On Legitimacy Feedback Mechanisms in Agent-Based Modeling of Civil Violence

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Epstein's agent-based model (ABM) of civil violence has been very popular and successful due to its formulation soundness, simplicity, and explanatory power. Variants of this model have been proposed for the simulation of different types of social conflict phenomena (worker protest, riots, or urban crime) and for investigating the effect of mechanisms that are not considered in the original model. In a previous work, we introduced imprisonment delay, "news impact," and legitimacy feedback effects in Epstein's ABM of civil violence. In this paper, we focused specifically on improving the formulation of legitimacy feedback. In the model presented herein, legitimacy varies as a function of subindicators identified in theoretical studies on legitimacy measurement. We considered four different functions for expressing the legitimacy—weighted average, geometric mean, exponentially decaying "system support," and "justification"—and two different feedback mechanisms: homogeneous (global) perceived legitimacy and heterogeneous (agent-dependent) perceived legitimacy. It was found that, for certain combinations of input parameters, the present model produced solutions with an initial period of calm with small bursts of rebellion, followed by a sudden large-scale rebellion after which intermittent bursts of rebellion occur (as in Epstein's model), where legitimacy drops never returning to the initial level. These results provide an alternative explanation of the mechanisms by which apparently stable authoritarian regimes, when facing an unexpected large-scale uprising, respond with repression and afterward struggle with intermittent bursts of rebellion because they are perceived as illegitimate. The present model can also be used to test theories on the aggregation of legitimacy indicators in a global legitimacy score.

1. INTRODUCTION

Social conflict is a fundamental characteristic of human societies and a driving force for their change.[1–3] The study of social conflict phenomena and civil violence is an important and timely topic in political science,[4,5] sociology,[2] social psychology,[3] and social simulation.[6]

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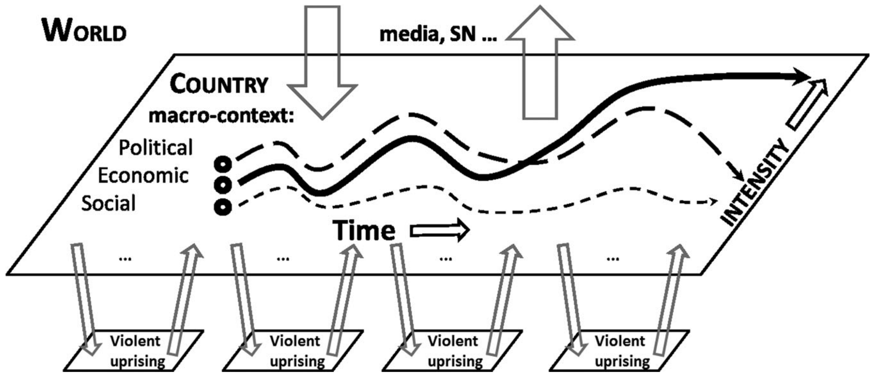

Figure 1. Evolution of the intensity of social conflict viewed as a complex and path-dependent process with micro–macro and feedback links.

Predicting the time evolution of the intensity of social conflict is a very complex and path-dependent process because it involves a large number of social context factors, multiple interactions, and different scales (of time, space, and proportion of the population involved).[7] This process may lead to a stable or unstable condition, as shown in Figure 1. Unstable processes may be gradual (escalation of tension or violence) or sudden (revolution or outbreak or armed conflict).[4,5]

Epstein and co-workers[8,9] introduced a very successful and popular agent-based model (ABM) for simulation of civil violence against a central authority or ethnic violence between two rival groups mediated by a central authority, with two types of agents, "citizens," and "cops," each with two simple rules ("move" and "behave"). This model successfully explains many features of civil violence processes such as intermittent bursts of rebellion, safe havens in peacekeeping, or the effects of sudden or gradual variations of the legitimacy and deterring capability of the central authority (expressed by the number of "cops").

Recently, we extended the Epstein et al. ABM of civil violence to consider (i) a time delay for imprisonment, (ii) "media" agents to simulate "news coverage," and (iii) a feedback mechanism for varying the legitimacy as a function of the number of arrests and violent episodes registered by the "media" agents.[7] The extended model produced large intermittent peaks of rebellion with a more realistic fine structure than those obtained with the original model, but the legitimacy feedback mechanism required a sounder foundation and further refinement.

In this work, we studied the effect of legitimacy feedback on the Epstein et al. ABM,[8,9] considering different functions for expressing the legitimacy in terms of the proportion of citizens that do not support the government and two feedback mechanisms: (i) homogeneous, in which all citizens share the same perceived legitimacy, and (ii) heterogeneous, in which the perceived legitimacy is an individual attribute. The improved formulation of the legitimacy feedback extended the explanatory power of the original model to other phenomena, such as the occurrence of a sudden, large, and unexpected uprising after a long period of apparent stability, followed by a struggle of the central authority to dominate successive bursts of rebellion due to irreversible loss of legitimacy, while preserving the simplicity of the original model.

The remainder of this paper is organized as follows. In Section 2, we present (i) an overview of ABM of civil violence, with emphasis on the Epstein et al. ABM and on our previous work on the inclusion of small-scale, "news impact" and legitimacy feedback effects in this model; and (ii) a review of legitimacy concepts, indicators, and (difficulty of) measurement. Section 3 contains the description of the present model, using a simplified subset of the "Overview, Design Concepts, and Details" (ODD) protocol.[10] In Section 4, we present the results of the model for different legitimacy functions and feedback mechanisms. Section 5 contains the discussion and Section 6 the conclusions.

2. THEORETICAL BACKGROUND

2.1. Epstein's ABM of Civil Violence

The ABM by Epstein et al.,[8] herein referred as Epstein's model,[9] is a very successful and popular model for simulation of rebellion against a central authority (Model I) or ethnic violence between rival groups mediated by a central authority (Model II). In this work, we will only consider Model I (civil violence). Epstein's ABM is an example of an "abstract model" since it is focused on describing large-scale emergent phenomena (generalized bursts of rebellion) in terms of a small number of agent types, rules, and parameters.[11] The model includes two types of agents, "citizens" and "cops," moving in a homogeneous torus space. Both types of agents have one movement rule M (move to a random empty cell within the agent's vision radius) and one action rule. The action rule for "citizen" agents is

Rule A: if $G - N > T$ be "active"; otherwise be "quiet"

where $G = H \cdot (1 - L)$ is the grievance, $H \sim U(0,1)$ is the perceived hardship, L is the perceived legitimacy of the central authority assumed equal for all agents, $N = R \cdot P$ is the net risk perception, where $R \sim U(0,1)$ is the risk aversion, P is the estimated arrest probability, and T is a threshold (assumed constant for all agents). The form of the arrest probability presented in Epstein's model is[8,9]

$$P = 1 - \exp[-k(C/A)_v], \tag{1}$$

where C and A are the number of "cops" and "active" citizens within the agent's vision radius v and $k = 2.3$ is the arrest constant, chosen so that $P = 0.9$ when $C = 1$ and $A = 1$. In several implementations of this model,[7,12,13] the estimated arrest probability is computed using the formula

$$P = 1 - \exp(-k \lfloor C/(A+1) \rfloor_v), \tag{2}$$

which leads to a sudden drop of P from 0.9 to zero when $C = A$ (in estimating the arrest probability, the agent counts itself as "active"). Notice that rule A is consistent with several features of microsituational theories of violence:[14] predisposition factors (via G and R), situational influence and deterrence (via P), and a "barrier" of tension fear (expressed by T). The action rule for "cop" agents is

Rule C: Inspect all sites within v' and arrest a random "active" citizen

where v' is the "cop" vision radius (which may be different from v). Arrested citizens are removed from the simulation space ("jailed") for a certain number of cycles, or jail term $J \sim U(0,J_{\max})$, where $J_{\max}$ is the maximum jail term set as an input variable. The jailing of citizens introduces a memory effect in the system, with a time scale proportional to $J_{\max}$.

