

Proyecto 2: Temperatura en un placa, estado estacionario

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Abstract

En el presente trabajo se determinará la temperatura final y en distintas etapas de una placa la cual se verá sujeta a un cambio térmico.

1 Introducción

Quizás la forma más precisa o la prueba más pura que se tenga acerca de la energía provenga del calor. En nuestro día cotidiano solemos vivir al lado de un sin fin de fenómenos donde el calor desempeña un papel fundamental: el encender nuestro coche, calentar un taza de café, taparnos del sol y abrigarnos del frío o incluso el simple hecho de que seamos seres termorreceptivos.

Se pueden tener un número infinito de objetos en un número infinito de estados y en un número infinito de lugares, algunos caen, otros chocan, unos se rompen y otros se combinan para crear otros objetos. Todos ellos contienen características totalmente distintas haciéndoles perfectamente desiguales, ninguno se repite ni tampoco ninguno comparte características entre sí; o al menos eso parece. Resulta que en realidad si comparten una característica y no sólo un par, todos los objetos, la infinita cantidad de objetos, comparten una cosa: la de tener temperatura(calor) y la de tener masa.

En el presente trabajo les hablaré del primero: calor.

2 Marco Teórico

Se tiene una placa cuadrada de lado n que se encuentra aislada en su totalidad a excepción de sus bordes donde se le aplica un calor ajustable y constante. La placa se encuentra totalmente aislada por lo que el único factor que se necesita tomar en cuenta son sus dimensiones x y y . Para lo siguiente se recurre a la *Ecuación de Laplace*:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (1)$$

Para su solución se utilizará se tratará la placa (por sus coordenadas) como una matriz. Considerando esto se tiene:

$$T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j} = 0 \quad (2)$$

Resolviendo la ecuación por método de *Gauss-Seidel* se tiene:

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4} \quad (3)$$

A partir de aquí se resuelve de manera iterativa para $j = 0$ hasta n y $i = 0$ hasta n .

Las soluciones se repiten hasta tener todos los errores relativos porcentuales " $(\epsilon_a)_{i,j}$ ", teniendo:

$$|(\epsilon_a)_{i,j}| = \left| \frac{T_{i,j}^{nuevo} - T_{i,j}^{anterior}}{T_{i,j}^{viejo}} \right| 100\% \quad (4)$$

3 Código

Se tiene como datos iniciales (ϵ_s) y el número de lados que en este caso son 4 y a los cuales llamaremos *up*, *down*, *right* y *left*. El programa escanea los datos de un archivo de texto llamado "datos.txt" y los utiliza como una matriz dinámica reservando memoria y configurando la matriz, por ejemplo, de la siguiente manera:

$$\begin{pmatrix} 0,4 & 1,4 & 2,4 & 3,4 & 4,4 \\ 0,3 & 1,3 & 2,3 & 3,3 & 4,3 \\ 0,2 & 1,2 & 2,2 & 3,2 & 4,2 \\ 0,1 & 1,1 & 2,1 & 3,1 & 4,1 \\ 0,0 & 1,0 & 2,0 & 3,0 & 4,0 \end{pmatrix}$$

Ejemplo de una matriz para la placa de lado $n * n$

4 Resultados

Una vez obtenidos los resultados para la matriz se imprimieron en archivos de texto los cuales demostraban el valor de su temperatura por cada sección de la matriz. Se graficó utilizando excel. Los espacios indican la temperatura que se alcanzó por cada coordenada siendo los de color rojo aquellos que son mayores y de color azulados aquellos que son menores. A continuación se mostrarán las gráficas y su evolución:

