Ministerie van Defensie

Department of Military Sciences

Student data	
Name:	
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General			
Course:	Probability & Statistics (Resit)	**Course code:**	P&S
Date:	August 25, 2025	**Time:**	10:00-13:00
Examiner:	Dr. ir. D.A.M.P. Blom	**Number of Pages:**	5
Number of Questions	5	**Total Points:**	100

General instructions
- All answers must be supported by a clear explanation. Answers such as “yes” or “no” without justification will receive no credit. - Round final answers to four decimal places, where applicable. - If you are unable to solve a subquestion, you are encouraged to make a reasonable assumption and proceed. Partial credit may still be awarded for correct methodology, even if intermediate answers are incorrect. - The use of a graphical calculator without a CAS (Computer Algebra System) is permitted. - No exam-related material may be taken out of the examination room. - Please write your name and PeopleSoft number on each page and number all pages of your answers (e.g., 1/5, 2/5, etc. if you hand in five answer sheets). - The use of electronic devices capable of sending, receiving or storing information (e.g., mobile phones, smartwatches) is strictly prohibited. These must be left outside the exam room or handed in to the examiner, switched off or in airplane mode. - Ensure your handwriting is legible. Illegible or unclear answers will not be graded. - Toilet visits are only allowed with prior permission from the examiner. - Upon leaving the examination room, all materials (exam paper, scrap paper, formula sheets) must be handed in to the examiner.

Grading
- The final grade of the Probability and Statistics course is entirely based on this exam. - The exam consists of five open-ended questions, each with subquestions. - The number of points available for each (sub)question is indicated in brackets. A total of 100 points can be earned. - Your final grade will be calculated by dividing the total points earned by 10. - A minimum final grade of 5.5 is required to pass the course.

Procedure after the exam
- Exam results will be published within ten working dates after the exam date. - If you have questions about the grading, you may contact the course coordinator within ten working days after the results have been released.

Good luck!

Problem 1 (20 points) A coastal rescue team is conducting search operations for missing swimmers lost at sea. Based on previous data and ocean current modeling, the joint probability density function (PDF) for the coordinates (X, Y) of a swimmer's location (in kilometers relative to the shore) is given by:

$$f_{X,Y}(x,y) = \begin{cases} C \cdot (3x+2y), & \text{if } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 3 \\ 0, & \text{otherwise,} \end{cases}$$

where C is a constant.

1a [6pt] Determine the value of C so that $f_{X,Y}(x,y)$ satisfies the properties of a valid joint probability density function.

Uitwerking

The joint probability density function $f_{X,Y}(x,y)$ is a valid joint probability density function if the following two conditions hold

1. First condition: $f_{X,Y}(x,y) \geq 0$ for all possible values of x and y, and
2. Second condition: $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx\, dy = 1$. (1pt)

The first condition holds if and only if $C(3x+2y)$ for $0 \leq x \leq 2$ and $0 \leq y \leq 3$, so we need $C \geq 0$. The second condition holds if (1pt)

$$\begin{aligned} 1 &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx\, dy \\ &= \int_0^3\int_0^2 C \cdot (3x+2y)\, dx\, dy \\ &= C \cdot \int_0^3 \left[\int_0^2 (3x+2y) dx\right] dy \\ &= C \cdot \int_0^3 \left[\frac{3}{2}x^2 + 2xy\right]_0^2 dy \\ &= C \cdot \int_0^3 6 + 4y\, dy \\ &= C \cdot \left[6y + 2y^2\right]_0^3 \\ &= C \cdot (18+18) = 36 \cdot C \end{aligned}$$ (3pt)

The constant C should be equal to $\frac{1}{36}$ in order for $f_{X,Y}(x,y)$ to satisfy the properties of a valid joint probability density function. (1pt)

1b [6pt] Compute the marginal PDFs $f_X(x)$ and $f_Y(y)$, for X and Y respectively.

Uitwerking

- The marginal PDF of X is given by

$$\begin{aligned} f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}\ dy &= \int_0^3 \frac{1}{36} \cdot (3x+2y)\ dy \\ &= \left[\frac{1}{12}xy + \frac{1}{36}y^2\right]_0^3 \\ &= \frac{1}{4}x + \frac{1}{4} \end{aligned}$$ (3pt)

- The marginal PDF of Y is given by

$$\begin{aligned} f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}\ dx &= \int_0^2 \frac{1}{36} \cdot (3x+2y)\ dx \\ &= \left[\frac{1}{24}x^2 + \frac{1}{18}xy\right]_0^2 \\ &= \frac{1}{9}y + \frac{1}{6} \end{aligned}$$ (3pt)

1c [8pt] Are X and Y independent random variables? Justify your answer with a computation.

