Homework 1

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Chapter 1

1.1

2. Which of these are propositions? What are the truth values of those that are propositions?

   a. Do not pass go.
   b. What time is it?
   c. There are no black flies in Maine.
   d. $4 + x = 5$.
   e. The moon is made of green cheese.
   f. $2n \geq 100$.

   Answer: c and e are both propositions, and both of their truth values are false.

4. What is the negation of each of these propositions?

   a. Jennifer and Teja are friends.
   b. There are 13 items in a baker's dozen.
   c. Abby sent more than 100 text messages every day.
   d. 121 is a perfect square.

   Answer:

   a. Jennifer and Teja are not friends.
   b. There aren't 13 items in a baker's dozen.
   c. Abby sent less than or equal to 100 text messages every day.
   d. 121 is not a perfect square.

6. Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

   a. Smartphone B has the most RAM of these three smartphones.
   b. Smartphone C has more ROM or a higher resolution camera than Smartphone B.
   c. Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
   d. If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
   e. Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

   Answer:

   a. True
   b. True
   c. False
   d. False
   e. False

1.2

For exercises 2 & 4, translate into propositional logic.

2. You can see the movie only if you are over 18 years old or you have the permission of a parent. Express your answer in terms of m: "You can see the movie," e: "You are over 18 years old," and p: "You have the permission of a parent."

   Answer: $m \rightarrow (e \vee p)$

4. To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service. Express your answer in terms of w: "You can use the wire- less network in the airport," d: "You pay the daily fee," and s: "You are a subscriber to the service."

   Answer: $w \rightarrow (d \vee s)$

1.3

6. Use a truth table to verify the first De Morgan law

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Answer:

p	q	$\neg p$	$\neg q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

8. Use De Morgan's laws to find the negation of each of the following statements

   a. Kwame will take a job in industry or go to graduate school.
   b. Yoshiko knows Java and calculus.
   c. James is young and strong.
   d. Rita will move to Oregon or Washington.

   Answer:

   a. Kwame will not take a job in industry and not go to graduate school.
   b. Yoshiko doesn't know Java or doesn't know calculus.
   c. James is not young or not strong.
   d. Rita will not move to Oregon and will not move to Washington.

10. Show that each of these conditional statements is a tautology by using truth tables.

   a. $[\neg p \wedge (p \vee q)] \to q$
   b. $[(p \to q) \wedge (q \to r)] \to (p \to r)$
   c. $[p \wedge (p \to q)] \to q$
   d. $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

   Answer:

a.

p	q	$\neg p$	$p \vee q$	$[\neg p \wedge (p \vee q)]$	$[\neg p \wedge (p \vee q)] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	T	T	T

b.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$[(p \to q) \wedge (q \to r)]$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

c.

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

d.

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

12 Show that each conditional statement in Exercise 10 is a tautology without using truth tables.

a. $[\neg p \wedge (p \vee q)] \to q$

b. $[(p \to q) \wedge (q \to r)] \to (p \to r)$

c. $[p \wedge (p \to q)] \to q$

d. $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

Answer:

a.

$$
\begin{aligned}
[\neg p \wedge (p \vee q)] \to q &\equiv \neg[\neg p \wedge (p \vee q)] \vee q && \text{(Equivalence from table)}\\
&\equiv [\neg(\neg p) \vee \neg(p \vee q)] \vee q && \text{(De Morgan's Law)}\\
&\equiv [p \vee (\neg p \wedge \neg q)] \vee q && \text{(Double negation and De Morgan's Law)}\\
&\equiv [(p \vee \neg p) \wedge (p \vee \neg q)] \vee q && \text{(Distributive Law)}\\
&\equiv [\mathbf{T} \wedge (p \vee \neg q)] \vee q && \text{(Negation Law)}\\
&\equiv [(\mathbf{T} \wedge p) \vee (\mathbf{T} \wedge \neg q)] \vee q && \text{(Distributive Law)}\\
&\equiv (p \vee \neg q) \vee q && \text{(Identity Law)}\\
&\equiv p \vee (\neg q \vee q) && \text{(Associativity)}\\
&\equiv p \vee \mathbf{T} && \text{(Negation Law)}\\
&\equiv \mathbf{T} && \text{(Domination Law)}
\end{aligned}
$$

b.

