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ECON 2316

Lecture 2:
Math + Econ Refresher
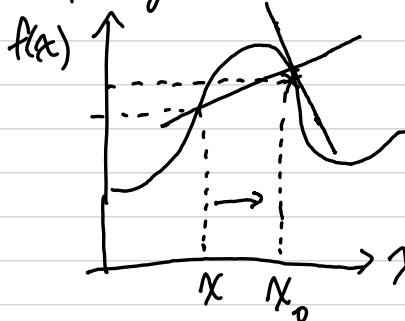
Calculus Review pt 1
↳ What we need for ECON 2316

today { - Derivatives — elasticities
{ - Find extrema — optimization, consumer choice

next { - partial derivatives
time { - Understand functions w/ multiple variables

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Taking derivatives
↳ what is a derivative
- slope of the tangent line
- instantaneous rate of change
- Denoted as $f'(x)$ or $\frac{df}{dx}$ or $\frac{d}{dx} f(x)$
↳ Finding the derivative

$f(x)$

x x_0

Let $m(x) = \dfrac{f(x) - f(x_0)}{x - x_0}$

as $x \to x_0$,
$m(x) \to$ slope at x_0

$f'(x_0) = \lim\limits_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0}$

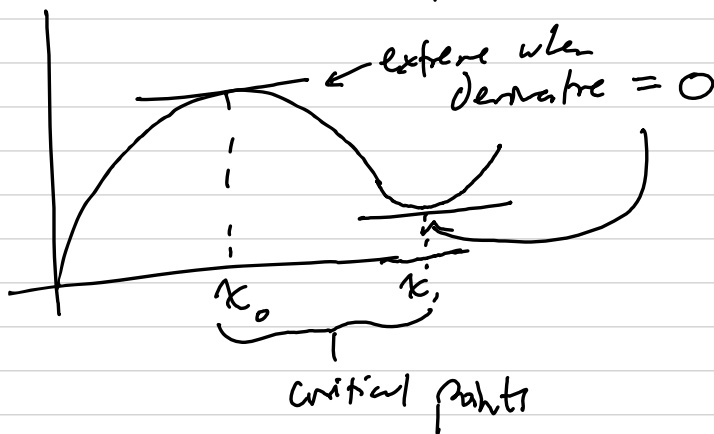- The derivative is the $\lim\limits_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0}$
or:

$\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Basic derivative rules

- Power rule $\frac{d}{dx} x^n = nx^{n-1}$

- Constant rule $\frac{\partial}{\partial x} n = 0$

- Sum rule $\frac{d}{dx}[f(x) + g(x)]$

$$= \left(\frac{d}{dx} f(x)\right) + \left(\frac{d}{dx} g(x)\right)$$

Optimization

- consumers maximize utility
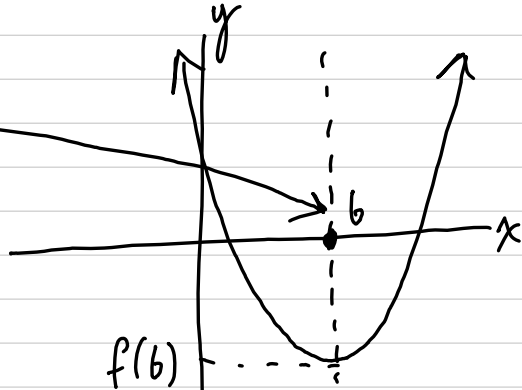- Consumers minimize expenditure
- firms maximize profits

extreme when
derivative $= 0$

x_0 x_1

critical points

Example - Critical points of $x^2 - 12x$

$f(x) = x^2 - 12x$
$f'(x) = 2x - 12 = 0$
$\quad\quad x = 6$

$f(6)$

$f''(x) = 2 > 0$ ∴ concave up ⟹ minimum

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Econ Refresher

Demand curve
  ↳ Law of Demand - price ↑ Q_s ↓
  ↳ Holds determinants of D constant

Q

← Prices increased, Q ↓

↑ P_f
  P_o

D
  Q

Q_f ← Q_o
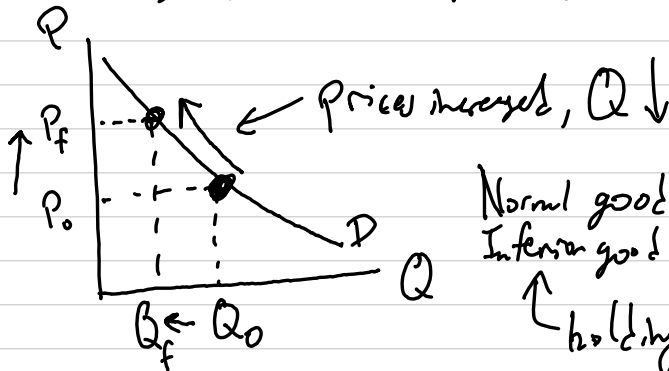
Normal good - income ↑, D ↑
Inferior good - income ↑, D ↓
  ↳ holding prices constant

