

```
function y = f
y = @(x) exp(x)-1.5-atan(x);
```

ans =

function\_handle with value:

@(x)exp(x)-1.5-atan(x)

```
function yp = fp  
yp = @(x) exp(x)-1/(x^2+1);
```

ans =

function\_handle with value:

$\text{@}(x)\exp(x)-1/(x^2+1)$

```

function T = newton(f,fp,x0,tol,Nmax)
X = zeros(Nmax,1);
X(1) = x0-f(x0)/fp(x0);
for k = 2:Nmax
    X(k) = X(k-1)-f(X(k-1))/fp(X(k-1));
    if abs(f(X(k))) < tol
        t = X(1:k);
        n = size(t,1);
        Iteration = zeros(n,1);
        x = zeros(n,1);
        F = zeros(n,1);
        for i = 1:n
            Iteration(i) = i;
            x(i) = X(i);
            F(i) = abs(f(X(i)));
        end
        R = table(Iteration,x,F,'VariableNames',{'Iteration','x','|f(x)|'});
        T = table(R,'VariableNames',{'Results for all Iterations'});
        return
    end
end
T = 'N/A';
disp('Newton method exceeds maximum iterations')

```

```
x0 = -7;
tol = 10^-10;
Nmax = 30;
newton(f,fp,x0,tol,Nmax)
```

ans =

5x1 table

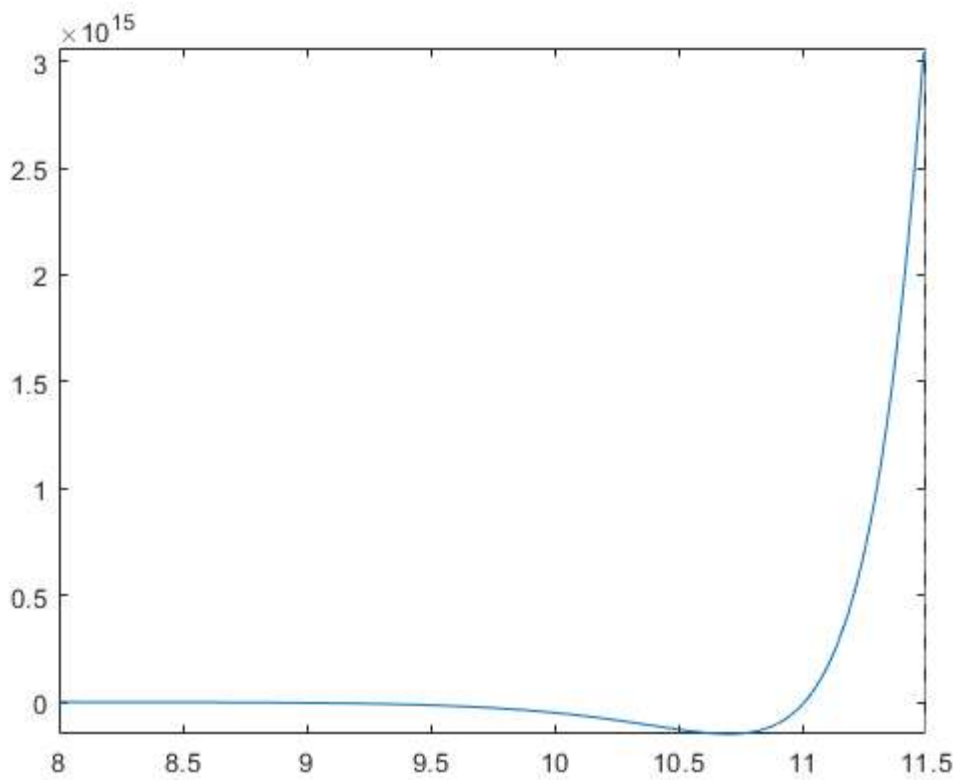
| Results for all Iterations |                       |                      |
|----------------------------|-----------------------|----------------------|
| Iteration                  | x                     | f(x)                 |
| 1.000000000000000e+00      | -1.06770961766400e+01 | 2.25666081098759e-02 |
| 2.000000000000000e+00      | -1.32791673756327e+01 | 4.36601933391256e-03 |
| 3.000000000000000e+00      | -1.40536558542692e+01 | 2.39019777052984e-04 |
| 4.000000000000000e+00      | -1.41011099568664e+01 | 7.99584812360976e-07 |
| 5.000000000000000e+00      | -1.41012697709394e+01 | 9.00834962180852e-12 |

```
function y = f
y = @(x) 3.^(3*x+1)-7*5.^(2*x);
fplot(y,[8,11.5])
```

