Homework 3

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Chapter 9

9.1

2. a. List all the ordered pairs in the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$.
   b. Display this relation graphically, as was done in Example 4.
   c. Display this relation in tabular form, as was done in Example 4.

   Answer:

   a. $\{(1,1),\ (1,2),\ (1,3),\ (1,4),\ (1,5),\ (1,6),\ (2,2),\ (2,4),\ (2,6),\ (3,3),\ (3,6),\ (4,4),\ (5,5)\ (6,6)\}$

4. Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in$ R if and only if

   a. a is taller than b.
   b. a and b were born on the same day.
   c. a has the same first name as b.
   d. a and b have a common grandparent.

   Answer:

   a. antisymmetric and transitive
   b. reflexive, symmetric, and transitive
   c. reflexive, symmetric, and transitive
   d. reflexive, symmetric

10. Give an example of a relation on a set that is

    a. both symmetric and antisymmetric.
    b. neither symmetric nor antisymmetric.

    Answer:

    a. A relation where the number maps to only itself
    b. Set {1,2,3,4} R = {(1,2), (2,1)}

9.3

2. Represent each of these relations on 1, 2, 3, 4 with a matrix (with the elements of this set listed in increasing order).

   a. {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}
   b. {(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)}
   c. {(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)}
   d. {(2, 4), (3, 1), (3, 2), (3, 4)}

   Answer:

   a. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   b. $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

   c. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

4. List the ordered pairs in the relations on $\{1, 2, 3, 4\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

   a. $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

   b. $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

   c. $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

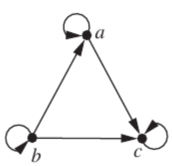
   Answer:

   a. {(1,1), (1,2), (1,4), (2,1), (2,3), (3,2), (3,3), (3,4), (4,1), (4,3), (4,4)}
   b. {(1,1), (1,2), (1,3), (2,2), (3,3), (3,4), (4,1) , (4,4)}
   c. {(1,2), (1,4), (2,1), (2,3), (3,2), (3,4), (4,1), (4,3)}

8. Determine whether the relations represented by the ma- trices in Exercise 4 are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.
   Answer:

   a. symmetric
   b. reflexive and antisymmetric
   c. irreflexive and symmetric

24. List order pairs in relation represented by the directed graph.



   Answer:
   $\{(a, a),\ (a, c),\ (b, b),\ (b, a),\ (b, c),\ (c, c)\}$

9.5

2. Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack.

   a. $\{(a, b) \mid a$ and b are the same age$\}$
   b. $\{(a, b) \mid a$ and b have the same parents$\}$
   c. $\{(a, b) \mid a$ and b share a common parent$\}$
   d. $\{(a, b) \mid a$ and b have met$\}$
   e. $\{(a, b) \mid a$ and b speak a common language$\}$

   Answer:

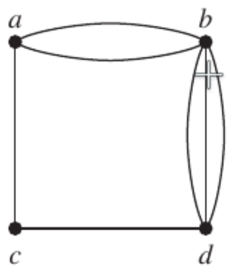
   a. equivalence relation

b. equivalence relation
c. missing transitive
d. missing transitive
e. missing transitive

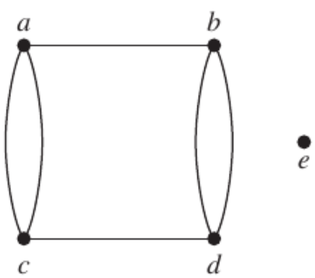
Chapter 10

10.1

For Exercises 4, 6 and 8, determine whether the graph shown has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph in Table 1 this graph is.

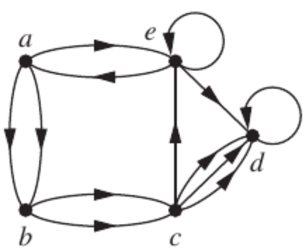
4. 

   Answer:
   Undirected with multiple edges. So it's a multigraph.

6. 

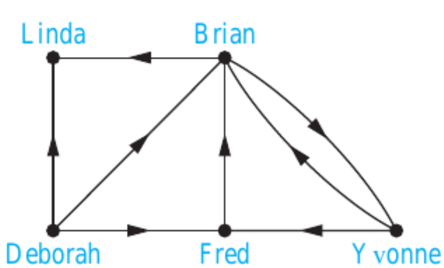
   a

   Answer:
   Undirected with multiple edges. It's a multigraph.

