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Demostraciones Diferenciación por Pasos ♡

→ Encontrar una combinación lineal de x_i, x_i+h y x_i+2h para aproximar $f''(x_i)$

Partimos de la serie de Taylor

Para $f(x_i+h)$

$$f(x_i+h) = f(x_i) + f'(x_i)h + f''(x_i)\frac{h^2}{2} + f'''(x_i)\left(\frac{h^3}{6}\right) + O(h^4)$$

Para $f(x_i+2h)$:

$$f(x_i+2h) = f(x_i) + 2hf'(x_i) + \underbrace{2h^2}_{=4h^2} f''(x_i) + f'''(x_i)\frac{4h^3}{3} + O(h^4)$$

⇒ Se toma la combinación $f(x_i+2h) - 2f(x_i+h)$ para cancelar el término de la primera derivada

$$\Rightarrow f(x_i+2h) - 2f(x_i+h) = \left(f(x_i) + 2hf'(x_i) + f''(x_i)2h^2 + f'''(x_i)\frac{4h^3}{3}\right) - 2\left(f(x_i) + f'(x_i) + f'''(x_i)\frac{h^2}{2} + f'''(x_i)\frac{h^3}{6}\right) + O(h^4)$$

$$\Rightarrow f(x_i) + f'(x_i) + 2h + f''\cancel{(x_i)}2h^2 + f'''(x_i)\frac{4h^3}{3}$$

$$- \cancel{2f(x_i)} - \cancel{2f'(x_i)h} - f''(x_i)h^2 - \frac{h^3}{3}f'''(x_i) + Oh^4$$

$$f(x_i+2h) - 2f(x_i+h) = -f(x_i) + h^2 f''(x_i) + h^3 f'''(x_i) + O(h^4)$$

⇒ Sumamos $f(x_i)$ a ambos lados (para cancelar $-f(x_i)$) y div. $\div h^2$

$$f(x_i+2h) - 2f(x_i+h) + f(x_i) = h^2 f''(x_i) + f'''(x_i)h^3 + Oh^4$$

$$\Rightarrow f''(x_i) = \frac{f(x_i+2h) - 2f(x_i+h) + f(x_i)}{h^2} - f'''(x_i)h + O(h^2)$$

$-hf'''(x_i)$

es de orden h ∴ error global ⇒ $O(h)$

$$\Rightarrow f''(x_i) = \frac{f(x_i+2h) - 2f(x_i+h) + f(x_i)}{h^2} + Oh \Big\}$$ 2º derivada hacia adelante

2º derivada hacia atrás.

→ Se emplean los mismos puntos x_i-2h, x_i-h y x_i pero hacia la requerida (restas).

$h = x_i - h$

$$f(x_i-h) = f(x_i) - \cancel{h}f'(x_i)h + f''(x_i)\frac{h^2}{2} - f'''(x_i)\frac{h^3}{6} + f''''(x_i)\frac{h^4}{24} + Oh^5$$

Para $f(x_i-2h)$

$$f(x_i-2h) = f(x_i) - 2hf'(x_i) + 2f''(x_i)h^2 - f'''(x_i)\frac{4h^3}{3} + f''''(x_i)\frac{2h^4}{3} + O(h^5)$$

$2h^2f''(x_i)$ viene de $\frac{(2h)^2}{2}f''(x_i) = 2h^2f''(x_i)$

→ Eliminamos $f'(x_i)$ para con $f(x_i-2h) - 2f(x_i-h)$:

$$f(x_i-2h) - 2f(x_i-h) = \left(f(x_i) - \cancel{2hf'}\, 2f'(x_i)h + 2f''(x_i)h^2 - f'''(x_i)\frac{4h^3}{3} + f''''(x_i)\frac{2h^4}{3}\right) - 2\left(f(x_i) - f'(x_i)h + f''(x_i)\frac{h^2}{2} - f'''(x_i)\frac{h^3}{6} + f''''(x_i)\frac{h^4}{24}\right) + O(h^5)$$

$$\Rightarrow f(x_i) - 2hf'(x_i) + 2f''(x_i)h^2 - f'''(x_i)\frac{4h^3}{3} + f''''(x_i)\frac{2h^4}{3} - 2f(x_i) + 2f'(x_i)h - f''(x_i)h^2 + f'''(x_i)\frac{h^3}{3} - f''''(x_i)\frac{h^4}{12} + O(h^5)$$

$$\Rightarrow f(x_i-2h) - 2f(x_i-h) = -f(x_i) + h^2f''(x_i) - f'''(x_i)h^3 + f''''(x_i)\frac{7h^4}{12} + O(h^5)$$

→ Sumamos $f(x_i)$ a ambos lados:

$$f(x_i-2h)-2f(x_i-h)+f(x_i) = f''(x_i)h^3 + f''''(x_i)\frac{7h^4}{12} + O(h^5)$$

→ Dividir $\div h^2$

$$\Rightarrow f''(x_i) = \frac{f(x_i-2h)-2f(x_i-h)+f(x_i)}{h^2} + f'''(x_i)h - f''''(x_i)\frac{7h^2}{12} + O(h^3)$$

$$\Rightarrow f''(x_i) = \frac{f(x_i-2h)-2f(x_i-h)+f(x_i)}{h^2} + Oh.$$ } Método/ec 2ª derivada hacia atrás.

2ª derivada centrada ☺.

Serie de Taylor y las sumamos

$$f(x_i+h) = f(x_i) + hf'(x_i) + f''(x_i)\frac{h^2}{2} + f'''(x_i)\frac{h^3}{6} + f''''(x_i)\frac{h^4}{24} + O(h^5)$$

$$f(x_i-h) = f(x_i) - f'(x_i)h + f''(x_i)\frac{h^2}{2} - f'''(x_i)\frac{h^3}{6} + f''''(x_i)\frac{h^4}{24} + Oh^5$$

$$f(x_i+h)+f(x_i-h) = 2f(x_i) + f''(x_i)h^2 + f''''(x_i)\frac{h^4}{12} + O(h^5)$$

→ Separamos $f''(x_i)$ restando $2f(x_i)$ a ambos lados

$$\Rightarrow f(x_i+h)+f(x_i-h)-2f(x_i) = f''(x_i)h^2 + f''''(x_i)\frac{h^4}{12} + O(h^5)$$

→ Dividiendo $\div h^2$

$$\Rightarrow f''(x_i) = \frac{f(x_i+h)+f(x_i-h)-2f(x_i)}{h^2} - \underbrace{f''''(x_i)\frac{h^2}{12} + O(h^5)}_{\text{es } O(h^2) \therefore \text{ orden } h^2 \text{ ☺}}$$

$$\Rightarrow f''(x_i) = \frac{f(x_i+h)+f(x_i-h)-2f(x_i)}{h^2} + O(h^2)$$

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