Cooperation and Coordination in Heterogeneous Populations with Interaction Diversity

INTRODUCTION

In this appendix, we implement the stability analysis for the various equilibrium points of the replicator equation:

$$\begin{aligned} \dot{x} &= x(1-x)(\pi_{WC} - \pi_{WD}), \\ \dot{y} &= y(1-y)(\pi_{SC} - \pi_{SD}). \end{aligned} \tag{S1}$$

These two equations describe how the fractions of WC and SC change over time. Specifically, $\pi_{WC} - \pi_{WD}$ represents the payoff difference between strategy WC and WD, while $\pi_{SC} - \pi_{SD}$ signifies the payoff difference between strategy SC and SD. The expressions for these payoff differences are presented as follows:

$$\begin{aligned} g_1(x,y) &= \pi_{WC} - \pi_{WD} \\ &= b_w \rho x + (1-\rho)\eta y - \alpha, \\ g_2(x,y) &= \pi_{SC} - \pi_{SD} \\ &= (1-\rho)(\alpha - \eta)x - \rho(b_s - \alpha)y + b_s - 2\alpha. \end{aligned} \tag{S2}$$

In these equations, the parameter ρ signifies the interaction scenario: $\rho = 1$ indicates solely intrapopulation interaction, while $\rho = 0$ indicates solely interpopulation interaction. The parameter η denotes the presence of gifting: $\eta = 0$ represents the scenario without gifting, while $\eta > 0$ denotes the scenario with gifting. Throughout our analysis, the parameters always satisfy the following ranking:

- $b_s > 2\alpha$, $b_s > b_w$, and $b_w > \alpha$.

APPENDIX A: STABILITY ANALYSIS OF REPLICATOR DYNAMICS WITHOUT INTERACTION DIVERSITY

The role of intrapopulation interaction

Firstly, we consider the solely intrapopulation scenario, *i.e.*, the condition of $\rho = 1$. Following this condition, the replicator equation Eq. S1 becomes to

$$\begin{aligned} \dot{x} &= x(1-x)(b_w x - \alpha), \\ \dot{y} &= y(1-y)(-(b_s - \alpha)y + b_s - 2\alpha). \end{aligned} \tag{S3}$$

It is evident that in the absence of coupling between weak and strong populations, the system of Eq.S3 returns to two symmetric population games. In the weak population, there exist two asymptotically stable states, including $x = 0$ and $x = 1$. The state to which the population converges depends on the initial fraction of cooperation, with a critical value at $x^* = \frac{\alpha}{b_w}$. In the strong population, a single asymptotically stable state exists at $y^* = \frac{b_s - 2\alpha}{b_s - \alpha}$, where cooperation coexists with defection. Whether cooperation-dominance ($y = 1$) or defection-dominance ($y = 0$) state is unstable.

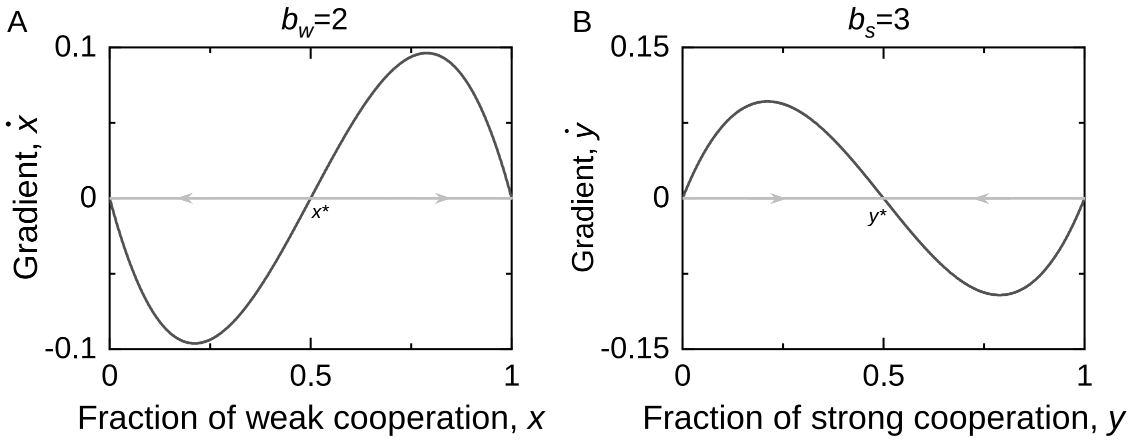

Figure S1: Phase diagram of equilibrium in solely intrapopulation interaction, given $\alpha = 1$.

The role of interpopulation interaction in the presence or absence of gifting

Secondly, we consider the solely interpopulation scenario, *i.e.*, the condition of $\rho = 0$. Following this condition, the replicator equation Eq. S1 becomes to

$$\begin{aligned} \dot{x} &= x(1-x)(\eta y - \alpha) \\ \dot{y} &= y(1-y)((\alpha - \eta)x + b_s - 2\alpha) \end{aligned} \tag{S4}$$

Lemma 1. *The system of Eq. S4 has the following equilibrium points:*

- $F_1 = (0,0)$*: $x = 0$ and $y = 0$, signifying the co-extinction of WC and SC.*
- $F_2 = (1,0)$*: $x = 1$ and $y = 0$, signifying a polarized state with the dominance of WC and the extinction of SC.*
- $F_3 = (0,1)$*: $x = 0$ and $y = 1$, signifying a polarized state with the extinction of WC and the dominance of SC.*
- $F_4 = (1,1)$*: $x = 1$ and $y = 1$, signifying a global cooperation and coordination state with the co-dominance of WC and SC.*

In the presence of gifting, there exists the fifth equilibrium point except for the above four equilibrium points.

