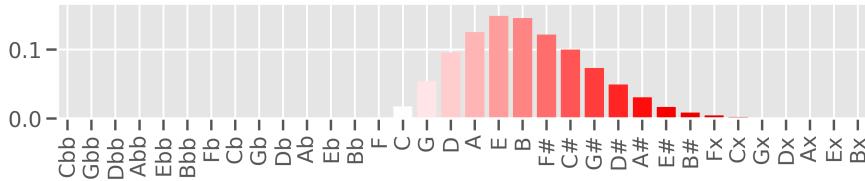


Supporting material for:
“Discovering tonal profiles with Latent Dirichlet Allocation”
Music & Science

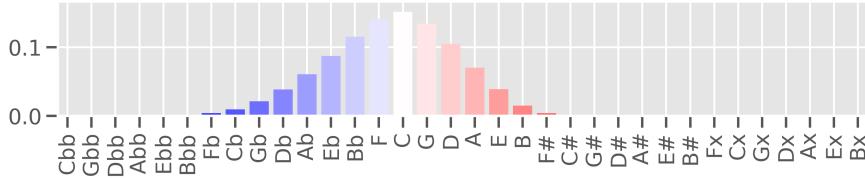
Fabian C. Moss & Martin Rohrmeier

June 14, 2021

1 Figures

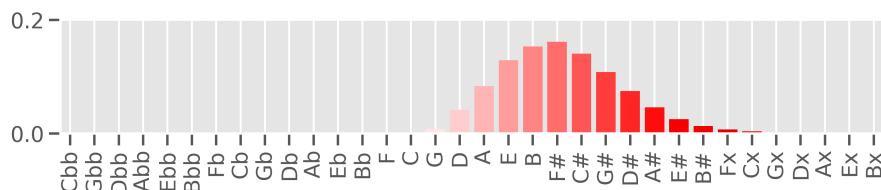


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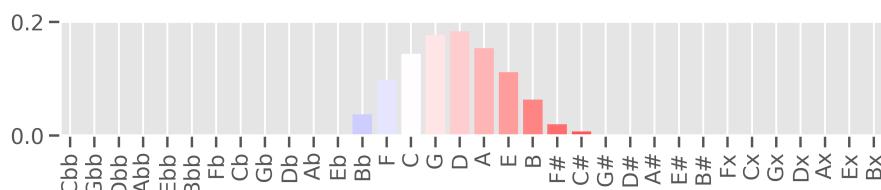


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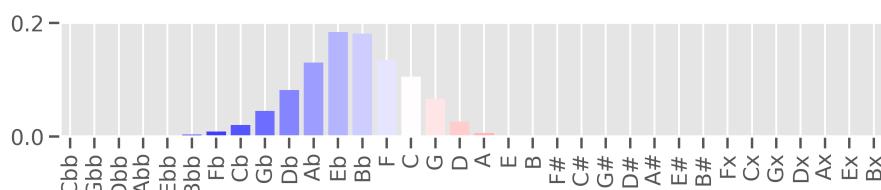
Figure 1: The note distributions for $K = 2$ topics.



(a) Topic 1 of 3.

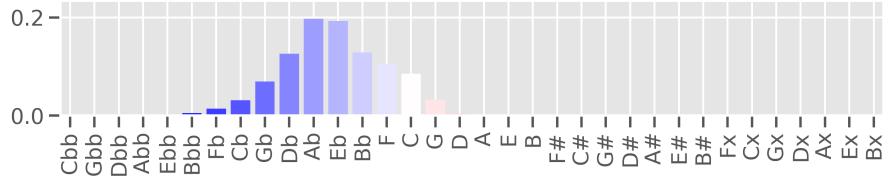


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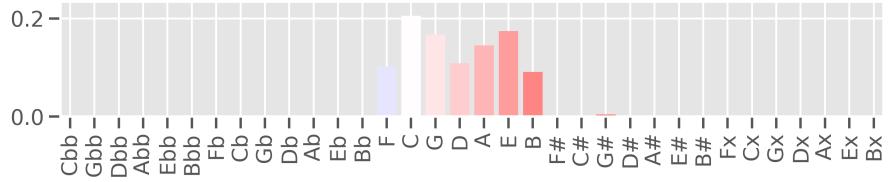


(c) Topic 3 of 3.

