# DCMIP2016, Part 1: Models and Equation Sets

### Contributions for the CSU Model

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## 3 Model Grids

[ALL] Add a short description of your model grid here.

The CSU model uses the geodesic grid described by Heikes et al. (2013). The grid is generated by recursive bisection, starting from an icosahedron, as illustrated in Fig. 1, which is

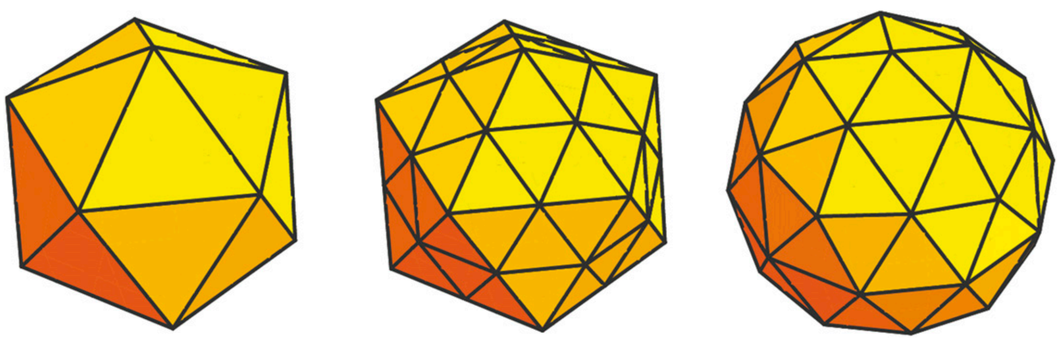

Fig. 1:

taken from Heikes et al. (2013). The vertices of the triangular faces generated in this way are used to define the centers of hexagonal and pentagonal cells, as shown in Fig. 2, which is also

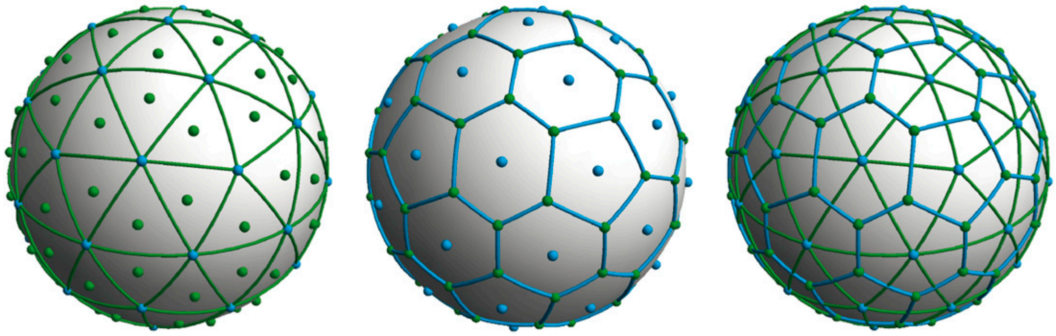

Fig. 2:

taken from Heikes et al. (2013). In this way we can generate arbitrarily fine grids. We then optimize the grid through a variational "tweaking" procedure described by Heikes et al. (2013).

| Grid | Number of grid cells | Average grid distance (km) | Ratio of shortest to longest grid spacing (%) | Ratio of smallest to largest grid size (%) |
|---|---|---|---|---|
| G0 | 12 | 6699.1 | 100 (100) | 100 (100) |
| G1 | 42 | 3709.8 | 88.1 (88.1) | 88.5 (88.5) |
| G2 | 162 | 1908.8 | 82.0 (84.8) | 91.6 (84.2) |
| G3 | 642 | 961.4 | 79.8 (83.9) | 94.2 (76.3) |
| G4 | 2562 | 481.6 | 79.0 (83.7) | 94.8 (74.1) |
| G5 | 10242 | 240.9 | 78.7 (83.6) | 95.0 (73.6) |
| G6 | 40962 | 120.4 | 78.6 (83.6) | 95.2 (73.4) |
| G7 | 163842 | 60.2 | 78.6 (83.6) | 95.2 (73.4) |
| G8 | 655362 | 30.1 | 78.6 (83.6) | 95.3 (73.4) |
| G9 | 2621442 | 15.0 | 78.6 (83.6) | 95.3 (73.4) |
| G10 | 10485762 | 7.53 | 78.6 (83.6) | 95.3 (73.4) |
| G11 | 41943042 | 3.76 | 78.6 (83.6) | 95.3 (73.4) |
| G12 | 167772162 | 1.88 | 78.6 (83.6) | 95.3 (73.4) |
| G13 | 671088642 | 0.94 | 78.6 (83.6) | 95.3 (73.4) |

Table 1: Some properties of the tweaked and raw (untweaked) grids. The properties of the raw grids are shown in parentheses. Averaged grid distance is the arithmetic average of the maximum and minimum of grid distances.

