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ECON 2316

Lecture 11
Cost Functions + Competitive Supply

Total cost: $TC = wL + rK$, $TC = VC + FC$ (in SR)
$TC = VC$ (in LR)

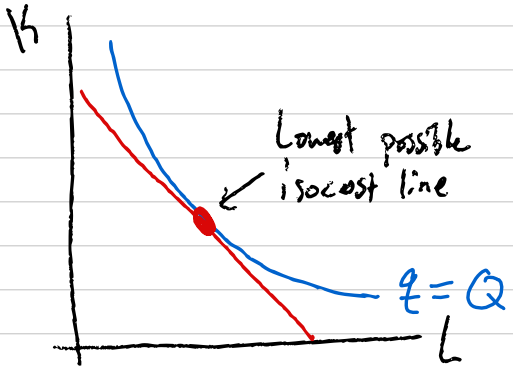
→ in terms of quantity q

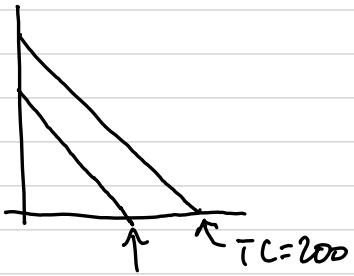
Recall: $MRTS = \frac{MP_L}{MP_K}$

Firm minimizing cost at $MRTS = \frac{w}{r}$

or ... $\frac{MP_L}{w} = \frac{MP_K}{r}$

Cost Minimization





Higher isocost curve ⇒ higher cost

$\frac{MP_L}{MP_K} = \frac{w}{r}$ Plug relationship into production function

Example:

Production function $q = K^{0.5} L^{0.5}$, $w = 10$, $r = 40$

Find optimal L, K for $q = 100$

$$MPL = K^{0.5} \cdot 0.5 L^{-0.5} = \frac{\partial q}{\partial L}$$

$$MPK = 0.5 K^{-0.5} \cdot L^{0.5} = \frac{\partial q}{\partial K}$$

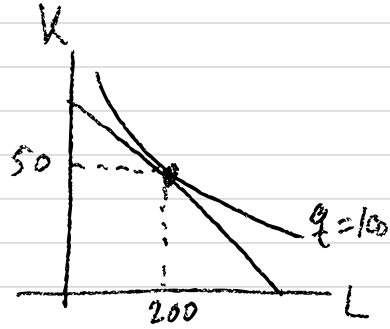
$$MRTS = \frac{K^{0.5} L^{-0.5}}{K^{-0.5} L^{0.5}} = \frac{K}{L}$$

$$\frac{K}{L} = \frac{w}{r} = \frac{10}{40}$$

$$40K = 10L \Rightarrow L = 4K$$

$$100 = K^{0.5} \cdot (4K)^{0.5}$$

$$= 2K \Rightarrow \boxed{K = 50, L = 200}$$



What is the total cost at this K, L?

$$TC = 10L + 40K$$

$$= 2000 + 2000$$

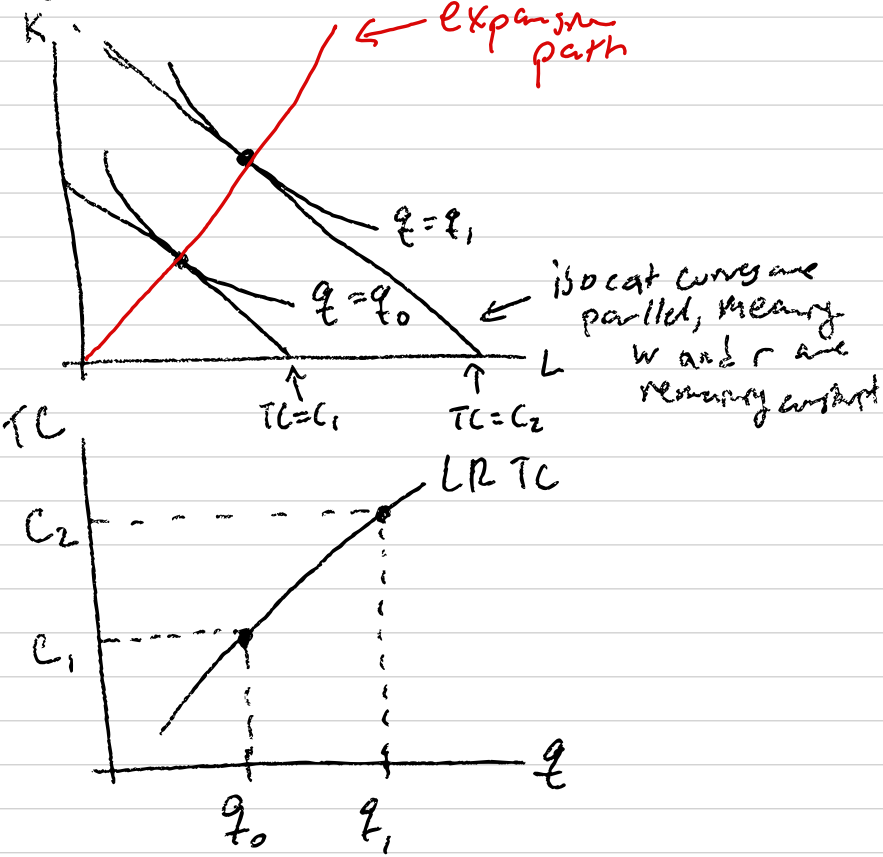
$$\boxed{= \$4000}$$

$\min wL + rK$ s.t. $f(K,L) = q_0$

Corresponding Lagrangian:

