Clase 12. Metodos Númericos.

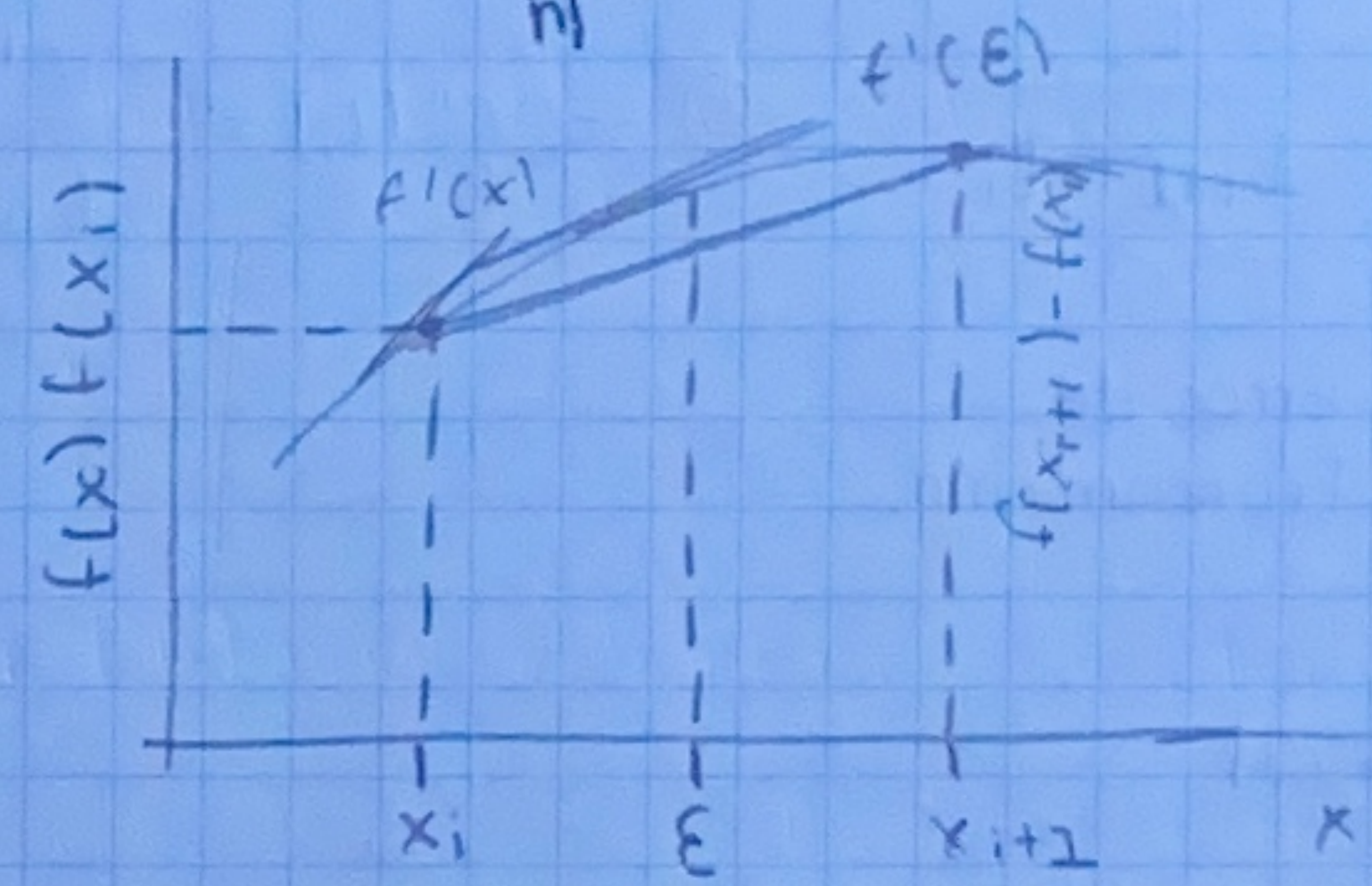
29-Agosto-2025.

Caso de serie de Taylor.

Inicio

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)(x_{i+1}-x_i)^2}{2!}$$
$$+ \frac{f^{(3)}(x_i)(x_{i+1}+x_i)^3}{3!} + \dots + \frac{f^{(n)}(x_i)(x_{i+1}-x_i)^n}{n!} + R_n$$

$$R_n = \frac{f^{(n+1)}(\xi)(x_{i+1}-x_i)^{n+1}}{(n+1)!}$$



Caso particular.

$f(x_{i+1}) \cong f(x_i)$

$R_0 = f'(\xi)(x_{i+1} - x_i)$

Aprox...

