Switching and cycling near heteroclinic networks as a piecewise-smooth dynamical system

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Supervised by Vivien Kirk and Claire Postlethwaite

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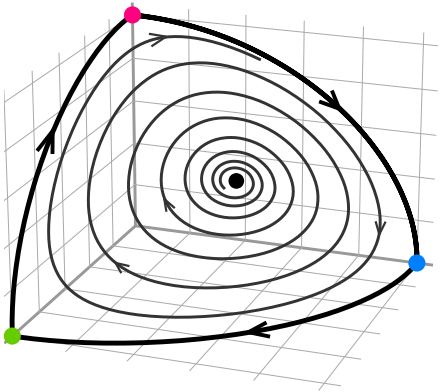
SCIENCE
DEPARTMENT OF MATHEMATICS

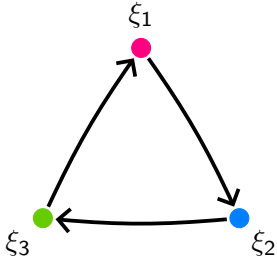
Competition between three species[1]

[First frame of animation here.]

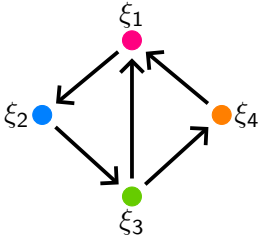
[1] May and Leonard, *SIAM J. Appl. Math.*, 1975

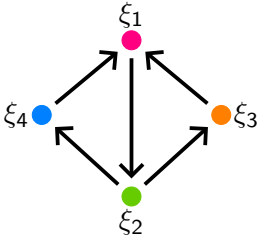
Heteroclinic cycles

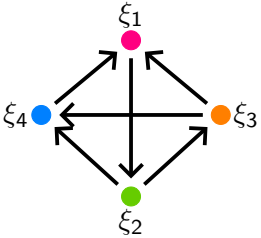




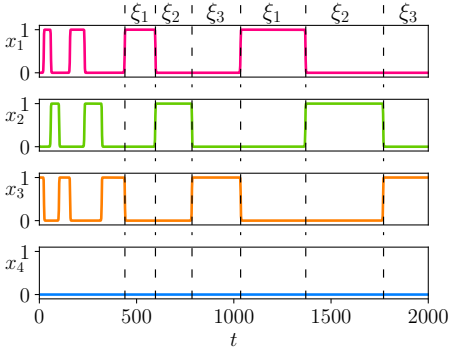
Heteroclinic networks

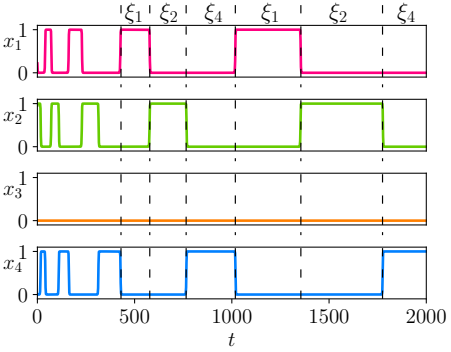


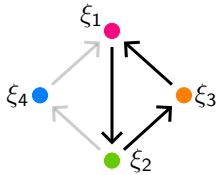


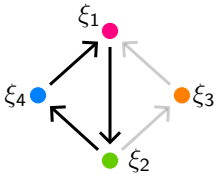


The Kirk-Silber network[2]