Epstein's model successfully describes many characteristics of large-scale civil violence processes, such as the occurrence of large intermittent bursts of rebellion (punctuated equilibrium), society's response to progressive or sudden drops of legitimacy of the central authority (stability in the first case and a burst of rebellion in the latter case), and the effect of progressive reduction of the number of "cops" (a sudden burst of rebellion due to the central authority neglecting its deterring capability). The strength of the model lies in its explanatory power, which derives from the simplicity of the formulation (rule A) and the choice of very relevant variables: grievance, legitimacy, and risk perception. Legitimacy is a key variable that determines the average level of grievance and its time variation has a large impact on the system's behavior. This raises an important question (suggested in Figure 1): how does the state of the system (i.e., the proportion of rebellious and/or jailed citizens) affect the legitimacy? Epstein et al.[8] suggest a mechanism for introducing endogenous legitimacy feedback in Model II (ethnic violence), but this approach was not implemented and further explored for the case of civil violence (Model I).

Because of its popularity and success, Epstein's model has been improved in different ways by various authors (see Lemos et al.[15] for a review). For instance, Kim and Hanneman[16] proposed and ABM of worker protest in which the grievance is expressed as a function of relative deprivation (RD) resulting from wage inequality, including group identity effects, which is a theoretically sound approach since RD is strongly correlated with the potential for conflict[17] and so is group grievance.[18] Fonoberova et al.[13] considered different forms of the arrest probability function and used empirical data to study the dynamics of urban crime for different sizes of the police force. However, none of these authors considered the problem of endogenous legitimacy feedback.

2.2. Previous Work: Introduction of Imprisonment Delay, "News Impact" and Legitimacy Feedback in Epstein's Model

In a previous work,[7] we proposed an extension of Epstein's model of civil violence including a time delay for imprisonment, a third type of agent called "media" to represent a "news impact factor," and a legitimacy feedback mechanism for varying the legitimacy as a function of the number of arrests and violent episodes (temporary "fights" or "confrontations" between rebellious agents and "cops") registered by the "media" agents. The arrest delay represents a time cost for "cops" to arrest citizens, which occurs in microscale processes, such as fights and physical confrontation in street protests, and in macroscale processes, such as information services or law-enforcing agents confirming and pursuing target subjects. It was found that for certain combinations of parameters, the extended model produced intermittent peaks of rebellion, which lasted longer and had a more complicated fine structure than those obtained with the original model, whereas for other combinations the system's long-term behavior changed from (complex) punctuated equilibrium to a state of permanent rebellion (equilibrium) with random fluctuations.

Figure 2 shows a comparison of the solutions showing intermittent bursts of rebellion obtained with Epstein's model and with the modified model with an imprisonment delay different from zero (top and middle figures), for two runs of the simulations described in Lemos et al.[7] This figure also shows a time series of the number of violent events in France in the period 1830–1960, extracted from Tilly et al.[19] It can be observed that the introduction of imprisonment delay changes the characteristics of the solutions by superposing small-scale fluctuations on the violence bursts and by increasing the duration of the latter. The bottom image in this figure (which is qualitatively similar to time series of other violent processes, such as terrorist attacks) shows that the solutions of the improved model are more realistic than those obtained with the original model.

The imprisonment delay also created opportunities for "media' agents to record episodes of confrontation. These agents also had one movement rule and one action rule, but instead of moving at random they were attracted to visible "fights," other "media" agents and "cops," and repelled from "quiet" citizens.[7] This simulates the agenda setting bias toward showing violence.[20] The action rule for "media" agents consisted of looking for "fighting" agents (citizens or "cops") within their vision radius, and "taking photographs" of those agents. The records taken by the "media" agents were then used in the formulation of legitimacy feedback.

The legitimacy feedback mechanism was implemented using the following expressions:

$$L^* = L_t - (N\text{arrests}_t / N\text{citizens}) - A_f \cdot (N\text{fights}_t / N\text{citizens}) \tag{3}$$

$$\Delta L = \left(L^* - L_0\right) \cdot \exp\left(-\alpha \cdot \Delta t\right) \tag{4}$$

$$L_{t+1} = \max\left(0, \min\left(L_0 + \Delta L, 1\right)\right) \tag{5}$$

where L_0 is the government legitimacy set as global parameter (as in Epstein's model), $N\text{arrests}_t$ and $N\text{fights}_t$ are the number of arrests and recorded fights at time t, A_f is an "audience factor" and α is a "memory constant" that allows for slower or faster decay of the legitimacy drop due to arrests and fights in subsequent time cycles. For simplicity, A_f was set to the number of sites within vision radius of "media" agents to avoid introducing another parameter. This formulation allows for a decay of legitimacy during violent outbursts, and for memory from past events in the "evening news" fading progressively, while ensuring that legitimacy