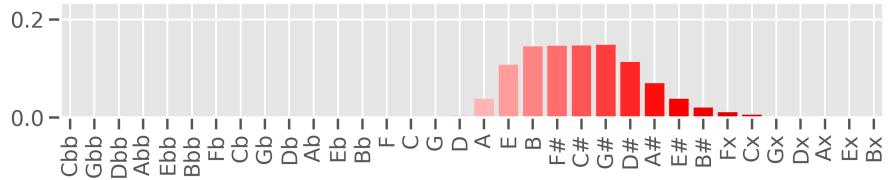
Figure 2: The note distributions for $K = 3$ topics.



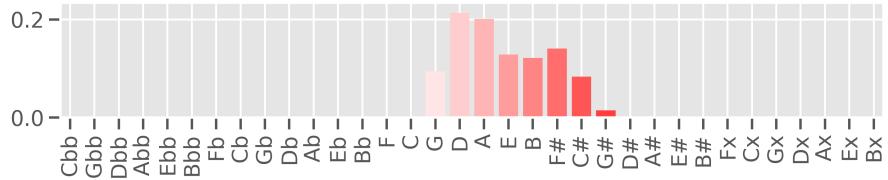
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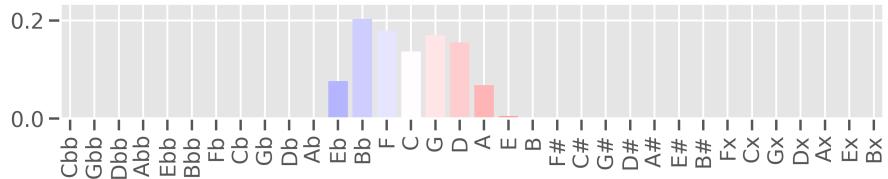
(b) Topic 2 of 5.



(c) Topic 3 of 5.

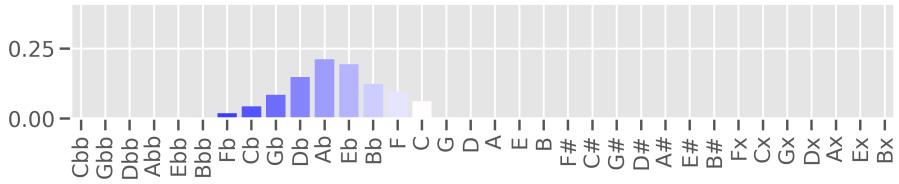


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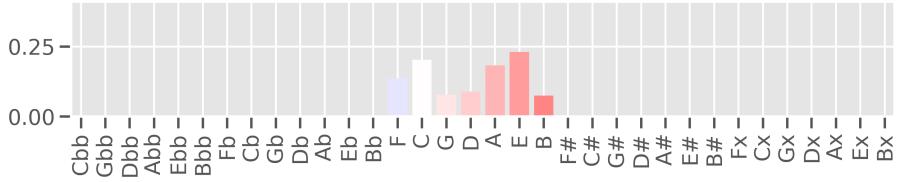


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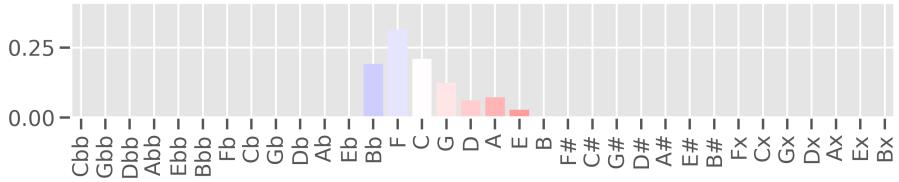
Figure 3: The note distributions for $K = 5$ topics.



