$$f[x_1,x_0] = \frac{f(x_1)-f(x_0)}{x_1-x_0}$$

$$f[x_2,x_1,x_0] = \frac{f[x_2,x_1]-f[x_1,x_0]}{x_2-x_0}$$

$$f(x_i,x_j,x_n) = \frac{f[x_i,x_j]-f[x_j,x_n]}{x_i-x_n}$$

Ejercicio

Calcula $f(2)$ usando interpolación lineal, cuadrática y cúbica

	x	f(x)
0	1	0
1	4	1.386294
2	5	1.609438
3	6	1.791752

✓ Lineal

$$f_1(x) = \frac{f(x_1)-f(x_0)}{x_1-x_0}(x-x_0)+f(x_0)$$

$$f_1(2) = \frac{1.386294-0}{4-1}(2-1)+0$$

$$f_1(2) = 0.462098$$

Cuadrática

$$f_2(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$$

$$b_0 = f(x_0) = 0$$

$$b_1 = \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{1.386294-0}{4-1} = 0.462098$$

$$b_2 = \frac{\frac{f(x_2)-f(x_1)}{x_2-x_1} - \frac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0} = \frac{\frac{1.609438-1.386294}{5-4} - \frac{1.386294-0}{4-1}}{5-1}$$

$$b_2 = \frac{0.223144-0.462098}{4} = -0.0597385$$

$$f_2(x) = 0 + 0.462098(2-1) + (-0.0597385)(2-1)(2-4)$$

$$f_2(2) = 0 + 0.462098 + 0.119477 = 0.581575$$

Cubica

$$f_3(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$f_3(x_3) = f(x_0) + \frac{f(x_1)-f(x_0)}{x_1-x_0}(x_2-x_0) + \left(\frac{\frac{f(x_2)-f(x_1)}{x_2-x_1} - \frac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0}\right)(x_2-x_0)(x_2-x_1)$$

$$+ b_3(x-x_0)(x-x_1)(x-x_2)$$

$b_3 =$

Polinomio de interpolación de Newton

$$f[x_n, x_{n-1}, x_1, x_0] = \frac{f[x_n, x_{n-1}, x_1] - f[x_n, x_{n-1}, x_2, x_0]}{x_n - x_0}$$

~~$f(x_3, x_2, x_1, x_0) = f[x_n, x_2]$~~

$$= \frac{f[x_n, x_{n-1}, \cdots, x_2] - f[x_n, x_{n-1}, \cdots, x_1]}{x_n - x_1} - \frac{f[x_n, x_{n-1}, \cdots x_2] - f[x_n, x_{n-1}, \cdots, x_3, x_0]}{x_n - x_0}$$

$$x_n - x_0$$

$$f(x_3, x_2, x_1, x_0) = \frac{f(x_3, x_2, x_1) - f(x_3, x_2, x_0)}{x_3 - x_0}$$

$$= \frac{\frac{f(x_3, x_2) - f(x_2, x_1)}{x_3 - x_1} - \frac{f(x_3, x_2) - f(x_3, x_0)}{x_3 - x_0}}{x_3 - x_0}$$

$$= \frac{\frac{f(x_3)-f(x_2)}{x_3-x_1} - \frac{f(x_3)-f(x_1)}{x_3-x_1} - \frac{f(x_3)-f(x_2)}{x_3-x_0} - \frac{f(x_3)-f(x_0)}{x_3-x_0}}{x_3-x_0}$$

$$\frac{\frac{f(x_3)-f(x_2)}{x_3-x_2} - \frac{f(x_3)-f(x_1)}{x_3-x_1} - \frac{f(x_3)-f(x_2)}{x_3-}}{x_3-x_1}$$

$$= \frac{\frac{f(x_3)-f(x_2)}{x_3-x_1} - \frac{f(x_2)-f(x_1)}{x_3-x_1} - \frac{f(x_3)-f(x_2)}{x_3-x_0} - \frac{f(x_2)-f(x_0)}{x_3-x_0}}{x_3-x_0} = b_3$$

$$= \frac{\frac{1.791752-1.609438}{6-4} - \frac{1.609438-1.386294}{6-4} - \frac{1.791752-1.609438}{6-1} - \frac{1.609438-0}{6-1}}{6-1}$$

$$= \frac{0.091157 - 0.111572 - 0.0364628 - 0.3218876}{5} = -0.07575308$$

$$f_3(2) = 0 + 0.462098(1) + (-0.05973385)(2-1)(2-4) + \frac{(-0.07575308)(1)}{(2-4)(2-5)}$$

$$= 0 + 0.462098 + 0.1194677 - 0.04545848$$

$$f_3(2) = 0.12705652 //$$