Simplified approach to the Jones calculus in retracing optical circuits

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The application of the Jones calculus to optical circuits in which counterpropagating light beams are present is discussed, with particular attention to the conventions associated with the Jones calculus for the description of the polarization state of an optical beam and to the reference system adopted. The reference system adopted here differs from that used by Jones, but it exploits a simpler formalism when used to describe complex optical systems where multiple reflections take place. The differences between these two reference systems are pointed out with particular attention to the behavior of nonreciprocal optical devices and reflectors. An application of the results to a simple optical circuit is presented.

1. Introduction

The evolution of the state of polarization (SOP) of a light beam propagating in an optical circuit can be represented as the trajectory of a point on the surface of the Poincarè sphere or calculated with the Jones or the Müller calculus. These computational methods handle the effects of optical devices inducing polarization changes well but with the implicit assumption that the direction of propagation of the light beam, i.e., the **k** vector, is fixed. Even in the simplest optical circuits this condition is seldom fulfilled because of reflections from mirrors or other optical surfaces. Counterpropagating light beams inside the same optical devices may add another difficulty to handling the problem. Jones himself made rules to deal with optical circuits in which the direction of light was reversed,[1] but these rules depend on the reference system used for the description of the optical circuit. Moreover the Jones matrices associated with particular optical devices, such as mirrors, also depend on the particular reference system adopted. Therefore, when a reference system is chosen, not only the matrix transformation rules but also the form of particular matrices must be consistent with it; otherwise the Jones calculus gives erroneous results. If a nonreciprocal device, such as a Faraday rotator, is inserted into an optical circuit, new problems arise when the light direction is reversed. Jones pointed out that some statements established for reciprocal optical systems did not hold in this case. Thus, with some constraints, it is possible to describe the behavior of such a system with simple manipulations of the transformation rules.

The reference system adopted here is slightly different from that used by Jones, but it is currently used in the technical literature and provides a simpler approach to the calculus of complex optical systems.

The aim of this research is to point out the differences between these two reference systems, in particular when specular reflectors and birefringent optical elements, reciprocal and/or nonreciprocal, are present. Few of the calculation rules given here are the translations in this new framework of the analogous properties given by Jones. Simple applications to practical circuits are described to show the differences between these two approaches.

2. Light-Propagation Reversal

It is well known that a sequence of optical devices acting on the polarization of an optical beam is characterized not only by the polarization properties of each device but also by their relative order inside the circuit. This property may be apparent in the various formalisms used to describe the polarization evolution in an optical scheme: For example, in the Jones calculus the multiplication order of the various matrices must be observed; otherwise erroneous results may be obtained. Consequently, if the light reverses its direction of propagation, it should be necessary to recalculate the Jones matrix representing the whole optical circuit. Concerning this problem, Jones[1] indicated that there was no need to

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recalculate the matrix but only to transpose the relative Jones matrix. But he also added that the statement is true only if all the optical devices of the circuit are reciprocal: Whenever one nonreciprocal element such as a Faraday rotator is present, the whole matrix must be recalculated. To carry out the calculus, we must know the matrix form of each device when the light direction is reversed: If this task is quite simple for elementary matrices, it is not trivial when complex matrices, describing devices in which both reciprocal and nonreciprocal effects take place simultaneously, are analyzed. The introduction of these rules enables us to extend the Jones formalism to these systems and greatly simplifies the entire calculus procedure.

The first step toward this objective is the definition of the reference system and the conventions used to define the SOP of the light and the orientations of the polarization-active devices: Here we adopt a right-handed Cartesian coordinate system. The optical elements are placed in an orderly sequence along the z axis (the light-propagation axis) with their plates lying in x–y oriented planes. The x axis is normal to the incidence plane defined by the y–z axes. We also adopt the convention that the optical beam always travels toward the observer.[2] The azimuth of a polarized light beam is measured counterclockwise starting from the x axis. One measures the azimuth of a wave plate, i.e., the angle between the fast axis and the x axis of our coordinate system, viewing the light beam passing through the optical device toward the observer.[2] If the light direction is reversed, we observe different behavior from each of the optical components, depending on the polarization properties of the device. The conventions assumed here are quite different from those defined by Jones. Following our definitions, for example, the right-handed optical rotator rotates the plane of polarization of the linearly polarized lightwave always on the right, independent of the direction of propagation of the light beam itself. On the contrary, in the Jones framework the same rotator rotates the light polarization right or left, depending on the light direction. Moreover the SOP of a light beam is defined completely by the two components of the relative Jones vector even when counterpropagating beams are present: There is no need to add another subscript indicating the light-traveling direction.[3]

