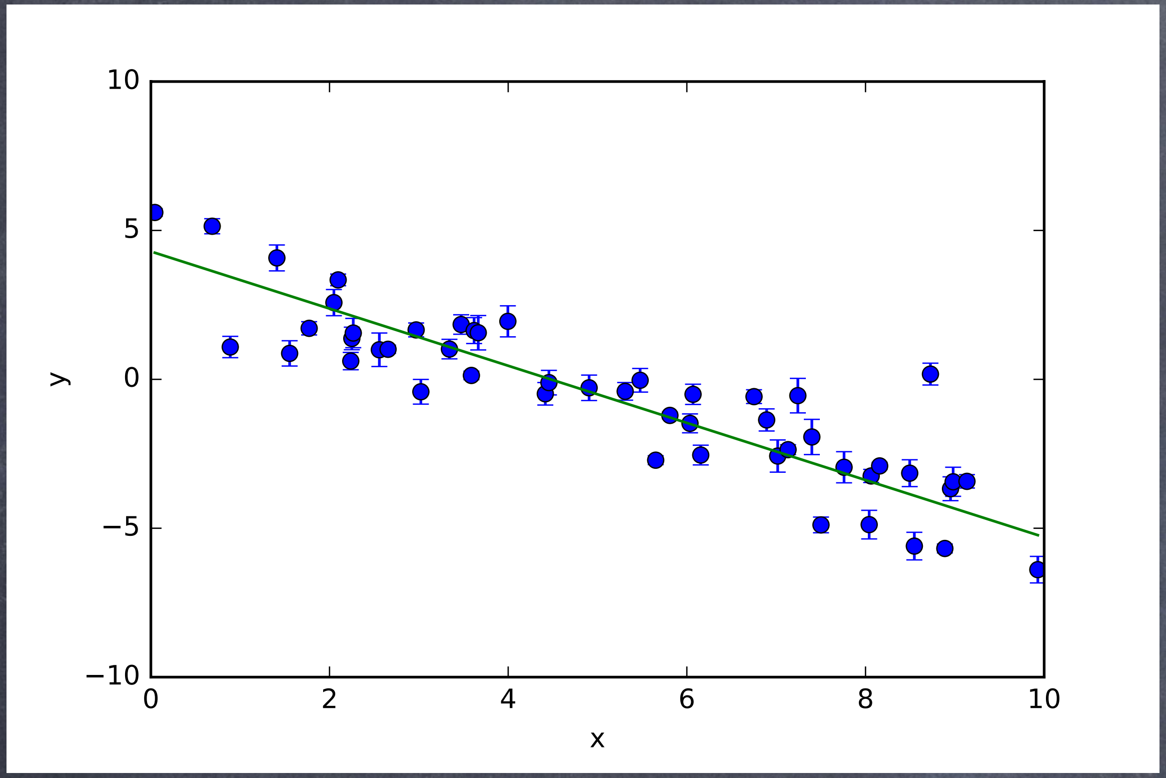
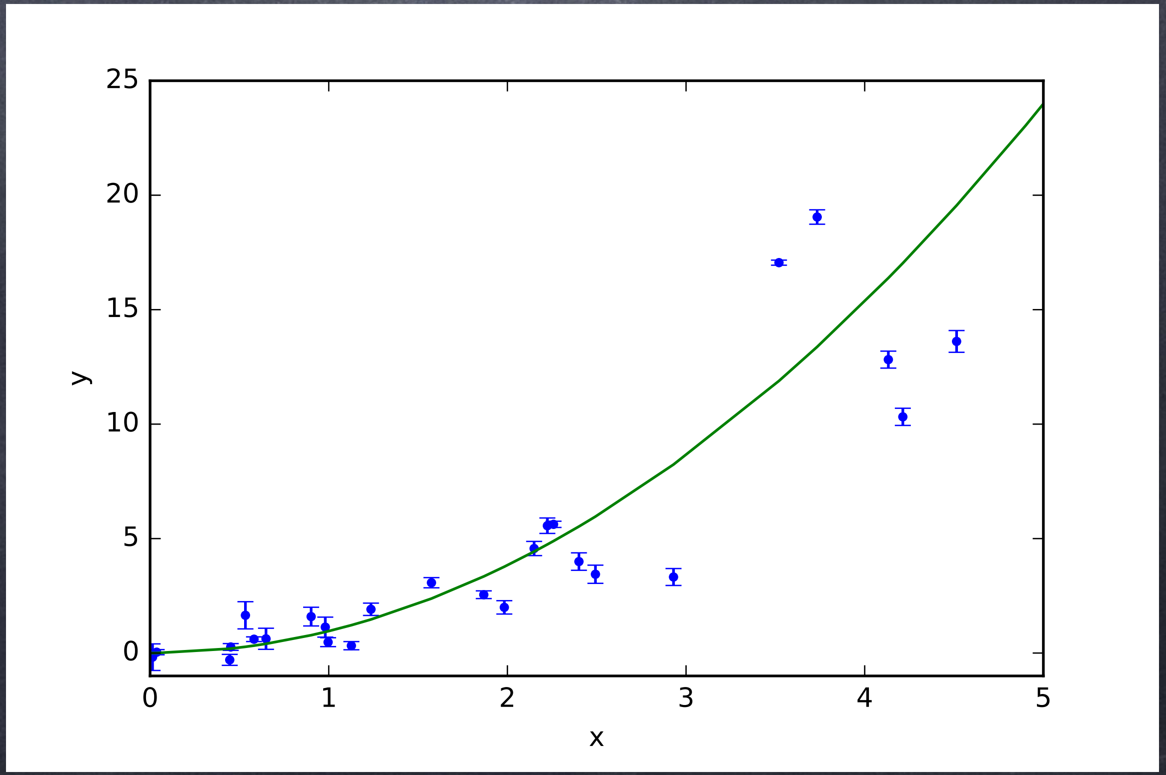
Parameter Estimation

- What do we do if we want to estimate the slope and y-intercept?



- Linear least square method

- What if data is not a straight line? And/or model is not linear, and/or if we have more than two free parameters? or more important what if I dont believe too much on the error bars*



Parameter Estimation. Level 0

- Least square method.

$$
\begin{aligned}
a &= \frac{\sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i\, y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \\
&= \frac{\bar{y}\left(\sum_{i=1}^{n} x_i^2\right) - \bar{x} \sum_{i=1}^{n} x_i\, y_i}{\sum_{i=1}^{n} x_i^2 - n\,\bar{x}^2} \\
b &= \frac{n \sum_{i=1}^{n} x_i\, y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \\
&= \frac{\left(\sum_{i=1}^{n} x_i\, y_i\right) - n\,\bar{x}\,\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\,\bar{x}^2}
\end{aligned}
$$

Where this comes from?

Minimize the residual

$$R^2 \equiv \sum \left[y_i - f(x_i, a_1, a_2, ..., a_n)\right]^2$$

assuming:

- A linear function f=ax+b.

- Errors are Gaussian and uncorrelated.

Minimization implies:

$$\frac{\partial R^2}{\partial a_i} = 0$$

Parameter Estimation. Level 1

- χ^2 **Minimization**

$$\frac{\partial \chi^2}{\partial \theta} = 0$$

$$\chi^2 = \sum (y_i - y(x_i, \theta))^2 / \sigma_{y_i}^2$$

θ : free parameters

σ_{y_i} : variance ony_i

We will see how least square, and minimum Chi^2 methods are just special cases of Inference.

Show that if y=mx+b, and you minimize with respect to m and b, you recover the expressions for the least square methods.