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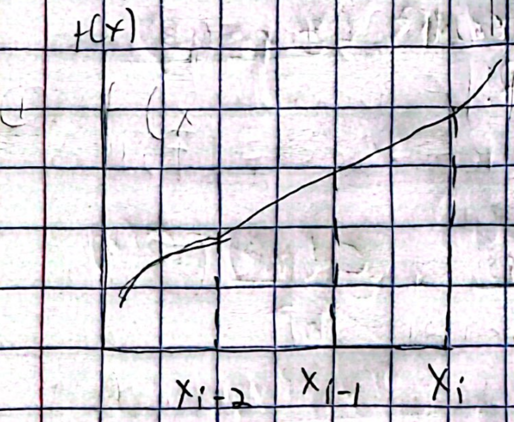
Segunda derivada por métodos de diferencias finitas

Hacia atrás



Ecuación

① $f(x_{i-1}) = f(x_i) + f'(x_i)\overbrace{(x_{i-1}-x_i)}^{h} + \frac{f''(x_i)(x_{i-1}-x_i)^2}{2!} + O(x_{i-1}-x_i)$

② $f(x_{i-2}) = f(x_i) + f'(x_i)\overbrace{(x_{i-2}-x_i)}^{2h} + \frac{f''(x_i)(x_{i-2}-x_i)^2}{2!} + O(x_{i-2}-x_i)$

Ecu ① $h = x_{i-1} - x_i$ Ecu ② $h = x_{i-2} - x_i$

- Multiplicamos por 2 la primera ecuación para que se elimine la primera derivada.

$-2f(x_{i-1}) = -2f(x_i) - 2f'(x_i)h - \frac{2f''(x_i)h^2}{2!} - O(2h^3)$

- Esta se suma con la segunda ecuación

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$$-2f(x_{i-1}) = -2f(x_i) - \cancel{2f'(x_i)h} - \frac{2f''(x_i)h^2}{2!} - O(2h^3)$$

$$+ \quad f(x_{i-2}) = f(x_i) + \cancel{f'(x_i)2h} + \frac{f''(x_i)2h}{2!} + O(2h^3)$$

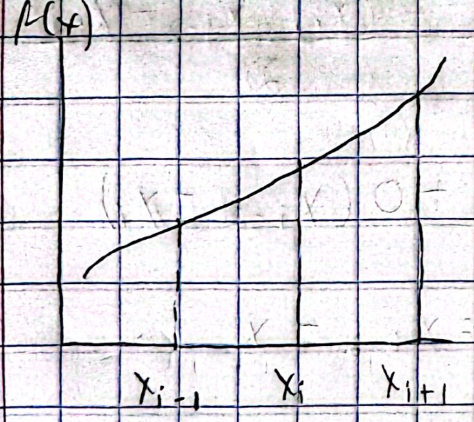
$$f(x_{i-2}) - 2f(x_{i-1}) = -f(x_i) + \frac{2f''(x_i)h^2}{2!} + O(2h^3)$$

· Despejamos a $f''(x_i)$

$$f''(x_i) = \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{h^2} + \cancel{\frac{O(h^3)}{h^2}} \; O(h)$$

$$\underbrace{f''(x_i) = \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{h^2} + O(h)}$$

Centrada



Ecuación

$$① \; f(x_{i-1}) = f(x_i) + f'(x_i)\overbrace{(x_{i+1} - x_i)}^{h} + \frac{f''(x_i)(x_{i-1} - x_i)^2}{2!} + O(x_{i-1} - x_i)^2$$

$$② \; f(x_{i+1}) = f(x_i) + f'(x_i)\overbrace{(x_{i+1} - x_i)}^{2h} + \frac{f''(x_i)(x_{i+1} - x_i)^2}{2!} + O(x_{i+1} - x_i)^2$$

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Ecu (1) $-h = x_{i-1} - x_i$ Ecu (2) $h = x_{i+1} - x_i$

Sumamos la ecuación primera ecuación para que se elimi

$$f(x_{i-1}) = f(x_i) - \cancel{f'(x_i)h} + \frac{f''(x_i)(h^2)}{2!} + O(h^3)$$

$$+ \quad f(x_{i+1}) = f(x_i) + \cancel{f'(x_i)h} + \frac{f''(x_i)h^2}{2!} + O(h^3)$$

$$f(x_{i-1}) + f(x_{i+1}) = 2f(x_i) + f''(x_i)h^2 + O(h^2)$$

$$f''(x_i)h^2 = f(x_{i+1}) + f(x_{i-1}) - 2f(x_i) + O(h^2)$$

$$f''(x_i) = \frac{f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)}{h^2} + O(h^2)$$

