Homework 1

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Chapter 1

1.1

2. Which of these are propositions? What are the truth values of those that are propositions?
   a. Do not pass go.
   b. What time is it?
   c. There are no black flies in Maine.
   d. 4 + x = 5.
   e. The moon is made of green cheese.
   f. 2n ≥ 100.

   Answer:c and e are both propositions, and both of their truth values are false.

4. What is the negation of each of these propositions?
   a. Jennifer and Teja are friends.
   b. There are 13 items in a baker's dozen.
   c. Abby sent more than 100 text messages every day.
   d. 121 is a perfect square.

   Answer:

   a. Jennifer and Teja are not friends.
   b. There aren't 13 items in a baker's dozen.
   c. Abby sent less than or equal to 100 text messages every day.
   d. 121 is not a perfect square.

6. Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.
   a. Smartphone B has the most RAM of these three smartphones.
   b. Smartphone C has more ROM or a higher resolution camera than Smartphone B.
   c. Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
   d. If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
   e. Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

   Answer:

   a. True
   b. True
   c. False
   d. False
   e. False

1.2

For exercises 2 & 4, translate into propositional logic.

2. You can see the movie only if you are over 18 years old or you have the permission of a parent. Express your answer in terms of m: "You can see the movie," e: "You are over 18 years old," and p: "You have the permission of a parent."

   Answer:$m \to (e \vee p)$

4. To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service. Express your answer in terms of w: "You can use the wire- less network in the airport," d: "You pay the daily fee," and s: "You are a subscriber to the service."

   Answer:$w \to (d \vee s)$

1.3

6. Use a truth table to verify the first De Morgan law

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Answer:

p	q	$\neg p$	$\neg q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

8. Use De Morgan's laws to find the negation of each of the following statements

   a. Kwame will take a job in industry or go to graduate school.
   b. Yoshiko knows Java and calculus.
   c. James is young and strong.
   d. Rita will move to Oregon or Washington.

   Answer:

   a. Kwame will not take a job in industry and not go to graduate school.
   b. Yoshiko doesn't know Java or doesn't know calculus.
   c. James is not young or not strong.
   d. Rita will not move to Oregon and will not move to Washington.

10. Show that each of these conditional statements is a tautology by using truth tables.

   a. $[\neg p \wedge (p \vee q)] \to q$
   b. $[(p \to q) \wedge (q \to r)] \to (p \to r)$
   c. $[p \wedge (p \to q)] \to q$
   d. $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

   Answer:

a.

p	q	$\neg p$	$p \vee q$	$[\neg p \wedge (p \vee q)]$	$[\neg p \wedge (p \vee q)] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	T	T	T

b.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$[(p \to q) \wedge (q \to r)]$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

c.

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

d.

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

12 Show that each conditional statement in Exercise 10 is a tautology without using truth tables.

a. $[\neg p \wedge (p \vee q)] \to q$

b. $[(p \to q) \wedge (q \to r)] \to (p \to r)$

c. $[p \wedge (p \to q)] \to q$

d. $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

Answer:

a.

$$
\begin{aligned}
[\neg p \wedge (p \vee q)] \to q &\equiv \neg[\neg p \wedge (p \vee q)] \vee q && \text{(Equivalence from table)}\\
&\equiv [\neg(\neg p) \vee \neg(p \vee q)] \vee q && \text{(De Morgan's Law)}\\
&\equiv [p \vee (\neg p \wedge \neg q)] \vee q && \text{(Double negation and De Morgan's Law)}\\
&\equiv [(p \vee \neg p) \wedge (p \vee \neg q)] \vee q && \text{(Distributive Law)}\\
&\equiv [\mathbf{T} \wedge (p \vee \neg q)] \vee q && \text{(Negation Law)}\\
&\equiv [(\mathbf{T} \wedge p) \vee (\mathbf{T} \wedge \neg q)] \vee q && \text{(Distributive Law)}\\
&\equiv (p \vee \neg q) \vee q && \text{(Identity Law)}\\
&\equiv p \vee (\neg q \vee q) && \text{(Associativity)}\\
&\equiv p \vee \mathbf{T} && \text{(Negation Law)}\\
&\equiv \mathbf{T} && \text{(Domination Law)}
\end{aligned}
$$

b.