It is possible to distinguish four basic properties of polarization-active devices: linear birefringence, circular birefringence (also called optical activity), linear dichroism, and circular dichroism. Following the conventions stated above, we can observe that optically active devices are insensitive to the direction of the light, because the handedness is a property of the material and therefore independent from external references. For optically active components the Jones matrix always maintains the same form. The same considerations apply to the circular dichroism. This does not hold for nonreciprocal devices, such as the Faraday rotator: The handedness is related to the relative orientation of the magnetic field and the propagation direction of the electromagnetic wave. For nonreciprocal rotators the associated Jones matrix must change its handedness when the light passes through the optical device in the opposite direction.

Following the convention to define the azimuth of a wave plate as seen when one looks through the birefringent element toward the light source, if light propagates along the opposite direction, the azimuth must change its sign, as shown in Fig. 1. To translate this statement into the Jones formalism, one must remember that matrix $\mathbf{M}$ associated with a linear birefringent wave plate of retardance β and azimuth ρ can be written as

$$\mathbf{M}(\beta, \rho) = \mathbf{S}(\rho)\mathbf{L}(\beta)\mathbf{S}(-\rho), \tag{1}$$

where $\mathbf{S}(\rho)$ is the rotation matrix,

$$\mathbf{S}(\rho) = \begin{bmatrix} \cos(\rho) & -\sin(\rho) \\ \sin(\rho) & \cos(\rho) \end{bmatrix}, \tag{2}$$

and $\mathbf{L}(\beta)$ is the Jones matrix associated with a linear birefringement plate of retardance β and azimuth $\rho = 0$:

$$\mathbf{L}(\beta) = \begin{bmatrix} \exp(i\beta/2) & 0 \\ 0 & \exp(-i\beta/2) \end{bmatrix}. \tag{3}$$

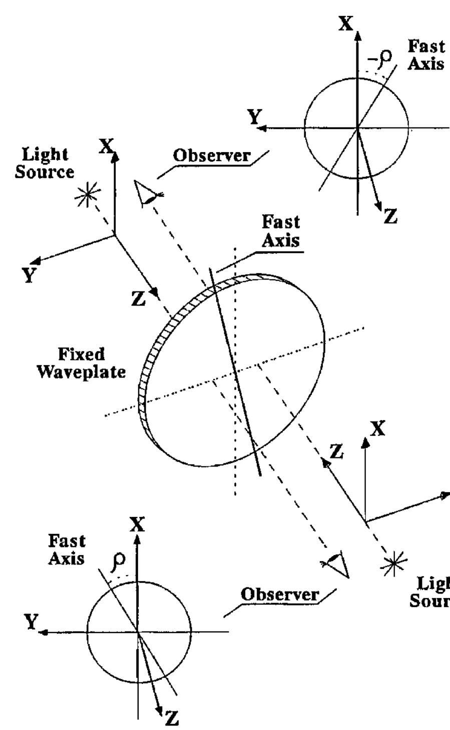

Fig. 1. Definition of the azimuth angle of a linear birefringent or linear dichroic wave plate: The observer always looks toward the light source through the wave plate. Reversing the light propagation causes one to observe the light source from the other side of the wave plate. As a result the azimuth angle also changes from ρ to $-\rho$.

The Jones matrix of the same optical element when the direction of the light beam is reversed is obtained simply from Eq. (1), changing ρ to $-\rho$. The same considerations apply to a linear dichroic optical medium, whose matrix is obtained by the substitution of $\mathbf{L}(\beta)$ into Eq. (3) with $\mathbf{D}(\mathrm{p})$, where $\mathbf{D}(\mathrm{p})$ is

$$\mathbf{D}(\mathrm{p}) = \begin{bmatrix} \exp(p) & 0 \\ 0 & \exp(-p) \end{bmatrix}. \tag{4}$$

The matrix must be corrected with the scalar $\exp(-p_{\mathrm{av}})$, which is the device amplitude transmittance averaged over the two principal dichroic axes.

Note the different results obtained here compared with the analogous statements given by Jones[1]: The discrepancies arise from the choice of the reference system and the convention stated for the definition of the SOP of a light beam. These differences do not appear in circuits in which the direction of propagation of light does not change.

