Chapter 2. Probability

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§2.2 Sample Spaces and the Algebra of Sets

▶ **Some terminologies:**

- **Experiment:** A repeatable procedure with a set of possible results.
- **Sample Outcome:**(Sample Point) Only one of the possible results of an experiment.
- **Sample Space:** the set of All the possible outcomes of an experiment. (Usually denoted by S)
  e.g. $\{H,T\}$
- **Event:** Null or one or more outcomes of an experiment.

▶ **Classical (naive) definition of probability:** Event $\subset S$

Suppose the outcomes of an experiment are all equally likely, and the total number of all possible outcomes is finite. notated w/ A, B, C, etc.

$$\boxed{\textbf{Probability of an event} = \frac{\text{Number of ways it can happen}}{\text{Total number of all possible outcomes}} = \frac{n(A)}{n(S)}}$$

The probability of an event is a real number in the interval $[0, 1]$.

Example 1. Experiment: Flipping a fair Coin once.

$S = \{H, T\}$

$A =$ landing heads $= \{H\}$ One way to achieve $\{H\}$

Event: Landing head.

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$

Number of ways it can happen = 1. Total number of possible outcomes = 2

Probability of 'landing head'$= \frac{1}{2}$.

Example 2. Experiment: Rolling a fair 6-sided die once.

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{5, 6\}$

Event: Rolling a number larger than 4 with a die.

Number of ways it can happen = 2. Total number of possible outcomes =6

Probability of 'Rolling a number larger than 4' $= \frac{2}{6} = \frac{1}{3}$.

$P(n > 4) = \frac{|A|}{|S|} = \frac{2}{6} = \frac{1}{3}$

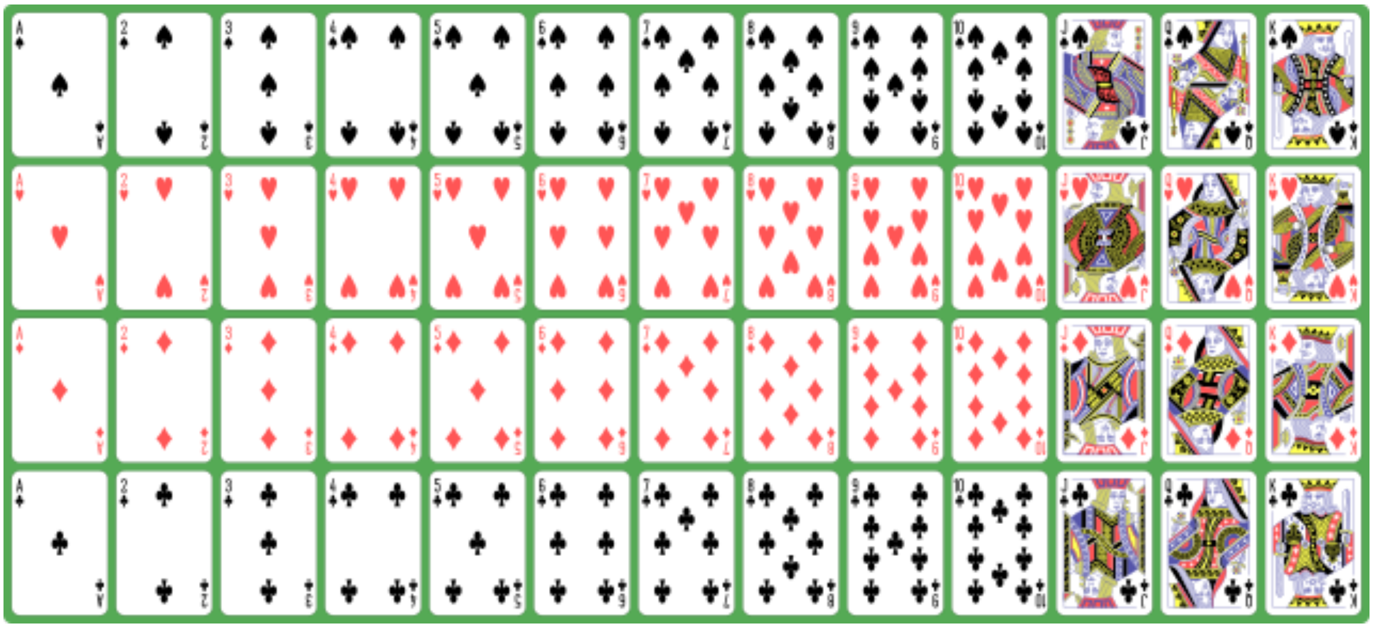
Thm $0 \le P(A) \le P(S)$

$\therefore \boxed{0 \le P(A) \le 1}$

Example 3. Randomly draw a card from a standard deck of cards.