ans =

function\_handle with value:

$@(x)3.^{(3*x+1)}-7*5.^{(2*x)}$



```
function yp = fp
yp = @(x) log(3)*3^(3*x+2)-14*log(5)*5^(2*x);
critical = fzero(yp,1)
```

```
critical =
```

```
1.070242694004809e+01
```

```
ans =
```

```
function_handle with value:
```

```
@(x)log(3)*3^(3*x+2)-14*log(5)*5^(2*x)
```

```

function T = newton(f,fp,x0,tol,Nmax)
X = zeros(Nmax,1);
X(1) = x0-f(x0)/fp(x0);
for k = 2:Nmax
    X(k) = X(k-1)-f(X(k-1))/fp(X(k-1));
    if abs((X(k)-X(k-1)))/abs(X(k-1)) < tol
        t = X(1:k);
        n = size(t,1);
        Iteration = zeros(n-1,1);
        x = zeros(n-1,1);
        F = zeros(n-1,1);
        for i = 1:n-1
            Iteration(i) = i;
            x(i) = X(i);
            F(i) = abs((X(i+1)-X(i)))/abs(X(i));
        end
        R = table(Iteration,x,F,'VariableNames',{'Iteration','x','Relative Error'});
        T = table(R,'VariableNames',{'Results for all Iterations'});
    end
end
T = 'N/A';
disp('Newton method exceeds maximum iterations')

```

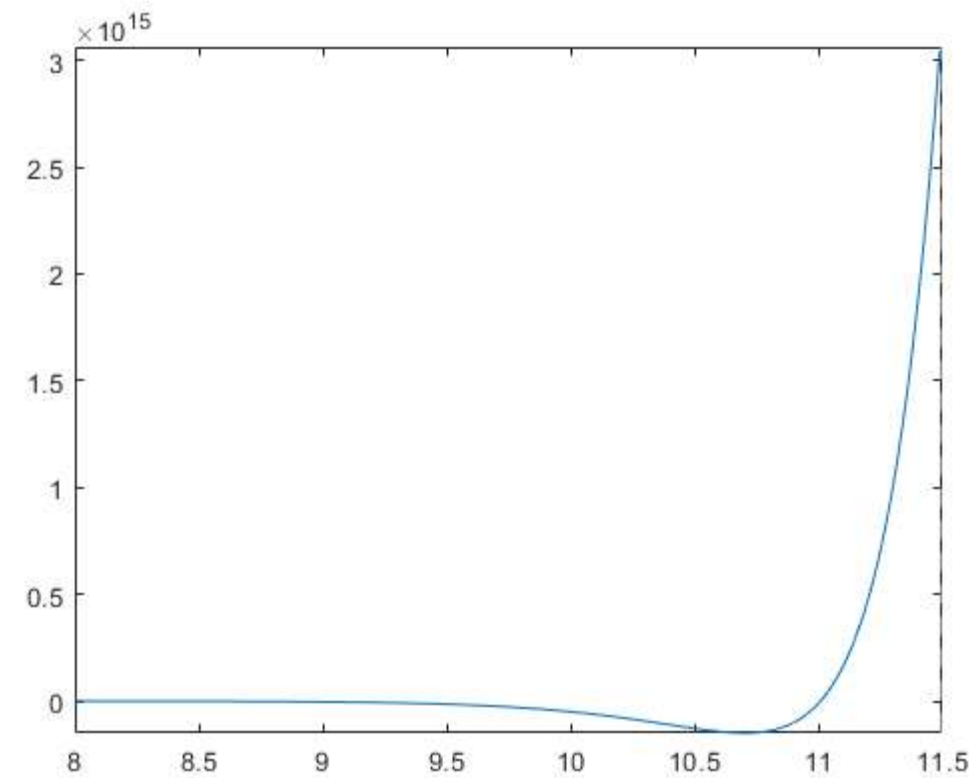
```
x0 = 11;
tol = 10^-10;
Nmax = 30;
newton(f,fp,x0,tol,Nmax)
```

