$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$
$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$
$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$

Usando Gauss-Jordan

$x_1 = 3.032100$
$x_2 = -2.480246$
$x_3 = 7.000375$

Usando código

Usando factorização LU

$$A = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \quad b = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$l_{21} = \frac{a_{21}}{a_{11}} = \cancel{\frac{0.1}{3}} = \cancel{\frac{1}{30}} = \frac{0.1}{3} = \frac{1}{30}$$

$$l_{31} = \frac{a_{31}}{a_{11}} = \frac{0.3}{3} = \frac{1}{10}$$

$$\cancel{L = \begin{bmatrix} 1 & 0 & 0 \\ 1/30 & 1 & 0 \\ & & 1 \end{bmatrix}}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/30 & 1 & 0 \\ 1/10 & \frac{-0.19}{7.0033} & 1 \end{bmatrix}$$

$$l_{32} = \frac{a_{32}}{a_{22}} = \frac{-0.19}{7.0033} = -0.02713$$

$$U = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.0033 & -22/75 \\ 0 & -0.19 & 10.02 \end{bmatrix}$$

$$\Rightarrow U = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.0033 & -22/75 \\ 0 & 0 & 10.0120 \end{bmatrix}$$

$$LU = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/30 & 1 & 0 \\ 1/10 & \frac{-0.19}{7.0033} & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ X_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.0033 & -22/75 \\ 0 & 0 & 10.0120 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.56 \\ 70.084 \end{bmatrix}$$

$$X_3 = \frac{70.0843}{10.0120} \cong 7.0000$$

$$X_2 = \left[-19.56 - \left(\frac{-22}{75} \right)(7.0000) \right] / 7.0033 \cong -2.4998$$

$$X_1 = \left[\cancel{70.084}\ 7.85 - (-0.2)(7.0000) - (-0.1)(-2.4998) \right] / 3$$

$$X_1 \cong 3.0000$$

Usando Gauss - Seidel

$X_1^0 = 0$

$X_2^0 = 0$

$X_3^0 = 0$

$$X_1^{(1)} = \frac{-\sum_{2 \le j \le 3} a_{1j} X_j^{(0)} + b_1}{a_{11}} = \frac{-(a_{12} X_2^0 + a_{13} X_3^0) + b_1}{a_{11}}$$

$$x_1^{(1)} = \frac{-[(-0.1)(0) + (-0.2)(0)] + 7.85}{3} = 2.6167$$

$$x_2^{(1)} = \frac{-\sum_{1\le j\le 1} a_{2j} x_j^{(1)} - \sum_{3\le j\le 3} a_{2j} x_j^{(0)} + b_2}{a_{22}} =$$

$$x_2^{(1)} = \frac{-(a_{21} x_1^{(1)}) - (a_{23} x_3^{(0)}) + b_2}{a_{22}} = \frac{-[(0.1)(2.6167)] - [(-0.3)(0)] + (-19.3)}{7}$$

$$x_2^{(1)} = -2.7945$$

$$x_3^{(1)} = \frac{-\sum_{1\le j\le 2} a_{3j} x_j^{(1)} + b_3}{a_{33}} = \frac{-[a_{31} x_1^{(1)} + a_{32} x_2^{(1)}] + b_3}{a_{33}}$$

$$x_3^{(1)} = \frac{-[(0.3)(2.6167) + (-0.2)(-2.7945)] + 71.4}{10} = 7.0056$$

$$x_1^{(2)} = \frac{-\sum_{2\le j\le 3} a_{1j} x_j^{(1)} + b_1}{a_{11}} = \frac{-(a_{12} x_2^{(1)} + a_{13} x_3^{(1)}) + b_1}{a_{11}}$$

$$x_1^{(2)} = \frac{-[(-0.1)(-2.7945) + (-0.2)(7.0056)] + 7.85}{3} = 3.1769$$

$$x_2^{(2)} = \frac{-\sum_{1\le j\le 1} a_{2j} x_j^{(2)} - \sum_{3\le j\le 3} a_{2j} x_j^{(1)} + b_2}{a_{22}} =$$

$$x_2^{(2)} = \frac{-(a_{21} x_1^{(2)}) - (a_{23} x_3^{(1)}) + b_2}{a_{22}}$$

$$x_2^{(2)} = \frac{-[(0.1)(3.1769)] - [(-0.3)(7.0056)] + (-19.3)}{7} = -2.5023$$

$$X_3^{(2)} = \frac{-\sum_{1 \le j \le 2} a_{3j} X_j^{(2)} + b_3}{a_{33}} = \frac{-[a_{31} X_1^{(2)} + a_{32} X_2^{(2)}] + b_3}{a_{33}}$$

