Universidad de Guanajuato.

División de Ciencias e Ingenierías.

Campus León.

Licenciatura en ingeniería en química sustentable.

UDA: Metodos numéricos.

Proyecto: Solución de ecuaciones diferenciales ordinarias.

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Planteamiento.

Se considera un sistema mecánico masa-resorte-amortiguador sujeto a una fuerza externa periódica. Este sistema representa un problema típico en dinámica estructural y vibraciones mecánicas.

Parametros.

- Masa del cuerpo: $m = 2$ kg
- Constante del resorte: $k = 8$ N/m
- Coeficiente de amortiguación:
- Fuerza externa aplicada: $F(t) = 2\cos(3t)$ N

La ecuación diferencial que gobierna el movimiento del sistema está dada por la segunda ley de Newton:

- $m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$

Sustituyendo los valores numéricos:

- $2\frac{d^2x}{dt^2} + \frac{dx}{dt} + 8x = 2\cos(3t)$

Para aplicar métodos numéricos, transformamos la EDO de segundo orden en un sistema de dos EDOs de primer orden mediante el cambio de variables:

Variables de estado:

- $y_1(t) = x(t)$ → Posición de la masa
- $y_2(t) = \dot{x}(t)$ → Velocidad de la masa

Sistema de ecuaciones resultante:

$$\frac{dy_1}{dt} = y_2$$

$$\frac{dy_2}{dt} = \cos(3t) - 0.5y_2 - 4y_1$$

El sistema parte del reposo con un desplazamiento inicial:

$$y_1(0) = 0.5 \text{ m}, y_2(0) = 0 \text{ m/s}$$

Parámetros de integración:

- Tiempo inicial: $t_0 = 0$ s
- Tiempo final: $t_f = 5$ s

- Tamaño de paso: $h = 0.1$ s
- Número de pasos: $N = 50$

Metodo de Euler.

Para el sistema:

$$y_1^{n+1} = y_1^n + h \cdot f_1(t_n, y_1^n, y_2^n)$$
$$y_2^{n+1} = y_2^n + h \cdot f_2(t_n, y_1^n, y_2^n)$$

Donde:

$$f_1(t, y_1, y_2) = y_2$$
$$f_2(t, y_1, y_2) = \cos(3t) - 0.5y_2 - 4y_1$$

Metodo de Runge-Kutta 2do orden.

Para cada paso:

$$\begin{aligned} k_{11} &= h \cdot f_1(t_n, y_1^n, y_2^n) \\ k_{12} &= h \cdot f_2(t_n, y_1^n, y_2^n) \\ k_{21} &= h \cdot f_1(t_n + h, y_1^n + k_{11}, y_2^n + k_{12}) \\ k_{22} &= h \cdot f_2(t_n + h, y_1^n + k_{11}, y_2^n + k_{12}) \\ y_1^{n+1} &= y_1^n + \frac{1}{2}(k_{11} + k_{21}) \\ y_2^{n+1} &= y_2^n + \frac{1}{2}(k_{12} + k_{22}) \end{aligned}$$

Metodo de Runge-Kutta 4to orden.

Para cada paso:

$$
\begin{aligned}
k_{11} &= h \cdot f_1(t_n, y_1^n, y_2^n) \\
k_{12} &= h \cdot f_2(t_n, y_1^n, y_2^n) \\
k_{21} &= h \cdot f_1\left(t_n + \frac{h}{2}, y_1^n + \frac{k_{11}}{2}, y_2^n + \frac{k_{12}}{2}\right) \\
k_{22} &= h \cdot f_2\left(t_n + \frac{h}{2}, y_1^n + \frac{k_{11}}{2}, y_2^n + \frac{k_{12}}{2}\right) \\
k_{31} &= h \cdot f_1\left(t_n + \frac{h}{2}, y_1^n + \frac{k_{21}}{2}, y_2^n + \frac{k_{22}}{2}\right) \\
k_{32} &= h \cdot f_2\left(t_n + \frac{h}{2}, y_1^n + \frac{k_{21}}{2}, y_2^n + \frac{k_{22}}{2}\right) \\
k_{41} &= h \cdot f_1(t_n + h, y_1^n + k_{31}, y_2^n + k_{32}) \\
k_{42} &= h \cdot f_2(t_n + h, y_1^n + k_{31}, y_2^n + k_{32}) \\
y_1^{n+1} &= y_1^n + \frac{1}{6}(k_{11} + 2k_{21} + 2k_{31} + k_{41}) \\
y_2^{n+1} &= y_2^n + \frac{1}{6}(k_{12} + 2k_{22} + 2k_{32} + k_{42})
\end{aligned}
$$

