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VOOR DE MARINE

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problem 1

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ⓐ $f_{x,y}(x,y) = \begin{cases} c \cdot (3x+2y) & , \text{if } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 3 \\ 0 & , \text{otherwise} \end{cases}$

$\iint f_{xy}(x,y) = 1$ thus

$\int_0^2 \int_0^3 c \cdot (3x+2y)\, dy dx =$

$c \int_0^2 \left(3xy + y^2 \Big|_0^3 \right) dx = c \int_0^2 9x + 9 \, dx$

$= c \left(4{,}5x^2 + 9x \Big|_0^2 \right)$

$= c \cdot 36$

$1 = 36c$ makes $c = \frac{1}{36}$

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(1b) $f_x(x) = \int f(x,y)\, dy = \int_0^3 c\,(3x+2y)\, dy$

$= c\left(3xy + y^2 \Big|_0^3 \right) = c\,(9x+9)$ with $c = \frac{1}{36}$

$f_{xy}(x) = \frac{1}{4}x + \frac{1}{4}$

$f_y(y) = \int f(x,y)\, dx = \int_0^2 c\,(3x+2y)\, dx$

$= c\left(\frac{3}{2}x^2 + 2yx \Big|_0^2 \right) = c(6+4y)$ with $c = \frac{1}{36}$

$f_y(y) = \frac{1}{6} + \frac{1}{9}y$

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(1c) if $f_{xy}(x,y) = f_x(x) \cdot f_y(y)$ then X and Y are independent

for example $x=0$ and $y=0$

$f_{xy}(x,y) = f_{xy}(0,0) = \frac{1}{36}\,(3(0)+2(0)) = 0$

$f_x(0) = \frac{1}{4}$ and $f_y(0) = \frac{1}{6}$ makes $f_x(0) f_y(0) = \frac{1}{24}$

$f_{xy}(x,y) \neq f_x(x) \cdot f_y(y)$ thus they depend on eachother

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problem 2

(a) $\mu = 6,3 = \bar{X}$

$\sigma = 0,4$

Sample of 12 soldiers

95 % confidence interval thus $\alpha = 0,05$ 5%

$[g_1 ; g_2] = \left[\bar{X} - Z_{1/2\alpha} \cdot \frac{\sigma}{\sqrt{n}} ; \bar{x} + Z_{1/2\alpha} \cdot \frac{\sigma}{\sqrt{n}}\right]$

$Z_{1/2\alpha} = \text{invNorm}(1 - \frac{1}{2}\alpha ; 0 ; 1) = 1,96$

$[g_1 ; g_2] = [6,074 ; 6,526]$

95% of the samples will fall in the range of 6,074 to 6,526. with a sample mean at 6,3.

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(b) $n = ?$

$\bar{X}$ of $\pm 0,1$ minutes $\Rightarrow E = 0,1 + 0,1 = 0,2$

95 % CI $\Rightarrow \alpha = 5\% = 0,05$

$Z \frac{1}{2}\alpha = 1,96$

$\sigma = 0,4$

$n \geq \left(\frac{Z_{\frac{1}{2}\alpha}\, \sigma}{E}\right)^2$ fill in; $\left(\frac{1,96 \cdot 0,4}{0,2}\right)^2 \approx 15,3664$

so $n \geq 15,3666$ lets it be n=16

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(2c) $H_0: \mu = 6,0$ minutes $S = \sigma_{mean} = 0,4$

$H_1: \mu > 6,0$ minutes $\alpha = 0,05$

This is a right tailed hypothesis test. I think we want to see if the soldiers make the military standard of 6 minutes so above is not sufficient

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(2d) Critical value ;

$\mu_{mean} = 6,3 \cdot \sqrt{12}$ $df = 12 - 1 = 11$

$\sigma_{mean} = 0,4$ (s)

area = 0,95 as lower is good /within standard

$t = \text{invT}(0,95, 11) \approx 1,796$

$t \cdot s/\sqrt{n} = 1,796 \cdot 0,4/\sqrt{12} = 0,2074$

$0,2074 > 0,05$ thus it is likely that mean is higher than 6 minutes.