Until now we have examined the behavior of the basic properties of the polarization-active devices when they are not mixed into the same material. The extension of the previous rules to this more complex case is a further step, and we use the $\mathbf{N}$-matrix formalism[1] to accomplish the task. Jones found eight basic $\boldsymbol{\Theta}$ matrices that can be combined, each with a suitable weight, to describe any optical material. If we apply the previous considerations to these basic matrices, we can obtain the behavior of any optical device, reciprocal or not, when the propagation direction of the light is reversed. The correctness of this approach can be tested directly from the definitions of an $\mathbf{N}$ matrix starting from the corresponding $\mathbf{M}$ matrix of a discrete optical device. Moreover the $\mathbf{N}$ matrices are transformed on rotation in the same way as the $\mathbf{M}$ matrices[1] when Eq. (1) is applied again. Let us examine an optical medium possessing all the basic polarization properties discussed above: The $\mathbf{N}$ matrix corresponding to such a material is given by the sum of the whole set of eight basic matrices and has the form

$$\mathbf{N} = \begin{bmatrix} -\kappa + \mathrm{p}_0 - i\eta + i\xi_0 & -\omega + p_{45} - iq + i\xi_{45} \\ \omega + p_{45} + iq + i\xi_{45} & -\kappa - p_0 - i\eta - i\xi_0 \end{bmatrix}. \tag{5}$$

The meaning of the terms is the same as given by Jones, and they are reported in Table 1. The linear birefringence per unit length ξ at azimuth ρ is obtained by a combination of the two terms ξ_0 and ξ_{45}, representing the contribution per unit length of linear birefringence at 0° and 45° with respect to the reference axis. The relationships between them are as follows:

Table 1. Meaning of the N-Matrix Coefficients, as given by Jones[a]

Coefficient	Definition
$\eta = 2\pi n/\lambda$	η, propagation constant; n, index of refraction.
$\kappa = 2\pi k/\lambda$	κ, amplitude absorption coefficient; k, extinction coefficient.
$\omega = \frac{1}{2}(\eta_r - \eta_l)$	ω, angle of rotation of the plane of polarization of linearly polarized light (in radians per unit thickness); η_r, η_l, propagation constants for right and left circularly polarized light.
$q = \frac{1}{2}(\kappa_l - \kappa_r)$	q, measure of the circular dichroism; κ_1, κ_r, absorption coefficients for left and right circularly polarized light.
$\xi_0 = \frac{1}{2}(\eta_y - \eta_x)$	ξ_0, measure of the part of linear birefringence parallel with the reference axes; η_y, η_x, principal propagation constants.
$p_0 = \frac{1}{2}(\kappa_y - \kappa_x)$	p_0, measure of the part of linear dichroism parallel with the reference axes; κ_y, κ_x, principal absorption coefficients.
$\xi_{45} = \frac{1}{2}(\eta_{-45} - \eta_{45})$	ξ_{45}, measure of the part of linear birefringence parallel with the bisectors of the reference axes; η_{-45}, η_{45}, principal propagation constants.
$p_{45} = \frac{1}{2}(\kappa_{-45} - \kappa_{45})$	p_{45}, measure of the part of linear dichroism parallel with the bisectors of the reference axes; κ_{-45}, κ_{45}, principal absorption coefficients.

[a] Ref. 1.

$$\xi = (\xi_0{}^2 + \xi_{45}{}^2)^{1/2},$$
$$\rho = \frac{1}{2}\arctan\left(\frac{\xi_{45}}{\xi_0}\right). \tag{6}$$

The same result holds for linear dichroism components p_0 and p_{45}.

If $\mathbf{N}$ does not depend on z, the general Jones matrix can be computed from Eq. (5) and is

$$\mathbf{M} = \exp(-i\eta - \kappa)\begin{bmatrix} \cosh Q_N z + (p_0 + i\xi_0)\dfrac{\sinh Q_N z}{Q_N} & (-\omega + p_{45} - iq + i\xi_{45})\dfrac{\sinh Q_N z}{Q_N} \\ (\omega + p_{45} + iq + i\xi_{45})\dfrac{\sinh Q_N z}{Q_N} & \cosh Q_N z - (p_0 + i\xi_0)\dfrac{\sinh Q_N z}{Q_N} \end{bmatrix},$$
$$Q_N{}^2 = (p_0 + i\xi_0)^2 + ({}_{45} + i\xi_{45})^2 - (\omega + iq)^2. \tag{7}$$

