

Homework 3

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Chapter 9

9.1

2. a. List all the ordered pairs in the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$.
b. Display this relation graphically, as was done in Example 4.
c. Display this relation in tabular form, as was done in Example 4.

Answer:

- a. $\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)\}$
4. Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if
- a is taller than b.
 - a and b were born on the same day.
 - a has the same first name as b.
 - a and b have a common grandparent.

Answer:

- antisymmetric and transitive
 - reflexive, symmetric, and transitive
 - reflexive, symmetric, and transitive
 - reflexive, symmetric
10. Give an example of a relation on a set that is
- both symmetric and antisymmetric.
 - neither symmetric nor antisymmetric.

Answer:

- A relation where the number maps to only itself
- Set $\{1, 2, 3, 4\}$ $R = \{(1, 2), (2, 1)\}$

9.3

2. Represent each of these relations on 1, 2, 3, 4 with a matrix (with the elements of this set listed in increasing order).
- $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$
 - $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$
 - $\{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$
 - $\{(2, 4), (3, 1), (3, 2), (3, 4)\}$

Answer:

a.
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b.
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

c.
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

d.
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4. List the ordered pairs in the relations on $\{1, 2, 3, 4\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a.
$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

b.
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

c.
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Answer:

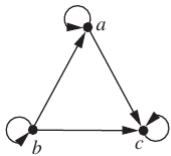
- a. $\{(1,1), (1,2), (1,4), (2,1), (2,3), (3,2), (3,3), (3,4), (4,1), (4,3), (4,4)\}$
- b. $\{(1,1), (1,2), (1,3), (2,2), (3,3), (3,4), (4,1), (4,4)\}$
- c. $\{(1,2), (1,4), (2,1), (2,3), (3,2), (3,4), (4,1), (4,3)\}$

8. Determine whether the relations represented by the matrices in Exercise 4 are reflexive, irreflexive, symmetric, anti-symmetric, and/or transitive.

Answer:

- a. symmetric
- b. reflexive and antisymmetric
- c. irreflexive and symmetric

24. List order pairs in relation represented by the directed graph.



Answer:

$$\{(a, a), (a, c), (b, b), (b, a), (b, c), (c, c)\}$$

9.5

2. Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack.
- a. $\{(a, b) \mid a \text{ and } b \text{ are the same age}\}$
 - b. $\{(a, b) \mid a \text{ and } b \text{ have the same parents}\}$
 - c. $\{(a, b) \mid a \text{ and } b \text{ share a common parent}\}$
 - d. $\{(a, b) \mid a \text{ and } b \text{ have met}\}$
 - e. $\{(a, b) \mid a \text{ and } b \text{ speak a common language}\}$

Answer:

- a. equivalence relation

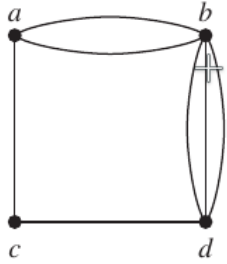
- b. equivalence relation
- c. missing transitive
- d. missing transitive
- e. missing transitive

Chapter 10

10.1

For Exercises 4, 6 and 8, determine whether the graph shown has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph in Table 1 this graph is.

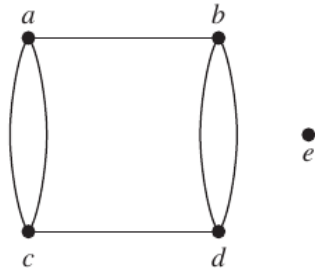
4.



Answer:

Undirected with multiple edges. So it's a multigraph.

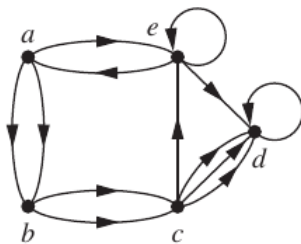
6.



Answer:

Undirected with multiple edges. It's a multigraph.

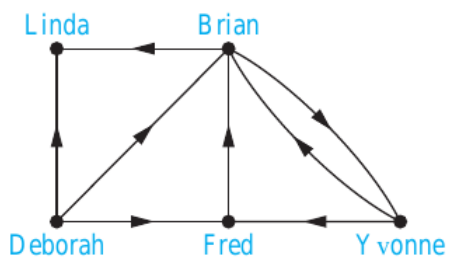
8.



Answer:

Directed with multiple edges and loops. It's a directed multigraph.

