# Module 2 Self Check

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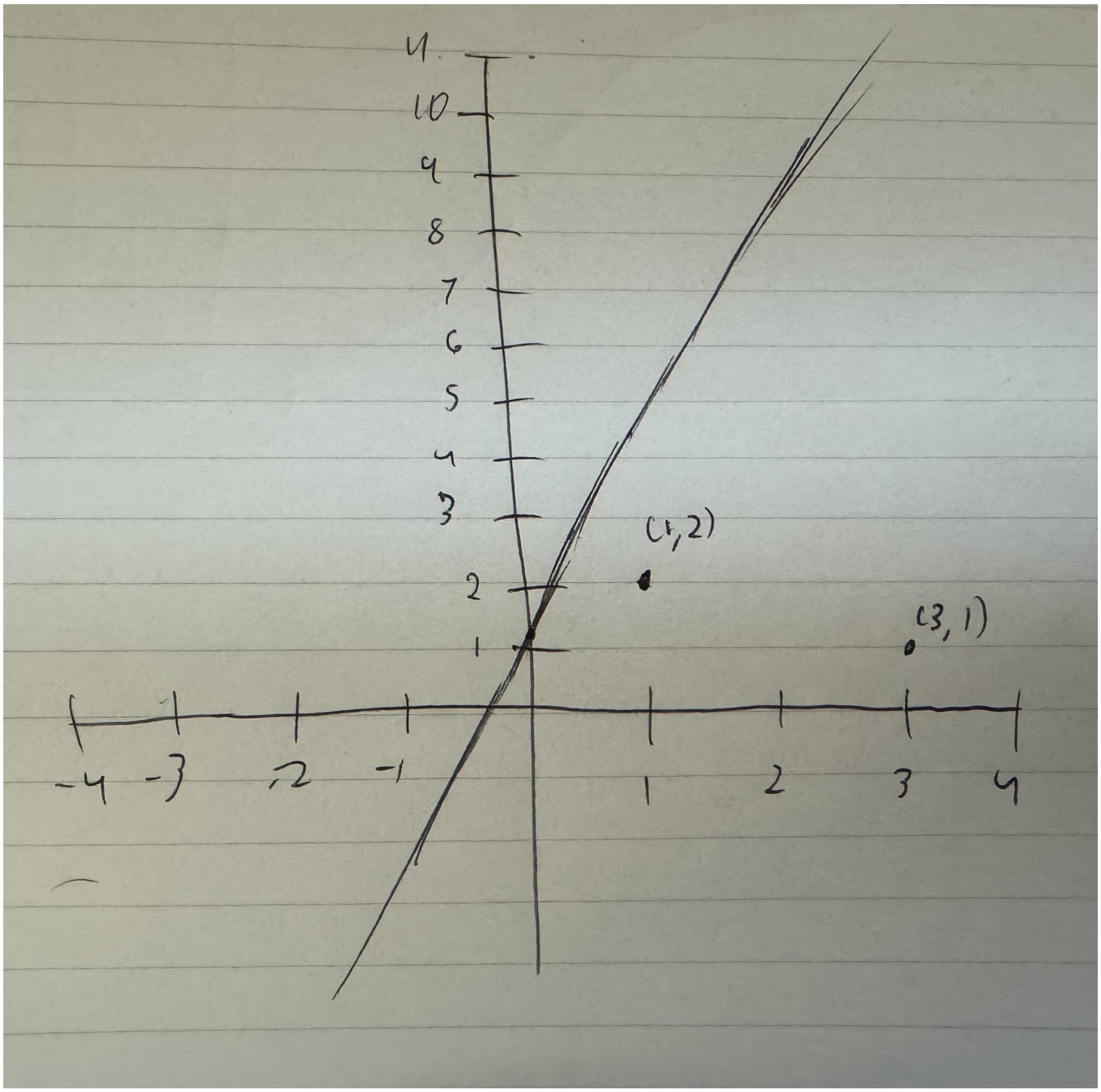
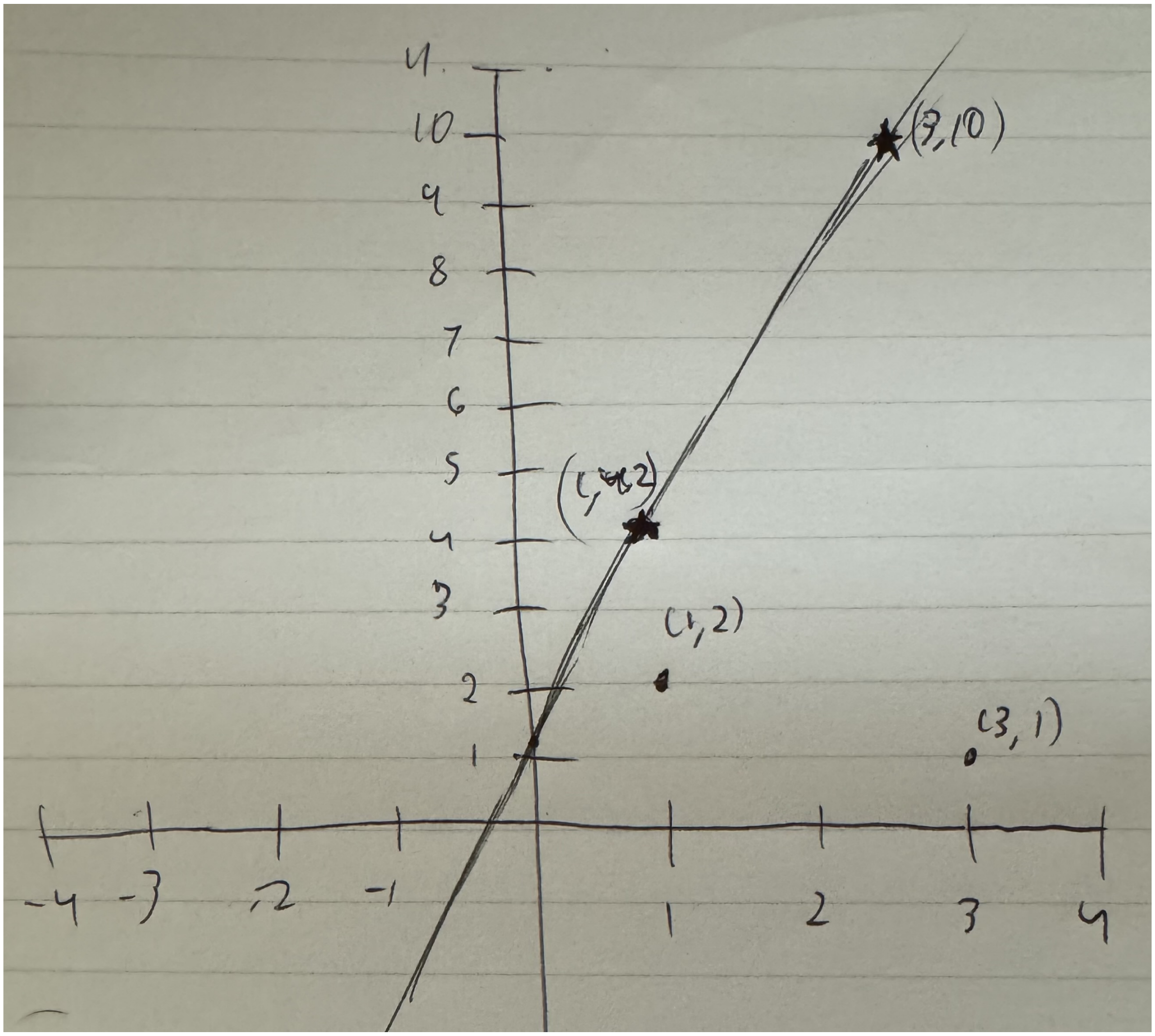
## 1 Linear Regression