Table 1 shows the first fourteen grids that are generated through this recursive procedure. All grids, whatever the resolution, have exactly twelve pentagonal cells, which come from the twelve vertices of the icosahedron that is used as a starting point for generating the grid. All the rest of the cells are hexagonal.

We choose the geodesic grid made of hexagons and a few pentagons. because all of the grid cells are (very nearly) the same size, so we say that the grid is “homogeneous.” In addition, the neighbors of each cell lie across cell walls; there are no neighbors across cell vertices, as does happen with quadrilateral and triangular cells. In this sense, the grid is also “isotropic.”

The CSU model defines all of its prognostic (i.e., time-stepped) variables at the cell centers; in other words, the grid is unstaggered. In particular, we predict the vertical component of the vorticity and the divergence of the horizontal wind vector at the cell centers. As discussed by Randall (1994), this “Z-grid” approach gives excellent dispersion properties for inertial gravity waves, and the model has no computational modes at all.

In order to horizontally advect various quantities on the grid, we need the normal component of the horizontal wind vector on each cell wall. The first step is to determine an stream function and velocity potential from the vorticity and divergence, respectively, by solving a pair of two-dimensional Poisson equations in each layer of the model, and on each time step. A multi-grid solver is used for this purpose. The stream function and velocity potential can be differentiated, using finite-difference methods, to obtain the required wind components.

## 4 Equation Sets

[ALL] Include the continuous equation set that you use for your model here.

The version of the model used here is based on the height coordinate and uses the nonhydrostatic “Unifed System” of equations, which was proposed by Arakawa and Konor (2009) and demonstrated in a limited-area model by Konor (2014). The Unified System filters vertically propagating sound waves, but allows the Lamb wave and includes compressibility effects that influence the phase speeds of long Rossby waves. Unlike conventional anelastic systems, the Unified System has no need for a reference state. This makes it suitable for use in global models.

As mentioned in Section 3, the CSU model predicts the vertical component of the vorticity and the divergence of the horizontal wind. In addition, the model predicts the potential temperature.

The Unified System divides the pressure, density, temperature, and the Exner function $\pi$ into quasi-static parts and the remainders. No approximation is involved in this step, which is purely a matter of definition. There is no need to define a quasi-static part of the potential temperature. The relevant equations are

$$p \equiv p_{qs} + \delta p \ ,$$
$$\rho \equiv \rho_{qs} + \delta \rho \ ,$$
$$\pi \equiv \pi_{qs} + \delta \pi \ ,$$
$$T \equiv T_{qs} + \delta T \ .$$

where the subscript $qs$ denotes the quasi-static portion of the field, and $\delta(\ )$ denotes the departure from the quasi-static state. As discussed by Arakawa and Konor (2009), the quasi-static parts of the pressure, density, temperature, and the Exner function can be obtained from the predicted potential temperature, by using the hydrostatic equation (with an upper boundary condition) and the equation of state.

The perturbation Exner function, $\delta\pi$ , is obtained by solving a three-dimensional Poisson equation:

$$\boldsymbol{\nabla}_H \cdot \left( \rho_{qs} c_p \theta \boldsymbol{\nabla}_H \delta\pi \right) + \frac{\partial}{\partial z} \left( \rho_{qs} c_p \theta \frac{\partial \delta\pi}{\partial z} \right) = forcing \ ,$$

with the lower and upper boundary conditions

$$\left( \frac{\partial \delta\pi}{\partial z} \right)_S = \left( \frac{\partial \delta\pi}{\partial z} \right)_T = 0 \ .$$

The solver uses a multi-grid approach in the horizontal and a tridiagonal method in the vertical.