$$
\begin{aligned}
[(p \to q) \wedge (q \to r)] \to (p \to r) &\equiv \neg[(p \to q) \wedge (q \to r)] \vee (p \to r) && \text{(Equivalence from table)}\\
&\equiv [\neg(p \to q) \vee \neg(q \to r)] \vee (p \to r) && \text{(De Morgan's Law)}\\
&\equiv [(p \wedge \neg q) \vee (q \wedge \neg r)] \vee (p \to r) && \text{(Equivalence from table)}\\
&\equiv [(p \wedge \neg q) \vee (q \wedge \neg r)] \vee (\neg p \vee r) && \text{(Equivalence from table)}\\
&\equiv [(p \wedge \neg q) \vee (\neg p \vee r)] \vee (q \wedge \neg r) && \text{(Commutative \& Associative Law)}\\
&\equiv [[(\neg p \vee r) \vee p] \wedge [(\neg p \vee r) \vee \neg q]] \vee (q \wedge \neg r) && \text{(Distributive Law)}\\
&\equiv [[(p \vee \neg p) \vee r] \wedge [(\neg p \vee r) \vee \neg q]] \vee (q \wedge \neg r) && \text{(Commutative \& Associative Law)}\\
&\equiv [(\mathbf{T} \vee r) \wedge [(\neg p \vee r) \vee \neg q]] \vee (q \wedge \neg r) && \text{(Negation Law)}\\
&\equiv [\mathbf{T} \wedge [(\neg p \vee r) \vee \neg q]] \vee (q \wedge \neg r) && \text{(Domination Law)}\\
&\equiv [(\neg p \vee r) \vee \neg q] \vee (q \wedge \neg r) && \text{(Domination Law)}\\
&\equiv [[(\neg p \vee r) \vee \neg q] \vee q] \wedge [[(\neg p \vee r) \vee \neg q] \vee \neg r] && \text{(Distributive Law)}\\
&\equiv [(\neg p \vee r) \vee (\neg q \vee q)] \wedge [(\neg p \vee \neg q) \vee (r \vee \neg r)] && \text{(Commutative \& Associative Law)}\\
&\equiv [(\neg p \vee r) \vee \mathbf{T}] \wedge [(\neg p \vee \neg q) \vee \mathbf{T}] && \text{(Negation Law)}\\
&\equiv \mathbf{T} \wedge \mathbf{T} && \text{Domination Law}\\
&\equiv \mathbf{T} && \text{(Identity Law)}
\end{aligned}
$$

c.

$$
\begin{aligned}
[p \wedge (p \to q)] \to q &\equiv \neg[p \wedge (p \to q)] \vee q && \text{(Equivalence from table)} \\
&\equiv [\neg p \vee \neg(p \to q)] \vee q && \text{(De Morgan's Law)} \\
&\equiv [\neg p \vee (p \wedge \neg q)] \vee q && \text{(Equivalence from table)} \\
&\equiv [(\neg p \vee p) \wedge (\neg p \vee \neg q)] \vee q && \text{(Distributive Law)} \\
&\equiv [\mathbf{T} \wedge (\neg p \vee \neg q)] \vee q && \text{(Negation Law)} \\
&\equiv (\neg p \vee \neg q) \vee q && \text{(Identity Law)} \\
&\equiv \neg p \vee (\neg q \vee q) && \text{(Associative Law)} \\
&\equiv \neg p \vee \mathbf{T} && \text{(Negation Law)} \\
&\equiv \mathbf{T} && \text{(Domination Law)}
\end{aligned}
$$

d.

$$
\begin{aligned}
[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r &\equiv [(p \vee q) \wedge [(p \to r) \wedge (q \to r)]] \to r && \text{(Associative Law)} \\
&\equiv [(p \vee q) \wedge [(p \vee q) \to r]] \to r && \text{(Equivalence from table)} \\
&\equiv \neg[(p \vee q) \wedge [\neg(p \vee q) \vee r]] \vee r && \text{(Equivalence from table)} \\
&\equiv [\neg(p \vee q) \vee \neg[\neg(p \vee q) \vee r]] \vee r && \text{(De Morgan's Law)} \\
&\equiv [\neg(p \vee q) \vee [(p \vee q) \wedge \neg r]] \vee r && \text{(De Morgan's + Double Negation)} \\
&\equiv [\neg(p \vee q) \vee (p \vee q) \wedge [\neg(p \vee q) \vee \neg r]] \vee r && \text{(Distributive Law)} \\
&\equiv [\mathbf{T} \wedge [\neg(p \vee q) \vee \neg r]] \vee r && \text{(Negation Law)} \\
&\equiv [\neg(p \vee q) \vee \neg r] \vee r && \text{(Identity Law)} \\
&\equiv \neg(p \vee q) \vee (\neg r \vee r) && \text{(Associative Law)} \\
&\equiv \neg(p \vee q) \vee \mathbf{T} && \text{(Negation Law)} \\
&\equiv \mathbf{T} && \text{(Domination Law)}
\end{aligned}
$$