Uitwerking

To show whether or not X and Y are independent random variables, we need to compute their covariance and check if it is equal to zero (independence) or not (not independence). We can compute the covariance as follows:

$$\text{Cov}(X,Y) = E[XY] - E[X] \cdot E[Y].$$ (1pt)

We first need to compute these quantities:

$$\begin{aligned} E[X] = \int_{-\infty}^{\infty} x f_X(x)\ dx &= \int_0^2 x \cdot \left(\frac{1}{4}x + \frac{1}{4}\right)\ dx \\ &= \int_0^2 \frac{1}{4}x^2 + \frac{1}{4}x\ dx \\ &= \left[\frac{1}{12}x^3 + \frac{1}{8}x^2\right]_0^2 = \frac{7}{6}. \end{aligned}$$ (2pt)

$$
\begin{aligned}
E[Y] = \int_{-\infty}^{\infty} y f_Y(y)\, dy &= \int_0^3 y \cdot \left(\frac{1}{9}y + \frac{1}{6}\right)\, dy \\
&= \int_0^3 \frac{1}{9}y^2 + \frac{1}{6}y\, dy \\
&= \left[\frac{1}{27}y^3 + \frac{1}{12}y^2\right]_0^3 = \frac{7}{4}. && \text{(2pt)}
\end{aligned}
$$

$$
\begin{aligned}
E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f_{X,Y}(x,y)\, dx\, dy &= \int_0^3 \int_0^2 \left(\frac{1}{12}x^2y + \frac{1}{18}xy^2\right)\, dx\, dy \\
&= \int_0^3 \left[\frac{1}{36}x^3y + \frac{1}{36}x^2y^2\right]_0^2\, dy \\
&= \int_0^3 \frac{2}{9}y + \frac{1}{9}y^2\, dy \\
&= \left[\frac{1}{9}y^2 + \frac{1}{27}y^3\right]_0^3 = 2. && \text{(2pt)}
\end{aligned}
$$

The covariance of X and Y is equal to $\mathrm{Cov}(X,Y) = E[XY] - E[X] \cdot E[Y] = 2 - \frac{7}{6} \cdot \frac{7}{4} = -\frac{1}{24} \neq 0$. Hence, X and Y cannot be independent random variables. (1pt)

Problem 2 (23 points) During CBRN readiness training, military standards require that personnel are able to don full protective gear in no more than 6.0 minutes. A commander collects data from a random sample of twelve soldiers. The sample mean donning time is 6.3 minutes. Assume donning times are normally distributed with a population standard deviation of 0.4 minutes.

2a [5pt] Construct a $95\,\%$-confidence interval for the true mean donning time μ. Interpret the result in context.

Uitwerking

As the true standard deviation $\sigma = 0.4$, we can use the normal distribution to compute the confidence interval. The z-value corresponding to a $95\,\%$ confidence level, i.e., $\alpha = 0,05$, is (in case of two-sided confidence intervals) equal to: (1pt)

$$\begin{aligned} z_{\alpha/2} &= \text{InvNorm}(\textbf{area} = 1 - \alpha/2; \mu = 0; \sigma = 1) \\ &= \text{InvNorm}(\textbf{area} = 0.975; \mu = 0; \sigma = 1) \\ &\approx 1,96. \end{aligned}$$ (1pt)

The $95\,\%$-confidence interval is then given by

$$\begin{aligned} &[\overline{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}; \overline{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}] \\ &= [6.3 - 1.96 \cdot \frac{0.4}{\sqrt{12}}; 6.3 + 1.96 \cdot \frac{0.4}{\sqrt{12}}] \\ &= [6.0737; 6.5263]. \end{aligned}$$ (2pt)

As the interval does not contain the value 6.0, with $95\,\%$ confidence we can say that the true average donning time is longer than 6.0 minutes. (1pt)

2b [4pt] What is the minimum sample size required to estimate the true mean donning time μ within ± 0.1 minutes, with $95\,\%$ confidence?

