Ejemplo 2

$f(x) \cos x$

Predecir el valor en $x = \pi/3$ usando la serie de taylor en torno a $x = \pi/4$ de orden cero hasta 4 calculando el residuo en cada caso. Usando $x_{i+1} = \pi/3$

$x_0 = \pi/4$

O	$f(x=1)$	R_n	$E = f_{(1)} -$ aproximación
1	0.71	-0.03	-0.207
2	0.52	-0.03	-0.021
3	0.49	0.00039	0.0027
4	0.49	0.00019	0.00013
5	0.50	-0.0000013	-0.00000075

Derivando

$f(x) = \cos(x)$

$f'(x) = -\operatorname{sen}(x)$

$f''(x) = -\cos(x)$

$f^{(3)}(x) = \operatorname{sen}(x)$

$f^{(4)}(x) = \cos(x)$

En $x_0 = 0$

$f'(0) = 1$

$f''(0) = 0.9$

$f'^{4}(0) = -2.4$

Residuo con $\xi = \pi/4$

$R_0 = -\operatorname{sen} \xi \, h_1 = -0.0341$

$R_1 = -\cos \xi / h^2_{2} = -0.0331$

$R_2 = \operatorname{sen} \xi / 6h^3 = 0.0039$

$R_3 = \cos \xi / 24h^4 = 0.00019$

$R_4 = -\operatorname{sen} \xi / 120h^5 = -0.0000019$

Fórmula polinomica de Taylor en $x = \pi/3$

$T(0)(1) = \cos(9) = 0.71$

$T(1)(1) = \cos(9) - \operatorname{sen}(9) = 0.5219$

$T(2)(1) = T_1 - \frac{\cos 9}{2} h^2 = 0.497$

$T(3)(1) = T_2 + \frac{\operatorname{sen} 9}{6} h^3 = 0.499$

$T(4)(1) = T_3 + \frac{\cos 9}{24} h^4 = 0.5$

Norma

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{k}(X_0)}{k!}(x - X_0)^k$$

Ejemplo 1

$f(x) = -0.14x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$

Predecir el valor en $x = 1$ con $h = 1$ usando la serie de Taylor de orden 0 hasta 4 calculando el residuo en cada caso.

$E_v = 0.5$ Derivadas

$f'(x) = -0.56x^3 - 0.45x^2 - x - 0.25$

$f''(x) = -1.68x^2 - 0.9x - 1$

$f'''(x) = -3.36x - 0.9$

$f''''(x) = -3.36$

$h = x - x_0$

$x_0 = x - h = 1 - 1 = 0$

$x = 1$ desde $x_0 = 0$

$k = 0$

$f(x_0) = \cancel{-0.14} \cancel{-0.15} \cancel{-0.5} + 1.2 = \cancel{0.41}$ 1.2

$k = 1$

$f'(x_0) = -\cancel{0.56} \cancel{-0.45} - 0.25 \cancel{-1.26}$

$f'(x_0) = -0.25$

$k = 2$

$f''(x_0) = \frac{-1}{2} = -0.5$

$k = 3$ $f'''(x_0) = -0.9$

Termino $3 = \frac{-0.9}{3!} = -0.15$

$k = 4$

$f''''(x_0) = -3.36$ Termino $4 = -0.14$

Sumando los Terminos

$T_4(1) = 0.16$ $R_{n+1}(x) = f(x) - T_n(x)$

$$= \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)^{n+1}$$

Residuo

$n = 0$ — $R_1 = -1.13$

$n = 1 = -0.4125$

$$R_{n(1)} = \frac{f^{(n+1)}(\xi)}{(n+1)!}$$

$n = 2 = -0.875$

$n = 3 = -0.35$

$n = 4 = -0.10$

$n = 5 = 0$

0	$f(x=1)$	R_n	$E = E_{(1)}$
1	1.2	-0.91	1
2	0.95	0.87	0.75
3	0.45	-0.35	0.25
4	0.30	-0.10	0.1
5	0.20	0	0