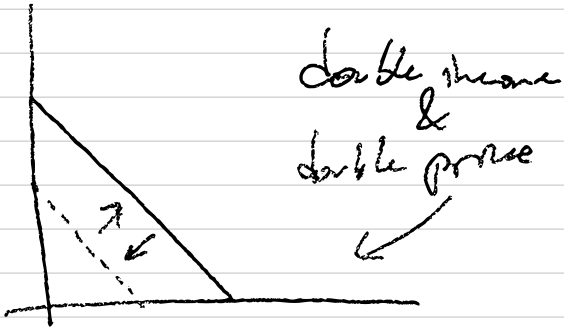
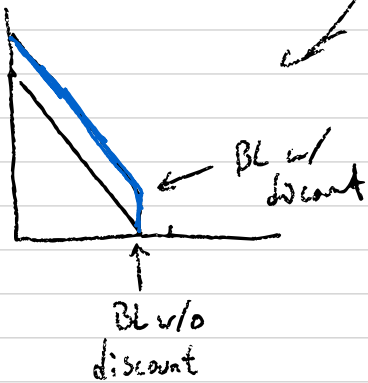
Dylan Black

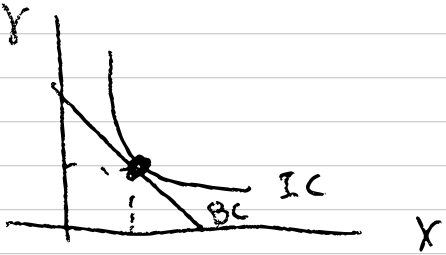
Lecture 6
Individual Demand



Suppose we get the first unit of y free



Consumer choice:
Consume on budget line & highest possible IC



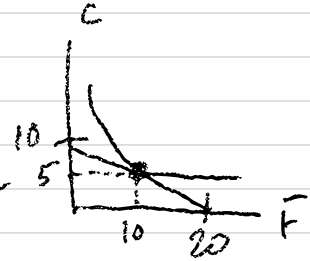
IC & BC must be tangent

- Slope of IC = MRS = $\frac{MU_x}{MU_y} = \frac{\left(\frac{\partial U}{\partial x}\right)}{\left(\frac{\partial U}{\partial y}\right)}$
- Slope of BC = $\frac{P_x}{P_y}$

$MRS = \frac{P_x}{P_y}$

$\therefore \boxed{\frac{MU_x}{MU_y} = \frac{P_x}{P_y}}$ ← gives us relationship between x, y for points on BC

Road Map:
- find MRS
- find P_x/P_y
- find relative quantities
- plug relative quantities into BC



Ex. $U(F,C) = FC$, BC: $200 = 10F + 20C$

$I = 200$
$P_F = 10$
$P_C = 20$

$MU_F = \frac{\partial U}{\partial F} = C$

$MU_C = \frac{\partial U}{\partial C} = F$

$MRS = \frac{C}{F}$,

$\frac{C}{F} = \frac{10}{20} \Rightarrow F = 2C$

$200 = 40C$
$\therefore C = 5$
$F = 10$

Ex 2: $U(F,C) = FC^2$, $200 = 10F + 20C$

$MU_F = C^2$

$MU_C = 2FC$

$MRS = \frac{C}{2F}$

$\frac{C}{2F} = \frac{1}{2} \Rightarrow C = F$

$\rightarrow 200 = 30C$

$\frac{200}{30} = C$

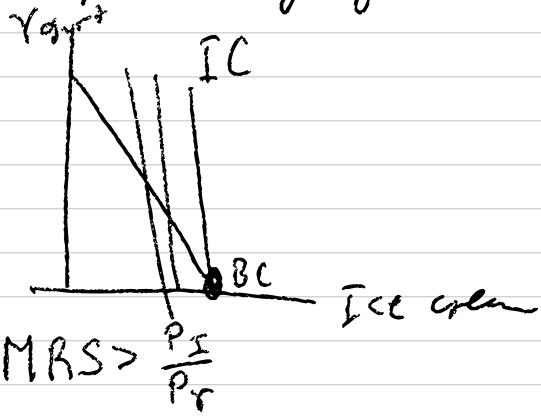
$\boxed{F = C = 6\frac{2}{3}}$

$MRS = \frac{P_x}{P_y} \Rightarrow \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$