Uitwerking

As the population standard deviation $\sigma = 0.4$ is known, we can use the normal distribution to compute the minimum sample size. Furthermore, we know that $\alpha = 0,05$ and the deviation is at most $a = 0,1$. The minimum required sample size (1pt) is then equal to

$$n \geq \left(\frac{z_{\alpha/2} \cdot \sigma}{a}\right)^2 = \left(\frac{1.96 \cdot 0.4}{0.1}\right)^2 \approx 61.4633. \qquad \text{(2pt)}$$

Notice that we need to round up to the nearest integer. At least 62 soldiers must be sampled to estimate the mean donning time within ± 0.1 minutes with $95\,\%$ confidence, assuming the standard deviation remains at 0.4 minutes. (1pt)

We would like to test if the true mean donning time is indeed up to military standards. For that purpose, a hypothesis test needs to be conducted. From the following questions onwards, we assume that the **sample** standard deviation (based on the twelve soldiers) is equal to 0.4 minutes, and the true standard deviation σ is unknown. Use a significance level $\alpha = 0,05$.

2c [4pt] State the null and alternative hypothesis of the hypothesis test. Explain your choice on the type of hypothesis test (left-sided, two-sided or right-sided).

Uitwerking

In this hypothesis test, we want to test a claim on the true mean donning time μ. The null hypothesis H_0 and the alternative hypothesis H_1 can be formulated as follows:

$$\begin{aligned} &H_0 : \mu \leq 6.0 && \text{(mean donning time conform military standard)} \\ &H_1 : \mu > 6.0 && \text{(mean donning time violate military standard)} \end{aligned} \qquad \text{(2pt)}$$

In this case, we work with a right-sided test (alternative hypothesis has sign $>$), since we take the military standard (within 6.0 minutes) as default setting and only reject the claim of satisfying this standard if the sample mean donning time is significantly larger than 6.0 minutes. (1pt)

2d [10pt] Perform the hypothesis test and state your conclusion based on the critical region.

Uitwerking

We use the null hypothesis H_0 and the alternative hypothesis H_1 from the previous subquestion. Furthermore, the sample mean $\overline{x} = 6.3$ and $s = 0.4$ are given, as well as the significance level $\alpha = 0, 05$.

As we work with a right-sided test, the critical region is of the form $[g, \infty)$. We want to compute the smallest value g such that the probability of a type-I error (rejecting H_0 while it is true) is equal to $\alpha = 0, 05$. (1pt) (1pt)

Under the null hypothesis, the true mean donning time $\overline{X}$ of twelve arbitrarily soldiers is normally distributed with $\mu = 6.0$ en standard deviation $\frac{\sigma}{\sqrt{12}}$. As σ is unknown and $n = 12 < 30$, we need to resort to the t-distribution, using the estimation $s = 0.4$. As we have a right-sided test, the corresponding t-value is equal to (1pt) (1pt)

$$t = \mathrm{InvT}(opp = 1 - \alpha; df = n - 1) = \mathrm{InvT}(opp = 0, 95; df = 11) \approx 1, 7959.$$ (1pt)

The associated boundary value g is then equal to

$$g = \mu + t \cdot \frac{s}{\sqrt{12}} = 6.0 + 1, 7959 \cdot \frac{0.4}{\sqrt{12}} \approx 6.2074.$$ (2pt)

The sample mean $\overline{x} = 6.3$ is larger than this boundary value $g \approx 6.2074$, so $\overline{x}$ lies in the critical region. Therefore, under this significance level α, the null hypothesis will be rejected. Based on the given sample, there is enough evidence that the true mean donning time is significantly longer than 6.0 minutes, therefore violating military standards. (1pt) (2pt)

Problem 3 (15 points) A set of n independent and identically distributed random variables $X_1, X_2, \ldots, X_n$ are drawn from a distribution with the probability density function:

$$f(x;\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \quad x \geq 0, \quad \theta > 0$$

3a [3pt] Write down the likelihood function $L(x_1, x_2, \ldots, x_n; \theta)$ given a sample of realizations of $X_1, X_2, \ldots, X_n$.

Uitwerking

The likelihood function can be written as follows:

$$\begin{aligned} L(x_1, x_2, \ldots, x_n; \theta) &= f(x_1;\theta) \cdot f(x_2;\theta) \ldots \cdot f(x_n;\theta) \\ &= \frac{1}{\theta} e^{-\frac{x_1}{\theta}} \cdot \frac{1}{\theta} e^{-\frac{x_2}{\theta}} \ldots \cdot \frac{1}{\theta} e^{-\frac{x_n}{\theta}} \\ &= (\frac{1}{\theta})^n e^{-\frac{\sum_{i=1}^n x_i}{\theta}} \end{aligned}$$

(3pt)

3b [7pt] Derive the maximum likelihood estimator (MLE) of the parameter θ.

