Clase 9. Metodos Numericos.

FECHA: 22 /Agosto /25.

Ejemplo 1.

$f(x) = -0.14x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$

Predecir el valor en $x=1$ con $h=1$ usando la serie de Taylor de orden cero hasta 4 calculando el residuo en cada caso.

$\xi = 0.5$

| 0 | f(x=1) | Rn | E = f(1) - aproximación |
|---|---|---|---|
| 1 | 1.2 | -0.91 | 0.20 - 1.20 = 1 |
| 2 | 0.95 | 0.87 | 0.20 - 0.95 = 0.75 |
| 3 | 0.45 | -0.35 | 0.20 - 0.45 = 0.25 |
| 4 | 0.30 | -0.10 | 0.20 - +0.1 |
| 5 | 0.20 | 0 | 0.20 0 |

$R_{n(1)} = \frac{f^{(n+1)}(\xi)}{(n+1)!} = (1-0)^{n-1}$

Derivadas.

$f'(x) = 0.4x^3 - 0.45x^2 - x - 0.25$

$f''(x) = -1.2x^2 - 0.4x - 1$

$f^{(3)}(x) = -2.4x - 0.4$

$f^{(4)}(x) = -2.4$

$f^{(5)}(x) = 0$

En $x_0 = 0$

$f'(0) = -1$

$f^2(0) = -0.9$

$f^4(0) = -2.4$

Residuos con $\xi = 0.5$

$R_0 = f'(0.5) = -0.4125$

$R^1 = f'' \frac{(0.5)}{2} = -0.875$

$R_2 = f^{(3)} \frac{(0.5)}{2} = -0.35$

$R_3 = \frac{f^{(4)}(0.5)}{24} = -0.10$

$R_4 = \frac{f^{(5)}(0.5)}{120} = 0$

Formula (polinomios de Taylor en x=1)

$T_n(1) = \sum_{k=0}^{n} \frac{f^k(0)}{k!} 1^k$

$T(0)(1) = 1.2$

$T(1)(1) = 1.2 - 0.25 = 0.95$

$T(2)(1) = 0.95 + (-1/2) = 0.45$

$T(3)(1) = 0.45 + \left(-\frac{0.9}{6}\right) = 0.30$

$T(4)(1) = 0.30 + \left(-\frac{2.4}{24}\right) = 0.20$

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Ejemplo 2.

$f(x) = \cos x$

Predecir el valor en $x = \frac{\pi}{3}$ usando la serie de Taylor entorno a $x = \frac{\pi}{4}$, de orden cero hasta 4, calculando el residuo en cada caso. Usando $x_{i+1} = \frac{\pi}{3}$

$x_i = \frac{\pi}{4}$

| O | f(x=1) | Rn | E = f(1) - aproximación |
|---|---|---|---|
| 1 | 0.71 | -0.03 | -0.207 |
| 2 | 0.52 | -0.03 | -0.021 |
| 3 | 0.49 | 0.00039 | 0.0022 |
| 4 | 0.49 | 0.00019 | 0.00013 |
| 5 | 0.50 | -0.0000013 | -0.0000075 |

$f(x) = \cos\left(\frac{\pi}{3}\right) = 0.5$

$R_{n(1)} = \frac{f^{(n+1)}(\xi)}{(n+1)!} h^{n+1}$

Derivadas.

$f(x) = \cos x$

$f'(x) = -\operatorname{sen} x$

$f''(x) = -\cos x$

$f^{(3)}(x) = \operatorname{sen} x$

$f^{(4)}(x) = \cos x$

En $x_0 = 0$

$f'(0) = -1$

$f^2(0) = -0.9$

$f^4(0) = -2.4$

Residuos con $\xi = \pi/24$

$R_0 = -\operatorname{sen} \xi\, h_1 = -0.0341$

$R_1 = -\cos \xi / 2h^2 = -0.0339$

$R_2 = \operatorname{sen} \xi / 6h^3 = 0.0089$

$R_3 = \cos \xi / 24h^4 = 0.00019$

$R_4 = -\operatorname{sen} \xi / 120h^5 = -0.0000019$

Formula (polinomios de Taylor en $x = \pi/3$)

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$a = \pi/4,\ h = \pi/12$

$T_{(0)}(1) = \cos a = 0.71$

$T_{(1)}(1) = \cos a - \operatorname{sen} a = 0.5219$

$T_{(2)}(1) = T_1 - \frac{\cos a}{2} h^2 = 0.497$

$T_{(3)}(1) = T_2 + \frac{\operatorname{sen} a}{6} h^3 = 0.499$

$T_{(4)}(1) = T_3 + \frac{\cos a}{24} h^4 = 0.500$

$\to T_0 + f' \cos h = 0.71 - 0.71 + 0.26$

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