$$X_3^{(2)} = \frac{-[(0.3)(3.1769) + (-0.2)(-2.5023)] + 71.4}{10} = \underline{6.9946}$$

$$X_1^{(3)} = \frac{-\sum_{2 \le j \le 3} a_{1j} X_j^{(2)} + b_1}{a_{11}} = \frac{-(a_{12} X_2^{(2)} + a_{13} X_3^{(2)}) + b_1}{a_{11}}$$

$$X_1^{(3)} = \frac{-[(-0.1)(-2.5023) + (-0.2)(6.9946)] + 7.85}{3} = \underline{2.9996}$$

$$X_2^{(3)} = \frac{-\sum_{1 \le j \le 1} a_{2j} X_j^{(3)} - \sum_{3 \le j \le 3} a_{2j} X_j^{(2)} + b_2}{a_{22}}$$

$$X_2^{(3)} = \frac{-(a_{21} X_1^{(3)}) - (a_{23} X_3^{(2)}) + b_2}{a_{22}}$$

$$X_2^{(3)} = \frac{-[(0.1)(2.9996)] - [(-0.3)(6.9946)] + (-19.3)}{7} = \underline{-2.5002}$$

$$X_3^{(3)} = \frac{-\sum_{1 \le j \le 2} a_{3j} X_j^{(3)} + b_3}{a_{33}} = \frac{-[(0.3)(2.9996) + (-0.2)(-2.5002)] + 71.4}{10}$$

$$X_3^{(3)} = \underline{7.0000}$$

$$X_1^{(4)} = \frac{-\sum_{2 \le j \le 3} a_{1j} X_j^{(3)} + b_1}{a_{11}} = \frac{-[(-0.1)(-2.5002) + (-0.2)(7.0000)] + 7.85}{3}$$

$$X_1^{(4)} = \underline{3.0000}$$

$$X_2^{(4)} = \frac{-\sum_{1 \le j \le 1} a_{2j} X_j^{(4)} - \sum_{3 \le j \le 3} a_{2j} X_j^{(3)} + b_2}{a_{22}}$$

$$x_2^{(4)} = \frac{-[(0.1)(3.0000)] - [(-0.3)(7.0000)] + (-19.3)}{7} = \underline{-2.5000}$$

$$x_3^{(4)} = \frac{-\sum_{1 \le j \le 2} a_{3j} x_j^{(4)} + b_3}{a_{33}} = \frac{-[(0.3)(3.0000) + (-0.2)(-2.5000)] + 71.4}{10}$$

$$x_3^{(4)} = \underline{7.0000}$$

El siguiente sistema de ecuaciones es utilizado para determinar concentraciones (1)
(c_i, en g/m^3) en una serie de reactores acoplados, como función de la cantidad de masa (g/día)
que entra a cada uno de ellos.

$$15c_1 - 3c_2 - c_3 = 3800$$
$$-3c_1 + 18c_2 - 6c_3 = 1200$$
$$-4c_1 - c_2 + 12c_3 = 2350$$

1 - Encontrar c_1, c_2 y c_3

Usando Gauss - Jordan

$$\left.\begin{aligned} c_1 &= 284.6566 \\ c_2 &= 135.3629 \\ c_3 &= 307.4136 \end{aligned}\right\}$$ Usando el código

Usando LU

$$A = \begin{bmatrix} 15 & -3 & -1 \\ -3 & 18 & -6 \\ -4 & -1 & 12 \end{bmatrix} \quad b = \begin{bmatrix} 3800 \\ 1200 \\ 2350 \end{bmatrix} \quad C = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3/15 & 1 & 0 \\ \frac{-4}{15} & \frac{-1.8}{17.4} & 1 \end{bmatrix}$$

$$l_{21} = \frac{a_{21}}{a_{11}} = \frac{-3}{15}$$
$$l_{31} = \frac{a_{31}}{a_{11}} = \frac{-4}{15}$$
$$l_{32} = \frac{a_{32}}{a_{22}} = \frac{-1.8}{17.4}$$

$$U = \begin{bmatrix} 15 & -3 & -1 \\ 0 & 17.4 & -6.2 \\ 0 & -1.8 & 176/15 \end{bmatrix} \Rightarrow \begin{bmatrix} 15 & -3 & -1 \\ 0 & 17.4 & -6.2 \\ 0 & 0 & \cancel{1715.51} \\ & & 11.09 \end{bmatrix}$$

$$LU = \begin{bmatrix} 15 & -3 & -1 \\ -3 & 18 & -6 \\ -4 & -1 & 12 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -3/15 & 1 & 0 \\ -4/15 & \frac{-1.8}{17.4} & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 3800 \\ 1200 \\ 2350 \end{bmatrix}$$