8. 

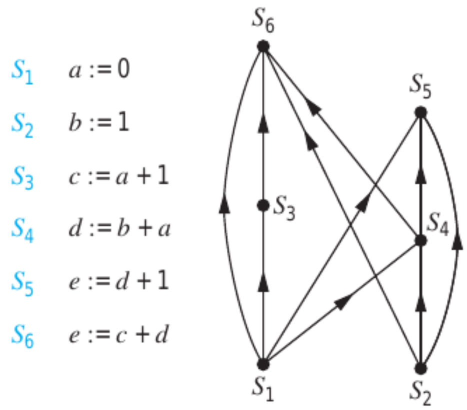
   Answer:
   Directed with multiple edges and loops. It's a directed multigraph.

18. Who can influence Fred and whom can Fred influence in the influence graph in Example 2?



Answer:
Fred can influence Brian, and he can be influenced by Yvonne and Deborah.

32. Which statements must be executed before S_6 is executed in the program in Example 8? (Use the precedence graph in Figure 10.)



Answer:
S_1, S_2, S_3, S_4

10.2

6. Show that the sum, over the set of people at a party, of the number of people a person has shaken hands with, is even. Assume that no one shakes his or her own hand.
Answer:
The sum must be even becuase the total goes up by 2 each time someone shakes another person's hand because it goes up by one for each person. Therefore the final total must be even.

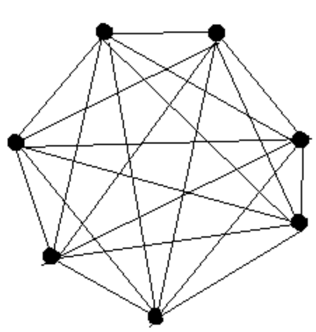
14. What does the degree of a vertex in the Hollywood graph represent? What does the neighborhood of a vertex represent? What do the isolated and pendant vertices represent?
Answer:
The degree represents the amount of actors that person has worked with. The neighborhood represents the set of specific actors they have worked with. The isolated vertices mean that the actor haven't worked with anyone. The pendant vertices represent the actors/actressses who have only done a movie with one other actor/actress.

20. Draw these graphs.

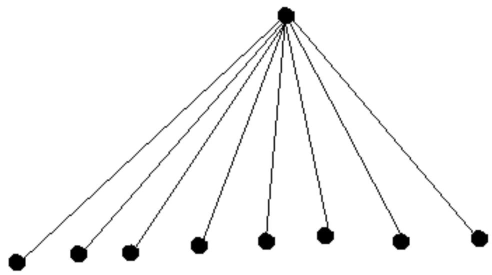
    a. K_7
    b. $K_{1,8}$
    c. $K_{4,4}$
    d. C_7
    e. W_7
    f. Q_4

Answer:

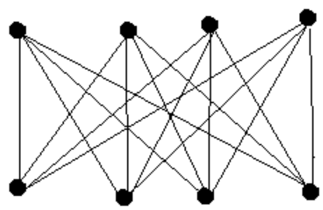
a.



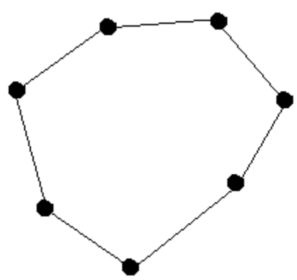
b.



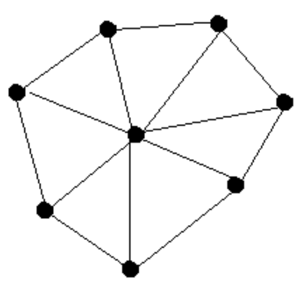
c.



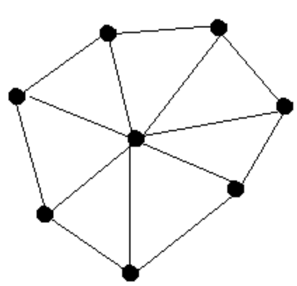
d.



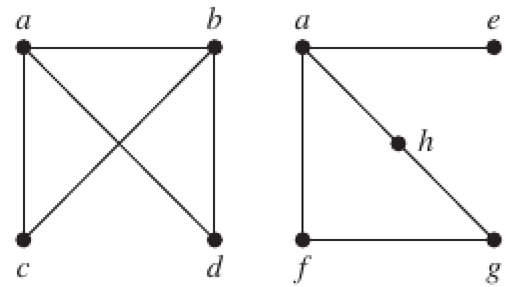
e.



