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Segunda Derivada finita hacia atrás

① $f(x_{i-1}) = f(x_i) + f'(x_i)(h) + \frac{f''(x_i)h^2}{2!}$

② $f(x_{i-2}) = f(x_i) + f'(x_i)(2h) + \frac{f''(x_i)}{2!}(2h)^2$

Multiplicamos ① (-2)

$-2f(x_{i-1}) = -2f(x_i) \cancel{-2f'(x_i)(h)} - \frac{2f''(x_i)h^2}{2}$

$+$

$f(x_{i-2}) = f(x_i) \cancel{+f'(x_i)(2h)} + \frac{f''(x_i)(2h)^2}{2!}$

$f(x_{i-2}) - 2f(x_{i-1}) = -f(x_i) + \frac{2f''(x_i)(h^2)}{2!}$

Despejamos $f''(x_i)$

$$\boxed{f''(x_i) = \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{h^2}}$$ Segunda Derivada hacia atrás

Segunda derivada centrada

① $f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + \frac{f'''(x_i)h^3}{3!}$

+

② $f(x_{i-1}) = f(x_i) - f'(x_i)h + \frac{f''(x_i)h^2}{2!} - \frac{f'''(x)h^3}{3!}$

$f(x_{i+1}) + f(x_{i-1}) = 2f(x_i) + \cancel{2}\frac{f''(x_i)h^2}{\cancel{2}}$

$$\boxed{f''(x_i) = \frac{f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)}{h^2}}$$

Segunda derivada centrada.

Demostracion de la tercera derivada

Se trunca la serie de Taylor hasta $f'''(x)$ para x_{i+1}, x_{i+2} y x_{i+3}

① $f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x)(x_{i+1}-x_i)^2}{2!} + \frac{f'''(x)(x_{i+1}-x_i)^3}{3!}$

② $f(x_{i+2}) = f(x_i) + f'(x_i)(x_{i+2} - x_i) + \frac{f''(x_i)(x_{i+2}-x_i)^2}{2!} + \frac{f'''(x_i)(x_{i+2}-x_i)^3}{}$

③ $f(x_{i+3}) = f(x_i) + f'(x_i)(x_{i+3} - x_i) + \frac{f''(x_i)(x_{i+3}-x_i)^2}{2!} + \frac{f'''(x_i)(x_{i+3}-x_i)^3}{3!}$

$x_{i+1} - x_i = h$

$x_{i+2} - x_i = 2h$

$x_{i+3} - x_i = 3h$

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Reescribimos

① $f(x_i+1) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + \frac{f'''(x_i)h^3}{3!} + O(h)$

② $f(x_i+2) = f(x_i) + 2f'(x_i)h + \frac{4f''(x_i)h^2}{2!} + \frac{8f'''(x_i)h^3}{3!} + O(h)$

③ $f(x_i+3) = f(x_i) + 3f'(x_i)h + \frac{9f''(x_i)h^2}{2!} + \frac{27f'''(x_i)h^3}{3!} + O(h)$

Sumamos ec. 1 y ec 3 y multiplicamos por -2 la ec. 2.

④ $f(x_i+3) + f(x_i+1) = 2f(x_i) + 4f'(x_i)h + \frac{10f''(x_i)h^2}{2!} + \frac{28f'''(x_i)h^3}{3!}$

⑤ $-2f(x_i+2) = -2f(x_i) - 4f'(x_i)h - \frac{8f''(x_i)h^2}{2!} - \frac{16f'''(x_i)h^3}{3!}$

Sumamos ec. 4 y 5.

$f(x_i+3) + f(x_i+1) - 2f(x_i+2) = \frac{2f''(x_i)h^2}{2!} + \frac{12f'''(x_i)h^3}{3!}$

$f(x_i+3) + f(x_i+1) - 2f(x_i+2) = f''(x_i)h^2 + 2f'''(x_i)h^3$

Se conoce $f''(x_i)h^2 = f(x_i+2) - 2f(x_i+1) + f(x_i)$

Sustituimos

$f(x_i+3) + f(x_i+1) - 2f(x_i+2) = f(x_i+2) - 2f(x_i+1) + f(x_i) + 2f'''(x_i)h^3$

Despejamos $f'''(x_i)$

$2f'''(x_i)h^3 = f(x_i+3) + f(x_i+1) - 2f(x_i+2) - f(x_i+2) + 2f(x_i+1) - f(x_i)$

$$2f'''(x_i)h^3 = f(x_{i+3}) + 3f(x_{i+1}) - 3f(x_{i+2}) - f(x_i)$$

$$\boxed{f'''(x_i) = \frac{f(x_{i+3}) + 3f(x_{i+1}) - 3f(x_{i+2}) - f(x_i)}{2h^3}}$$