DCMIP2016, Part 1: Models and Equation Sets

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Abstract. This paper provides a comprehensive review of the design of modern non-hydrostatic atmospheric dynamical cores, including relevant equation sets, numerical stabilization techniques and idealized physics routines.

1 Introduction

INSTRUCTIONS FOR AUTHORS

Fill in text in section 3, 4, 5, 6, 7 and 8 below.

2 Notation

2.1 List of Symbols

Table 1 lists the symbols used in this paper.

Table 1. List of symbols used in this manuscript

Symbol	Description
λ	Longitude (in radians)
φ	Latitude (in radians)
z	Height with respect to mean sea level (set to zero)
p_s	Surface pressure (p_s of moist air if $q > 0$)
Φ_s	Surface geopotential
z_s	Surface elevation with respect to mean sea level (set to zero)
u	Zonal wind
v	Meridional wind
w	Vertical velocity
ω	Vertical pressure velocity
δ	Divergence
ζ	Relative vorticity
p	Pressure (pressure of moist air if $q > 0$)
ρ	Total air density
ρ_d	Dry air density
T	Temperature
T_v	Virtual temperature
Θ	Potential temperature
Θ_v	Virtual potential temperature
q	Specific humidity
P_{ls}	Large-scale precipitation rate
q_c	Cloud water mixing ratio
q_r	Rain water mixing ratio

2.2 List of Physical Constants

A list of physical constants which are used throughout this document is given in Table 2. Constants which are specific to each test case are similarly tabulated at the beginning of each section.

2.3 Great Circle Distance

The great circle distance is used throughout the document and is given by

$$R_c(\lambda_1,\varphi_1;\lambda_2,\varphi_2) = a\arccos\left(\sin\varphi_1\sin\varphi_2 + \cos\varphi_1\cos\varphi_2\cos(\lambda_1-\lambda_2)\right). \tag{1}$$

Table 2. A list of physical constants used in this document.

Constant	Description	Value
a_{ref}	Radius of the Earth	6.37122×10^6 m
Ω_{ref}	Rotational speed of the Earth	7.292×10^{-5} s^{-1}
g_c	Gravitational acceleration	9.80616 m s^{-2}
p_0	Reference pressure	1000 hPa
c_p	Specific heat capacity of dry air at constant pressure	1004.5 J kg^{-1} K^{-1}
c_v	Specific heat capacity of dry air at constant volume	717.5 J kg^{-1} K^{-1}
R_d	Gas constant for dry air	287.0 J kg^{-1} K^{-1}
R_ν	Gas constant for water vapor	461.5 J kg^{-1} K^{-1}
κ	Ratio of R_d to c_p	$2/7$
ε	Ratio of R_d to R_ν	0.622
M_v	Constant for virtual temperature conversion	0.608
ρ_{water}	Reference density of water	1000 kg m^{-3}

3 Model Grids

[ALL] Add a short description of your model grid here.

3.1 Latitude-longitude grid

3.2 Cubed-sphere grid

The equiangular cubed-sphere grid (Sadourny, 1972; Ronchi et al., 1996) consists of six Cartesian patches arranged along the faces of a cube which is then inflated onto a spherical shell. More information on this choice of grid can be found in Ullrich (2014). On the equiangular cubed-sphere grid, coordinates are given as (α, β, p), with central angles $\alpha, \beta \in [-\frac{\pi}{4}, \frac{\pi}{4}]$ and panel index $i_p \in \{1,2,3,4,5,6\}$. By convention, we choose panels 1–4 to be along the equator and panels 5 and 6 to be centered on the northern and southern pole, respectively.

3.3 Icosahedral grid

3.4 Centroidal Voronoi tessellation grid

4 Equation Sets

[ALL] Include the continuous equation set that you use for your model here.

4.1 Tempest

The continuity, momentum and thermodynamic equations can be written as:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}), \tag{2}$$

$$\frac{\partial \mathbf{u}}{\partial t} = -\nabla(K + \Phi) - \theta \nabla \Pi + \boldsymbol{\eta} \times \mathbf{u}, \tag{3}$$

$$\frac{\partial \theta_v}{\partial t} = -\mathbf{u} \cdot \nabla \theta_v, \tag{4}$$

in terms of Kinetic energy $K = \mathbf{u} \cdot \mathbf{u}$, geopotential $\Phi = g_c z$ and absolute vorticity $\boldsymbol{\eta} = \boldsymbol{\zeta} + \boldsymbol{\Omega}$, which consists of relative vorticity $\boldsymbol{\zeta} = \nabla \times \mathbf{u}$ and planetary vorticity $\boldsymbol{\Omega}$. The Exner pressure is related to the prognosed density and potential temperature via

$$\Pi = c_p \left(\frac{p_0}{p} \right)^{R_d/c_p} = c_p \left(\frac{R_d \rho \theta_v}{p_0} \right)^{R_d/c_v}. \tag{5}$$

