Metodos Numéricos

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Ejemplo del uso de la serie de Taylor

$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$.

Predecir el valor en $x=1$ con $h=1$ cuando la serie de Taylor de orden cero haata 4, y calculando el residuo en cada caso

$f(0) = -0.1(0)^4 - 0.15(0)^3 - 0.5(0)^2 - 0.25(0) + 1.2 = 1.2$.

Derivadas.

$f'(x) = -0.4x^3 - 0.45x^2 - 1x - 0.25$
$f''(x) = -1.2x^2 - 0.9x - 1$
$f'''(x) = -2.4x - 0.9$
$f^{iv}(x) = -2.4$
$f^{v}(x) = 0$.

n=0

$f(0) = 1.2$

$Rn = \frac{f'(0.5)(h)^1}{1!}$

$\cancel{Rn=}\ f'(0.5) = -0.4(0.5)^3 - 0.45(0.5)^2 - (0.5) - 0.25$

$f'(0.5) = -0.9125$

$Error = f(1) - 1.2$

$f(1) = -0.1(1)^4 - 0.15(1)^3 - 0.5(1) - 0.25(1) + 1.2 = 0.2$

$Error = 0.2 - 1.2 = -1$

n=1

$f(0) = 1.2 + \frac{f'(0)h^1}{1!}$

$f'(0) = -0.4(0)^3 - 0.45(0)^2 - 0 - 0.25$
$f'(0) = -0.25$

$f(0) = 1.2 - 0.25 = 0.95$

$Rn = \frac{f''(0.5)h^2}{2!}$

$\cancel{Rn=}\ f''(0.5) = -1.2(0.5)^2 - 0.9(0.5) - 1$

$f''(0.5) = -1.75$

$Rn = \frac{(-1.75)(1)^2}{2!} = -0.875$

$Error = \cancel{0.2 - 1.2 = 0.95}$
$0.2 - 0.95 = -0.75$

n=2.

$f(0) = 1.2 + \frac{f'(0)h^1}{1!} + \frac{f''(0)h^2}{2!}$

$f''(0) = -1.2(0)^2 - 0.9(0) - 1$
$f''(0) = -1$

$f(0) = 1.2 - 0.25 - \frac{1}{2} = 0.45$

n	f(1)	Rn	Error
0	1.2	-0.91	-1
1	0.95	-0.85	-0.75
2	0.45	-0.35	-0.25
3	0.3	-0.1	-0.1
4	0.2	0	0

$$R_n = \frac{f^{(3)}(0.5)\,h^3}{3!}$$

$$f'''(0.5) = -2.4(0.5) - 0.9$$
$$f'''(0.5) = -2.1.$$

$$R_n = \frac{-2.1(1)^3}{3!} = -0.35$$

$$\text{Error} = 0.2 - 0.45 = -0.25$$

n = 3.

$$f(0) = 1.2 - 0.25 + 0.45 + \frac{f'''(0)\,h^3}{3!}$$

$$f'''(0) = -2.4(0) - 0.9 = -0.9 \cancel{-0.825}$$

$$f(0) = 1.2 - 0.25 + 0.45 + \frac{(-0.9)(1)^3}{3!}$$

$$f(0) = 1.2 - 0.25 + 0.45 - 0.15 = \cancel{1.25}\ \cancel{0.25} = 0.3$$

$$R_n = \frac{f^{(4)}(0.5)\,h^4}{4!}$$

$$f^{IV}(0.5) = -2.4$$

$$R_n = \frac{(-2.4)(1)^4}{4!}$$

$$R_n = -0.1$$

$$\text{Error} = \cancel{0.2} \cancel{-1.25} \; 0.2 - 0.3 = -0.1$$

n = 4

$$f(0) = 1.2 - 0.25 - 0.5 - 0.15 - \frac{f^{IV}(0)\,h^4}{4!}$$

$$f^{IV}(0) = -2.4 \qquad \frac{-2.4}{4!} = -0.1$$

$$f(0) = 1.2 - 0.25 - 0.5 - 0.15 - 0.1 = 0.2$$

$$f^{V} = 0$$

$$R_n = \frac{(0)1^5}{5!} = 0$$

$$\text{Error} = 0.2 - 0.2$$

$f(x) = \cos(x)$

Predecir el valor de $x = \pi/3$ cuando la serie de Taylor entorno a $x = \pi/4$ de orden cero hasta 4, calculando el residuo en cada caso.