$f(x_{i+1}) \cong f(x_i)$

$R_0 = f'(x_i)h + f''(x_i)h^2 + f^{(3)}(x_i)h^3 + \dots$

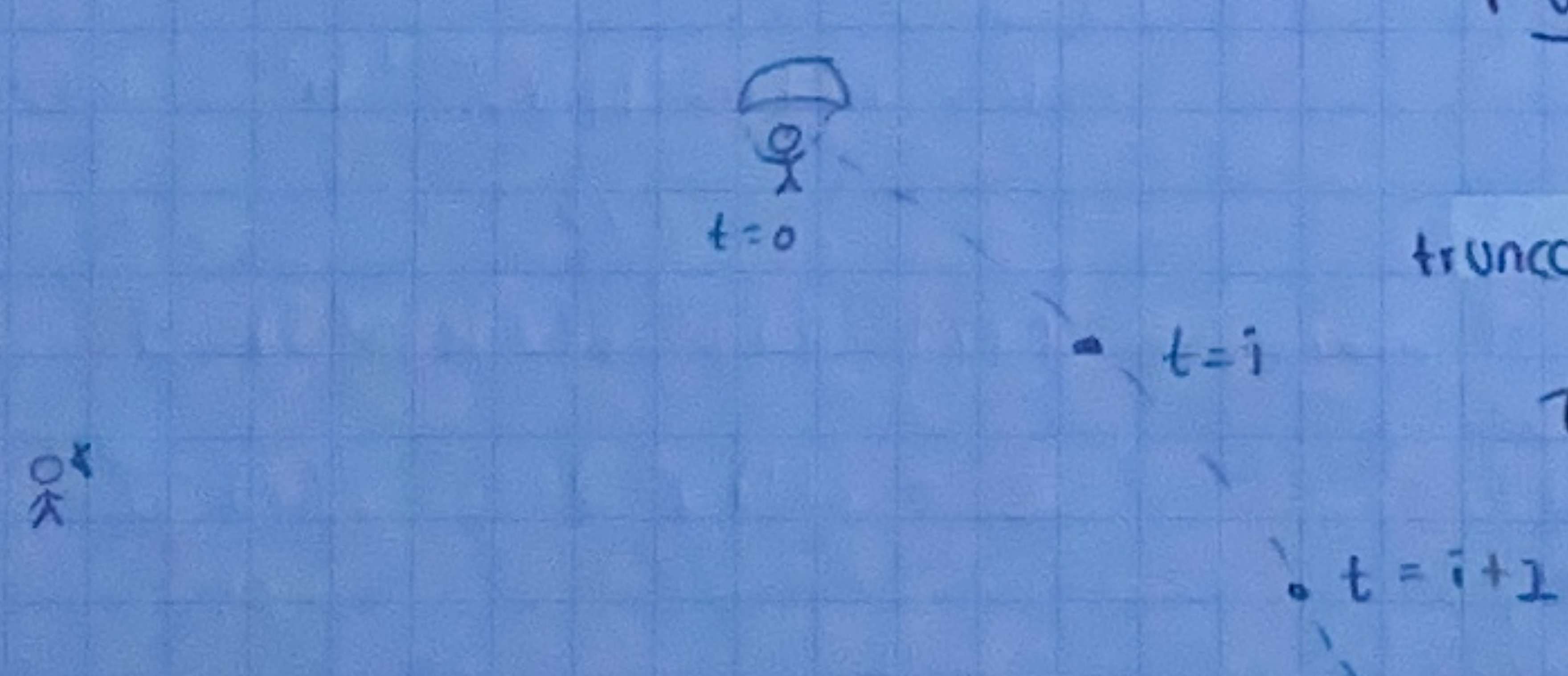
si aproximo R_0 truncamiento a partir del primer término

$R_0 \cong f'(x_i)h$

De la serie de Taylor ∴

$$f(x_{i+1}) \cong f(x_i) + f'(x_i)h \Rightarrow f'(x_i) \cong \frac{f(x_{i+1}) - f(x_i)}{h}$$

Caso del paracaidas.



$$v(t_{i+1}) = v(t_i) + v'(t_{i+1})(t_{i+1} - t_i) + \frac{v''(t_i)(t_{i+1}-t_i)^2}{2!} + \dots + R_n \quad — ①$$

truncamiento la serie en $n=1$

② $$v(t_{i+1}) = v(t_i) + v'(t_i)(t_{i+1} - t_i) + R_1$$

$$R_1 = \frac{v'(\xi)(t_{i+1} - t_i)^2}{2!} \quad - ③$$

Aprox...

de la ecuación 2...

④ $$-v'(t_i) = \overbrace{\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}}^{\approx v'(t_i)}$$

$$\underbrace{\frac{-R_1}{t_{i+1} - t_i}}$$

Error de Truncamiento.

⑤ $$-v''(t_i) = -\frac{v'(t_i)}{t_{i+1} - t_i}$$

Sustituyendo ⑤ en ③

$$R_1 = \frac{v'(\varepsilon)\overbrace{(t_{i+1} - t_i)}^{h}}{2!}$$

h pequeño — error ~~grande~~ pequeño

h grande — error ~~pequeño~~ grande.

$$R_1 = O(t_{i+1} - t_i)$$
$$= \underline{\underline{O}}(h)$$

orden → O

en general

$$R_n = O((t_{i+1} - t_i)^n)$$
$$= O(h^n)$$

Caso de función de Taylor para una función que...