$$
\begin{aligned}
[(p \to q) \wedge (q \to r)] \to (p \to r) &\equiv \neg[(p \to q) \wedge (q \to r)] \vee (p \to r) && \text{(Equivalence from table)}\\
&\equiv [\neg(p \to q) \vee \neg(q \to r)] \vee (p \to r) && \text{(De Morgan's Law)}\\
&\equiv [(p \wedge \neg q) \vee (q \wedge \neg r)] \vee (p \to r) && \text{(Equivalence from table)}\\
&\equiv [(p \wedge \neg q) \vee (q \wedge \neg r)] \vee (\neg p \vee r) && \text{(Equivalence from table)}\\
&\equiv [(p \wedge \neg q) \vee (\neg p \vee r)] \vee (q \wedge \neg r) && \text{(Commutative \& Associative Law)}\\
&\equiv [[(\neg p \vee r) \vee p] \wedge [(\neg p \vee r) \vee \neg q]] \vee (q \wedge \neg r) && \text{(Distributive Law)}\\
&\equiv [[(p \vee \neg p) \vee r] \wedge [(\neg p \vee r) \vee \neg q]] \vee (q \wedge \neg r) && \text{(Commutative \& Associative Law)}\\
&\equiv [(\mathbf{T} \vee r) \wedge [(\neg p \vee r) \vee \neg q]] \vee (q \wedge \neg r) && \text{(Negation Law)}\\
&\equiv [\mathbf{T} \wedge [(\neg p \vee r) \vee \neg q]] \vee (q \wedge \neg r) && \text{(Domination Law)}\\
&\equiv [(\neg p \vee r) \vee \neg q] \vee (q \wedge \neg r) && \text{(Domination Law)}\\
&\equiv [[(\neg p \vee r) \vee \neg q] \vee q] \wedge [[(\neg p \vee r) \vee \neg q] \vee \neg r] && \text{(Distributive Law)}\\
&\equiv [(\neg p \vee r) \vee (\neg q \vee q)] \wedge [(\neg p \vee \neg q) \vee (r \vee \neg r)] && \text{(Commutative \& Associative Law)}\\
&\equiv [(\neg p \vee r) \vee \mathbf{T}] \wedge [(\neg p \vee \neg q) \vee \mathbf{T}] && \text{(Negation Law)}\\
&\equiv \mathbf{T} \wedge \mathbf{T} && \text{Domination Law}\\
&\equiv \mathbf{T} && \text{(Identity Law)}
\end{aligned}
$$

c.

$$\begin{aligned}
[p \wedge (p \to q)] \to q &\equiv \neg[p \wedge (p \to q)] \vee q && \text{(Equivalence from table)}\\
&\equiv [\neg p \vee \neg(p \to q)] \vee q && \text{(De Morgan's Law)}\\
&\equiv [\neg p \vee (p \wedge \neg q)] \vee q && \text{(Equivalence from table)}\\
&\equiv [(\neg p \vee p) \wedge (\neg p \vee \neg q)] \vee q && \text{(Distributive Law)}\\
&\equiv [\mathbf{T} \wedge (\neg p \vee \neg q)] \vee q && \text{(Negation Law)}\\
&\equiv (\neg p \vee \neg q) \vee q && \text{(Identity Law)}\\
&\equiv \neg p \vee (\neg q \vee q) && \text{(Associative Law)}\\
&\equiv \neg p \vee \mathbf{T} && \text{(Negation Law)}\\
&\equiv \mathbf{T} && \text{(Domination Law)}
\end{aligned}$$

d.

$$\begin{aligned}
[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r &\equiv [(p \vee q) \wedge [(p \to r) \wedge (q \to r)]] \to r && \text{(Associative Law)}\\
&\equiv [(p \vee q) \wedge [(p \vee q) \to r]] \to r && \text{(Equivalence from table)}\\
&\equiv \neg[(p \vee q) \wedge [\neg(p \vee q) \vee r]] \vee r && \text{(Equivalence from table)}\\
&\equiv [\neg(p \vee q) \vee \neg[\neg(p \vee q) \vee r]] \vee r && \text{(De Morgan's Law)}\\
&\equiv [\neg(p \vee q) \vee [(p \vee q) \wedge \neg r]] \vee r && \text{(De Morgan's + Double Negation)}\\
&\equiv [\neg(p \vee q) \vee (p \vee q) \wedge [\neg(p \vee q) \vee \neg r]] \vee r && \text{(Distributive Law)}\\
&\equiv [\mathbf{T} \wedge [\neg(p \vee q) \vee \neg r]] \vee r && \text{(Negation Law)}\\
&\equiv [\neg(p \vee q) \vee \neg r] \vee r && \text{(Identity Law)}\\
&\equiv \neg(p \vee q) \vee (\neg r \vee r) && \text{(Associative Law)}\\
&\equiv \neg(p \vee q) \vee \mathbf{T} && \text{(Negation Law)}\\
&\equiv \mathbf{T} && \text{(Domination Law)}
\end{aligned}$$

1.4

6. Let N (x) be the statement "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.

   a. $\exists x N(x)$
   b. $\forall x N(x)$
   c. $\neg\exists x N(x)$
   d. $\exists x \neg N(x)$
   e. $\neg\forall x N(x)$
   f. $\forall x \neg N(x)$

   Answer:

   a. There exists a student in my school who has visited North Dakota.
   b. Every student in my school has visted North Dakota.
   c. No student in my school has visted North Dakota.
   d. At least one student in my school has not visted North Dakota.
   e. At least one student in my school has not visted North Dakota.
   f. No student in my school has visted North Dakota.

8. Translate these statements into English, where R(x) is "x is a rabbit" and H (x) is "x hops" and the domain consists of all animals.

   a. $\forall x(R(x) \to H(x))$
   b. $\forall x(R(x) \wedge H(x))$
   c. $\exists x(R(x) \to H(x))$
   d. $\exists x(R(x) \wedge H(x))$

Answer:

a. Every rabbit hops.
b. All animals are rabbits who hop.
c. There exists an animal that if they are a rabbit, then they hop.
d. Some rabbits hop.

