6.4 Type I and Type II Errors

Because sample data are used to decide to reject a null hypothesis or not, it is possible that an incorrect decision can be made.

In reality, the null hypothesis may actually be true or may not actually be true. Regardless of actual truth, a decision to reject the null hypothesis or not is made. Thus, there are two possibilities for a correct decision and two for an incorrect decision:

H_0 is true but we reject it

		True State of Nature	
		The null hypothesis is true	The null hypothesis is false
Decision	We decide to reject the null hypothesis	**Type I error** (rejecting a true null hypothesis) *P*(type I error) = α	Correct decision $1 - \beta$
	We fail to reject the null hypothesis	Correct decision $1 - \alpha$	**Type II error** (failing to reject a false null hypothesis) *P*(type II error) = β

- We reject the null hypothesis when it is true. This is an incorrect decision and results in a **Type I error**.
- We reject the null hypothesis when it is false. This is a correct decision.
- We do not reject the null hypothesis when it is true. This is a correct decision.
- We do not reject the null hypothesis when it is false. This is an incorrect decision and results in a **Type II error**.

we reject H_0 when Prob of getting z^* is lower than our P of getting a type I error

■ Probability

- **α = P(Type I Error) = P(Reject H_0 | H_0 is true) = "Level of significance" of the test.**
- **Power of the test = $1 - \beta$**
- **Confidence level = $1 - \alpha$**
- **β = P(Type II Error) = P(Accept H_0 | H_0 is false)**

Type I → we reject H_0 but it is true

Type II → we accept H_0 but it is false

Truth

Start
- H_0 is true
  - α Reject H_0 → error → Type I
  - $1-\alpha$ Accept H_0 → correct
- H_0 is false
  - $1-\beta$ Reject H_0 → correct
  - β Accept H_0 → error → Type II

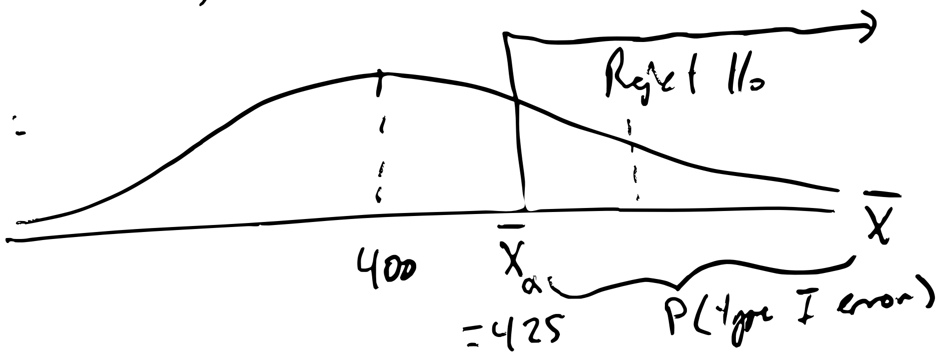
6.4 Type I and Type II Errors

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

■ **Example 1:** A company wants to see if an ad campaign will increase sales of a product. They will assume σ =100 per day. If they use $H_0: \mu = 400$ and they will reject this hypothesis if a sample of 50 days finds an average of more than 425 (so reject H_0 if $\bar{X} > 425$), then find P(type I error).

$= \alpha$

$H_0: \mu = 400, \; H_a: \mu > 400$



425 is critical value in nonstandard form

0.0385

P(type I error) = α = P(reject H_0 | H_0 is true)

= P(reject H_0 | $\mu = 400$)

= P($\bar{X} > 425$ | $\mu = 400$) = normalcdf(425, 10^{99}, 400, $\frac{\sigma}{\sqrt{n}}$)

■ **Example 2:** If H_0: $\mu = 30$ is tested against H_1: $\mu > 30$ using $n = 16$ observations (normally distributed) and if $1 - \beta = 0.85$ when $\mu = 34$, what does α equal? Assume that σ = 9.