Hint: consider the natural logarithm (ln) of the likelihood function (the log-likelihood).

Uitwerking

The maximum likelihood estimator can be obtained by first considering the log-likelihood, as it results in easier computations later on:

$$\begin{aligned} \ell(x_1, x_2, \ldots, x_n; \theta) &= \ln(\frac{1}{\theta})^n e^{-\frac{\sum_{i=1}^n x_i}{\theta}} \\ &= \ln(\frac{1}{\theta})^n + \ln e^{-\frac{\sum_{i=1}^n x_i}{\theta}} \\ &= -n \cdot \ln\theta - \frac{\sum_{i=1}^n x_i}{\theta} \end{aligned}$$

(3pt)

The maximum likelihood estimator (MLE) can then be found by optimizing the log-likelihood function, i.e., taking the derivative of the likelihood function and setting it equal to zero: Notice that the log-likelihood function has the same optimal solutions as the likelihood function, as the logarithm is a strictly increasing function and hence preserving optimality.

$$\frac{d\ell(x_1, x_2, \ldots, x_n; \theta)}{d\theta} = -\frac{n}{\theta} + \frac{\sum_{i=1}^n x_i}{\theta^2} = 0 \qquad \text{(1pt)}$$

Solving the equation gives us:

$$-\frac{n}{\theta} + \frac{\sum_{i=1}^n x_i}{\theta^2} = 0 \Rightarrow \frac{\sum_{i=1}^n x_i}{\theta^2} = \frac{n}{\theta} \Rightarrow \hat{\theta}_{\text{MLE}} = \frac{\sum x_i}{n} = \overline{x}. \qquad \text{(2pt)}$$

The maximum likelihood estimator of θ is the sample mean $\hat{\theta}_{\text{MLE}} = \frac{\sum_{i=1}^n X_i}{n} = \overline{X}$. (1pt)

3c [5pt] Show that the sample mean $\overline{X} = \frac{(X_1+X_2+\ldots+X_n)}{n}$ is an unbiased estimator for the parameter θ.

Uitwerking

We need to show that $E[\frac{\sum_{i=1}^n X_i}{n}] = \theta$. Notice that the given probability density function is equal to the density function of a exponential distribution with parameter $\lambda = \frac{1}{\theta}$. (2pt)

This means that each X_i has an expected value of $E[X_i] = \frac{1}{\lambda} = \theta$. (1pt)

Therefore, we have that

$$E[\frac{\sum_{i=1}^n X_i}{n}] = \frac{1}{n}\sum_{i=1}^n E[X_i] = \frac{1}{n}(\theta + \theta + \ldots + \theta) = \frac{1}{n} \cdot n\theta = \theta,$$

which finishes the proof. (2pt)

Problem 4 (20 points) An air force unit is testing a new type of radar used for detecting enemy drones. The manufacturer, Thales, claims that the radar has a $70\,\%$ success rate in detecting a drone. To test this claim, 1000 independent trials are conducted. In each trial, the radar attempts to detect 4 drones. The number of drones detected per trial is recorded as follows:

Number of detected drones	**Observed frequencies**
0	15
1	105
2	290
3	360
4	230
Total:	1000

To test the claim with the manufacturer, we would like to test whether the observed data indeed fits a binomial distribution using a chi-square goodness-of-fit test.

4a [2pt] Which values for the parameters n and p should we use for our hypothesis test. Explain your answer.

Uitwerking

Notice that in each trial, the radar attempts to detect 4 drones, so the value of $n = 4$. Furthermore, the radar has a $70\,\%$ success rate, so the binomial success (1pt)
probability $p = 0,7$. (1pt)

Note: the value of n is NOT equal to 1000. This number only indicates how many times a binomial random variable is being observed!

4b [3pt] State the null hypothesis H_0 and the alternative hypothesis H_1 of the hypothesis test. What does rejecting H_0 tell us about the distribution of the number of detected drones?

Uitwerking

The null hypothesis H_0 and the alternative hypothesis H_1 of this test can be formulated as follows:

H_0 : The number of detected drones follows a binomial distribution

with $n = 4$ and $p = 0,7$. (1pt)

H_1 : The number of detected drones does not follow a binomial distribution

with $n = 4$ and $p = 0,7$. (1pt)

Rejecting the null hypothesis only tells us that the number of detected drones does not follow this specific distribution. It can still be binomially distributed, but possibly with a different value for the success probability p. (1pt)

4c [3pt] Calculate the expected frequencies under the null hypothesis H_0.

