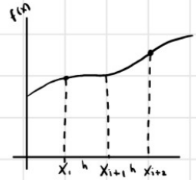
03/ Septiembre /2025

① $-2f(x_{i+1}) = -2f(x_i) - 2f'(x_i)h - 2\frac{f''(x_i)\,h^2}{2!} - O(2h^3)$

$h = x_{i+1} - x_i$
$= x_{i+2} - x_{i+1}$

② + $f(x_{i+2}) = f(x_i) + f'(x_i)\overbrace{(x_{i+2}-x_i)}^{2h} + \frac{f''(x_i)\overbrace{(x_{i+2}-x_i)}^{2h}{}^2}{2!} + O(2h^3)$




$f(x_{i+2}) - 2f(x_{i+1}) = -f(x_i) + \frac{\cancel{2}f''(x_i)h^2}{\cancel{2!}} + O(-h^3)$

hacia atrás es (−)

Diferencias finitas.

$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + \frac{O(h^3)}{\cancel{h^2}}^{O(h)}$ Expresión hacia adelante

$f''(x_i) = \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{h^2} + O(h)$ Expresión hacia atrás

$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{2h^2} + O(h)$ centrada

ó $= \frac{f(x_{i+1}) - f(x_i) - f(x_i) + f(x_{i-1})}{h \cdot h} + Oh$

$f''(x_i) = \frac{\frac{f(x_{i+1}) - f(x_i)}{h} - \frac{f(x_i) - f(x_{i-1})}{h}}{h} = \frac{f'(x_i) - f(x_i)}{h}$

m s pr ci

* TAREA completar como se llega?

De las 3 expresiones demostrar una de la tercera derivada {atrás, adelante o central}
y demostrar la de atrás y la central de la 2da derivada.

MARTES.

## a) 2da derivada hacia átras

$h = x_i - x_{i-1} = x_{i+1} - x_i$

$f(x_{i-1}) = f_i - f_i'h + \frac{1}{2}f_i''h^2 - \frac{1}{6}f_i^{(3)}h^3 + \frac{1}{24}f_i^{(4)}h^4 + \dots$

$f(x_{i-2}) = f_i - 2f_i'h + \frac{1}{2}f_i''(2h)^2 - \frac{1}{6}f_i^{(3)}(2h)^3 + \frac{1}{24}f_i^{(4)}(2h)^4 + \dots$

$f_{i-1} = f_i - f_i'h + \frac{1}{2}f_i''h^2 - \frac{1}{6}f_i^{(3)}h^3 + \frac{1}{24}f_i^{(4)}h^4 + \dots$

$f_{i-2} = f_i - 2f_i'h + 2f_i''h^2 - \frac{8}{6}f_i^{(3)}h^3 + \frac{16}{24}f_i^{(4)}h^4 \dots$

combinación: $f_i - 2f_{i-1} + f_{i-2}$

los term. $f_i'$ cancelan

$f_i - 2f_{i-1} + f_{i-2} = (1-2+1)f_i + (0)f_i'h + (-2\cdot\frac{1}{2}+2)f_i''h^2$
$+ (-2\cdot(-\frac{1}{6}) + (-\frac{8}{6}))f_i^{(3)}h^3 +$ term. orden $h^4 \dots$

Calculando coef.:

- Coef. de $f_i''h^2$: $-1+2=1 \rightarrow +f_i''h^2$
- Coef. de $f_i^{(3)}h^3$: $\frac{2}{6} - \frac{8}{6} = -1 \rightarrow -f_i^{(3)}h^3$.

Por tanto

$f_i - 2f_{i-1} + f_{i-2} = f_i''h^2 - f_i^{(3)}h^3 + O(h^4)$ *Divid. por $h^2$

$\frac{f_i - 2f_{i-1} + f_{i-2}}{h^2} = f_i'' - hf_i^{(3)} + O(h^2)$

## b) 2da derivada centrada.

$$f_{i+1} = f_i + f_i' h + \frac{1}{2} f_i'' h^2 + \frac{1}{6} f_i^{(3)} h^3 + \frac{1}{24} f_i^{(4)} h^4 + \ldots$$
$$f_{i-1} = f_i - f_i' h + \frac{1}{2} f_i'' h^2 - \frac{1}{6} f_i^{(3)} h^3 + \frac{1}{24} f_i^{(4)} h^4 + \ldots$$

Sumando $f_{i+1} - 2f_i + f_{i-1}$:

$f_i'$, $f_i^{(3)}$ se cancelan $\rightarrow h^2 f_i''$, el sig. término no nulo es de orden $h^4$

$$f_{i+1} - 2f_i + f_{i-1} = f_i'' h^2 + \frac{h^4}{12} f_i^{(4)} + O(h^6)$$

Div. $h^2$: $$\frac{f_{i+1} - 2f_i + f_{i-1}}{h^2} = f_i'' + \frac{h^2}{12} f_i^{(4)} + O(h^4)$$

## c) Diferencia centrada para la 3ra derivada.

$$f_{i+2} = f_i + 2h f_i' + 2h^2 f_i'' + \frac{8}{6} h^3 f_i^{(3)} + \frac{16}{24} h^4 f_i^{(4)} + \ldots$$
$$f_{i+1} = f_i + h f_i' + \frac{1}{2} h^2 f_i'' + \frac{1}{6} h^3 f_i^{(3)} + \frac{1}{24} h^4 f_i^{(4)} + \ldots$$
$$f_{i-1} = f_i - h f_i' + \frac{1}{2} h^2 f_i'' - \frac{1}{6} h^3 f_i^{(3)} + \frac{1}{24} h^4 f_i^{(4)} + \ldots$$
$$f_{i-2} = f_i - 2h f_i' + 2h^2 f_i'' - \frac{8}{6} h^3 f_i^{(3)} + \frac{16}{24} h^4 f_i^{(4)} + \ldots$$

comb. $N \equiv f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2}$

Calculamos término a término:

- Ctes. : $1 - 2 + 2 - 1 = 0$
- $f_i$ - términos: $2h - 2h - 2h + 2h = 0$
- $f_i''$ - términos: $2h^2 - 2\left(\frac{1}{2}h^2\right) + 2\left(\frac{1}{2}h^2\right) - 2h^2 = 0$
- $f_i^{(3)}$ - términos: $\frac{8}{6}h^3 - 2\left(\frac{1}{6}h^3\right) + 2\left(-\frac{1}{6}h^3\right) - \left(-\frac{8}{6}h^3\right)$

$$= 8/6\, h^3 - 2/6\, h^3 - 2/6\, h^3 + 8/6\, h^3 = 12/6\, h^3 = 2h^3$$

$$N = 2h^3 f_i^{(3)} + O(h^5)$$

$$f^{(3)}(x_i) \approx \frac{f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2}}{2h^3}$$ con error $O(h^2)$

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