$H_0: \mu = 30$ vs $H_a: \mu > 30$

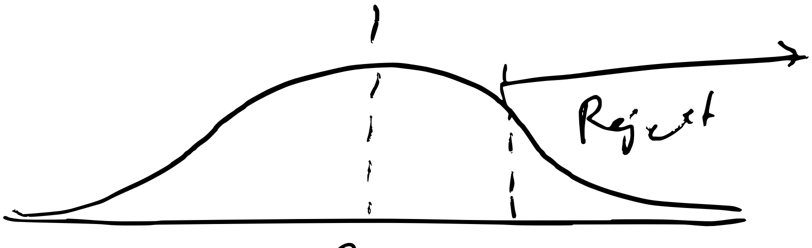
$n = 16, \; \sigma = 9$

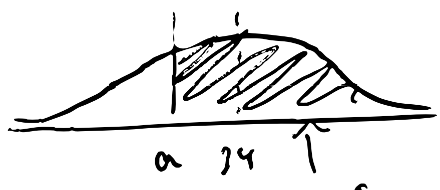
$1 - \beta = 0.85 \Rightarrow \beta = 0.15$ when $\mu = 34$

$1 - \beta$

= P(reject H_0 | H_a is true)

= P($\bar{x} > a$ | $\mu = 34$)





a = invNorm(0.85, 34, $\frac{9}{\sqrt{16}}$, RIGHT)

$= 31.668$

α = normalcdf(31.668, 10^{99}, 30, $\frac{9}{\sqrt{16}}$)

$= 0.2292$

6.4 Type I and Type II Errors

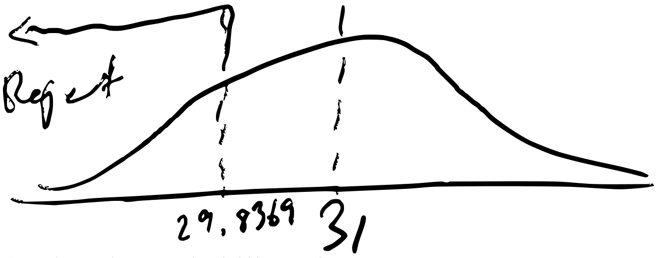
■ **Example 3:** Boston wants to see if the mean age in the city has decreased. They will assume $\sigma = 10$ and use a sample of 200 to test at the 5% level of significance. Using Ho: $\mu = 31$ and Ha: $\mu < 31$:

a) Find the critical values and a rejection test.

$n = 200$ $\quad H_0: \mu = 31$ vs. $H_a: \mu < 31$

$\alpha = 0.05$

$X_\alpha = \text{invNorm}(0.05, 31, \frac{10}{\sqrt{200}}, \text{LEFT}) = 29.8369$



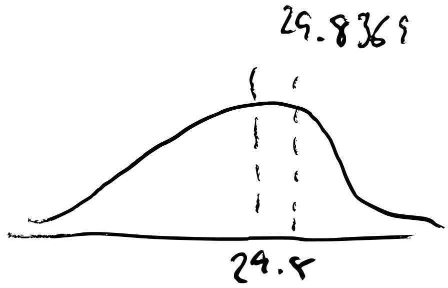
Reject if $\bar{X} < 29.8369$

Accept if $\bar{X} \geq 29.8369$

b) Give the probability of a type I error.

$P(\text{type I error}) = \alpha = 0.05$

c) Find the probability of a type II error if the real mean is 29.5.



$P(\text{type II error}) = P(\text{accept } H_0 \mid H_0 \text{ is false})$

$= P(\bar{X} \geq 29.8369 \mid \mu = 29.5)$

$\boxed{= 0.315}$

If $\mu = 29.5$, there is a 31.5% chance we will incorrectly accept H_0 with our current rejection test

d) What is the power of the test if the real mean is 29.5? H_a is true

$\text{Power} = P(\text{reject } H_0 \mid H_0 \text{ is false})$

$= P(\bar{X} < 29.84 \mid \mu = 29.5)$

$\boxed{= 0.685}$

6.4 Type I and II Errors

Errors in Hypothesis Tests

- Type I error:

  The symbol α is used to represent the probability of a type I error.

- Type II error:

  The symbol β is used to represent the probability of a type II error.

Example 1: The average energy intake for 2-year-old children is believed to be 1286 kcal. A study will look at a random sample of 94 to see if a particular subgroup is less. If the standard deviation is 256 and they will test at the 5% level of significance.

(a) Give your null and alternate hypotheses, find the critical values, and give a rejection test.

$H_0: \mu = 1286$, $H_a: \mu < 1286$

$X^* = \text{invNorm}(0.05, 1286, \frac{256}{\sqrt{94}}, \text{LEFT})$

$= 1242.57$

Reject if $\bar{X} < 1242.57$

(b) What is the probability of a type I error?