- $F_5 = (\frac{2\alpha - b_s}{\alpha - \eta}, \frac{\alpha}{\eta})$*: $x^* = \frac{2\alpha - b_s}{\alpha - \eta}$ and $y^* = \frac{\alpha}{\eta}$, signifying the co-existence of WC, WD, SC, and SD. Note that this interior equilibrium point exists if and only if $0 < x^* < 1$ and $0 < y^* < 1$.*

Proof. By solving $\dot{x} = 0$ and $\dot{y} = 0$, we can derive the above equilibrium points. □

Without gifting, the condition $\dot{x} \leq 0$ always holds due to $\alpha > 0$, and the equality sign is met when $x = 0$ or $x = 1$. Therefore, adding perturbation around the equilibrium will always drive the weak population to a defection state, *i.e.*, $x = 0$. Meanwhile, the condition $\dot{y} \geq 0$ always holds due to $b_s - 2\alpha + \alpha x > 0$, and the equality sign is met when $y = 0$ or $y = 1$. Therefore, adding perturbation around the equilibrium will always drive the strong population to a cooperation state, *i.e.*, $y = 1$. In total, the equilibrium point $(0,1)$ emerges as the sole asymptotically stable state. When we consider the presence of gifting, the scenario becomes considerably more intricate, and we showcase the following three theorems to elucidate it:

Theorem 1. *In the scenario of solely interpopulation interaction with gifting, i.e., $\eta > 0$, the equilibrium point $(1,1)$ is asymptotically stable if $\alpha < \eta < b_s - \alpha$.*

Proof. The Jacobian of equilibrium point $(1,1)$ is

$$J(1,1) = \begin{bmatrix} \alpha-\eta & 0 \\ 0 & \alpha+\eta-b_s \end{bmatrix}. \tag{S5}$$

The trace and determinant of equilibrium point $(1,1)$ are calculated by $Tr = J[1,1] + J[2,2]$ and $\Delta = J[1,1]J[2,2]$. To satisfy the conditions $Tr < 0$ and $\Delta > 0$ simultaneously, it is necessary that $J[1,1] < 0$ and $J[2,2] < 0$. Therefore, the equilibrium point $(1,1)$ is an asymptotically stable state if $\alpha < \eta < b_s - \alpha$.

□

Theorem 2. *In the scenario of solely interpopulation interaction with gifting, i.e., $\eta > 0$, the equilibrium point $(0,1)$ is asymptotically stable if $\eta < \alpha$, given $b_s > 2\alpha$.*

Proof. The Jacobian of equilibrium point $(0,1)$ is

$$J(0,1) = \begin{bmatrix} \eta-\alpha & 0 \\ 0 & 2\alpha-b_s \end{bmatrix}. \tag{S6}$$

The trace and determinant of equilibrium point $(0,1)$ are calculated by $Tr = J[1,1] + J[2,2]$ and $\Delta = J[1,1]J[2,2]$. To satisfy the conditions $Tr < 0$ and $\Delta > 0$ simultaneously, it is necessary that $J[1,1] < 0$ and $J[2,2] < 0$. Therefore, the equilibrium point $(0,1)$ is an asymptotically stable state if $\eta < \alpha$, given $b_s > 2\alpha$.

□

Theorem 3. *In the scenario of solely interpopulation interaction with gifting, i.e., $\eta > 0$, the equilibrium point $(\frac{2\alpha-b_s}{\alpha-\eta}, \frac{\alpha}{\eta})$ is neutrally stable with all the boundary equilibrium points are unstable if $\eta > b_s - \alpha$.*

Proof. The Jacobian of equilibrium point $(\frac{2\alpha-b_s}{\alpha-\eta}, \frac{\alpha}{\eta})$ is

$$J(\frac{2\alpha-b_s}{\alpha-\eta}, \frac{\alpha}{\eta}) = \begin{bmatrix} 0 & -\frac{(2\alpha-b_s)(\alpha+\eta-b_s)\eta}{(\alpha-\eta)^2} \\ -\frac{\alpha(\alpha-\eta)^2}{\eta^2} & 0 \end{bmatrix}. \tag{S7}$$

The trace and determinant are calculated by $Tr = 0$ and $\Delta = -J[1,2]J[2,1]$. If $\Delta > 0$, the eigenvalues of the Jacobian matrix are pure imaginary numbers, leading to a neutrally stable state. Therefore, when $\eta > b_s - \alpha$, the equilibrium point $(\frac{2\alpha-b_s}{\alpha-\eta}, \frac{\alpha}{\eta})$ is neutrally stable. The condition $\eta > b_s - \alpha$ contradicts the stability conditions for $(0,1)$ and $(1,1)$, rendering both $(0,1)$ and $(1,1)$ unstable. The trace and determinant of the Jacobian matrix for equilibrium point $(0,0)$ are $Tr = b_s - 3\alpha$ and $\Delta = -\alpha(b_s - 2\alpha)$. Since $\Delta < 0$ holds consistently, the equilibrium point $(0,0)$ is unstable. The trace and determinant of the Jacobian matrix for equilibrium point $(1,0)$ are $Tr = \alpha$ and $\Delta = \alpha(b_s - \alpha - \eta)$. Since $Tr > 0$ holds consistently, the equilibrium point $(1,0)$ is unstable. □

