Ejemplo. 1

$f(x) = -0.14x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$

Predecir el valor en $x=1$ con $h=1$ usando la serie de Taylor de orden cero hasta 4 calculando el residuo en cada caso.

$\xi = 0.5$

0	$f(x=1)$	R_n	$E = f(1) -$ aproximación
1	1.2	−0.91	0.20 − 1.20 = 1
2	0.95	0.87	0.20 − 0.95 = 0.75
3	0.45	−0.35	0.20 − 0.45 = 0.25
4	0.30	−0.10	0.20 −0.1
5	0.20	0	0.20 0

$R_{n(1)} = \frac{f^{(n+1)}(\xi)}{(n+1)!} = (1-0)^{n-1}$

Derivadas

$f'(x) = 0.4x^3 - 0.45x^2 - x - 0.25$

$f''(x) = -1.2x^2 - 0.9x - 1$

$f^{(3)}(x) = -2.4x - 0.9$

$f^{(4)}(x) = -2.4$

$f^{(5)}(x) = 0$

En $x_0 : 0$

$f'(0) = -1$

$f^2(0) = -0.9$

$f^4(0) = -2.4$

Residuos con $\xi = 0.5$

$R_0 = f'(0.5) = -0.4125$

$R_1 = \frac{f''(0.5)}{2} = -0.875$

$R_2 = \frac{f^{(3)}(0.5)}{2} = -0.35$

$R_3 = \frac{f^{(4)}(0.5)}{24} = -0.10$

$R_4 = \frac{f^{(5)}(0.5)}{120} = 0$

Fórmula (polinomios de Taylor

$T_n(1) = \sum_{k=0}^{n} \frac{f^k(0)}{k!} 1^k$ en $x=1$

$T_{(0)}(1) = 1.2$

$T_{(1)}(1) = 1.2 - 0.25 = 0.95$

$T_{(2)}(1) = 0.95 + {}^{-1}/_{2} = 0.45$

$T_{(3)}(1) = 0.45 + \frac{-0.9}{6} = 0.30$

$T_{(4)}(1) = 0.30 + \frac{-2.4}{24} = 0.20$

Ejemplo 2.

Predecir el valor en $x=\pi/3$ usando la serie de Taylor entorno a $x=\pi/4$, de orden cero hasta 4, calculando el residuo en cada caso. Usando $x_{i+1}=\pi/3$

$x_i=\pi/4$

O	f(x=1)	Rn	E = F(1) - aproximación
1	0.71	-0.03	-0.207
2	0.52	-0.03	-0.021
3	0.49	0.00039	0.0022
4	0.49	0.00019	0.00013
5	0.50	-0.0000013	-0.0000075

$f(x)=\cos\left(\frac{\pi}{3}\right)=0.5$

$R_n(1)=\frac{f^{(n+1)}(\xi)}{(n+1)!}h^{n+1}$

Derivadas

$f(x)=\cos x$

$f'(x)=-\operatorname{sen} x$

$f''(x)=-\cos x$

$f^{(3)}(x)=\operatorname{sen} x$

$f^{(4)}(x)=\cos x$

En $x_0:0$

$f'(0)=-1$

$f^{2}(0)=-0.9$

$f^{4}(0)=-2.4$

Residuos con $\xi=\pi/24$

$R_0=-\operatorname{sen}\xi h_1=-0.0341$

$R_1=-\cos\xi/2\,h^2=-0.0339$

$R_2=\operatorname{sen}\xi/6\,h^3=0.00039$

$R_3=\cos\xi/24\,h^4=0.00019$

$R_4=-\operatorname{sen}\xi/120\,h^5=-0.0000019$

Fórmula (polinomios de Taylor en $x=\pi/3$

$T_n(x)=\sum_{k=0}^{n}\frac{f^{(k)}(a)}{k!}(x-a)^k$

$a=\pi/4,\ h=\frac{\pi}{12}$

$T_{(0)}(1)=\cos a=0.71$

$T_{(1)}(1)=\cos a-\operatorname{sen} a=0.5219$

$T_{(2)}(1)=T_1-\frac{\cos a}{2}h^2=0.497$

$T_{(3)}(1)=T_2+\frac{\operatorname{sen} a}{6}h^3=0.499$

$T_{(4)}(1)=T_3+\frac{\cos a}{24}h^4=0.500$

$T_0+f'(a)h=0.71-0.71\cdot 0.26$