Homework 3

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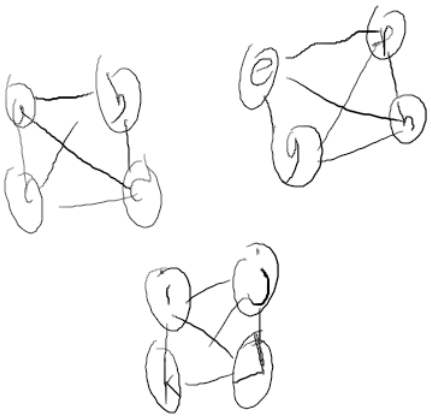
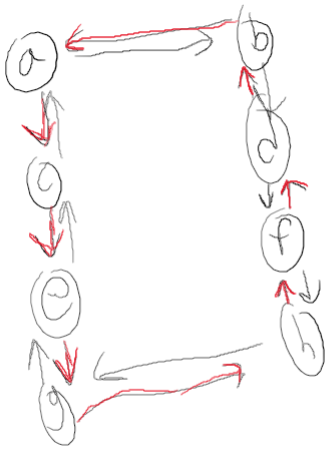
Chapter 13

R-13.1 Draw a simple undirected graph G that has 12 vertices, 18 edges, and 3 connected components. Why would it be impossible to draw G with 3 connected components if G had 66 edges?
Answer:

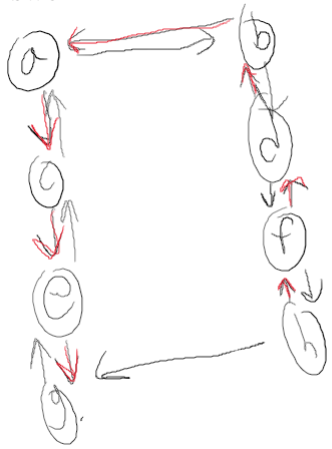


If we consider the maximum amount of edges that G can have with 12 vertices, we see that it would be $\binom{12}{2} = 66$. But if there are three connected components, then that is equivalent to removing at least two edges from this theoretical graph and so the max number of edges would be 64.

R-13.3 Draw a simple connected directed graph with 8 vertices and 16 edges such that the in-degree and out-degree of each vertex is 2. Show that there is a single (nonsimple) cycle that includes all the edges of your graph, that is, you can trace all the edges in their respective directions without ever lifting your pencil. (Such a cycle is called an Euler tour.)
Answer:



R-13.4 Repeat the previous problem and then remove one edge from the graph. Show that now there is a single (nonsimple) path that includes all the edges of your graph. (Such a path is called an Euler path.)
Answer:



R-13.5 Bob loves foreign languages and wants to plan his course schedule for the following years. He is interested in the following nine language courses: LA15, LA16, LA22, LA31, LA32, LA126, LA127, LA141, and LA169. The course prerequisites are:

- LA15: (none)
- LA16: LA15
- LA22: (none)
- LA31: LA15
- LA32: LA16, LA31
- LA126: LA22, LA32
- LA127: LA16
- LA141: LA22, LA16
- LA169: LA32

Find the sequence of courses that allows Bob to satisfy all the prerequi- sites.
Answer: LA22 -> LA15 -> LA16 -> LA32 -> LA126 -> LA127 -> LA141

R-13.6 Suppose we represent a graph G having n vertices and m edges with the edge list structure. Why, in this case, does the insertVertex function run in O(1) time while the eraseVertex function runs in O(m) time?
Answer:
The insertVertex function runs in O(1) time because the vertices are stored in a doubly linked list, so insertion O(1). The eraseVertex function takes O(m) time because you have to check each edge and remove the ones incident to the vertex you are removing.

R-13.7 Let G be a graph whose vertices are the integers 1 through 8, and let the adjacent vertices of each vertex be given by the table below:

Vertex	Adjacent Vertices
1	(2, 3, 4)
2	(1, 3, 4)
3	(1, 2, 4)
4	(1, 2, 3, 6)
5	(6, 7, 8)
6	(4, 5, 7)
7	(5, 6, 8)
8	(5, 7)

Assume that, in a traversal of G, the adjacent vertices of a given vertex are returned in the same order as they are listed in the table above.

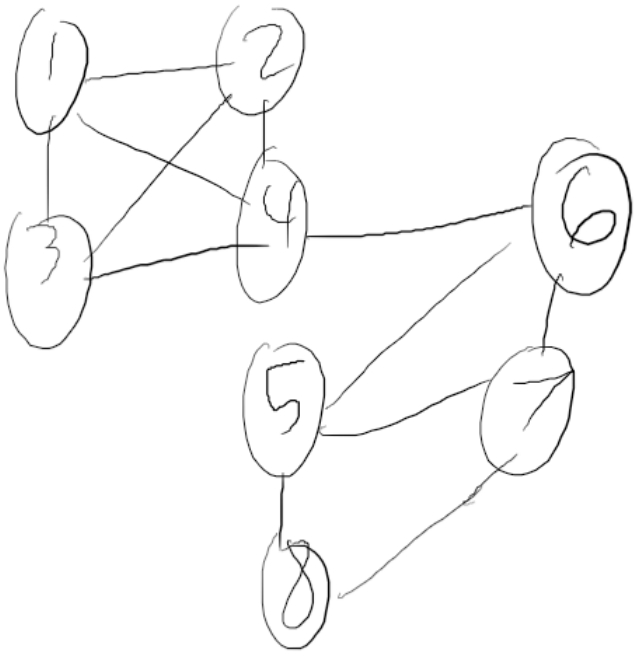
a. Draw G.

b. Give the sequence of vertices of G visited using a DFS traversal starting at vertex 1.

c. Give the sequence of vertices visited using a BFS traversal starting at vertex 1.

