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Ejemplo 1

$f(x) = -0.1 4x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$

Predecir el valor en $x=1$ con $h=1$ usando la serie de Taylor de orden 0 hasta 4 calculando el residuo en cada caso. $\mathcal{E}=0.5$

0	f(x=1)	Rn	E = f(1) - aprox.
1	1.2	-0.91	0.2 - 1.2 = 1
2	0.95	0.87	0.2 - 0.95 = 0.75
3	0.45	-0.35	0.2 - 0.45 = 0.25
4	0.3	-0.10	0.2 - 0.1
5	0.2	0	0.2 0

Derivadas

$f'(x) = 0.41x^3 - 0.45x^2 - x - 0.25$

$f''(x) = -1.2x^2 - 0.4x - 1$

$f'''(x) = -2.4x - 0.9$

$f''''(x) = -2.4$

$f'''''(x) = 0$

En $x_0 = 0$

$f'(0) = -1$

$f''(0) = -0.9$

$f''''(0) = -2.4$

Residuos con $\mathcal{E}=0.5$ $\quad R_n(1) = \dfrac{f^{(n+1)}(\mathcal{E})}{(n+1)!} = (1-0)^{n-1}$

$R_0 = f'(0.5) = -0.9125$

$R_1 = f''(0.5)/2 = -0.875$

$R_2 = f^{(3)}(0.5/2 = -0.35$

$R_3 = f^{(4)}(0.5/24 = -0.1$

$R_4 = f^{(5)}(0.5)/120 = 0$

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Fórmula (polinomios de Taylor) $T_n(1)=\sum_{K=0}^{n}\frac{f^K(0)\,1^K}{K!}$ en $x=1$

$T_{(0)}(1)=1.2$
$T_{(1)}(1)=1.2-0.25=0.95$
$T_{(2)}(1)=0.95+(-1/2)=0.45$
$T_{(3)}(1)=0.45+(-0.9/6)=0.3$
$T_{(4)}(1)=0.30+(-2.4/24)=0.2$

Ejemplo 2. $f(x)=\cos x$

Predecir el valor en $x=\pi/3$ usando Taylor entorno a $x=\pi/4$, de orden 0 a 4, calculando el residuo en cada cas. Usando $x_{i+1}=\pi/3$
$x_i=\pi/4$

O	f(x=1)	Rn	E= f(1)-aprox.
1	0.71	-0.03	-0.207
2	0.52	-0.03	-0.021
3	0.49	0.00039	0.0022
4	0.49	0.00014	0.00013
5	0.5	-0.0000013	-0.0000075

$f(x)=\cos(\frac{\pi}{3})=0.5$

Derivadas
$f(x)=\cos(x)$
$f'(x)=-\operatorname{Sen}(x)$
$f''(x)=-\cos(x)$
$f'''(x)=\operatorname{Sen}(x)$
$f^{(4)}(x)=\cos(x)$

En $x_0=0$
$f'(0)=-1$
$f^2(0)=-0.9$
$f4(0)=-2.4$

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Residuos con $\varepsilon = \pi/24$ $\qquad R_{n+1} = \dfrac{f^{(n+1)}(\varepsilon)\, h^{n+1}}{(n+1)!}$

$R_0 = -\operatorname{sen}\varepsilon\, h_1 = -0.0341$

$R_1 = -\cos\varepsilon/2h^2 = -0.0339$

$R_2 = \operatorname{sen}\varepsilon/6h^3 = 0.00039$

$R_3 = \cos\varepsilon/24h^4 = 0.00019$

$R_4 = -\operatorname{sen}\varepsilon/120h^5 = -0.0000019$

Fórmula (polinomios de Taylor en $x = \pi/3$)

$$T_n(x) = \sum_{k=0}^{n} \frac{f(k)(a)}{k!}(x-a)^k \qquad a = \pi/4 \quad h = \pi/12$$

$T(0)(1) = \cos a = 0.71$

$T(1)(1) = \cos a - \operatorname{sen} a = 0.26$

$T(2)(1) = T_1 - (\cos a)/2 \cdot h^2 = 0.497$

$T(3)(1) = T_2 + (\operatorname{sen} a)/6 \cdot h^3 = 0.499$

$T(4)(1) = T_3 + (\cos a)/24 \cdot h^4 = 0.5$