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Segunda y tercera derivada por método de diferencias finitas

* Segunda derivada hacia atrás

$$① \left(F(x_{i-1}) = F(x_i) - F'(x_i)(x_i - x_{i-1}) + \frac{F''(x_i)(x_i - x_{i-1})^2}{2!} + O(\sim h)^3 \right)(-2) \quad h = x_i - x_{i-1}$$

$$② F(x_{i-2}) = F(x_i) - F'(x_i)\overbrace{(x_i - x_{i-2})}^{2h} + \frac{F''(x_i)(x_i - x_{i-2})^2}{2!} + O(\sim h)^3 \quad h = x_i - x_{i-2}$$

Resolviendo el sistema para $F''(x_i)$

$$-2F(x_{i-1}) = -2F(x_i) + 2F'(x_i)(h) - \frac{2F''(x_i)(h)^2}{2!}$$

$$F(x_{i-2}) = F(x_i) - F'(x_i)(2h) + \frac{F''(x_i)(2h)^2}{2!}$$

$$F(x_{i-2}) - 2F(x_{i-1}) = -F(x_i) + F''(x_i)h^2$$

$$\boxed{F''(x_i) = \frac{F(x_{i-2}) - 2F(x_{i-1}) + F(x_i)}{h^2}} + O(h)$$

* Segunda derivada centrada

$$① F(x_{i+1}) = F(x_i) + F'(x_i)(x_{i+1} - x_i) + F''(x_i)\frac{(x_{i+1} - x_i)^2}{2!}$$

$$② F(x_{i-1}) = F(x_i) - F'(x_i)(x_i - x_{i-1}) + F''(x_i)\frac{(x_i - x_{i-1})^2}{2!}$$

$$F(x_{i+1}) + F(x_{i-1}) = 2F(x_i) + 2F''(x_i)\frac{(h)^2}{2!}$$

$$F''(x_i) = \frac{F(x_{i+1}) + F(x_{i-1}) - 2F(x_i)}{\cancel{2}\left(\frac{h^2}{2!}\right)} \Rightarrow \boxed{F''(x_i) = \frac{F(x_{i+1}) + F(x_{i-1}) - 2F(x_i)}{h^2}}$$

* Tercera derivada hacia atrás

$$① F(x_{i-1}) = F(x_i) - F'(x_i)(x_i - x_{i-1}) + \frac{F''(x_i)(x_i - x_{i-1})^2}{2!} - \frac{F'''(x_i)(x_i - x_{i-1})^3}{3!}$$

$$② F(x_{i-2}) = F(x_i) - F'(x_i)(x_i - x_{i-2}) + \frac{F''(x_i)(x_i - x_{i-2})^2}{2!} - \frac{F'''(x_i)(x_i - x_{i-2})^3}{3!}$$

$$③ F(x_{i-3}) = F(x_i) - F'(x_i)(x_i - x_{i-3}) + \frac{F''(x_i)(x_i - x_{i-3})^2}{2!} - \frac{F'''(x_i)(x_i - x_{i-3})^3}{3!}$$

$x_i - x_{i-1} =$
$x_i - x_{i-2} =$
$x_i - x_{i-3}$

① $F(x_{i-1}) - f(x_i) = -f'(x_i)(h) + \frac{f''(x_i)(h)^2}{2!} - \frac{f'''(x_i)(h)^3}{3!}$

② $F(x_{i-2}) - f(x_i) = -f'(x_i)(2h) + \frac{f''(x_i)(2h)^2}{2!} - \frac{f'''(x_i)(2h)^3}{3!}$

③ $F(x_{i-3}) - F(x_i) = -f'(x_i)(3h) + \frac{f''(x_i)(3h)^2}{2!} - \frac{f'''(x_i)(3h)^3}{3!}$

Multiplicar ① por 2 y restamos ②

$$2[F(x_{i-1}) - f(x_i)] = -2\cancel{f'(x_i)(h)} + f''(x_i)h^2 - \frac{f'''(x_i)(h)^3}{3}$$

$$- \quad F(x_{i-2}) - F(x_i) = -\cancel{2f'(x_i)h} + 2f''(x_i)h^2 - \frac{4f'''(x_i)h^3}{3}$$

$$2[f(x_{i-1}) - f(x_i)] - f(x_{i-2}) = -f''(x_i)h^2 + f'''(x_i)h^3 \quad ④$$

Multiplicamos ① por 3 y restamos ③

$$3[f(x_{i-1}) - f(x_i)] = -3\cancel{f'(x_i)(h)} + \frac{3f''(x_i)(h)^2}{2} - \frac{f'''(x_i)h^3}{2}$$

$$- \quad f(x_{i-3}) - f(x_i) = -\cancel{f'(x_i)(3h)} + \frac{f''(x_i)(3h)^2}{2} - \frac{f'''(x_i)(3h)^3}{2}$$

$$3[f(x_{i-1}) - f(x_i)] - [f(x_{i-3}) - f(x_i)] = -3f''(x_i)h^2 + 4f'''(x_i)h^3 \quad ⑤$$

Multiplicamos ④ por 3 aquí $A = f(x_{i-1}) - f(x_i)$, $I = f(x_{i-2}) - f(x_i)$
y restamos 5 $Q = f(x_{i-3}) - f(x_i)$

$$- \quad 3(2A - I) = -\cancel{3f''(x_i)h^2} + 3f'''(x_i)h^3$$

$$(3A - Q) = -\cancel{3f''(x_i)h^2} + 4f'''(x_i)h^3$$

$$6A - 3I - 3A + Q = -f'''(x_i)h^3 \quad \Rightarrow \quad 3A - 3I + Q = -f'''(x_i)h^3$$

$$f'''(x_i) = -\frac{3A - 3I + Q}{h^3}$$

$$f'''(x_i) = -\frac{3[f(x_{i-1}) - f(x_i)] - 3[f(x_{i-2}) - f(x_i)] + [f(x_{i-3}) - f(x_i)]}{h^3}$$

$$f'''(x_i) = -\frac{3f(x_{i-1}) - 3f(x_i) - 3f(x_{i-2}) + 3f(x_i) + f(x_{i-3}) - f(x_i)}{h^3} =$$

$$f'''(x_i) = -\frac{3f(x_{i-1}) - 3(x_{i-2}) - f(x_i) + f(x_{i-3})}{h^3}$$

$$F'''(x_i) = \frac{F(x_i) - 3F(x_{i-1}) + 3F(x_{i+2}) - F(x_{i-3})}{h^3}$$