1. 


line y = 1.3 + 2.9x with plotted points (1,2) and (3,1)

2. Calculate $\hat{y}$ for (1,2) and (3,1)
   $\hat{y} = 1.3 + 2.9(1.0) = 4.2$
   $\hat{y} = 1.3 + 2.9(3.0) = 10.0$

3. 


   line y = 1.3 + 2.9x with plotted points (1,2) and (3,1) and estimated points (1, 4.2) and (3, 10)

4. Calculate MSE

   MSE $= \frac{1}{2n}\sum_i (y_i - \hat{y}_i)^2$

   First Point: $(4.2 - 2.0)^2 = 4.84$
   Second point: $(10.0 - 1.0)^2 = 81.0$

   MSE $= \frac{1}{4}(4.84 + 81.0) = 21.46$

   (Utilized the error calculation in the Pseudocode documentation, slightly different than the MSE equation I had learned in previous classes but we have been told to ignore what we have learned previously)

5. Calculate One Adjustment

   We already know that $\theta_0$ is 1.3 and $\theta_1$ is 2.9. We also know that the MSE is 21.46.

   Utilizing this equation:

   $\frac{\partial J}{\partial \theta_j} = \frac{1}{n}\sum_i (\hat{y}_i - y_i)x_{ij}$

   we can calculate the derivatives:

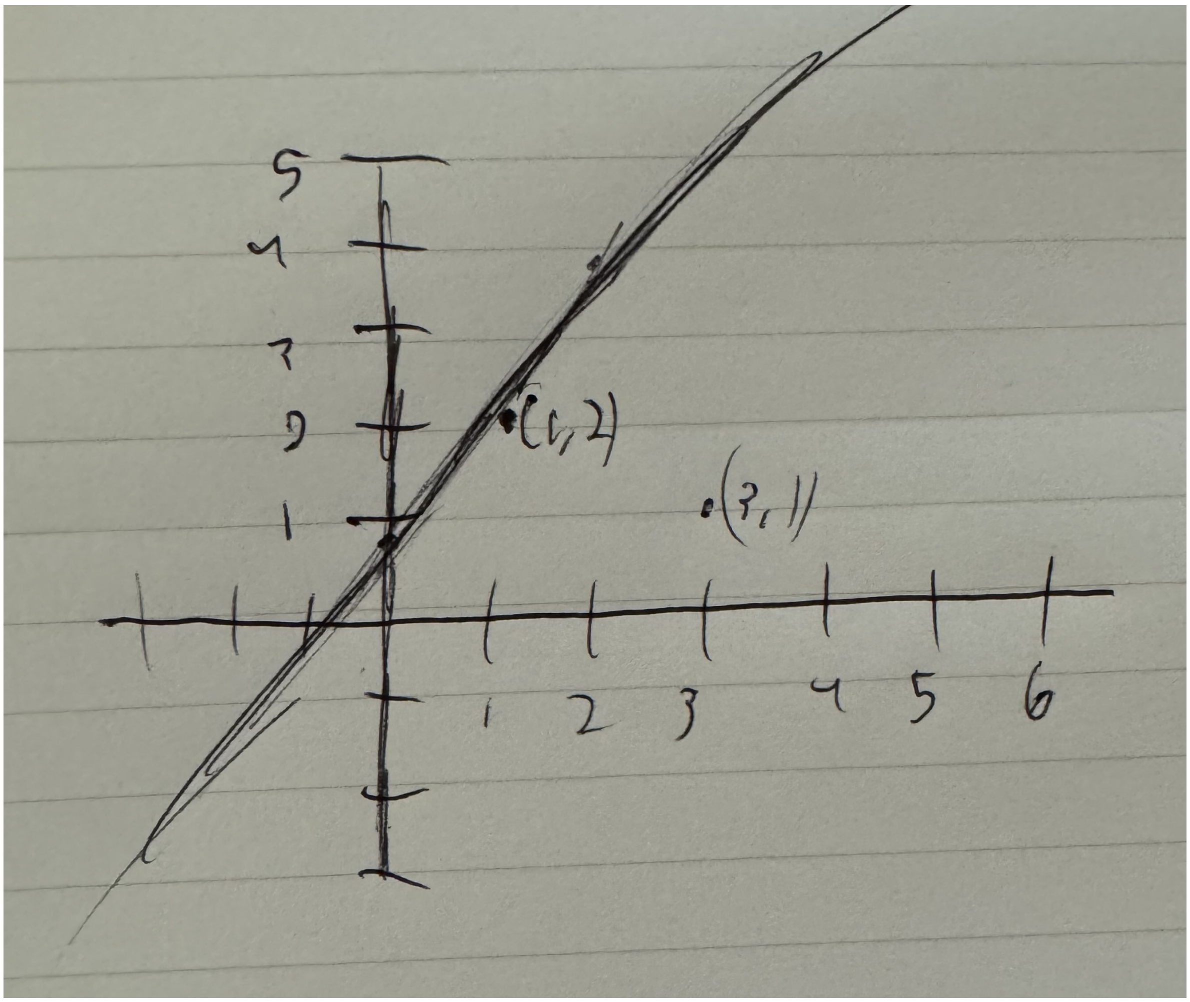
$\frac{\partial J}{\partial \theta_0} = \frac{1}{2}\left[(\hat{y}_1 - y_1)\cdot 1 + (\hat{y}_2 - y_2)\cdot 1\right]$
$= \frac{1}{2}\left[(4.2-2)\cdot 1 + (10-1)\cdot 1\right]$
$= \frac{1}{2}\left[2.2 + 9.0\right] = \frac{1}{2} \times 11.2 = 5.6$