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Problem 3

ⓐ $f(x;\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$ $\quad x \geq 0 \quad \theta > 0$

$L(x_1, x_2 \dots x_n; \theta = \Pi \frac{1}{\theta^n} e^{\frac{-\sum_{i=1}^{n} x_i}{\theta}}$

$= \theta^{-n} e^{\frac{-\sum x_i}{\theta}}$

ⓑ $\ln(L(x_1, x_2 \dots x_n; \theta) = -n \log(\theta) - \sum x_i / \theta$

$\hat{\theta} \Rightarrow -n \log(\theta) - \frac{\sum x_i}{\theta^n} = 0$

first $d/d\theta = d/d\theta(-n \log(\theta) - \frac{\sum x_i}{\theta})$

$= \frac{-n}{\theta} + \frac{\sum x_i}{\theta^2}$

$d/d\theta = 0 ; \quad \frac{-n}{\theta} + \frac{\sum x_i}{\theta^2} = 0$

$\sum x_i = n \cdot \theta \rightarrow \hat{\theta} = \frac{1}{n} \sum_{i=1}^{n} x_i$ (MLE of $\hat{\theta}$)

ⓒ $E(\bar{x}) = E(\frac{1}{n} \sum x_i) = \frac{1}{n} \sum E(x_i) = \frac{1}{n} \cdot n\theta = \theta$

$\bar{x}$ is thus an unbiased estimator of θ

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4 ⓐ 2

$n = 4$ 4 drones can be detected per trail

$p = 70\% = 0{,}7$ success rate detecting a drone

for every drone the possibility is 70% 4drones per attempt

ⓑ 1

$H_0: \chi^2 \nsim Bin(4; 0{,}7)$

$H_1: \chi^2 \sim Bin(4; 0{,}7)$

Rejecting H_0 means the distribution based on observed frequencies does not ~~fits a binom distribution~~

ⓒ

0 Drones ; $4 \text{ nCr } 0 \cdot 0{,}7^0 (1-0{,}7)^{4-0} = 0{,}0081$

1 Drone ; $4 \text{ ncr } 1 \cdot 0{,}7^1 (1-0{,}7)^3 = 0{,}0756$

2 Drones ; $4 \text{ ncr } 2 \cdot 0{,}7^2 (1-0{,}7)^2 = 0{,}2646$

3 Drones ; $4 \text{ ncr } 3 \cdot 0{,}7^3 (1-0{,}7)^1 = 0{,}4116$

4 Drones ; $4 \text{ ncr } 4 \cdot 0{,}7^4 (1-0{,}7)^0 = 0{,}2401$

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frequencies in % = { 0,0081 8,1 times 0 Drones detected
0,0756 ; 75,6 times 1 Drone detected
0,2646 264,6 times 2 Drones detected
0,4116 411,6 times 3 Drones detected
0,2401 240,1 times 4 Drones detected

ⓓ 2

Expected	observed	$(O_i - E_i)^2 / E_i$
8,1	15	5,8778
75,6	105	11,437
264,6	290	2,438
411,6	360	6,469
240,01	230	0,4175

χ^2 $df = 3-1-0 = 2$ as we guessed 0 parameters

$p(\bar{x} \leq \mu \mid H_0) \leq 0{,}05 \cdot \frac{1}{2}$

~~$2p(\bar{x} \leq \mu \mid H_0) \leq 0{,}05$~~

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(e) Based on the Expected and observed table we could make the assumption that the Number of detected drones follow a binomial distribution as the Values are not far apart, error is low.

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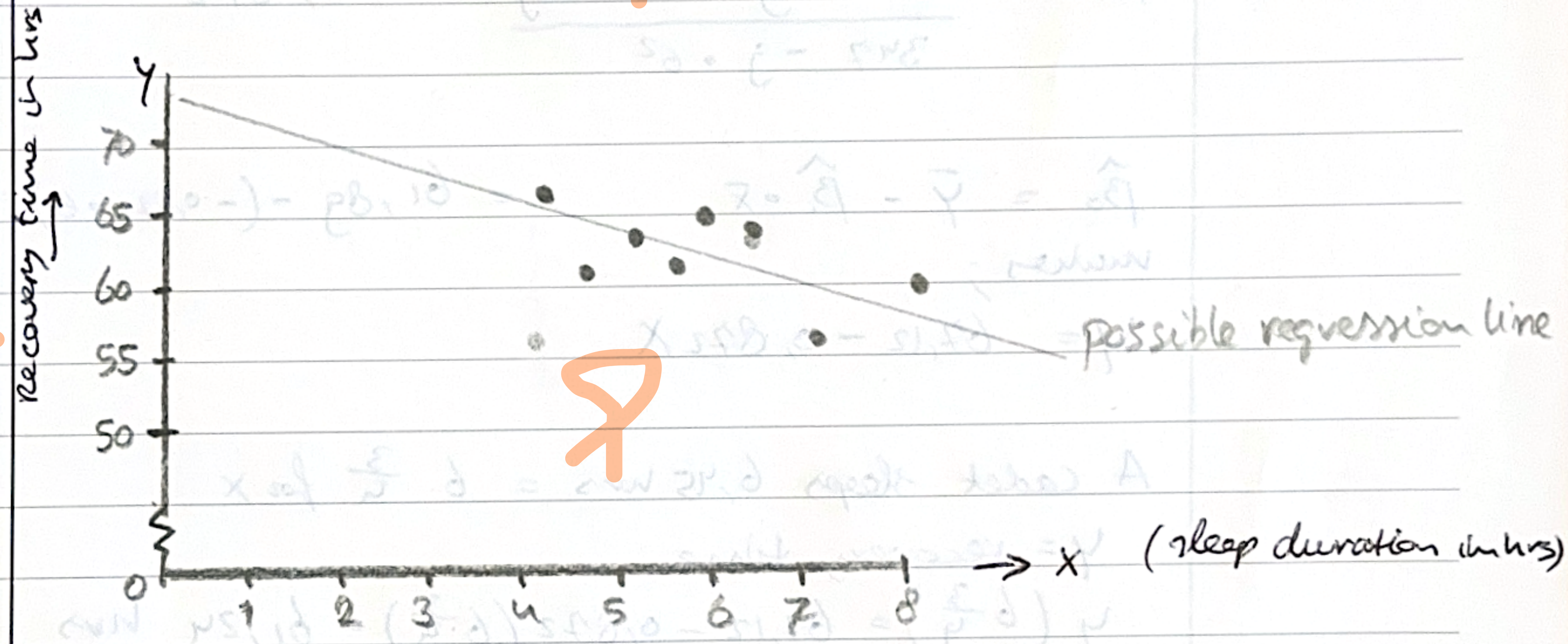
Problem 5 ⓐ

