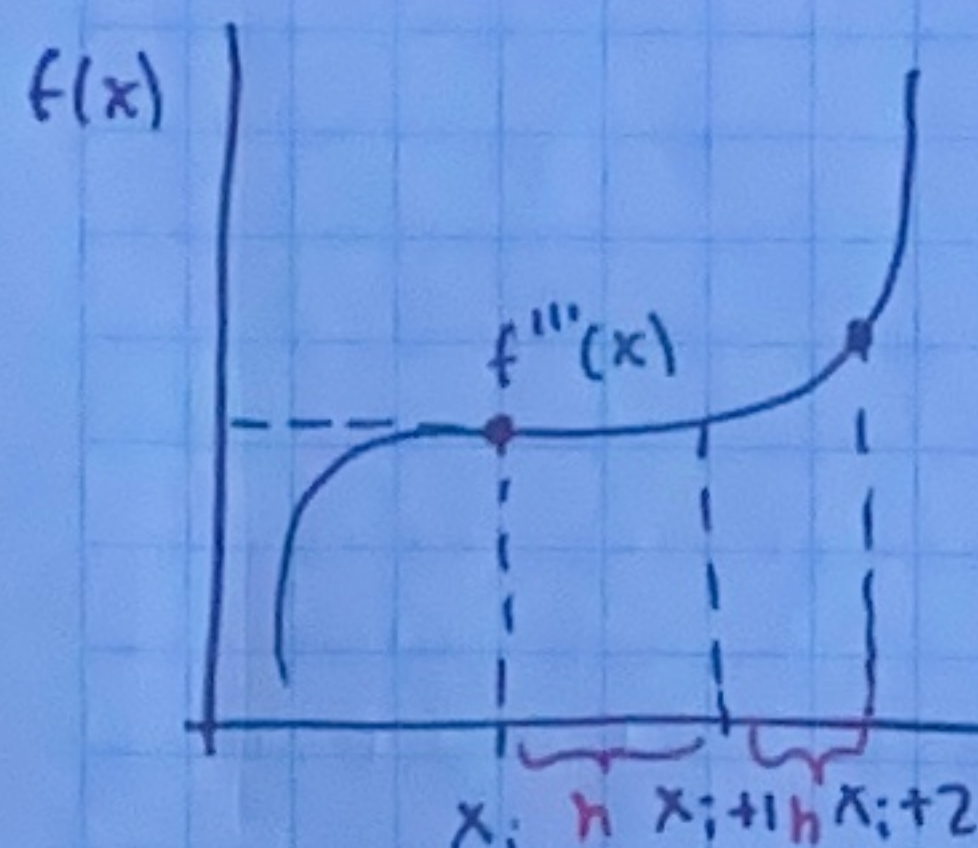


Tarea. Demostrar la de atrás y la central de la 2da derivada.

Tarea. Encontrar para la tercera derivada del tema. una de las tres derivadas

Martes  
09-09-25



$$f(x_{i+2}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + O(h^3)$$

$$f(x_{i+2}) = f(x_i) + f'(x_i)(x_{i+2} - x_i) + \frac{f''(x_i)(x_{i+2} - x_i)^2}{2!} + O(h^3)$$

$$\textcircled{1} -2f(x_{i+1}) = -2f(x_i) - 2f'(x_i)h - \frac{2f''(x_i)h^2}{2!} - O(h^3) \quad h = x_{i+2} - x_i$$

$$\textcircled{2} f(x_{i+2}) = f(x_i) + f'(x_i)(x_{i+2} - x_i) + \frac{f''(x_i)(x_{i+2} - x_i)^2}{2!} + O(h^3)$$

$$f(x_{i+2}) - 2f(x_{i+1}) = -f(x_i) + \frac{f''(x_i)h^2}{2!} + O(h^3)$$

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + \frac{O(h^3)}{h^2} = \frac{O(h^3)}{h^2} + O(h)$$

Segunda diferencial hacia adelante.

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + O(h)$$

Segunda diferencial hacia atrás.

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + O(h)$$

$$\text{ó } f''(x_i) = \frac{f(x_{i+2}) - f(x_{i+1}) - f(x_i) + f(x_{i-1}))}{h \cdot h} + O(h^2)$$

segunda diferencial central.

$$f''(x_i) = \frac{\frac{f(x_{i+2}) - f(x_{i+1})}{h} - \frac{f(x_{i+1}) - f(x_i)}{h}}{h} = \frac{f'_+(x_i) - f'_-(x_i)}{h}$$

Nuevo tema. Propagación del error.

