

Ana Isabel Esquivel Castro 436578

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Predice el valor de $x=1$ con $h=1$ usando la serie de Taylor de orden 0 hasta 4 y calculando el residuo en cada caso.

n	$f(x)=1$	P_n	$E = f(x) - \text{aproximación}$
0	1.2	0.9125	$1 - f(x) = 0.2 - 1.2 = -1$ $= -0.75$
1	0.95	-0.875	$0.2 - 0.95$ $= -0.25$
2	0.45	-0.35	$0.2 - 0.45$ $= -1$
3	0.3	-0.1	$0.2 - 0.3$ $= 0$
4	0.2	0	$0.2 - 0.2$ $= 0$

Función original: $-0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$

$$P_n = \frac{f^{(n)}(0.5)(h)^n}{n!}$$

$$P_n = \frac{f^{(n+1)}(x_i)h^{(n+1)}}{(n+1)!} \quad \left. \begin{array}{l} \text{Error de} \\ \text{truncamiento} \end{array} \right\}$$

$$① P_n(0) = \frac{f'(0.5)(h)^0}{1!} = \frac{f(0.5)}{1!} = 0.5$$

Para $n=0$

② Tomamos un punto medio entre 0 y 1 = 0.5

y sustituimos en la primera derivada de la función original =

$$f'(x) = -0.4x^3 - 0.45x^2 - 1x - 0.25$$

$$③ f(0.5) = -0.4(0.5)^3 - 0.45(0.5)^2 - (0.5) - 0.25 = 0.9125$$

$$④ f(1) = -0.1(1)^4 - 0.15(1)^3 - 0.5(1)^2 - 0.25(1) + 1.2 = 0.2$$

$n=1$

$$P_n(0) = \frac{f''(0.5)(h)^1}{1!} =$$

$$f'(0) = -0.4(0)^3 - 0.45(0)^2 - (0) - 0.25 = -0.25$$

P_1

$$f(0) = 1.2 + \frac{f'(0)h^1}{1!}$$

$$f(0) = 1.2 + \frac{-0.25(1)^1}{1!} = 0.95$$

P_n = Como en la anterior fue la 1ª derivada, en esta es la 2ª derivada

$$= P_n(0) = \frac{f''(0.5)h^2!}{2!}$$

$$f''(0.5) = -1.2x^2 - 0.9x - 1 = -1.2(0.5)^2 - 0.9(0.5) - 1 = -1.75$$

$$P_n = \frac{-1.75(1)^2!}{2!} = -0.875$$

$$Error = 0.2 - 0.95 = -0.75$$

$n=2$

$$f'(0) = -1.2x^2 - 0.9x - 1 = -1.2(0)^2 - 0.9(0) - 1 = -1$$

$$f'(0) = 1.2 + \frac{f'(0)h^1}{1!} + \frac{f''(0)h^2!}{2!} =$$

$$Error = 0.2 - 0.45 = -0.25$$

$$f''(0) = 1.2 + (-0.25) + (-1) = 0.45 //$$

$$P_n(0) = \frac{f'''(0.5)h^3!}{3!}$$

$$f'''(0) = -2.4x - 0.9 = -2.4(0.5) - 0.9 = 0.21$$

$$P_n = \frac{0.21(1)^3!}{3!} = -0.35$$

$n=3$

$$f''(0) = -2.4(0) - 0.9 = -0.9$$

$$f'''(0) = \frac{-0.9(1)^3}{3!} = -0.15$$

$$f''(0) = 0.45 - 0.15 = 0.3$$

$$P_n = \frac{f^{(4)}(h)^4!}{4!}$$

$$f^{(4)} = \frac{-2.4(1)^4!}{4!} = -0.1$$

$$n=4$$

$$f^{IV}(0) = 1.2 + \frac{f(0)(h)^1}{1!} + \frac{f'(0)(h)^2}{2!} + \frac{f''(0)(h)^3}{3!} + \frac{f^{IV}(0)(h)^4}{4!}$$

$$f^{IV}(0) = -0.1$$

$$f^{IV}(0) = 0.30 - 0.1 = 0.20$$

$$R_n = \frac{f^{(5)}(h)}{5!} = \frac{0(1)}{5!} = 0.$$

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