Answer:

a.



b. 1, 2, 3, 4, 6, 5, 7, 8

c. 1,2, 3, 4, 6, 5, 7, 8

R-13.8 Would you use the adjacency list structure or the adjacency matrix struc- ture in each of the following cases? Justify your choice.

a. The graph has 10,000 vertices and 20,000 edges, and it is important to use as little space as possible.

b. The graph has 10,000 vertices and 20,000,000 edges, and it is important to use as little space as possible.

c. You need to answer the query isAdjacentTo as fast as possible, no matter how much space you use.

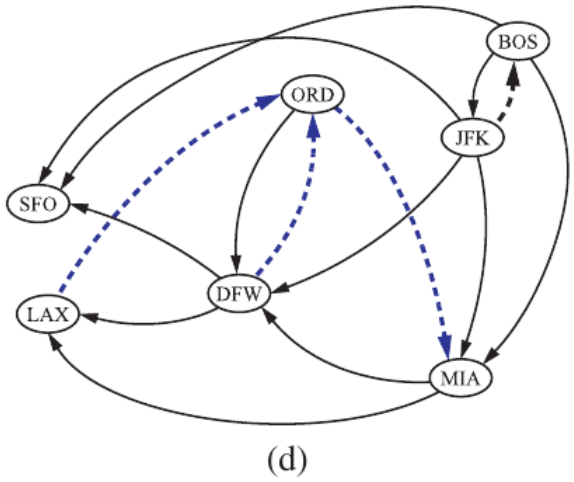
Answer:

a. Adjacency list because the space usage is O(n + m) where n is vertices and m is edges, while the adjacency matrix has a space usage of O(n^2) where n is vertices. So $10,000 + 20,000 = 30,000 < 100,000,000 = 10,000^2$

b. Still adjacency list since $20,010,000 < 10,000^2$ and thus the list will take less space.

c. Adjacency matrix since its supports isAdjacentTo is O(1) time.

R-13.11 Compute a topological ordering for the directed graph drawn with solid edges in Figure 13.8(d).



(d)

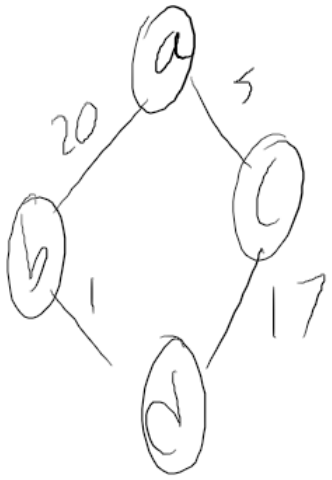
Answer:
BOS, JFK, MIA, ORD, DFW, SFO, LAX

R-13.29 How many edges are in the transitive closure of a graph that consists of a simple directed path of n vertices?
Answer:
Vertex 1 will have n-1 edges, since it can reach every other vertex, vertex 2 will have n-2 unique edges and so on until vertex n-1 which has unique 1 edge and vertex n will have no new/unique edges. So the final result is $\sum_{i=1}^{n-1} i = \frac{(n-1)(n)}{2}$.

C-13.16 Consider the following greedy strategy for finding a shortest path from vertex start to vertex goal in a given connected graph.

1. Initialize path to start.
2. Initialize VisitedVertices to start.
3. If start=goal, return path and exit. Otherwise, continue.
4. Find the edge (start,v) of minimum weight such that v is adjacent to start and v is not in VisitedVertices.
5. Add v to path.
6. Add v to VisitedVertices.
7. Set start equal to v and go to step 3.

Does this greedy strategy always find a shortest path from start to goal? Either explain intuitively why it works, or give a counter example.
Answer:
No, this greedy algorithm does not work. For example with this graph:



If we start at vetex a and want to reach d, it will go to c since $5 < 20$ and then go to d. However, that path has a total weight of 22 and if it went to b then d, it would have a better total weight of 21.

C-13.21 NASA wants to link n stations spread over the country using communication channels. Each pair of stations has a different bandwidth available, which is known a priori. NASA wants to select n - 1 channels (the minimum possible) in such a way that all the stations are linked by the channels and the total bandwidth (defined as the sum of the individual bandwidths of the channels) is maximum. Give an efficient algorithm for this problem and determine its worst-case time complexity. Consider the weighted graph G = (V, E), where V is the set of stations and E is the set of chanels between the stations. Define the weight w(e) of an edge e in E as the bandwidth of the corresponding channel.

Answer:

You can use a variation of the PrimJarnik minimum spanning tree algorithm. Insead of setting the iniital distance/key to infinity you can set it to negative infinity even negative 1 assuming no negative bandwidth. Then for the edge relaxtion you will update the value when the weight is greater than the key. This will have the same running time of $O(m \log n)$.

Chapter 14

14.6 Consider an initially empty memory cache consisting of four pages. How many page misses does the LRU algorithm incur on the following page request sequence: (2, 3, 4, 1, 2, 5, 1, 3, 5, 4, 1, 2, 3)?

Answer:

5 page misses.

14.7 Consider an initially empty memory cache consisting of four pages. How many page misses does the FIFO algorithm incur on the following page request sequence: (2, 3, 4, 1, 2, 5, 1, 3, 5, 4, 1, 2, 3)

Answer:

3 page misses.