APPENDIX B: STABILITY ANALYSIS OF REPLICATOR DYNAMICS WITH INTERACTION DIVERSITY

In scenarios with interaction diversity, *i.e.*, $0 < \rho < 1$, we meet complicated evolutionary dynamics:

$$\begin{aligned} \dot{x} &= x(1-x)(b_w\rho x + (1-\rho)\eta y - \alpha), \\ \dot{y} &= y(1-y)((1-\rho)(\alpha-\eta)x - \rho(b_s-\alpha)y + b_s - 2\alpha). \end{aligned} \tag{S8}$$

Lemma 2. *The system of Eq. S8 has the following equilibrium points:*

- $F_1 = (0,0)$*,* $F_2 = (1,0)$*,* $F_3 = (0,1)$*, and* $F_4 = (1,1)$*.*
- $F_5 = (\frac{\alpha}{b_w\rho}, 0)$*:* $x_1^* = \frac{\alpha}{b_w\rho}$ *and* $y = 0$*, signifying the existence of WC in the extinction of SC. Note that this equilibrium exists if and only if* $0 < x_1^* < 1$*.*

- $F_6 = (\frac{\eta\rho+\alpha-\eta}{b_w\rho}, 1)$*:* $x_2^* = \frac{\eta\rho+\alpha-\eta}{b_w\rho}$ *and* $y = 1$*, signifying the existence of WC in the dominance of SC. Note that this equilibrium exists if and only if* $0 < x_2^* < 1$*.*

- $F_7 = (0, \frac{b_s-2\alpha}{\rho(b_s-\alpha)})$*:* $x = 0$ *and* $y_1^* = \frac{b_s-2\alpha}{\rho(b_s-\alpha)}$*, signifying the existence of SC in the extinction of WC. Note that this equilibrium exists if and only if* $0 < y_1^* < 1$*.*

- $F_8 = (1, \frac{\rho(\eta-\alpha)+b_s-\eta-\alpha}{\rho(b_s-\alpha)})$*:* $x = 1$ *and* $y_2^* = \frac{\rho(\eta-\alpha)+b_s-\eta-\alpha}{\rho(b_s-\alpha)}$*, signifying the existence of SC in the dominance of WC. Note that this equilibrium exists if and only if* $0 < y_2^* < 1$*.*

- $F_9 = (x^*, y^*)$*:* $x^* = \frac{\rho\alpha(b_s-\alpha)+\eta(2\alpha-b_s)(1-\rho)}{(1-\rho)^2\eta(\alpha-\eta)+\rho^2 b_w(b_s-\alpha)}$ *and* $y^* = \frac{\alpha(1-\rho)(\alpha-\eta)+b_w\rho(b_s-2\alpha)}{(1-\rho)^2\eta(\alpha-\eta)+\rho^2 b_w(b_s-\alpha)}$*, signifying the co-existence of WC, WD, SC, and SD. Note that this equilibrium exists if and only if* $0 < x^* < 1$ *and* $0 < y^* < 1$*.*

Proof. By solving $\dot{x} = 0$ and $\dot{y} = 0$, we can derive the above equilibrium points. □

Given the condition of $b_s > 2\alpha$, $b_s > b_w$, and $b_w > \alpha$, we showcase the following theorems:

Theorem 4. *In the absence of gifting, i.e.,* $\eta = 0$*, the equilibrium point* $(1, 1)$ *is asymptotically stable if* $b_s > \frac{\alpha}{1-\rho}$ *and* $b_w > \frac{\alpha}{\rho}$*. In the presence of gifting, i.e.,* $\eta > 0$*, the equilibrium point* $(1, 1)$ *is asymptotically stable if* $b_s > \frac{\alpha+\eta(1-\rho)}{1-\rho}$ *and* $b_w > \frac{\eta(\rho-1)+\alpha}{\rho}$*.*

Proof. The Jacobian of equilibrium point $(1, 1)$ is

$$J(1,1) = \begin{bmatrix} \eta(\rho-1) - b_w\rho + \alpha & 0 \\ 0 & (b_s-\eta)\rho - b_s + \eta + \alpha \end{bmatrix}. \tag{S9}$$

The trace and determinant of equilibrium point $(1, 1)$ are calculated by $Tr = J[1,1] + J[2,2]$ and $\Delta = J[1,1]J[2,2]$. To satisfy the conditions $Tr < 0$ and $\Delta > 0$ simultaneously, it is necessary that $J[1,1] < 0$ and $J[2,2] < 0$. Therefore, in the absence of gifting, *i.e.*, $\eta = 0$, the equilibrium point $(1, 1)$ is an asymptotically stable state if $b_s > \frac{\alpha}{1-\rho}$ and $b_w > \frac{\alpha}{\rho}$. In the presence of gifting, *i.e.*, $\eta > 0$, the equilibrium point $(1, 1)$ is an asymptotically stable state if $b_s > \frac{\alpha+\eta(1-\rho)}{1-\rho}$ and $b_w > \frac{\eta(\rho-1)+\alpha}{\rho}$. □

Theorem 5. *In the absence of gifting, i.e.,* $\eta = 0$*, the equilibrium point* $(0, 1)$ *is asymptotically stable if* $b_s > \frac{(2-\rho)\alpha}{1-\rho}$*. In the presence of gifting, i.e.,* $\eta > 0$*, the equilibrium point* $(0, 1)$ *is asymptotically stable if* $\eta < \frac{\alpha}{1-\rho}$ *and* $b_s > \frac{(2-\rho)\alpha}{1-\rho}$*.*