1.4

6. Let N (x) be the statement “x has visited North Dakota,” where the domain consists of the students in your school. Express each of these quantifications in English.

   a. $\exists x N(x)$
   b. $\forall x N(x)$
   c. $\neg \exists x N(x)$
   d. $\exists x \neg N(x)$
   e. $\neg \forall x N(x)$
   f. $\forall x \neg N(x)$

   Answer:

   a. There exists a student in my school who has visited North Dakota.
   b. Every student in my school has visted North Dakota.
   c. No student in my school has visted North Dakota.
   d. At least one student in my school has not visted North Dakota.
   e. At least one student in my school has not visted North Dakota.
   f. No student in my school has visted North Dakota.

8. Translate these statements into English, where R(x) is “x is a rabbit” and H (x) is “x hops” and the domain consists of all animals.

   a. $\forall x(R(x) \to H(x))$
   b. $\forall x(R(x) \wedge H(x))$
   c. $\exists x(R(x) \to H(x))$
   d. $\exists x(R(x) \wedge H(x))$

Answer:

a. Every rabbit hops.
b. All animals are rabbits who hop.
c. There exists an animal that if they are a rabbit, then they hop.
d. Some rabbits hop.

14. Determine the truth value of each of these statements if the domain consists of all real numbers.

a. $\exists x(x^3 = -1)$
b. $\exists x(x^4 < x^2)$
c. $\forall x((-x)^2 = x^2)$
d. $\forall x(2x > x)$

a. True, x = -1
b. True, x = 0.1
c. True, squaring returns a positive result
d. False, x = -1 returns false.

1.5

2. Translate these statements into English, where the domain for each variable consists of all real numbers.

a. $\exists x \forall y(xy = y)$
b. $\forall x \forall y((((x \geq 0) \wedge y < 0)) \rightarrow (x - y > 0)$
c. $\forall x \forall y \exists z(x = y + z)$

Answer:

a. There exists an x such that for all y, $xy = y$, x and y are real.
b. For all pairs of x and y, both real, if $(x \geq 0)$ and $(y < 0)$, then $x - y > 0$.
c. Let x, y and z be real numbers; for all pairs x and y there exists a z such that $x = y + z$.

6. Let C(x, y) mean that student x is enrolled in class y, where the domain for x consists of all students in your school and the domain for y consists of all classes being given at your school. Express each of these statements by a simple English sentence.

a. $C(\text{Randy Goldberg}, \text{CS 252})$
b. $\exists x C(x, \text{Math 695})$
c. $\exists y C(\text{Carol Sitea}, y)$
d. $\exists x(C(x, \text{Math 222}) \wedge C(x, \text{CS 252}))$
e. $\exists x \exists y \forall z((x = y) \wedge (C(x, z) \rightarrow C(y, z)))$
f. $\exists x \exists y \forall z((x = y) \wedge (C(x, z) \leftrightarrow C(y, z)))$

Answer:

a. Randy Goldberg is enrolled in CS 252.
b. There exists a student in my school that is enrolled in Math 695.
c. Carol Sitea is taking some class at my school.
d. There exists a student who is enrolled in Math 22 and CS 252.

10. Let $F(x, y)$ be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a. Everybody can fool Fred.
b. Evelyn can fool everybody.
c. Everybody can fool somebody.

d. There is no one who can fool everybody.
e. Everyone can be fooled by somebody.
f. No one can fool both Fred and Jerry.
g. Nancy can fool exactly two people.
h. There is exactly one person whom everybody can fool.
i. No one can fool himself or herself.
j. There is someone who can fool exactly one person besides himself or herself.

