

Gauss-Jordan

$$\begin{aligned} 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\ 0.7x_1 + 7x_2 - 0.3x_3 &= -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 &= 71.4 \end{aligned}$$

$$\begin{aligned} \left[\begin{array}{ccc|c} 3 & -0.1 & -0.2 & 7.85 \\ 0.7 & 7 & -0.3 & -19.3 \\ 0.3 & -0.2 & 10 & 71.4 \end{array} \right] & \xrightarrow{F_1/3} & \left[\begin{array}{ccc|c} 1 & -0.0333 & -0.0667 & 2.6167 \\ 0.7 & 7 & -0.3 & -19.3 \\ 0.3 & -0.2 & 10 & 71.4 \end{array} \right] & \xrightarrow{\begin{array}{l} F_2 - 0.7F_1 \\ F_3 - 0.3F_1 \end{array}} & \left[\begin{array}{ccc|c} 1 & -0.0333 & -0.0667 & 2.6167 \\ 0 & 7.0233 & -0.2533 & -21.1317 \\ 0 & -19 & 10.02 & 70.615 \end{array} \right] \end{aligned}$$

$$\xrightarrow{F_2 \cdot 0.0233} \left[\begin{array}{ccc|c} 1 & -0.0333 & -0.0667 & 2.6167 \\ 0 & 1 & -0.0361 & -3.0089 \\ 0 & -19 & 10.02 & 70.615 \end{array} \right] \xrightarrow{\begin{array}{l} F_1 + 0.0333F_2 \\ F_3 + 0.19F_2 \end{array}} \left[\begin{array}{ccc|c} 1 & 0 & -0.0629 & 2.5165 \\ 0 & 1 & -0.0361 & -3.0089 \\ 0 & 0 & 10.0131 & 70.0432 \end{array} \right]$$

$$\xrightarrow{F_3/10.0131} \left[\begin{array}{ccc|c} 1 & 0 & -0.0629 & 2.5165 \\ 0 & 1 & -0.0361 & -3.0089 \\ 0 & 0 & 1 & 6.9977 \end{array} \right] \xrightarrow{\begin{array}{l} F_1 + 0.0629F_3 \\ F_2 + 0.0361F_3 \end{array}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2.9919 \\ 0 & 1 & 0 & -2.7565 \\ 0 & 0 & 1 & 6.9977 \end{array} \right] \quad \begin{aligned} x_1 &= 2.9919 \\ x_2 &= -2.7565 \\ x_3 &= 6.9977 \end{aligned}$$

Factorización LU

$$A = \left[\begin{array}{ccc} 3 & -0.1 & -0.2 \\ 0.7 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{array} \right] \quad \begin{aligned} L_{21} &= \frac{0.7}{3} = 0.2333 \\ L_{31} &= \frac{0.3}{3} = 0.1 \end{aligned} \quad \xrightarrow{F_2 - L_{21}F_1} \left[\begin{array}{ccc} 3 & -0.1 & -0.2 \\ 0 & 7.0233 & -0.2533 \\ 0 & -0.19 & 10.2 \end{array} \right]$$

$$L_{32} = \frac{0.19}{7.0233} = 0.027 \quad \xrightarrow{F_3 - L_{32}F_2} \left[\begin{array}{ccc} 3 & -0.1 & -0.2 \\ 0 & 7.0233 & -0.2533 \\ 0 & 0 & 10.0132 \end{array} \right] = U \quad L = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ L_{21} & 1 & 0 & 0 & 1 & 0 \\ L_{31} & L_{32} & 1 & 0 & 0 & 1 \end{array} \right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0.2333 & 1 & 0 & 0 & 1 & 0 \\ 0.1 & 0.027 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$Ly = B$$

$$y_1 = 7.85$$

$$\begin{aligned} 0.2333y_1 + y_2 &= -19.3 & 0.2333(7.85) + y_2 &= -19.3 & 1.82905 + y_2 &= -19.3 & y_2 &= -21.1317 \\ 0.1y_1 - 0.026y_2 + y_3 &= 71.4 & 0.1(7.85) - 0.026(-21.1317) + y_3 &= 71.4 & 0.82905 + y_3 &= 71.4 & y_3 &= 70.0432 \end{aligned}$$

$$Ux = y$$

$$\begin{aligned} 10.0132x_3 &= 70.0432 & \frac{70.0432}{10.0132} &= 6.9977 = x_3 \\ 7.0233x_2 - 0.2533x_3 &= -21.1317 & 7.0233x_2 - 0.2533(6.9977) &= -21.1317 & x_2 &= -2.7565 \\ 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 & 3x_1 - 0.1(-2.7565) - 0.2(6.9977) &= 7.85 & x_1 &= 2.9919 \end{aligned}$$

Gauss-Seidel

$$\begin{aligned} x_1 &= (7.85 + 0.1x_2 + 0.2x_3)/3 \\ x_2 &= (-19.3 - 0.7x_1 + 0.3x_3)/7 \\ x_3 &= (71.4 - 0.3x_1 + 0.2x_2)/10 \end{aligned}$$

Valores iniciales $x_1(0)=0, x_2(0)=0, x_3(0)=0$

$i=1$

$$\begin{aligned} x_1 &= (7.85 + 0 + 0)/3 = 2.6167 \\ x_2 &= (-19.3 - 0.7(2.6167) + 0)/7 = -3.0089 \\ x_3 &= (71.4 - 0.3(2.6167) + 0.2(-3.0089))/10 = 7.0033 \end{aligned}$$

$i=2$

$$\begin{aligned} x_1 &= (7.85 + 0.1(-3.0089) + 0.2(7.0033))/3 = 2.9906 \\ x_2 &= (-19.3 - 0.7(2.9906) + 0.3(7.0033))/7 = -2.7997 \\ x_3 &= (71.4 - 0.3(2.9906) + 0.2(-2.7997))/10 = 7.0002 \end{aligned}$$

$i=3$

$$\begin{aligned} x_1 &= (7.85 + 0.1(-2.7997) + 0.2(7.0002))/3 = 2.9917 \\ x_2 &= (-19.3 - 0.7(2.9917) + 0.3(7.0002))/7 = -2.7568 \\ x_3 &= (71.4 - 0.3(2.9917) + 0.2(-2.7568))/10 = 6.9997 \end{aligned}$$

$i=4$

$$\begin{aligned} x_1 &= (7.85 + 0.1(-2.7568) + 0.2(6.9997))/3 = 2.9919 \\ x_2 &= (-19.3 - 0.7(2.9919) + 0.3(6.9997))/7 = -2.7565 \\ x_3 &= (71.4 - 0.3(2.9919) + 0.2(-2.7565))/10 = 6.9998 \end{aligned}$$