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Segunda derivada: Fórmula hacia atrás

$f''(x_i)$ usando puntos hacia izquierda $f(x_i), f(x_{i-1}), f(x_{i-2})$

$f(x_i) = f(x_i)$

$f(x_{i-1}) = f(x_i - h) = f(x_i) - f'(x_i)h + \frac{f''(x_i)}{2}h^2 - \frac{f'''(x_i)}{6}h^3 + O(h^4)$

$f(x_{i-2}) = f(x_i - 2h) = f(x_i) - 2f'(x_i)h + 2f''(x_i)h^2 - \frac{4f'''(x_i)}{3}h^3 + O(h^4)$

Eliminamos $f'(x_i)$ y $f(x_i)$

Encontrar A, B, C

$$Af(x_i) + Bf(x_{i-1}) + Cf(x_{i-2}) = f''(x_i)h^2 + O(h^P)$$

$$Af(x_i) + B\left[f(x_i) - f'(x_i)h + \frac{f''(x_i)}{2}h^2 - \frac{f'''(x_i)}{6}h^3\right] + C\left[f(x_i) - 2f'(x_i)h + 2f''(x_i)h^2 - \frac{4f'''(x_i)}{3}h^3\right] = f''(x_i)h^2 + O(h^P)$$

Agrupamos

$f(x_i) : A + B + C$

$f'(x_i) : -Bh - 2Ch$

$f''(x_i) : \frac{B}{2}h^2 + 2Ch^2$

$f'''(x_i) - \frac{B}{6}h^3 - \frac{4C}{3}h^3$

Eliminamos y obtener

1. $A + B + C = 0$ eliminar $f(x_i)$
2. $-B - 2C = 0$ ... $f'(x_i)$
3. $\frac{B}{2} + 2C = 1$ Obtener $f''(x_i)h^2$

2 y 3 sustituimos

$$\frac{-2C}{2} + 2C = -C + 2C = C = 1 \rightarrow B = -2$$

De: $A - 2 + 1 = 0 \rightarrow A = 1$

$$f(x_i) - 2f(x_{i-1}) + f(x_{i-2}) = f''(x_i)h^2 + \left[-\frac{-2}{6} - \frac{4}{3}\right]f'''(x_i)h^3 + O(h^4) = f''(x_i)h^2$$

$$+ \left(\frac{1}{3} - \frac{4}{3}\right)f'''(x_i)h^3 + O(h^4)$$

$$= f''(x_i)h^2 - f'''(x_i)h^3 + O(h^4)$$

$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2} + f'''(x_i)h + O(h^2)$$

Error $O(h)$

○ Segunda derivada, formula centrada.

puntos simetricos: $f(x_{i-1})$, $f(x_i)$, $f(x_{i+1})$

· $f(x_{i+1}) = f(x_i+h) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2}h^2 + \frac{f'''(x_i)}{6}h^3 + \frac{f^4(x_i)}{24}h^4 + O(h^5)$

· $f(x_{i-1}) = f(x_i-h) = f(x_i) - f'(x_i)h + \frac{f''(x_i)}{2}h^2 - \frac{f'''(x_i)}{6}h^3 + \frac{f^4(x_i)}{24}h^4 + O(h^5)$

Sumamos

$$f(x_{i+1}) + f(x_{i-1}) = 2f(x_i) + f''(x_i)h^2 + \frac{f^4(x_i)}{12}h^4 + O(h^5)$$

$$f(x_{i+1}) - 2f(x_i) + f(x_{i-1}) = f''(x_i)h^2 + \frac{f^4(x_i)}{12}h^4 + O(h^5)$$

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2} - \frac{f^{(4)}(x_i)}{12}h^2 + O(h^3)$$

○ Tercera derivada: Formula centrada

· Usaremos formula simetrica de 5 puntos

$f(x_{i+2}) = f(x_i) + 2f'(x_i)h + 2f''(x_i)h^2 + \frac{4f'''(x_i)}{3}h^3 + \frac{2f^{(4)}(x_i)}{3}h^4 + O(h^5)$