Answer:

a. $\forall x F(x, \text{Fred})$
b. $\forall x F(\text{Evelyn}, x)$
c. $\forall x \exists y F(x, y)$
d. $\forall x \exists y \neg F(x, y)$
e. $\forall x \exists y F(y, x)$
f. $\forall x(\neg F(x, \text{Fred}) \vee \neg F(x, \text{Jerry}))$
g. $\exists x \exists y(x \neq y \wedge F(\text{Nancy}, x) \wedge F(\text{Nancy}, y))$
h. $\exists x \forall y(F(y, x) \wedge \forall z F(F(y, z) \rightarrow z = x))$
i. $\forall x \neg F(x, x)$
j. $\exists x \exists y(x \neq y \wedge F(x, y) \wedge \forall z F(x, z) \rightarrow z = y)$

1.6

2. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

If George does not have eight legs, then he is not a spider.
George is a spider

$\therefore$ George has eight legs

Answer: Yes the conclusion is true because of modus tollens.

4. What rule of inference is used in each of these arguments?

a. Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
b. It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
c. Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
d. Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.
e. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Answer:

a. Simplification
b. Disjunctive syllogism
c. Modus ponens
d. Addition
e. Hypothetical syllogism

8 What rules of inference are used in this argument? "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

Answer: Modus ponens

1.7

6. Use a direct proof to show that the product of two odd numbers is odd.

   Proof. Let x and y be odd. This means that $x = 2k + 1$, $y = 2s + 1$; k, s $\in \mathbb{Z}$.
   Then:

$$\begin{aligned} x * y &= (2k+1) \cdot (2s+1) \\ &= 4ks + 2k + 2s + 1 \\ &= 2(2ks + k + s) + 1 \\ &= 2r + 1 \end{aligned}$$

   Which is an odd number where $r = 2ks + k + s$. ■

18. Prove that if n is an integer and 3n + 2 is even, then n is even using

    a. a proof by contraposition.
    b. a proof by contradiction.

    a. *Proof.* Assume that n is odd, so n = 2k + 1 Then:

$$\begin{aligned} 3n + 2 &= 3(2k+1) + 2 \\ &= 6k + 3 + 2 \\ &= 6k + 4 + 1 \\ &= 2(3k+2) + 1 \\ &= 2r + 1 \end{aligned}$$

    Where $r \in \mathbb{Z}$
    So when n is odd then, 3n + 2 is odd is true, and the contrapositive that when 3n + 2 is even, then n is even is also true. ■

    b. *Proof.* Assume that $3n+2$ is even and n is odd. Because n is odd there is an integer k such that $n = 2k+1$ Then:

$$\begin{aligned} 3n + 2 &= 3(2k+1) + 2 \\ &= 6k + 3 + 2 \\ &= 6k + 4 + 1 \\ &= 2(3k+2) + 1 \\ &= 2r + 1 \end{aligned}$$

    So $3n + 2 = 2r + 1$ where $r = 3k + 1$. Which shows $3n + 2$ is odd. We have 3n + 2 both even and odd which is a contradiction, which means our assumption that n is odd is false thus n is even. ■

24. Show that at least three of any 25 days chosen must fall in the same month of the year.

    Proof. Assume that no more than 2 of any 25 days chosen must fall in the same month of the year. Then because we have 12 months in a year at most 24 days can be chosen as each month can have at most 2 days, however this contradicts the premise that there are 25 days chosen so at least 3 of any 25 days must fall in the same month of the year ■

28. Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$

    Proof. ($\leftarrow$) WTS: $(m = n \vee m = -n) \to m^2 = n^2$
    Let $m = n$ then $m^2 = m \cdot m = n \cdot n = n^2$
    Let $m = -n$, then $m^2 = m \cdot m = -n \cdot -n = n^2$

($\rightarrow$) WTS: $m^2 = n^2 \rightarrow (m = n \vee m = -n)$
Assume $m \neq n$ and $m \neq -n$

$m \neq n$	Premise
$m \cdot m \neq n \cdot m$	Multiply both sides by m
$m^2 \neq m \cdot n \neq n \cdot n \neq n^2$	True from premise
$m \neq -n$	
$m \cdot m \neq m \neq -n$	
$m^2 \neq m \cdot -n \neq -n \cdot -n \neq n^2$	Same steps as above

Contrapositive is true so the original statement is also true.
Have shown both implications is to so the biconditional is true. ■