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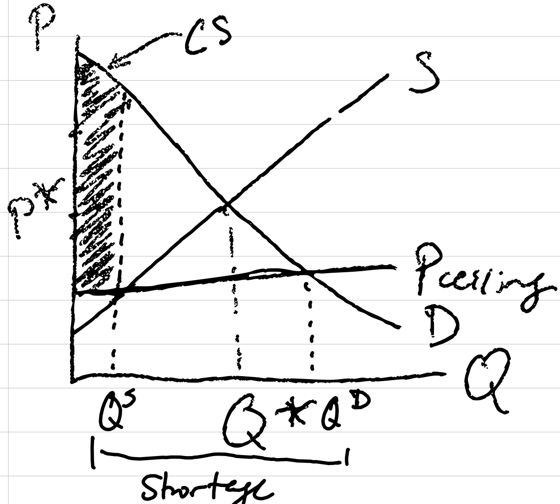
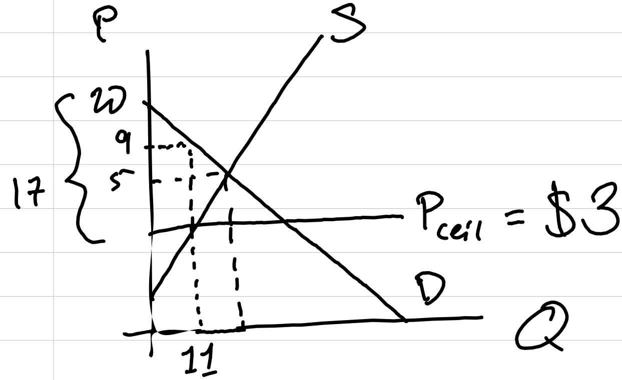
Lecture 3:
Math Refresher 7, Elasticity

Market Mechanisms



Example

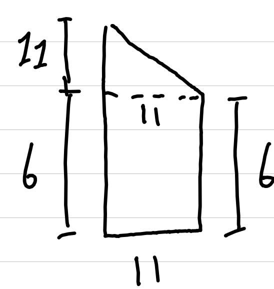
$$Q^s = 5 + 2P$$
$$Q^d = 20 - P$$

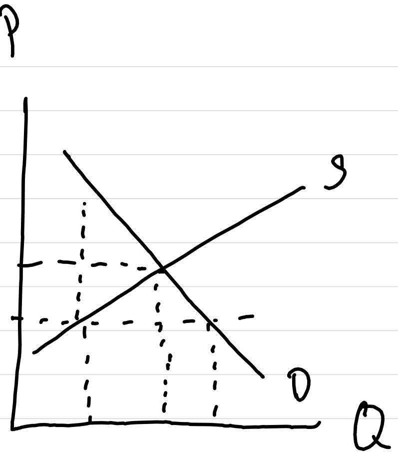


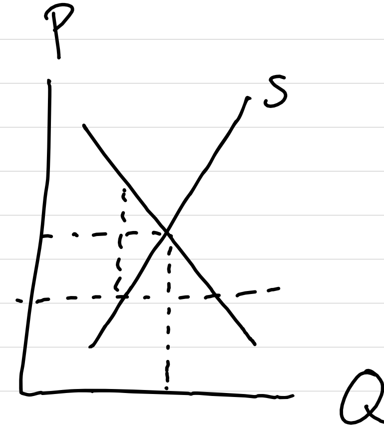
$$CS = \frac{1}{2}(11)^2 + (6 \cdot 11)$$

$$= \frac{121}{2} + 66 = 60.5 + 60$$

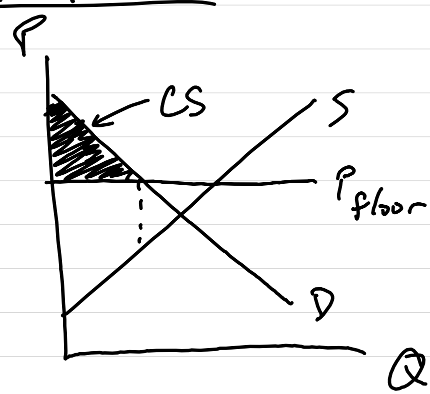
$$\boxed{= 125.5}$$







Price Floor



Math: Partial Derivatives

Functions w/ multiple variables: $z = f(x, y)$

- derivative w.r.t. x - just treat y like a constant and vice versa

- Notation - $\frac{\partial f}{\partial x}$

Ex. $f(x,y) = 2x^2 + y^3$

1) Find $\frac{\partial f}{\partial x} = 4x + 0 = 4x$

$$f(x,y) = 2x^2 + y^3$$

2) $$\frac{\partial f}{\partial y} = 3y^2$$

$$f(x,y) = 4x^2 y^3$$

$$\frac{\partial f}{\partial x} = 8xy^3, \quad \frac{\partial f}{\partial y} = 12y^2x^2$$

Elasticity

Elasticity - $\% \Delta$ one variable resulting from a $1\% \Delta$ in another
- "sensitivity"