A standard deck of playing cards with four suites: Club, Diamond, Spade, and Heart. For each suit, there are 13 values: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. There are $4 \times 13 = 52$ cards. (**No joker cards.**)

S=



If you draw a card randomly, the probability of getting a face (J, or Q, or K) is

A={J, Q, K for each Suit}

|A|=12 |S|=52

$$P(Face) = \frac{12}{52} = \frac{6}{26} = \frac{3}{13}$$

If you draw a card randomly, the probability of getting an Ace is

$$P(Ace) = \frac{4}{52} = \frac{1}{13}$$

We need to use the **basic set theory** to study probability.

Definition.

- A **set** S is simply a collection of (possibly infinitely many) things.
- If a is an **element** of a set S, we write $\boxed{a \in S.}$ If a is NOT an element of S, then we write $\boxed{a \notin S.}$
- A **subset** A of S is a set whose elements are in S, denoted as $\boxed{A \subset S.}$

Every set S has at least 2 subsets, itself and the $\boxed{\text{empty set } \emptyset.}$

each set has exactly 2^n subsets

Extra examples

$n(A) = n$

↳ A has n elements

Thm A has 2^n subsets

Proof
$A = \{ i_1, i_2, i_3, \ldots i_n \}$

0 or 1   0 or 1   0 or 1   ...

2 possibilities · 2 · 2 ... n times $= \boxed{2^n}$

Countable Sets

1) Finite countable sets Ex. $\{1, 2, 3\}$, $\{5, 6, 7\}$
2) Infinite countable sets Ex. $\{1, 2, 3, 4, \ldots\}$

1-to-1 mapping to $\mathbb{N}$

For example, the sample space S of an experiment is the set of all possible outcomes. The event is a subset of S.

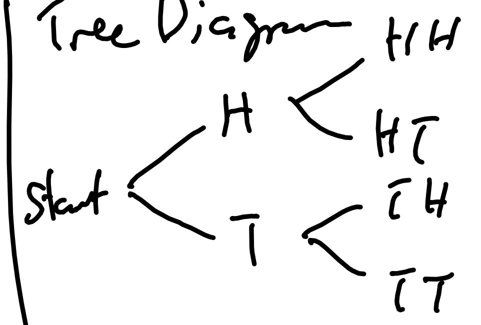
Example 4. Experiment: Flipping (tossing) a coin twice.

The sample space $S =$ $\{HH, HT, TH, TT\}$

The event of 'landing head only once' is $A =$ $\{HT, TH\}$

The probability $P(A) =$ $\frac{2}{4} = \frac{1}{2}$

Tree Diagram



Example 5. Experiment: Flipping (tossing) a coin n times. The size of the sample space is 2^n.

$2 \cdot 2 \cdot 2 \dots$ n flips

Example 6. Experiment: Rolling a 6-sided die.

The sample space $S = \{1, 2, 3, 4, 5, 6\}$.

The event of 'Rolling a number larger than 4' is $A = \{5, 6\}$ which is a **subset** of S, denoted as $A \subset S$.

The probability of A is $P(A) = 2/6 \approx 0.333$

$N = \{1, 2, 3, 4, 5, 6\}$

Example 7. Rolling two 6-sided dice (one red, one blue) once. We can win if we obtain total number larger than 8. What is the probability we can win?

Cartesian Product, $N \times N = S$, $|S| = |N|^2$

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\}; A = \left\{ \begin{array}{cccc} & & & \\ & & & \\ & & & (3,6) \\ & & (4,5) & (4,6) \\ & (5,4) & (5,5) & (5,6) \\ (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\}$$

So, the probability of event A is $\frac{10}{36} = \frac{5}{18}$

What is the event B that the sum of the two faces showing equal 9? What is the event C that absolute of the difference of the two faces showing equal 3?

$B =$ $4/36 = 2/18 = 1/9$
$C =$ $6/36 = 1/6$

More generally, if we roll a dice n times, the size of the sample space is 6^n.

All sample spaces in the above two examples are **finite** sets, i.e., there are a finite number of elements in each set. For a finite set S, the number of element in S is called the **cardinality** of S.

In general, a set can contain infinitely many elements.

Example 8. (Countable set)

Experiment: Tossing a coin until we get a head.

Sample Space: $S = \{H,\ TH,\ TTH,\ TTTH,\ \ldots\ldots\}$. infinite countable set

Event: Getting a head with no more than 3 tosses, $A = \{H,\ TH,\ TTH\}$.

A **countable** infinite set has a one-to-one correspondence to the set of natural numbers $\mathbb{N}$.

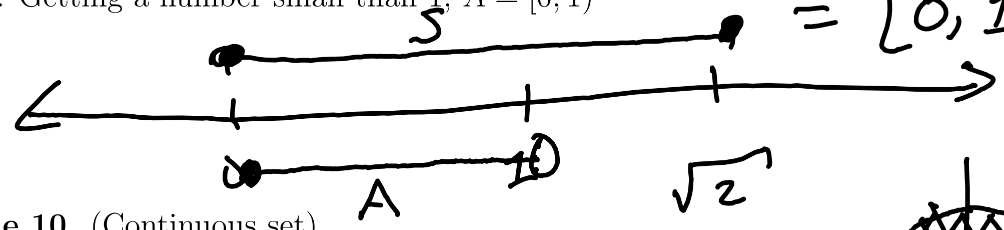
Discrete set means finite or countable set.