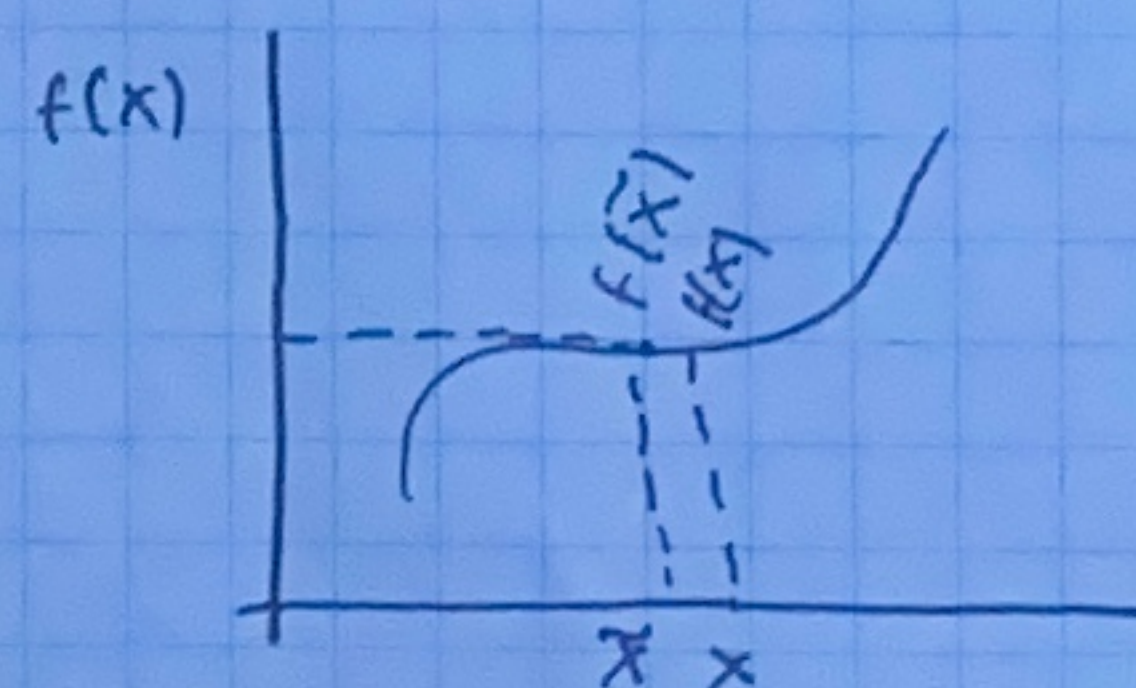
$$\Delta f(\bar{x}) = |f(x) - f(\bar{x})| = |f'(\bar{x})| \Delta x \approx h$$

Usando la serie de Taylor

$$f(x) = f(\bar{x}) + f'(\bar{x})(x - \bar{x}) + \frac{f''(\bar{x})(x - \bar{x})^2}{2!} + \dots$$

$$f(x) \approx f(\bar{x}) + f'(\bar{x})(x - \bar{x})$$

$$f(x) - f(\bar{x}) \approx f'(\bar{x})(x - \bar{x})$$





Ejemplo. sea  $\tilde{x} = 2.5$   $\Delta x = 0.01$

¿Cuál es el error resultante para  $f(x) = x^3$ ?

$$\Delta f(\tilde{x} = 2.5) = f'(\tilde{x}) \Delta x$$

$$= 3(2.5)^2 \cdot 0.01 = 0.1875$$

$$f(2.5) = 15.625 \pm 0.1875$$

Nuevo tema. Estabilidad y condición.

$$f(x) = f(\tilde{x}) + f'(\tilde{x})(x - \tilde{x})$$

Error relativo en f.

$$\frac{f(x) - f(\tilde{x})}{f(x)} = \frac{f'(\tilde{x})(x - \tilde{x}) \cdot \tilde{x}}{f(x) \cdot \tilde{x}} \approx \frac{f'(\tilde{x})(\tilde{x})}{f(\tilde{x})} \cdot \frac{(x - \tilde{x})}{\tilde{x}}$$

Error relativo en x.

$$\text{Condición} = \frac{f'(\tilde{x}) \tilde{x}}{f(\tilde{x})}$$

Tabla de error estimado relacionado con las operaciones matemáticas comunes usando número inexactos  $\tilde{u}$  y  $\tilde{v}$

Operación.		Error estimado.
Adición	$\Delta(\tilde{u} + \tilde{v})$	$\Delta\tilde{u} + \Delta\tilde{v}$
Restación	$\Delta(\tilde{u} - \tilde{v})$	$\Delta\tilde{u} + \Delta\tilde{v}$
Multiplicación	$\Delta(\tilde{u} \times \tilde{v})$	$ \tilde{u}  \Delta\tilde{v} +  \tilde{v}  \Delta\tilde{u}$
División	$\Delta\left(\frac{\tilde{u}}{\tilde{v}}\right)$	$\frac{ \tilde{u}  \Delta\tilde{v} +  \tilde{v}  \Delta\tilde{u}}{ \tilde{v} ^2}$