$$\begin{bmatrix} 15 & -3 & -1 \\ 0 & 17.4 & -6.2 \\ 0 & 0 & 11.91 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 3800 \\ 1960 \\ 3566.09 \end{bmatrix}$$

$$C_3 = 299.42$$

$$C_2 = \frac{1960 - (-6.2)(299.42)}{17.4} = 219.33$$

$$C_1 = \frac{3800 - (-1)(299.42) - (-3)(219.33)}{15} = 317.16$$

Usando Gauss - Seidel

$\cancel{X}_1^{(0)} = 200$

$C_2^{(0)} = 200$

$C_3^{(0)} = 200$

$$C_1^{(1)} = \frac{-\sum_{2\le j\le 3} a_{1j} C_j^{(0)} + b_1}{a_{11}} = \frac{-[(-3)(200) + (-1)(200)] + 3800}{15} = 306.67$$

$$C_2^{(1)} = \frac{-\sum_{1\le j\le 1} a_{2j} C_j^{(1)} - \sum_{3\le j\le 3} a_{2j} C_j^{(0)} + b_2}{a_{22}} = \frac{-[(-3)(306.67)] - [(-6)(200)] + 1200}{18}$$

$$C_2^{(1)} = 184.44$$

$$C_3^{(1)} = \frac{-\sum_{1\le j\le 2} a_{3j} C_j^{(1)} + b_3}{a_{33}} = \frac{-[(-4)(306.67) + (-1)(184.44)] + 2350}{12} = 313.43$$

$$C_1^{(2)} = \frac{-\sum_{2\le j\le 3} a_{1j} C_j^{(1)} + b_1}{a_{11}} = \frac{-[(-3)(184.44) + (-1)(313.43)] + 3800}{15} = 311.12$$

$$C_2^{(2)} = \frac{-\sum_{1\le j\le 1} a_{2j} C_j^{(2)} - \sum_{3\le j\le 3} a_{2j} C_j^{(1)} + b_2}{a_{22}} = \frac{-[(-3)(311.12)] - [(-6)(313.43)] + 1200}{18}$$

$C_2^{(2)} = 223.00$

$$C_3^{(2)} = \frac{-\sum_{1 \le j \le 2} a_{3j} C_j^{(2)} + b_3}{a_{33}} = \frac{-[(-4)(311.12) + (-1)(223.00)] + 2350}{12} = 318.12$$

$$C_1^{(3)} = \frac{-\sum_{2 \le j \le 3} a_{1j} C_j^{(2)} + b_1}{a_{11}} = \frac{-[(-3)(223) + (-1)(318.12)] + 3800}{15} = 319.14$$

$$C_2^{(3)} = \frac{-\sum_{1 \le j \le 1} a_{2j} C_j^{(3)} - \sum_{3 \le j \le 3} a_{2j} C_j^{(2)} + b_2}{a_{22}} = \frac{-[(-3)(319.14)] - [(-6)(318.12)] + 1200}{18}$$

$C_2^{(3)} = 225.90$

$$C_3^{(3)} = \frac{-\sum_{1 \le j \le 2} a_{3j} C_j^{(3)} + b_3}{a_{33}} = \frac{-[(-4)(319.14) + (-1)(225.90)] + 2350}{12} = 321.04$$

$$C_1^{(4)} = \frac{-\sum_{2 \le j \le 3} a_{1j} C_j^{(3)} + b_1}{a_{11}} = \frac{-[(-3)(225.9) + (-1)(321.04)] + 3800}{15} = 319.92$$

$$C_2^{(4)} = \frac{-\sum_{1 \le j \le 1} a_{2j} C_j^{(4)} - \sum_{3 \le j \le 3} a_{2j} C_j^{(3)} + b_2}{a_{22}} = \frac{-[(-3)(319.92)] - [(-6)(321.04)] + 1200}{18}$$

$C_2^{(4)} = 227$

$$C_3^{(4)} = \frac{-\sum_{1 \le j \le 2} a_{3j} C_j^{(4)} + b_3}{a_{33}} = \frac{-[(-4)(319.92) + (-1)(227.00)] + 2350}{12} = 321.39$$

$$C_1^{(5)} = \frac{-\sum_{2 \le j \le 3} a_{1j} C_j^{(4)} + b_1}{a_{11}} = \frac{-[(-3)(227) + (-1)(321.39)] + 3800}{15} = 320.16$$

$$C_2^{(5)} = \frac{-\sum_{1 \le j \le 1} a_{2j} C_j^{(5)} - \sum_{3 \le j \le 3} a_{2j} C_j^{(4)} + b_2}{a_{22}} = \frac{-[(-3)(320.16)] - [(-6)(321.39)] + 1200}{18} = 227.16$$

$$C_3^{(5)} = \frac{-\sum_{1 \le j \le 2} a_{3j} C_j^{(5)} + b_3}{a_{33}} = \frac{-[(-4)(320.16) + (-1)(227.16)] + 2350}{12} = 321.48$$