Example 9. (Continuous set) – set of values that are measured

Experiment: Pick a real number randomly from 0 to $\sqrt{2}$.

Sample Space: $S = [0, \sqrt{2}]$.

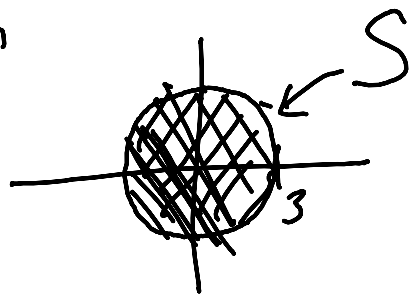
Event A: Getting a number small than 1; $A = [0, 1)$ $\{x \in S \mid x < 1\} = [0,1) \subset S$



Example 10. (Continuous set)

Experiment: Drop a point in a disc of radius 3.

Sample Space: $S = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 3^2\}$

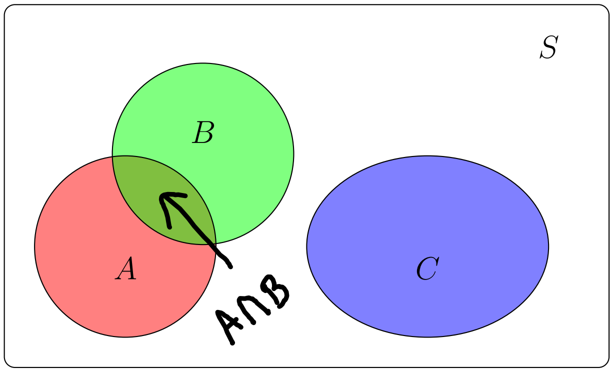


Event A: Get a point (x, y) such that $x \geq 0$ and $y \geq 0$.

$$A = \{(x, y) \in \overset{S}{\cancel{\mathbb{R}^2}} \mid x \geq 0, y \geq 0\}$$

▶ The **Venn diagram** is a useful visual aid of sets.

The set (or sample space) S is represented as a rectangle and subsets (or events) A, B, C are circles or ellipses.



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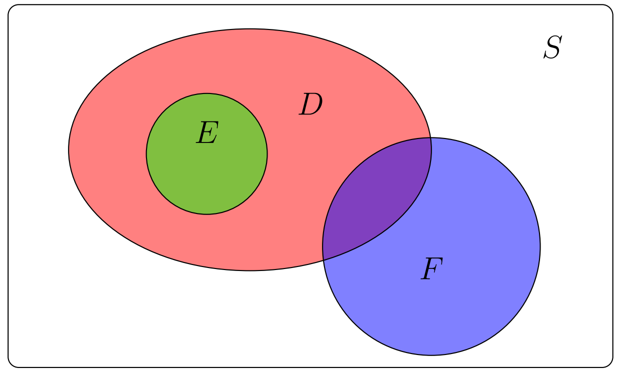


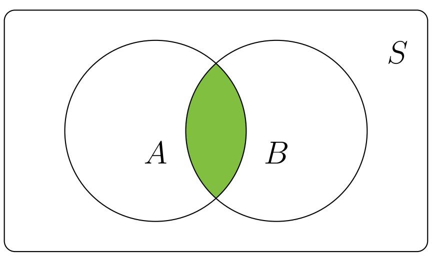
▶ **Basic operations on sets** (or events).

1. Intersection

Definition.

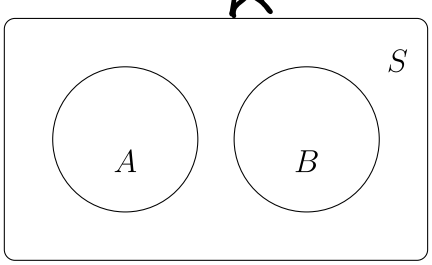
The **intersection** of events A and B, (denoted as $A \cap B$), is the event that whose outcomes belong to both A and B, that is, $A \cap B$ is the event that "both A and B occur".

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$



M.E.

Events A and B are called mutually **exclusive** (**disjoint**) if A and B have no common outcome, i.e., $A \cap B = \emptyset$. A



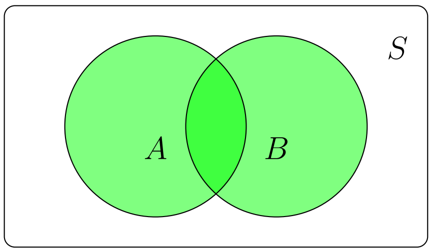
2. Union

Definition.