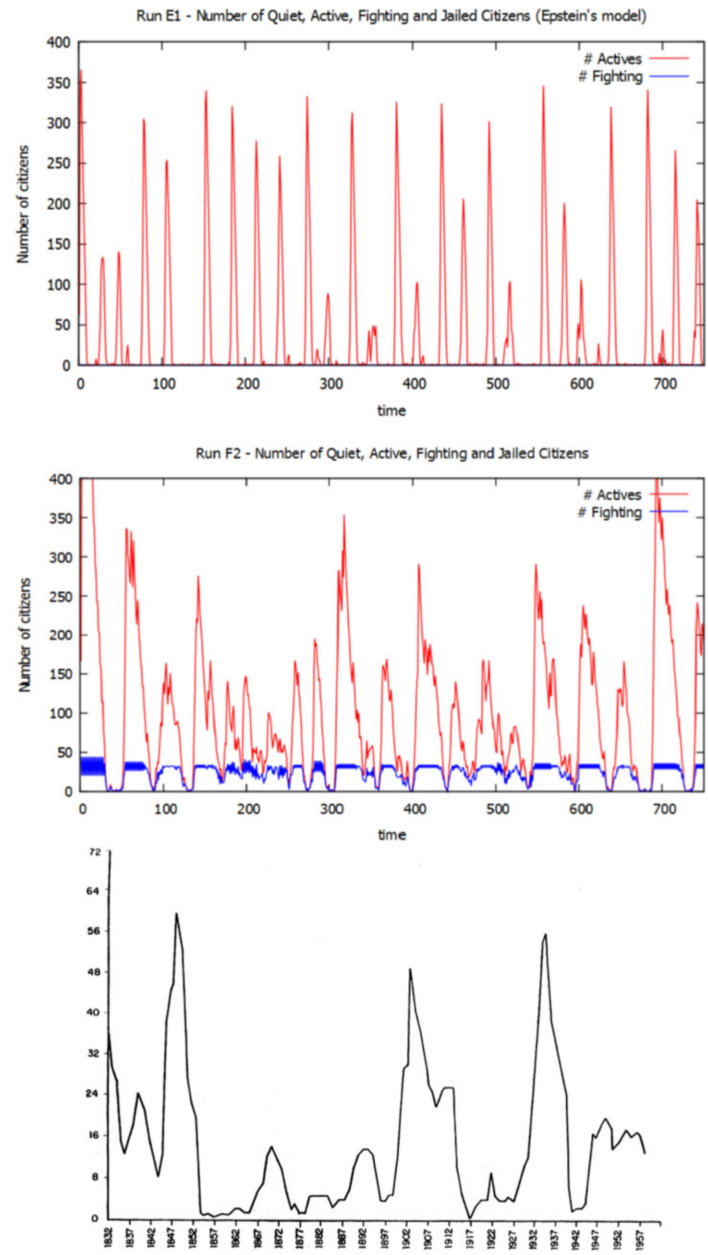

Figure 2. Solutions from two simulations in Lemos et al.,[7] showing the reference case with Epstein's model (top) for $L = 0.82$ and $J_{\max} = 30$, and an illustrative case with the extended model (middle) with $L = 0.82$, $J_{\max} = 60$, imprisonment delay equal to one, and legitimacy decay constant $\alpha = 0.5$; and a time series of the number of violent events in France in the period 1830–1960 (source: Tilly et al.,[19] p. 56).

remains in the interval [0,1]. Introduction of this feedback mechanism lead to sudden legitimacy drops after large bursts of rebellion, with legitimacy recovering to its initial value with a time scale controlled by the parameter α. This form of legitimacy feedback had a significant impact in the results of the simulations, with larger average numbers of "active" and "jailed" citizens than with Epstein's model, and changes in the system's long-term behavior for certain values of α. Although plausible, the legitimacy feedback mechanism defined by Equations (3)–(5) has the following drawbacks:

- It is based on plausible hypotheses but lacks further support from theories on the meaning and measurement of legitimacy.[20,21,22]
- It requires the introduction of a third time scale parameter ($T_l = 1/\alpha$).
- Legitimacy is considered as a global variable, whereas in reality it is a subjective perception.[22] Therefore, it should be set as an attribute of "citizen" agents and its global value should be found by averaging over the whole population.

In the present work, we developed alternative forms of introducing legitimacy feedback in Epstein's model with the goals of (i) minimizing the number of additional parameters, (ii) avoiding the introduction of new artificial time scales, and (iii) keeping the model as simple as possible. We also considered theoretical studies on indicators and measures of legitimacy to provide a sounder foundation for the proposed legitimacy formulae.

2.3. Legitimacy Concepts, Indicators, and Measurement

Legitimacy is the value that confers recognition and acceptance of the exercise of power by an authority or institution.[20–22] The concept of legitimacy is a central one in political science, but is also a *latent concept* because it cannot be measured directly.[20,23] Dogan[21] mentions the difficulty of measuring legitimacy from opinion polls (because these often measure other factors related to legitimacy but not legitimacy itself), the relationship between legitimacy and coercion (the higher the latter, the lower the former) and the role of trust in institutions and effectiveness of government in sustaining legitimacy. There is no generally accepted formula for evaluating the legitimacy of states. Beetham[24,25] considers that legitimacy is a multidimensional and multilayered concept, with three main components, namely, legality, normative justifiability, and expressed consent. Gilley[20,22] devised a system for ranking the legitimacy of countries based on this three-component conception, using a set of subindicators related to three subtypes of legitimacy: views of legality (related to the formality and laws), views of justification (related to moral beliefs), and acts of consent (related to behavior). Table I (adapted from Appendix 1B of Gilley[22]) summarizes the indicators considered and the data sources from which the subindicators were evaluated for 72 different countries.[20] Attitude indicators relate to the beliefs and motivations of citizens, whereas behavioral indicators relate to actions.

Gilley considers two types of variables, constitutive and substitutive, as legitimacy indicators. In theory, constitutive variables conceptually define the higher order concept (legitimacy), whereas substitutive variables are chosen based on how they correlate to the latent concept (see Gilley,[22] Appendix A1).

Table I. Legitimacy indicators related to legality, justification, and consent (adapted from Gilley[22]).

Legitimacy subtype	Indicator	Source	Constitutive or substantive	Attitude or behavior
Views of legality	Evaluation of state respect for human rights	World Values Survey 1999–2002 Question 173	Constitutive	Attitude
	Confidence in police	World Values Survey 1999–2002 Question 152	Substitutive	Attitude
	Confidence in civil service	World Values Survey 1999–2002 Question 156	Substitutive	Attitude
Views of justification	Satisfaction with democratic development	World Values Survey 1999–2002 Question 168	Constitutive	Attitude
	Evaluation of current political system	World Values Survey 1999–2002 Question 163A	Constitutive	Attitude
	Satisfaction with operation of democracy	Global Barometer Regional Surveys, 2001–2002 EuroBarometer 2001 EuroCandidate, 2002	Constitutive	Attitude
	Use of violence in civil protest	World Handbook of Political and Social Indicators IV, 1996–2000	Substitutive	Behaviour
Acts of consent	Voter turnout	International Institute for Democracy and Electoral Assistance, 1996–2002	Constitutive	Behaviour
	Quasi-voluntary taxes	International Monetary Fund, Government Finance Yearbook 1996–2002	Substitutive	Behaviour

The raw scores for the indicators were first transformed to a common scale 0–10, then aggregated within each subtype and finally aggregated across subtypes. For within subtype aggregation, Gilley used a simple average of the related indicators. Aggregation across subtypes raises more fundamental theoretical questions: are all subtypes equally important? If they are not, should legitimacy be evaluated using a weighted average, or using a formula that requires all three subtypes to be nonzero for nonzero legitimacy?

These are fundamental questions in theoretical studies on the meaning and measure of legitimacy (see Gilley,[22] Appendix 1F). Assuming that the global legitimacy score should be calculated using the arithmetic mean or a weighted average of the three subtypes implies the rejection of theories based on the hypothesis that all subtypes are necessary, and acceptance of theories advocating that one subtype may be sufficient (in the case of a weighted average score). Gilley discusses this question at length and considers that "The inclusion of justification and consent in our conceptualization is thus a rejection of sufficiency of legality, . . . " and ". . . the most

Table II. Subindicators, weights in Gilley's legitimacy score and related ABM variables selected for modeling legitimacy feedback.