14. Determine the truth value of each of these statements if the domain consists of all real numbers.

    a. $\exists x(x^3 = -1)$
    b. $\exists x(x^4 < x^2)$
    c. $\forall x((-x)^2 = x^2)$
    d. $\forall x(2x > x)$

    a. True, x = -1
    b. True, x = 0.1
    c. True, squaring returns a positive result
    d. False, x = -1 returns false.

1.5

2. Translate these statements into English, where the domain for each variable consists of all real numbers.

    a. $\exists x \forall y(xy = y)$
    b. $\forall x \forall y((((x \geq 0) \wedge y < 0)) \rightarrow (x - y > 0)$
    c. $\forall x \forall y \exists z(x = y + z)$

    Answer:

    a. There exists an x such that for all y, $xy = y$, x and y are real.
    b. For all pairs of x and y, both real, if $(x \geq 0)$ and $(y < 0)$, then $x - y > 0$.
    c. Let x, y and z be real numbers; for all pairs x and y there exists a z such that $x = y + z$.

6. Let C(x, y) mean that student x is enrolled in class y, where the domain for x consists of all students in your school and the domain for y consists of all classes being given at your school. Express each of these statements by a simple English sentence.

    a. $C(\text{Randy Goldberg}, \text{CS 252})$
    b. $\exists x C(x, \text{Math 695})$
    c. $\exists y C(\text{Carol Sitea}, y)$
    d. $\exists x(C(x, \text{Math 222}) \wedge C(x, \text{CS 252}))$
    e. $\exists x \exists y \forall z((x = y) \wedge (C(x, z) \rightarrow C(y, z)))$
    f. $\exists x \exists y \forall z((x = y) \wedge (C(x, z) \leftrightarrow C(y, z)))$

    Answer:

    a. Randy Goldberg is enrolled in CS 252.
    b. There exists a student in my school that is enrolled in Math 695.
    c. Carol Sitea is taking some class at my school.
    d. There exists a student who is enrolled in Math 22 and CS 252.

10. Let $F(x, y)$ be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

    a. Everybody can fool Fred.
    b. Evelyn can fool everybody.
    c. Everybody can fool somebody.

d. There is no one who can fool everybody.
e. Everyone can be fooled by somebody.
f. No one can fool both Fred and Jerry.
g. Nancy can fool exactly two people.
h. There is exactly one person whom everybody can fool.
i. No one can fool himself or herself.
j. There is someone who can fool exactly one person besides himself or herself.

Answer:

a. $\forall x F(x, \text{Fred})$
b. $\forall x F(\text{Evelyn}, x)$
c. $\forall x \exists y F(x, y)$
d. $\forall x \exists y \neg F(x, y)$
e. $\forall x \exists y F(y, x)$
f. $\forall x(\neg F(x, \text{Fred}) \vee \neg F(x, \text{Jerry}))$
g. $\exists x \exists y(x \neq y \wedge F(\text{Nancy}, x) \wedge F(\text{Nancy}, y))$
h. $\exists x \forall y(F(y, x) \wedge \forall z F(F(y, z) \rightarrow z = x))$
i. $\forall x \neg F(x, x)$
j. $\exists x \exists y(x \neq y \wedge F(x, y) \wedge \forall z F(x, z) \rightarrow z = y)$

1.6

2. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

	If George does not have eight legs, then he is not a spider.
	George is a spider
$\therefore$	George has eight legs

Answer: Yes the conclusion is true because of modus tollens.

4. What rule of inference is used in each of these arguments?

a. Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
b. It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
c. Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
d. Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.
e. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Answer:

a. Simplification
b. Disjunctive syllogism
c. Modus ponens
d. Addition
e. Hypothetical syllogism

8 What rules of inference are used in this argument? “No man is an island. Manhattan is an island. Therefore, Manhattan is not a man.”

Answer: Modus ponens

1.7

6. Use a direct proof to show that the product of two odd numbers is odd.

   Proof. Let x and y be odd. This means that $x = 2k + 1$, $y = 2s + 1$; k, s $\in \mathbb{Z}$.
   Then:

$$\begin{aligned} x * y &= (2k + 1) \cdot (2s + 1) \\ &= 4ks + 2k + 2s + 1 \\ &= 2(2ks + k + s) + 1 \\ &= 2r + 1 \end{aligned}$$

   Which is an odd number where $r = 2ks + k + s$. ■

18. Prove that if n is an integer and 3n + 2 is even, then n is even using

    a. a proof by contraposition.
    b. a proof by contradiction.

    a. *Proof.* Assume that n is odd, so n = 2k + 1 Then:

$$\begin{aligned} 3n + 2 &= 3(2k + 1) + 2 \\ &= 6k + 3 + 2 \\ &= 6k + 4 + 1 \\ &= 2(3k + 2) + 1 \\ &= 2r + 1 \end{aligned}$$