The **union** of A and B, (denoted as $A \cup B$), is the event whose outcomes belong to either A or B (or both).

$$A \cup B = \{x \mid x \in A \ \text{ or } \ x \in B\}$$

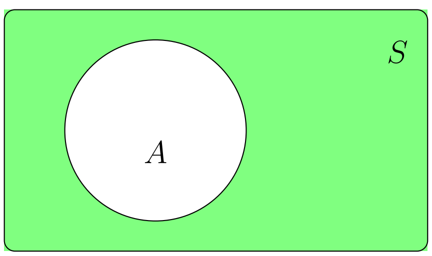
So, $A \cup B$ means that "A or B occurs".



3. Complement

The **complement** of an event A, denoted as A^c (or A^C), is the event whose outcomes in S not belong to A.

$$A^c = \{x \in S \mid x \notin A\}$$



Some quick formulas

$$A \cup A^c = S, \qquad A \cap A^c = \emptyset, \qquad (A^c)^c = A,$$
$$A \cup B = B \cup A, \qquad A \cap B = B \cap A.$$

Example 11. Rolling a 6-sided die once.

The sample space $S = \{1, 2, 3, 4, 5, 6\}$

The event of 'Rolling a number larger than 4' is $A = \{5, 6\}$

The event of 'Rolling even number' is $B = \{2, 4, 6\}$

The event of 'Rolling number which is even **and** larger than 4' is the intersection $A \cap B = \{6\}$

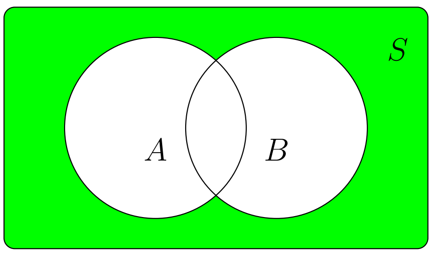
The event of 'Rolling number which is even **or** larger than 4' is the union $A \cup B = \{2, 4, 5, 6\}$

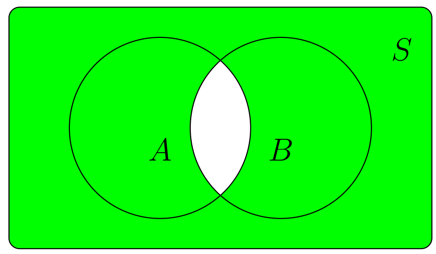
The event of 'Rolling number which is **not** even' is the complement $A^C = \{1, 3, 5\}$

Theorem. **DeMorgan's Law**

1. $(A \cup B)^c = A^c \cap B^c$ $(\neg(A \vee B) \equiv \neg A \wedge \neg B)$
2. $(A \cap B)^c = A^c \cup B^c$ $(\neg(A \wedge B) \equiv \neg A \vee \neg B)$

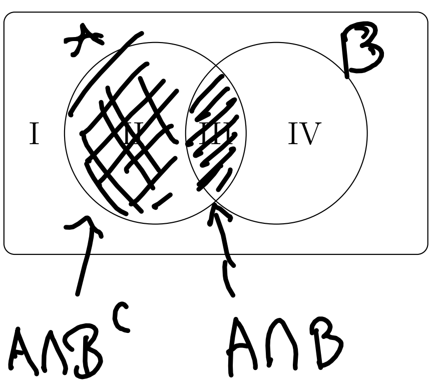
DeMorgan's Law in Venn diagram.

1. 

2. 

Example 12. Prove that $A = (A \cap B) \cup (A \cap B^c)$.

For complicated questions, it is better to label the diagram by disjoint parts I, II, III, IV.



$A \cap B = \mathrm{III}$

$A \cap B^c = \mathrm{II}$

$\mathrm{II} \cup \mathrm{III} = A$ $\blacksquare$

Here, $A \cap B^c$ means that "A occurs but B does not occur".

Example 13. Sketch the regions in xy-plane $\mathbb{R}^2$ corresponding $A \cup B$ and $A \cap B$.

$$A = \{(x, y) \mid x^2 + y^2 \leq 4\}$$

$$B = \{(x, y) \mid 0 \leq x < 4, \ -2 \leq y \leq 2\}$$

Do not be confused with the Venn diagram.