$\frac{\partial J}{\partial \theta_1} = \frac{1}{2}\left[(\hat{y}_1 - y_1)\cdot x_1 + (\hat{y}_2 - y_2)\cdot x_2\right]$
$= \frac{1}{2}\left[(4.2-2)\cdot 1 + (10-1)\cdot 3\right]$
$= \frac{1}{2}\left[2.2 + 27.0\right] = \frac{1}{2} \times 29.2 = 14.6$

Finally We can Update the Thetas According to the Pseudocode (new_thetas[ j] = thetas[ j] – alpha * derivative( j, thetas, data)) :

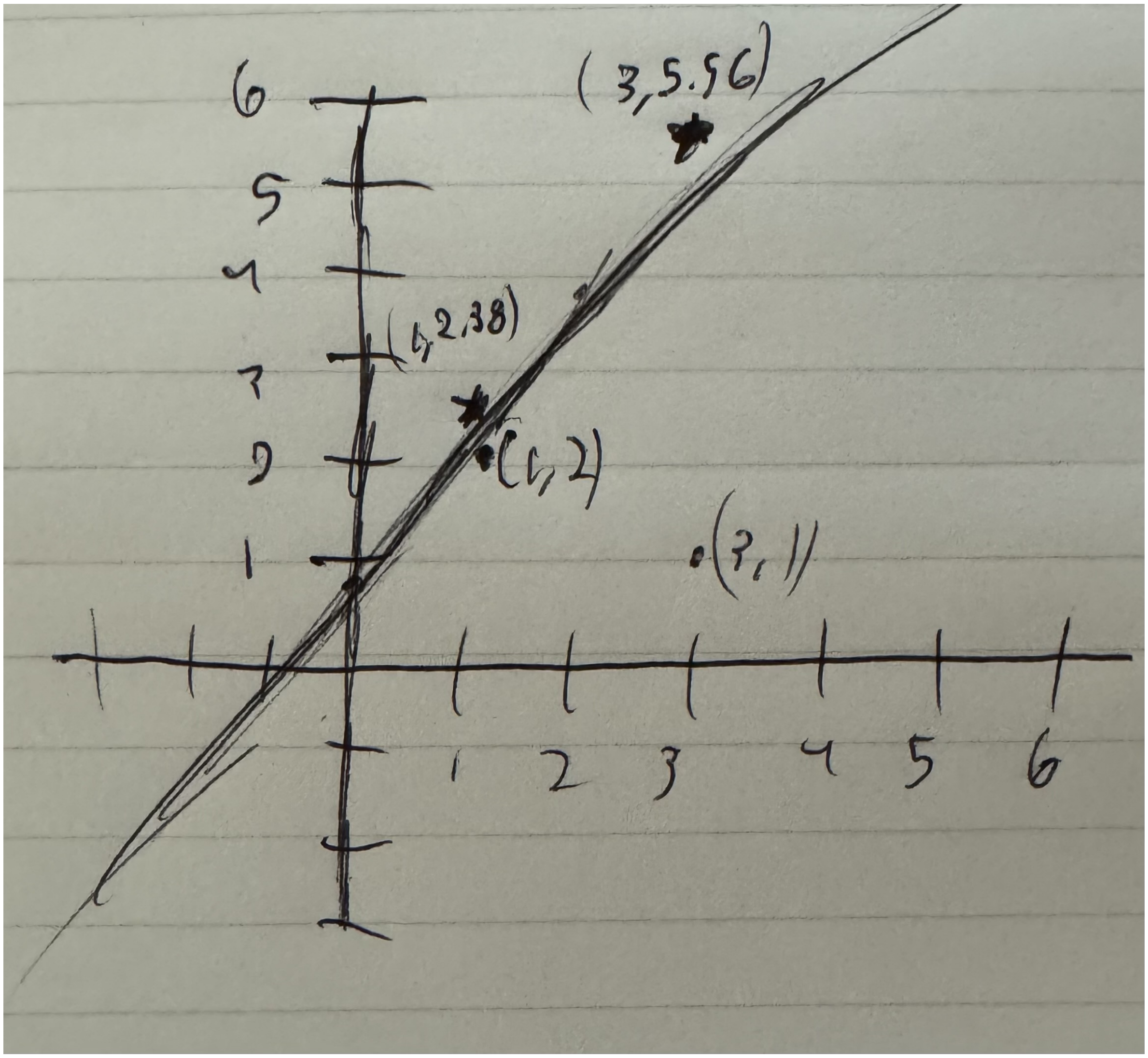
$\theta_0^{\text{new}} = \theta_0^{\text{old}} - \alpha \times \frac{\partial J}{\partial \theta_0}$
$= 1.3 - 0.1 \times 5.6 = 1.3 - 0.56 = 0.74 = \theta_0$

$\theta_1^{\text{new}} = \theta_1^{\text{old}} - \alpha \times \frac{\partial J}{\partial \theta_1}$
$= 2.9 - 0.1 \times 14.6 = 2.9 - 1.46 = 1.44 = \theta_1$

6. 


   line y = 0.74 + 1.44x with plotted points (1,2) and (3,1) and estimated points (1, 4.2) and (3, 9)