    Where $r \in \mathbb{Z}$
    So when n is odd then, 3n + 2 is odd is true, and the contrapositive that when 3n + 2 is even, then n is even is also true. ■

    b. *Proof.* Assume that $3n+2$ is even and n is odd. Because n is odd there is an integer k such that $n = 2k+1$ Then:

$$\begin{aligned} 3n + 2 &= 3(2k + 1) + 2 \\ &= 6k + 3 + 2 \\ &= 6k + 4 + 1 \\ &= 2(3k + 2) + 1 \\ &= 2r + 1 \end{aligned}$$

    So $3n + 2 = 2r + 1$ where $r = 3k + 1$. Which shows $3n + 2$ is odd. We have 3n + 2 both even and odd which is a contradiction, which means our assumption that n is odd is false thus n is even. ■

24. Show that at least three of any 25 days chosen must fall in the same month of the year.

    Proof. Assume that no more than 2 of any 25 days chosen must fall in the same month of the year. Then because we have 12 months in a year at most 24 days can be chosen as each month can have at most 2 days, however this contradicts the premise that there are 25 days chosen so at least 3 of any 25 days must fall in the same month of the year ■

28. Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$

    Proof. $(\leftarrow)$ WTS: $(m = n \vee m = -n) \rightarrow m^2 = n^2$
    Let $m = n$ then $m^2 = m \cdot m = n \cdot n = n^2$
    Let $m = -n$, then $m^2 = m \cdot m = -n \cdot -n = n^2$

($\rightarrow$) WTS: $m^2 = n^2 \rightarrow (m = n \lor m = -n)$
Assume $m \neq n$ and $m \neq -n$

$$
\begin{aligned}
m \neq n & \qquad \text{Premise} \\
m \cdot m \neq n \cdot m & \qquad \text{Multiply both sides by m} \\
m^2 \neq m \cdot n \neq n \cdot n \neq n^2 & \qquad \text{True from premise} \\
m \neq -n & \\
m \cdot m \neq m \neq -n & \\
m^2 \neq m \cdot -n \neq -n \cdot -n \neq n^2 & \qquad \text{Same steps as above}
\end{aligned}
$$

Contrapositive is true so the original statement is also true.
Have shown both implications is to so the biconditional is true. ∎

Chapter 2

2.1

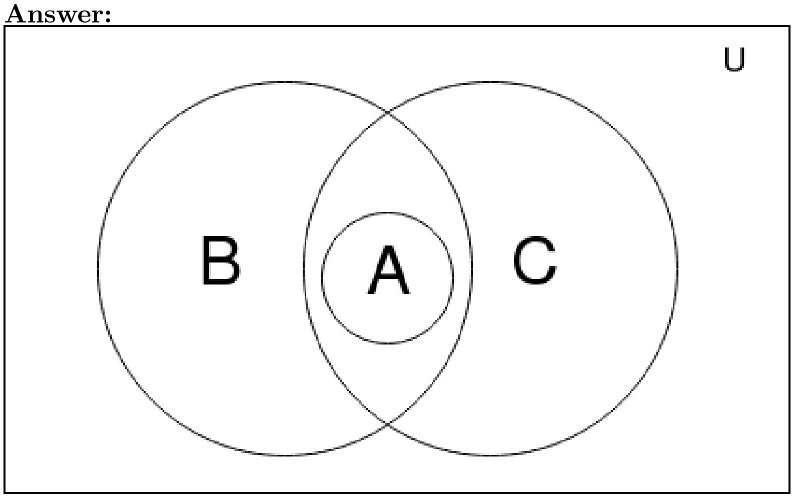
10. Determine whether these statements are true or false.

   a. $\emptyset \in \{\emptyset\}$
   b. $\emptyset \in \{\emptyset, \{\emptyset\}\}$
   c. $\{\emptyset\} \in \{\emptyset\}$
   d. $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
   e. $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
   f. $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

   Answer:

   a. True
   b. True
   c. False
   d. True
   e. True
   f. True
   g. False

16. Use a Venn diagram to illustrate the relationship $A \subset B$ and $A \subset C$
   Answer:



24. Determine whether each of these sets is the power set of a set, where a and b are distinct elements.

   a. $\emptyset$

b. $\{\emptyset, \{a\}\}$
c. $\{\emptyset, \{a\}, \{\emptyset, a\}\}$
d. $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Answer:

1. No
2. Yes, $P(\{a\})$
3. No
4. Yes, $P(\{a, b\})$

32. Let $A = \{a, b, c\}, B = \{x, y\}, \text{and} C = \{0, 1\}$. Find

a. $A \times B \times C$.
b. $C \times B \times A$.
c. $C \times A \times B$.
d. $B \times B \times B$.