Prove $A = (A \cap B) \cup (A \cap \overline{B})$

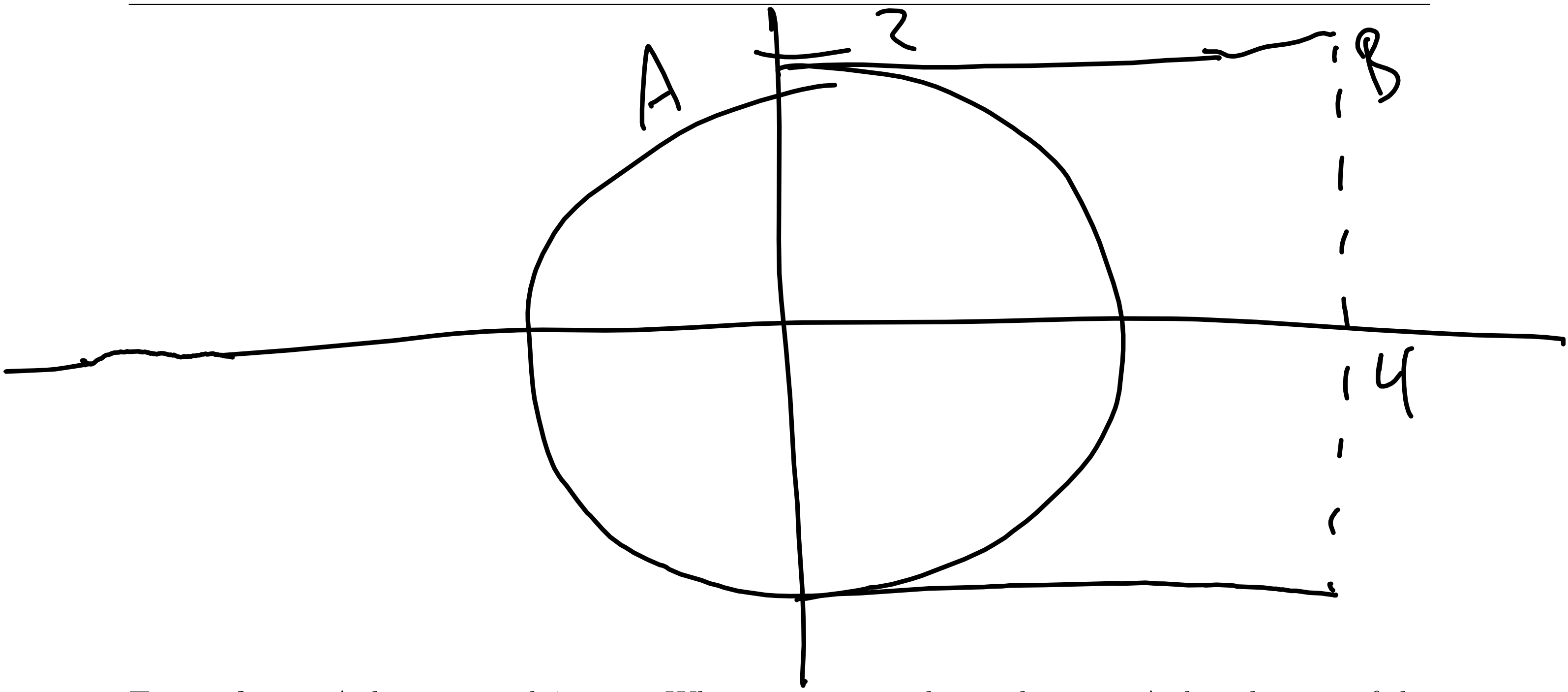
$$= [(A \cap B) \cup A] \cap [(A \cap B) \cup \overline{B}]$$

$$= [\cancel{(A \cup A)}^{A} \cap (B \cup A)] \cap [A \cup \overline{B}] \cancel{\cap (B \cup \overline{B})}$$

$$= [B \cup A] \cap [A \cup \overline{B}]$$

$$= [A \cup B] \cap [A \cup \overline{B}]$$

trivially A



Example 14. A dice is tossed 4 times. What outcomes make up the event A that the sum of the four face results showing equal 5? How many outcomes in the sample space?

$A = \{(1,1,1,2), (1,1,2,1), (1,2,1,1), (2,1,1,1)\}$

The size of the sample space is 6^4

Example 15. Three events A, B, and C. Find the following events using union, intersection and complement.

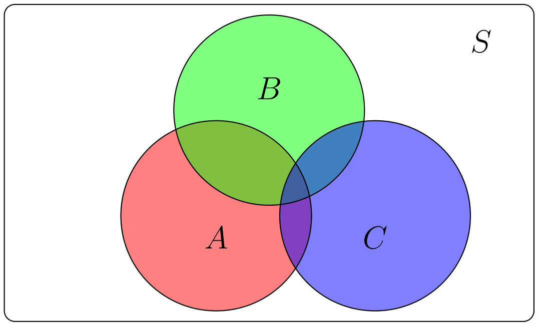
(1) Only B occurs. $B \cap A^c \cap C^c$

(2) exactly one event occurs. $\longrightarrow (A \cap B^c \cap C^c) \cup (B \cap A^c \cap C^c) \cup (C \cap A^c \cap B^c)$

(3) Only A and B occur. $A \cap B \cap C^c$

(1) Only B occurs= B occurs **and** A does not occur **and** C does not occur.
The answer is $B \cap A^c \cap C^c$ or $B \cap (A \cup C)^c$
(2) Exactly one event occurs= only A occurs **or** only B occurs **or** only C occurs.
So the answer is $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$
(3) Only A and B occur= A and B occur **and** C does not occur.
So the answer is $A \cap B \cap C^c$

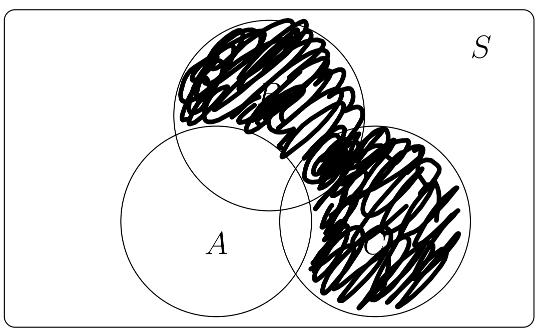
The Venn diagram is very helpful for understanding this kind of questions.