7. Calculate $\hat{y}$ for (1,2) and (3,1)
   $\hat{y} = 0.79 + 1.59(1.0) = 2.38$
   $\hat{y} = 0.79 + 1.59(3.0) = 5.56$

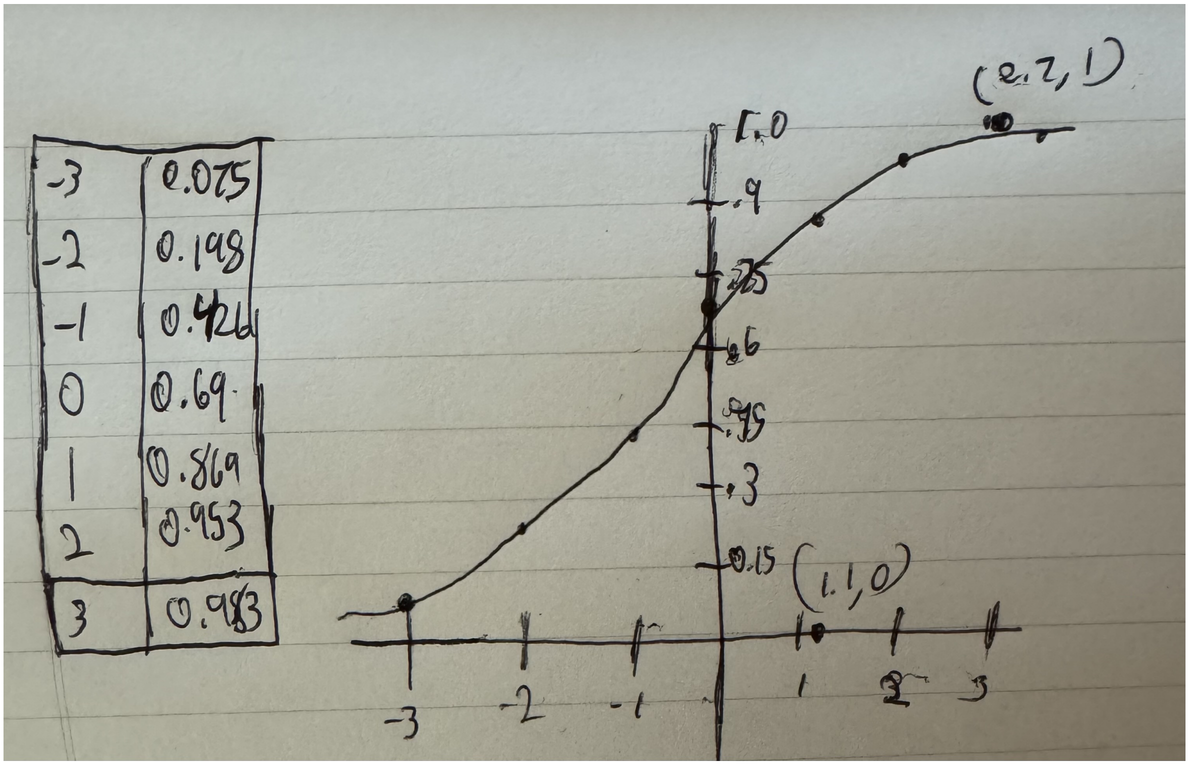


line y = 0.79 + 1.59x with plotted points (1,2) and (3,1) and estimated points (1, 2.38) and (3, 5.56)

Note: I don't own anything that would allow for different colors, I utilized stars for the new points

# 2 Logistic Regression

1. Draw Graph




2. Calculate Error Let's set a new variable z as $z = \theta_0 + \theta_1 * x$.

   For x = 1.1 we get $z = 0.8 + 1.1 * 1.1 = 2.01$ so $\hat{y}$
   $\hat{y} = \frac{1}{1+e^{-2.01}} \approx 0.881$

   For x = 2.7 we get $z = 0.8 + 1.1 * 2.7 = 3.77$ so $\hat{y}$
   $\hat{y} = \frac{1}{1+e^{-3.77}} \approx 0.976$

   With that we have the information for the log loss of each datapoint:

   Log-Loss $= -[- * \log(0.881) + (1-0) * \log(1-0.881)] = 2.13$

   Log-Loss $= -[- * \log(0.976) + (1-1) * \log(1-0.976)] = 0.024$

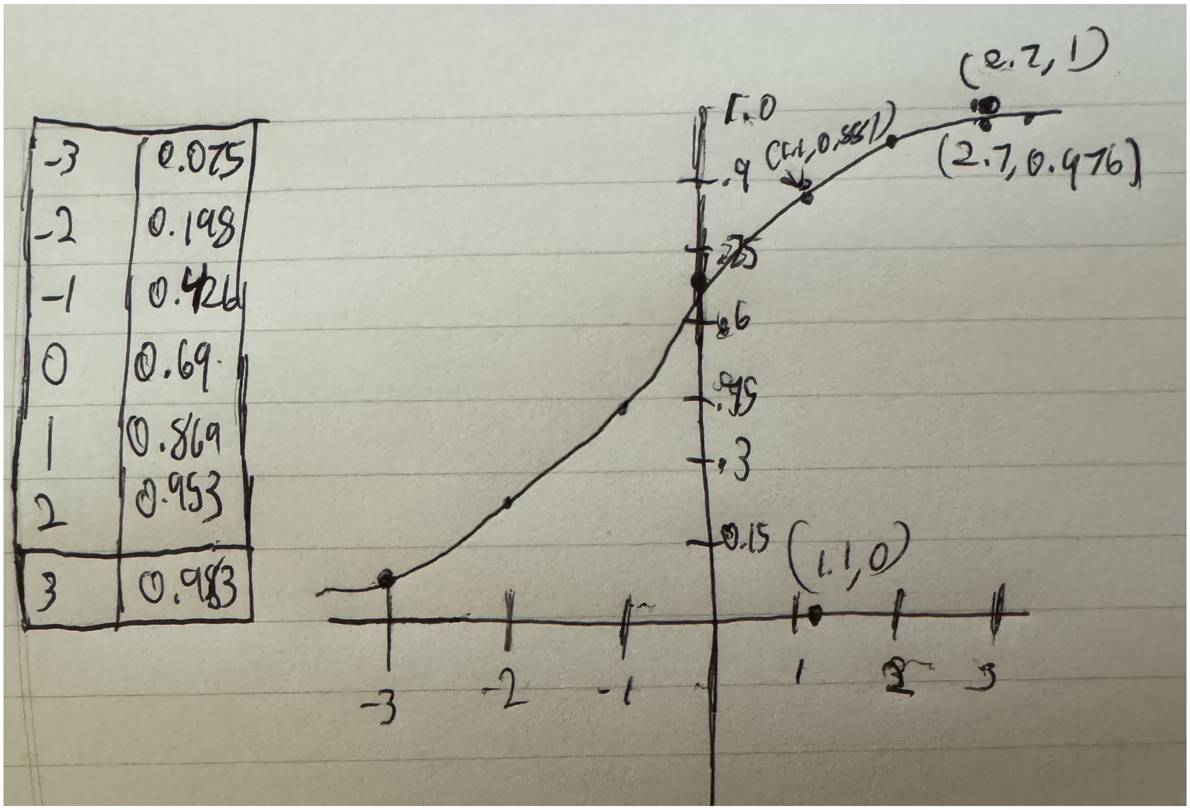
   We can then average them together to get the log loss of the dataset:

   $\frac{2.13+0.024}{2} \approx 1.077$

3. Plot points the estimated points

   For $x_1 = 1.1$ :
   $z = \theta_0 + \theta_1 \cdot x_1 = 0.8 + 1.1 \cdot 1.1 = 0.8 + 1.21 = 2.01$
   $\hat{y} = \frac{1}{1+e^{-2.01}} = \frac{1}{1+e^{2.01}} \approx \frac{1}{1+7.47} \approx 0.881$

   For $x_1 = 2.7$ :
   $z = \theta_0 + \theta_1 \cdot x_1 = 0.8 + 1.1 \cdot 2.7 = 0.8 + 2.97 = 3.77$
   $\hat{y} = \frac{1}{1+e^{-3.77}} = \frac{1}{1+e^{3.77}} \approx \frac{1}{1+43.43} \approx 0.976$




4. Calculate one adjustment

   We already know that $\theta_0$ is 0.8 and $\theta_1$ is 1.1.

   Utilizing this equation:

   $\frac{\partial J}{\partial \theta_j} = \frac{1}{n}\sum_i(\hat{y}_i - y_i)x_{ij}$

   we can calculate the derivatives:

   $\frac{\partial J}{\partial \theta_0} = \frac{1}{2}\left[(\hat{y}_1 - y_1)\cdot 1 + (\hat{y}_2 - y_2)\cdot 1\right]$
   $= \frac{1}{2}\left[(0.881 - 0)\cdot 1 + (0.976 - 1)\cdot 1\right]$
   $= \frac{1}{2}\left[0.881 + (-0.024)\right]$
   $= \frac{1}{2} \times 0.857 = 0.4285$