Indicator	Subtype	Weight	Related variables
Satisfaction with democratic development	Justification	$\frac{1}{12}$	$\frac{n_{\text{quiet}}}{N}$
Evaluation of current political system	Justification	$\frac{1}{12}$	$\frac{n_{\text{quiet}}}{N}$
Satisfaction with operation of democracy	Justification	$\frac{1}{12}$	$\frac{n_{\text{quiet}}}{N}$
Use of violence in civil protest	Justification	$\frac{1}{4}$	$\frac{n_{\text{active}}}{N}, \frac{n_{\text{fighting}}}{N}, \frac{n_{\text{jailed}}}{N}$

N is the total number of citizens, and n_{quiet}, n_{active}, n_{fighting}, and n_{jailed} are the number of citizens in each of the four possible states.

enduring criticism of traditional theories of legitimacy mounted in the past 25 years has concerned the inadequacy of legal and consent subtypes and the overarching importance of moral justification. Justification has been the most underestimated subtype of legitimacy, both in the real world of politics and the virtual world of scholarship." (Gilley,[22] Appendix 1F). Based on these considerations and on statistical correlations between the subtypes, this author proposes a legitimacy score with 50% weight for "justification" and 25% for "legality" and "acts of consent."

To introduce legitimacy feedback in the ABM of civil violence, based on the theoretical approach described above, we have to answer the following questions:

- Which legitimacy subindicators are the most important and should be included explicitly?
- How can these subindicators be written as functions of the emergent properties of the system?
- How should these functions be combined to evaluate the time-dependent legitimacy?

Table II shows our selection of subindicators and the dependent variables of the ABM that can be used to express them. Notice that all subindicators are related to justification, and that the sum of their weights represents one half of the legitimacy score. Since there is no universally accepted theory for measuring legitimacy, we will consider four different functions for expressing the legitimacy in terms of these subindicators to gain insights on their relative merits and explanatory power. The analytic form of these functions is presented in Section 3.

3. MODEL DESCRIPTION

In this section, we present a description of the ABM. Following our previous work, the model was implemented in NetLogo,[26] using as starting point the "Rebellion" NetLogo Library model example [12] (which is an implementation of Epstein's model).

3.1. Synopsis

The ODD protocol[10] is a very thorough and useful method for describing ABM. In this work, we will present a simplified description based on this specification,

Table III. Simplified ODD description of the extended Epstein model I (civil violence) with legitimacy feedback.

ODD item	Description
Purpose	Study the effect of legitimacy feedback in Epstein's ABM of civil violence, using different functions for expressing the legitimacy and two different updating mechanisms: global society and agent-based
Entities, state variables, and scales	Agents: Two types of reactive agents, "citizens" and "cops," with one "move" and one "behave" rule for each type (same as in Epstein's model, but with a different "behave" rule for "cop" agents in the case of nonzero arrest delay). Scenario: Homogeneous 2D torus space (no boundaries, consistent with an "abstract" model). Scales: – Whole artificial society, undetermined time step and patch size (consistent with an "abstract" model) – Spatial scale(s) in units of patch size: vision radius for "citizens" and "cops" – Time scale(s) in units of time step size: "fight duration" F_d and "maximum jail term" $J_{\max}$.
Process overview and scheduling	All agents activated once per model cycle in random order; "nonfighting" agents execute their "move" and "behave" rule; "fighting" agents are immobilized for F_d cycles, after which the "fighting" citizen is jailed and the arresting "cop" becomes mobile again
Submodels	Legitimacy feedback (Section 3.3)

since the model has a relatively simple structure similar to that already described in our previous work.[7] Table III shows a summary of the model features. The details of the implementation are presented in the following sections.

3.2. Model Entities

Figure 3 shows the class diagram for the "observer" (i.e., the model user), "citizen," and "cop" agents. The "observer" box shows the global input variables that must be supplied by the user and the two NetLogo procedures that implement the model.[12] The initial densities for the two agent types, duration of the simulation (maximum number of time steps), vision radius, initial (reference) government legitimacy, maximum jail term, and fight duration are numeric parameters. The "legitimacy-feedback" variable (F_L) is a list with three strings, "none," "global," and "agents," used to define the type of legitimacy feedback (constant legitimacy as in the original model, variable homogeneous legitimacy, and variable heterogeneous legitimacy, respectively).

3.2.1. Citizen Agent Specification

"Citizen" agents can be in one of four possible states: "quiet," "active," "fighting," or "jailed." Agents that are not "fighting" or "jailed" change state between "quiet" and "active," and have the same "move" (M) and "behave" (A) rules as in

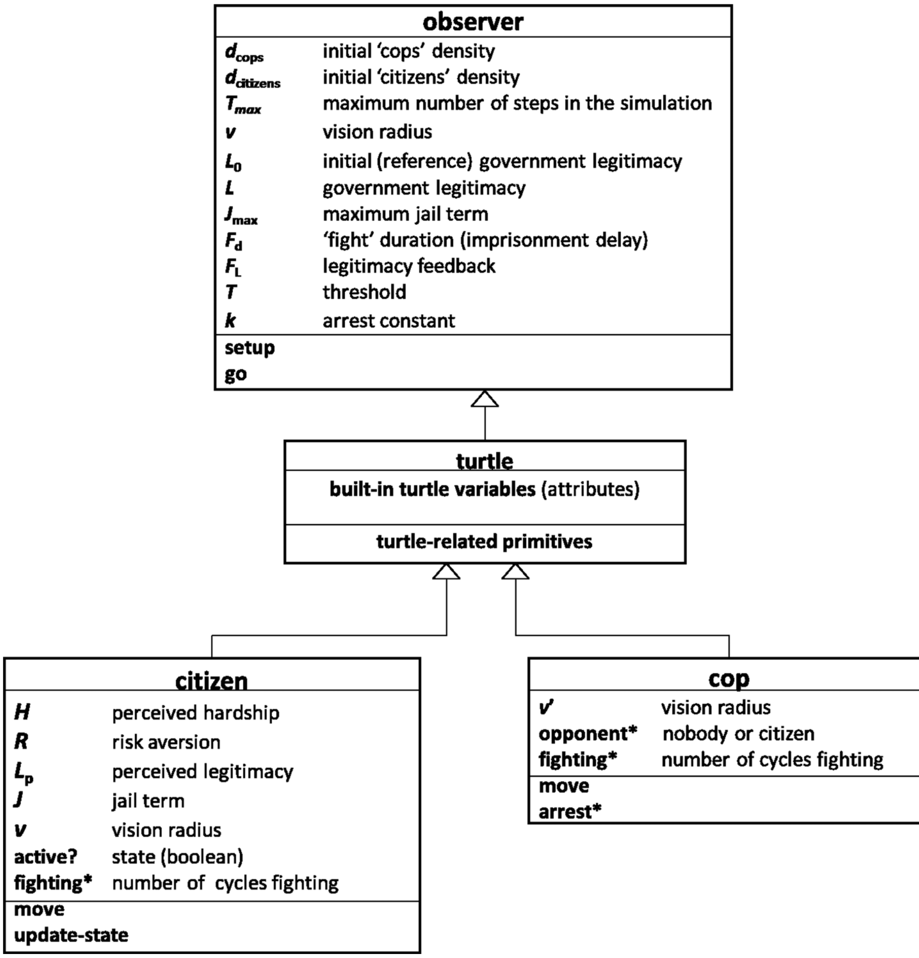

Figure 3. Class diagram for the "observer," "citizen," and "cop" agent types in the NetLogo implementation. The "observer" and "turtle" agents are specific of the NetLogo system.[26] The "citizen" and "cop" agent types are subclasses of the generic NetLogo "turtle" agent type. The agents' attributes and methods that result from extensions or modifications of Epstein's model are marked by an asterisk.