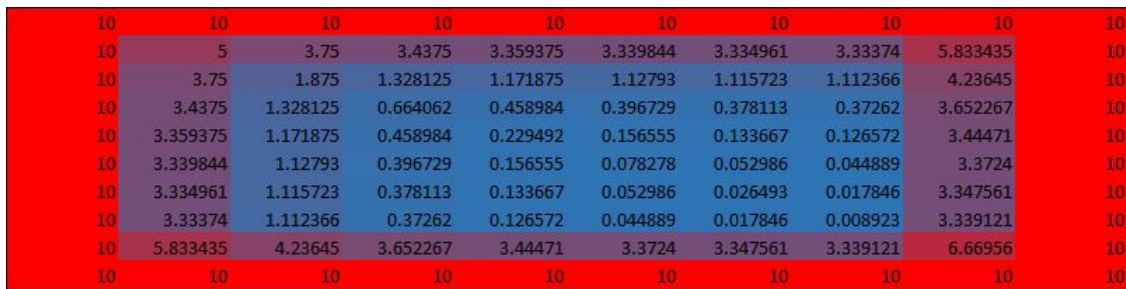


Figure 1: Inicio del proceso. Resultado 1

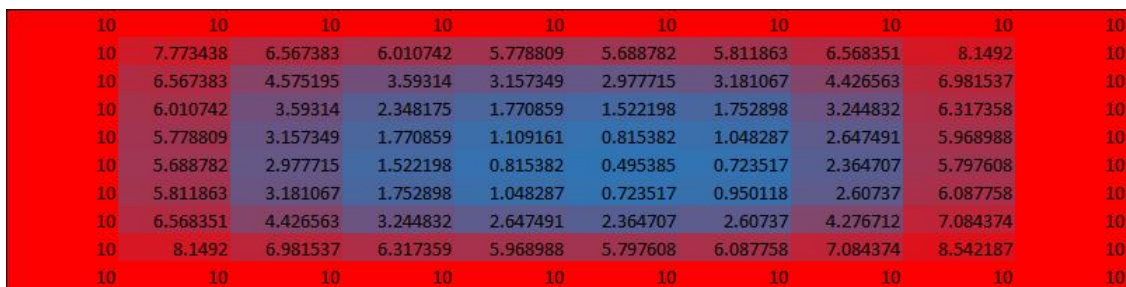


Figure 2: Resultado 2

10	10	10	10	10	10	10	10	10	10
10	8.283691	7.217407	6.647339	6.373367	6.290736	6.510038	7.271451	8.563247	10
10	7.217407	5.405273	4.389534	3.877869	3.717967	4.101867	5.399921	7.570131	10
10	6.647339	4.389534	3.080196	2.397356	2.170901	2.641472	4.251561	6.94767	10
10	6.373367	3.877869	2.397356	1.606369	1.330235	1.835679	3.605234	6.587628	10
10	6.290736	3.717967	2.170901	1.330235	1.026876	1.544345	3.388639	6.516006	10
10	6.510038	4.101867	2.641472	1.835679	1.544345	2.075858	3.957242	6.889406	10
10	7.271451	5.399921	4.251561	3.605234	3.388639	3.957242	5.520808	7.738101	10
10	8.563247	7.570132	6.94767	6.587628	6.516006	6.889405	7.7381	8.86905	10
10	10	10	10	10	10	10	10	10	10

Figure 3: Resultado 3

10	10	10	10	10	10	10	10	10	10
10	8.608704	7.665329	7.107058	6.818915	6.76173	7.033762	7.749232	8.829842	10
10	7.665329	6.027431	5.023139	4.489345	4.38096	4.864029	6.108738	7.971562	10
10	7.107058	5.023139	3.710247	2.994215	2.836721	3.446997	5.02716	7.396587	10
10	6.818916	4.489345	2.994215	2.162225	1.965375	2.640488	4.410979	7.080893	10
10	6.76173	4.38096	2.836721	1.965375	1.75486	2.464961	4.337297	7.076899	10
10	7.033762	4.864029	3.446997	2.640488	2.464961	3.211102	4.989653	7.451163	10
10	7.749233	6.108738	5.02716	4.410979	4.337297	4.989653	6.363877	8.171022	10
10	8.829841	7.971562	7.396587	7.080893	7.076899	7.451163	8.171022	9.085511	10
10	10	10	10	10	10	10	10	10	10

