) -> (a -> IO c)`

## Type Class **Monad**

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- These sequencing functions follow an obvious pattern:

```
(>=>) :: (a -> b) -> (b -> c) -> (a -> c)
(>=>) :: (a -> Maybe b) -> (b -> Maybe c) -> (a -> Maybe c)
(>=>) :: (a -> Exc b) -> (b -> Exc c) -> (a -> Exc c)
(>=>) :: (a -> NonDet b) -> (b -> NonDet c) -> (a -> NonDet c)
(>=>) :: (a -> ST b) -> (b -> ST c) -> (a -> ST c)
(>=>) :: (a -> IO b) -> (b -> IO c) -> (a -> IO c)

(>=>) :: (a -> m b) -> (b -> m c) -> (a -> m c)
```

- To instantiate `(>=>)` for different type constructors `m`, build a type class whose instances are **type constructors** (i.e. `m` is of kind `* -> *`).



## Type Class **Monad** (Or: Why Haskell's Logo is $\gg=$ )

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- Type class **Monad** ( $m$  has kind  $* \rightarrow *$ ):

```
class Applicative m => Monad m where
  return :: a -> m a           -- lifting
  (>>=)  :: m a -> (a -> m b) -> m b  -- sequencing ("bind")
```

$$x \xrightarrow{\text{return}} \boxed{x} \qquad \boxed{x} \xrightarrow{>>= g} \boxed{y} \quad \Bigg\} \equiv g \ x \quad \left( \begin{array}{l} g \text{ creates} \\ y \text{ and new} \\ \text{structure} \end{array} \right)$$

- NB:** Once a value is inside the monadic  $m$ -structure, class **Monad** does not let it “escape” again.

## `(>>=)` vs. `(>=>)`

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- **Bind** `(>>=)` is simpler/does less than `(>=>)`:
  - `f >=> g` involves *two* applications (of `f` and `g`). Instead, `s >>= g` receives structure `s`, extracts its value `x`, and routes `x` to `g` (*one* application).
  - Nevertheless, both are closely related.  
`(>=>)` may be expressed in terms of `(>>=)`:

```

(>=>) :: Monad m => (a -> m b) -> (b -> m c) -> (a -> m c)
f >=> g = \x -> f x >>= g
                └─
                :: m b

```

- `(>=>)` is also known as **Kleisli composition**.

## Lifting via `return`

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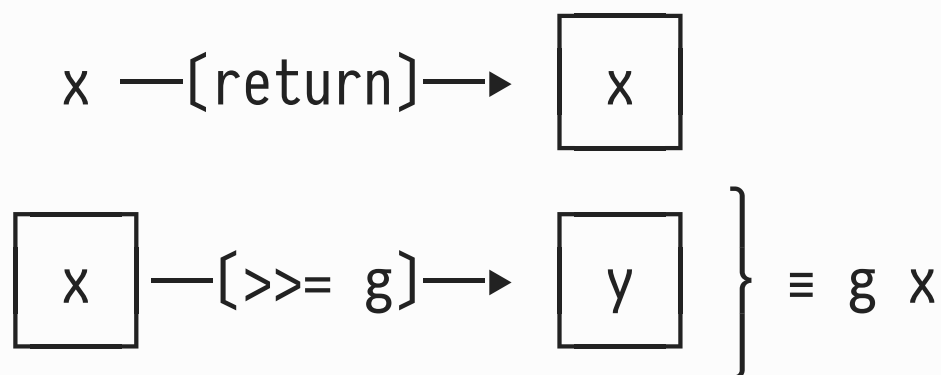
- **Lifting** creates a trivial context/structure (recall `pure`):

Monad	lifting ( <code>return x</code> )
Maybe	Just x
Exc	Right x
NonDet	[x]
ST	\s -> (s,x)

- **Example:** Make `Maybe` an instance of `Monad`:

```
instance Monad Maybe where
  return x = Just x
```

```
Nothing >>= g = Nothing
Just x   >>= g = g x
```



## The **Functor**/**Applicative**/**Monad** Tower

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- The type classes **Functor**, **Applicative**, and **Monad** are obviously closely related:

```
class Functor m      => Applicative m where ...
class Applicative m => Monad m        where ...
```

- Monad**, **Applicative**, **Functor** form a “tower of abstractions”:

<b>Monad</b> (return, (>>=))	
-----	
<b>Applicative</b> (pure, (<*>))	
-----	
<b>Functor</b> (fmap)	

expressiveness

▲ *high*

|

| *low*

- Monadic bind `>=>` can create entirely **new contexts/structure**:

$(\$)$  ::  $(a \rightarrow b) \rightarrow a \rightarrow b$   
 $(<\$>)$  :: **Functor**  $m \Rightarrow (a \rightarrow b) \rightarrow m a \rightarrow m b$   
 $(<*>)$  :: **Applicative**  $m \Rightarrow m (a \rightarrow b) \rightarrow m a \rightarrow m b$

$$(>>=) :: \text{Monad } m \Rightarrow m \ a \rightarrow \underbrace{(a \rightarrow m \ b)} \rightarrow m \ b$$

This function can generate  
a new context  $m$ !