Epstein's Model I:

Rule M: move to a random empty site within the vision radius v

Rule A: if $G - N > T$ be "active"; otherwise be "quiet"

where $G = H \cdot (1 - L_p)$ is the level of grievance, $N = R{\cdot}P$ is the net risk perception, T (constant exogenous variable) is a threshold, $H \sim U(0,1)$ is the (endogenous) perceived hardship, $L_p \in [0,1]$ is the "perceived government legitimacy", $R \sim U(0,1)$ is the (endogenous) risk aversion, and P is the estimated arrest probability given by

Equation (2). If F_L is set to “none” or “global,” the value of L_p is set equal to the value of the global variable L. Otherwise, it is computed using the same function used to evaluate the legitimacy, with the global number of agents of each type and at each state replaced by the corresponding numbers of agents within the agent’s vision radius (see Section 3.3). “Active” agents engaged by one “cop” agent change state to “fighting” if $F_d > 0$ (imprisonment delay different from zero), or “jailed” if $F_d = 0$. “Fighting” agents are immobilized for F_d cycles before they are “jailed.” “Jailed” agents are removed from the simulation space for J cycles, after which they are reinserted in a random empty site within the simulation space with their state set to “quiet.”

3.2.2. Cop Agent Specification

“Cop” agents can be in one of two possible states, “nonfighting” or “fighting.” “Nonfighting” cops have the same “move” (M) and “behave” (C) rules as in Epstein’s Model I:

Rule M: move to a random empty site within the vision radius v'

Rule C: Inspect all sites within v' and arrest a random “active” citizen

Although the vision radius can be set differently for “citizen” and “cop” agents, we used a common value in all simulations. If $F_d = 0$, “cop” agents immediately arrest (“jail”) one “active” citizen (within their vision radius). If $F_d > 0$ and they find one “suspect,” they mark it as “opponent” and start “fighting” with that agent for F_d cycles. During the “fight,” the enforcing “cop” and its opponent are immobilized. At the end of the “fight” the “active” citizen is “jailed” for $J \sim U(0, J_{\max})$ cycles. This implementation of imprisonment delay can be interpreted as a representation of the cost for law-enforcing agents to arrest suspects (“cop” agents cannot search for targets and fight simultaneously) and introduces small scale effects in the dynamics, which increase the realism of the solutions (Figure 2).

3.3. Legitimacy Feedback

To implement legitimacy feedback, we need to define functions for expressing the perceived legitimacy in terms of the model’s dependent variables. We first defined auxiliary functions for expressing the three subtypes of legitimacy considered by Gilley,[22] namely “views of legality,” “views of justification,” and “acts of consent”:

$$L_{\text{leg}} = \frac{n_{\text{quiet}}}{N} \tag{6}$$

$$\begin{aligned} L_{\text{just}} = &\frac{1}{2}\left(1 - \frac{n_{\text{active}} + n_{\text{fighting}}}{N}\right) \\ &+ \frac{1}{2}\left(1 - \exp\left(-\ln(2)/2 \cdot \left\lfloor \frac{N}{n_{\text{active}} + n_{\text{fighting}} + n_{\text{jailed}} + 1} \right\rfloor\right)\right) \end{aligned} \tag{7}$$

$$L_{\text{consent}} = L_{\text{leg}} \tag{8}$$

where in (7) we used the arithmetic mean for subtype aggregation as in Gilley[22] and in (8) we assumed that only "quiet" citizens "express consent." "Views of legality" and "acts of consent" were represented by the proportion of "supporters" of the central authority (i.e., "quiet" citizens). The first term in Equation (7) represents "political violence" and the second "system support." The form of the second term in Equation (7) implies that support to the central authority decays in steps, in such a way that it drops from 0.5 to zero when 50% of the citizens are nonsupporters. After defining these auxiliary functions, we considered four different functions for expressing the legitimacy: (i) weighted average; (ii) geometric mean; (iii) "system support," and (iv) "justification." The first two were chosen to test the difference between sum and product aggregation, whereas the third and fourth were introduced to study the effect of considering only "system support" and "justification," which are very significant to the global score. The expressions for the legitimacy functions are

$$L_{\text{wa}} = L_0 \cdot \left(\frac{1}{4} \left(L_{\text{leg}} + L_{\text{consent}} \right) + \frac{1}{2} L_{\text{just}} \right) \tag{9}$$

$$L_{\text{gm}} = L_0 \cdot \left(L_{\text{leg}} \cdot L_{\text{just}} \cdot L_{\text{consent}} \right)^{\frac{1}{3}} \tag{10}$$

$$L_{\text{ss}} = L_0 \cdot \left(1 - \exp\left(-\ln(2)/2 \cdot \left\lfloor \frac{N}{n_{\text{active}} + n_{\text{fighting}} + n_{\text{jailed}} + 1} \right\rfloor \right) \right) \tag{11}$$

$$L_j = L_0 \cdot L_{\text{just}} \tag{12}$$

where L_{wa} is weighted average, L_{gm} is geometric mean, L_{ss} is system support, and L_{j} is "justification."

3.4. Process Overview and Scheduling

The model is implemented in two procedures, `setup` and `go`, with the following operation sequences:

> `setup`: clears all variables from the previous simulation, initializes the global variables, creates the agents list, and opens the output file used for postprocessing simulation results.
> `go`: implements the main cycle which consists of testing for termination and closing the output file at the end of the simulation, updating the global perceived legitimacy, running the "move" and "behave" rule for all "nonfighting' agents, decrementing F_d for all "fighting" agents and printing the cycle information variables (number of "citizens" in each state and global legitimacy).

Table IV. Values of the global parameters used in all simulations.