Verify the DeMorgan's law:

1. $(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$ which means none of the three events occurs.
2. $(A \cap B \cap C)^c = A^c \cup B^c \cup C^c$ which means not all three events occur.

Example 16. Find $A^c \cap (B \cup C)$ in the Venn diagram.



§2.3 The Probability Function

Recall that we defined sample space and event of an experiment.

- **Sample Space S:** Set of all the possible outcomes.
- **Event $A \subset S$:** Subset of the sample space.

Recall the classical definition of probability: Suppose the outcomes of an experiment are all equally likely, and the sample space is finite.

Probability of an event $A = P(A) = \dfrac{\text{Cardinality of } A}{\text{Cardinality of } S} = \dfrac{\#(A)}{\#(S)} = \dfrac{|A|}{|S|}$

Example 1. Rolling two 6-sided dice (one red, one blue) once. Let A be the event that the difference (absolute value) of the two numbers is 1. What is the probability of A?

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\};$$

So, the probability of event A is $P(A) = \frac{10}{36} = \frac{5}{18}$

▶ In 1930s, Kolmogorov gave a modern axiomatic definition of the probability function P.

Definition. Definition of Probability Function

A **probability function** P assigns a real number to any event of a sample space.
If the sample space S is infinite, the probability function satisfies the following axioms.

- **Axiom 1.** $P(A) \geq 0$ for any event A.
- **Axiom 2.** $P(S) = 1$.
- **Axiom 3.** For any two mutually exclusive (disjoint) events A and B,

$$P(A \cup B) = P(A) + P(B)$$

- **Axiom 4.** Let $A_1, A_2, A_3, \ldots,$ be events over S.
If any two of them are mutually exclusive, then (ie. all are disjoint)

$$P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$$

$A_i \cap A_j = \emptyset$ $\forall i,j$

$A_1 \cup A_2 \cup A_3 \ldots$

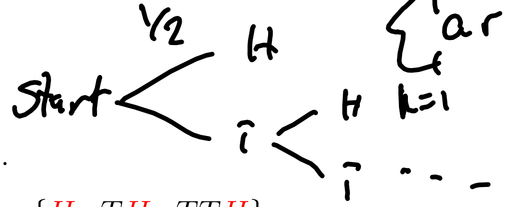
Remark: In Kolmogorov's definition, conditions of equally likely and finite are NOT needed any more.

Recall $\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$ if $|r|<1$

Example 2. Flip a biased coin once, with $P(Head) = 1/3$ and $P(Tail) = 2/3$.

Example 3. (Countable set)

Experiment: Tossing a fair coin until we get a head.



Sample Space: $S = \{H,\ TH,\ TTH,\ TTTH,\ \dots\dots\}$.

Event: Getting a head with no more than 3 tosses, $A = \{H,\ TH,\ TTH\}$.

What is the probability of A? (infinite S, not equally-likely.)

$P(S) = P(H) + P(TH) \cdots$
$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

$P(A) = P(H) + P(TH) + P(TTH)$
$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$

► Some properties can be derived easily from Kolmogorov's axioms. They are extremely important in solving problems.

Theorem 1. $P(A^C) = 1 - P(A)$.

$S = A \cup A^c$ where $A \cap A^c = \emptyset$

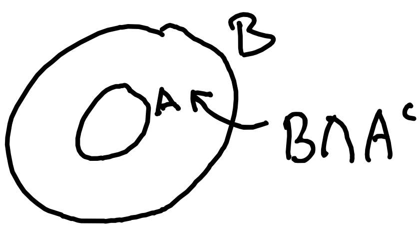
$P(S) = P(A \cup A^c) = P(A) + P(A^c) = 1 \Rightarrow P(A^c) = 1 - P(A)$ $\blacksquare$

Theorem 2. $P(\emptyset) = 0$.

$P(\emptyset) = P(S^c) = 1 - P(S) = 1 - 1 = 0$ $\blacksquare$

Theorem 3. If $A \subset B$ then $P(A) \le P(B)$.

Proof: $B = A \cup (B \cap A^c)$ where A and $B \cap A^c$ are disjoint.



