1.04 0.130 0.78

Ejemplo: $f(x) = \cos(x)$ Predecir el valor en $x = \pi/3$ usando la serie de Taylor entorno a $x = \pi/4$, de orden 0 hasta 4 y calculando el residuo en cada caso.

$h = \pi/12$ $x_i = \pi/4$ $x_{i+1} = \pi/3$ $\xi = \frac{7}{24}\pi$

$f(\pi/3) = \cos(x) = 0.5$

$f(x) = \cos(x)$
$f'(x) = -\operatorname{sen} x$
$f''(x) = -\cos x$
$f'''(x) = \operatorname{sen} x$
$f^{(4)}(x) = \cos(x)$

n	$f(x=\pi/3)$	R_n	$E = f(\pi/3)$ aprox
0	0.71	-0.20	-0.21
1	0.53	-0.020	-0.03
2	0.51	0.00237	-0.01
3	0.51	0.00119	-0.01
4	0.50	-0.00813	0

$[f(x_i)]$ $f(\pi/4) = \cos(\pi/4) = 0.71$ $R_n = \frac{f^{(0+1)}(\frac{7}{24}\pi)(\pi/12)^{0+1}}{(0+1)!} = -0.20$

$[f'(x_i)h]$ $f'(\pi/4) = [-\operatorname{sen}(\pi/4)](\pi/12) = -0.1851$ $R_n = \frac{f^{(2)}(\frac{7}{24}\pi)(\pi/12)^2}{2!} = -0.020$

$f(x_{i+1}) = 0.71 - 0.18 = 0.53$

$\left[\frac{f''(x_i)h^2}{2!}\right]$ $f''(\pi/4) = \frac{[-\cos(\pi/4)](\pi/12)^2}{2!} = -0.024$ $R_n = \frac{f^{(3)}(\frac{7}{24}\pi)(\pi/12)^3}{3!} = 2.37 \times 10^{-3}$

$f(x_{i+1}) = 0.53 - 0.024 = 0.51$

$\left[\frac{f'''(x_i)h^3}{3!}\right]$ $f'''(\pi/4) = \frac{[\operatorname{sen}(\pi/4)](\pi/12)^3}{3!} = 2.11 \times 10^{-3}$ $R_n = \frac{f^{(4)}(\frac{7}{24}\pi)(\pi/12)^4}{4!} = 1.19 \times 10^{-4}$

$f(x_{i+1}) = 0.51$ $2.11 \times 10^{-3} = 0.51$

$\left[\frac{f^{(4)}(x_i)h^4}{4!}\right]$ $f^{(4)}(\pi/4) = \frac{[\cos(\pi/4)][\pi/12]^4}{4!} = 1.38 \times 10^{-4}$

$f(x_{i+1}) = 0.50$

$R_n = \frac{f^{(5)}(\frac{7}{24}\pi)(\pi/12)^5}{5!} = -8.13 \times 10^{-6}$

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Ejemplo: $f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$. Predecir el valor en $x=1$, con $h=1$ usando la serie de Taylor de orden cero hasta 4, y calculando el residuo en cada paso.

$x_{i+1} = 1$ $x_i = 0$ $\xi = 0.5$

$h = 1$

$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$

$f'(x) = -0.4x^3 - 0.45x^2 - x - 0.25$

$f''(x) = -1.2x^2 - 0.9x - 1$

$f^{(3)}(x) = -2.4x - 0.9$

$f^{(4)}(x) = -2.4$

$f(1) = -0.1(1)^4 - 0.15(1)^3 - 0.5(1)^2 - 0.25(1) + 1.2$

$f(1) = 0.2$

n	$f(x=1)$	R_n	$E = f(1) - \text{aprox}$
0	1.2	-0.91	-1
1	0.95	-0.87	-0.75
2	0.45	-0.35	-0.25
3	0.30	-0.1	-0.1
4	0.20	0	0

$[f(x_i)]$ $f(x) = -0.1(0)^4 - 0.15(0)^3 - 0.5(0)^2 - 0.25(0) + 1.2 = 1.2$

$R_n = \frac{f^{(0+1)}(0.5)(1)^{0+1}}{(0+1)!} = \frac{(-0.9125)(1)^{0+1}}{1!}$

$R_n = -0.91$

$[f'(x_i)h]$ $f'(0) = -0.4(0)^3 - 0.45(0)^2 - 0 - 0.25 = -0.25(1) = -0.25$

$R_n = \frac{f^{(1+1)}(0.5)(1)^{1+1}}{(1+1)!} = -0.87$

$f(x_{i+1}) = 1.2 - 0.25 = 0.95$

$\left[\frac{f''(x_i)h^2}{2!}\right]$ $f''(0) = \frac{-1.2(0)^2 - 0.9(0) - 1\,(1)^2}{2!} = -0.5$

$R_n = \frac{f^{(2+1)}(0.5)(1)^{2+1}}{(2+1)!} = -0.35$

$f(x_{i+1}) = 0.95 - 0.5 = 0.45$

$\left[\frac{f^{(3)}(x_i)h^3}{3!}\right]$ $f^{(3)}(0) = \frac{-2.4(0) - 0.9\,(1)^3}{3!} = -0.15$

$R_n = \frac{f^{(3+1)}(0.5)(1)^{3+1}}{(3+1)!} = -0.1$

$f(x_{i+1}) = 0.45 - 0.15 = 0.30$

$\left[\frac{f^{(4)}(x_i)h^4}{4!}\right]$ $f^{(4)}(0) = \frac{1^4(-2.4)}{4!} = -0.1$

$R_n = \frac{f^{(4+1)}(0.5)(1)^{4+1}}{(4+1)!} = 0$

$f(x_{i+1}) = 0.30 - 0.1 = 0.20$