Answer:

a. $A \times B \times C = \{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$
b. $C \times B \times A = \{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$
c. $C \times A \times B = \{(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y), (1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c, x), (1, c, y)\}$
d. $B \times B \times B = \{(x, x, x), (x, x, y), (x, y, x), (x, y, y), (y, x, x), (y, x, y), (y, y, x), (y, y, y)\}$

2.2

2. Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B.

a. The set of sophomores taking discrete mathematics in your school
b. The set of sophomores at your school who are not taking discrete mathematics
c. The set of students at your school who either are sophomores or are taking discrete mathematics
d. The set of students at your school who either are not sophomores or are not taking discrete mathematics

a. $A \cap B$
b. $A - B$
c. $A \cup B$
d. $\overline{A} \cup \overline{B}$

4. Let $A = \{a, b, c, d, e\} \text{and} B = \{a, b, c, d, e, f, g, h\}$.
Find.

a. $A \cup B$
b. $A \cap B$
c. $A - B$
d. $B - A$

Answer:

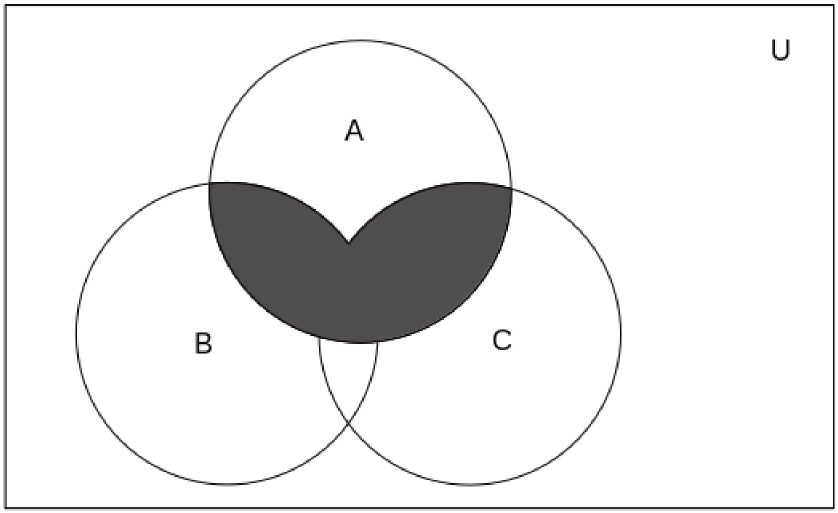
a. $\{a, b, c, d, e, f, g, h\}$
b. $\{a, b, c, d, e\}$
c. $\emptyset$
d. $\{f, g, h\}$

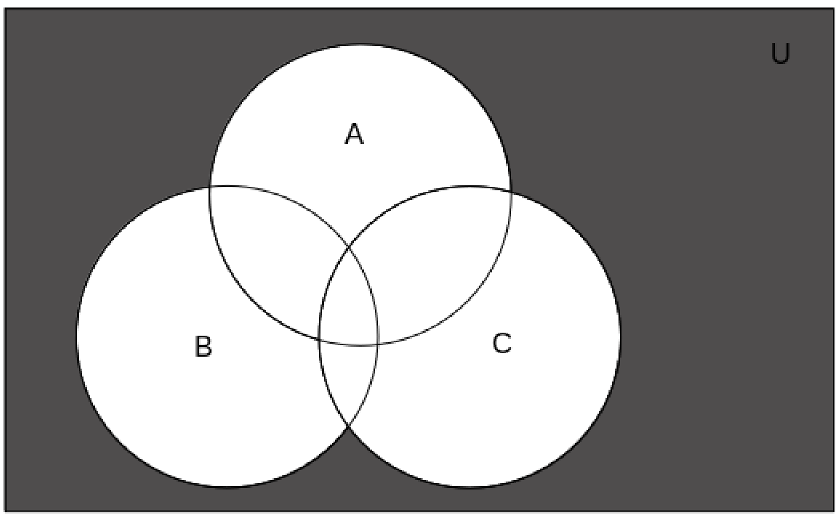
26. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

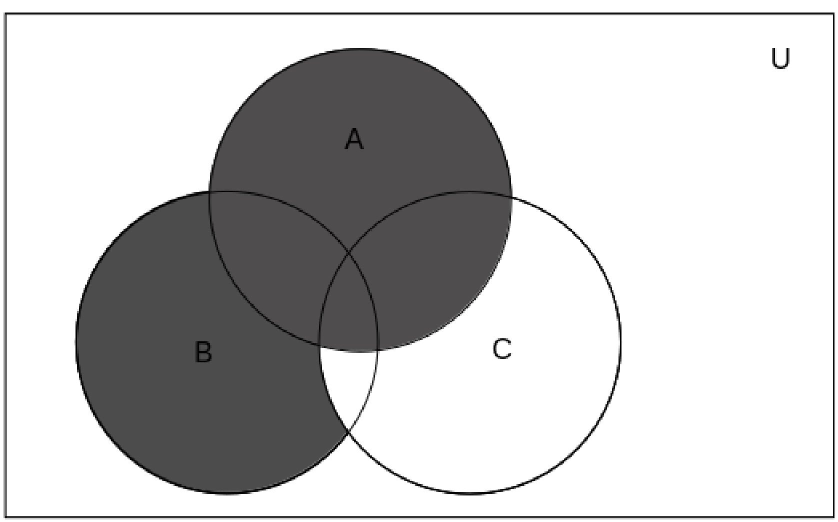
a. $A \cap (B \cup C)$

b. $\overline{A} \cap \overline{B} \cap \overline{C}$

c. $(A - B) \cup (A - C) \cup (B - C)$

a. 

b. 

c. 