4.2 Tracer transport

Lagrangian form:

$$\frac{dq}{dt} = 0. \tag{6}$$

Non-conservative Eulerian form:

$$\frac{\partial q}{\partial t} = -\mathbf{u} \cdot \nabla q. \tag{7}$$

Flux form:

$$\frac{\partial}{\partial t}(\rho q) = -\nabla \cdot (\rho q \mathbf{u}). \tag{8}$$

4.3 Height-based coordinates

Define Unstaggered, Lorenz and Charney-Phillips staggering here

4.4 Mass-based coordinates

5 Diffusion and Stabilization

[ALL] Include explicit diffusion and stabilization techniques that you have applied in the dynamical core here.

5.1 Scalar viscosity

Scalar viscosity (direct):

$$\frac{ds}{dt} = \ldots + \nu \nabla \cdot \nabla s. \tag{9}$$

Scalar viscosity (conservative):

$$\frac{d}{dt}(\rho q) = \ldots + \nu \nabla \cdot (\rho \nabla q). \tag{10}$$

5.2 Smagorinsky eddy viscosity

5.3 Vector viscosity, divergence and vorticity damping

Vector viscosity:

$$\frac{d\mathbf{u}}{dt} = \ldots + \nu \nabla^2 \mathbf{u} \tag{11}$$

Divergence damping:

$$\frac{d\mathbf{u}}{dt} = \ldots + \nu_{div} \nabla (\nabla \cdot \mathbf{u}) \tag{12}$$

Vorticity damping:

$$\frac{d\mathbf{u}}{dt} = \ldots + \nu_{vort} \nabla \times (\nabla \times \mathbf{u}) \tag{13}$$

5.4 Hyperviscosity

Repeated application of the scalar and vector viscosity operators

6 Filters and Fixers

[ALL] Include explicit filters and fixers that you have utilized in the dynamical core here.

6.1 Mass borrowing (positive definite preservation)

6.2 Mass fixers

6.3 Energy fixers

7 Temporal Discretizations

[ALL] Describe the time-stepping scheme employed by your dynamical core here.

7.1 Runge-Kutta

7.1.1 Ullrich-Kinnmark-Gray 5 step 3rd order scheme

Explicit terms are evolved using a Runge-Kutta method which supports a large stability bound for spatial discretizations with purely imaginary eigenvalues. This particular scheme is based on Kinnmark and Gray (1984a, b) and takes the form

$$
\begin{aligned}
\psi^{(1)} &= \psi^{(0)} + \tfrac{\Delta t}{5} f(\psi^{(0)}),\\
\psi^{(2)} &= \psi^{(0)} + \tfrac{\Delta t}{5} f(\psi^{(1)}),\\
\psi^{(3)} &= \psi^{(0)} + \tfrac{\Delta t}{3} f(\psi^{(2)}),\\
\psi^{(4)} &= \psi^{(0)} + \tfrac{2\Delta t}{3} f(\psi^{(3)}),\\
\psi^{(5)} &= -\tfrac{1}{4}\psi^{(0)} + \tfrac{5}{4}\psi^{(1)} + \tfrac{3\Delta t}{4} f(\psi^{(4)}).
\end{aligned}
\tag{14}
$$

7.2 Semi-Implicit time integration

8 Dynamical Cores

In this section provide a short description (approximately 0.5 pages) of the dynamical core, focusing on unique features or design specifications. Do not include information on the physical parameterizations used by the modeling system. Make reference to the model grid employed from section 3, the specific equation set being discretized by the model in section 4, explicit numerical techniques for diffusion and stabilization in section 5, filters and fixers in section 6 and the temporal discretization in section 7.