Parameter	Value
Grid size (torus space)	40×40
Number of "citizens"	1120
Number of "cops"	64
Vision radius v, v' (units of patch cell width)	7
Maximum jail term $J_{\max}$ (time steps)	30
Duration (maximum number of time steps)	5000
Threshold T	0.1
Arrest constant k	2.3
Hardship H	$\sim U(0,1)$
Risk aversion R	$\sim U(0,1)$

4. RESULTS

We studied the effect of legitimacy feedback by comparing the results obtained with Epstein's ABM model with those obtained using different legitimacy functions and global or agent-averaged perceived legitimacy. Our focus was on (i) the characterization of the system's long-term behavior, (ii) the identification of tipping points, and (iii) how the improved model extends the explanatory power of Epstein's ABM. In the simulations reported herein, we kept the same values of grid size, number of "citizens" and "cops," vision radius, and maximum jail term used in other works[7,8] for better comparison. We set the imprisonment delay to zero in all simulations to isolate legitimacy feedback as the only deviation with respect to Epstein's model. Table IV shows the values of the global parameters common to all simulations.

We considered three initial values of the government legitimacy, $L_0 = 0.85$, $L_0 = 0.87$, and $L_0 = 0.89$ and for each case we performed simulations with the four legitimacy functions and two aggregation forms (homogeneous and heterogeneous perceived legitimacy). It was found that the case $L_0 = 0.89$ provided the richest and most interesting opportunities for further exploration, so we selected it for further study and consideration herein.[a] For $T = 1/10$ and $L_0 = 0.89$, the original model produces equilibrium solutions without rebellion peaks, so we used this result as reference case (R21E in Table V). We ran simulations with the four legitimacy functions and with the two forms of legitimacy feedback, with a total of ten simulations for each case. In this way, we could compare the influence of the legitimacy function and assess the importance of nonlinear effects. For each case, we analyzed the long-term behavior of the system and, for the cases with bursts of rebellion, the peak interval (waiting time between peaks), the size of these outbursts (maximum number of "active" agents), and the time for arrival of the first rebellion peak. Following Epstein et al.,[8] we considered as peaks of rebellion the upsurges of violence with more than 50 "active" citizens.

[a]Notice that $L_0 = 0.89$ is very close to the limiting value for which rebellion peaks cannot occur. In fact, for $T = 1/10$, $L_0 = 9/10$ (more generally, for $L_0 \in [9/10,1]$), and $H \sim U(0,1)$, we have $G = H \cdot (1 - L) \sim U(0,1/10)$, and the condition $G - N > T$ will be false for all "citizen" agents, even if $N = 0$.

Table V. Legitimacy function, feedback mechanism, and system long-term behavior for all simulated cases with $L_0 = 0.89$.

Case	Legitimacy function	Legitimacy feedback	Long-term behavior
R21E (reference)	–	"None"	Stationary, no rebellion
R21G	Weighted average	"Global" (homogeneous)	Tipping point: Stationary ↔ Intermittent busts of violence
R21A	Weighted average	"Agents" (heterogeneous)	Intermittent bursts of violence
R22G	Geometric mean	"Global" (homogeneous)	Intermittent bursts of violence
R22A	Geometric mean	"Agents" (heterogeneous)	Intermittent bursts of violence
R23G	System support	"Global" (homogeneous)	Stationary, no rebellion
R23A	System support	"Agents" (heterogeneous)	Intermittent bursts of violence
R24G	Justification	"Global" (homogeneous)	Stationary, no rebellion
R24A	Justification	"Agents" (heterogeneous)	Tipping point: Stationary ↔ Intermittent busts of violence

Table VI. Mean and standard deviation of the peak interval, peak size, and mean time for the first peak, and % of time with rebellion and absolute calm for the simulations with $L_0 = 0.89$.

Case	Mean peak interval	Standard peak interval	Mean peak size	Std peak size	% time with rebellion (Nactive > 50)	% time with absolute calm (Nactive = 0)	Mean time for first peak	Standard of time for the first peak
R21G	21.02	46.57	267	137	50	25	819.80	705.81
R21A	15.89	10.89	233	123	69	2	26.70	11.25
R22G	13.56	10.42	205	113	69	5	224.30	176.80
R22A	13.18	10.12	206	111	72	1	29.00	41.18
R23G	–	–	–	–	0	94	–	–
R23A	9.64	7.31	158	74	72	3	137.80	116.76
R24G	–	–	–	–	0	94	–	–
R24A	25.01	17.40	341	171	56	13	70.50	57.30

Table V shows the legitimacy function, the legitimacy feedback mechanism and the resulting long-term behavior for all simulated cases with $L_0 = 0.89$. This table shows that there were three types of long-term behavior: stationary (calm), alternating periods of calm and intermittent rebellion (near tipping points), and intermittent bursts of rebellion. It also shows that both the legitimacy function and the feedback mechanism are important for determining the nature of the solutions.

Table VI shows the mean value and standard deviation of the interval (waiting time) between peaks, peak size, and time of occurrence of the first rebellion peak, as well as the proportions of the time with rebellion and absolute calm, for the simulations with $L_0 = 0.89$. Cases R21G and R24A, with alternating periods of calm and rebellion, had larger peaks (most notably for case R24A), longer peak intervals (larger than the mean jail term 15), and longer time for arrival of the first peak (which was exceptionally long for case R21G) than the cases with intermittent peaks of rebellion (R21A, R22G, R22A, and R23A). The results in this table show that solutions with these two types of behavior (intermittent peaks of rebellion and alternating periods of calm and violence bursts) have markedly different qualitative characteristics.

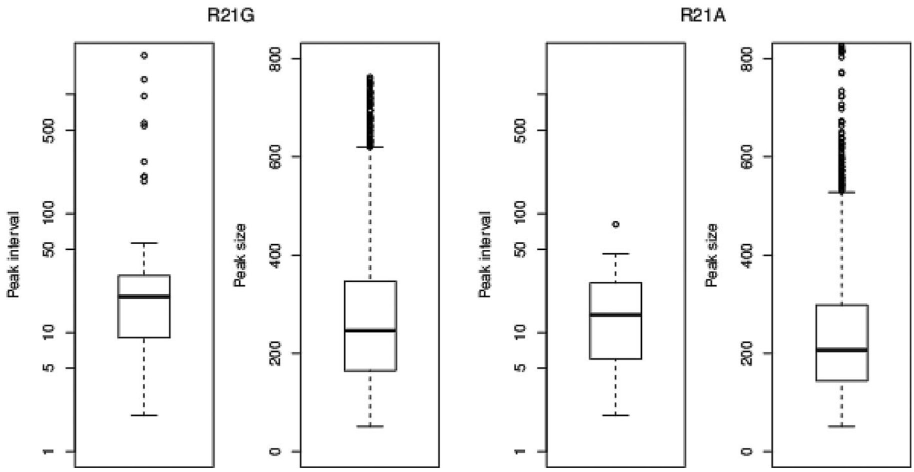

Figure 4. Boxplots of peak interval and peak size distributions for cases R21G (two plots on the left) and R21A (two plots on the right). The vertical scale for the peak interval is logarithmic.

Figure 4 shows a graphical illustration of this difference by comparing boxplots of the distributions of peak interval and size for cases R21G and R21A (which differ only in the feedback mechanism). Case R21G is remarkable for the extreme outliers of the peak interval. This characteristic and the extremely high mean time for the occurrence of the first peak indicate that the solutions of case R21G had a very interesting behavior, which was worthwhile examining in further detail.