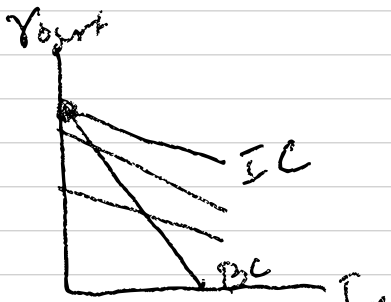
$\Rightarrow \boxed{\frac{MU_x}{P_x} = \frac{MU_y}{P_y}}$

Corner solutions

↳ Optimize utility by consuming at all of 1 good



$MRS > \frac{P_I}{P_Y}$

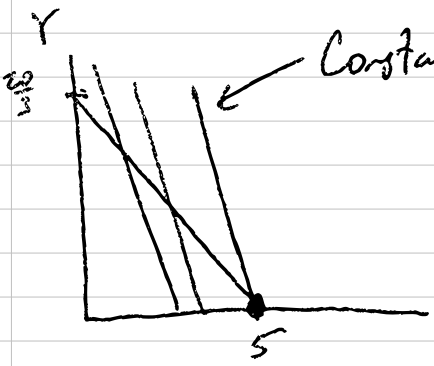


$MRS < \frac{P_I}{P_Y}$

$U(X,Y) = 2X + Y$, BL: $20 = 4x + 3y$

$-MRS = 2$

$-\text{Price ratio} = \frac{4}{3}$ ← 2 is never $= \frac{4}{3}$, ∴ prob. NOT a corner solution



$MRS > \frac{P_x}{P_y}$ at all points, so consume at all x,

$\boxed{5x, 0y}$

Constrained optimization 2

Problem: $\max_{x,y} U(x,y)$
s.t. $P_x x + P_y y = I$

The corresponding Lagrangian is:

$$L = U(x,y) + \lambda(I - P_x x - P_y y)$$

First order conditions:

$$\frac{\partial L}{\partial x} = \frac{\partial U(x,y)}{\partial x} - \lambda P_x = 0$$

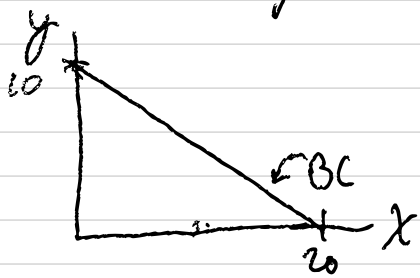
$$\frac{\partial L}{\partial y} = \frac{\partial U}{\partial Y} - \lambda P_y = 0$$

$$\frac{\partial L}{\partial \lambda} = I - P_x x - P_y Y = 0$$

Lagrange multiplier: roc of the maximum value of the utility function as constraint is relaxed

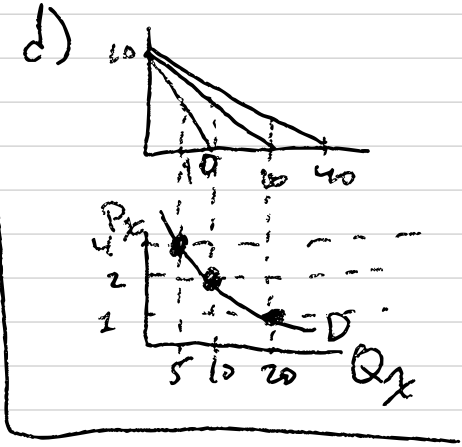
$I = 40,\ P_x = 2,\ P_y = 4$

a) $40 = 2x + 4y$



b) $-\text{slope} = \frac{P_x}{P_y} = \frac{1}{2}$

d)



c) $U(x,y) = 2xy \Rightarrow MRS = \frac{y}{x}$

$\frac{y}{x} = \frac{1}{2} \Rightarrow 2y = x$

$\boxed{y = 5,\ x = 10}$