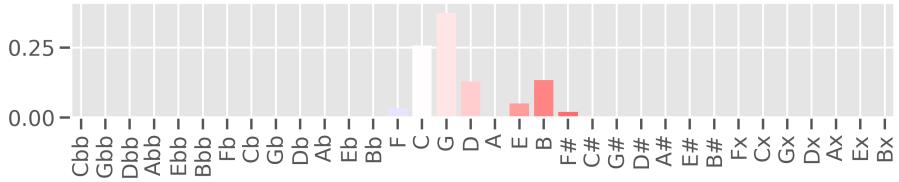
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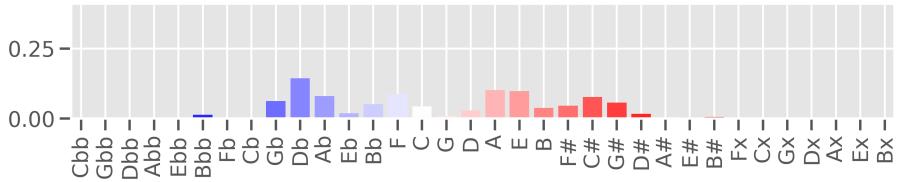
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(c) Topic 3 of 10.

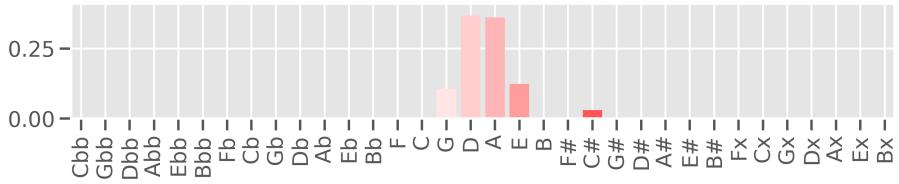


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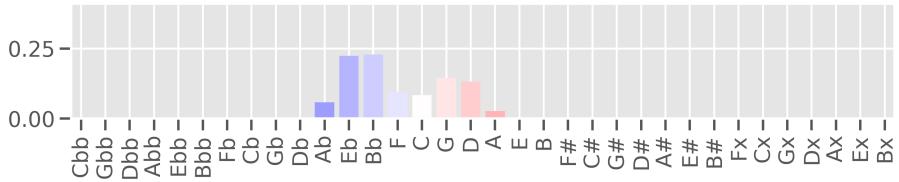


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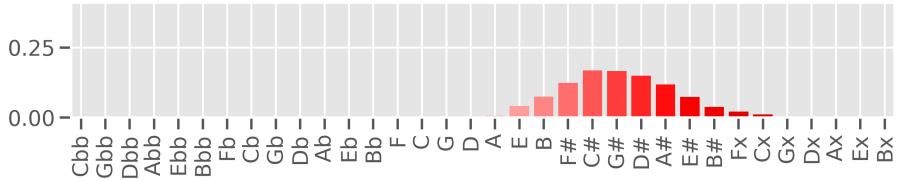
Figure 4: The note distributions for $K = 10$ topics.



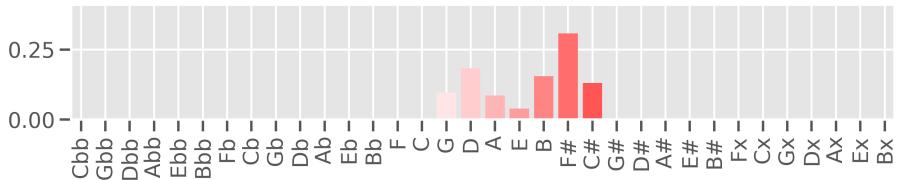
(f) Topic 6 of 10.