$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2}h^2 + \frac{f'''(x_i)}{6}h^3 + \frac{f^{(4)}(x_i)}{24}h^4 + O(h^5)$

$f(x_{i-1}) = f(x_i) - f'(x_i)h + \frac{f''(x_i)}{2}h^2 - \frac{f'''(x_i)}{6}h^3 + \frac{f^{(4)}(x_i)}{24}h^4 + O(h^5)$

$f(x_{i-2}) = f(x_i) - 2f'(x_i)h + 2f''(x_i)h^2 - \frac{4f'''(x_i)}{3}h^3 + \frac{2f^{(4)}(x_i)}{3}h^4 + O(h^5)$

Combinamos eliminar $f(x_i), f'(x_i), f''(x_i), f^{(4)}(x_i)$. Buscamos A, B, C, D, E

$$Af(x_i-2) + Bf(x_i-1) + Cf(x_i) + Df(x_i+1) + Ef(x_i+2) = f'''(x_i)h^3 + O(h^5)$$

Sustituimos expansiones y agrupamos

$f(x_i)$: $A + B + C + D + E$

$f^{(4)}(x_i)$: $\left(\frac{2A}{3} + \frac{B}{24} + \frac{D}{24} + \frac{2E}{3}\right)h^4$

$f'(x_i)$: $(-2A - B + D + 2E)h$

$f''(x_i)$: $\left(2A + \frac{B}{2} + \frac{D}{2} + 2E\right)h^2$

$f'''(x_i)$: $\left(-\frac{4A}{3} - \frac{B}{6} + \frac{D}{6} + \frac{4E}{3}\right)h^3$

Queremos

1. $A + B + C + D + E = 0$
2. $-2A - B + D + 2E = 0$
3. $2A + \frac{B}{2} + \frac{D}{2} + 2E = 0$
4. $-\frac{4A}{3} - \frac{B}{6} + \frac{D}{6} + \frac{4E}{3} = 1$
5. $\frac{2A}{3} + \frac{B}{24} + \frac{D}{24} + \frac{2E}{3} = 0$

Resolviendo sale $A = -E$, $B = -D$, $C = 0$

② $-2A - B - B + 2(-A) = -2A - 2B - 2A = -4A - 2B = 0 \rightarrow 2A + B = 0$

$B = -2A$

④ $-\frac{4A}{3} - \frac{-2A}{6} + \frac{-2A}{6} + \frac{4(-A)}{3} = \frac{-4A}{3} + \frac{A}{3} - \frac{A}{3} - \frac{4A}{3} = \frac{-8A}{3} = 1 \rightarrow A = -\frac{3}{8}$

Entonces $B = -2\left(-\frac{3}{8}\right) = \frac{3}{4}$, $D = -B = -\frac{3}{4}$, $E = -A = \frac{3}{8}$

Verificamos

$$2\left(-\frac{3}{8}\right) + \frac{3/4}{2} + \frac{-3/4}{2} + 2\left(\frac{3}{8}\right) = -\frac{6}{8} + \frac{3}{8} - \frac{3}{8} + \frac{6}{8} = 0$$

$$-\frac{3}{8}f(x_i-2) + \frac{3}{4}f(x_i-1) - \frac{3}{4}f(x_i+1) + \frac{3}{8}f(x_i+2) = f'''(x_i)h^3 + O(h^5)$$

$$f'''(x_i) = \frac{-f(x_i-2) + 2f(x_i-1) - 2f(x_i+1) + f(x_i+2)}{2h^3} + O(h^2)$$

multiplicando $\cdot 8$: $-3f_{i-2} + 6f_{i-1} - 6f_{i+1} + 3f_{i+2} = 8f'''h^3$

Dividimos $2h^3$

$$f'''(x_i) = \frac{f(x_i-2) - 2f(x_i-1) + 2f(x_i+1) - f(x_i+2)}{2h^3} + O(h^2)$$