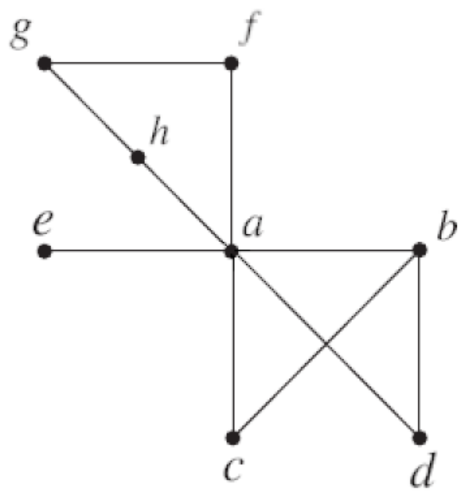
f.



58. Find the union of the given pair of simple graphs. (Assume edges with the same endpoints are the same.)



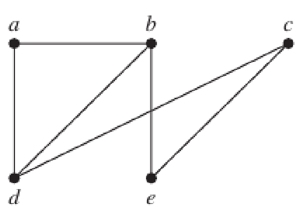
Answer:



10.3

For 2 and 4 use an adjacency list to represent the given graph.

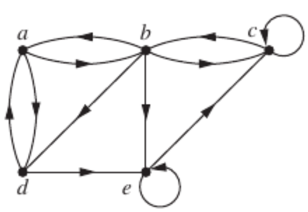
2.



Answer:

Vertex	Adjacent Vertices
a	$b,\ d$
b	$a,\ d,\ e$
c	$d,\ e$
d	$a,\ b,\ c$
e	$b,\ c$

4.



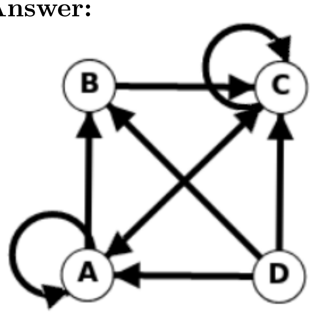
Answer:

Vertex	Adjacent Vertices
a	$b,\ d$
b	$a,\ c,\ d,\ e$
c	$b,\ c$
d	$a,\ e$
e	$c,\ e$

6. Represent the graph in Exercise 2 with an adjacency matrix.
Answer:
$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

8. Represent the graph in Exercise 4 with an adjacency matrix.
Answer:
$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

12. Draw a graph with the given adjacency matrix.
Answer:

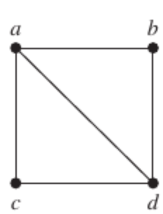


14. Represent given graph with an adjacency matrix.

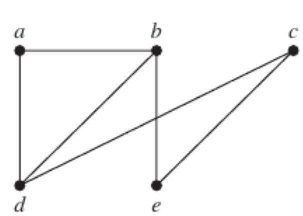
Answer:
$$\begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}$$

26. Use an incidence matrix to represent the graphs in Exercises 1 and 2.

a)



b)



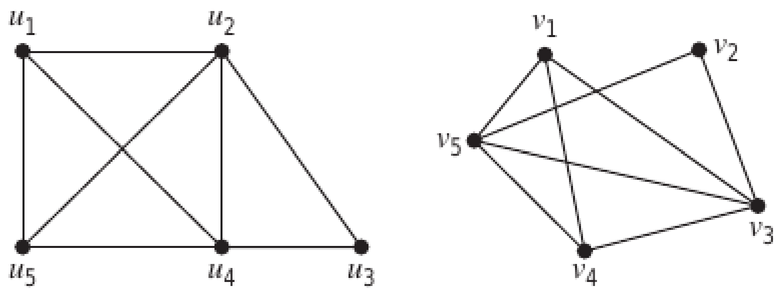
Answer:

a. $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

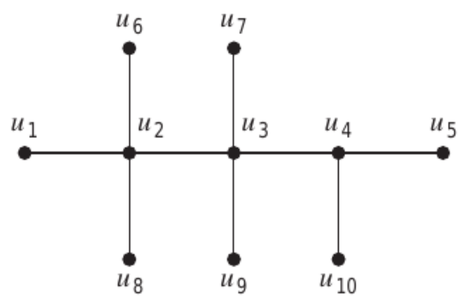
In Exercises 38 and 42 determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

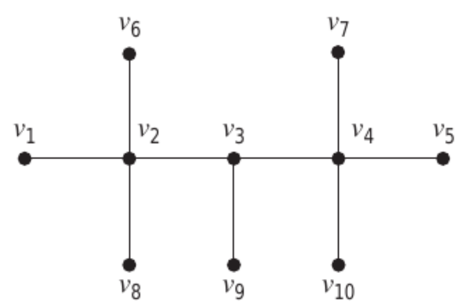
38.



