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Tarea_09_Sep.

$h = x_{i+1} - x_i$
$= x_{i+2} - x_{i+1}$

① $-2f(x_{i+1}) = -2f(x_i) - 2f'(x_i)h - 2\frac{f''(x_i)h}{2!} - O(2h^3)$

② $+f(x_{i+2}) = f(x_i) + f'(x_i)\overbrace{(x_{i+2} - x_i)}^{2h} + \frac{f''(x_i)\overbrace{(x_{i+2} - x_i)}^{2h}{}^2}{2!} + O(2h^3)$

$f(x_{i+2}) - 2f(x_{i+1}) = -f(x_i) + \frac{\cancel{2}f''(x_i)h^2}{\cancel{2!}} + O(-h^2)$

hacia atrás es (-)

Diferencias finitas.

$f''(x_i) = \frac{f(x_i+2) - 2f(x_i+1) + f(x_i)}{h^2} + \frac{\cancel{O(h^2)}^{O(h)}}{\cancel{h^2}}$ Expresión hacia adelante

$f''(x_i) = \frac{f(x_i-2) - 2f(x_i-1) + f(x_i)}{h^2} + O(h)$ Expresión hacia atrás

$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{2h^2} + O(h)$ centrada

$f''(x_i) = \frac{f(x_i)\cdot f}{}$

$\sigma = \frac{f(x_i+1) - f(x_i) - f(x_i) + f(x_i-1)}{\cancel{h} h h} + O(h)$

a) 2da derivada hacia atrás.

$h = x_i - x_{i-1} = x_{i+1} - x_i$

$f(x_i-1) = f_i - f_i'h + \frac{1}{2}f_i''h^2 - \frac{1}{6}f_i^{(3)}h^3 + \frac{1}{24}f_i^{(4)}h^4 + \ldots$

$f(x_i-2) = f_i - 2f_i'h + \frac{1}{2}f_i''(2h)^2 - \frac{1}{6}f_i^{(3)}(2h)^3 + \frac{1}{24}f^{(4)}(2h)^4 + \ldots$

$f_{i-1} = f_i - f_i' + \frac{1}{2}f_i''h^2 - \frac{1}{6}f_i^{(3)}h^3 + \frac{1}{24}f_i^{(4)}h^4 + \ldots$

$f_{i-2} = f_i - 2f_i'h + 2f_i''h^2 - \frac{8}{6}f_i^{(3)}h^3 + \frac{16}{24}f^{(4)}h^4 \ldots$

Conbinación: $f_i - 2f_{i+1} + f_{i-2}$

$f_i - 2f_{i-1} + f_{i-2} = (1-2+1)f_i + (0)f_i'h + (-2\cdot\frac{1}{2} + 2)f_i''h^2$
$+ (-2(-\frac{1}{6}) + (-\frac{8}{6})f_i^{(3)}h^3 +$ termino orden $h^4 \ldots$

Calculando coeficiente

- Coeficiente de $f_i''h^2 - 1 + 2 = 1 \rightarrow +f_i''h^2$
- Coeficiente de $f_i^{(3)}h^3 = \frac{2}{6} - \frac{8}{6} = -1 \rightarrow -f_i^{(3)}h^3$

Por lo tanto

$f_i - 2f_{i-1} + f_{i-2} = f_i''h^2 - f_i^{(3)}h^3 + O(h^4) \xrightarrow{\div h^2} \frac{f_i - 2f_{i-1} + f_{i-2}}{h^2}$

$= f_i'' - hf_i^{(3)} + O(h^2)$

b) 2da derivada centrada.

$$f_{i+1} = f_i + f_i' h + \frac{1}{2} f_i'' h^2 + \frac{1}{6} f_i^{(3)} h^3 + \frac{1}{24} f_i^{(4)} h^4 + \dots$$
$$f_{i-1} = f_i - f_i' h + \frac{1}{2} f_i'' h^2 - \frac{1}{6} f_i^{(3)} h^3 + \frac{1}{24} f_i^{(4)} h^4 + \dots$$

Sumando $f_{i+1} - 2f_i + f_{i-1}$:

f_i', $f_i^{(3)}$ se cancela $\longrightarrow$ $h^2 f_i''$, el siguiente termino no nulo es de orden h^4

$$f_{i+1} - 2f_i + f_{i-1} = f_i'' h^2 + \frac{h^4}{12} f_i^{(4)} + O(h^6)$$

Dividimos por h^2: $\dfrac{f_{i+1} - 2f_i + f_{i-1}}{h^2} = f_i'' + \dfrac{h^2}{12} f_i^{(4)} + O(h^4)$

c) Diferencia centrada para la 3ra derivada.

$$f_{i+2} = f_i + 2h f_i' + 2h^2 f_i'' + \frac{8}{6} h^3 f_i^{(3)} + \frac{16}{24} h^4 f_i^{(4)} + \dots$$
$$f_{i+1} = f_i + h f_i' + \frac{1}{2} h^2 f_i'' + \frac{1}{6} h^3 f_i^{(3)} + \frac{1}{24} h^4 f_i^{(4)} + \dots$$
$$f_{i-1} = f_i - h f_i' + \frac{1}{2} h^2 f_i'' - \frac{1}{6} h^3 f_i^{(3)} + \frac{1}{24} h^4 f_i^{(4)} + \dots$$
$$f_{i-2} = f_i - 2h f_i' + 2h^2 f_i'' - \frac{8}{6} h^3 f_i^{(3)} + \frac{16}{24} h^7 f_i^{(4)} + \dots$$

combinatoria. $N \equiv f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2}$

Calculamos termino a término:

- Constante $1 - 2 + 2 - 1 = 0$

f_i' — términos: $2h - 2h - 2h + 2h = 0$

f_i'' — términos: $2h^2 - 2(\frac{1}{2}h^2) + 2(\frac{1}{2}h^2) - 2h^2 = 0$

f_i''' — términos: $\frac{8}{6}h^3 - 2(\frac{1}{6}h^3) + 2(-\frac{1}{6}h^2) - (-\frac{8}{6}h^3)$

$$= \frac{8}{6}h^3 - \frac{2}{6}h^3 - \frac{2}{6}h^3 + \frac{8}{6}h^3 = \frac{12}{6}h^3 = 2h^3$$

$$N = 2h^3 f_i^{(3)} + O(h^5)$$

$$f^{(3)}(x_i) \approx \frac{f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2}}{2h^3}$$ con error $O(h^2)$