Sleep duration will be variable x, meaning the independent and y will be Recovery time. As the sleep duration will impact the Recovery time.

ⓑ



ⓒ

See Sepa paper

$$R(x,y) = \frac{\text{cov}(x,y)}{\sqrt{\text{VAR}(x)\cdot \text{VAR}(y)}}$$

Wrong...

$\text{cov}(x,y) = E(x \cdot y) - E(x)E(y) = 3837{,}89 - 6 \cdot 61{,}89 \approx 3466{,}55$

$\text{VAR}(x) = E(x^2) - E(x)^2 = 38{,}56 - 6^2 \approx 2{,}56$

$\text{VAR}(y) = E(y^2) - E(y)^2 = 3837{,}89 - 61{,}89^2 \approx 7{,}52$

fill in;

$$R(x,y) = \frac{3466{,}55}{\sqrt{2{,}56 \cdot 7{,}52}} \approx$$

5d $y = a + b \cdot x$ for now $y = \hat{\beta}_0 + \hat{\beta}_1 x$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i - n \cdot \bar{x} \cdot \bar{y}}{\sum_{i=1}^{n} x_i^2 - n \cdot \bar{x}^2}$$

$n = 9$

$\bar{x} = 6$

$\bar{y} \approx 61{,}89$

$\sum x_i y_i = 3322$

$\sum x_i^2 = 347$

$$\hat{\beta}_1 = \frac{3322 - 9 \cdot 6 \cdot 61{,}89}{347 - 9 \cdot 6^2} = -0{,}872$$ Thus b

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x} = 61{,}89 - (-0{,}872 \cdot 6) = 67{,}12$ thus a

makes;

$y = 67{,}12 - 0{,}872\,x$

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A cadet sleeps 6;45 hrs $= 6\frac{3}{4}$ for x

y = recovery time;

$y(6\frac{3}{4}) = 67{,}12 - 0{,}872(6\frac{3}{4}) = 61{,}24$ hrs

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5ⓒ $r(x,y) = \dfrac{\sum_{i=1}^{n} x_i y_i - n \cdot \bar{x} \cdot \bar{y}}{\sqrt{(\sum_{i=1} x_i^2 - n \cdot \bar{x}^2)(\sum y_i^2 - n \cdot \bar{y}^2)}}$

$r(x,y) = \dfrac{3322 - 9 \cdot 6 \cdot 61{,}89}{\sqrt{(347 - 9 \cdot 6^2)(34541 - 9 \cdot 61{,}89^2)}}$

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$= -0{,}509$

There is a negative and medium relationship between X (sleep duration) and Y (recovery time)

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5d

x	y	xy	x^2	y^2
4,0	66	264	16	4356
4,5	61	274,5	20,25	3721
5,0	63	315	25	3969
5,5	62	341	30,25	3844
6,0	65	390	36	4225
6,5	64	416	42,25	4096
7,0	57	399	49	3249
7,5	59	442,5	56,25	3481
8,0	60	480	72	3600
Σ= 54	Σ 557	Σ= 3322	Σ 342	Σ 34541

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5c

$E(x) = \mu = \frac{\Sigma x_i}{9} = 6$

$E(y) = 61,89$

$E(xy) = \frac{\Sigma xy}{9} = 369\frac{1}{9}$

$E(x^2) = \frac{\Sigma x_i^2}{9} = 38,56$

$E(y^2) = \frac{34541}{9} = 3837,89$

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