Figure 5 shows the time history of the number of "active" citizens and the legitimacy for two simulations of the R21G case, showing a very interesting behavior: switching between periods of calm with all "citizens" quiet and legitimacy constant at its maximum value, and periods of intermittent rebellion. In the first case, a long period of recurring rebellion peaks set in after an initial short period of calm, followed by a long period of calm, and then by another period of turmoil. In the second case, rebellion peaks began after an exceptionally long initial period of calm. This indicates that for this combination of input parameters, initial legitimacy, and legitimacy function, the feedback mechanism drives the system across a tipping point between equilibrium and complex regime, so that the system does not have well-defined long-term behavior. For heterogeneous weighted average legitimacy (case R21A), the system's long-term behavior changed to intermittent bursts of rebellion. Thus, nonlinear effects are significant and the introduction of diversity in the perceived legitimacy eliminated intermittent periods of equilibrium.

For geometric mean legitimacy (cases R22G and R22A), the solutions showed intermittent peaks of rebellion, after an initial period of calm with variable duration. In fact, one of the simulations showed equilibrium with only small spikes of rebellion (below the threshold of 50 "active" agents), even after 2000 cycles. The geometric mean legitimacy function lead to larger drops of legitimacy, more instability, and more frequent peaks of rebellion than the weighted average legitimacy function.

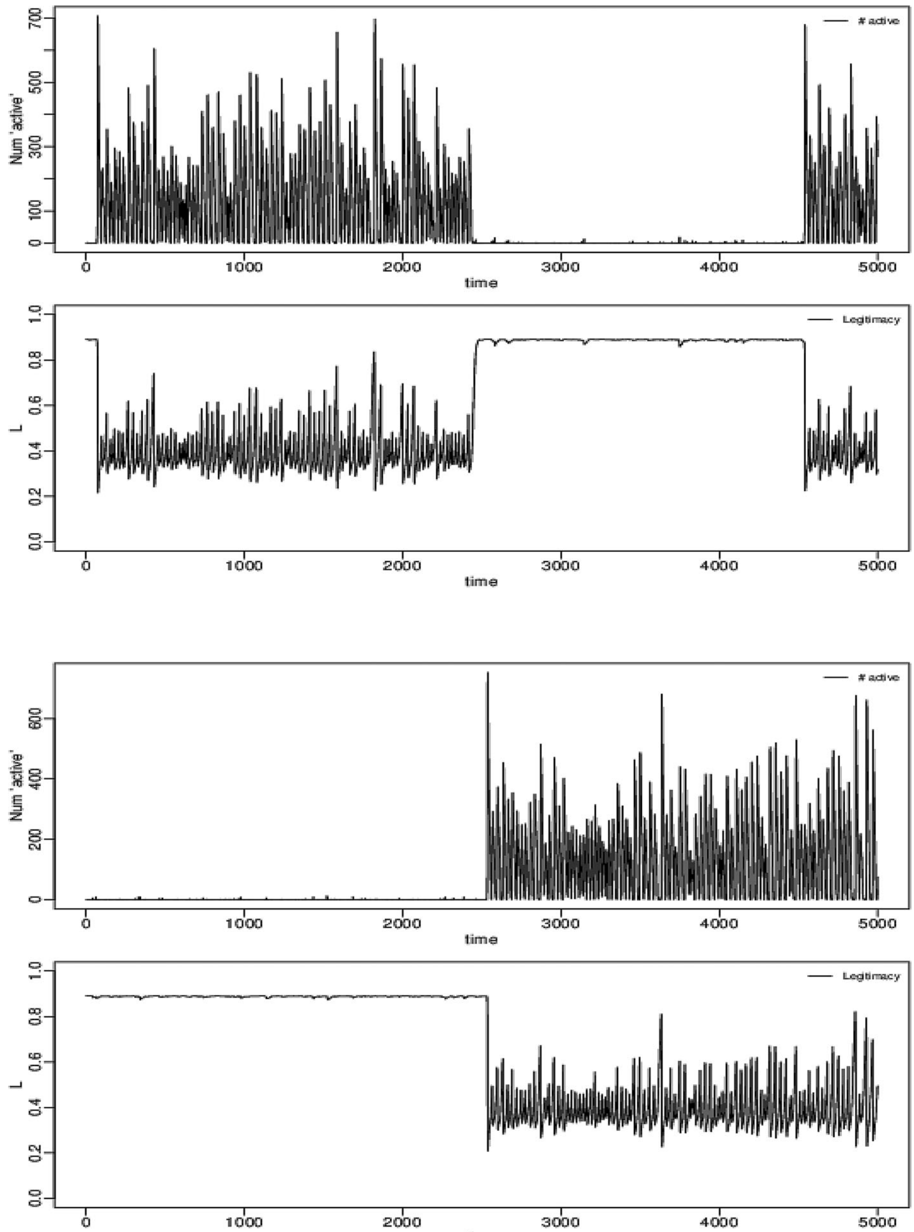

Figure 5. Time history of the number of "active" citizens and legitimacy for two simulations of the R21G case.

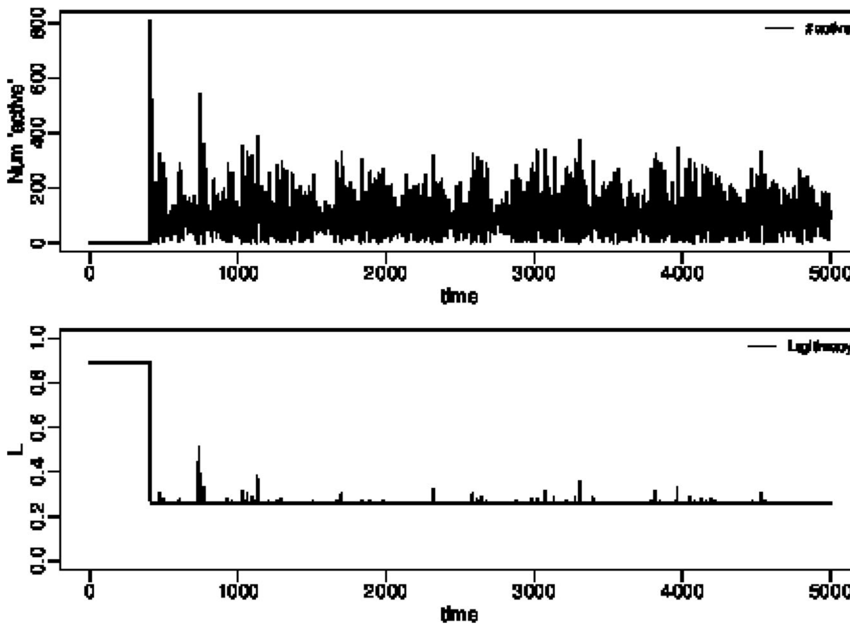

Figure 6. Time history of the number of "active" citizens (top) and legitimacy (bottom) for one of the simulations of the R23A case.