Resultados Euler.

```
   i      t (s)     y1 (pos)     y2 (vel)
0  0.000000  0.500000  0.000000
1  0.100000  0.500000  -0.100000
2  0.200000  0.490000  -0.199466
3  0.300000  0.470053  -0.302959
4  0.400000  0.439757  -0.413672
5  0.500000  0.398390  -0.532655
6  0.600000  0.345125  -0.658305
7  0.700000  0.279294  -0.786160
8  0.800000  0.200678  -0.909054
9  0.900000  0.109773  -1.017612
10  1.000000  0.008012  -1.101048
11  1.100000  -0.102093  -1.148199
12  1.200000  -0.216913  -1.148700
13  1.300000  -0.331783  -1.094176
14  1.400000  -0.441201  -0.979347
15  1.500000  -0.539135  -0.802925
16  1.600000  -0.619428  -0.568204
17  1.700000  -0.676248  -0.283273
18  1.800000  -0.704576  0.039188
19  1.900000  -0.700657  0.382528
20  2.000000  -0.662404  0.727135
21  2.100000  -0.589691  1.051757
22  2.200000  -0.484515  1.335032
23  2.300000  -0.351012  1.557109
24  2.400000  -0.195301  1.701231
25  2.500000  -0.025178  1.755125
26  2.600000  0.150335  1.712103
27  2.700000  0.321545  1.571760
28  2.800000  0.478721  1.340199
29  2.900000  0.612741  1.029772
30  3.000000  0.715718  0.658322
31  3.100000  0.781550  0.248006
32  3.200000  0.806351  -0.176237
33  3.300000  0.788727  -0.588434
34  3.400000  0.729884  -0.963423
35  3.500000  0.633542  -1.278632
36  3.600000  0.505678  -1.515671
37  3.700000  0.354111  -1.661591
38  3.800000  0.187952  -1.709733
39  3.900000  0.016979  -1.660078
40  4.000000  -0.149029  -1.519106
41  4.100000  -0.300939  -1.299154
42  4.200000  -0.430855  -1.017347
43  4.300000  -0.532590  -0.694194
44  4.400000  -0.602009  -0.351963
45  4.500000  -0.637205  -0.012973
46  4.600000  -0.638502  0.302050
47  4.700000  -0.608297  0.575430
48  4.800000  -0.550754  0.793693
49  4.900000  -0.471385  0.948329
50  5.000000  -0.376552  1.036108
En t = 5.000000 segundos:
  Posicion (y1) = -0.376552 m
  Velocidad (y2) = 1.036108 m/s
```

Resultados Runge-Kutta 2do orden.