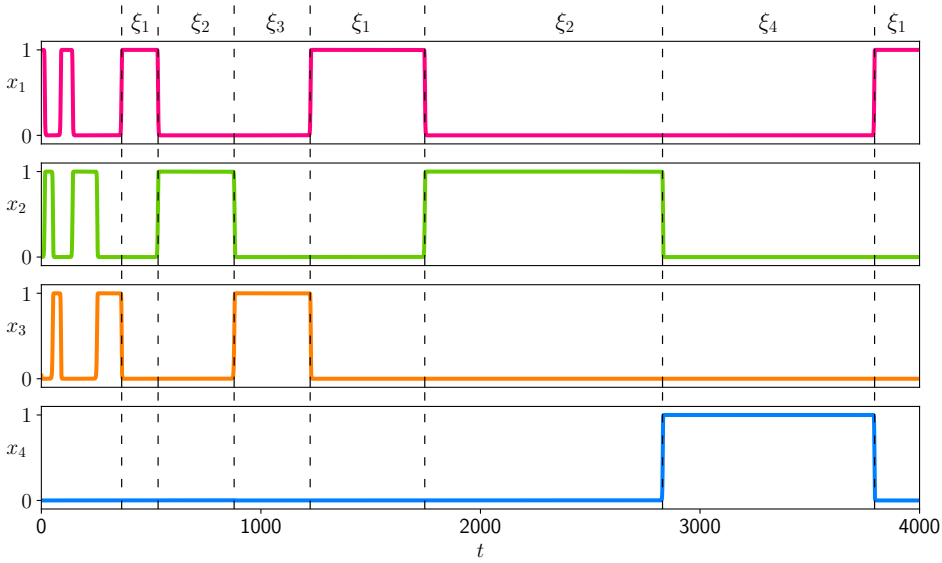




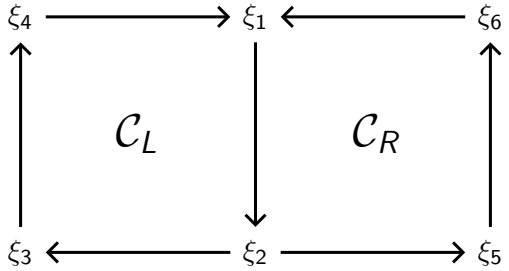


[2] Kirk and Silber, *Nonlinearity*, 1994

Switching near the Kirk-Silber network

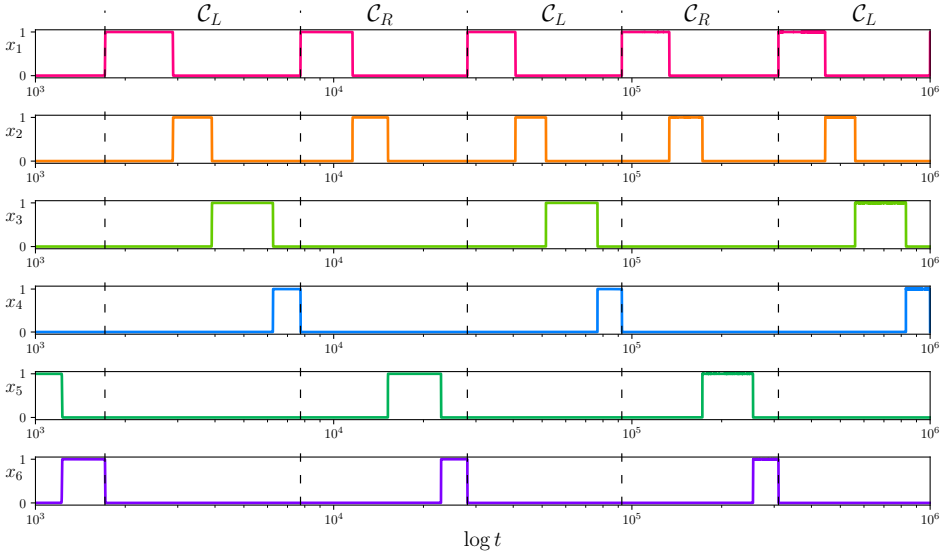


Podvigina's 2-cycle network[3]

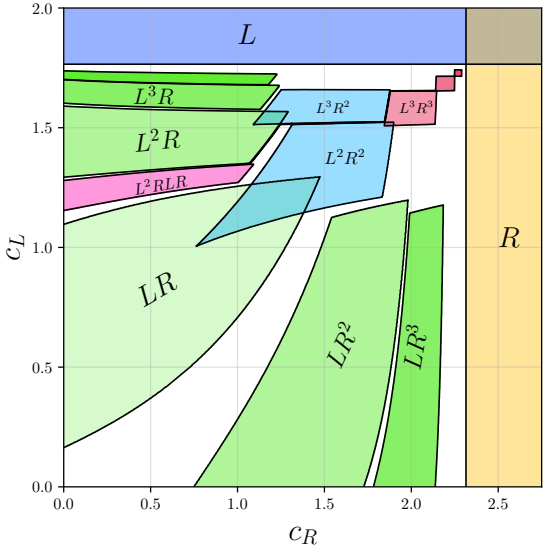


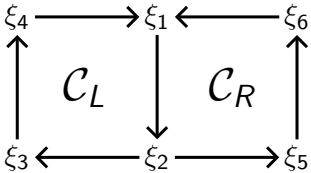
[3] Podvigina, *arXiv:2107.09982*, 2021

Dynamics near Podvigina's 2-cycle network



Partial bifurcation set of Podvigina's 2-cycle network





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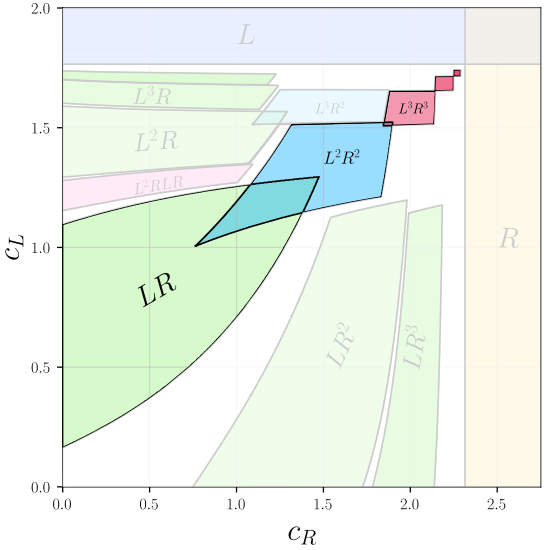