$P(B) = P(A) + \underbrace{P(B \cap A^c)}_{\ge 0}$

$P(B) - P(A) = P(B \cap A^c) \ge 0$

$\Rightarrow P(B) \ge P(A) \Rightarrow P(A) \le P(B)$

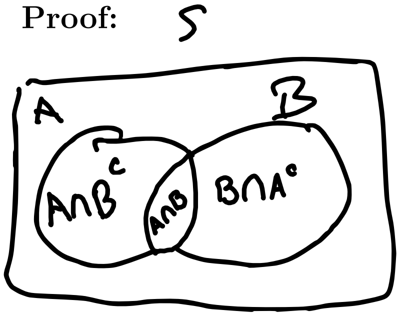
Theorem 4. For every event A, we have $P(A) \le 1$.

$A \subset S$

Proof: By Thm 3, $P(A) \leq P(S) = 1$

Theorem 5. $P(A) = P(A \cap B^C) + P(A \cap B)$.

Proof:



$A = (A \cap B^c) \cup (A \cap B)$

$P(A) = P(A \cap B^c) + P(A \cap B)$

Theorem 6. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Inclusion - Exclusion Principle $P(A \cup B)$

Proof:

$A = (A \cap B^c) \cup (A \cap B) \Rightarrow P(A) = P(A \cap B^c) + P(A \cap B)$

$B = (A \cap B) \cup (A^c \cap B) \Rightarrow P(B) = P(A \cap B) + P(A^c \cap B)$ +

$A \cup B = (A \cap B^c) \cup (A \cap B) \cup (A^c \cap B)$ $P(A \cup B) + P(A \cap B)$

Example 4. Let A and B be two events on S. Suppose $P(A) = 0.5$, $P(B) = 0.6$ and $P((A \cap B)^c) = 0.8$. Answer the following questions:

1. What is the probability that **only** A occurs?

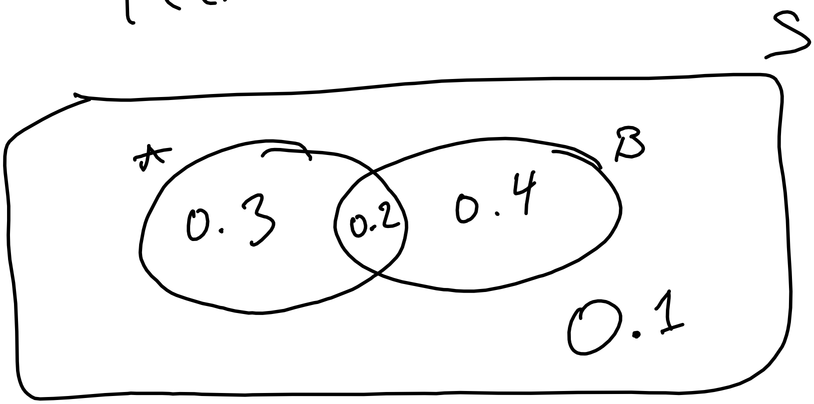
2. What is the probability that A or B occurs?

3. What is the probability that both A and B occur?

$P(A) = 0.5$

$P(B) = 0.6$

$P((A \cap B)^c) = 0.8 \Rightarrow P(A \cap B) = 0.2$



$P(\text{Only } A) = P(A) - P(A \cap B)$

$= 0.5 - 0.2 = 0.3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.6 - 0.2$

$= 1.1 - 0.2 = 0.9$

$P(A \cap B) = 0.2 \quad \because A \cap B = ((A \cap B)^c)^c$

$\Rightarrow P(A \cap B) = 1 - P((A \cap B)^c)$

$= 1 - 0.8 = 0.2$

4. What is the probability that A or B occurs, but not both occurs?

$p(A \cap B^c) + p(A^c \cap B) = 0.7$

5. What is the probability that neither A nor B occurs?

$p((A \cup B)^c) = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

Example 5. Draw 2 cards from a standard deck. What is the probability that the first card is larger than the second card.

A = first is larger
B = Second card larger
C = equal

$S = A \dot{\cup} B \dot{\cup} C$
$p(S) = p(A) + p(B) + p(C)$
$A = \{(3\diamondsuit, 2\heartsuit) \ldots\}$, $B = \{$invert all in $A\}$
$C = \{(10, 10$

see next page

Example 6. A fair coin is tossed four times. What is the probability that at most three heads will occur?

$n(S) = 2^4$
A = 3 or less H in 4 tosses

$\underline{H}\ \underline{H}\ \underline{H}\ \underline{T}$

$p(A) = 1 - p(A^c) = 1 - \frac{1}{16} = \frac{15}{16}$

Example 7. Rolling two 6-sided dice (one red, one blue) once. Find the probability that the first roll is 1, **or** the absolute value of the difference is 1.