i	t (s)	y1 (pos)	y2 (vel)	k11	k12	k21	k22
0	0.0	0.500000	0.000000	---	---	---	---
1	0.100000	0.500000	0.000000	0.000000	-0.100000	-0.010000	-0.099466
1	0.100000	0.495000	-0.099733				
2	0.200000	0.495000	-0.099733	-0.009973	-0.097480	-0.019721	-0.101616
2	0.200000	0.480153	-0.199281				
3	0.300000	0.480153	-0.199281	-0.019928	-0.099563	-0.029884	-0.106987
3	0.300000	0.455246	-0.302556				
4	0.400000	0.455246	-0.302556	-0.030256	-0.104810	-0.040737	-0.113392
4	0.400000	0.419750	-0.411657				
5	0.500000	0.419750	-0.411657	-0.041166	-0.111081	-0.052274	-0.118223
5	0.500000	0.373030	-0.526310				
6	0.600000	0.373030	-0.526310	-0.052631	-0.115823	-0.064213	-0.118773
6	0.600000	0.314608	-0.643608				
7	0.700000	0.314608	-0.643608	-0.064361	-0.116383	-0.075999	-0.112584
7	0.700000	0.244428	-0.758091				
8	0.800000	0.244428	-0.758091	-0.075809	-0.110351	-0.086844	-0.097765
8	0.800000	0.163102	-0.862150				
9	0.900000	0.163102	-0.862150	-0.086215	-0.095873	-0.095802	-0.073261
9	0.900000	0.072093	-0.946716				
10	1.000000	0.072093	-0.946716	-0.094672	-0.071909	-0.101862	-0.039037
10	1.000000	-0.026174	-1.002189				
11	1.100000	-0.026174	-1.002189	-0.100219	-0.038420	-0.104061	0.003840
11	1.100000	-0.128314	-1.019479				
12	1.200000	-0.128314	-1.019479	-0.101948	0.003552	-0.101593	0.053225
12	1.200000	-0.230084	-0.991091				
13	1.300000	-0.230084	-0.991091	-0.099109	0.051912	-0.093918	0.106043
13	1.300000	-0.326598	-0.912113				
14	1.400000	-0.326598	-0.912113	-0.091211	0.103651	-0.080846	0.158521
14	1.400000	-0.412626	-0.781027				
15	1.500000	-0.412626	-0.781027	-0.078103	0.155076	-0.062595	0.206510
15	1.500000	-0.482975	-0.600234				
16	1.600000	-0.482975	-0.600234	-0.060023	0.202122	-0.039811	0.245855
16	1.600000	-0.532893	-0.376246				
17	1.700000	-0.532893	-0.376246	-0.037625	0.240719	-0.013553	0.272781
17	1.700000	-0.558481	-0.119496				
18	1.800000	-0.558481	-0.119496	-0.011950	0.267165	0.014767	0.284258
19	1.900000	-0.557073	0.156216	0.015622	0.278488	0.043470	0.278316
19	1.900000	-0.527527	0.434618				
20	2.000000	-0.527527	0.434618	0.043462	0.272751	0.070737	0.254275
20	2.000000	-0.470427	0.698131				
21	2.100000	-0.470427	0.698131	0.069813	0.249281	0.094741	0.212861
21	2.100000	-0.388150	0.929202				
22	2.200000	-0.388150	0.929202	0.092920	0.208786	0.113799	0.156216
22	2.200000	-0.284791	1.111703				
23	2.300000	-0.284791	1.111703	0.111170	0.153354	0.126506	0.087768
23	2.300000	-0.165953	1.232264				
24	2.400000	-0.165953	1.232264	0.123226	0.086340	0.131860	0.011995
24	2.400000	-0.038409	1.281432				
25	2.500000	-0.038409	1.281432	0.128143	0.012127	0.129356	-0.065908
25	2.500000	0.090340	1.254541				
26	2.600000	0.090340	1.254541	0.125454	-0.064200	0.119034	-0.140439
26	2.600000	0.212584	1.152222				
27	2.700000	0.212584	1.152222	0.115222	-0.137249	0.101497	-0.206226
27	2.700000	0.320944	0.980484				
28	2.800000	0.320944	0.980484	0.098048	-0.201756	0.077873	-0.258462
28	2.800000	0.408905	0.750375				
29	2.900000	0.408905	0.750375	0.075038	-0.253010	0.049737	-0.293310
29	2.900000	0.471292	0.477215				
30	3.000000	0.471292	0.477215	0.047722	-0.287242	0.018997	-0.308217
30	3.000000	0.504651	0.179486				
31	3.100000	0.504651	0.179486	0.017949	-0.301948	-0.012246	-0.302139
31	3.100000	0.507502	-0.122558				
32	3.200000	0.507502	-0.122558	-0.012256	-0.296096	-0.041865	-0.275635
32	3.200000	0.480442	-0.408423				
33	3.300000	0.480442	-0.408423	-0.040842	-0.270224	-0.067865	-0.230827
33	3.300000	0.426088	-0.658948				
34	3.400000	0.426088	-0.658948	-0.065895	-0.226407	-0.088536	-0.171236
34	3.400000	0.348873	-0.857770				
35	3.500000	0.348873	-0.857770	-0.085777	-0.168087	-0.102586	-0.101499
35	3.500000	0.254692	-0.992563				
36	3.600000	0.254692	-0.992563	-0.099256	-0.099802	-0.109237	-0.026989
36	3.600000	0.150445	-1.055959				
37	3.700000	0.150445	-1.055959	-0.105596	-0.026813	-0.108277	0.046622
37	3.700000	0.043509	-1.046054				

Resultados Runge-Kutta 4to orden.