8.1 Tempest

[ULLRICH]

The Tempest model (Ullrich, 2014; Guerra and Ullrich, 2016) uses a horizontal spectral element discretization and vertical staggered nodal finite element method based on the cubed-sphere grid with terrain-following height-based coordinate. The standard Eulerian equations are employed with moist density ρ, thermodynamic closure θ_v and tracer density ρq. These continuous equations are given in section 4.1. The implementation includes both fully explicit time integration, using the UKG53 scheme described in section 7.1.1, and implicit-explicit options, where horizontal terms are explicitly discretized and vertical terms are treated implicitly. Scalar hyperviscosity is employed for ρ, θ and tracer variables via repeated application of (9). Vector hyperviscosity is also applied by decomposing the horizontal vector Laplacian into divergence damping (12) and vorticity damping (13) terms. Both viscosity operations are applied after the completion of all Runge-Kutta sub-cycles.

8.2 High-Order Method Modeling Environment (HOMME)

[HALL]

8.3 Model for Prediction Across Scales (MPAS)

[SKAMAROCK]

8.4 Colorado State University Model (CSU)

[RANDALL]

8.5 Geophysical Fluid Dynamics Laboratory FV Cubed (GFDL-FV3)

[HARRIS]

8.6 Chombo

[JOHANSEN]

8.7 Naval Research Laboratory NEPTUNE Model

[VINER, REINECKE]

8.8 Global Environmental Multiscale (GEM) Model

[LEE]

8.9 Ocean-Land-Atmosphere Model (OLAM)

[WALKO]

8.10 DYNAMICO

[DUBOS]

8.11 ECMWF PentaRei Finite Volume Model

[KUEHNLEIN]

8.12 Icosahedral Non-hydrostatic (ICON) Model

[GIORGETTA]

8.13 Nonhydrostatic ICosahedral Atmospheric Model (NICAM) Model

[MIURA]

9 Idealized Physical Parameterizations

9.1 Kessler Physics

The cloud microphysics update according to the following equation set:

$$\frac{\Delta\theta}{\Delta t} = -\frac{L}{c_p\pi}\left(\frac{\Delta q_{vs}}{\Delta t} + E_r\right) \tag{15}$$

$$\frac{\Delta q_v}{\Delta t} = \frac{\Delta q_{vs}}{\Delta t} + E_r \tag{16}$$

$$\frac{\Delta q_c}{\Delta t} = -\frac{\Delta q_{vs}}{\Delta t} - A_r - C_r \tag{17}$$

$$\frac{\Delta q_r}{\Delta t} = -E_r + A_r + C_r - V_r\frac{\partial q_r}{\partial z}, \tag{18}$$

where L is the latent heat of condensation, A_r is the autoconversion rate of cloud water to rain water, C_r is the collection rate of rain water, E_r is the rain water evaporation rate, and V_r is the rain water terminal velocity.

The pressure follows from the equation of state

$$p = \rho R_d T(1 + 0.61q_v) \tag{19}$$

with p the pressure, ρ the density of moist air, R_d the gas constant for dry air, T the temperature and q_v the mixing ratio of water vapor. The equation is rewritten as a nondimensional pressure Π equation.

$$\pi = \left(\frac{p}{p_0}\right)^{\frac{R_d T}{cp}} \tag{20}$$

To determine the saturation vapor mixing ratio the Teten's formula is used,

$$q_{vs}(p,T) = \left(\frac{380.0}{p}\right)\exp\left(17.27 \times \frac{T - 273.0}{T - 36.0}\right) \tag{21}$$

The autoconvection rate (A_r) and collection rate (C_r) follow Kessler parametrization and are defined by:

$$A_r = k_1(q_c - a) \tag{22}$$

$$C_r = k_2 q_c q_r^{0.875} \tag{23}$$

With $k_1 = 0.001\mathrm{s}^{-1}$, $a = 0.001\mathrm{g.g}^{-1}$ and $k_2 = 2.2\mathrm{s}^{-1}$

Deriving from Klemp and Wilhelmson (1978) description of cloud water,rain water and water vapor mixing ratios. they are define as followed:

$$q_c^{n+1} = \max(q_c^r - \Delta q_r, 0) \tag{24}$$

$$q_r^{n+1} = \max(q_r^r - \Delta q_r + S, 0) \tag{25}$$

where S is the sedimentation term and Δq_r is defined as

$$\Delta q_r = q_c^n - \frac{q_c^n - \Delta \mathrm{t}\, \max(A_r, 0)}{1 + \Delta \mathrm{t} C_r} \tag{26}$$

The Rain evaporation equation is defined similarly to Ogura and Takahashi (1971) description:

$$E_r = \frac{1}{\rho} \frac{\left(1 - \frac{q_v}{q_{vs}}\right) C(\rho q_r)^{0.525}}{5.4 \times 10^5 + \frac{2.55 \times 10^6}{p q_{vs}}} \tag{27}$$