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Tercera derivada por método de diferencias finitas

Hacia adelante

① $f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)(x_{i+1} - x_i)^2}{2!}$

$+ \frac{f'''(x_i)(x_{i+1} - x_i)^3}{3!} + O(h)$

② $f(x_{i+2}) = f(x_i) + f'(x_i)(x_{i+2} - x_i) + \frac{f''(x_i)(x_{i+2} - x_i)^2}{2!}$

$+ \frac{f'''(x_i)(x_{i+2} - x_i)^3}{3!} + O(h)$

③ $f(x_{i+3}) = f(x_i) + f'(x_i)(x_{i+3} - x_i) + \frac{f''(x_i)(x_{i+3} - x_i)^2}{2!}$

$+ \frac{f'''(x_i)(x_{i+3} - x_i)^3}{3!} + O(h)$

$h = x_{i+1} - x_i \qquad 2h = x_{i+2} - x_i \qquad 3h = x_{i+3} - x_i$

Sustituimos a h en las tres ecuaciones

① $f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + \frac{f'''(x_i)h^3}{3!} + O(h)$

② $f(x_{i+2}) = f(x_i) + f'(x_i)2h + \frac{f''(x_i)(2h)^2}{2!} + \frac{f'''(x_i)(2h)^3}{3!}$

$+ O(h)$

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D 09 M 09 A 2025

③ $f(x_{i+3}) = f(x_i) + f'(x_i)(3h) + \frac{f''(x_i)(3h)^2}{2!} + \frac{f'''(x_i)(3h)^3}{3!} + O(h)$

Sumamos la ecuación ① y ③

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)(h)^2}{2!} + \frac{f'''(x_i)(h)^3}{3!} + O(h)$$

$$+\quad f(x_{i+3}) = f(x_i) + f'(x_i)(3h) + \frac{f''(x_i)(3h)^2}{2!} + \frac{f'''(x_i)(3h)^3}{3!} + O(h)$$

$$f(x_{i+1}) + f(x_{i+3}) = 2f(x_i) + 4f'(x_i)h + \frac{10f''(x_i)h^2}{2!} + \frac{28f'''(x_i)}{3!} + O(h)$$

Multiplicamos la ecuación por -2 para eliminar 1era derivada

$$-2f(x_{i+2}) = -2f(x_i) - 4f'(x_i)h - \frac{8f''(x_i)h^2}{2!} - \frac{16f'''(x_i)h^3}{3!} + O(h)$$

Sumamos estos dos resultados

$$f(x_{i+1}) + f(x_{i+3}) = \cancel{2f(x_i)} + \cancel{4f'(x_i)h} + \frac{10f''(x_i)h^2}{2!} + \frac{28f'''(x_i)h^3}{3!} + O(h)$$

$$+\quad -2f(x_{i+2}) = \cancel{-2f(x_i)} - \cancel{4f'(x_i)h} - \frac{8f''(x_i)h^2}{2!} - \frac{16f'''(x_i)h^3}{3!} + O(h)$$

$$f(x_{i+1}) + f(x_{i+3}) - 2f(x_{i+2}) = \frac{2f''(x_i)h^2}{2!} + \frac{12f'''(x_i)h^3}{3!} + O(h)$$

Resolviendo el factorial nos da

$$f(x_{i+1}) + f(x_{i+3}) - 2f(x_{i+2}) = f''(x_i)h^2 + 2f'''(x_i)h^3 + O(h)$$

Como conocemos

$$f''(x_i)h^2 = f(x_{i+2}) - 2f(x_{i+1}) + f(x_i) + O(h)$$

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Scribe

Sustituimos

$$f(x_{i+1}) + f(x_{i+3}) - 2f(x_{i+2}) = f(x_{i+2}) - 2f(x_{i+1}) + f(x_i) + 2f'''(x_i)h^3 + O(h)$$

$$2f'''(x_i)h^3 = f(x_{i+3}) - 3f(x_{i+2}) + 3f(x_{i+1}) - f(x_i) + O(h)$$

$$f''(x_i) = \frac{f(x_{i+3}) - 3f(x_{i+2}) + 3f(x_{i+1}) - f(x_i)}{2h^3} + O(h)$$