Proof. The Jacobian of equilibrium point $(0, 1)$ is

$$J(0,1) = \begin{bmatrix} -\eta\rho + \eta - \alpha & 0 \\ 0 & b_s(\rho-1) + \alpha(2-\rho) \end{bmatrix}. \tag{S10}$$

The trace and determinant of equilibrium point $(0, 1)$ are calculated by $Tr = J[1,1] + J[2,2]$ and $\Delta = J[1,1]J[2,2]$. To satisfy the conditions $Tr < 0$ and $\Delta > 0$ simultaneously, it is necessary that $J[1,1] < 0$ and $J[2,2] < 0$. Therefore, in the absence of gifting, *i.e.*, $\eta = 0$, the equilibrium point $(0, 1)$ is an asymptotically stable state if $b_s > \frac{(2-\rho)\alpha}{1-\rho}$. In the presence of gifting, *i.e.*, $\eta > 0$, the equilibrium point $(0, 1)$ is an asymptotically stable state if $\eta < \frac{\alpha}{1-\rho}$ and $b_s > \frac{(2-\rho)\alpha}{1-\rho}$. □

Theorem 6. *In the absence of gifting, i.e.,* $\eta = 0$*, the equilibrium point* $(1, 0)$ *is unstable. In the presence of gifting, i.e.,* $\eta > 0$*, the equilibrium point* $(1, 0)$ *is asymptotically stable if* $b_w > \frac{\alpha}{\rho}$ *and* $\eta > \frac{b_s-(1+\rho)\alpha}{1-\rho}$*.*

Proof. The Jacobian of equilibrium point $(1, 0)$ is

$$J(1,0) = \begin{bmatrix} \alpha - b_w\rho & 0 \\ 0 & \eta(\rho-1) + b_s - (\rho+1)\alpha \end{bmatrix}. \tag{S11}$$

The trace and determinant of equilibrium point $(1, 0)$ are calculated by $Tr = J[1,1] + J[2,2]$ and $\Delta = J[1,1]J[2,2]$. To satisfy the conditions $Tr < 0$ and $\Delta > 0$ simultaneously, it is necessary that $J[1,1] < 0$ and $J[2,2] < 0$. Therefore, in the absence of gifting, *i.e.*, $\eta = 0$, the equilibrium point $(1, 0)$ is unstable as $J[2,2] > 0$ hold consistently. In the presence of gifting, *i.e.*, $\eta > 0$, the equilibrium point $(1, 0)$ is an asymptotically stable state when $b_w > \frac{\alpha}{\rho}$ and $\eta > \frac{b_s-(1+\rho)\alpha}{1-\rho}$. □

Theorem 7. *In the absence of gifting, i.e., $\eta = 0$, the equilibrium point $(0, \frac{b_s-2\alpha}{\rho(b_s-\alpha)})$ is asymptotically stable, given $0 < y_1^* < 1$. In the presence of gifting, i.e., $\eta > 0$, the equilibrium point $(0, \frac{b_s-2\alpha}{\rho(b_s-\alpha)})$ is asymptotically stable if $\eta < \frac{\rho\alpha(b_s-\alpha)}{(1-\rho)(b_s-2\alpha)}$, given $0 < y_1^* < 1$.*

Proof. The Jacobian of equilibrium point $(0, \frac{b_s-2\alpha}{\rho(b_s-\alpha)})$ is

$$J(0, y_1^*) = \begin{bmatrix} \frac{\rho\alpha^2+((-b_s+2\eta)\rho-2\eta)\alpha-(-1+\rho)b_s\eta}{\rho(b_s-\alpha)} & 0 \\ \frac{(-\alpha+\eta)((-\rho+2)\alpha+b_s(-1+\rho))(-1+\rho)(b_s-2\alpha)}{\rho^2(b_s-\alpha)^2} & \frac{-((-\rho+2)\alpha+(-1+\rho)b_s)(b_s-2\alpha)}{\rho(b_s-\alpha)} \end{bmatrix}. \tag{S12}$$

The trace and determinant of equilibrium point $(0, \frac{b_s-2\alpha}{\rho(b_s-\alpha)})$ are calculated by $Tr = J[1,1] + J[2,2]$ and $\Delta = J[1,1]J[2,2]$. To satisfy the conditions $Tr < 0$ and $\Delta > 0$ simultaneously, it is necessary that $J[1,1] < 0$ and $J[2,2] < 0$. Therefore, in the absence of gifting, *i.e.*, $\eta = 0$, the equilibrium point $(0, \frac{b_s-2\alpha}{\rho(b_s-\alpha)})$ is an asymptotically stable state as $J[1,1] < 0$ and $J[2,2] < 0$ always hold, given $0 < y_1^* < 1$. In the presence of gifting, *i.e.*, $\eta > 0$, the equilibrium point $(0, \frac{b_s-2\alpha}{\rho(b_s-\alpha)})$ is an asymptotically stable state if $\eta < \frac{\rho\alpha(b_s-\alpha)}{(1-\rho)(b_s-2\alpha)}$, given $0 < y_1^* < 1$. □

Theorem 8. *In the absence of gifting, i.e., $\eta = 0$, the equilibrium point $(1, \frac{b_s-\alpha-\alpha\rho}{\rho(b_s-\alpha)})$ is asymptotically stable if $\frac{\alpha}{b_w} < \rho < 1$, given $0 < y_2^* < 1$. In the presence of gifting, i.e., $\eta > 0$, the equilibrium point $(1, \frac{\rho(\eta-\alpha)+b_s-\eta-\alpha}{\rho(b_s-\alpha)})$ is asymptotically stable if $b_w > \frac{\alpha-(1-\rho)\eta y_2^*}{\rho}$, given $0 < y_2^* < 1$.*