Price Elasticity of Demand

$$E_P = \frac{\% \Delta Q_d}{\% \Delta P}$$

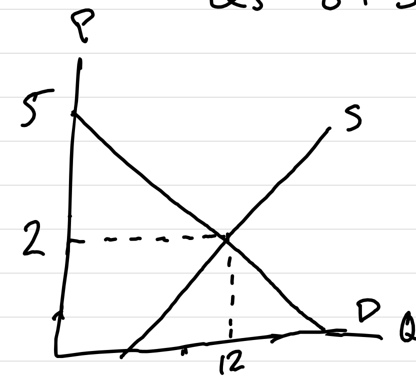
$$\frac{\Delta Q}{\Delta P} \cdot \frac{\frac{1}{Q}}{\frac{1}{P}}$$

$$= \frac{\frac{Q_2 - Q_1}{Q}}{\frac{P_2 - P_1}{P}} = \frac{\Delta Q_d}{\Delta P} \cdot \frac{P}{Q_d}$$

Assuming continuous demand:

$$\boxed{E_P = \frac{P}{Q} \frac{\partial Q}{\partial P}}$$

Example: $Q_d = 20 - 4P$
$Q_s = 6 + 3P$

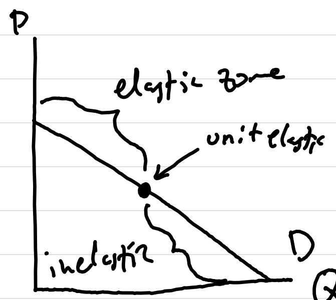


E_p^D @ Equilibrium?

$20 - 4P = 6 + 3P$
$P = 2$
$Q = 12$

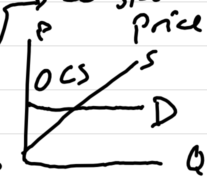
$$E_p^D = \frac{P}{Q} \cdot \frac{\partial Q}{\partial P}$$
$$= \frac{2}{12} \cdot (-4)$$
$$= \frac{-8}{12} = \boxed{-0.6\overline{6}}$$

relatively inelastic

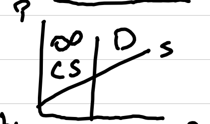


E_p is between 0 and $-\infty$

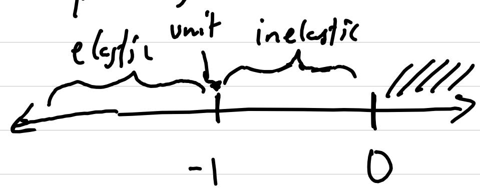
$E_p < -1$ $(|E_p| > 1) \Rightarrow$ elastic
$\rightarrow E_p = -\infty$, perfectly elastic $\rightarrow$ constant price

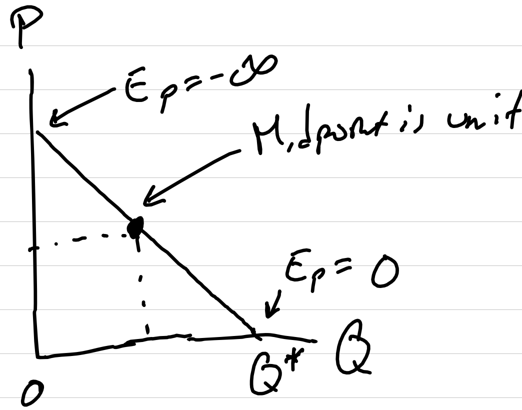


$E_p > -1$ $(|E_p| < 1) \Rightarrow$ inelastic
$\rightarrow E_p = 0 \Rightarrow$ perfectly inelastic $\rightarrow$ constant quantity demanded

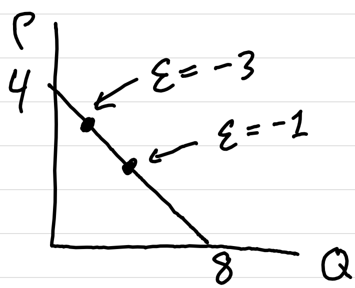


$E_p = -1$, unit elastic





$Q_d = 8 - 2P$



E_p^P @ $P = 2$

$$E_p^D = \frac{P}{Q}\frac{dQ}{dP}$$

$$= \frac{2}{4} \cdot (-2)$$

$$= -1$$

E_p^D @ $P = 3$

$$E_p^D = \frac{3}{2} \cdot (-2)$$

$$= -3$$

Short run vs. Long run

↳ PED may fluctuate w/ time

In the short run - one of the inputs (or more) are fixed

Ex. Gasoline + Automobiles

- Demand for gas is more elastic in the long run than in the short run
  - In the long run you can buy an electric car

- Demand for cars is less elastic in the long run
  - can defer but will eventually have to buy

Income elasticity of demand $= \frac{I}{Q} \cdot \frac{\partial Q}{\partial I}$

Cross price ED $= \frac{P_a}{Q_b} \cdot \frac{\partial Q_b}{\partial P_a}$

PES: $\frac{P}{Q_s} \cdot \frac{\partial Q_s}{\partial P}$

$Q_d = 100 - 5P$, $Q_s = 20 + 3P$

a) $100 - 5P = 20 + 3P$

$80 - 8P = 0$

$P = 10, \ Q = 50$

b) $E_d = \frac{P}{Q} \frac{dQ}{dP}$

$= \frac{10}{50} \cdot (-5)$

$= -1$

c) $\frac{P}{Q} \cdot \frac{dQ}{dP} = -3$

$\frac{P}{100 - 5P} \cdot -5 = -3$

$\frac{-5P}{100 - 5P} = \frac{-3}{1}$

$-5P = -300 + 15P$

$-20P = -300$

$P = \$15$