Figure 4: Resultado 4

10	10	10	10	10	10	10	10	10	10
10	8.832664	7.991788	7.45846	7.177383	7.148026	7.440322	8.094726	9.016572	10
10	7.991788	6.507463	5.541379	5.023484	4.968065	5.491031	6.64612	8.26482	10
10	7.458461	5.541379	4.267797	3.572557	3.488249	4.161732	5.653854	7.749892	10
10	7.177384	5.023485	3.572557	2.768966	2.663141	3.425203	5.124312	7.487775	10
10	7.148026	4.968065	3.488249	2.663141	2.564051	3.384413	5.143819	7.520689	10
10	7.440322	5.491031	4.161732	3.425203	3.384413	4.187033	5.786473	7.869546	10
10	8.094726	6.64612	5.653854	5.124312	5.143819	5.786473	6.978747	8.483451	10
10	9.016572	8.26482	7.749892	7.487775	7.520689	7.869546	8.483451	9.241726	10
10	10	10	10	10	10	10	10	10	10

Figure 5: Resultado 5

10	10	10	10	10	10	10	10	10	10
10	8.995894	8.240455	7.739804	7.477829	7.471554	7.764328	8.356754	9.155394	10
10	8.240455	6.890917	5.9805	5.499738	5.487643	6.014956	7.072596	8.494471	10
10	7.739804	5.980501	4.776528	4.13337	4.111471	4.801371	6.187043	8.042322	10
10	7.477829	5.499738	4.13337	3.398256	3.374745	4.17121	5.747462	7.827618	10
10	7.471554	5.487643	4.111471	3.374745	3.379579	4.22041	5.818758	7.878981	10
10	7.764328	6.014956	4.801371	4.17121	4.220411	5.003442	6.417623	8.195013	10
10	8.356755	7.072596	6.187043	5.747462	5.818759	6.417624	7.450537	8.721819	10
10	9.155394	8.494471	8.042322	7.827619	7.878981	8.195013	8.721819	9.360909	10
10	10	10	10	10	10	10	10	10	10

Figure 6: Resultado 6

10	10	10	10	10	10	10	10	10	10
10	9.120228	8.437737	7.974016	7.736327	7.747074	8.029696	8.564421	9.264723	10
10	8.437737	7.209119	6.364851	5.930548	5.951012	6.463669	7.427401	8.683611	10
10	7.974017	6.364851	5.249111	4.672346	4.699869	5.380447	6.649408	8.290159	10
10	7.736327	5.930548	4.672346	4.023546	4.068551	4.854218	6.2875	8.114161	10
10	7.747074	5.951012	4.699869	4.068551	4.144481	4.955225	6.384832	8.173502	10
10	8.029696	6.463669	5.380448	4.854218	4.955225	5.686424	6.929202	8.456131	10
10	8.564422	7.427401	6.649408	6.287501	6.384832	6.929202	7.82551	8.910638	10
10	9.264723	8.683612	8.290159	8.114161	8.173502	8.456131	8.910637	9.455318	10
10	10	10	10	10	10	10	10	10	10

Figure 7: Resultado 7

10	10	10	10	10	10	10	10	10	10
10	9.218868	8.600501	8.17542	7.963261	7.985992	8.253521	8.736411	9.355005	10
10	8.600501	7.482676	6.709438	6.324014	6.368386	6.857439	7.731717	8.84422	10
10	8.17542	6.709438	5.690892	5.18458	5.250491	5.902889	7.053066	8.502862	10
10	7.963261	6.324014	5.18458	4.626565	4.718939	5.466138	6.75455	8.357728	10
10	7.985992	6.368386	5.250491	4.718939	4.837082	5.593619	6.862718	8.419145	10
10	8.253521	6.857439	5.902889	5.466138	5.593619	6.261411	7.351443	8.670306	10
10	8.736411	7.731718	7.053067	6.75455	6.862719	7.351442	8.131041	9.064166	10
10	9.355006	8.84422	8.502862	8.357729	8.419145	8.670305	9.064166	9.532084	10
10	10	10	10	10	10	10	10	10	10