Answer:
These graphs are isomorphic with a function f that has $f(u_1) = v_1,\ f(u_2) = v_5,\ f(u_3) = v_2,\ f(u_4) = v_3,\ f(u_5) = v_4$

42.





Answer:
They are not isomorphic because $deg(u_4) = 3$ so it must map to v_3 since it is the only one in the second graph with the same degree. However it can't map to v_3 since it must also be adjacent to exactly one vertex of degree 4 but v_3 is adjacent to two degree 4 vertices.

10.4

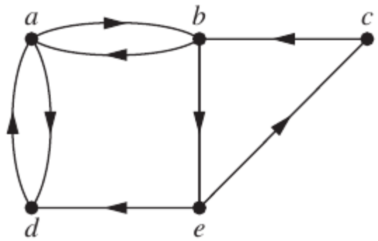
2. Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?

a) $a,\ b,\ e,\ c,\ b$

b) $a,\ d,\ a,\ d,\ a$

c) $a,\ d,\ b,\ e,\ a$

d) $a,\ b,\ e,\ c,\ b,\ d,\ a$



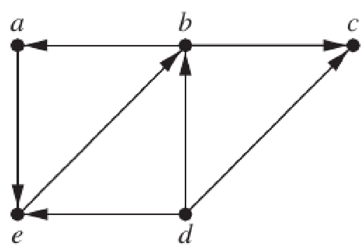
Answer:

a. Simple path, length: 4
b. Not simple path, length: 4
c. Not a path
d. Not a path

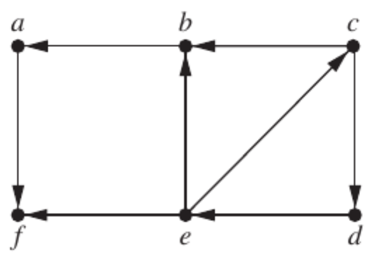
8. What do the connected components of a collaboration graph represent?
Answer:
They represent the largest set of people who have worked together or there is a way to get from one person to another via the other people in the set.

14. Find the strongly connected components of each of these graphs.

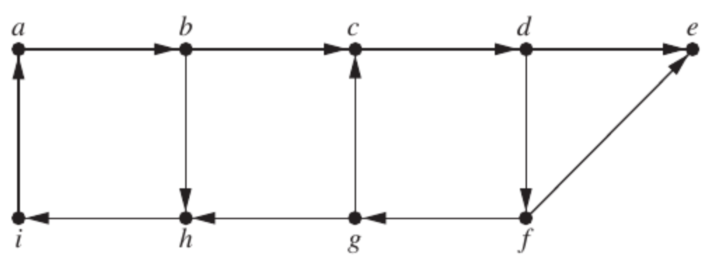
a.



b.



c.

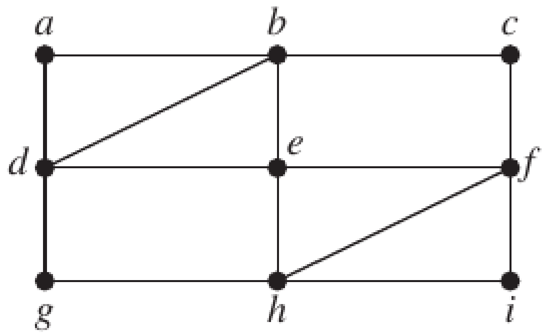


Answer:

a. $\{a, b, e\}$, $\{c\}$, $\{d\}$
b. $\{c, d, e\}$, $\{a\}$, $\{b\}$, $\{f\}$
c. $\{a, b, c, d, f, g, h, i\}$, $\{e\}$