When the light-propagation direction is reversed, for reciprocal material we obtain the new $\mathbf{N}_r$ matrix,

$$\mathbf{N}_r = \begin{bmatrix} -\kappa + p_0 - i\eta + i\xi_0 & -\omega - p_{45} - iq - i\xi_{45} \\ \omega - p_{45} + iq - i\xi_{45} & -\kappa - p_0 - i\eta - i\xi_0 \end{bmatrix}, \tag{8}$$

and the associated Jones matrix,

$$M_r = \exp(-i\eta - \kappa)\begin{bmatrix} \cosh Q_{N_r}z + (p_0 + i\xi_0)\dfrac{\sinh Q_{N_r}z}{Q_{N_r}} & (-\omega - p_{45} - iq - i\xi_{45})\dfrac{\sinh Q_{N_r}z}{Q_{N_r}} \\ (\omega - p_{45} + iq - i\xi_{45})\dfrac{\sinh Q_{N_r}z}{Q_{N_r}} & \cosh Q_{N_r}z - (p_0 + i\xi_0)\dfrac{\sinh Q_{N_r}z}{Q_{N_r}} \end{bmatrix},$$
$${Q_{N_r}}^2 = (p_0 + i\xi_0)^2 + (-p_{45} - i\xi_{45})^2 - (\omega + iq)^2 = Q_N^2. \tag{9}$$

A comparison of Eqs. (9) with Eqs. (7) shows the general transformation rule that holds for the most general Jones matrix when the light travels in the opposite direction in an optical reciprocal material, with the initial conventions stated for the definition of the light polarization and of the reference system. This rule is shown more clearly in expression (10):

$$\begin{bmatrix} m_1 & m_4 \\ m_3 & m_2 \end{bmatrix} \Rightarrow \begin{bmatrix} m_1 & -m_3 \\ -m_4 & m_2 \end{bmatrix}. \tag{10}$$

When nonreciprocity is inserted into the optical path, the task is slightly complicated and the result depends on the kind of birefringence and/or dichroism present in the optical material.

If a nonreciprocal optical activity, such as a Faraday effect, is induced in optical material that possesses no reciprocal optical activity or circular dichroism by itself, but only linear birefringence and/or linear dichroism, the result is quite simple, as shown in the following expression:

$$\begin{bmatrix} m_1 & m_4 \\ m_3 & m_2 \end{bmatrix} \Rightarrow \begin{bmatrix} m_1 & -m_4 \\ -m_3 & m_2 \end{bmatrix}. \tag{11}$$

When this transformation rule is used, there is no need to recalculate the whole Jones matrix. If reciprocal optical activity and/or circular dichroism are already present in the optical medium, we must recalculate the Jones matrix, adding or subtracting the nonreciprocal effect to the intrinsic reciprocal polarization properties of the material.

The rule in expression (10) can also be extended to a train of reciprocal devices. This can be shown with a direct calculation similar to that used by Jones to prove the reversibility theorem in his reference system. This is not true for expression (11). Whenever a nonreciprocal device is present in an optical circuit, it is possible to apply expression (11) to the nonreciprocal device, but the Jones matrix corresponding to the whole circuit must be recalculated.

To conclude, note that, with the conventions stated by Jones, when the light direction is inverted in a reciprocal optical circuit, we have only to transpose the Jones matrix associated with the system to obtain the proper Jones matrix. In our reference system we must not only transpose the Jones matrix but also change the sign of the two terms m_3 and m_4. When a nonreciprocal optical activity is introduced into a linear birefringent and/or linear dichroic medium, we need only change the sign of these two terms again to obtain the Jones matrix for the reverse propagation direction.

3. Mirrors

The optical devices examined now are normally used in transmission to change the polarization characteristics of the light passing through them: A mirror is used in reflection to change the direction of propagation of a light beam. This property can be translated into a Jones matrix too, but actually the form of the matrix depends on the particular reference system used for the description of the circuit. Consequently it is necessary to choose a particular reference system before writing the mirror matrix, because the various forms are not interchangeable. If we follow the convention described in Section 2, the Jones matrix of an ideal mirror is

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{12}$$

This matrix has the same form as a half-wave plate,[4,5] oriented with an azimuth of 0° or 90°, i.e., with its fast axis parallel to the x or to the y axis of the reference system. In Fig. 2 the effect of a specular reflection on the SOP of a linearly polarized light beam is shown: The fast axis of the half-wave plate associated with the ideal mirror is parallel to the axis orthogonal to the incidence plane. (As to the p- and s-polarization definitions, this axis is parallel to the s orientation.)