Figure 8: Resultado 8

10	10	10	10	10	10	10	10	10	10
10	9.300251	8.739586	8.353071	8.16577	8.196919	8.447693	8.883604	9.431956	10
10	8.739587	7.724512	7.023122	6.685464	6.747579	7.207469	7.99709	8.982977	10
10	8.353071	7.023122	6.103851	5.666593	5.759	6.371418	7.40648	8.686796	10
10	8.16577	6.685464	5.666593	5.192766	5.313746	6.008333	7.158814	8.566189	10
10	8.196918	6.747578	5.759	5.313746	5.453683	6.146536	7.268985	8.62637	10
10	8.447692	7.207469	6.371418	6.008333	6.146536	6.748989	7.70483	8.848842	10
10	8.883604	7.99709	7.40648	7.158815	7.268984	7.70483	8.384498	9.191356	10
10	9.431955	8.982977	8.686796	8.566189	8.626369	8.848842	9.191356	9.595678	10
10	10	10	10	10	10	10	10	10	10

Figure 9: Resultado 9

10	10	10	10	10	10	10	10	10	10
10	9.369793	8.861844	8.512684	8.348766	8.386009	8.61927	9.012079	9.498764	10
10	8.861844	7.942483	7.311121	7.018515	7.092749	7.520132	8.230417	9.103994	10
10	8.512684	7.311121	6.488857	6.114785	6.223175	6.78953	7.716389	8.846643	10
10	8.348766	7.018515	6.114784	5.714265	5.849864	6.486186	7.509437	8.745613	10
10	8.386009	7.092748	6.223174	5.849864	5.9982	6.62559	7.616557	8.802753	10
10	8.61927	7.520132	6.78953	6.486186	6.62559	7.16521	8.003777	8.999472	10
10	9.012079	8.230417	7.71639	7.509438	7.616557	8.003777	8.597567	9.29818	10
10	9.498764	9.103994	8.846643	8.745613	8.802753	8.999472	9.29818	9.64909	10
10	10	10	10	10	10	10	10	10	10

Figure 10: Resultado 10

10	10	10	10	10	10	10	10	10	10
10	9.430922	8.971522	8.657852	8.515594	8.556904	8.772279	9.125364	9.55734	10
10	8.971521	8.141321	7.576636	7.324941	7.406288	7.799628	8.436344	9.210082	10
10	8.657852	7.576636	6.845711	6.527023	6.643176	7.161345	7.988442	8.986034	10
10	8.515594	7.324941	6.527022	6.188443	6.329001	6.906343	7.814239	8.900757	10
10	8.556904	7.406287	6.643176	6.329001	6.477296	7.041352	7.91553	8.953939	10
10	8.772279	7.799628	7.161345	6.906343	7.041351	7.522564	8.258783	9.127726	10
10	9.125365	8.436344	7.988442	7.814239	7.91553	8.258783	8.778481	9.388824	10
10	9.55734	9.210082	8.986034	8.900757	8.953939	9.127726	9.388824	9.694412	10
10	10	10	10	10	10	10	10	10	10

Figure 11: Resultado 11

10	10	10	10	10	10	10	10	10	10
10	9.485762	9.071234	8.790866	8.668179	8.711686	8.90917	9.225714	9.608949	10
10	9.071234	8.323935	7.821363	7.605713	7.690051	8.049229	8.618366	9.303337	10
10	8.790867	7.821363	7.174193	6.902882	7.02082	7.491209	8.227462	9.107889	10
10	8.668178	7.605712	6.902881	6.615941	6.7551	7.275475	8.079806	9.035409	10
10	8.711686	7.690051	7.02082	6.7551	6.898226	7.402948	8.173869	9.084251	10
10	8.90917	8.049228	7.491208	7.275475	7.402949	7.830866	8.477736	9.237703	10
10	9.225714	8.618367	8.227462	8.079805	8.173869	8.477736	8.93328	9.466349	10
10	9.60895	9.303338	9.107889	9.035408	9.08425	9.237703	9.466349	9.733174	10
10	10	10	10	10	10	10	10	10	10