Proof. The Jacobian of equilibrium point $(1, \frac{b_s-\alpha-\alpha\rho}{\rho(b_s-\alpha)})$ is

$$J(1, y_2^*) = \begin{bmatrix} (1-2x)g_1(x,y) & 0 \\ y(1-y)\frac{\partial g_2(x,y)}{\partial x} & (1-2y)h_2(x,y) + y(1-y)\frac{\partial g_2(x,y)}{\partial y} \end{bmatrix}. \tag{S13}$$

The trace and determinant of equilibrium point $(1, \frac{\rho(\eta-\alpha)+b_s-\eta-\alpha}{\rho(b_s-\alpha)})$ are calculated by $Tr = J[1,1] + J[2,2]$ and $\Delta = J[1,1]J[2,2]$. To satisfy the conditions $Tr < 0$ and $\Delta > 0$ simultaneously, it is necessary that $J[1,1] < 0$ and $J[2,2] < 0$. Therefore, in the absence of gifting, *i.e.*, $\eta = 0$, the equilibrium point $(1, \frac{b_s-\alpha-\alpha\rho}{\rho(b_s-\alpha)})$ is an asymptotically stable state if $\frac{\alpha}{b_w} < \rho < 1$, given $0 < y_2^* < 1$. In the presence of gifting, *i.e.*, $\eta > 0$, the equilibrium point $(1, \frac{\rho(\eta-\alpha)+b_s-\eta-\alpha}{\rho(b_s-\alpha)})$ is an asymptotically stable state if $b_w > \frac{\alpha-(1-\rho)\eta y_2^*}{\rho}$, given $0 < y_2^* < 1$. □

Theorem 9. *In the absence of gifting, i.e., $\eta = 0$, the equilibrium point $(\frac{\alpha}{b_w\rho}, \frac{b_s b_w\rho-2\alpha b_w\rho-\alpha^2\rho+\alpha^2}{b_w\rho^2(b_s-\alpha)})$ is unstable. In the presence of gifting, i.e., $\eta > 0$, the equilibrium point (x^*, y^*) is asymptotically stable if $b_w\rho x^*(1-x^*) - \rho(b_s-\alpha)y^*(1-y^*) < 0$ and $\rho^2 b_w(b_s-\alpha) + (1-\rho)^2\eta(\alpha-\eta) < 0$.*

Proof. In the absence of gifting, *i.e.*, $\eta = 0$, the Jaccobian is given as follows:

$$J^* = \begin{bmatrix} b_w\rho x(1-x) & 0 \\ (1-\rho)\alpha y(1-y)) & -\rho(b_s-\alpha)y(1-y) \end{bmatrix}_{\left(\frac{\alpha}{b_w\rho}, \frac{b_s b_w\rho-2\alpha b_w\rho-\alpha^2\rho+\alpha^2}{b_w\rho^2(b_s-\alpha)}\right)}. \tag{S14}$$

The trace and determinant of equilibrium point $(\frac{\alpha}{b_w\rho}, \frac{b_s b_w\rho-2\alpha b_w\rho-\alpha^2\rho+\alpha^2}{b_w\rho^2(b_s-\alpha)})$ are calculated by $Tr = J[1,1] + J[2,2]$ and $\Delta = J[1,1]J[2,2]$. To satisfy the conditions $Tr < 0$ and $\Delta > 0$ simultaneously, it is necessary that $J[1,1] < 0$ and $J[2,2] < 0$. Given $0 < \frac{\alpha}{b_w\rho} < 1$, $J[1,1] > 0$ holds consistently. Therefore, in the absence of gifting, the equilibrium point $(\frac{\alpha}{b_w\rho}, \frac{b_s b_w\rho-2\alpha b_w\rho-\alpha^2\rho+\alpha^2}{b_w\rho^2(b_s-\alpha)})$ is unstable.

In the presence of gifting, *i.e.*, $\eta = 0$, the Jaccobian is

$$J(x^*, y^*) = \begin{bmatrix} b_w\rho x(1-x) & (1-\rho)\eta x(1-x) \\ (1-\rho)(\alpha-\eta)y(1-y)) & -\rho(b_s-\alpha)y(1-y) \end{bmatrix}. \tag{S15}$$

The trace and determinant of equilibrium state (x^*, y^*) are calculated by $Tr = J[1,1] + J[2,2] = b_w\rho x^*(1-x^*) - \rho(b_s-\alpha)y^*(1-y^*)$ and $\Delta = J[1,1]J[2,2] - J[1,2]J[2,1] = x^*(1-x^*)y^*(1-y^*)(-\rho^2 b_w(b_s-1) - (1-\rho)^2\eta(1-\eta))$. Therefore, in the presence of gifting, the equilibrium point (x^*, y^*) is an asymptotically stable state when $Tr < 0$ and $\Delta > 0$. □

Theorem 10. *Regardless of the presence or absence of gifting, the equilibrium points* $(0,0)$, $(\frac{\alpha}{b_w\rho},0)$, *and* $(\frac{\eta\rho-\eta+\alpha}{b_w\rho},1)$ *are unstable.*

Proof. The Jacobian of equilibrium point $(0,0)$ is

$$J(0,0)=\begin{bmatrix} -\alpha & 0 \\ 0 & b_s-2\alpha \end{bmatrix}. \tag{S16}$$

The trace and determinant of the equilibrium point $(0,0)$ are given by $b_s-3\alpha$ and $\alpha(2\alpha-b_s)$, respectively. In the condition where $b_s>2\alpha$, it becomes impossible to satisfy $Tr<0$ and $\Delta>0$ simultaneously. Consequently, the equilibrium point $(0,0)$ is unstable.