A mirror has several interesting properties that we can find in its associated Jones matrix. When the direction of the light beam is reflected, i.e., the incident and the reflected rays are interchanged, the matrix form does not change. A general elliptically polarized light beam is reflected with the opposite handedness, and the azimuth angle is symmetrically

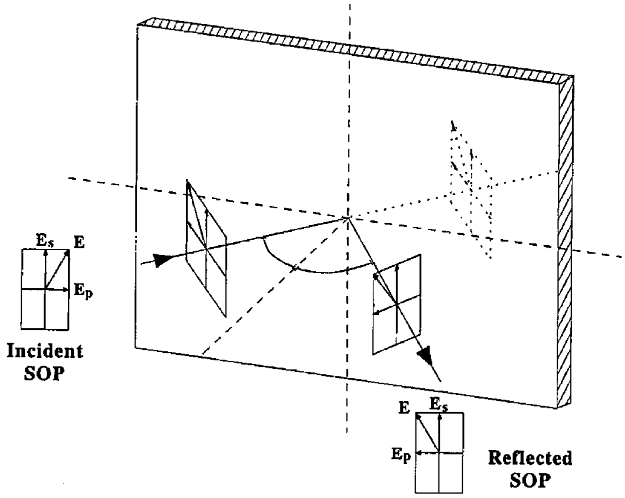

Fig. 2. E_s component of the electric field of the optical beam incident on an ideal mirror reflected without change. The component lying in plane of incidence E_p is reversed in sign. When partial reflections at interfaces between different optical media occur, various effects may change the relative phases and amplitude of the reflected E_s and E_p.

reflected with respect to this axis. Another interesting property is that two specular reflections cancel any effect onto the SOP of the light beam:

$$\mathbf{MM} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{13}$$

This result can be extended to any case in which an even number of specular reflections take place. A simple experimental setup to show this interesting property is shown in Fig. 3. Figure 3A represents a classical scheme used to suppress reflections from a shiny surface. The light impinging onto a linear polarizer is converted to a circularly polarized light beam by the quarter-wave plate whose fast axis is 45°

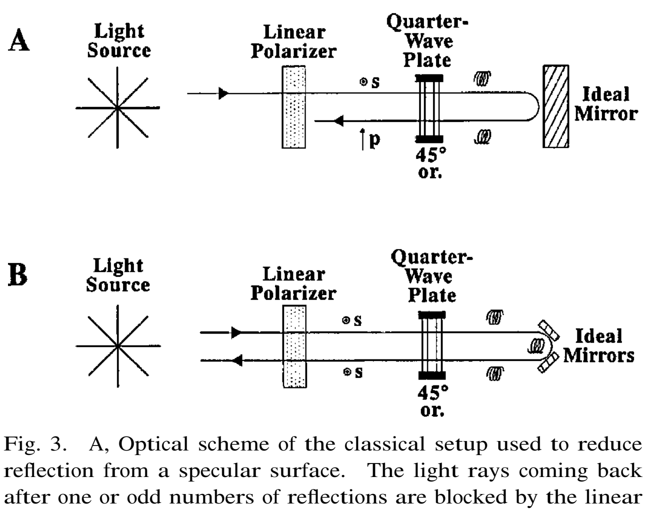

Fig. 3. A, Optical scheme of the classical setup used to reduce reflection from a specular surface. The light rays coming back after one or odd numbers of reflections are blocked by the linear polarizer. If the optical beam is backreflected by two or any even number of reflections, as in B, it is not blocked by the linear analyzer and the scheme is not able to isolate the source from the reflected light.

oriented with respect to the polarizer axis. The reflected beam possesses the opposite handedness and is converted by the plate to the linear polarization state orthogonal to the transmission axis of the linear polarizer. The train of Jones matrices representing the optical setup and the matrix representing the whole system are shown by

$$\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{14}$$

For ideal components the result is the null matrix: A total elimination of the reflected light has occurred.

We obtain a different result using the configuration in Fig. 3B, in which the light beam returns after two ideal reflections have occurred. The form of the Jones matrix representing this setup is given by

$$\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \tag{15}$$

It is equivalent to an ideal linear polarizer, so the intensity of the reflected light is equal to that of the light passing through the polarizer, and no suppression of specular reflections takes place at all: The two different behaviors repeat themselves for odd or even specular reflections.