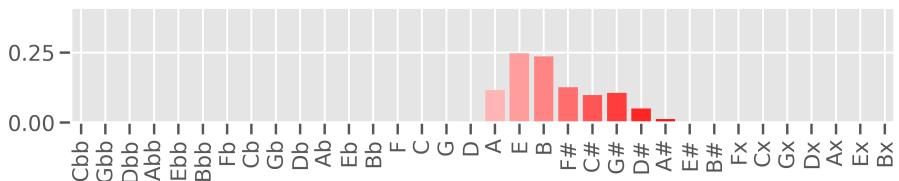
(g) Topic 7 of 10.



(h) Topic 8 of 10.

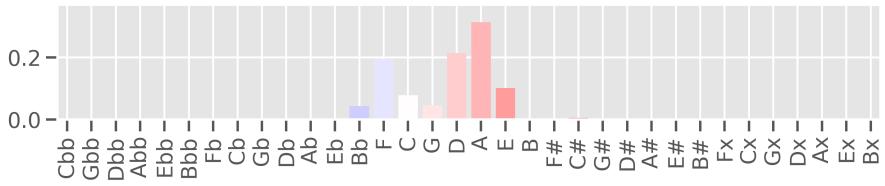


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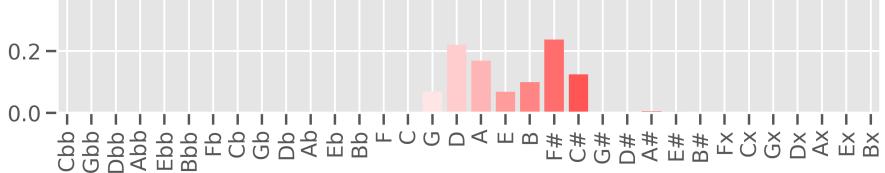


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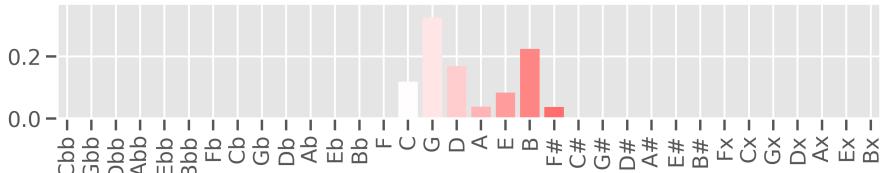
Figure 4: The note distributions for $K = 10$ topics (cont.).



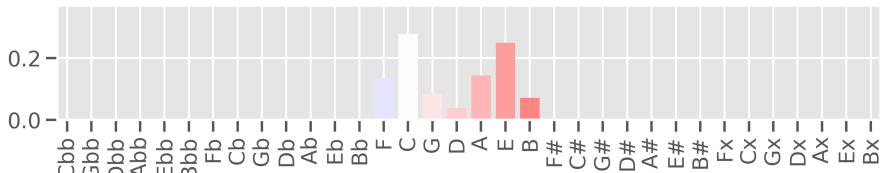
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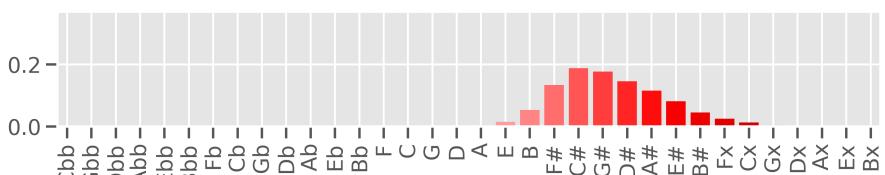
(b) Topic 2 of 12.



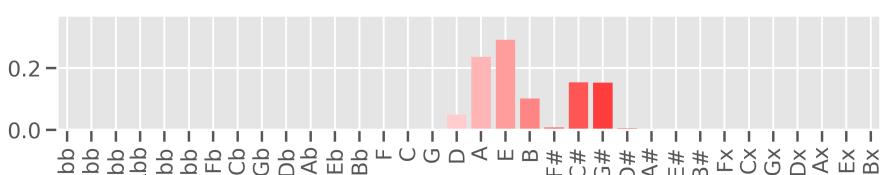
(c) Topic 3 of 12.



(d) Topic 4 of 12.

