Homework 3

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Chapter 9

9.1

2. a. List all the ordered pairs in the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$.
   b. Display this relation graphically, as was done in Example 4.
   c. Display this relation in tabular form, as was done in Example 4.

   Answer:

   a. $\{(1,1),\ (1,2),\ (1,3),\ (1,4),\ (1,5),\ (1,6),\ (2,2),\ (2,4),\ (2,6),\ (3,3),\ (3,6),\ (4,4),\ (5,5)\ (6,6)\}$

4. Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in$ R if and only if

   a. a is taller than b.
   b. a and b were born on the same day.
   c. a has the same first name as b.
   d. a and b have a common grandparent.

   Answer:

   a. antisymmetric and transitive
   b. reflexive, symmetric, and transitive
   c. reflexive, symmetric, and transitive
   d. reflexive, symmetric

10. Give an example of a relation on a set that is

    a. both symmetric and antisymmetric.
    b. neither symmetric nor antisymmetric.

    Answer:

    a. A relation where the number maps to only itself
    b. Set {1,2,3,4} R = {(1,2), (2,1)}

9.3

2. Represent each of these relations on 1, 2, 3, 4 with a matrix (with the elements of this set listed in increasing order).

   a. {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}
   b. {(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)}
   c. {(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)}
   d. {(2, 4), (3, 1), (3, 2), (3, 4)}

   Answer:

   a. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   b. $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

   c. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

4. List the ordered pairs in the relations on {1, 2, 3, 4} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a. $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Answer:

a. {(1,1), (1,2), (1,4), (2,1), (2,3), (3,2), (3,3), (3,4), (4,1), (4,3), (4,4)}

b. {(1,1), (1,2), (1,3), (2,2), (3,3), (3,4), (4,1) , (4,4)}

c. {(1,2), (1,4), (2,1), (2,3), (3,2), (3,4), (4,1), (4,3)}

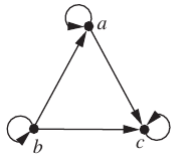
8. Determine whether the relations represented by the ma- trices in Exercise 4 are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.
Answer:

a. symmetric

b. reflexive and antisymmetric

c. irreflexive and symmetric

24. List order pairs in relation represented by the directed graph.



Answer:
$\{(a,a),\ (a,c),\ (b,b),\ (b,a),\ (b,c),\ (c,c)\}$

9.5

2. Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a. $\{(a,b) \mid a$ and b are the same age$\}$

b. $\{(a,b) \mid a$ and b have the same parents$\}$

c. $\{(a,b) \mid a$ and b share a common parent$\}$

d. $\{(a,b) \mid a$ and b have met$\}$

e. $\{(a,b) \mid a$ and b speak a common language$\}$

Answer:

a. equivalence relation

b. equivalence relation

c. missing transitive

d. missing transitive

e. missing transitive

Chapter 10

10.1

10.2

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10.4

10.5

Chapter 11

11.1

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Chapter 12

12.1

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