$\frac{df}{dt} = \lim_{\Delta t \to 0} \frac{\Delta f}{\Delta t}$

Notación:

t_i = tiempo i-esimo

t_{i+1} = tiempo i-esimo mas uno

$X_i = X(t_i)$: coordenada X al tiempo t_i

$X_{i+1} = X(t_{i+1})$ - coordenada X al tiempo t_{i+1}

$$\frac{df}{dt} \cong \frac{\Delta f}{\Delta t} = \frac{f(t_{i+1}) - f(t_i)}{t_{i+1} - t_i} = \frac{f_{i+1} - f_i}{t_{i+1} - t_i}$$

$$R_n = \int_a^x \frac{(x-a)^n f^{(n+1)}(a')\,da'}{n!}$$

a' : variable muda

otra expresión es:

$$R_n = \frac{f^{(n+1)}(\xi_i)(x-a)^{n+1}}{(n+1)!} \longrightarrow$$ Forma de Lagrange del residuo

Notacion

$$f(X_{i+1}) = f(X_i) + f'(X_i)h + \frac{f''(X_i)h^2}{2!} + \frac{f^{(3)}(X_i)h^3}{3!} + \frac{f^{n}(X_i)h^n}{n!} + \overset{R_n}{\frac{f^{(n+1)}(X_i)h^{(n+1)}}{(n+1)!}}$$

$$f_{i+1} = f_i + f'_i h + f'_i \frac{h^2}{2!} + \frac{f^3_i h^3}{3!} \ldots$$

Serie de Taylor

$f(x_{i+1}) = f(x_i) + f'(x_i)\underbrace{(x_{i+1} - x_i)}_{h} + f''(x_i)\frac{(x_{i+1}-x_i)^2}{2!} + f^3(x_i)\frac{(x_{i+1}-x_i)^3}{3!} + \ldots$
$\ldots + f^n(x_i)\frac{(x_{i+1}-x_i)^n}{n!} + f^{(n+1)}(\xi)\frac{(x_{i+1}-x_i)^{n+1}}{(n+1)!}$

$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$
$f(1) = 0.2$
Predecir el valor en $x=1$ con $h=1$ usando la serie de Taylor de orden 0 hasta 4, y calculando el ~~resultado~~ residuo en cada uno $\xi = 0.5$
$x_{i+1} = 1$
$x_i = 0$

n	$f(x=1)$ aproximación	R_n	$E = f(1) - aprox$
0	1.2	-0.91	-1
1	0.95	-0.87	-0.75
2	0.45	-0.35	-0.25
3	0.3	-0.1	-0.1
4	0.2	0	0

$f'(x) = -0.4x^3 - 0.45x^2 - x - 0.25$
$f''(x) = -1.2x^2 - 0.9x - 1$
$f'''(x) = -2.4x - 0.9$
$f^4(x) = -2.4$
$f^5(x) = 0$

$f(0) = 1.2$
$f'(0) = -0.25$
$f''(0) = -1$
$f'''(0) = -0.9$
$f^4(0) = -2.4$

Aproximaciones

Orden 0
$f(1) \approx f(0) = 1.2$

Orden 1
$f(1) \approx f(0) + f'(0)h = 1.2 + (-0.25)(1) = 0.95$

Orden 2
$f(1) \approx f(0) + f'(0)h + \frac{f''(0)h^2}{2!} = 1.2 + (-0.25)(1) + \frac{(-1)(1)^2}{2} = 0.45$

Orden 3
$f(1) \approx \text{Orden } 2 + \frac{f'''(0)h^3}{3!} = 0.45 + \frac{(-0.9)(1)^3}{6} = 0.45 - 0.15 = 0.3$

Orden 4
$f(1) \approx \text{orden } 3 + \frac{f^4(0)h^4}{4!} = 0.3 + \frac{(-2.4)(1)^4}{24} = 0.2$

~~Error rel Orden 0~~

E_R orden $0 = 0.2 - 1.2 = -1$
E_R orden $1 = 0.2 - 0.95 = -0.75$
E_R orden $2 = 0.2 - 0.3 = -0.1$
E_R orden $4 = 0.2 - 0.2 = 0$

R_n
$f'(0.5) = -0.91$
$f''(0.5) = -1.75$
$f'''(0.5) = -2.1$
$f^4(0.5) = -2.4$

$R(n=0) = \frac{-0.91}{1}(1) = -0.91$
$R(n=1) = \frac{-1.75}{2}(1) = -0.87$
$R(n=2) = \frac{-2.1}{6}(1) = -0.35$
$R(n=3) = \frac{-2.4}{24}(1) = -0.1$
$R(n=4) = \frac{0}{120}(1) = 0$