2.3

8. Find these values.

a. $\lfloor 1.1 \rfloor$

b. $\lceil 1.1 \rceil$
c. $\lfloor -0.1 \rfloor$
d. $\lceil -0.1 \rceil$
e. $\lceil 2.99 \rceil$
f. $\lfloor -2.99 \rfloor$
g. $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$
h. $\lceil \lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil + \frac{1}{2} \rceil$

Answer:

a. 1
b. 2
c. -1
d. 0
e. 3
f. -2
g. 1
h. 2

10. Determine whether each of these functions from $\{a, b, c, d\}$ to itself is one-to-one.

a. $f(a) = b, f(b) = a, f(c) = c, f(d) = d$
b. $f(a) = b, f(b) = b, f(c) = d, f(d) = c$
c. $f(a) = d, f(b) = b, f(c) = c, f(d) = d$

Answer:

a. Yes
b. No
c. No

12. Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if

a. $f(m, n) = 2m - n$
b. $f(m, n) = m^2 - n^2$
c. $f(m, n) = m + n + 1$
d. $f(m, n) = |m| - |n|$
e. $f(m, n) = m^2 - 4$

Answer:

a. Yes, to get any $x \in \mathbb{Z}$ you can do $f(0, -x)$
b. No, for example it cannot map to 1
c. Yes, to get any $x \in \mathbb{Z}$ you can do $f(x, -1)$
d. Yes, to get any $x \in \mathbb{Z}^+$ you can do $f(x, 0)$ and to get any $y \in \mathbb{Z}^-$ you can do $f(0, y)$ and to get $f(m, n) = 0$ you can do $f(0, 0)$.
e. No, function cannot map to anything less than -4.

20 Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ that is

a. One-to-one, but not onto.
b. Onto, but not one-to-one.
c. Both onto and one-to-one.(Not the identity)
d. Neither one-to-one or onto

Answer:

a. $f(x) = x + 1$

b. $f(x) = \begin{cases} 0 & x = 0 \\ x - 1 & x \geq 1 \end{cases}$

c. $f(x) = \begin{cases} 1 & x = 0 \\ 0 & x = 1 \\ x & x > 1 \end{cases}$

d. $f(x) = 1$

2.4

2. What is the term a_8 of the sequence $\{a_n\}$ if a_n equals

   a. 2^{n-1}
   b. 7
   c. $1 + (-1)^n$
   d. $-(-2)^n$

   Answer:

   a. 128
   b. 7
   c. 2
   d. -256

4. What are the terms a_0 , a_1 , a_2 , and a_3 of the sequence $\{a_n\}$, where a_n equals

   a. $(-2)^n$
   b. 3
   c. $7 + 4^n$
   d. $2^n + (-2)^n$

   Answer:

   a. 1, -2, 4, -8
   b. 3, 3, 3, 3
   c. 8, 11, 24, 71
   d. 2, 0, 8, 0

6. List the first 10 terms of each of these sequences.

   a. the sequence obtained by starting with 10 and obtaining each term by subtracting 3 from the previous term
   b. the sequence whose nth term is the sum of the first n positive integers
   c. the sequence whose nth term is $3^n - 2^n$
   d. the sequence whose nth term is $\lfloor \sqrt{n} \rfloor$
   e. the sequence whose first two terms are 1 and 5 and each succeeding term is the sum of the two previous terms
   f. the sequence whose nth term is the largest integer whose binary expansion (defined in Section 4.2) has n bits (Write your answer in decimal notation.)
   g. the sequence whose terms are constructed sequentially as follows: start with 1, then add 1, then multiply by 1, then add 2, then multiply by 2, and so on
   h. the sequence whose nth term is the largest integer k such that $k! \leq n$

   Answer:

   a. 10, 7, 4, 1, -2, -5, -8, -11, -14, -17
   b. 1, 3, 6, 10, 15, 21, 28, 36, 45, 55
   c. 1, 5, 19, 65, 211, 665, 2059, 6305, 19171, 58025

d. 1, 1, 1, 2, 2, 2, 2, 2, 3, 3

e. 1, 5, 6, 11, 17, 28, 45, 73, 118, 191

f. 1, 3, 7, 15, 31, 63, 127, 255, 511, 1023

g. 1, 2, 2, 4, 8, 11, 33, 37, 148, 153

h. 1, 2, 2, 2, 2, 3, 3, 3, 3, 3

2.6

2. Find $A + B$

a. $A = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 2 & 2 \\ 0 & -2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 3 & 5 \\ 2 & 2 & 3 \\ 2 & -3 & 0 \end{bmatrix}$

b. $A = \begin{bmatrix} -1 & 0 & 5 & 6 \\ -4 & -3 & 5 & -2 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 9 & -3 & 4 \\ 0 & -2 & -1 & 2 \end{bmatrix}$

Answer:

a. $A + B = \begin{bmatrix} 0 & 3 & 9 \\ 1 & 4 & 5 \\ 2 & -5 & -3 \end{bmatrix}$

b. $A + B = \begin{bmatrix} -4 & 9 & 2 & 10 \\ -4 & -5 & 4 & 0 \end{bmatrix}$

4. Find the product AB, where

a. $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & 2 \\ 2 & 1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 & 2 & 3 \\ -1 & 0 & 3 & -1 \\ -3 & -2 & 0 & 2 \end{bmatrix}$

c. $A = \begin{bmatrix} 0 & -1 \\ 7 & 2 \\ -4 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -1 & 2 & 3 & 0 \\ -2 & 0 & 3 & 4 & 1 \end{bmatrix}$