$p(\text{first } 1 \cup \text{abs diff } 1)$
$= p(\text{first } 1) + p(\text{abs diff } 1) - p(\text{first} \cap \text{diff})$

$n(\text{first} \cap \text{diff}) = 1$ $\{(1,2)\}$

$n(\text{first } 1) = 6$, $n(S) = 36$

$n(\text{abs diff } 1) = 10$

$(1,2), (2,3), (3,4), (4,5), (5,6)$

$= \frac{6}{36} + \frac{10}{36} - \frac{1}{36}$

$\boxed{= \frac{15}{36} = \frac{5}{12}}$

Ex. 5 cont

$$A = \{(3\diamondsuit, 2\heartsuit), \ldots\}$$

$$B = \{(2\heartsuit, 3\diamondsuit), \ldots\}$$

$n(A) = n(B)$ by symmetry

$$\Downarrow$$

$$P(A) = P(B)$$

$$1 = 2P(A) + P(C)$$

$$C = \Big\{ \Big(\overset{\text{1st}}{\boxed{\text{any}}}, \overset{\text{2nd}}{\boxed{\text{3 options}}} \Big)$$

$$P(C) = \frac{52}{52} \cdot \frac{3}{51} = \frac{3}{51}$$

$$1 = 2P(A) + \frac{3}{51}$$

$$\frac{48}{51} = 2P(A) \Rightarrow \boxed{P(A) = \frac{24}{51}}$$

▶ **Counting and Probability***

For a finite sample space with all equally likely outcomes, it is still very important to use the classical definition of probability to compute classical examples.

A very basic principle of counting is the **multiplication rule**:

If operation A can be performed in m different ways and operation B in n different ways, the **sequence (operation A, operation B)** can be performed in $m \cdot n$ different ways.

Example 8. When we buy a cup of smoothie, we can choose Large, Medium, or Small for the cup, then choose Banana, Chocolate, Strawberry, Vanilla for the flavor.

How many ways we can buy a cup of smoothie?

3 sizes × 4 flavors = 12 smoothies

Definition.

The number of ways to *arrange* k objects of a set of n distinct elements (**permutations**), repetitions not allowed, is denoted by the symbol ${}_nP_k$, or P_k^n, or $P(n,k)$,

$$P_k^n = n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

order matters

Example 9. What is the probability that we get the word NBA if we arrange the letters A, B, N randomly?

$|S| = {}_3P_3 = \frac{3!}{0!} = 6$

$|A| = 1 \Rightarrow \{NBA\}$

$P(A) = \frac{|S|}{|A|} = \frac{1}{6}$

Example 10. What is the probability that at least two students in our class (~~70~~ 30 students) share the same birthday?

Example - 6 people (A, B, C, D, E, F)

1) # of ways to choose pres, VP, + secretary

$$6P3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = 6 \cdot 5 \cdot 4 = 120$$

2) Choose committy of size 3

$$\binom{6}{3} = \frac{6!}{3!(6-3)!} = \frac{6!}{3! \cdot 3!} = \frac{6 \cdot 5 \cdot 4}{3!}$$

$$= \frac{120}{6} = 20$$

event A

Solution:

$P(\text{at least 2 in 30 share a birthday}) = 1 - P(A^c)$ 1 yr = 365

order matters b/c days are distinguishable

$P(A^c) = P(\text{nobody shares a birthday})$ $= 1 - \frac{365P30}{365^{30}}$

# of ways to choose 30 distinct days $= 365P_{30}$

Definition.

The number of ways to *choose* a subset of k objects from n distinct objects (**combinations**), denoted by C_k^n or $\binom{n}{k}$, or $C(n,k)$,

order doesn't matter

$$\binom{n}{k} = \frac{n!}{(n-k)!k!} = \frac{nPk}{k!}$$

Example 11. Five cards are drawn from a standard deck. What is the probability of getting a "royal flush"(10, J, Q, K, A of the same suit)?

$n(S) = \binom{52}{5} =$

$n(A) = 4$

$P(\text{flush}) = \frac{4}{\binom{52}{5}} = 0.00000154$

Example 12. Five cards are drawn from a standard deck. What is the probability of getting a "straight flush"(5 cards in order of the same suit, e.g., A,2,3,4,5 from club or 10, J, Q, K, A from heart)?

$\binom{52}{5}$ ways to pick

$n(A) = 10 \cdot 4$

$P(\text{straight flush}) = \frac{40}{\binom{52}{5}} = 0.0000154$

Example 13. Five cards are drawn from a standard deck. What is the probability that there are exactly 3 diamonds?

Solution: In a standard deck, there are 13 diamonds and 39 non-diamonds. Let A be the event that exactly 3 diamonds in 5 random cards.

$\binom{52}{5}$ ways to pick

$\binom{13}{3}$ ways to pick diamonds, $\binom{39}{2}$ ways to pick non

$|A| = \binom{13}{3}\binom{39}{2}$

Hypergeometric probability (Sec. 3.7)

+ + 5

$$\frac{{}_{13}C_3 \cdot {}_{39}C_2}{{}_{52}C_5}$$

add to 52