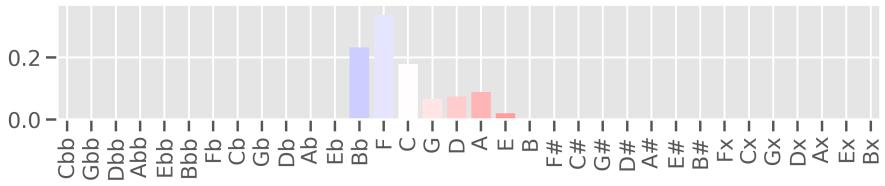


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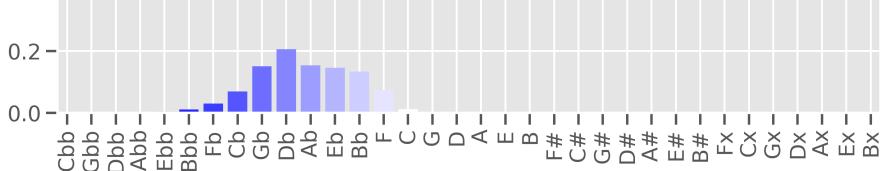


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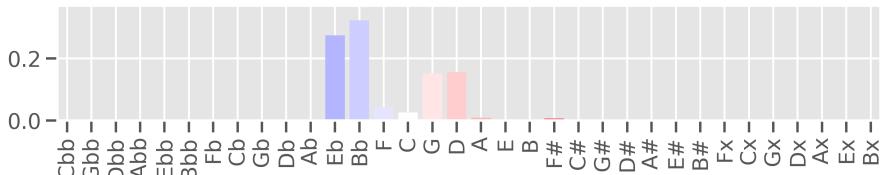
Figure 5: The note distributions for $K = 12$ topics.



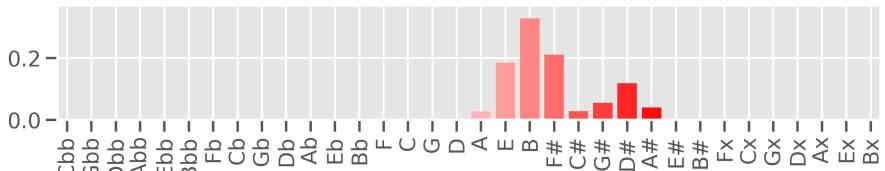
(g) Topic 7 of 12.



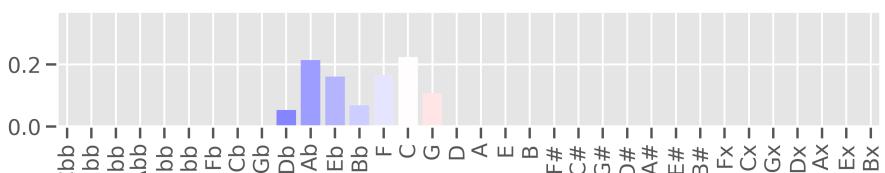
(h) Topic 8 of 12.



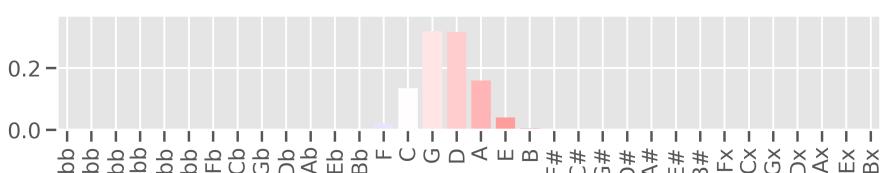
(i) Topic 9 of 12.



(j) Topic 10 of 12.



(k) Topic 11 of 12.



(l) Topic 12 of 12.

Figure 5: The note distributions for $K = 12$ topics (cont.).