Answer:

a. $AB = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}$

b. $AB = \begin{bmatrix} 4 & -1 & -7 & 6 \\ -7 & -5 & 6 & 5 \\ -5 & 0 & 7 & 3 \end{bmatrix}$

c. $AB = \begin{bmatrix} 2 & 0 & -3 & -4 & -1 \\ 24 & -7 & 20 & 29 & 2 \\ -10 & 4 & -17 & -24 & -3 \end{bmatrix}$

26. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Find

a. $A \vee B$.

b. $A \wedge B$

c. $A \odot B$.

Answer:

a. $A \vee B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

b. $A \wedge B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

c. $A \odot B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Chapter 3

3.1

4. Describe an algorithm that takes as input a list of n integers and produces as output the largest difference obtained by subtracting an integer in the list from the one following it.
   Answer:
   Go through the list starting at first element ending at the second to last element and sent the max_difference to zero. Take the absolute value of the difference between the element you are on and the next one. Compare the value to max_difference and replace if it is larger.

12. Describe an algorithm that uses only assignment statements that replaces the triple (x, y, z) with (y, z, x). What is the minimum number of assignment statements needed?
    Answer:
    temp = z
    z = x
    x = y
    y = temp

    Min is 4 statements

34. Use the bubble sort to sort 6, 2, 3, 1, 5, 4, showing the lists obtained at each step.
    Answer:
    2, 3, 1, 5, 6
    2, 1, 3, 5, 6
    1, 2, 3, 5, 6

3.2

2. Determine whether each of these functions is $\mathcal{O}(x^2)$.

   a. $f(x) = 17x + 11$
   b. $f(x) = x^2 + 1000$
   c. $f(x) = x \log x$
   d. $f(x) = x^4/2$
   e. $f(x) = 2^x$
   f. $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

   Answer:

   a. Yes
   b. Yes
   c. Yes
   d. No
   e. No
   f. Yes

24. Suppose that you have two different algorithms for solving a problem. To solve a problem of size n, the first algorithm uses exactly $n^2 \cdot 2^n$ operations and the second algorithm uses exactly $n!$ operations. As n grows, which algorithm uses fewer operations?

    Answer: $n^2 \cdot 2^n$ grows faster as n grows.

3.3

2. Give a big-$\mathcal{O}$ estimate for the number additions used in this segment of an algorithm.

$t := 0$
$for i := 1$ to n
 for $j := 1$ to n
  $t := t + i + j$

Answer: $\mathcal{O}(n^2)$

16. What is the largest n for which one can solve within a day using an algorithm that requires $f(n)$ bit operations, where each bit operation is carried out in 10^{-11} seconds, with these functions $f(n)$?

  a. $\log n$
  b. $1000n$
  c. n^2
  d. $1000n^2$
  e. n^3
  f. 2^n
  g. 2^{2n}
  h. 2^{2^n}

Answer:
Note: Max bit operations in a day is $8.64 \cdot 10^{15}$

  a. $2^{(8.64*10^{15})}$
  b. $8.64 * 10^{12}$
  c. 92951600
  d. 2939387
  e. 205197
  f. 52
  g. 26
  h. 5

18. How much time does an algorithm take to solve a problem of size n if this algorithm uses $2n^2 + 2^n$ operations, each requiring 10^{-9} seconds, with these values of n?

  a. 10
  b. 20
  c. 50
  d. 100

Answer:

  a. $1.224 \cdot 10^{-6}$ s
  b. 0.001049376 s
  c. 1125899.906847624 s
  d. $1.2676506002282295 \cdot 10^{21}$ s

Chapter 4

4.1

6. Show that if a, b, c, and d are integers, where $a, b \neq 0$, such that $a \mid c$ and $b \mid d$, then $ab \mid cd$
   Answer:

   Proof. WTS: $ab|cd$ or that $cd/ab \in \mathbb{Z}$
   Let $a, b, c, d \in \mathbb{Z}$ such that $a \mid c$ and $b \mid d$
   $cd/ab = c/a \cdot d/b$
   But since a divides c and b divides be we are multiplying two integers so cd/ab is an integer, thus ab divides cd. ■

10. What are the quotient and remainder when

    a. 44 is divided by 8?
    b. 777 is divided by 21?
    c. -123 is divided by 19?
    d. -1 is divided by 23?
    e. -2002 is divided by 87?
    f. 0 is divided by 17?
    g. 1,234,567 is divided by 1001?
    h. -100 is divided by 101?

