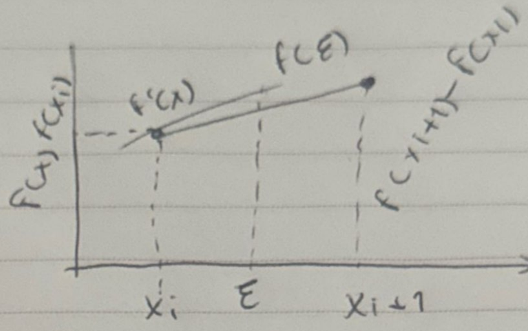
29/Agosto/2025.

Caso de serie de Taylor.

INICIO

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)(x_{i+1} - x_i)^2}{2!} + \frac{f^{(3)}(x_i)(x_{i+1} - x_i)^3}{3!} + \dots + \frac{f^{(n)}(x_i)(x_{i+1} - x_i)^n}{n!} + R_n$$

$$R_n = \frac{f^{(n+1)}(\xi)(x_{i+1} - x_i)^{n+1}}{(n+1)!}$$




caso particular.

$f(x_{i+1}) \cong f(x_i)$

$R_0 = f'(\xi)(x_{i+1} - x_i)$

Aprox...

$f(x_{i+1}) \cong f(x_i)$

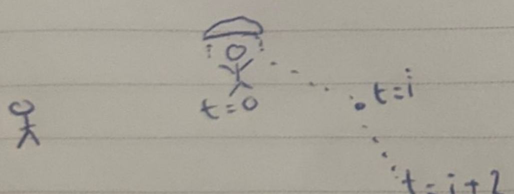
$R_0 = f'(x_i)h + f''(x_i)h^2 + f^{(3)}(x_i)h^3 + \dots$

Si aproximo $R_0$ truncamiento a partir del primer término $R_0 \cong f'(x_i)h$

De la serie de Taylor:

$f(x_{i+1}) \cong f(x_i) + f'(x_i)h \rightarrow f'(x_i) \cong f(x_{i+1}) - f(x_i)/h$

Caso paracaídas




$v(t_{i+1}) = v(t_i) + v(t_{i+2})(t_{i+2} - t_i)$ → (1)

$+ v''(t_i)(t_{i+2} - t_i)^2/2! + \dots + R_n$

truncamiento de la serie en n=1.

$v(t_{i+1}) = v(t_i) + v'(t_i)(t_{i+1} + t_i) + R$ (2)

$R_1 = v'(\xi)(t_{i+2} - t_i)^2/2!$ (3)

Aprox. ec. (2)

(4) $\underbrace{v'(t_i)}_{} = v(t_{i+2}) - v(t_i)/t_{i+2} - t_i$

$\underbrace{-R_1/t_{i+2} - t_i}_{\text{error de truncamiento.}}$

(5) $v''(t_i) = -v'(t_i)/t_{i+2} - t_i$

sust. (5) en (3)

$R_1 = v'(\xi)\overbrace{(t_{i+2} - t_i)}^{h}$

h pequeño - error pequeño

$R_1 = O(t_{i+2} - t_i)$

$= O(h)$

orden → O.

en general

$R_n = O((t_{i+1} - t_i)^n) = O(h^n)$

caso de función de Taylor para una función que...

Sea $f(x) = x^m$ donde m puede ser $m = 1, 2, 3, 4, \ldots$ en el rango $x = 1$ a $2$. la aprox. usando la serie de Taylor de primer orden.

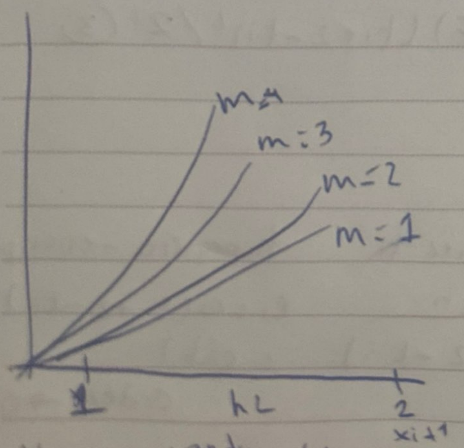
$f(x_{i+1}) \cong f(x_i) + f'(x_i)h = x_i^m + m x_i^{m-1} h$

$R_i = f''(x_i)h^2/2! + f^{(3)}(x_i)h^3/3! + \ldots$

| h | m | $f(x_{i+1}=2)$ | $f$ aprox $(x_{i+1}=2)$ | R |
|---|---|---|---|---|
| 1 | 1 | 2 | 2 | 0 |
| | 2 | 4 | 3 | 1 |
| | 3 | 8 | 4 | 4 |
| | 4 | 16 | 5 | 11 |
| 0.5 | 4 | 5.0625 | 3 | 2.0625 |
| 0.25 | 4 | 2.44196 | 2 | 0.4914 |
| 0.1 | 4 | 1.4641 | 1.4 | 0.064 |
| 0.01 | 4 | 1.0406 | 1.04 | 0.0006 |

$R = m(m-1)x_i^{m-2}h^2/2! + m(m+1)(m-2)x_i^{m-3}h^3/3! + \ldots$

$f(2) = (1)^2 + (2)(1)(1) = 3$ $\quad R_1 = 2(1)1^0 1^2/2! + 2(1)(0)^0/3! \ldots = 1$




Vo comentando la m se va a ir haciendo una curva.