With ventilation factor C define as

$$C_r = 1.6 + 124.9(\rho q_r)^{0.2046} \tag{28}$$

The liquid water terminal velocity is similar to Soong and Ogura (1973) description with a mean density adjustment as suggested by Kessler (1969):

$$V_r = 36349(\rho q_r)^{0.1346} \left(\frac{\rho}{\rho_0}\right)^{-\frac{1}{2}} \tag{29}$$

9.2 Simplified Mixing in the Planetary Boundary Layer

The forcing by the planetary boundary layer is described in Reed and Jablonowski (2012) and is partly reproduced here. To parameterize the surface fluxes that impact the zonal velocity u, the meridional velocity v and moisture q we start with the time rate of change equations

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial \rho\, \overline{w'u'}}{\partial z} \tag{30}$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho} \frac{\partial \rho\, \overline{w'v'}}{\partial z} \tag{31}$$

$$\frac{\partial q}{\partial t} = -\frac{1}{\rho} \frac{\partial \rho\, \overline{w'q'}}{\partial z}. \tag{32}$$

Potential temperature, as opposed to temperature, is used in the boundary layer parameterization because the vertical profile of the potential temperature is a suitable indicator of static stability. This adds the time rate of change equation

$$\frac{\partial \Theta}{\partial t} = -\frac{1}{\rho} \frac{\partial \rho\, \overline{w'\Theta'}}{\partial z}. \tag{33}$$

Here u', v', w', Θ' and q' symbolize the deviations of the zonal velocity, meridional velocity, vertical velocity, potential temperature and specific humidity from their averages, respectively. The average is indicated by an overbar. Note, assuming pressure is held constant (which is a common assumption in physical parameterizations), the potential temperature time tendency can be converted back to a temperature tendency of the following form

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho} \left(\frac{p}{p_0}\right)^{\kappa} \frac{\partial \rho\, \overline{w'\Theta'}}{\partial z}. \tag{34}$$

with the reference pressure $p_0 = 1000$ hPa.

The turbulent mixing is characterized by a constant vertical eddy diffusivity to represent Ekman-like profiles of boundary layers

$$\overline{w'u'} = -K_m \frac{\partial u}{\partial z} \tag{35}$$

$$\overline{w'v'} = -K_m \frac{\partial v}{\partial z} \tag{36}$$

$$\overline{w'\Theta'} = -K_E \frac{\partial \Theta}{\partial z} \tag{37}$$

$$\overline{w'q'} = -K_E \frac{\partial q}{\partial z}. \tag{38}$$

Here, K_m is the eddy diffusivity coefficient for momentum and K_E is the eddy diffusivity coefficient for energy and set equal to that for water vapor. In order to calculate the eddy diffusivity coefficients, the eddy diffusivity is matched to that for the surface flux calculated in Appendix **??** at the lowermost model level. To allow for a smooth transition above the boundary layer ($p_{top} = 850$ hPa) the diffusivity coefficients for momentum taper to zero as

$$\begin{aligned} K_m &= C_d|\boldsymbol{v}_a|z_a && \text{for } p > p_{top} \\ K_m &= C_d|\boldsymbol{v}_a|z_a \exp\left(-\left[\tfrac{p_{top}-p}{p_{strato}}\right]^2\right) && \text{for } p \leq p_{top}. \end{aligned} \tag{39}$$

Here the constant p_{strato} determines the rate of decrease and is set to 100 hPa. K_E is defined by

$$\begin{aligned} K_E &= C_E|\boldsymbol{v}_a|z_a && \text{for } p > p_{top} \\ K_E &= C_E|\boldsymbol{v}_a|z_a \exp\left(-\left[\tfrac{p_{top}-p}{p_{strato}}\right]^2\right) && \text{for } p \leq p_{top}. \end{aligned} \tag{40}$$

We suggest implementing the boundary layer scheme with an implicit temporal discretization to avoid numerical instabilities. The details of this discretization are somewhat complicated, and so we refer to implementation details in Appendix D of Reed and Jablonowski (2012). In addition, we supply the DCMIP modeling groups with the complete “simple-physics” package as used in the model CAM which can serve as a template routine.

10 Conclusions

TEXT

Author contributions. TEXT

Acknowledgements. [Include a complete list of DCMIP2016 student participants here along with sponsors]

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