Uitwerking

We can evaluate the expected frequencies (under H_0) by using the total of 500 independent trials and the binomial distribution with parameters $n = 4$ and $p = 0,7$.

Number of detected drones	**Observed frequencies**	**Expected frequencies**
0	15	$1000 \cdot \text{binompdf}(n = 4; p = 0,7; k = 0) \approx 8,1$
1	105	$1000 \cdot \text{binompdf}(n = 4; p = 0,7; k = 1) \approx 75,6$
2	290	$1000 \cdot \text{binompdf}(n = 4; p = 0,7; k = 2) \approx 264,6$
3	360	$1000 \cdot \text{binompdf}(n = 4; p = 0,7; k = 3) \approx 411,6$
4	230	$1000 \cdot \text{binompdf}(n = 4; p = 0,7; k = 4) \approx 240,1$
Total:	1000	1000

(3pt)

4d [9pt] Perform the hypothesis test at a significance level $\alpha = 0,05$ by computing the p-value.

Uitwerking

The test statistic for a chi-square goodness-of-fit test is given by:

$$X^2 = \frac{(O_0 - E_0)^2}{E_0} + \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \frac{(O_3 - E_3)^2}{E_3} + \frac{(O_4 - E_4)^2}{E_4}$$ (1pt)

where O_i and E_i are respectively the observed and expected frequencies for value $i = 0, 1, 2, 3, 4$. We can use the table of observed and expected frequencies to calculate the observed test statistic

$$\begin{aligned}\chi^2 &= \frac{(15 - 8,1)^2}{8,1} + \ldots + \frac{(230 - 240,1)^2}{240,1} \\ &\approx 26,643.\end{aligned}$$ (3pt)

Under the null hypothesis, the test statistic X^2 follows a chi-square distribution with $\text{df} = 5 - 1 = 4$ degrees of freedom. Hence, we can calculate the p-value as the probability of observing a test statistic larger than χ^2: (1pt)

$$\begin{aligned}p = P(\chi^2 > 26,643) &= \chi^2\text{cdf}(\text{lower} = 26,643; \text{upper} = 10^{99}; \text{df} = 4) \\ &\approx 2,3471 \cdot 10^{-5}.\end{aligned}$$ (2pt)

As the p-value is (much) smaller than the significance level α, we reject the null hypothesis. There is ample evidence that the number of detected drones does not follow a binomial distribution with $n = 4$ and $p = 0,8$. (2pt)

4e [3pt] Interpret the result of the hypothesis test using data from the tables of observed and expected frequencies.

Uitwerking

If we consider the table of observed and expected frequencies, we see that the observed frequencies are higher than expected for 0, 1 or 2 detected drones, and lower than expected for 3 or 4 detected drones. It is therefore likely that the (2pt) success probability of the radar is smaller than what the manufacturer claims it to be. (1pt)

Problem 5 (22 points) In a joint project between physical training instructors of the Royal Military Academy (KMA) and the medical service, the relationship is investigated between average sleep duration of a cadet (in hours) and the recovery time after intense physical exertion (in hours).

Recovery time is measured using a muscle soreness index; a cadet is considered fully recovered when this index falls below a certain threshold.

For nine cadets, the average sleep duration and recovery time after intense exertion are measured.

Sleep duration (in hours)	$4,0$	$4,5$	$5,0$	$5,5$	$6,0$	$6,5$	$7,0$	$7,5$	$8,0$
Recovery time (in hours)	66	61	63	62	65	64	57	59	60