Substitutes — P_s ↑, Q_{orig} ↑
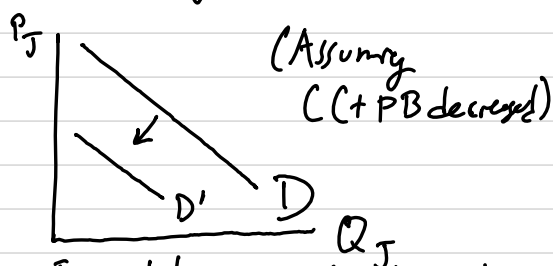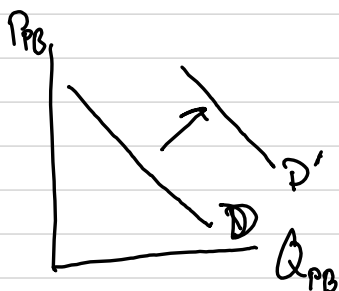Complement - P_c ↑, Q_{orig} ↓

Complements vs. Substitutes

Suppose PB and J and cream cheese

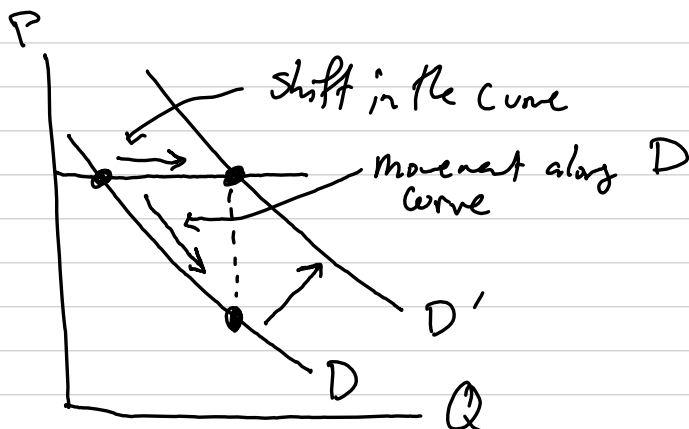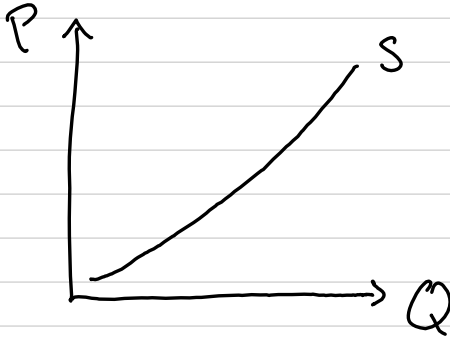| PB+J complement J+CC complement PB+CC substitute	Price of CC ↑ D for PB ↑ D for Jelly

(Assume
CC + PB decreased)

P_{PB}

D'

D

Q_{PB}

P_J

D'

D

Q_J

In reality, probably doesn't
change, but ignoring PB, it
decreases

P

Shift in the curve

movement along D
curve

D'

D

Q

Market Supply Curve

    ↳ Law of Supply — $P\uparrow$, $Q_s\uparrow$
    ↳ holds determinants of supply

P ↑
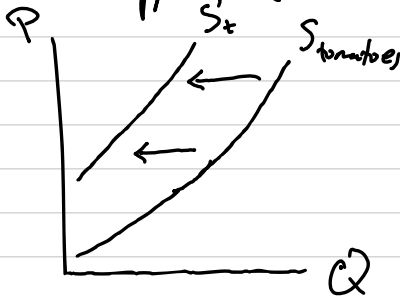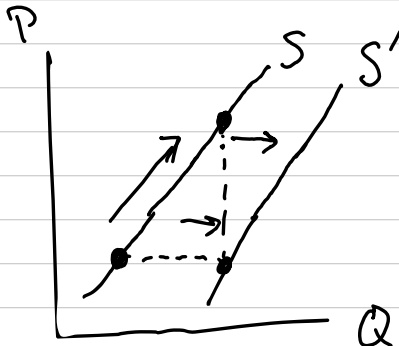   S

Determinants
    ↳ cost of inputs
    ↳ Technology
    ↳ substitutes in production