Figure 12: Resultado 12

10	10	10	10	10	10	10	10	10	10
10	9.535617	9.162604	8.913036	8.807609	8.851707	9.031662	9.314744	9.65452	10
10	9.162604	8.491983	8.046231	7.861693	7.945862	8.271774	8.779329	9.385435	10
10	8.913036	8.046231	7.474556	7.243253	7.358856	7.783391	8.437605	9.214613	10
10	8.807609	7.861692	7.243252	6.999176	7.132933	7.59977	8.311663	9.152632	10
10	8.851707	7.945862	7.358855	7.132933	7.267941	7.718111	8.39794	9.197069	10
10	9.031662	8.271774	7.783392	7.59977	7.718111	8.097923	8.666712	9.332533	10
10	9.314745	8.779329	8.437604	8.311663	8.39794	8.666712	9.06653	9.53306	10
10	9.65452	9.385435	9.214612	9.152632	9.197069	9.332533	9.53306	9.76653	10
10	10	10	10	10	10	10	10	10	10

Figure 13: Resultado 13

10	10	10	10	10	10	10	10	10	10
10	9.581303	9.24658	9.025105	8.934627	8.978038	9.141139	9.393747	9.694796	10
10	9.24658	8.646406	8.25194	8.093921	8.175648	8.469877	8.921666	9.457768	10
10	9.025105	8.25194	7.747596	7.549888	7.660465	8.041929	8.622468	9.308217	10
10	8.934626	8.09392	7.549887	7.34141	7.467396	7.884775	8.514454	9.254935	10
10	8.978037	8.175646	7.660464	7.467396	7.592753	7.993348	8.592896	9.295091	10
10	9.141139	8.469876	8.041928	7.884774	7.993348	8.33003	8.830498	9.414662	10
10	9.393747	8.921666	8.622467	8.514453	8.592896	8.830498	9.181779	9.590742	10
10	9.694796	9.457768	9.308217	9.254935	9.295091	9.414662	9.590743	9.795372	10
10	10	10	10	10	10	10	10	10	10

Figure 14: Resultado 14

10	10	10	10	10	10	10	10	10	10
10	9.62329	9.3237	9.127567	9.049882	9.091667	9.238823	9.463821	9.730397	10
10	9.3237	8.78782	8.439226	8.30366	8.381417	8.645959	9.047504	9.521529	10
10	9.127566	8.439225	7.994556	7.825023	7.928942	8.270536	8.785178	9.390411	10
10	9.049881	8.303659	7.825023	7.64621	7.76317	8.135377	8.692097	9.344399	10
10	9.091666	8.381416	7.928941	7.763169	7.878259	8.23414	8.762957	9.380505	10
10	9.238822	8.645958	8.270535	8.135376	8.23414	8.532319	8.972929	9.486044	10
10	9.46382	9.047503	8.785177	8.692096	8.762957	8.972929	9.281836	9.640813	10
10	9.730397	9.521529	9.39041	9.344399	9.380505	9.486044	9.640813	9.820406	10
10	10	10	10	10	10	10	10	10	10

Figure 15: Resultado 15

10	10	10	10	10	10	10	10	10	10
10	9.66185	9.394309	9.220854	9.154045	9.193571	9.325838	9.525935	9.761867	10
10	9.394309	8.916767	8.608959	8.492361	8.565208	8.802272	9.158729	9.577752	10
10	9.220854	8.608959	8.216991	8.071126	8.167511	8.472585	8.928455	9.462651	10
10	9.154045	8.492361	8.071126	7.917148	8.024574	8.355849	8.847916	9.422768	10
10	9.193571	8.565208	8.167509	8.024573	8.129356	8.445121	8.911617	9.455107	10
10	9.325838	8.802271	8.472584	8.355848	8.44512	8.709024	9.09713	9.548262	10
10	9.525934	9.158728	8.928454	8.847914	8.911617	9.09713	9.368971	9.68441	10
10	9.761866	9.577751	9.462651	9.422768	9.455107	9.548263	9.68441	9.842205	10
10	10	10	10	10	10	10	10	10	10