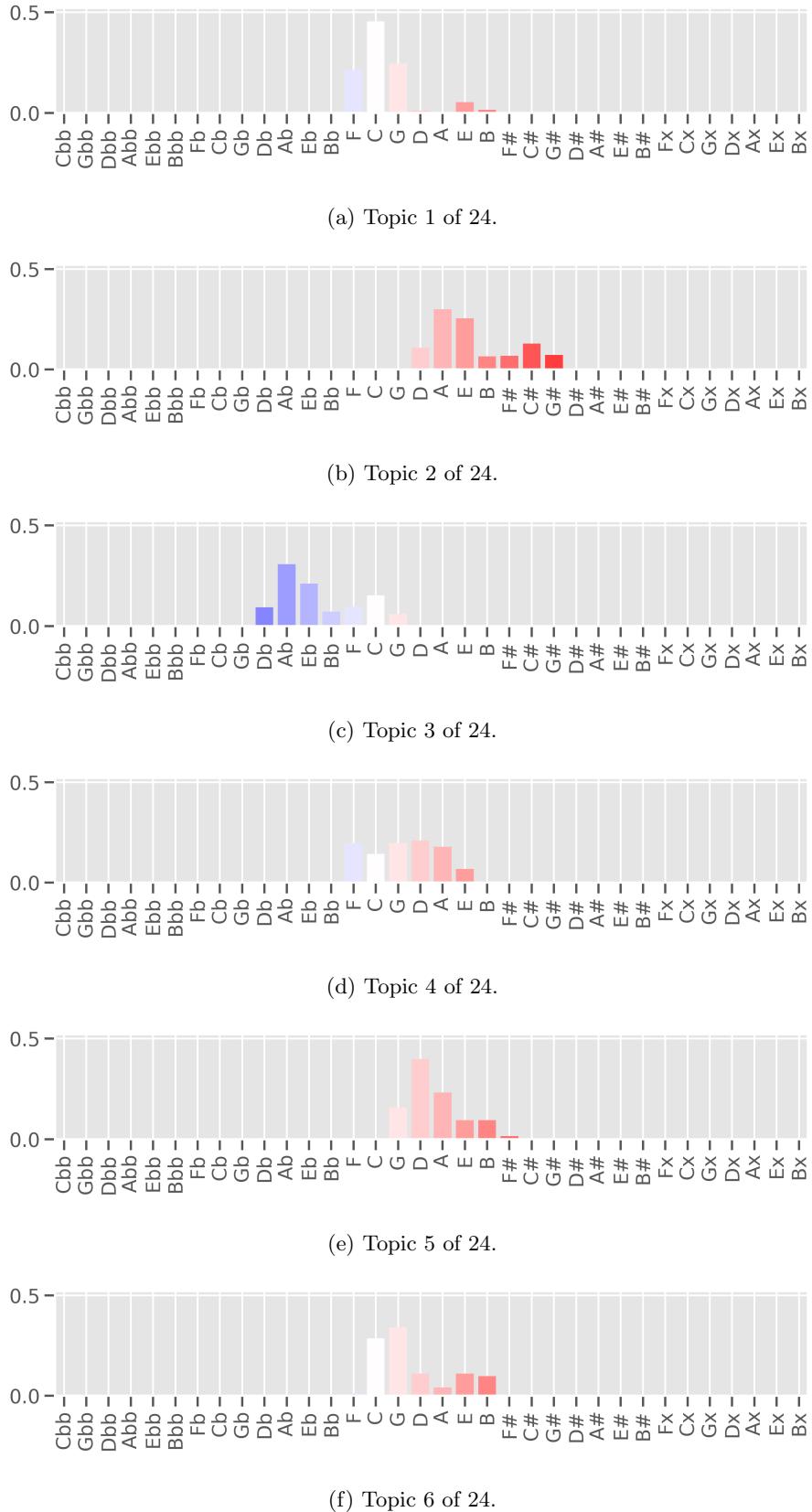
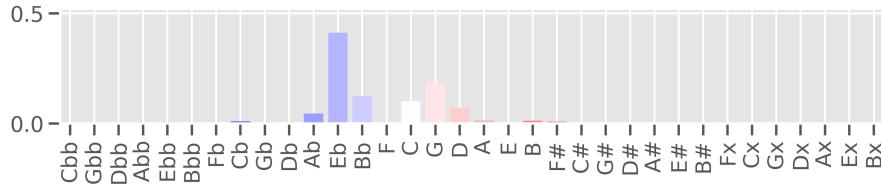
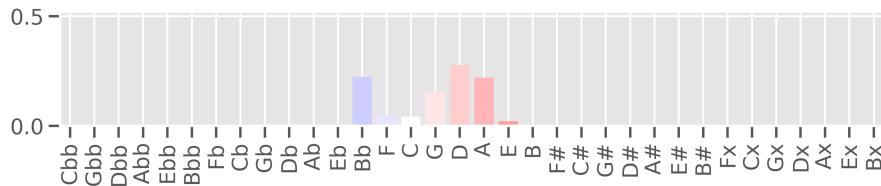


