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Ejemplo 1

$x_{i+1} = 1$

$x_i = 0$

$\xi = 0.5$

$$F(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$$

Predecir el valor en $x=1$ con $h=1$ usando la serie de Taylor de orden 0 hasta 4, y calculando el residuo en cada caso

$$F(x_{i+1}) = F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!} + \frac{F^{(3)}(x_i)h^3}{3!} \dots$$

$$R_n = \frac{F^{(n+1)}(x_i)h^{n+1}}{(n+1)!}$$

n	$F(x=1)$	R_n	$E = F(1) - \text{aprox}$
0	$F(x_i) = F(0) = \cancel{-0.1(0)^4} \cancel{-0.15(0)^3} \cancel{-0.5(0)^2} \cancel{-0.25(0)} + 1.2$ $F(0) = 1.2$	$F'(\xi) = F'(0.5) = -0.91$	-1
1	$F(x_i) + F'(x_i)h = F(0) + F'(0)h$ $1.2 + (\cancel{-0.4(0)^3} \cancel{-0.45(0)^2} \cancel{-0} - 0.25)$ $1.2 + (-0.25) = 0.95$	$F^2(\xi) = \frac{F''(0.5)}{2!} = -0.87$	-0.75
2	$F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!} = 1.2 - 0.25 + (-1.2x^2 - 0.9x - 1)\frac{1}{2!}$ $1.2 - 0.25 + (-1\cancel{.2(0)^2} \cancel{-0.9(0)} - 1)\left(\frac{1}{2!}\right)$ $1.2 - 0.25 + (-0.5) = 0.45$	$F^3(\xi) = \frac{F'''(0.5)}{3!} = -0.35$	-0.25
3	$F(x_i) + F(x_i)h + \frac{F''(x_i)h^2}{2!} + \frac{F'''(x_i)h^3}{3!}$ $= 1.2 - 0.25 - 0.5 - 2\cancel{.4(0)} - 0.9\left(\frac{1}{3!}\right) = 0.3$	$F^4(\xi) = \frac{F''''(0.5)}{4!} = -0.1$	-0.1
4	$F(x_i) + F(x_i)h + \frac{F''(x_i)h^2}{2!} + \frac{F'''(x_i)h^3}{3!} + \frac{F''''(x_i)h^4}{4!}$ $= 1.2 - 0.25 - 0.5 - 0.15 - 2.4\left(\frac{1}{4!}\right) = 0.2$	$F''(\xi) = \frac{F'''''(0.5)}{5!} = 0$	0

$$F'(x) = -0.4x^3 - 0.45x^2 - x - 0.25$$

$$F''(x) = -1.2x^2 - 0.9x - 1$$

$$F'''(x) = -2.4x - 0.9$$

$$F''''(x) = -2.4$$

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Ejemplo 2

$F(x) = \cos(x)$

Predecir el valor en $x = \pi/3$ usando la serie de Taylor entorno a $x = \pi/4$, de orden cero hasta 4 y calculando el residuo en cada caso.

$$F(x_{i+1}) = F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!} + \frac{F^{(3)}(x_i)h^3}{3!} \ldots$$

$$R_n = \frac{F^{(n+1)}(x_i)\xi^{n+1}}{(n+1)!} \qquad x_{i+1} = \pi/3 \quad x_i = \pi/4 \qquad h = \pi/12 \quad \xi = \frac{7\pi}{24}$$

n	$F(\pi/3)$	R_n	$E = F(\pi/3) - \text{aprox}$
0	$F(x_i) = F(\pi/4) = \cos\left(\frac{\pi}{4}\right) = 0.71$	$F'(\xi) = -\text{sen}\left(\frac{7\pi}{24}\right)\left(\frac{\pi}{12}\right) = -0.21$	-0.21
1	$F(x_i) + F'(x_i)h = 0.71 + (-\text{sen}\,\frac{\pi}{4})(\pi/12)$ $0.71 + (-0.19) = 0.52$	$F''(\xi) = \frac{-\cos\left(\frac{7\pi}{24}\right)\left(\frac{\pi}{12}\right)^2}{2!} = -0.021$	-0.02
2	$F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!}$ $0.71 - 0.19 + (-\cos\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right)^2/2! = 0.4977$	$F'''(\xi) = \text{sen}\left(\frac{7\pi}{24}\right)\left(\frac{\pi}{12}\right)^3/3! = 2.37 \times 10^{-3}$	0.0023
3	$F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!} + \frac{F'''(x_i)h^3}{3!}$ $0.71 - 0.19 - 0.024 + (\text{sen}\,\pi/4)(\pi/12)^3/3!$ $0.71 - 0.19 - 0.024 + 2.11 \times 10^{-3} = 0.498$	$F''''(\xi) = \cos\left(\frac{7\pi}{24}\right)\left(\frac{\pi}{12}\right)^4/4! = 1.19 \times 10^{-4}$	0.0002
4	$F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!} + \frac{F'''(x_i)h^3}{3!} + \frac{F''''(x_i)h^4}{4!}$ $0.71 - 0.19 - 0.024 + 2.11 \times 10^{-3} + (1.38 \times 10^{-4})$ $= 0.50$	$F'''''(\xi) = \frac{-\text{sen}\left(\frac{7\pi}{24}\right)\left(\frac{\pi}{12}\right)^5}{5!} = -8.13 \times 10^{-6}$	0

$F'(x) = -\text{sen}(x)$

$F''(x) = -\cos(x)$

$F'''(x) = \text{sen}(x)$

$F''''(x) = \cos(x)$

$F'''''(x) = -\text{sen}(x)$