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Segunda diferencia atrás

$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2} + O(h)$$

① $f(x_{i-1}) = f(x_i) - f'(x_i)h + \frac{f''(x_i)h^2}{2!} - \frac{f'''(x_i)h^3}{3!} + O(h^4)$

$f(x_{i-2}) = f(x_i) - f'(x_i)2h + \frac{f''(x_i)(2h)^2}{2!} - \frac{f'''(x_i)(2h)^3}{3!} + O(h^4)$

$f(x_{i-2}) = f(x_i) - 2f'(x_i)h + \frac{4f''(x_i)h^2}{2} - \frac{8f'''(x_i)h^3}{6} + O(h^4)$

② $f(x_{i-2}) = f(x_i) - 2f'(x_i)h + 2f''(x_i)h^2 - \frac{4}{3}f'''(x_i)h^3 + O(h^4)$

Multiplicamos ① por 2

③ $2f(x_{i-1}) = 2f(x_i) - 2f'(x_i)h + f''(x_i)h^2 - \frac{1}{3}f'''(x_i)h^3 + O(h^4)$

Restamos ③ de ②

$$\begin{array}{r} f(x_{i-2}) = f(x_i) - 2f'(x_i)h + 2f''(x_i)h^2 \\ - \; 2f(x_{i-1}) = 2f(x_i) - 2f'(x_i)h + f''(x_i)h^2 + O(h^3) \\ \hline f(x_{i-2}) - 2f(x_{i-1}) = -f(x_i) + f''(x_i)h^2 + O(h^3) \end{array}$$

$$f''(x_i)h^2 = f(x_{i-2}) - 2f(x_{i-1}) + f(x_i) + O(h^3)$$

$$f''(x_i) = \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{h^2} + O(h) \; //$$

Segunda derivada centrada

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2} + O(h^2)$$

① $f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + \frac{f'''(x_i)}{3!}h^3 + \frac{f^{(4)}(x_i)}{4!}h^4 + O(h^5)$

② $f(x_{i-1}) = f(x_i) - f'(x_i)h + \frac{f''(x_i)}{2!}h^2 - \frac{f'''(x_i)}{3!}h^3 + \frac{f^{(4)}(x_i)}{4!} + O(h^5)$

Sumamos ① y ②

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \frac{f'''(x_i)}{3!}h^3 + \frac{f^{(4)}(x_i)}{4!}h^4 + O(h^5)$$

$+$

$$f(x_{i-1}) = f(x_i) - f'(x_i)h + \frac{f''(x_i)}{2!}h^2 - \frac{f'''(x_i)}{3!}h^3 + \frac{f^{(4)}(x_i)}{4!}h^4 + O(h^5)$$

$$f(x_{i+1}) + f(x_{i-1}) = 2f(x_i) + f''(x_i)h^2 + \frac{f^{(4)}(x_i)}{12}h^4 + O(h^5)$$

$$f''(x_i)h^2 = f(x_{i+1}) + f(x_{i-1}) - 2f(x_i) + \frac{f^{(4)}(x_i)}{12}h^4 + O(h^5)$$

$$f''(x_i) = \frac{f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)}{h^2} + \frac{f^{(4)}(x_i)}{12}h^2 + O(h^3)$$

$$f''(x_i) = \frac{f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)}{h^2} + O(h^2)$$

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Forma alternativa

$$f''(x_i) \cong \frac{\frac{f(x_{i+1}) - f(x_i)}{h} - \frac{f(x_i) - f(x_{i-1})}{h}}{h}$$

$$\frac{f(x_{i+1}) - f(x_i)}{h} - \frac{f(x_i) - f(x_{i-1})}{h}$$

Simplificamos

$$\frac{1}{h}\left[f(x_{i+1}) - f(x_i) - f(x_i) - f(x_{i-1})\right] = \frac{1}{h}\left[f(x_{i+1}) - 2f(x_i) + f(x_{i-1})\right]$$

Dividimos entre h

$$\frac{1}{h^2}\left[f(x_{i+1}) - 2f(x_i) + f(x_{i-1})\right]$$

Segunda derivada (Tasa de cambio de la derivada)

$$f''(x_i) \simeq \frac{f'(x_{i+1/2}) - f'(x_{i-1/2})}{h}$$

$$f''(x_i) \simeq \frac{\frac{f(x_{i+1}) - f(x_i)}{h} - \frac{f(x_i) - f(x_{i-1})}{h}}{h}$$