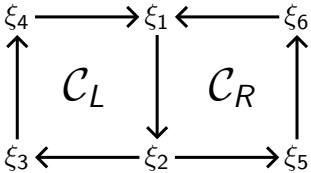

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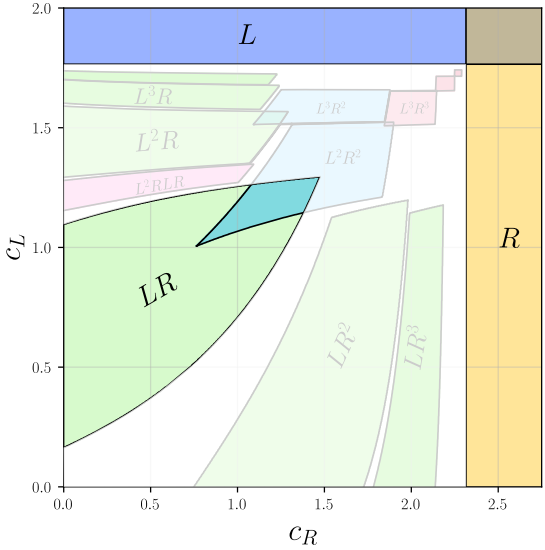

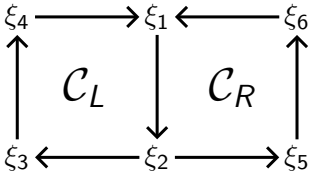

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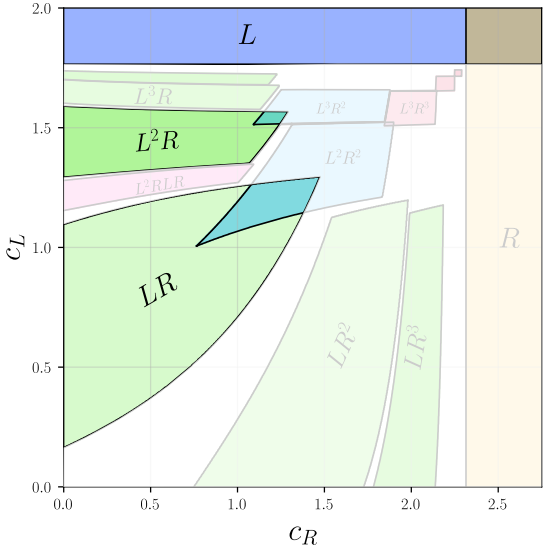

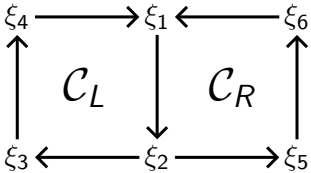

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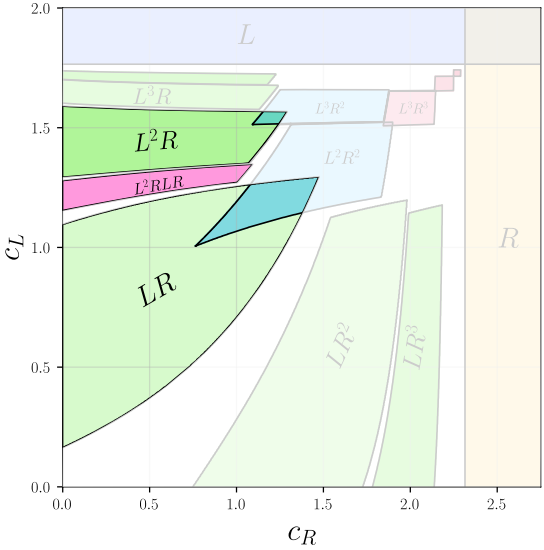
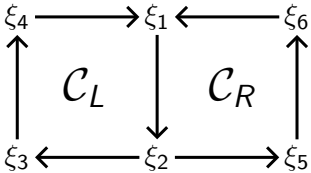
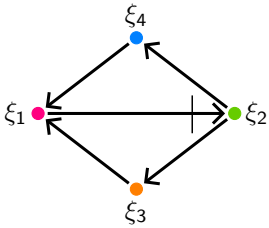
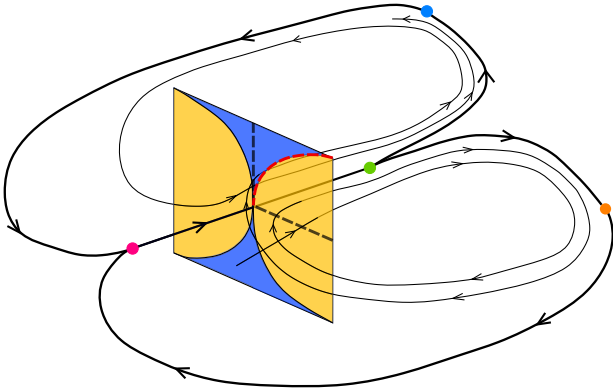


