Ejercicio (4)

Tenemos la distribución:

$$\chi^2 = \frac{\sum(y_i - y(x_i,\theta))^2}{\sigma_{y_i}^2} \quad ; \quad \frac{\partial \chi^2}{\partial \theta} = 0$$

La ecuación de la recta: $y = ax + b$, por lo tanto los parámetros $\theta$ son $a$ y $b$

$$\frac{\partial \chi^2}{\partial a} = \frac{2\sum(y_i - (x_i,\theta)x_i)}{\sigma^2} = 0$$

$$\frac{\partial \chi^2}{\partial b} = \frac{2\sum(y_i - y(x_i,\theta))}{\sigma^2} = 0$$

$$\Longrightarrow \begin{cases} ① \sum(y_i - ax_i - b)x_i = 0 \\ ② \sum(y_i - ax_i - b) = 0 \end{cases}$$

$\Rightarrow$ De (2): $\quad b = \sum y_i - a\sum x_i \quad (3)$

De (1): $\quad \sum y_i x_i - a\sum x_i^2 - b\sum x_i = 0 \quad (4)$

Sustituyendo (3) en (4)

$$\sum y_i x_i - a\sum x_i^2 - \frac{1}{n}\left(\sum y_i - a\sum x_i\right)\sum x_i = 0$$

$$n\sum y_i x_i - an\sum x_i^2 - \sum y_i \sum x_i + a\sum x_i \sum_i = 0$$

Despejando a de la expresión anterior:

$$a = \frac{n\sum y_i x_i - \sum y_i \sum x_i}{n\sum x_i^2 - (\sum x_i)^2}$$

Sustituyendo lo anterior en (3)

$$b = \frac{1}{n}\left(\sum y_i - \left[\frac{n\sum y_i x_i - \sum y_i \sum x_i}{n\sum x_i^2 - (\sum x_i)^2}\right]\sum x_i\right.$$

$$= \frac{\sum y_i \sum x_i^2 - \frac{1}{n}\sum y_i(\sum x_i)^2 - \sum y_i x_i \sum x_i + \frac{1}{n}\sum y_i(\sum x_i)^2}{n\sum x_i^2 - (\sum x_i)^2}$$

$$b = \frac{\sum y_i \sum x_i^2 - \sum y_i x_i}{n\sum x_i^2 - (\sum x_i)^2}$$