i	t	y1	y2	k11	k12	k21	k22	k31	k32	k41	k42
0	0.000000	0.500000	0.000000								
1	0.100000	0.495062	-0.098362	0.000000	-1.000000	-0.050000	-0.986229	-0.049311	-0.976573	-0.097657	-0.976110
2	0.200000	0.480328	-0.196691	-0.098362	-0.975731	-0.147148	-0.986554	-0.147690	-0.976526	-0.196015	-0.997829
3	0.300000	0.455591	-0.298949	-0.196691	-0.997630	-0.246572	-1.026998	-0.248040	-1.016288	-0.298319	-1.051325
4	0.400000	0.420332	-0.407293	-0.298949	-1.051277	-0.351513	-1.089245	-0.353412	-1.077783	-0.406728	-1.115276
5	0.500000	0.373932	-0.521506	-0.407293	-1.115323	-0.463059	-1.149332	-0.464759	-1.137329	-0.521026	-1.164173
6	0.600000	0.315926	-0.638747	-0.521506	-1.164240	-0.579718	-1.180690	-0.580541	-1.168637	-0.638370	-1.171531
7	0.700000	0.246259	-0.753612	-0.638747	-1.171532	-0.697323	-1.157473	-0.696621	-1.146109	-0.753358	-1.113223
8	0.800000	0.165533	-0.858539	-0.753612	-1.113077	-0.809266	-1.057856	-0.806505	-1.048105	-0.858423	-0.970618
9	0.900000	0.075184	-0.944489	-0.858539	-0.970256	-0.907052	-0.866952	-0.901887	-0.859832	-0.944522	-0.733189
10	1.000000	-0.022410	-1.001859	-0.944489	-0.732564	-0.981118	-0.579067	-0.973443	-0.575579	-1.002047	-0.400328
11	1.100000	-0.123927	-1.021522	-1.001859	-0.399422	-1.021830	-0.199037	-1.011811	-0.200052	-1.021864	0.017818
12	1.200000	-0.225200	-0.995902	-1.021522	0.018989	-1.020573	0.257480	-1.008648	0.251328	-0.996389	0.500603
13	1.300000	-0.321423	-0.919963	-0.995902	0.501991	-0.970802	0.764820	-0.957661	0.753230	-0.920579	1.018220
14	1.400000	-0.407451	-0.792021	-0.919963	1.019741	-0.868976	1.289170	-0.855505	1.272237	-0.792740	1.534003
15	1.500000	-0.478159	-0.614273	-0.792021	1.535553	-0.715243	1.791320	-0.702455	1.769570	-0.615064	2.007521
16	1.600000	-0.528847	-0.393005	-0.614273	2.008976	-0.513824	2.230053	-0.502771	2.204437	-0.393830	2.398157
17	1.700000	-0.555642	-0.138413	-0.393005	2.399389	-0.273035	2.565888	-0.264710	2.537732	-0.139232	2.668866
18	1.800000	-0.555867	0.135930	-0.138413	2.669754	-0.004925	2.764801	-0.000173	2.735727	0.135160	2.789752
19	1.900000	-0.528332	0.413953	0.135930	2.790195	0.275439	2.801607	0.276010	2.773420	0.413272	2.741140
20	2.000000	-0.473535	0.678230	0.413953	2.741063	0.551006	2.662666	0.547086	2.637216	0.677674	2.515825
21	2.100000	-0.393729	0.911299	0.678230	2.515194	0.803990	2.347642	0.795612	2.326679	0.910898	2.120304
22	2.200000	-0.292859	1.097043	0.911299	2.119126	1.017256	1.870148	1.004807	1.855181	1.096817	1.574818
23	2.300000	-0.176355	1.222016	1.097043	1.573146	1.175700	1.257182	1.159902	1.249350	1.221978	0.912210
24	2.400000	-0.050812	1.276603	1.222016	0.910136	1.267523	0.547379	1.249385	0.547347	1.276751	0.175641
25	2.500000	0.076445	1.255892	1.276603	0.173297	1.285268	-0.211792	1.266014	-0.203898	1.256214	-0.584630
26	2.600000	0.197855	1.160190	1.255892	-0.587089	1.226537	-0.967655	1.207509	-0.952270	1.160665	-1.315159
27	2.700000	0.306160	0.995135	1.160190	-1.317561	1.094312	-1.666486	1.076866	-1.644588	0.995731	-1.963577
28	2.800000	0.394916	0.771368	0.995135	-1.965752	0.896848	-2.257839	0.882243	-2.230879	0.772047	-2.482849
29	2.900000	0.458967	0.503800	0.771368	-2.484637	0.647136	-2.698673	0.636434	-2.668476	0.504520	-2.835145
30	3.000000	0.494815	0.210527	0.503800	-2.836413	0.361979	-2.956929	0.355953	-2.925552	0.211245	-2.995000
31	3.100000	0.500878	-0.088526	0.210527	-2.995654	0.060744	-3.014223	0.059816	-2.983802	-0.087853	-2.951485
32	3.200000	0.477594	-0.373119	-0.088526	-2.951475	-0.236100	-2.867440	-0.231898	-2.840026	-0.372529	-2.709177
33	3.300000	0.427375	-0.624243	-0.373119	-2.708504	-0.508544	-2.529060	-0.499572	-2.506461	-0.623765	-2.287856
34	3.400000	0.354413	-0.825569	-0.624243	-2.286572	-0.738571	-2.026201	-0.725553	-2.009844	-0.825227	-1.720933
35	3.500000	0.264345	-0.964690	-0.825569	-1.719135	-0.911526	-1.398434	-0.895491	-1.389261	-0.964495	-1.052747
36	3.600000	0.163813	-1.034034	-0.964690	-1.050572	-1.017219	-0.694570	-0.999419	-0.692965	-1.033987	-0.334950
37	3.700000	0.059950	-1.031385	-1.034034	-0.332564	-1.050662	0.031329	-1.032467	0.025557	-1.031478	0.377711
38	3.800000	-0.040161	-0.959975	-1.031385	0.380129	-1.012378	0.724357	-0.995167	0.711950	-0.960190	1.031853
39	3.900000	-0.130021	-0.828122	-0.959975	1.034124	-0.908269	1.333230	-0.893314	1.315411	-0.828434	1.579784
40	4.000000	-0.204185	-0.648464	-0.828122	1.581741	-0.749035	1.814419	-0.737401	1.792785	-0.648843	1.983319
41	4.100000	-0.258644	-0.436851	-0.648464	1.984825	-0.549223	2.135606	-0.541684	2.111988	-0.437265	2.216778
42	4.200000	-0.291078	-0.210986	-0.436851	2.217733	-0.325964	2.278164	-0.322943	2.254476	-0.211403	2.268889
43	4.300000	-0.300970	0.011059	-0.210986	2.269241	-0.097524	2.238460	-0.099063	2.216537	0.010668	2.143464
44	4.400000	-0.289563	0.212332	0.011059	2.143210	0.118220	2.027871	0.112453	2.009322	0.211991	1.858786
45	4.500000	-0.259685	0.378366	0.212332	1.857971	0.305231	1.671528	0.295909	1.657609	0.378093	1.445764
46	4.600000	-0.215435	0.498301	0.378366	1.444477	0.450590	1.205896	0.438660	1.197416	0.498107	0.945036
47	4.700000	-0.161786	0.565683	0.498301	0.943405	0.545472	0.675421	0.532072	0.672687	0.565570	0.403286
48	4.800000	-0.104115	0.578855	0.565683	0.401461	0.585756	0.128535	0.572110	0.131344	0.578817	-0.130926
49	4.900000	-0.047727	0.540932	0.578855	-0.132783	0.572215	-0.386623	0.559523	-0.378949	0.540960	-0.611412
50	5.000000	0.002609	0.459363	0.540932	-0.613142	0.510275	-0.826395	0.499613	-0.814932	0.459439	-0.998344