10.5

2. Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

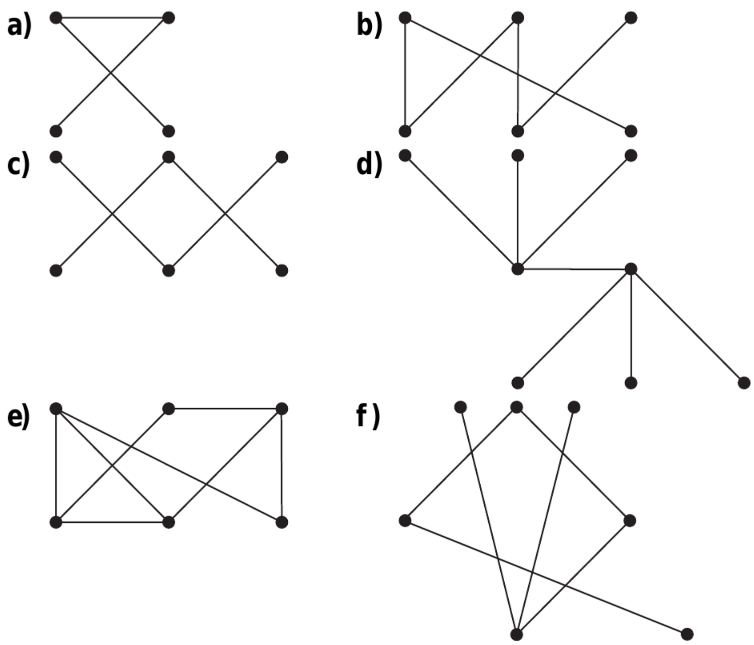


Answer:
An Euler circuit exists since all vertices have an even degree. (a, b, e, d, g, h, f, i, h, e, f, c, b, d, a)

Chapter 11

11.1

2. Which of these graphs are trees?



Answer:

a) Tree

b) Tree

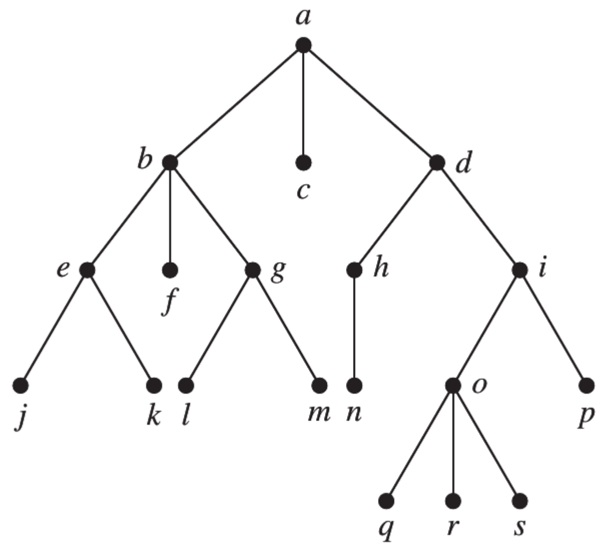
c) Not a tree

d) Tree

e) Not a tree

f) Tree

4. Answer these questions about the rooted tree illustrated.



a. Which vertex is the root?
b. Which vertices are internal?
c. Which vertices are leaves?
d. Which vertices are children of j ?
e. Which vertex is the parent of h?
f. Which vertices are siblings of o?
g. Which vertices are ancestors of m?
h. Which vertices are descendants of b?

Answer:

a. a
b. a, b, d, e, g, h, i, o
c. $c, f, j, k, l, m, n, p, q, r, s$
d. j has no children.
e. d
f. p
g. g, b, a
h. e, f, g, j, k, l, m

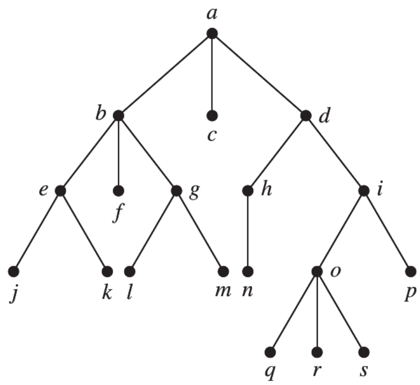
6. Is the rooted tree in Exercise 4 a full m-ary tree for some positive integer m?
Answer:
No, it's not a full m-ary tree because there are some internal nodes who have a different ammount of children. For example, b has 3 children but "i" only has 2.

10. Draw the subtree of the tree in Exercise 4 that is rooted at

a) a. b) c. c) e.

