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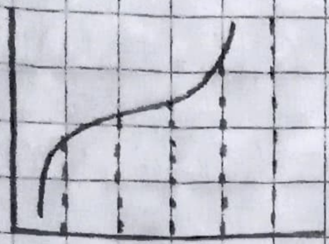
Segunda derivada con solo 2 puntos

Diferencia Finita hacia atrás



$$F(x_{i-1}) = F(x_i) - F'(x_i)\overbrace{(x_i - x_{i-1})}^{h} + F''(x_i)\frac{\overbrace{(x_i - x_{i-1})^2}^{h}}{2!} + O(\sim h)^3$$

$$F(x_{i-2}) = F(x_i) - F'(x_i)\overbrace{(x_i - x_{i-2})}^{2h} + F''(x_i)\frac{(x_i - x_{i-2})^2}{2!} + O(\sim h)^3$$

Multiplicando la primera ecuación por 2 para eliminar términos

$$-2F(x_{i-1}) = -2F(x_i) + \cancel{2F'(x_i)h} - 2F''(x_i)\frac{h^2}{2!} - O(\sim h^3)$$

$+$

$$F(x_{i-2}) = \cancel{F(x_i)} - \cancel{F'(x_i)(2h)} + F''(x_i)\frac{(2h)^2}{2!} + O(\sim h^3)$$

$$F(x_{i-2}) - 2F'(x_{i-1}) = -F(x_i) + \cancel{2}F''(x_i)\frac{h^2}{\cancel{2!}} + \frac{O(h^3)}{h^2} \to O(h)$$

$$F''(x_i) = \frac{F(x_{i-2}) - 2F(x_{i-1}) + F(x_i)}{h^2} + O(h)$$

Diferencia Finita centrada

Sumando las ecuaciones finitas hacia atrás y adelante

$$F(x_{i+1}) = F(x_i) + \cancel{F'(x_i)(x_{i+1} - x_i)} + F''(x_i)\frac{(x_{i+1} - x_i)^2}{2!} + O(x_{i+1} - x_i)^3$$

$+$

$$F(x_{i-1}) = F(x_i) \cancel{- F'(x_i)(x_i - x_{i-1})} + F''(x_i)\frac{(x_i - x_{i-1})^2}{2!} - O(x_i - x_{i-1})^3$$

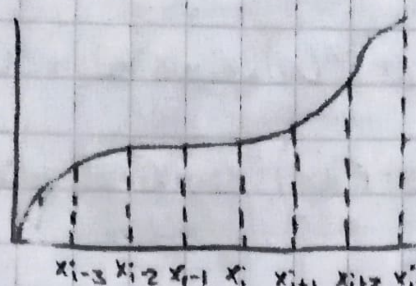
$$F(x_{i+1}) + F(x_{i-1}) = 2F(x_i) + 2F''(x_i)\frac{(h)^2}{2!} + \frac{O(h^3)}{\cancel{h}}$$

$$F''(x_i) = \frac{F(x_{i+1}) + F(x_{i-1}) - 2F(x_i)}{\cancel{2}\left(\frac{h^2}{\cancel{2!}}\right)} + O(h^2)$$

$$F''(x_i) = \frac{F(x_{i+1}) + F(x_{i-1}) - 2F(x_i)}{h^2} + O(h^2)$$

Tercera Derivada con solo dos puntos

Diferencia Finita hacía adelante



1) $$F'(x_{i+1}) = F(x_i) + F''(x_i)h + \frac{F''(x_i)h^2}{2!} + \frac{F^{(3)}(x_i)h^3}{3!}$$

2) $$F(x_{i+2}) = F'(x_i) + F(x_i)(2h) + \frac{F''(x_i)(2h)^2}{2!} + \frac{F^{(3)}(x_i)(2h)^3}{3!}$$

$$F(x_{i+3}) = F(x_i) + F(x_i)(3h) + \frac{F''(x_i)(3h)^2}{2!} + \frac{F^{(3)}(x_i)(3h)^3}{3!} \quad (3)$$

Multiplicando la ecuación 1 por 3 y la 2 por -3 y sumando las 3 ecuaciones

$$F(x_{i+3}) - 3F(x_{i+2}) + 3F(x_{i+1}) =$$

$$F(x_i) + F'(x_i)(3h) + \frac{F''(x_i)(3h)^2}{2!} + \frac{F^{(3)}(x_i)(3h)^3}{3!} + 3F(x_i) - 3F'(x_i)(2h) -$$

$$\frac{3F''(x_i)(2h)^2}{2!} - \frac{3F^{(3)}(x_i)(2h)^3}{3!} - 3F(x_i) + 3F'(x_i)(h) + \frac{3F''(x_i)(h)^2}{2!} +$$

$$\frac{3F^{(3)}(x_i)(h)^3}{3!}$$

Eliminando terminos

$$F(x_{i+3}) - 3F(x_{i+2}) + 3F(x_{i+1}) = F(x_i) - \cancel{3F(x_i)} + \cancel{3F(x_i)}$$

$$\cancel{F'(x_i)(3h)} - \cancel{3F'(x_i)(2h)} + \cancel{3F'(x_i)(h)}$$

$$\underbrace{\frac{F''(x_i)(3h)^2}{\cancel{2!}} - \frac{3F''(x_i)(2h)^2}{\cancel{2!}} + \frac{3F''(x_i)(h)^2}{\cancel{2!}}}$$

$$\cancel{F''(x_i)(h)^2} - \cancel{F''(x_i)(2h)^2} + \cancel{F''(x_i)(h)^2}$$

$3! = 6$
$2! = 2$

$$\underbrace{\frac{F^{(3)}(x_i)(3h)^2}{\cancel{3!}} - \frac{3F^{(3)}(x_i)(2h)^2}{\cancel{3!}} + \frac{3F^{(3)}(x_i)(h)^2}{\cancel{3!}}}$$

$$\frac{F^{(3)}(x_i)(h)^3}{\cancel{2}} - \frac{F^{(3)}(x_i)(\cancel{2}h)^3}{\cancel{2}} + \frac{F^{(3)}(x_i)\cancel{(h)^3}}{\cancel{2}}$$

Despejando $F^{(3)}(x_i)$

$$F^{(3)}(x_i) = \frac{F(x_{i+3}) - 3F(x_{i+2}) + 3F(x_{i+1}) - F(x_i)}{h^3}$$