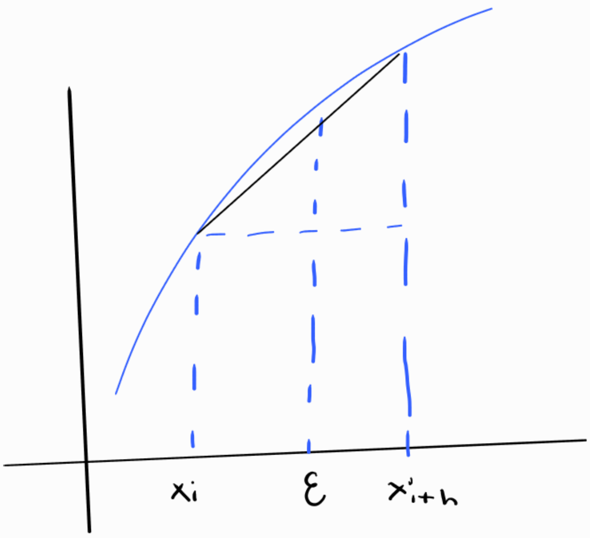
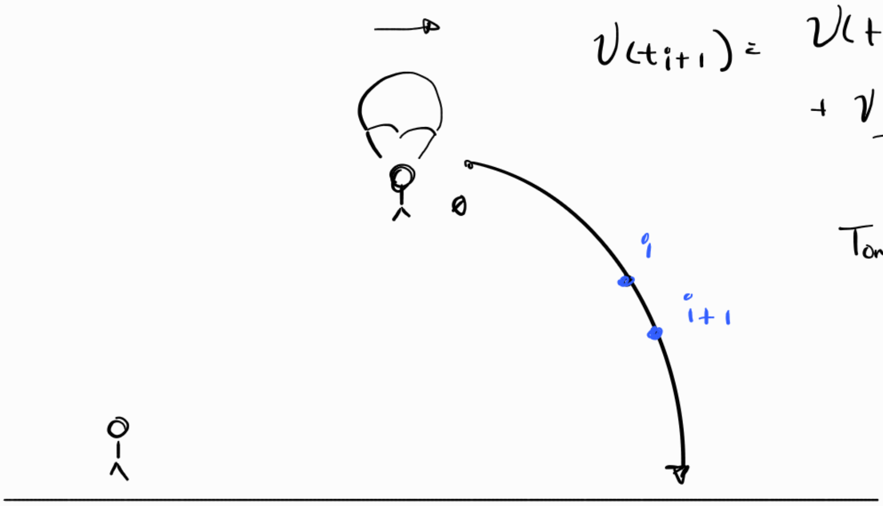
Teorema del punto medio

$$f'(x) \approx \frac{f(x_{i+1}) + f(x)}{h}$$





$$v(t_{i+1}) = v(t_i) + v'(t_{i+1})(t_{i+1}-t_i) + \frac{v''(t_i)(t_{i+1}-t_i)^2}{2!} + \dots + R_n \quad (1)$$

Tomando en $n=1$ (2)

$$v(_{i+1}) = v(t_i) + v'(t_i)(t_{i+1}-t_i) + R_1$$

$$R_1 = \frac{v''(\varepsilon)(t_{i+1}-t_i)^2}{2} \quad (3)$$

$\rightarrow$ Aprox. de la ec. 2

$$v'(t_i) = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1}-t_i} \quad (4) \approx v'(t_i)$$

$$\underbrace{\frac{-R_i}{t_{i+1}-t_i}}_{\text{Error de truncamiento}}$$

$$v''(t_i) = \frac{-v'(t_i)}{(t_{i+1} - t_i)} \quad ⑤$$

Sustituimos ⑤ en ③

$$R_1 = \frac{v'(\varepsilon)(t_{i+1} - t_i)}{2!}$$

El error cometido es de orden de magnitud h

$$R_1 = O(t_{i+1} - t_i)$$
$$= O(h)$$
$$R_n = O(n) = \frac{f^{(n+1)}(\varepsilon)(x_{i+1} - x)^{n+1}}{(n+1)!}$$

En general R_n es de orden h^n

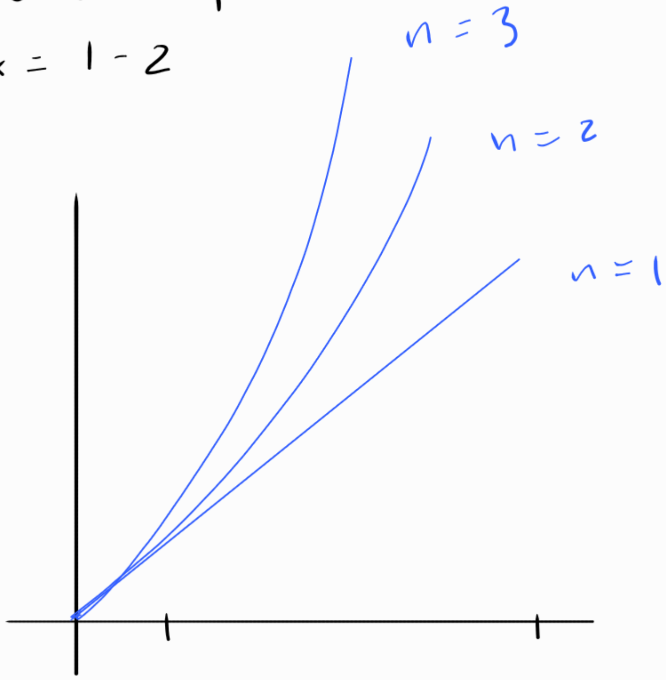
$$R_n = O((t_{i+1} - t_i)^n)$$
$$= O(h^n)$$

$$v(t_{+i}) = v(t_i) + v'(t_{i+1})(t_{i+1} - t_i)$$

Sea $f(x) = x^n$

donde n puede ser $n = 1, 2, 3, 4 \ldots$ en el rango

$x = 1 - 2$



La aproximación usando la serie de taylor de primer orden

$$f(x_{i+1}) \cong f(x_i) + f'(x_i)h$$

$$R_1 = \frac{f''(x_i)h^2}{2!} + \frac{f^{(3)}(x_i)h^3}{3!}$$

$$f(x_{i+1}) = x_i^n + nx_i^{n-1}h \qquad \text{Donde } i+1 = 2$$

$$R_1 = \frac{n(n-1)(x_i^{n-2})h^2}{2!} + \frac{n(n-1)(n-2)(x_i^{n-3})h^3}{3!}$$

n	$f^V(x_{i+1})$	aprox. $f(x_{i+1})$	R
1	2	$1+1=2$	$\emptyset$
2	4	$1+2=3$	1
3	8	$1+3=4$	$3+1=4$
4	16	5	10

$n=4 \quad x_i=1 \quad x_{i+1}=2$

$$f^V(x_{i+1}) = f^V(2) = 2^4 = 16$$

$$f(x_{i+1}) = 1+4 = 5$$

$$R_n = \frac{4(3)(1^2)1^2}{2!} = 6 + \frac{4(3)(2)(1^1)(1^3)}{3!} = 4$$

h	n	$f^{V}_{x_{i+1}}$	$f(x_{i+1})$	R
0.5	4	5.0625	3	2
0.25		2.4414	2	0.4375
0.1		1.4641	1.4	0.064
0.01		1.0406	1.04	0.000604