critical =  
  
1.070242694004809e+01

ans =

3x1 table

| Results for all Iterations |                      |                      |
|----------------------------|----------------------|----------------------|
| Iteration                  | x                    | Relative Error       |
| 1.00000000000000e+00       | 1.10097380401553e+01 | 2.71672393864384e-05 |
| 2.00000000000000e+00       | 1.10094389359663e+01 | 2.64952589170358e-08 |
| 3.00000000000000e+00       | 1.10094386442684e+01 | 1.83937334349971e-14 |





```
function Y = FF
Y{1} = @(x,y) 7*x^3-10*x-y-1;
Y{2} = @(x,y) 8*y^3-11*y+x-1;
```

ans =

1×2 cell array

{@(x,y)7\*x^3-10\*x-y-1}    {@(x,y)8\*y^3-11\*y+x-1}

```
function A = JF
A{1,1} = @(x,y) 21*x^2-10;
A{1,2} = @(x,y) -1;
A{2,1} = @(x,y) 1;
A{2,2} = @(x,y) 24*y^2-11;
```

ans =

2x2 cell array

|                   |          |                   |
|-------------------|----------|-------------------|
| {@(x,y)21*x^2-10} | {        | @(x,y)-1}         |
| {                 | @(x,y)1} | {@(x,y)24*y^2-11} |

```

function S = NNewton(FF,JF,X0,Nmax,tol)
[~,m] = size(FF);
N = zeros(m,Nmax);
Jac = zeros(m,m);
B = zeros(m,1);
Init = FF;
L = JF;
for i = 1:m
    B(i) = -feval(Init{i},X0(1),X0(2));
    for k = 1:m
        Jac(i,k) = feval(L{i,k},X0(1),X0(2));
    end
end
X = Jac\B;
N(:,1) = X+X0;
for p = 2:Nmax
    for i = 1:m
        B(i) = -feval(Init{i},N(1,p-1),N(2,p-1));
        for k = 1:m
            Jac(i,k) = feval(L{i,k},N(1,p-1),N(2,p-1));
        end
    end
    N(:,p) = Jac\B+N(:,p-1);
    if norm(B,inf) < tol
        t = N(:,1:p);
        n = size(t,2);
        Iteration = zeros(n-1,1);
        x = zeros(n-1,1);
        y = zeros(n-1,1);
        F = zeros(n-1,1);
        for i = 1:n-1
            Iteration(i) = i;
            x(i) = N(1,i);
            y(i) = N(2,i);
            for r = 1:m
                B(r) = -feval(Init{r},N(1,i),N(2,i));
            end
            F(i) = norm(B,inf);
        end
        R = table(Iteration,x,y,F,'VariableNames',{'Iteration','x','y','||f(x,y)||'});
        S = table(R,'VariableNames',{'Results for all Iterations'});
        return
    end
end
S = 'N/A';
disp('Newton method exceeds maximum iterations')