→ Q

Substitutes in Production

Ex. gardener can grow cucumbers or tomatoes
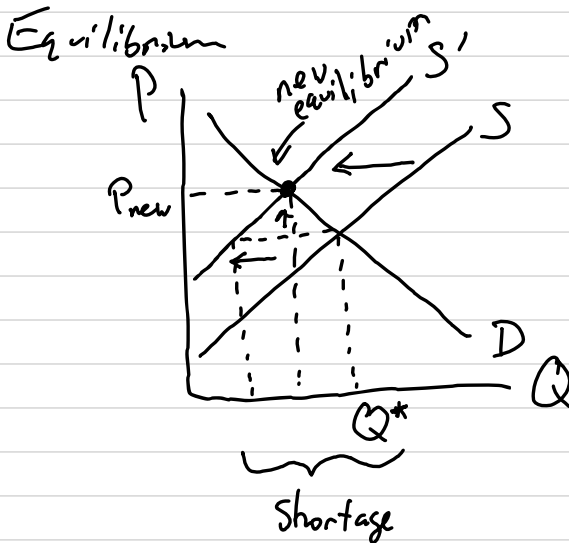   Suppose $P_{cucumber}$ increases ⌝

P | S_t' $S_{tomatoes}$
    ←
    ←

↳ "I wanna sell more cucumbers"

→ Q

P | S   S'

→ Q

Equilibrium / market clearing price
$\hookrightarrow Q_d = Q_s$
Disequilibrium
$\hookrightarrow$ Shortage ~ $Q_d > Q_s$
$\hookrightarrow$ Surplus - $Q_d < Q_s$

Equilibrium

P

new equilibrium S'

S

P_{new}

D

Q

Q^*

Shortage

Consumer Surplus - difference between willing to pay vs. market price

P

CS = area below D and above P^* up to Q^*

P

P^*

D

Q

Q^*
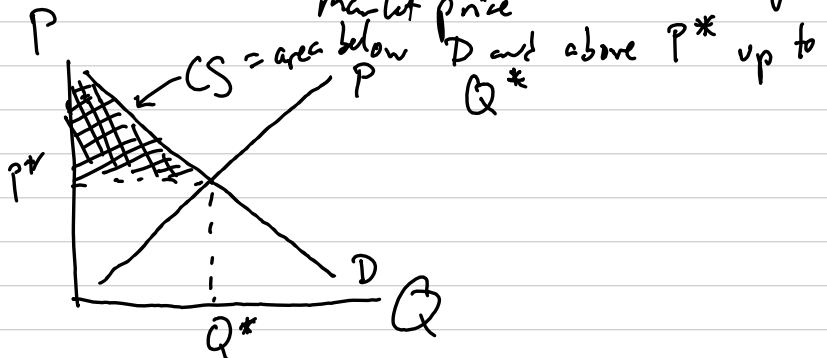
Practice Questions

1) $f(x) = x^3 - 12x$ in $(-5, 3)$

$f'(x) = 3x^2 - 12$, $f''(x) = 6x$

$f''(2) = 12 \Rightarrow$ conc up

$f''(-2) = -12 \Rightarrow$ conc down

$3x^2 - 12 = 0$
$3(x^2 - 4) = 0$
$x = \pm 2$

@ $x = 2$, min
@ $x = -2$, max
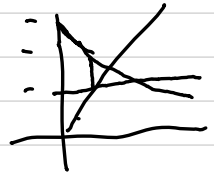
2) $Q^s = 5 + 2P$, $Q^d = 20 - P$

a) $5 + 2P = 20 - P$
$3P = 15$
$P = \$5$, $Q = 15$ units

b) $Q^s = 11$, $Q^d = 17$

c) $Q^s(3) = 11$
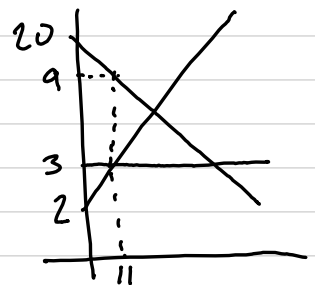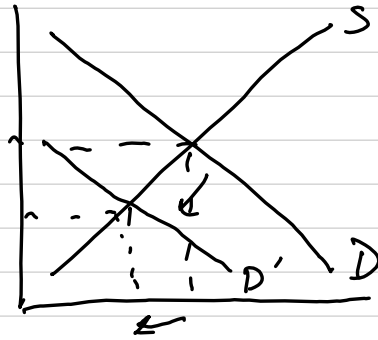$Q = 5 + 2P$

$20 - P = 11$
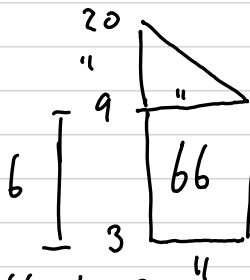
$\frac{1}{2}(11)(11)$
$= 5.5(11)$
$= 5.5(10 + 1)$
$= 55 + 5.5$
$= 60.5$

$CS = 126.5 = 66 + 60.5$

d) $P_{substitute} \downarrow$, $D\downarrow$

decrease