When the Jones conventions are used, a few interesting differences arise when the mirrors are considered. First, the matrix form of the mirror is the 2×2 identity matrix. Substituting this matrix into Eqs. (14) and (15) and using the proper conventions, we can easily see that Eq. (14) gives the correct result, whereas Eq. (15) fails to do so. To obtain the correct result from Eq. (15), we must set the matrix form of a double-bounce corner equal to Eq. (12), and this matrix form cannot be obtained by a simple multiplication of two identity matrices. Moreover, let us consider again a double reflection, obtained by two single specular reflections, but where the outcoming beam propagates in the same direction as the input light and is only parallel shifted. In this occurrence the matrix representing the system must be the identity matrix and the output beam has the same SOP as the input beam.

Therefore the matrix form of a system composed of two mirrors depends on the relative direction of propagation of the output light beam with respect to the input light beam. In the Jones reference system it is always necessary to specify the direction of propagation of the light to write properly the matrix form of an optical system and to define the SOP of the light beam. Moreover a distinction between normal and oblique incidence on a reflecting surface is necessary.[6]

From an operative point of view the reference system adopted here is simpler than the previous one:

Every composed matrix is always obtained by multiplication of the matrices of the various components, independent of the direction of the light beam and the incidence angle.

For a nonideal reflection the matrix coefficient may vary because of the physical nature of the interface at which the reflection takes place. An immediate example is given by the partial reflection at the interface between two dielectric media with different refractive indices: The reflection coefficient will vary for a *p*- or *s*-polarized light beam, depending on incidence angle Θ_i. The matrix expression is given by

$$\begin{bmatrix} R_s & 0 \\ 0 & -R_p \end{bmatrix}. \tag{16}$$

The two reflection coefficients, R_s and R_p, are given by the Fresnel formulas

$$R_s = \frac{\sin(\Theta_i - \Theta_t)}{\sin(\Theta_i + \Theta_t)},$$
$$R_p = \frac{\tan(\Theta_i - \Theta_t)}{\tan(\Theta_i + \Theta_t)}, \tag{17}$$

where Θ_t is the angle of refraction in the second medium. As usual, the common phase factors are not shown in expression (16). A similar result for stratified media can be found in Ref. 7. Also, different reflection coefficients are used to describe reflections from anisotropic and/or chiral media; see, for example, Refs. 8 and 9.

A common optical scheme in which all the previous considerations must be used is the so-called retracing optical circuit. A typical setup consists of placing a retroreflector at one end of the circuit, causing the light beam to travel forward and backward along the same optical path. The introduction just before the retroreflector of a Faraday rotator tuned for 45° of rotation power realizes a remarkable effect. Namely, it reflects the incident light beam with a SOP orthogonal to the input light. This property permits the compensation of reciprocal birefringences[10] and dichroisms present in the circuit, and it was used to realize fiber-optic sensors of various types.[11,12] The Jones matrix associated with the so-called mirrored Faraday rotator (MFR), or orthoconjugate reflector, also has different forms, depending on the reference system. In the framework adopted here the MFR matrix has the following form:

$$\mathrm{MFR} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}. \tag{18}$$

This equation holds if we substitute the simple mirror with a cat's-eye retroreflector or any device that reflects the light back after an odd number of reflections, as noted above. If we use as a retroreflector a square corner prism or a device with an even number of reflections, we will not obtain Eq. (18) and definitively lose the ability to reflect an orthogonal SOP beam.

4. Conclusions

The application of the Jones calculus to optical circuits where the propagation direction of a light beam is reversed has been examined. The connections among the transformation rules of the Jones matrices, the particular reference system adopted, and the conventions used to define the SOP of the light have also been shown. For a few devices, such as mirrors and MFR's, the matrix form itself depends on the reference system. The transformation rules that are valid for a reference system often used in the technical literature, but different from the Jones approach, have been given together with the Jones matrix of a mirror and an MFR. Although the two reference systems give the same results if applied properly, from an operative point of view the proposed one presents a simpler approach to the calculus of complex optical circuits, where multiple reflections take place and counterpropagating light beams are present.

An extension of the transformation rules for an optical material where nonreciprocal optical activity coexists with linear birefringence and linear dichroism has also been given.

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