```

```
X0 = [1,-2]';
tol = 10^-10;
Nmax = 30;
NNewton(FF,JF,X0,Nmax,tol)
```

ans =

6×1 table

| Iteration             | Results for all Iterations |                       | f(x,y)               |
|-----------------------|----------------------------|-----------------------|----------------------|
|                       | x                          | y                     |                      |
| 1.000000000000000e+00 | 1.22649572649573e+00       | -1.50854700854701e+00 | 1.06436604847738e+01 |
| 2.000000000000000e+00 | 1.18417792260737e+00       | -1.26355206353424e+00 | 2.05548142543646e+00 |
| 3.000000000000000e+00 | 1.18569965180473e+00       | -1.18836372934502e+00 | 1.68036807384419e-01 |
| 4.000000000000000e+00 | 1.18607182928602e+00       | -1.18103989121078e+00 | 1.52666942097612e-03 |
| 5.000000000000000e+00 | 1.18607512100179e+00       | -1.18097211482281e+00 | 1.30204003134793e-07 |
| 6.000000000000000e+00 | 1.18607512128382e+00       | -1.18097210904148e+00 | 2.66453525910038e-15 |

```
X0 = [2,2]';  
tol = 10^-10;  
Nmax = 30;  
NNewton(FF,JF,X0,Nmax,tol)
```

ans =

6×1 table

| Iteration             | Results for all Iterations |                      | f(x,y)               |
|-----------------------|----------------------------|----------------------|----------------------|
|                       | x                          | y                    |                      |
| 1.000000000000000e+00 | 1.54728977904944e+00       | 1.49944364965824e+00 | 1.10233778559015e+01 |
| 2.000000000000000e+00 | 1.34344401308963e+00       | 1.24759205209909e+00 | 2.15480722841390e+00 |
| 3.000000000000000e+00 | 1.29431470817365e+00       | 1.16769734481065e+00 | 1.87046191417671e-01 |
| 4.000000000000000e+00 | 1.29130690831615e+00       | 1.15922584245951e+00 | 2.00636934008402e-03 |
| 5.000000000000000e+00 | 1.29129333824564e+00       | 1.15913206944806e+00 | 2.44637659463365e-07 |
| 6.000000000000000e+00 | 1.29129333758699e+00       | 1.15913205796458e+00 | 3.10862446895044e-15 |

```
function Y = FF
Y{1} = @(x,y,z) sin(x)+x*exp(y-2)+x+y^2+cos(z)-4;
Y{2} = @(x,y,z) x^4+8*x*y+3*z+10*z*y-3*pi/2-10*pi;
Y{3} = @(x,y,z) cos(x^2+z^2)-y*exp(z)-cos((pi^2)/4)+2*exp(pi/2);
```

ans =

1×3 cell array

Column 1

{@(x,y,z)sin(x)+x\*exp(y-2)+x+y^2+cos(z)-4}

Column 2

{@(x,y,z)x^4+8\*x\*y+3\*z+10\*z\*y-3\*pi/2-10\*pi}

Column 3

{@(x,y,z)cos(x^2+z^2)-y\*exp(z)-cos((pi^2)/4)+2\*exp(pi/2)}

```

function A = JF
A{1,1} = @(x,y,z) cos(x)+exp(y-2)+1;
A{1,2} = @(x,y,z) x*exp(y-2)+2*y;
A{1,3} = @(x,y,z) -sin(z);
A{2,1} = @(x,y,z) 4*x^3+8*y;
A{2,2} = @(x,y,z) 8*x+10*z;
A{2,3} = @(x,y,z) 3+10*y;
A{3,1} = @(x,y,z) -2*x*sin(x^2+z^2);
A{3,2} = @(x,y,z) -exp(z);
A{3,3} = @(x,y,z) -2*z*sin(x^2+z^2)-y*exp(z);

```

ans =

3×3 cell array

Columns 1 through 2

|                                 |                                |
|---------------------------------|--------------------------------|
| {@(x,y,z)cos(x)+exp(y-2)+1}     | {@(x,y,z)x*exp(y-2)+2*y}       |
| {            @(x,y,z)4*x^3+8*y} | {            @(x,y,z)8*x+10*z} |
| {@(x,y,z)-2*x*sin(x^2+z^2)}     | {            @(x,y,z)-exp(z)}  |

