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Fecha de entrega

9	09	25

Tarea 2^{da} y 3^{er} derivada

hacia atrás

Realizar para el caso x_{i-2} y x_{i-1}

$$f(x_i-2) = f(x_i) + f'(x_i)(x_{i-2}-x_i) + \frac{f''(x_i)(x_{i-2}-x_i)^2}{2!} + O(h)$$
$$f(x_i-1) = f(x_i) + f'(x_i)(x_{i-1}-x_i) + f''(x_i)\frac{(x_{i-1}-x_i)^2}{2!} + O(h)$$

① $$f(x_i-2) = f(x_i) + f''(x_i)(x_{i+1}-x_i) + \frac{f''(x_i)(x_{i+1}-x_i)^2}{2!} + O(h)^2$$
$$f(x_i-2) = f(x_i) + 2f'(x_i)h + \frac{4f''(x_i)h^2}{2!} + O(h)$$

② $$f(x_i-1) = f(x_i) + f'(x_i)(-h) + \frac{f''(x_i)(-h)^2}{2!} + O(h)$$
$$f(x_i-1) = f(x_i) - f'(x_i)h + \frac{f''(x_i)h^2}{2!} + O(h)$$

Podemos multiplicar por (-2) y realizar suma

$$-2f(x_i-1) = -2f(x_i) + 2f'(x_i)h - f''(x_i)h^2 + O(h)$$
$$f(x_i-2) = f(x_i) - 2f'(x_i)h + 2f''(h_i)h^2 + O(h)$$

$$f(x_i-2) - 2f(x_i-1) = -f(x_i) + f''(x_i)h^2 + O(h)$$

$$f''(x_i) = \frac{f(x_i-2) - 2f(x_i-1) + f(x_i)}{h^2} + O(h)$$

- Centrada
  considerando x_{i+1} y x_{i-1}

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)(x_{i+1} - x_i)^2}{2!} + O(h^2)$$

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + O(h)^2$$

$$f(x_{i-1}) = f(x_i) + f'(x_i)(x_{i-1} - x_i) + \frac{f''(x_i)(x_{i-1} - x_i)^2}{2!} + O(h)$$

$$f(x_{i-1}) = f(x_i) - f'(x_i)h + \frac{f''(x_i)(h^2)}{2!} + O(h)^2$$

$$f(x_{i+1}) + f(x_{i-1}) = 2f(x_i) + f'(x_i)h - f(x_i)h + \frac{2f''(x_i)h^2}{2!} + O(h^2)$$

$$f''(x_i) = \frac{f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)}{h^2} + O(h)^2$$

3ra derivada

Caso hacia adelante

Consideramos

$x_{i+1} - x_i = h$

$x_{i+2} - x_i = 2h$

$x_{i+3} - x_i = 3h$

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + \frac{f'''(x_i)h^3}{3!} + O(h)$$

$$f(x_{i+2}) = f(x_i) + 2f'(x_i)h + \frac{4f''(x_i)h^2}{2!} + \frac{8f'''(x_i)h^3}{3!} + O(h)$$

$$f(x_{i+3}) = f(x_i) + 3f'(x_i)h + \frac{9f''(x_i)h^2}{2!} + \frac{27f'''(x_i)h^3}{3!} + O(h)$$

Podemos multiplicar por ... para aplicar ...

1 y Ec 4 (...)

$$f(x_i+3)+f(x_i+1)=2f(x_i)+4f'(x_i)h+\frac{10f''(x_i)h^2}{2!}+\frac{28f'''(x_i)h^3}{3!}+O(h)$$

Ec. 5

$$-2f(x_i+2)=-2f(x_i)-4f'(x_i)h-\frac{8f''(x_i)h^2}{2!}-\frac{16f'''(x_i)h^3}{3!}+O(h)$$

Sumando ambas ecuaciones obtenemos

$$f(x_i+3)+f(x_i+1)-2f(x_i+2)=\frac{2f''(x_i)h^2}{2!}+\frac{12f'''(x_i)h^3}{3!}+O(h)$$

$$f(x_i+3)+f(x_i+1)-2f(x_i+2)=f''(x_i)h^2+2f'''(x_i)h^3+O(h)$$

Como conocemos f'' reemplazamos

$$f(x_i+3)+f(x_i+1)-2f(x_i+2)=f(x_i+2)-2f(x_i+1)+f(x_i)+2f'''(x_i)h^3+O(h)$$

$$2f'''(x_i)h^3=f(x_i+3)+f(x_i+1)-2f(x_i+2)-f(x_i+2)+2f(x_i+1)-f(x_i)+O(h)$$

$$f'''(x_i)=\frac{f(x_i+3)-3f(x_i+2)+3f(x_i+1)-f(x_0)}{2h^3}+O(h)$$