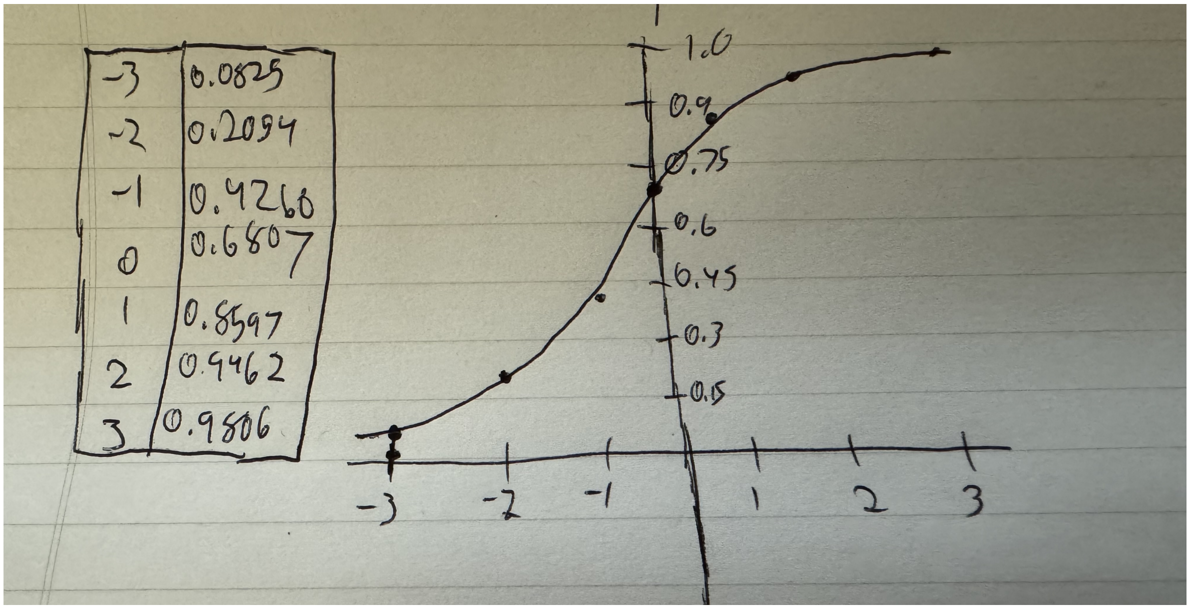
   $\frac{\partial J}{\partial \theta_1} = \frac{1}{2}\left[(\hat{y}_1 - y_1)\cdot x_1 + (\hat{y}_2 - y_2)\cdot x_2\right]$
   $= \frac{1}{2}\left[(0.881 - 0)\cdot 1.1 + (0.976 - 1)\cdot 2.7\right]$
   $= \frac{1}{2}\left[0.9691 + (-0.0648)\right]$
   $= \frac{1}{2} \times 0.9043 = 0.45215$

   Finally, we can update the thetas according to the pseudocode (new_thetas[ j] = thetas[ j] − alpha × derivative( j, thetas, data)):

   $\theta_0^{\text{new}} = \theta_0^{\text{old}} - \alpha \times \frac{\partial J}{\partial \theta_0}$
   $= 0.8 - 0.1 \times 0.4285$
   $= 0.8 - 0.04285 \approx 0.757 = \theta_0$ (True Value was 0.75715 but for the next part I wanted to actually be able to draw points)

   $\theta_1^{\text{new}} = \theta_1^{\text{old}} - \alpha \times \frac{\partial J}{\partial \theta_1}$
   $= 1.1 - 0.1 \times 0.45215$
   $= 1.1 - 0.045215 \approx= 1.055 = \theta_1$ (True Value was 1.054785 but for the next part I wanted to actually be able to draw points)

5. Draw new graph




6. Add new points

   For $x_1 = 1.1:$
   $z = \theta_0 + \theta_1 \cdot x_1 = 0.757 + 1.055 \cdot 1.1 = 0.757 + 1.1605 = 1.9175$
   $\hat{y} = \frac{1}{1+e^{-1.9175}} = \frac{1}{1+e^{1.9175}} \approx \frac{1}{1+6.803} \approx 0.8548$

   For $x_1 = 2.7:$
   $z = \theta_0 + \theta_1 \cdot x_1 = 0.757 + 1.055 \cdot 2.7 = 0.757 + 2.8485 = 3.6055$
   $\hat{y} = \frac{1}{1+e^{-3.6055}} = \frac{1}{1+e^{3.6055}} \approx \frac{1}{1+36.845} \approx 0.9737$