

$$\begin{aligned} 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 &= 71.4 \end{aligned}$$

Resolver usando Gauss-Jordan, Factorización LV y Gauss-Seidel

Gauss-Jordan

$$x_1 = 3$$

$$x_2 = -2.5$$

$$x_3 = 7$$

$$A = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \quad b = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Factorización LV

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0.1/3 & 1 & 0 \\ 0.3/3 & -0.19/2.003 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.003 & -2.99 \\ 0 & -0.2 & 10 \end{bmatrix} \quad L_{21} = \frac{0.1}{3} \quad L_{31} = \frac{0.3}{3}$$

$$L_{32} = \frac{-0.19}{7.003}$$

$$\begin{bmatrix} 0.1 & 7 & -0.3 \\ 0.1 & -3.33 \times 10^{-3} & -6.66 \times 10^{-7} \\ 0 & 7.003 & -0.299 \end{bmatrix}$$

$$U = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.003 & -0.299 \\ 0 & -0.19 & 10.02 \end{bmatrix}$$

$$\begin{bmatrix} 0.3 & -0.2 & 10 \\ 0.3 & -0.01 & -0.02 \\ 0 & -0.19 & 10.02 \end{bmatrix}$$

$$U = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.003 & -0.299 \\ 0 & -0.19 & 10.02 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -0.19 & 10.02 \\ 0 & -0.19 & 8.11 \times 10^{-3} \\ 0 & 0 & 10.01 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -0.19 & 8.11 \times 10^{-3} \\ 0 & 0 & 10.01 \end{bmatrix}$$

$$\Rightarrow U = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.003 & -0.299 \\ 0 & 0 & 10.01 \end{bmatrix}$$

$$LU = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix}$$

$$a_{11} = (3)(1) + (0)(0) + (0)(0) = 3$$

$$a_{12} = (1)(-0.1) + (0)(7.003) + (0)(0) = -0.1$$

$$a_{13} = (1)(-0.2) + (0)(-0.299) + (0)(10.01) = -0.2$$

$$a_{21} = \frac{(0.1)}{3}(3) + (1)(0) + (0)(0) = 0.1$$

$$a_{22} = \frac{(0.1)}{3}(-0.1) + (1)(7.003) + (0)(0) = 6.99 \approx 7$$

$$a_{23} = \frac{(0.1)}{3}(-0.2) + (1)(-0.299) + (0)(10.01) = -0.3$$

$$a_{31} = \frac{(0.3)}{3}(3) + \frac{(-0.19)}{7.003}(0) + (1)(0) = 0.3$$

$$a_{32} = \frac{(0.3)}{3}(-0.1) + \frac{(-0.19)}{7.003}(7.003) + (1)(0) = -0.2$$

$$a_{33} = \frac{(0.3)}{3}(-0.2) + \frac{(-0.19)}{7.003}(-0.299) + (1)(10.01) = 9.99 \approx 10$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.1/3 & 1 & 0 \\ 0.3/3 & -0.1/7.003 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

$$y_1 = b_1 = 7.85$$

$$y_2 = b_2 - L_{21}y_1 = -19.3 - \frac{(0.1)}{3}(7.85) = -19.56$$

$$x_3 = \frac{y_3}{U_{33}} \Rightarrow x_3 = \frac{70.08}{10.01} = 7.00$$

$$y_3 = b_3 - L_{31}y_1 - L_{32}y_2 = 71.4 - (0.3)(7.85) - \frac{(-0.19)}{7.003}(-19.56)$$

$$x_2 = \frac{y_2 - U_{23}x_3}{-U_{22}} \Rightarrow x_2 = \frac{-19.56 - (-0.299)(7)}{7.003} = 70.08$$

$$x_2 = -2.49$$

$$x_1 = \frac{y_1 + U_{12}x_2 + U_{13}x_3}{U_{11}} \Rightarrow x_1 = \frac{7.85 + (-0.1)(-2.49) + (-0.2)(7)}{3} = 3.00$$

Gauss-Seidel

$$x_1^{k=1} = -\frac{(a_{12}x_2^0 + a_{13}x_3^0) + b_1}{a_{11}} = -\frac{((-0.1)(0) + (-0.2)(0)) + 7.85}{3} = \frac{157}{60} = 2.61$$

$$x_2^{k=1} = -\frac{(a_{21}x_1^0 + a_{23}x_3^0) + b_2}{a_{22}} = -\frac{((0.1)(2.61) + (-0.3)(0)) + (-19.3)}{7} = -2.79$$

$$x_3^{k=1} = -\frac{(a_{31}x_1^0 + a_{32}x_2^0) + b_3}{a_{33}} = -\frac{((0.3)(2.61) + (-0.2)(-2.79)) + 71.4}{10} = 7.005$$

$$x_1^{k=2} = -\frac{(a_{12}x_2^0 + a_{13}x_3^0) + b_1}{a_{11}} = -\frac{((-0.1)(-2.79) + (-0.2)(7.005)) + 7.85}{3} = 2.99$$

$$x_2^{k=2} = -\frac{(a_{21}x_1^0 + a_{23}x_3^0) + b_2}{a_{22}} = -\frac{((0.1)(2.99) + (-0.3)(7.005)) + (-19.3)}{7} = -2.5$$

$$x_3^{k=2} = -\frac{(a_{31}x_1^0 + a_{32}x_2^0) + b_3}{a_{33}} = -\frac{((0.3)(2.99) + (-0.2)(-2.49)) + 71.4}{10} = 7.000$$

$$x_1^{k=3} = -\frac{(a_{12}x_2^0 + a_{13}x_3^0) + b_1}{a_{11}} = -\frac{((-0.1)(-2.49) + (-0.2)(7.18)) + 7.85}{3} = 3.01$$

$$x_2^{k=3} = -\frac{(a_{21}x_1^0 + a_{23}x_3^0) + b_2}{a_{22}} = -\frac{((0.1)(3.01) + (-0.3)(7.18)) + (-19.3)}{7} = -2.5$$

$$x_3^{k=3} = -\frac{(a_{31}x_1^0 + a_{32}x_2^0) + b_3}{a_{33}} = -\frac{((0.3)(3.01) + (-0.2)(-2.49)) + 71.4}{10} = 6.99 \approx 7$$