Demostración de Fórmulas (Expresiones)

Derivadas por Taylor

Segunda diferencial hacia atrás

$$f''(x_i) = \frac{f(x_i) - 2f(x_i - 1) + f(x_i - 2)}{h^2} - Oh$$

(1) $$f(x_i - 1) = f(x_i) - f'(x_i)h + \frac{f''(x_i)h^2}{2!} - \frac{f'''(x_i)h^3}{3!} + O(h^4)$$

$$f(x_i - 2) = f(x_i) - 2f'(x_i)h + \frac{4f''(x_i)h^2}{2!} - \frac{f'''(x_i)(2h)^3}{6} + O(h^4)$$

(2) $$f(x_i - 2) = f(x_i) - 2f'(x_i)h + 2f''(x_i h^2) - \frac{4}{3} f'''(x_i)h^3 + O(h^4)$$

Multiplicar ec. (1) por 2

(3) $$2f(x_i - 1) = 2f(x_i) - 2f'(x_i)h + f''(x_i)h^2 - \frac{1}{3} f'''(x_i)h^3 + O(h$$

Restándole a (3), (2)

$$f(x_i - 2) - 2f(x_i - 1) = -f(x_i) + f''(x_i)h^2 + O(h^3)$$

$$f''(x_i) = \frac{f(x_i - 2) - 2f(x_i - 1) + f(x_i)}{h^2} + O(h)$$

Segunda Derivada Centrada

$$f''(x_i) = \frac{f(x_i+1) - 2f(x_i) + f(x_i-1)}{h^2} + O(h^2)$$

$$① \; f(x_i+1) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + \frac{f'''(x_i)h^3}{3!} + \frac{f^{(4)}(x_i)h^4}{4!} + O(h^5)$$

$$② \; f(x_i-1) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} - \frac{f'''(x_i)h^3}{3!} + \frac{f^{(4)}(x_i)h^4}{4!}$$

Sumando ① y ②

$$f(x_i+1) + f(x_i-1) = 2(f(x_i)) + f''(x_i)h^2 + \frac{f^{(4)}(x_i)h^4}{12} + O(h^5)$$

$$f''(x_i) = \frac{f(x_i+1) + (f(x_i-1)) - 2f(x_i)}{h^2} + \frac{f^{(4)}(x_i)h^4}{12} + O(h^5)$$

$$f''(x_i) = \frac{f(x_i+1) + f(x_i-1) - 2f(x_i)}{h^2} + O(h^2)$$