$$\Phi = wL + rK + \lambda[q_0 - f(K,L)]$$

Deriving the Cost Curve



Back to the Example:

for any q, what is TC?

$TC = 10L + 40K$, $L = 4K$

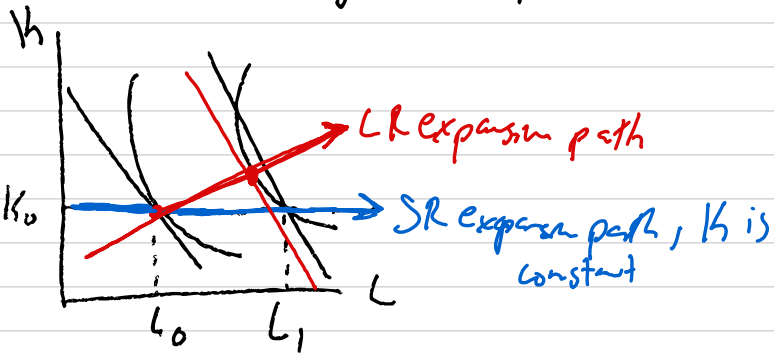
$q = K^{0.5} \cdot (4K)^{0.5} = 2K$, $K = \frac{q}{2}$, $L = 2q$

$TC = 10(2q) + 40\left(\frac{q}{2}\right)$

$TC = 20q + 20q \Rightarrow \boxed{TC = 40q}$ ← LR cost function

Short Run

In the SR, cost may not be optimized



Back to the Example 2:

In the short run, where we set $K = 50$, what is TC when $q = 200$? $q = K^{0.5}L^{0.5}$, $w = 10$, $r = 40$

$200 = 50^{0.5}L^{0.5} \Rightarrow L = \left(\frac{200}{50^{0.5}}\right)^2 = \frac{40000}{50} = 800$

$TC = wL + rK = 10(800) + 40(50)$

$= 8000 + 2000 = \boxed{\$10000}$

(In the LR, when K is flexible, cost @ $q = 200$ is $8000)

Short Run Total Cost Function: for any q,

$L = \frac{q^2}{50}$, $TC = 10\left(\frac{q^2}{50}\right) + 40 \cdot 50$

$TC = \frac{q^2}{5} + 2000$

$q = 50^{0.5} L^{0.5}$

$L^{0.5} = \frac{q}{50^{0.5}} \Rightarrow L = \frac{q^2}{50}$

↑ variable cost! ↑ fixed cost!

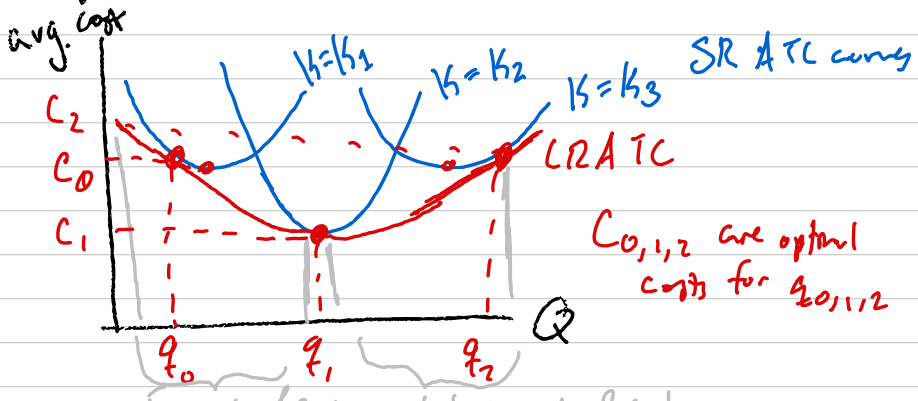
L in LR, only VC

SAC (SR avg cost)
- Avg cost of production when K is fixed

LAC (LR avg cost)
- Avg cost of production when all inputs are variable
- Lowest S.R avg total cost for every level of capital

LMC (LR marginal cost) ↑

Average Cost Curves in LR vs. SR



Economies of Scale
- Subdivision / Specialization
- "Bulk discounts"
- High upfront fixed cost

Diseconomies of Scale
- Coordination inefficiency
- Market power / monopsony
- Incentive problems (asymmetric information)

Dynamic Changes in Cost

MC and AC can fall over time