The Jacobian of equilibrium point $(\frac{\alpha}{b_w\rho},0)$ is

$$J(\frac{\alpha}{b_w\rho},0)=\begin{bmatrix} \frac{(b_w\rho-\alpha)\alpha}{b_w\rho} & \frac{\alpha(-b_w\rho+\alpha)(-1+\rho)\eta}{b_w^2\rho^2} \\ 0 & \frac{(1-\rho)\alpha^2+((-2b_w+\eta)\rho-\eta)\alpha+b_sb_w\rho}{b_w\rho} \end{bmatrix}. \tag{S17}$$

The trace and determinant of equilibrium point $(\frac{\alpha}{b_w\rho},0)$ are calculated by $Tr=J[1,1]+J[2,2]$ and $\Delta=J[1,1]J[2,2]$. To satisfy the conditions $Tr<0$ and $\Delta>0$ simultaneously, it is necessary that $J[1,1]<0$ and $J[2,2]<0$. Given $0<x_1^*<1$, $J[1,1]>0$ holds consistently. Therefore, regardless of the absence or presence of gifting, the equilibrium point $(\frac{1}{b_w\rho},0)$ is unstable.

The Jacobian of equilibrium point $(\frac{\eta\rho-\eta+\alpha}{b_w\rho},1)$ is

$$J(\frac{\eta\rho-\eta+\alpha}{b_w\rho},1)=\begin{bmatrix} (1-2x)g_1(x,y)+b_w\rho x(1-x) & x(1-x)\frac{\partial g_1(x,y)}{\partial y} \\ 0 & (1-2y)g_2(x,y) \end{bmatrix}. \tag{S18}$$

The trace and determinant of equilibrium point $(\frac{\eta\rho-\eta+\alpha}{b_w\rho},1)$ are calculated by $Tr=J[1,1]+J[2,2]$ and $\Delta=J[1,1]J[2,2]$. To satisfy the conditions $Tr<0$ and $\Delta>0$ simultaneously, it is necessary that $J[1,1]<0$ and $J[2,2]<0$. Given $0<x_2^*<1$, $J[1,1]>0$ holds consistently. Therefore, regardless of the absence or presence of gifting, the equilibrium point $(\frac{\eta\rho-\eta+\alpha}{b_w\rho},1)$ is unstable. □

Corollary 1. *In the absence of gifting, there exists a bi-stable state of* $(1,1)$ *and* $(0,1)$ *when* $b_s>\frac{(2-\rho)\alpha}{1-\rho}$ *and* $b_w>\frac{\alpha}{\rho}$*; a bi-stable state of* $(1,1)$ *and* $(0,\frac{b_s-2\alpha}{\rho(b_s-\alpha)})$ *when* $max(2\alpha,\frac{\alpha}{1-\rho})<b_s<\frac{(2-\rho)\alpha}{1-\rho}$ *and* $b_w>\frac{\alpha}{\rho}$*; a bi-stable state of* $(1,1)$ *and* $(1,\frac{b_s-\alpha-\alpha\rho}{\rho(b_s-\alpha)})$ *when* $b_s<\frac{\alpha}{1-\rho}$, $\rho>\frac{1}{2}$, *and* $b_w>\frac{\alpha}{\rho}$.

Proof. The conditions for the bi-stable state of $(1,1)$ and $(0,1)$ are summarized from theorems 4 and 5. The conditions for the bi-stable state of $(1,1)$ and $(0,\frac{b_s-2\alpha}{\rho(b_s-\alpha)})$ are summarized from theorems 4 and 7. The conditions for the bi-stable state of $(1,1)$ and $(1,\frac{b_s-\alpha-\alpha\rho}{\rho(b_s-\alpha)})$ are summarized from theorems 4 and 8. □

APPENDIX C: AGENT-BASED SIMULATION FOR FINITELY LARGE POPULATIONS

In a multi-agent system comprising weak and strong populations, each population consists of N agents. Initially, every agent has the opportunity to choose between cooperation or defection. With their chosen strategy, a randomly selected agent, denoted as i, obtains an expected payoff through interactions with $N-1$ other agents. When considering interaction diversity, these interacting agents encompass, on average, a fraction of ρ from the same population and a fraction of $1-\rho$ from a different population. Subsequently, agents update their strategies following the Fermi process [1]. Each agent imitates the strategy of one agent from the same population, with the probability determined by the Fermi function:

$$W=\frac{1}{1+e^{-(\mathcal{P}_j-\mathcal{P}_i)/K}}, \tag{S19}$$

where $\mathcal{P}_i$ and $\mathcal{P}_i$ represent the expected payoff of focal agent i and counterpart j, respectively. These simulation results are obtained with parameters set at $N=1000$ and $K=0.01$.