## Expressiveness: **Monad** > **Applicative** > **Functor**

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- A **Monad**  $m$  can do whatever an **Applicative** can.

Proof: Express **Applicative**'s operations in terms of **Monad**'s:

```

pure x     $\stackrel{\text{def}}{=}$     return x

      :: (a -> b)      :: a      :: b
      |               |         |
u <*> v     $\stackrel{\text{def}}{=}$     u >>= \f -> v >>= \x -> return (f x)
      |               |         |
      :: m (a -> b)   :: m a      :: m b
  
```

- Given this and  $\text{fmap } f \ v = \text{pure } f \ <*> v$ ,  $m$  also is a **Functor**:

```

fmap f v     $\stackrel{\text{def}}{=}$     return f >>= \f -> v >>= \x -> return (f x)
  
```

## A Dose of Syntactic Sugar: **do**-Notation

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- Looong chains of  $e_1 \gg= \backslash x \rightarrow e_2$  are typical for monadic code, yet hard to read.
- *Imperative* programming languages have introduced established notation for programs that
  1. perform computation  $e_1$ , name the result  $x$ , and then
  2. perform computation  $e_2$  (which may refer to variable  $x$ ):

```
x := e1;  
e2;          /* e2 may use x */
```

- Haskell offers **syntactic sugar** that can mimic such sequential programs (but indeed is much more general): **do-notation**.

## do-Notation

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- Haskell's **do**-notation ( $e$ : expression of type  $m\ a$ ,  $es$ : sequence of **;**-separated expressions  $e_1; e_2; \dots; e_n$ ):

```
do { e }           ≡ e
do { x <- e; es }   ≡ e >>= \x -> do { es }
do { e; es }        ≡ e >>= \_ -> do { es }
do { let v = x; es } ≡ let v = x in do { es }  -- a pure let
```

- May use 2D layout instead of  $\{ \dots \}$  and **;**.
- ⚠ Syntactically mimics imperative statements, but still is an expression that computes a monadic value:  $\text{do } \{ \dots \} :: m\ b$ .



## Example ①: Interactively Copy a File (do-Notation)

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- Using `do`-notation leads to almost Python-like  syntax:

```
copyFile :: FilePath -> FilePath -> IO Int
```

```
copyFile from to = do  
  content <- readFile from  
  writeFile to content  
  return (length content)
```

```
main :: IO ()
```

```
main = do  
  putStrLn "Which file do you want to copy?"  
  from <- getLine  
  putStrLn "Where do you want to copy it to?"  
  to <- getLine  
  n <- copyFile from to  
  putStrLn ("Copied " ++ show n ++ " bytes.")
```

## Example ②: Interactively Copy a File (>>= Explicit)

---

```
copyFile :: FilePath -> FilePath -> IO Int
```

```
copyFile from to =
```

```
  readFile from          >>= \content ->
```

```
  writeFile to content >>= \_      ->      -- ← may use (>>)
```

```
  return (length content)
```

```
main :: IO ()
```

```
main =
```

```
  putStrLn "Which file do you want to copy?" >>= \_ ->
```

```
  getLine >>= \from ->
```

```
  putStrLn "Where do you want to copy it to?" >>= \_ ->
```

```
  getLine >>= \to ->
```

```
  copyFile from to >>= \n ->
```

```
  putStrLn ("Copied " ++ show n ++ " bytes.")
```

## ! Do Not Let **do** Fool You

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- This is **not** imperative programming in Haskell. Beware of actually thinking in , , **JS**, ... terms.
- **Example:** Is this correct Haskell? What is the output?

```
f :: IO Integer
f = do
  putStrLn "Running f ..."
  return 1893
  return 1904

main :: IO ()
main = do
  x <- f
  putStrLn (show x)
```

## Input/Output

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- But how can I/O possibly be pure? Reading a string with `getLine` may well yield different results on each execution.