Column 3

|   |
|---|
| {                                @(x,y,z)-sin(z)} |
| {                                @(x,y,z)3+10*y}  |
| {@(x,y,z)-2*z*sin(x^2+z^2)-y*exp(z)}              |

```

function S = NNewton(FF,JF,X0,Nmax,tol)
[~,m] = size(FF);
N = zeros(m,Nmax);
Jac = zeros(m,m);
B = zeros(m,1);
Init = FF;
L = JF;
for i = 1:m
    B(i) = -feval(Init{i},X0(1),X0(2),X0(3));
    for k = 1:m
        Jac(i,k) = feval(L{i,k},X0(1),X0(2),X0(3));
    end
end
X = Jac\B;
N(:,1) = X+X0;
for p = 2:Nmax
    for i = 1:m
        B(i) = -feval(Init{i},N(1,p-1),N(2,p-1),N(3,p-1));
        for k = 1:m
            Jac(i,k) = feval(L{i,k},N(1,p-1),N(2,p-1),N(3,p-1));
        end
    end
    N(:,p) = Jac\B+N(:,p-1);
    if norm(B,inf) < tol
        t = N(:,1:p);
        n = size(t,2);
        Iteration = zeros(n-1,1);
        x = zeros(n-1,1);
        y = zeros(n-1,1);
        z = zeros(n-1,1);
        F = zeros(n-1,1);
        for i = 1:n-1
            Iteration(i) = i;
            x(i) = N(1,i);
            y(i) = N(2,i);
            z(i) = N(3,i);
            for r = 1:m
                B(r) = -feval(Init{r},N(1,i),N(2,i),N(3,i));
            end
            F(i) = norm(B,inf);
        end
        R = table(Iteration,x,y,z,F,'VariableNames',{'Iteration','x','y','z','||f(x,y,z)||'});
        S = table(R,'VariableNames',{'Results for all Iterations'});
        return
    end
end
S = 'N/A';
disp('Newton method exceeds maximum iterations')

```



```
X0 = [1,2,-5]';  
tol = 10^-10;  
Nmax = 30;  
NNewton(FF,JF,X0,Nmax,tol)
```

Newton method exceeds maximum iterations

ans =

'N/A'

```
X0 = [0.1,1.9,1.5]';  
tol = 10^-10;  
Nmax = 30;  
NNewton(FF,JF,X0,Nmax,tol)
```

ans =

4x1 table

| Iteration            | Results for all Iterations |                      |                      |  | f(x,y,z)             |
|----------------------|----------------------------|----------------------|----------------------|--|----------------------|
|                      | x                          | y                    | z                    |  |                      |
| 1.00000000000000e+00 | -1.17585376743411e-02      | 2.00918204708973e+00 | 1.57192345359389e+00 |  | 5.73623114326729e-02 |
| 2.00000000000000e+00 | -8.48326577447198e-05      | 2.00005494005840e+00 | 1.57078528033005e+00 |  | 7.48438150644404e-04 |
| 3.00000000000000e+00 | -3.78931176463670e-09      | 2.00000000223725e+00 | 1.57079632601818e+00 |  | 4.33507665320576e-08 |
| 4.00000000000000e+00 | -4.33128410779509e-16      | 2.00000000000000e+00 | 1.57079632679490e+00 |  | 1.77635683940025e-15 |

2c.) The real solution is  $x = \frac{\log(7) - \log(3)}{3 \log(3) - 2 \log(5)}$ .

2d.) Plotting  $f$  shows a function with a horizontal asymptote at  $x=0$ , and a root at around 11.0094. We can choose any initial  $x_0$  greater than 10.70243 ( $f'=0$ ) to find the root of  $f$  with newton's method. Choosing any value lower than this will result in a negative answer where the function approaches zero due to the horizontal asymptote. In this case, the newton method will give increasingly negative values based on the Nmax and tolerance inputs.