    Answer:

    a. Q = 5, r = 4
    b. Q = 37, r = 0
    c. Q = -7, r = 10
    d. Q = -1, r =22
    e. Q = 24, r = 86
    f. Q = 0, r = 0
    g. Q = 1233, r = 324
    h. Q = -1, r = 1

28. Decide whether each of these integers is congruent to 3 modulo 7.

    a. 37
    b. 66
    c. -17
    d. -67

    Answer:

    a. No
    b. Yes
    c. No
    d. Yes

4.2

2. Convert the decimal expansion of each of these integers to a binary expansion.

   a. 321
   b. 1023
   c. 100632

   Answer:

   a. 1 0010 0001
   b. 11 1111 1111
   c. 1010 1111 1000 1000

4. Convert the binary expansion of each of these integers to a decimal expansion.

   a. $(1\ 1011)_2$
   b. $(10\ 1011\ 0101)_2$
   c. $(11\ 1011\ 1110)_2$
   d. $(111\ 1100\ 0001\ 1111)_2$

   Answer:

   a. $16 + 8 + 2 + 1 = 27$
   b. $512 + 128 + 32 + 16 + 4 + 1 = 693$
   c. $512 + 256 + 128 + 32 + 16 + 8 + 4 + 2 = 958$
   d. $16384 + 8192 + 4096 + 2048 + 1012 + 16 + 15 = 31763$

6. Convert the binary expansion of each of these integers to an octal expansion.

   a. $(1111\ 0111)_2$
   b. $(1010\ 1010\ 1010)_2$
   c. $(111\ 0111\ 0111\ 0111)_2$
   d. $(101\ 0101\ 0101\ 0101)_2$

   Answer:

   a. $(367)_8$
   b. $(5252)_8$
   c. $(73567)_8$
   d. $(54545)_8$

8. Convert $(\text{BADFACED})_{16}$ from its hexadecimal expansion to its binary expansion.
   Answer: $(1011\ 1010\ 1101\ 1111\ 1010\ 1100\ 1110\ 1101)_2$

22. Find the sum and product of each of these pairs of numbers. Express your answers as a base 3 expansion.

    a. $(112)_3$, $(210)_3$
    b. $(2112)_3$, $(12021)_3$
    c. $(20001)_3$, $(1111)_3$
    d. $(120021)_3$, $(2002)_3$

    Answer:

    a. sum: $(1022)_3$ product: $(10220)_3$
    b. sum: $(21210)_3$ product: $(111020122)_3$
    c. sum: $(21112)_3$ product: $(22221111)_3$
    d. sum: $(122100)_3$ product: $(1011122112)_3$

4.3

2. Determine whether each of these integers is prime.
   a. 19
   b. 27
   c. 93
   d. 101
   e. 107
   f. 113

   Answer:
   a. Yes
   b. No
   c. No
   d. Yes
   e. Yes
   f. Yes

a. 4. Find the prime factorization of each of these integers.
   a 39
   b 81
   c 101
   d 143
   e 289
   f 899

   Answer:
   a. $3 \cdot 13$
   b. 3^4
   c. 101
   d. $11 \cdot 13$
   e. 17^2
   f. $29 \cdot 31$

16. Determine whether the integers in each of these sets are pairwise relatively prime.
   a. 21, 34, 55
   b. 14, 17, 85
   c. 25, 41, 49, 64
   d. 17, 18, 19, 23

   Answer:
   a Yes
   b No
   c Yes
   d Yes

24. What are the greatest common divisors of these pairs of integers?
   a. $2^2 \cdot 3^3 \cdot 5^5,\ 2^5 \cdot 3^3 \cdot 5^2$
   b. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13,\ 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$
   c. $17,\ 17^{17}$

d. $2^2 \cdot 7,\ 5^3 \cdot 13$
e. $0, 5$
f. $2 \cdot 3 \cdot 5 \cdot 7,\ 2 \cdot 3 \cdot 5 \cdot 7$

Answer:

a. $2^2 \cdot 3^3 \cdot 5^2$
b. $2 \cdot 3 \cdot 11$
c. 17
d. 1
e. 5
f. $2 \cdot 3 \cdot 5 \cdot 7$

26. What is the least common multiple of each pair in Exercise 24?

a. $2^2 \cdot 3^3 \cdot 5^5,\ 2^5 \cdot 3^3 \cdot 5^2$
b. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13,\ 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$
c. $17,\ 17^{17}$
d. $2^2 \cdot 7,\ 5^3 \cdot 13$
e. $0, 5$
f. $2 \cdot 3 \cdot 5 \cdot 7,\ 2 \cdot 3 \cdot 5 \cdot 7$

Answer:

a. $2^5 \cdot 3^3 \cdot 5^5$
b. $2^{11} \cdot 3^9 \cdot 5 \cdot 7 \cdot 11 \cdot 17^{14}$
c. 17^{17}
d. $2^2 \cdot 5^3 \cdot 7 \cdot 13$
e. Undefined
f. $2 \cdot 3 \cdot 5 \cdot 7$

4.6

2. Encrypt the message STOP POLLUTION by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters.

a. $f(p) = (p + 4) \mod 26$
b. $f(p) = (p + 21) \mod 26$
c. $f(p) = (17p + 22) \mod 26$

Answer:

a. wxst tsppyxmsr
b. nojk kjggpodji
c. qhar rabbyhcaj

4. Decrypt these messages that were encrypted using the Caesar cipher.

a. EOXH MHDQV
b. WHVW WRGDB
c. HDW GLP VXP

Answer:

1. BLUE JEANS
2. TEST TODAY
3. EAT DIMSUM