### 💡 Key Idea:

Decouple the **pure construction** of I/O actions from their **effectful execution**.

1. Construct (complex) I/O action of type `IO a` using `getLine`, `putStrLn`, `>>=`, ... **No I/O or side effects happened yet.**
2. Perform constructed `IO a` action: **side effects will occur** and value of type `a` is returned. *Only* the `IO a` action returned by function `main` is performed by Haskell's runtime system.

## Roll Your Own I/O DSL

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- GHC's internal representation of IO actions is opaque. Likewise, the internals of the Haskell runtime—the interpreter for these IO actions—are hidden.
- To better see what is going on, build our own versions of both in terms of a simple **IO action DSL**:
  1. The *constructors* build IO actions (e.g., printing/reading strings). Supply a **Monad** instance for the constructors to use **do**-notation to conveniently sequence complex IO actions.
  2. The *observer* **runIO** translates the resulting IO action into a regular Haskell **IO a** action that can then be performed with side effects.

## Monad Laws

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- Haskell presently cannot enforce (or even check) this, but `return` and `>=>` (or `>=>`) are expected to behave “sanely”:

<code>return &gt;=&gt; g</code>	<code>≡</code>	<code>g</code>	(left identity)
<code>f &gt;=&gt; return</code>	<code>≡</code>	<code>f</code>	(right identity)
<code>(f &gt;=&gt; g) &gt;=&gt; h</code>	<code>≡</code>	<code>f &gt;=&gt; (g &gt;=&gt; h)</code>	(associativity)

- $\Rightarrow$  (`>=>`, `return`) forms a monoid over the carrier of functions `a -> m b`.

*“A monad is just a monoid in the category of endofunctors.  
What's the problem?”* — James Iry

- Category is *Hask* (objects: Haskell types, arrows: functions)
- `m` is the endofunctor: `m b` is a Haskell type again

## Merging Monads: Monad Transformers

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The monads we have seen so far are specialists in representing computations with *a single* type of side effect:

Monad	represents computation that...
<code>Maybe a</code>	may yield an <code>a</code> or return no result at all
<code>Either e a</code>	throws an exception <code>e</code> or succeeds with <code>a</code>
<code>NonDet a ([a])</code>	yields one of many possible <code>a</code>
<code>ST a</code>	reads/modifies current state, then returns <code>a</code>
<code>IO a</code>	performs input/output, then returns <code>a</code>
<code>Reader e a</code>	reads environment <code>e</code> , then returns <code>a</code>

- Can we build a monad to represent computations that perform I/O **and** may potentially throw an exception?
  - Do we need to start from scratch?
  - Or can we **combine the `Either` and `IO` monads** systematically?


## Either + IO: ① Verifying E-Mail Addresses

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- Start with a computation that “verifies” a given e-mail address (does it contain exactly one '@')?

```
data LoginError = InvalidEmail
  deriving Show

checkEmail :: String -> Either LoginError String
checkEmail email = case splitOn "@" email of
    [_name, _domain] -> Right email
    _                  -> Left InvalidEmail
```

- Case `Left e` represents an exception that subsequent computations need to handle ( `monad-trans-1.hs`).




## Either + IO: ② Adding I/O

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- Go interactive: ask for e-mail address, then verify it:

```
askForEmail :: IO (Either LoginError String)
askForEmail = do
  putStrLn "Enter e-mail address:"
  email <- getLine
  return (checkEmail email)
```

- As long as exception-generating computation and I/O do not interact, we are doing fine.
  - Here: perform all I/O, *then* verify e-mail (and we're done).
- If, instead, exception-generating computation and I/O mix, the code quickly gets nested and messier ( [monad-trans-2.hs](#)).

## **Either + IO:** ③ Build a Custom Combined Monad

---

We are operating in the **combined** (or: **stacked**) monad  
`IO (Either LoginError String)`. Generalized:

```
-- A computation that performs I/O and then either  
-- ❶ fails with an exception (of type e) or  
-- ❷ succeeds and returns with a result (of type a):
```

```
newtype EitherIO e a = EitherIO { runEitherIO :: IO (Either e a) }
```

- Recall that this `newtype` declaration defines two type conversions (at no runtime cost):

```
EitherIO      :: IO (Either e a) -> EitherIO e a  
runEitherIO :: EitherIO e a      -> IO (Either e a)
```

## **Either + IO:** ③ Build a Custom Combined Monad

---

- The essence (behavior) of the combined monad:

