Metodo de descomposición LU

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$
$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{0.1}{3} & 1 & 0 \\ \frac{0.3}{3} & \frac{-0.19}{7} & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \quad L = \begin{bmatrix} 3 \\ 19 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$U = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.00 & -0.30 \\ 0 & 0 & 10.01 \end{bmatrix}$$

$$l_{21} = \frac{0.1}{3}$$
$$l_{31} = \frac{0.3}{3}$$
$$l_{32} = \frac{-0.19}{7}$$

$F_2 - l_{21} F_1$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \rightarrow \begin{pmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.00 & -0.30 \\ 0.3 & -0.2 & 10 \end{pmatrix} \rightarrow \begin{pmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.00 & -0.30 \\ 0 & -0.19 & 10.02 \end{pmatrix}$$

$$\begin{matrix} 0.1 & 7 & -0.3 \\ \frac{3 \cdot 0.1}{3} & -3.33\times10^{-3} & -6.66\times10^{-3} \\ 0 & 7.00 & -0.30 \end{matrix} \qquad \begin{matrix} 0.3 & -0.2 & 10 \\ \frac{0.3}{3}\times3 & \frac{0.3}{3}\times-0.1 & \frac{0.3}{3}\cdot-0.2 - \frac{-0.19}{7}\cdot0 \\ 0 & -0.19 & 10.02 \end{matrix} \qquad \begin{matrix} 0 & -0.19 & 10.02 \\ \frac{-0.19}{7}\cdot0 & \frac{-0.19}{7}\cdot7 & \frac{-0.19}{7}\cdot-0.3 \\ 0 & -0.19 & 10.01 \end{matrix}$$

$$\begin{pmatrix} 3 & -0.1 & -0.2 \\ 0 & 7.00 & -0.30 \\ 0 & 0 & 10.01 \end{pmatrix} = U$$

$$LU = \begin{matrix} (1)(3)+(0)(0)+(0)(0)=3 & (1)(-0.1)+(0)(7)+(0)(0)=-0.1 & (1)(-0.2)+(0)(-0.30)+(0)(10.01)=-0.2 \\ \frac{0.1}{3}(3)+(1)(0)+(0)(0)=0.1 & \frac{0.1}{3}(-0.1)+(1)(7)+(0)(0)=6.99 & \frac{0.1}{3}(-0.2)+(1)(-0.30)+(0)(10.01)=0.30 \\ \frac{0.3}{3}(3)+\frac{-0.19}{7}(0)+(1)(0)=0.3 & \frac{0.3}{3}(-0.1)+\frac{-0.19}{7}(7.00)+1(0)=-0.2 & \frac{0.3}{3}(-0.2)+\frac{-0.19}{7}(-0.3)+(1)(10.01)=9.99 \end{matrix}$$

$$Ly = b \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0.1/3 & 1 & 0 \\ 0.3/3 & -0.19/7 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

$y_1 = b_1 = 7.85$ $\qquad y_2 = b_2 - l_{21} y_1 = -19.3 - \frac{(0.1)}{3}(7.85) = -19.56$

$y_3 = b_3 - l_{31} y_1 - l_{32} y_2 = 71.4 - \left(\frac{0.3}{3}\right)(7.85) - \left(\frac{-0.19}{7.00}\right)(-19.3) = 70.01$

$$Ux \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0 & 7 & -0.30 \\ 0 & 0 & 10.01 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.56 \\ 70.01 \end{bmatrix}$$

$x_3 = \frac{y_3}{U_{33}} \Rightarrow \frac{70.01}{10.01} = 6.99 \approx 7.00$

$x_2 = \frac{y_2 - U_{23} x_3}{U_{22}} = \frac{-19.56 - (-0.30)(7)}{7.00} = -2.49 \approx -2.5$

$x_1 = \frac{y_1 - U_{12} x_2 - U_{13} x_3}{U_{11}} = \frac{7.85 - (-0.1)(-2.5) - (-0.2)(7)}{3} = 3$