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ECON 2316

Lecture 17
Game Theory

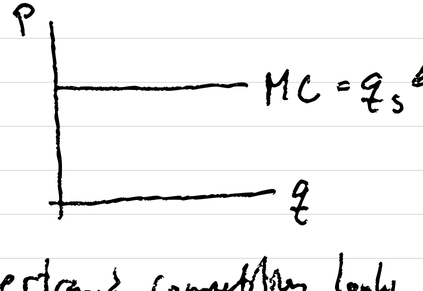
Bertrand Duopoly
- compete in prices

Example:
$P = 30 - Q$, $Q = q_1 + q_2$

So $P = 30 - q_1 - q_2$
$MC_1 = MC_2 = 4$
↳ $P = 4$, $Q = 26$
So (maybe) $q_1 = q_2 = 13$

$P = 4$ b/c each firm has no incentive to price below MC, but will keep undercutting the other

Suppose it's a perfect competition
↳ Draw one firm's supply curve
$MC = 4$



also market price b/c we add supply horizontally

Bertrand competition looks like perfect competition
- firms don't want Bertrand comp if products are homogeneous

To solve Bertrand models, just think of perfect competition

Positive profits are possible
- when products are not perfect substitutes
  - Similar to monopolistic competition but with barriers
- or Cournot competition

Bertrand examples - # consumers can be easily changed
- Breakfast cereals - choose prices

Cournot Competition
ex. airlines,
- choose q then stick w/ it

Numerical example
$P = 30 - Q$, $Q = q_1 + q_2$
$= 30 - q_1 - q_2$
$TC_1 = TC_2 = 0$

a) Bertrand P & Q?
$MC = 0, \Rightarrow \boxed{P = 0,\ Q = 30}$ $\Big\}$ $\pi = 0$ since $P = 0$
↳ in Bertrand, $P = MC$

b) Monopoly P & Q?
$Q = Q$, $P = 30 - Q$

$TR = (30 - Q)Q = 30Q - Q^2 \Rightarrow MR = 30 - 2Q \overset{\text{set}}{=} 0$
$= MC$

$\boxed{Q = 15,\ P = 15} \Rightarrow \pi = PQ - TC = 225$

c) Cournot Competition

$P = 30 - Q = 30 - q_1 - q_2$

$TR_1 = Pq_1 = (30 - q_1 - q_2) q_1 = 30q_1 - q_1^2 - q_1 q_2$

$MR_1 = \frac{\partial TR_1}{\partial q_1} = 30 - 2q_1 - q_2$

Set $MR_1 = MC_1 = 0$

$30 - 2q_1 - q_2 = 0$

$\Rightarrow 30 - q_2 = 2q_1 \Rightarrow q_1 = 15 - \frac{q_2}{2}$

firm 1's reaction function

Firms are identical, so

$q_2 = 15 - \frac{q_1}{2}$

Nash Equilibrium is when both equations hold

$q_1 = 15 - \frac{q_2}{2}, \quad q_2 = 15 - \frac{q_1}{2}$ } solve system

Here, because firms are identical, it must be that

$q_1 = q_2$, so $q_1 = q_2 = 10$

$= q^*$

@ equilibrium

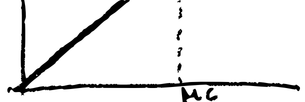
$\Rightarrow \boxed{Q = 20 \quad P = 10}$

π for each firm:

$\pi_1 = Pq_1 - TC_1 = 10(10) = 100$

$\pi_2 = \pi_1 = 100$

at market



Bertrand (identical goods)

Bertrand (differentiated products)

Stackelberg Competition

- firms compete in quantity, but move sequentially
- Solved using backwards induction
  - first mover picks q knowing other firm's reaction function

Example: Assume firm 1 moves first

$q_2 = 15 - \frac{q_1}{2}$ ← firm 2's reaction function

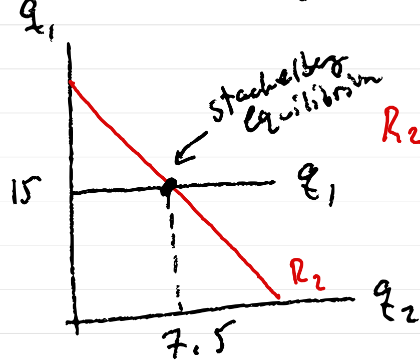
$$\rightarrow \pi_1 = Pq_1 - TC = (30 - q_1 - q_2)(q_1)$$

$$TR = \left(30 - q_1 - \left(15 - \frac{q_1}{2}\right)\right)(q_1)$$

$$= 15q_1 - \frac{q_1^2}{2}$$

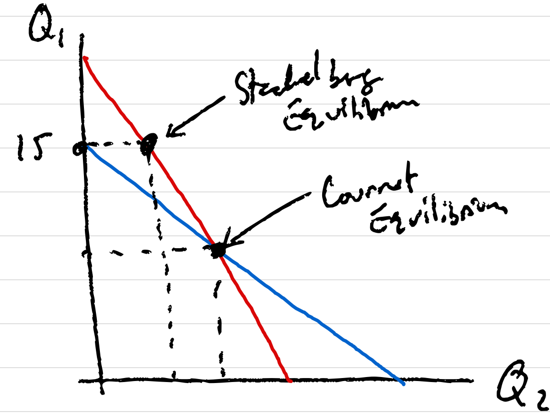
$$MR_1 = 15 - q_1 = MC_1 = 0$$

$q_1 = 15$, q_2 reacts and picks 7.5



$R_2 = q_2 = 15 - \frac{q_1}{2}$

First Mover Advantage in Stackelberg



Example:

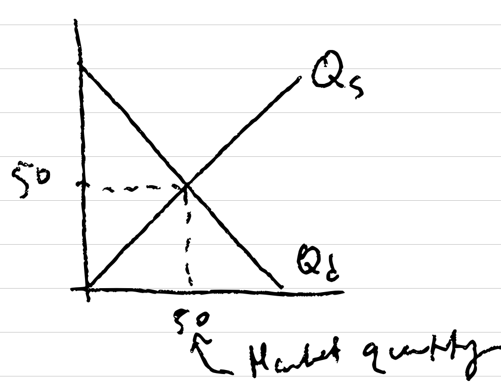
$$Q^D = 100 - P, \quad TC = 10 + q^2 \Rightarrow MC = 2q$$

a) Bertrand Competition

1 firm's supply $= q_s = \frac{P}{2}$ (from $P = MC$)

↳ 2 firms $\Rightarrow Q^S = 2q_s = P$

$Q^s = Q^d \Rightarrow P = 100 - P \Rightarrow \boxed{P = 50}$ ← Bertrand Price



b) $Q_d = 100 - P$

Find reaction functions
Each q & P

$$-P = Q_d - 100$$
$$P = 100 - Q_d$$

$$P = 100 - q_1 - q_2$$

$$TR_1 = (100 - q_1 - q_2)$$