Ejercicio 2

Predice el valor en $x=1$ y $h=1$ usando la serie de Taylor de orden 0 hasta 4 y calculando el residuo en cada caso

$n \quad f(x) = \quad x_{i+1} = \pi/3 \; ; \; x_i = \pi/4.$

La serie de Taylor se construye alrededor del punto conocido de x_i, no de el punto que se desea evaluar.

n	$f(x) = \pi/4$	R_n	$E = f(x) -$ aproximado
0	$\sin\frac{\pi}{4} = 0.707$	0.185	0
1	1.41	-2.2×10^{-2}	0.707
2	0.707	-3.79×10^{-5}	0
3	0	-3.165×10^{-16}	-0.707
4	0.707	8.44×10^{-73}	0

Derivadas

$f(x) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} = 0.707$

$f'(x) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$f''(x) = -\operatorname{sen}\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$f^{3}(x) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$f^{(4)}(x) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

R_n es evaluado en $\frac{\pi}{12}$ (lugar entre $\frac{\pi}{3}$ y $\frac{\pi}{4}$)

$R_n = \frac{f'(x)(h)^{n+1}!}{(n+1)!} = \frac{\frac{\sqrt{2}}{2}\left(\frac{\pi}{12}\right)^{1!}}{1!} = 0.185$

$R_n = \frac{f''(x)(h)^{n+1}!}{(n+1)!} = \frac{-\frac{\sqrt{2}}{2}\left(\frac{\pi}{12}\right)^{2!}}{2!} = 0.0242$

-2.42×10^{-2}

$$Rn^3 = \frac{-\frac{\sqrt{2}}{2}\left(\frac{\pi}{12}\right)^{3!}}{3!} = -3.79 \times 10^{-5}$$

$$Rn^5 = \frac{\frac{\sqrt{2}}{2}\left(\frac{\pi}{12}\right)^{5!}}{5!} = 8.44 \times 10^{-73}$$

$$Rn^4 = \frac{+\frac{\sqrt{2}}{2}\left(\frac{\pi}{12}\right)^{4!}}{4!} = -3.165 \times 10^{-16}$$

$$E = 0.707$$

$$\sin \frac{\pi}{4} = 0.707$$