2- ¿Cuánto se reduce la concentración del reactor 3 si la tasa de masa de entrada a los reactores 1 y 2 se reducen en 500 y 200 g/día?

$$15c_1 - 3c_2 - c_3 = \cancel{3800}\ 500$$
$$-3c_1 + 18c_2 - 6c_3 = 200$$
$$-4c_1 - c_2 + 12c_3 = 2350$$

Usando Gauss-Jordan

$$\left.\begin{array}{l} c_1 = 52.7120 \\ c_2 = 58.3949 \\ c_3 = 220.6060 \end{array}\right\} \text{Usando código}$$

Usando LU (sólo cambiamos b)

$$\begin{bmatrix} 1 & 0 & 0 \\ -3/15 & 1 & 0 \\ -4/15 & \frac{-1.8}{17.4} & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 500 \\ 200 \\ 2350 \end{bmatrix}$$

$$\begin{bmatrix} 15 & -3 & -1 \\ 0 & 17.4 & -6.2 \\ 0 & 0 & 11.91 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 500 \\ 300 \\ 2514.37 \end{bmatrix}$$

$$c_3 = \frac{2514.37}{11.91} = 211.11$$

$$c_2 = \frac{300 - (-6.2)(211.11)}{17.4} = 92.46$$

$$c_1 = \frac{500 - (-3)(92.46) - (-1)(211.11)}{15} = 65.90$$

Usando Gauss-Seidel

$$c_1^{(0)} = 50$$
$$c_2^{(0)} = 50$$
$$c_3^{(0)} = 50$$

$$c_1^{(1)} = \frac{-\sum_{2 \le j \le 3} a_{1j} c_j^{(0)} + b_1}{a_{11}} = \frac{-[(-3)(50) + (-1)(50)] + 500}{15} = 46.67$$

③

$$C_2^{(1)} = \frac{-\sum_{1\le j\le 1} a_{2j} C_j^{(1)} - \sum_{3\le j\le 3} a_{2j} C_j^{(0)} + b_2}{a_{22}} = \frac{-[(-3)(46.67)] - [(-6)(50)] + 300}{18} = \underline{41.11}$$

$$C_3^{(1)} = \frac{-\sum_{1\le j\le 2} a_{3j} C_j^{(1)} + b_3}{a_{33}} = \frac{-[(-4)(46.67) + (-1)(41.11)] + 2350}{12} = \underline{214.82}$$

$$C_1^{(2)} = \frac{-\sum_{2\le j\le 3} a_{1j} C_j^{(1)} + b_1}{a_{11}} = \frac{-[(-3)(41.11) + (-1)(214.82)] + 500}{15} = \underline{55.88}$$

$$C_2^{(2)} = \frac{-[(-3)(55.88)] - [(-6)(214.82)] + 300}{18} = \underline{95.12}$$

$$C_3^{(2)} = \frac{-[(-4)(55.88) + (-1)(95.12)] + 2350}{12} = \underline{222.39}$$

$$C_1^{(3)} = \frac{-[(-3)(95.12) + (-1)(222.39)] + 500}{15} = \underline{67.18}$$

$$C_2^{(3)} = \frac{-[(-3)(67.18)] - [(-6)(222.39)] + 300}{18} = \underline{101.99}$$

$$C_3^{(3)} = \frac{-[(-4)(67.18) + (-1)(101.99)] + 2350}{12} = \underline{226.72}$$

$$C_1^{(4)} = \frac{-[(-3)(101.99) + (-1)(226.72)] + 500}{15} = \underline{68.85}$$

$$C_2^{(4)} = \frac{-[(-3)(68.85)] - [(-6)(226.72)] + 300}{18} = \underline{103.71}$$

$$C_3^{(4)} = \frac{-[(-4)(68.85) + (-1)(103.71)] + 2350}{12} = \underline{227.43}$$

$$C_1^{(5)} = \frac{-[(-3)(103.71) + (-1)(227.43)] + 500}{15} = \underline{69.24}$$

$$C_2^{(5)} = \frac{-[(-3)(69.24)] - [(-6)(227.43)] + 300}{18} = \underline{104.02}$$

$$C_3^{(5)} = \frac{-[(-4)(69.24) + (-1)(104.02)] + 2350}{12} = \underline{227.58}$$