Figure 16: Resultado 16

10	10	10	10	10	10	10	10	10	10
10	9.697155	9.458694	9.305425	9.247839	9.284721	9.403233	9.580957	9.789678	10
10	9.458694	9.033827	8.762151	8.661581	8.729021	8.940891	9.257013	9.627336	10
10	9.305425	8.762151	8.416638	8.29072	8.379225	8.651105	9.054671	9.526194	10
10	9.247839	8.661581	8.290719	8.157646	8.25552	8.549915	8.984743	9.49151	10
10	9.284721	8.729021	8.379224	8.255518	8.350319	8.630219	9.0418	9.520393	10
10	9.403232	8.940891	8.651104	8.549913	8.630219	8.863674	9.205677	9.60262	10
10	9.580956	9.257013	9.05467	8.984742	9.0418	9.205677	9.445044	9.722467	10
10	9.789677	9.627335	9.526194	9.49151	9.520393	9.60262	9.722467	9.861234	10
10	10	10	10	10	10	10	10	10	10

Figure 17: Resultado 17

10	10	10	10	10	10	10	10	10	10
10	9.729347	9.517149	9.381784	9.332022	9.366069	9.471979	9.629667	9.814251	10
10	9.51715	9.13965	8.899914	8.81292	8.874776	9.063719	9.343849	9.671074	10
10	9.381784	8.899914	8.595317	8.486277	8.566919	8.808805	9.165897	9.582121	10
10	9.332022	8.81292	8.486277	8.370897	8.459513	8.720819	9.105007	9.55188	10
10	9.366068	8.874776	8.566919	8.459513	8.544866	8.792789	9.155967	9.577617	10
10	9.471979	9.063718	8.808805	8.720819	8.792789	8.999233	9.300715	9.6502	10
10	9.629667	9.343847	9.165897	9.105007	9.155967	9.300715	9.511591	9.755756	10
10	9.814251	9.671073	9.58212	9.55188	9.577617	9.6502	9.755756	9.877878	10
10	10	10	10	10	10	10	10	10	10

Figure 18: Resultado 18

10	10	10	10	10	10	10	10	10	10
10	9.758574	9.570002	9.450484	9.407368	9.438531	9.53298	9.67277	9.83596	10
10	9.570003	9.234959	9.02342	8.94796	9.004282	9.17248	9.420555	9.70966	10
10	9.450484	9.02342	8.754848	8.660156	8.733189	8.948096	9.263945	9.631371	10
10	9.407368	8.94796	8.660156	8.559834	8.639677	8.871392	9.210796	9.604946	10
10	9.43853	9.004282	8.733189	8.639677	8.716233	8.935706	9.256208	9.627838	10
10	9.532978	9.172478	8.948095	8.871392	8.935706	9.118211	9.384052	9.691912	10
10	9.67277	9.420554	9.263945	9.210795	9.256208	9.384052	9.569904	9.784924	10
10	9.83596	9.709659	9.631371	9.604946	9.627838	9.691912	9.784924	9.892462	10
10	10	10	10	10	10	10	10	10	10

Figure 19: Resultado 19

10	10	10	10	10	10	10	10	10	10
10	9.983585	9.971007	9.963289	9.960767	9.963128	9.969528	9.978745	9.989372	10
10	9.971007	9.948792	9.935162	9.930707	9.934879	9.946182	9.962462	9.981231	10
10	9.963289	9.935162	9.917904	9.912265	9.917548	9.931861	9.952474	9.976236	10
10	9.960767	9.930707	9.912264	9.90624	9.911887	9.927184	9.949212	9.974606	10
10	9.963128	9.934878	9.917547	9.911887	9.917196	9.931572	9.952272	9.976136	10
10	9.969527	9.946182	9.931861	9.927184	9.931572	9.943453	9.96056	9.98028	10
10	9.978745	9.962462	9.952474	9.949212	9.952272	9.96056	9.972491	9.986246	10
10	9.989372	9.981231	9.976236	9.974606	9.976135	9.98028	9.986246	9.993123	10
10	10	10	10	10	10	10	10	10	10

Figure 20: Resultado 20.Equilibrio Térmico.

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