

Notación

$$\frac{df}{dt} = \lim_{t \rightarrow 0} \frac{\Delta f}{\Delta t}$$

$$\frac{df}{dt} \approx \frac{\Delta f}{\Delta t} = \frac{f(t_{i+1}) - f(t_i)}{t_{i+1} - t_i}$$

Serie de Taylor

$$f(x) = f(a) + f'(a)(x-a) + f''(a) \frac{(x-a)^2}{2!} + \dots + \textcircled{R_n}$$

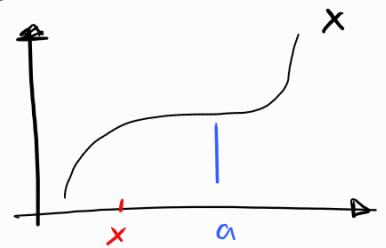
= Residuo

$$R_n = \int_a^x \frac{(x-a)^n f^{(n+1)}(a)}{n!} da$$

a: Variable muda

Otra expresión es

$$R_n = \frac{f^{(n+1)}(\xi) (x-a)^{n+1}}{n+1}$$



↳ Forma de Lagrange de Residuos

$$h = x_{i+1} - x_i$$

$$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$$

Predicir el valor en $x=1$ con $h=1$ usando la serie de Taylor de orden 0 hasta 4, calculando el residuo en cada caso

n	$f(x+1)$	R_n
0	1.2	-0.91
1	$(1.2) + (-0.25) = 0.95$	-0.875
2	0.45	-0.35
3	0.30	-0.1
4	0.2	0

Como $h=1$ f. que $(x_{i+1} - x_i) = 1$

$$\begin{array}{l} x_{i+1} = 1 \\ x_i = 0 \end{array} \quad \begin{array}{l} \text{Se sigue la forma} \\ \frac{f^{(n+1)}(\xi)(x_{i+1} - x_i)^n}{(n+1)!} \end{array}$$

$n=0$

funcion

$$\frac{f^{(0)}(0)1}{0!} = 1.2$$

Residuo

$$\frac{f^{(1)}(0.5)(1)}{1!} = -0.91$$

$n=1$

funcion

$$\frac{f^{(1)}(0)1}{1!} = -0.25$$

$$\frac{f^{(2)}(0.5)}{2!} = \frac{-1.2x^2 - 0.9x - 1}{2!} \Big|^{0.5} = -0.875$$

$$n=2 \quad \frac{f^{(2)}(0)(1)}{2!} = \frac{-1.2x^2 - 0.9x - 1}{2!} \Big|_0^1 = -0.5$$

$$\frac{f^{(3)}(0.5)}{3!} = \frac{-2.4x - 0.9}{3!} \Big|_0^{0.5} = -0.35$$

$$n=3 \quad \frac{f^{(3)}(0)(1)}{3!} = \frac{-2.4x - 0.9}{3!} \Big|_0^1 = -0.15$$

$$n \quad \frac{f^{(4)}(0.5)}{4!} = \frac{-2.4}{4!} \Big|_0^{0.5} = -0.1$$

$$n=4$$

$$// \quad // = -0.1$$

$$n \quad \frac{f^{(5)}(0.5)}{5!} = \frac{0}{5!} = 0$$

Predice el valor de $f(x)$ en $x = \pi/3$ en torno a $\frac{\pi}{4}$ en orden $O a 4$ calculando el residuo de cada uno

$$x_{i+1} = \pi/3$$

$$x_i = \pi/4$$

$$h = \frac{\pi}{3} - \frac{\pi}{4}$$

$$h = 15^\circ$$

n	$f(x_{i+1})$	R_n
0	0.71	0.13
1		
2		
3		
4		

$$n=1$$

$$f(\pi/4) = 0.71$$

$$\frac{f'(\pi/4)(1)}{1!} = \sin(\pi/4) = 0.13$$

$$n=2$$