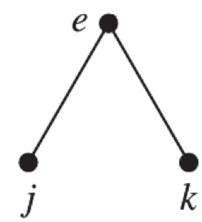
Answer:

a)



b)

c)

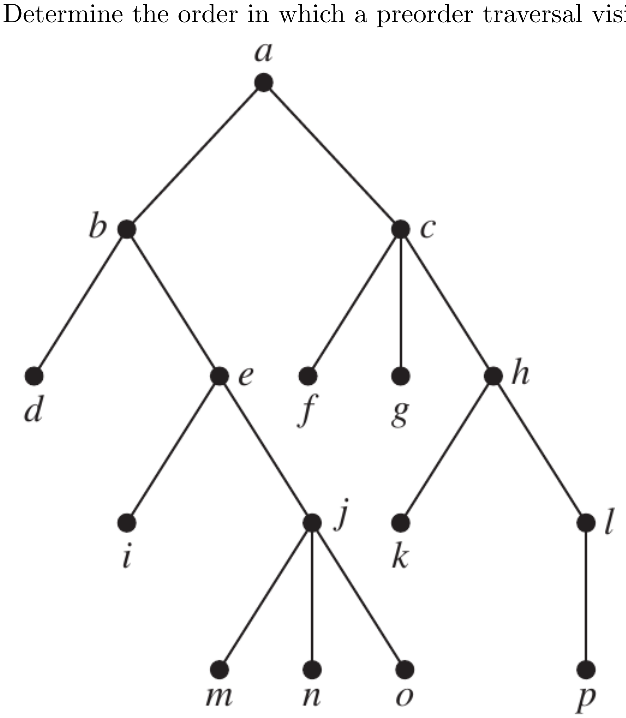


18. How many vertices does a full 5-ary tree with 100 internal vertices have?
Answer:
$n = mi + 1 = 5 \cdot 100 + 1 = 501$ vertices.

20. How many leaves does a full 3-ary tree with 100 vertices have?
Answer:
$l = n - (n-1)/m = 100 - 99/3 = 67$ leaves.

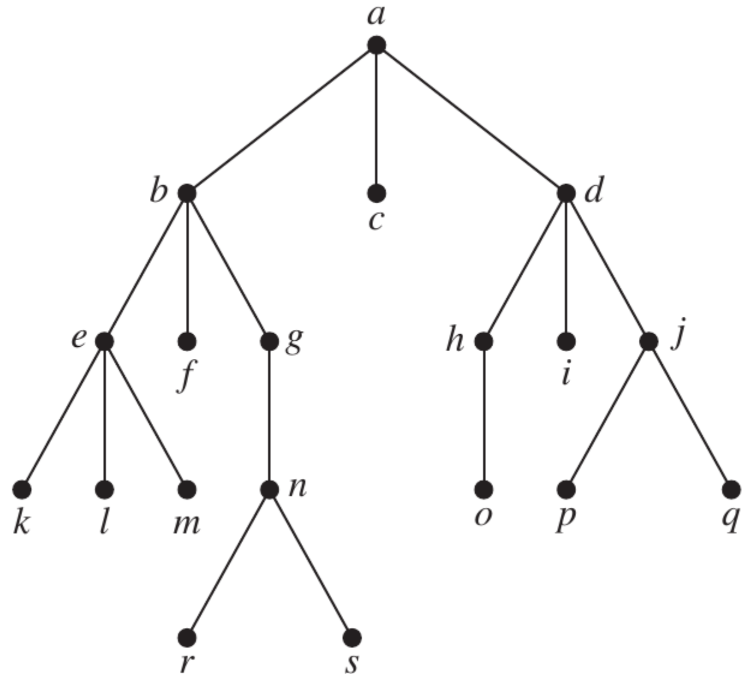
11.3

8. Determine the order in which a preorder traversal visits the vertices of the given ordered rooted tree.



Answer:
$a,\ b,\ d,\ e,\ i,\ j,\ m,\ n,\ o,\ c,\ f,\ g,\ h,\ k,\ l,\ p$

12. In which order are the vertices of this ordered rooted tree visited using an inorder traversal?



Answer:
$k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q$

14. In which order are the vertices of the ordered rooted tree in Exercise 8 visited using a postorder traversal?
Answer:
$d, i, m, n, o, j, e, b, f, g, k, p, l, h, c, a$

16.

11.4

Chapter 12

12.1

12.2

12.3