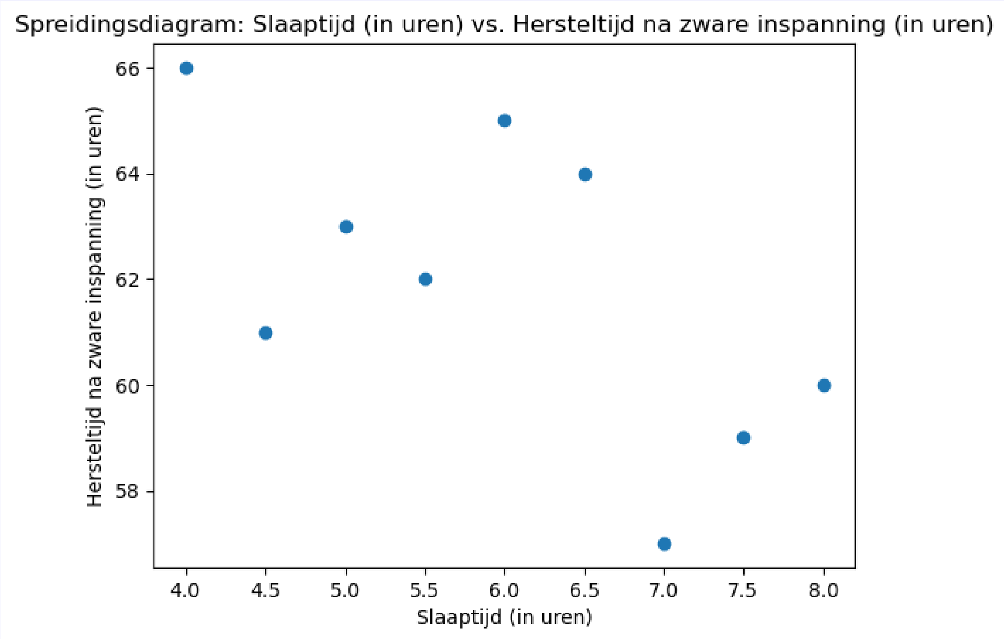
5a [2pt] If we want to perform a regression analysis, which variable would be the dependent variable Y and which variable the independent variable X?

Uitwerking

In regression analysis, it makes the most sense to take sleep time as the independent variable X and recovery time as the dependent variable Y. A longer recovery time does not explain why someone sleeps longer, but the other way around might make sense. (2pt)

5b [5pt] Draw the corresponding scatter plot based on your answer to subquestion (a).

Uitwerking



5c [8pt] Calculate Pearson's correlation coefficient $r(x, y)$. What can you conclude about the relationship between the two variables?

Uitwerking

We start by calculating Pearson's correlation coefficient:

$$r(x, y) = \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\sqrt{(\overline{x^2} - \overline{x}^2) \cdot (\overline{y^2} - \overline{y}^2)}}$$

We use the following table to find the necessary values:

x	y	xy	x^2	y^2
4	66	264	16	4356
4,5	61	274,5	20,25	3721
5	63	315	25	3969
5,5	62	341	30,25	3844
6	65	390	36	4225
6,5	64	416	42,25	4096
7	57	399	49	3249
7,5	59	442,5	56,25	3481
8	60	480	64	3600
$\overline{x} = 6$	$\overline{y} = 61,8889$	$\overline{xy} = 369,1111$	$\overline{x^2} = 37,6667$	$\overline{y^2} = 3837,8889$

(4pt)

Now we compute Pearson's correlation coefficient as follows:

$$\begin{aligned} r(x,y) &= \frac{\overline{x \cdot y} - \overline{x} \cdot \overline{y}}{\sqrt{(\overline{x}^2 - \overline{x^2}) \cdot (\overline{y}^2 - \overline{y^2})}} \\ &= \frac{369,1111 - 6 \cdot 61,8889}{\sqrt{(6^2 - 37,6667) \cdot (61,8889^2 - 3837,8889)}} \\ &= \frac{-2,2222}{3,5717} \\ &\approx -0,6222. \end{aligned}$$

(3pt)

The correlation coefficient is clearly negative and not close to zero, indicating a fairly strong negative trend. However, there is still some uncertainty in the exact linear relationship. (1pt)

5d [7pt] Calculate the regression line $Y = a + b \cdot X$ by computing the coefficients a and b. Based on this regression line, give a statistically sound prediction for the recovery time of a cadet who has slept for 6 hours and 45 minutes.

Uitwerking

In order to compute the coefficients a and b, we reuse the table from subquestion (a). We have that

$$
\begin{aligned}
b &= \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\overline{x^2} - (\overline{x})^2} \\
&= \frac{369,1111 - 6 \cdot 61,8889}{37,6667 - (6)^2} \\
&= \frac{-2,2222}{1,6667} \approx -1,3333 \\
a &= \overline{y} - b \cdot \overline{x} && \text{(2pt)} \\
&= 61,8889 - (-1,3333 \cdot 6) \\
&\approx 69,8889. && \text{(2pt)}
\end{aligned}
$$

The formula of the regression line is equal to $Y = 69,8889 - 1,3333 \cdot X$. A statistically sound prediction of the recovery time of a cadet who slept for 6 hours and 45 minutes is found by filling in $X = 6,75$: This results in a predicted recovery time of $Y = 69,8889 - 1,3333 \cdot 6,75 = 60,88887$ hours, or just under 61 hours. (1pt) (2pt)