n = 0

$f(\pi/3) = \cos(\pi/3) = 1/2$

$R_n = \frac{f'(\pi/24)(\pi/3 - \pi/4)^0}{1!}$

$R_n = \frac{f'(\pi/24)}{1!}$

$f'(\pi/24) = -\operatorname{Sen}(\pi/24) = -0.13$

$R_n = \frac{-0.13}{1!} = -0.13$

Error = $\cancel{f(\pi/4) - f(\pi/3)}$

Error = $f(\pi/4) - f(\pi/3)$

Error = $0.707 -$

$0.5 - 0.707 = -0.21$

n = 1

$f(\pi/3) = 1/2 + \frac{f'(\pi/3)(\pi/24)^1}{1!}$

$f(\pi/3) = 1/2$

$f'(\pi/3) = -\operatorname{Sen}(\pi/3) = -0.866$

$f(\pi/3) = \frac{1}{2} + \frac{(-0.866)(\pi/24)}{1!}$

$f(\pi/3) = \frac{1}{2} - 0.113 = 0.3866$

Derivadas

$f(x) = \cos(x)$
$f'(x) = -\operatorname{sen}(x)$
$f''(x) = -\cos(x)$
$f'''(x) = \operatorname{sen}(x)$
$f^{IV}(x) = \cos(x)$
$f^{V}(x) = -\operatorname{sen}(x)$

$R_n = \frac{f^2(\pi/24)(\pi/12)}{2!}$

$f''(x) = -\cos(x)$
$f''(\pi/24) = -\cos(\pi/24)$
$f''(\pi/24) = -0.991$

$R_n = \frac{(-0.991)(\pi/12)^2}{2!}$

$R_n = \cancel{-0.0339}\ -0.0339$

Error = $0.5 - 0.3866$
Error = 0.1134

n = 2

$f(\pi/3) = 1/2 - 0.113 + \frac{f''(\pi/3)(\pi/24)^2}{2!}$

$f''(\pi/3) = -\cos(\pi/3) = -1/2$

$f(\pi/3) = \frac{1}{2} - 0.113 + \frac{(0.5)(\pi/24)^2}{2!}$

$f(\pi/3) = 0.3827$

$R_n = \frac{f^3(\pi/24)(\pi/12)^3}{3!}$

$f'''(\pi/24) = Sen(\pi/24) = 0.130$

$Rn = \frac{(0.130)(\pi/12)^3}{3!} = 3.88 \times 10^{-4}$

$Error = 0.5 - \cancel{0.4??}^{0.3827} = 0.1173.$

n = 3.

$f(\pi/3) = 1/2 - 0.113 - \cancel{0.3827}\ 4.28 \times 10^3 + \frac{f'''(\pi/3)(\pi/12)^3}{3!}$

$f'''(\pi/3) = Sen(\pi/3) = 0.866.$

$f(\pi/3) = 1/2 - 0.113 - 4.28 \times 10^{-3} + \frac{(0.866)(\pi/12)^3}{3!}$

$f(\pi/3) = 1/2 - 0.113 - 4.28 \times 10^{-3} + 2.589 \times 10^{-3} = 0.385$

$f^{IV}(\pi/24) = Cos(\pi/24) = 0.99$

$Rn = \frac{f^{IV}(\pi/24)(\pi/12)^4}{4!} = \frac{(0.99)(\pi/12)^4}{4!} = 1.93 \times 10^{-4}$

$Error = 0.5 - 0.385 = 0.115$

n = 4

$f(\pi/4) = 1/2 - 0.113 - 4.28 \times 10^3 + 2.589 \times 10^{-3} + f^{IV}(\pi/4)($