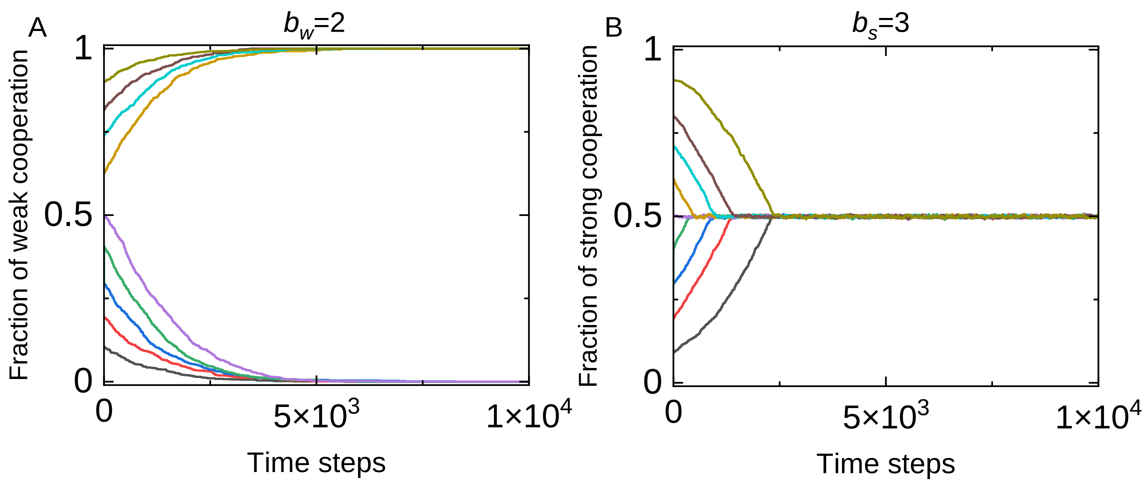

Figure S2: Time series of cooperation in scenarios with solely intrapopulation interaction, given $\alpha = 1$ and $\rho = 1$. These findings serve to validate the theoretical results in Figure S1.

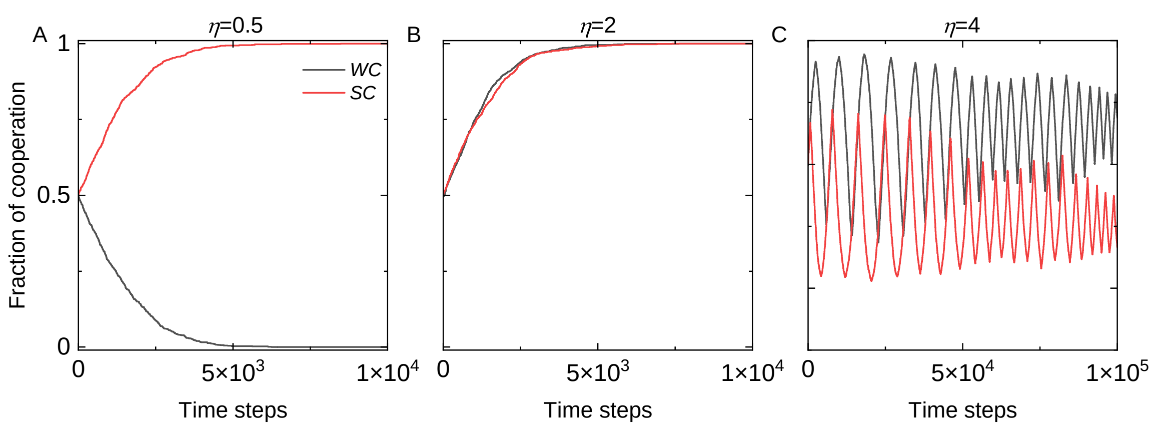

Figure S3: Time series of cooperation in scenarios with solely interpopulation interaction in the presence of gifting, given $\alpha = 1$, $\rho = 0$, $b_s = 4$, and $b_w = 2$. These findings validate the theoretical results in Fig. 2.