Resultado final: y1(5.000000) = 0.002609, y2(5.000000) = 0.459363

Conclusión.

El análisis de los tres métodos numéricos aplicados al sistema masa-resorte-amortiguador muestra una evolución natural en precisión y complejidad. El método de Euler destaca por su simplicidad conceptual, mientras que Runge-Kutta de segundo orden ofrece un equilibrio entre complejidad y mejora en exactitud. Runge-Kutta de cuarto orden representa el nivel más alto de precisión, capturando fielmente la dinámica oscilatoria del sistema.

La elección del método depende del contexto de aplicación, recursos disponibles y nivel de precisión requerido, desde exploraciones iniciales hasta análisis detallados. Cada método encuentra su utilidad según las demandas específicas del problema, ilustrando cómo el aumento gradual en complejidad algorítmica conduce a representaciones más precisas de sistemas dinámicos gobernados por ecuaciones diferenciales.

Referencias.

- Chapra, S. C., & Canale, R. P. (2015). *Numerical Methods for Engineers* (7th ed.). McGraw-Hill Education.

- Burden, R. L., & Faires, J. D. (2011). *Numerical Analysis* (9th ed.). Brooks/Cole, Cengage Learning.
- Kelly, S. G. (2012). *Mechanical Vibrations: Theory and Applications*. Cengage Learning.