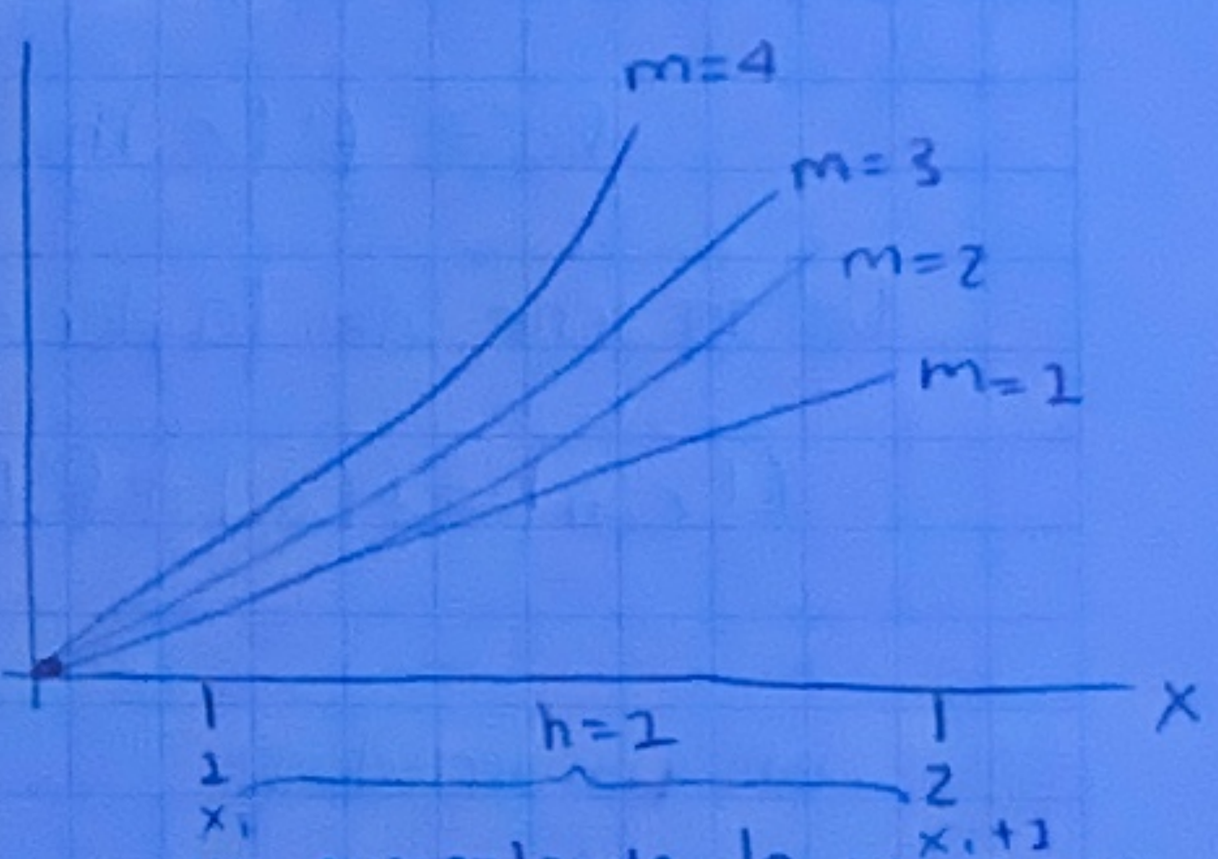
Sea

$$f(x) = x^n$$

donde m puede ser m = 1, 2, 3, 4, ... en el rango x = 1 a 2.
La aproximación usando la serie de taylor de primer orden.

$$f(x_{i+1}) \cong f(x_i) + f'(x_i)h = x_i^m + m x_i^{m-1} h$$

$$R_1 = \frac{f''(x_i)h^2}{2!} + \frac{f^{(3)}(x_i)h^3}{3!} + \dots$$



va aumentando la m se va a ir haciendo una curva

h	m	$f^{v}(x_{i+1}=2)$	aprox $f(x_{i+1}=2)$	R
1	1	2	2	0
	2	4	3	1
	3	8	4	4
	4	16	5	11
0.5	4	5.0625	3	2.0625
0.25	4	2.44140	2	0.4414
0.1	4	1.4641	1.4	0.064
0.01	4	1.0406	1.04	0.0006

$$R = \frac{m(m-1)x_i^{m-2}h^2}{2!} + \frac{m(m-1)(m-2)x_i^{m-3}h^3}{3!} + \dots$$

m=2 ⟶ $f(2) = (1)^2 + (2)(1)(1) = 3$

$$R_1 = \frac{2(1)1^0 1^2}{2!} + \frac{\cancel{2(1)(0)1^0}}{3!} \dots$$
$$= 1$$