**instance Monad (EitherIO e) where**

trivial (= no) I/O, computation succeeds


return x = **EitherIO** \$ (return<sub>io</sub> . **Right**) x

---

$x \gg= f = \text{EitherIO } \$ \underbrace{\text{runEitherIO } x \gg=_{\text{io}} \text{either}}_{\substack{\text{perform I/O,} \\ \text{then yield } \text{Either } e \ a}} \underbrace{(\text{return}_{\text{io}} . \text{Left})}_{\substack{\text{1 computation failed,} \\ \text{trivial I/O}}} (\text{runEitherIO } . f) \underbrace{\text{}}_{\substack{\text{2 computation succeeded,} \\ \text{continue with } f}}$

## ④ Lifting Values Into the Combined `EitherIO` Monad

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-  To run existing code in the new `EitherIO` monad, we need to convert/lift `IO a` (and `Either e a`) values into `EitherIO e a` values. Otherwise, type errors ( $\Leftarrow$ ) occur:

```
askForEmail :: EitherIO LoginError String
askForEmail = do
  putStrLn "Enter e-mail address:"  $\Leftarrow^1$  :: IO ()
  email <- getLine                   $\Leftarrow^1$  :: IO String
  return (checkEmail email)          $\Leftarrow^2$  :: IO (Either Lo...r String)
```

- $\Leftarrow^1$ : Has type `IO a` but we need `EitherIO e a`.
- $\Leftarrow^2$ : Has type `IO (Either e a)` but we need `EitherIO e a`.

#### ④ Lifting Values Into the Combined `EitherIO` Monad

---

- Both **liftings** (`Either e a → EitherIO e a ← IO a`) are readily defined and help to write `EitherIO` computations concisely:

```
liftEither :: Either e a -> EitherIO e a
liftEither x = EitherIO (return x)
                        |
                        :: IO (Either e a)

liftIO :: IO a -> EitherIO e a
liftIO x = EitherIO (fmap Right x)
                |
                :: IO (Either e a) [recall: IO is a functor]
```

- That is all we need to rewrite our login/password application code ( `monad-trans-3.hs`).

## ⑤ Throwing and Catching Exceptions

---

- In `userLogin`, `liftEither (Left exc)` is used to raise (or “throw”) exception `exc`. Capturing that idiom:

```
throw :: e -> EitherIO e a
throw exc = liftEither (Left exc)
```

- As a counterpart to `throw`, “catch” exceptions thrown by a computation and invoke a **handler** to cope with the exception:

```
computation that may
  throw an exception
      |
catch :: EitherIO e a -> (e -> EitherIO e a) -> EitherIO e a
                        |
                        handler (receives
                                exception as input)
```

## ⑤ Throwing and Catching Exceptions

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- Exception handlers allow to model complex application behavior.

### Example:

❶ Attempt user login.

❷ Success?  $\begin{cases} \text{Yes, logged in.} \\ \text{First failure?} \begin{cases} \text{Yes, offer 2nd attempt. Goto ❶.} \\ \text{No, abort. Print error message.} \end{cases} \end{cases}$

- Implementation ( [monad-trans-4.hs](#)):

```
loginDialogue :: EitherIO LoginError String
```

```
loginDialogue = do
```

```
    email <- userLogin `catch` wrongPassword `catch` abortLogin  
    liftIO $ putStrLn ("Logged in as " ++ email)  
    return email
```

## ⑥ Generalizing From `IO` to Any Monad `m`

---

- Abstract from `IO` and combine `Either` with any monad `m`. We arrive at the monad transformer `EitherT e m a`:

```
-- Run a computation in monad m and then either  
-- ① fail with an exception (of type e) or  
-- ② succeed and return with a result (of type a):
```

```
newtype EitherT e m a = EitherT { runEitherT :: m (Either e a) }
```

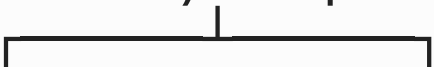
- See the Haskell `transformers` library for a collection of such monad transformers.
  - Example: An implementation of `EitherT` is found in `Control.Monad.Trans.ExceptT`.






## ⑥ Generalizing From **IO** to Any **Monad m**

**instance Monad m => Monad (EitherT e m) where**

trivial *m*-computation, computation succeeds

return x = **EitherT** \$  (return<sub>m</sub> . **Right**) x

**x >>= f = EitherT \$**  runEitherT x >>= <sub>m</sub> either  (return<sub>m</sub> . **Left**)  
(runEitherT . f) 

1 computation failed,  
trivial *m*-computation

2 computation succeeded,  
continue with f

- See  [monad-trans-5.hs](#).