$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$
$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

Usando Gauss-Jacobi

$x_1 = 3.032100$
$x_2 = -2.480246$
$x_3 = 7.000375$

} Usando código

Usando fatoração LU

$$A = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \qquad b = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$l_{21} = \frac{a_{21}}{a_{11}} = \frac{0.1}{3} = \frac{1}{30} \approx \frac{0.1}{3} = \frac{1}{30}$$

$$l_{31} = \frac{a_{31}}{a_{11}} = \frac{0.3}{3} = \frac{1}{10}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/30 & 1 & 0 \\ 1/10 & -0.19/7.0033 & 1 \end{bmatrix}$$

$$l_{32} = \frac{a_{32}}{a_{22}} = \frac{-0.19}{7.0033} = -0.02713$$

$$U = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.0033 & -22/75 \\ 0 & -0.19 & 10.02 \end{bmatrix}$$

$$\Rightarrow U = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.0033 & -22/75 \\ 0 & 0 & 10.0120 \end{bmatrix}$$

$$LU = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/30 & 1 & 0 \\ 1/10 & \frac{-0.19}{7.0033} & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.0033 & -22/75 \\ 0 & 6 & 10.0120 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.56 \\ 70.0843 \end{bmatrix}$$

$$X_3 = \frac{70.0843}{10.0120} \approx 7.0000 \checkmark$$

$$X_2 = \left[-19.56 - \left(\frac{-22}{75} \right)(7.0000) \right] / 7.0033 \approx -2.4998 \checkmark$$

$$X_1 = \left[7.85 - (-0.2)(7.0000) - (-0.1)(-2.4998) \right] / 3$$

$$X_1 \approx 3.0000 \checkmark$$

Usando Gauss - Seidel

$X_1^0 = 0$

$X_2^0 = 0$

$X_3^0 = 0$

$$X_i^{(1)} = \frac{-\sum_{2 \le j \le 3} a_{ij} X_j^{(0)} + b_i}{a_{11}} = \frac{-(a_{12} X_2^0 + a_{13} X_3^0) + b_1}{a_{11}}$$

$$x_1^{(1)} = -\frac{[(-0.1)(0) + (-0.2)(0)]}{3} + 7.85 = 2.6167$$

$$x_2^{(1)} = -\frac{\sum_{1 \le j \le 1} a_{2j} x_j^{(1)} - \sum_{3 \le j \le 3} a_{2j} x_j^{(0)} + b_2}{a_{22}} =$$

$$x_2^{(1)} = -\frac{(a_{21} x_1^{(1)}) - (a_{23} x_3^{(0)}) + b_2}{a_{22}} = -\frac{[(0.1)(2.6167)] - [(-0.3)(0)] + (-19.3)}{7}$$

$$x_2^{(1)} = -2.7945$$

$$x_3^{(1)} = -\frac{\sum_{1 \le j \le 2} a_{3j} x_j^{(1)} + b_3}{a_{33}} = -\frac{[a_{31} x_1^{(1)} + a_{32} x_2^{(1)}] + b_3}{a_{33}}$$

$$x_3^{(1)} = -\frac{[(0.3)(2.6167) + (-0.2)(-2.7945)] + 71.4}{10} = 7.0056$$

$$x_1^{(2)} = -\frac{\sum_{2 \le j \le 3} a_{1j} x_j^{(1)} + b_1}{a_{11}} = -\frac{(a_{12} x_2^{(1)} + a_{13} x_3^{(1)}) + b_1}{a_{11}}$$

$$x_1^{(2)} = -\frac{[(-0.1)(-2.7945) + (-0.2)(7.0056)] + 7.85}{3} = 3.1769$$

$$x_2^{(2)} = -\frac{\sum_{1 \le j \le 1} a_{2j} x_j^{(2)} + \sum_{3 \le j \le 3} a_{2j} x_j^{(1)} + b_2}{a_{22}} =$$

$$x_2^{(2)} = -\frac{(a_{21} x_1^{(2)}) - (a_{23} x_3^{(1)}) + b_2}{a_{22}}$$

$$x_2^{(2)} = -\frac{[(0.1)(3.1769)] + [(-0.3)(7.0056)] + (-19.3)}{7} = -2.5023$$

$$x_3^{(1)} = -\underbrace{\sum_{1 \le j \le 2} a_{3j} x_j^{(2)}}_{a_{31}} + b_3 = -\underbrace{\left[a_{31} x_1^{(2)} + a_{32} x_2^{(2)}\right]}_{a_{32}} + b_3$$

$$x_3^{(1)} = -\frac{\left[(0.3)(3.1769) + (-0.2)(-2.5023)\right]}{10} + 71.4 = 6.9946$$