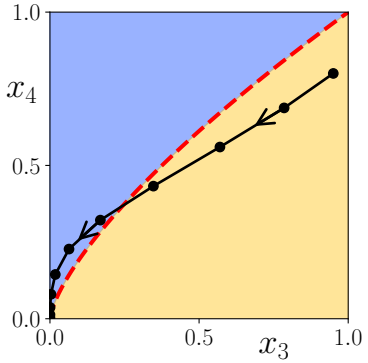


Poincaré sections

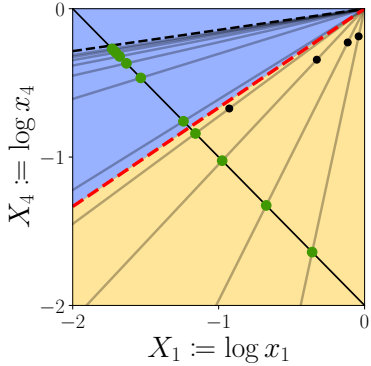




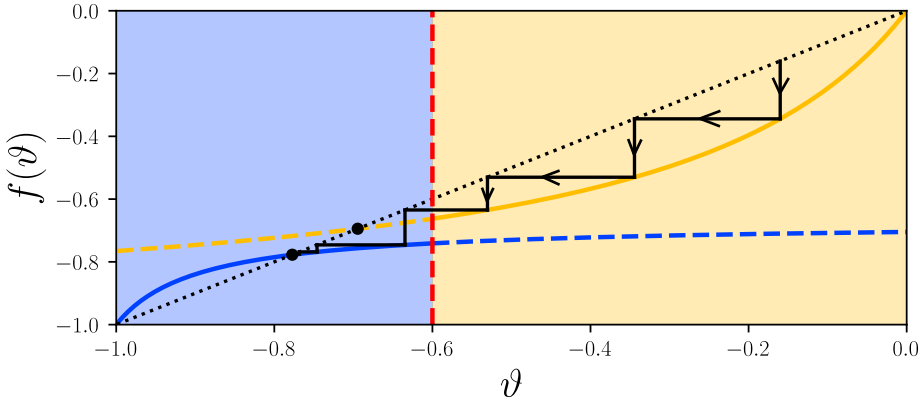
Return maps



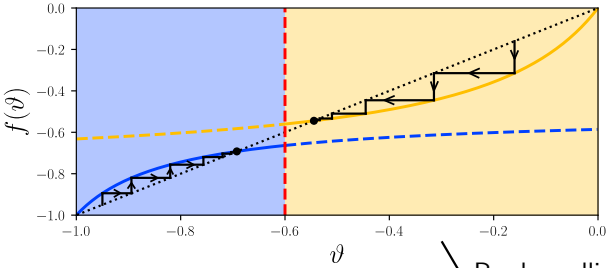
Take logarithms



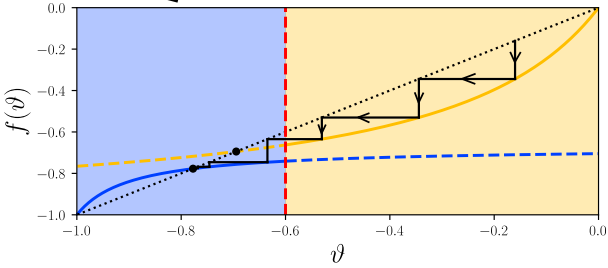
Projected map



Projected map



Border-collision bifurcation



Research goals

- Can we use the theory of PWS dynamical systems to explain the Farey-like concatenation of stable root sequences observed in many heteroclinic networks?

- Similarly, can we use this theory to explain other features of these bifurcation sets, such as the chains of stability regions and shrinking points?

- Little is known about possible dynamics near large heteroclinic networks. Can we classify possible dynamics near classes of these networks based on the topology and symmetries of its representation as a graph?