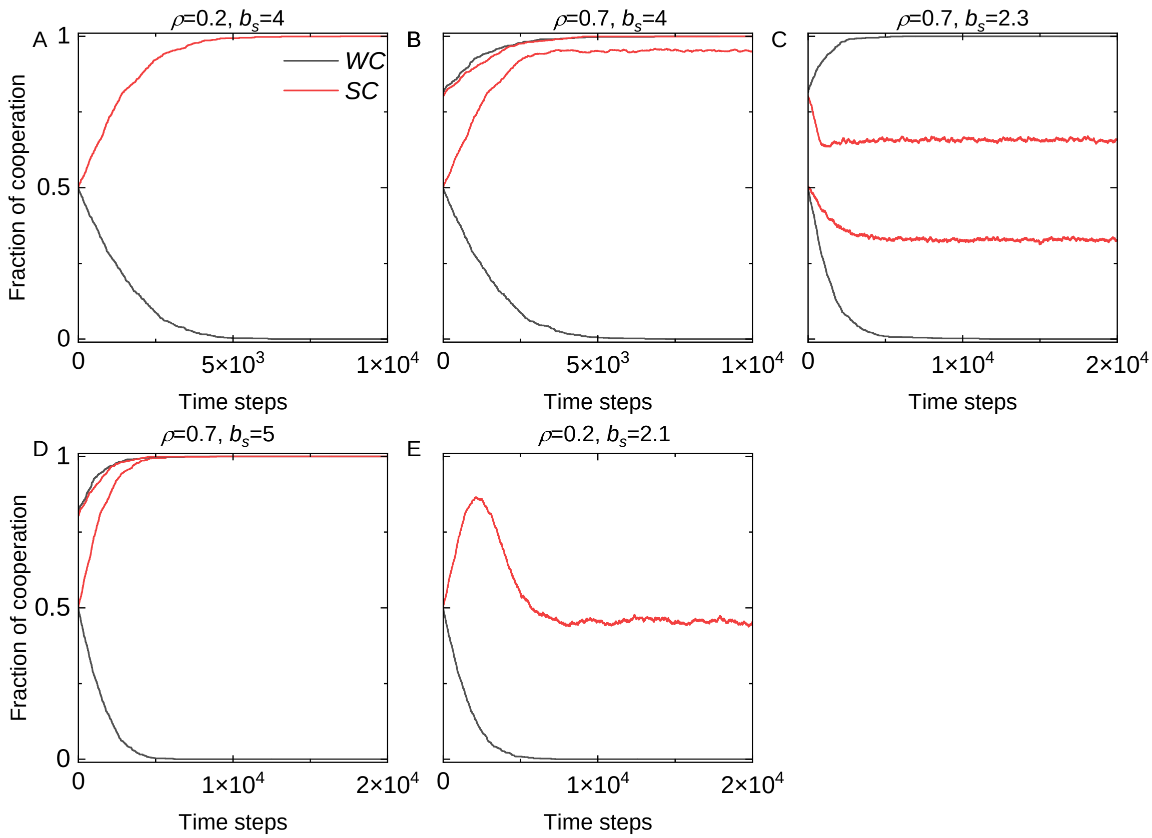

Figure S4: Time series of cooperation in scenarios with interaction diversity in the absence of gifting, given $\alpha = 1$ and $b_w = 2$. (A) and (E) show the monostable state, while (B), (C), and (D) show the bi-stable state. These findings validate the theoretical results in Fig. 3.

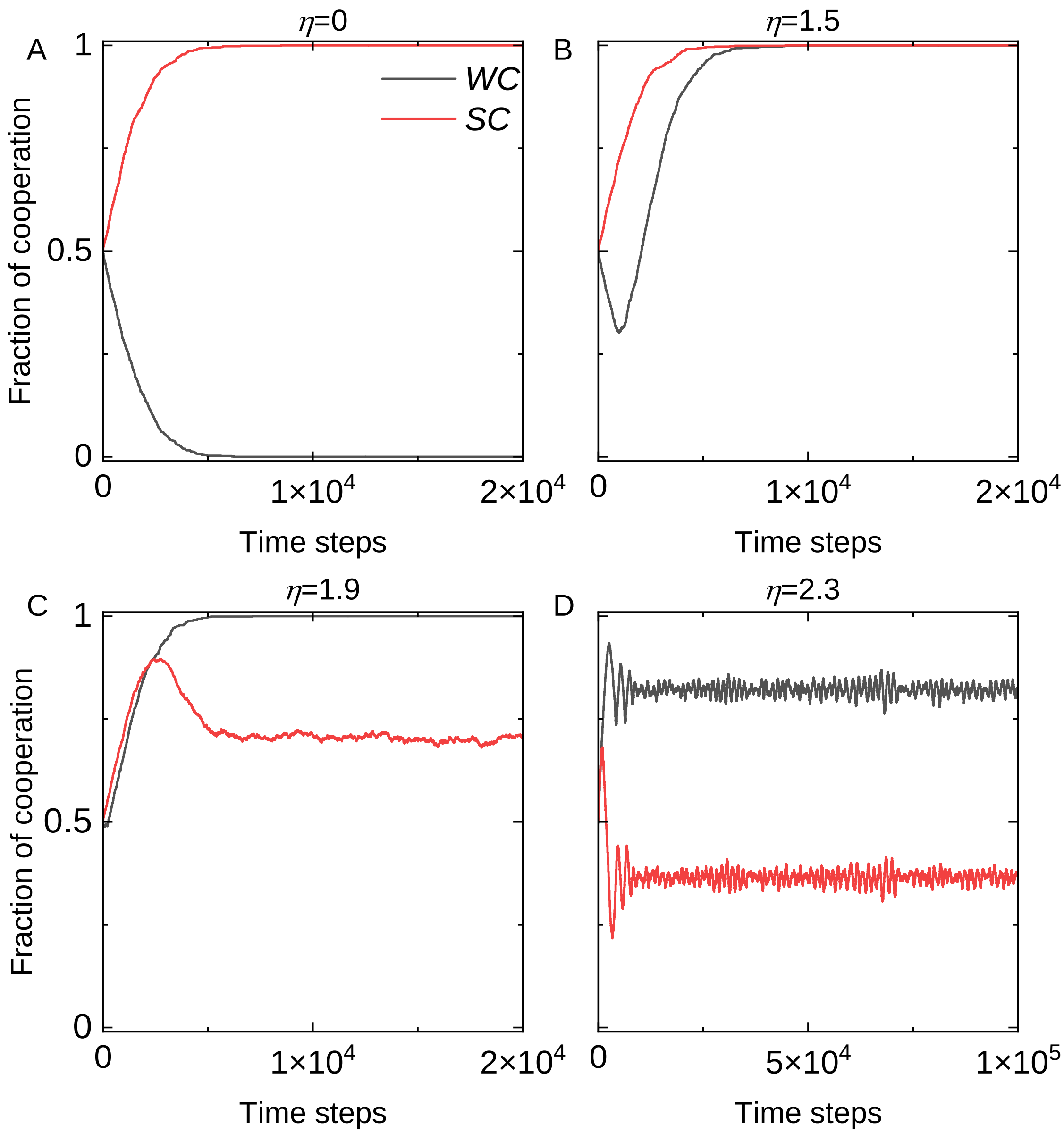

Figure S5: Time series of cooperation in scenarios with interaction diversity in the presence of gifting, given $\alpha = 1$, $\rho = 0.2$, $b_s = 3$, and $b_w = 2$. These findings validate the theoretical results in Fig. 4.

References

[1] Christoph Hauert and György Szabó. Game theory and physics. *American Journal of Physics*, 73(5):405–414, 2005.