When legitimacy was calculated using Equation (10), cases R23G and R23A, the system's long-term behavior was different for homogeneous and heterogeneous legitimacy. For homogeneous legitimacy, the system remained at equilibrium with only small uprisings that were quickly suppressed by the "cops," and the legitimacy remained constant. For heterogeneous legitimacy, the solutions showed a very interesting behavior (Figure 6): after an initial period of calm with constant legitimacy, a sudden large peak of rebellion occurred. After this event, the system's behavior changed to intermittent bursts of rebellion, legitimacy dropped to a lower value and then remained almost constant, without ever recovering, except for some sporadic and very small fluctuations. This variation of the legitimacy was very different from the saw-tooth-like variation of the previous cases and was due to the particular characteristics of the function defined by Equation (11). This is another example in which diversity of the perceived legitimacy induced instability.

When legitimacy was calculated using Equation (12), cases R24G and R24A, the system was at equilibrium without rebellion peaks for homogeneous perceived legitimacy, and intermittent peaks of rebellion alternating with periods of calm for heterogeneous perceived legitimacy. In the latter case, the system was also near a tipping point with longer and larger rebellion peaks than in the other cases studied.

It is also interesting to notice that the two cases in which the system alternated between periods of calm and periods of intermittent rebellion (R21G and R24A) were those in which the waiting time between successive peaks and the mean size of the rebellion bursts were clearly higher than in the other cases. In contrast, the cases in which the waiting time was relatively small (R22A and R23A) also had comparatively small peak sizes.

5. DISCUSSION

The results presented above show how the introduction of legitimacy feedback increases both the complexity and variety of the solutions' behavior and the explanatory power of Epstein's ABM of civil violence. For a reference case in which Epstein's original model gave equilibrium solutions with no rebellion peaks, our model produced equilibrium solutions, solutions with intermittent bursts of rebellion, and solutions with indeterminate long-term behavior, depending on the form of the legitimacy function and on the feedback mechanism (homogeneous or heterogeneous perceived legitimacy). Complexity is present in two forms of emergent patterns, the intermittent bursts of violence and the intermittent alternation between periods of calm and turmoil, none of which can be directly deduced from the interaction rules, the form of the feedback mechanism, or the form of the legitimacy function.

Of the four legitimacy functions used in the simulations, the weighted average legitimacy inspired in Gilley's formulation for a global state legitimacy score produced solutions with intriguing behavior, notably for homogenous perceived legitimacy: intermittent alternation between (sometimes very long) periods of calm with periods of intermittent bursts of rebellion. Also, the initial period of calm may be exceptionally long. This indicates that legitimacy is indeed a critical parameter in Epstein's model associated with possible crossing of tipping points, so that the system does not have well-defined long-term behavior. This provides a possible explanation of apparently stable regimes that suddenly face unanticipated turmoil, recover to a period of calm, which again can lead to another period of turmoil, and so on. In these situations, the legitimacy variations showed a saw-tooth-like pattern during periods of rebellious bursts to become constant during periods of calm (the central authority intermittently regains its initial legitimacy). Use of the geometric mean function produced solutions with intermittent bursts of rebellion for both homogeneous and heterogeneous perceived legitimacy. Once again, this result may have practical implications: legitimacy theories that advocate that no legitimacy subtype can be zero imply different tipping points than those supporting a weighted average approach. Finally, the simulations with the exponentially decaying "system support" function produced solutions with an initial period of calm with occasional small episodes of violence and constant legitimacy, followed by a large upsurge of violence and a sudden drop of legitimacy, and intermittent bursts of rebellion afterwards. In contrast with the weighted average case, legitimacy never recovered. This pattern provides an explanation for the phenomenon of apparently stable authoritarian regimes suppressing small bursts of rebellion and then facing a massive unexpected uprising, after which they struggle to dominate rebellion and never recover their initial legitimacy.

Heterogeneous perceived legitimacy induced more instability than homogenous perceived legitimacy. An explanation for this nonlinear effect is that agents with perceived legitimacy below average have higher grievance than they would in the case of homogeneous perceived legitimacy, and thus induce activation cascades more easily.

6. CONCLUSIONS

In this work, we studied the effect of introducing endogenous legitimacy feedback in Epstein's ABM of civil violence against a central authority (Model I), with two types of agents ("citizens" and "cops"). We considered three subtypes of legitimacy—views of legality, views of justification, and acts of consent—based on theoretical studies on measurement of legitimacy, and formulated four different functions for expressing the legitimacy in terms of the model's dependent variables (number of supporters and number of nonsupporters of the central authority): weighted average, geometric mean, exponentially decaying "system support" and "justification." We also considered two feedback mechanisms: homogeneous perceived legitimacy, in which all agents share the same perception value, and heterogeneous perceived legitimacy, in which legitimacy is an individual attribute and the global perceived legitimacy is an emergent property (evaluated by averaging over all "citizen" agents).

It was found that for homogeneous weighted average legitimacy the system's long-term behavior was indeterminate, oscillating between intermittent periods of calm with constant legitimacy and periods with peaks of rebellion. For heterogeneous weighted average legitimacy and for homogeneous and heterogeneous geometric mean legitimacy, the system's time evolution was characterized by an initial period of calm followed by a large burst of rebellion and successive intermittent peaks of rebellion thereafter. Legitimacy remained constant during the initial period of calm, suffered a large and sudden discontinuous drop associated with the outbreak of rebellion, and recovered in a saw-tooth-like manner. For the exponentially decaying "system support" the solutions showed (i) a period of calm with no active agents, punctuated by a few sporadic and small peaks of violence, (ii) a sudden, unanticipated, and large outburst of rebellion, and (iii) intermittent and overlapping peaks of rebellion. Legitimacy suffered a sudden discontinuous drop as in the other cases, but instead of recovering it remained constant at its lowest value.

These results represent a major improvement with respect to our previous formulation of legitimacy feedback[7] in three aspects: simplicity, soundness, and explanatory power. The present formulation is simpler, as it dispenses the introduction of a new type of agent, does not depend on assuming a particular form of the legitimacy variations, and does not require an extra timescale parameter. It is also sounder because the functions for describing the legitimacy variations are based on theoretical studies on the measurement of legitimacy.[20,22] Finally, the present model provides a simple and straightforward explanation of the phenomenon of apparently stable authoritarian regimes facing an unexpected large-scale uprising, responding with repression, and struggling afterwards with intermittent bursts of rebellion (after the initial peak of violence, too many citizens are arrested or "active," resulting in the government severely losing support and being perceived as illegitimate).

The model presented herein can be used to test theories on the aggregation of legitimacy subtypes via simulation, in complement to the traditional approaches based on statistical analyses of datasets.[20,22] It can also be improved by incorporating context-specific information obtained from such analyses in the legitimacy feedback functions, and using these improved functions to simulate the risk of different countries suffering large-scale uprisings.

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