$\alpha = 0.05 = P(\text{type I error}) = P(\text{rejecting } H_0 \mid H_0 \text{ is true})$

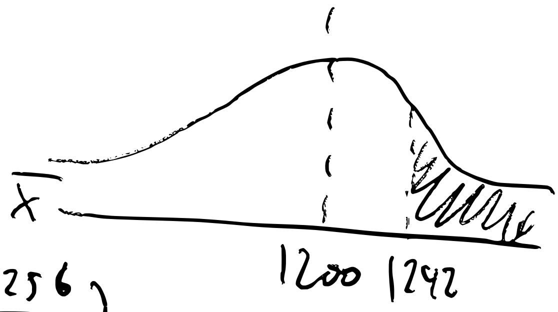
(c) Find the probability of a type II error if the real mean is 1200.

$\beta = P(\text{accept } H_0 \mid H_a \text{ is true})$

$= P(\bar{X} \geq 1242.57 \mid \mu = 1200)$

$= \text{normalcdf}(1242.57, 10^{99}, 1200, \frac{256}{\sqrt{94}})$

$= 0.053$



(d) Find the power of the test if the real mean is 1200.

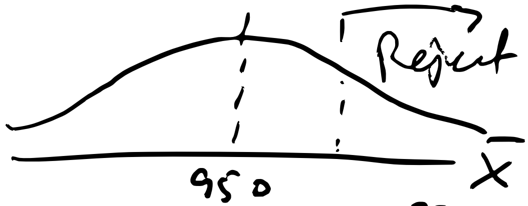
$\text{Power} = 1 - \beta = 1 - 0.053 = 0.947$

6.4 Type I and II Errors

Example 2: The 2000 Census found that the average rent for apartments in a city was \$950. The city will believe that the average rent has increased (and increase funding for rent subsidies) if a random sample of 500 apartments finds the average rent is more than \$990. If the standard deviation is \$320,

960

(a) find the probability of a type I error.



$\sigma = \frac{320}{\sqrt{500}}$

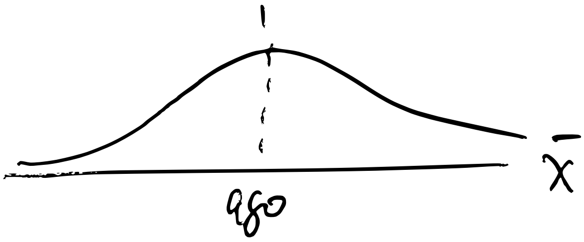
$\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true})$

$= P(\bar{X} > 990 \mid \mu = 950)$

$= \text{normalcdf}(990, 10^{99}, 950, \frac{320}{\sqrt{500}})$

$= 0.00259$

(b) find the probability of a type II error if the real mean is \$980.



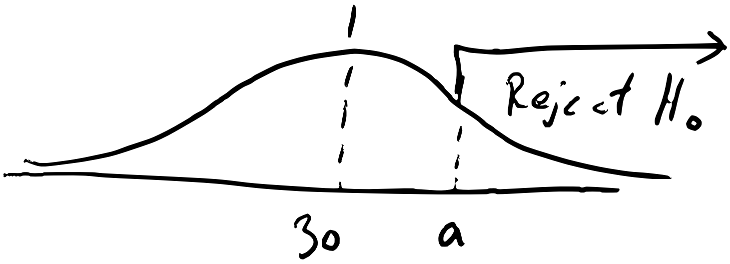
$\beta = P(\text{accept } H_0 \mid H_0 \text{ is false})$

$= P(\bar{X} < 990 \mid \mu = 980)$

$= \text{normalcdf}(-10^{99}, 990, 980, \frac{320}{\sqrt{500}})$

$= 0.757$

Example 3: If H_0 : $\mu = 30$ tested against H_1 : $\mu > 30$ using n = 16 observations (normally distributed) and if $\beta = 0.15$ when $\mu = 34$, what does α equal? Assume that $\sigma = 4$.



$\boxed{\begin{aligned}\alpha &= \text{normalcdf}(32.96, 10^{99}, 30, \tfrac{4}{\sqrt{16}}) \\ &= 0.00154\end{aligned}}$

$\beta = 0.15 \Rightarrow 1 - \beta = 0.85$

$0.85 = P(\text{reject } H_0 \mid H_a \text{ is true})$

$= P(\bar{X} > a \mid \mu = 34)$

$a = \text{invNorm}(0.85, 34, \frac{4}{\sqrt{16}}, \text{RIGHT})$

$= 32.96$