The continuous versions of the vorticity and divergence equations are

$$\left( \frac{\partial \zeta}{\partial t} \right)_z + \nabla_z \cdot (\zeta \mathbf{v}) + \hat{\mathbf{k}} \cdot \nabla_z \times \left( w \frac{\partial \mathbf{v}}{\partial z} \right) = -\hat{\mathbf{k}} \cdot \nabla_z \times \left[ \theta \nabla_z \left( \pi_{qs} + \delta\pi \right) \right] + \hat{\mathbf{k}} \cdot \nabla_z \times \mathbf{F}$$

$$\left( \frac{\partial D}{\partial t} \right)_z - J(\eta, \chi) - \nabla_z \cdot (\eta \nabla \psi) + \nabla_z \cdot \left( w \frac{\partial \mathbf{v}}{\partial z} \right) = -\nabla_z \cdot \left[ \theta \nabla_z \left( \pi_{qs} + \delta\pi \right) \right] + \nabla_z \cdot \mathbf{F}$$

Here $\mathbf{v}$ is the horizontal wind vector, $\hat{\mathbf{k}}$ is a unit vector pointing upward, $\mathbf{F}$ is the horizontal component of the friction vector, $D$ is the divergence, and $J(A,B) \equiv \hat{\mathbf{k}} \cdot (\nabla A \times \nabla B)$ is the Jacobian operator.

As discussed earlier, we solve a pair of Poisson equations to obtain a stream function and velocity potential from the vorticity and divergence, respectively. We then differentiate the stream function and velocity potential to obtain the rotational and divergent parts of the horizontal wind vector, respectively.

Finally, we have to determine the vertical velocity. The quasi-static density satisfies a continuity equation of the form

$$\frac{\partial \rho_{qs}}{\partial t} = -\nabla_H \cdot \left(\rho_{qs}\mathbf{v}\right) - \frac{\partial\left(\rho_{qs} w\right)}{\partial z} \ .$$

Recall, however, that the quasi-static density can be determined diagnostically from the potential temperature. This means that the continuity equation given above can be solved for the vertical velocity. To evaluate the local time-rate-of-change term, we use the quasi-static density from earlier time steps.

## 5 Diffusion and Stabilization

[ALL] Include explicit diffusion and stabilization techniques that you have applied in the dynamical core here.

We implement horizontal diffusion using a biharmonic $\nabla_z^4(\ )$ operator applied directly to potential temperature, absolute vorticity and divergence, and the tracer.

## 6 Filters and Fixers

[ALL] Include explicit filters and fixers that you have utilized in the dynamical core here.

We do not use any filters or fixers.

## 7 Temporal Discretizations

[ALL] Describe the time-stepping scheme employed by your dynamical core here.

All prognostic variables are advanced in time using the 3rd-order Adams-Bashforth scheme.

## 8 Dynamical Cores

In this section provide a short description (approximately 0.5 pages) of the dynamical core, focusing on unique features or design specifications. Do not include information on the physical parameterizations used by the modeling system. Make reference to the model grid employed from section 3, the specific equation set being discretized by the model in section 4, explicit numerical techniques for diffusion and stabilization in section 5, filters and fixers in section 6 and the temporal discretization in section 7.

The CSU model uses an optimized geodesic grid to discretize the sphere, with height as the vertical coordinate. The model is based on the non-hydrostatic Unified System of equations proposed by Arakawa and Konor (2009), which filters vertically propagating sound waves but allows the Lamb wave and does not require a reference state. The horizontal wind field is determined by predicting the vertical component of the vorticity and the divergence of the horizontal wind, and then solving a pair of two-dimensional Poisson equations for a stream function and velocity potential. Time-differencing is based on the third-order Adams-Bashforth scheme. Horizontal diffusion is included in the form of a $\nabla_z^4(\ )$ operator acting on the vorticity, divergence, potential temperature, and tracer.

## References

Arakawa, A, and C. S. Konor, 2009: Unification of the anelastic and quasi-hydrostatic systems of equations. *Mon. Wea. Rev.,* **137,** 710-726.

Heikes, R. P., D. A. Randall, and C. S. Konor, 2013: Optimized icosahedral grids: Performance of finite-difference operators and multigrid solver. *Mon. Wea. Rev.*, **141**, 4450-4469. doi: http://dx.doi.org/10.1175/MWR-D-12-00236.1.

Konor, C. S., 2014: Design of a Dynamical Core Based on the Nonhydrostatic ‘‘Unified System’’ of Equations. *Mon. Wea. Rev.*, **142**, 364-385.