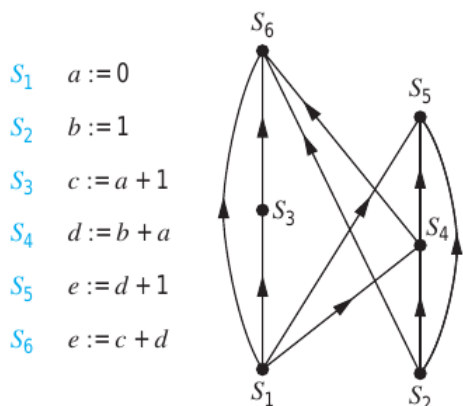
18. Who can influence Fred and whom can Fred influence in the influence graph in Example 2?



Answer:

Fred can influence Brian, and he can be influenced by Yvonne and Deborah.

32. Which statements must be executed before S_6 is executed in the program in Example 8? (Use the precedence graph in Figure 10.)

**Answer:**

S_1, S_2, S_3, S_4

10.2

6. Show that the sum, over the set of people at a party, of the number of people a person has shaken hands with, is even. Assume that no one shakes his or her own hand.

Answer:

The sum must be even because the total goes up by 2 each time someone shakes another person's hand because it goes up by one for each person. Therefore the final total must be even.

14. What does the degree of a vertex in the Hollywood graph represent? What does the neighborhood of a vertex represent? What do the isolated and pendant vertices represent?

Answer:

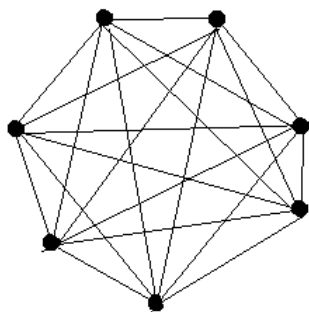
The degree represents the amount of actors that person has worked with. The neighborhood represents the set of specific actors they have worked with. The isolated vertices mean that the actor haven't worked with anyone. The pendant vertices represent the actors/actresses who have only done a movie with one other actor/actress.

20. Draw these graphs.

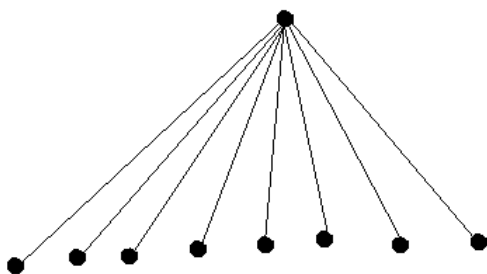
- K_7
- $K_{1,8}$
- $K_{4,4}$
- C_7
- W_7
- Q_4

Answer:

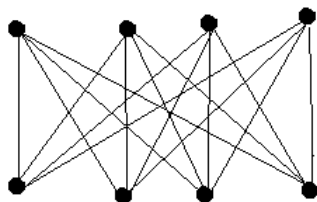
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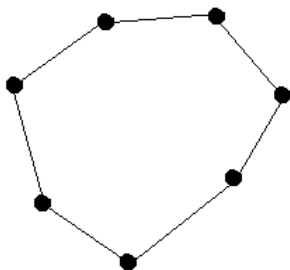
b.



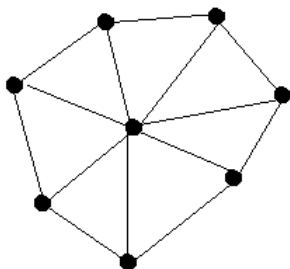
c.



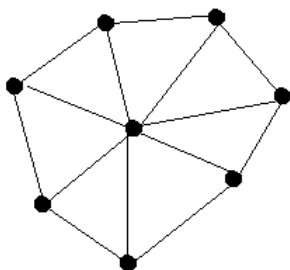
d.



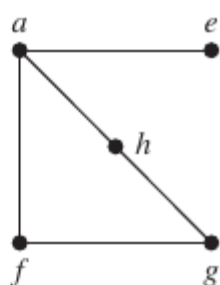
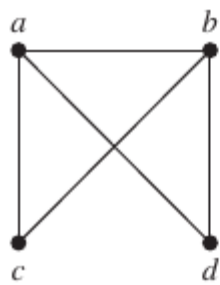
e.



f.



58. Find the union of the given pair of simple graphs. (Assume edges with the same endpoints are the same.)



10.3

10.4

10.5

Chapter 11

11.1

11.3

11.4

Chapter 12

12.1

12.2

12.3