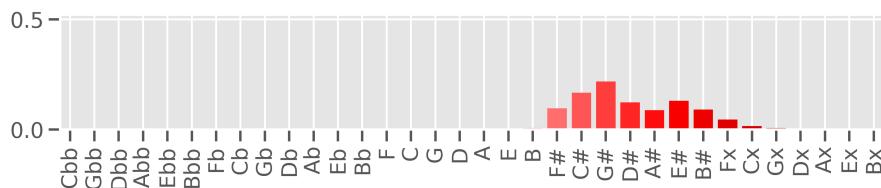
Figure 6: The note distributions for $K = 24$ topics.



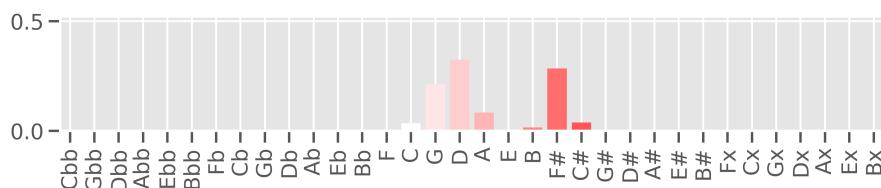
(g) Topic 7 of 24.



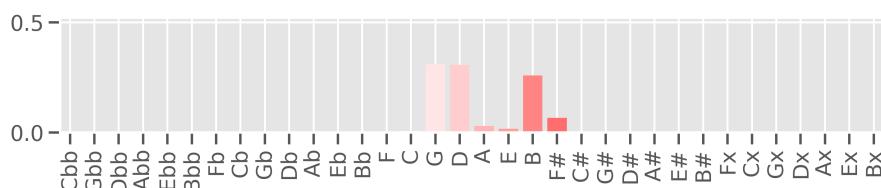
(h) Topic 8 of 24.



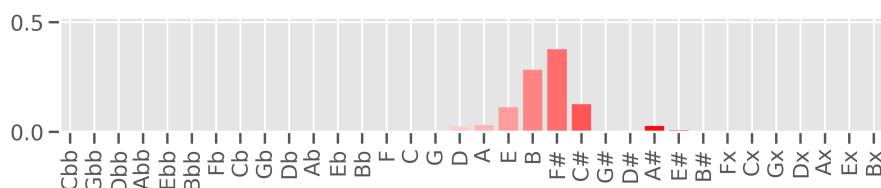
(i) Topic 9 of 24.



(j) Topic 10 of 24.

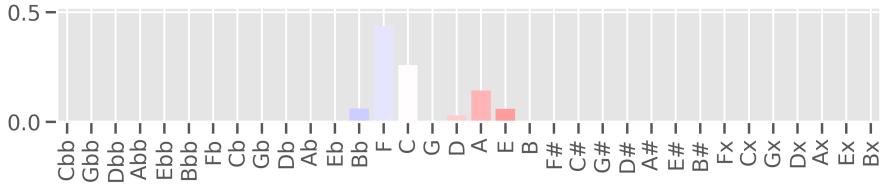


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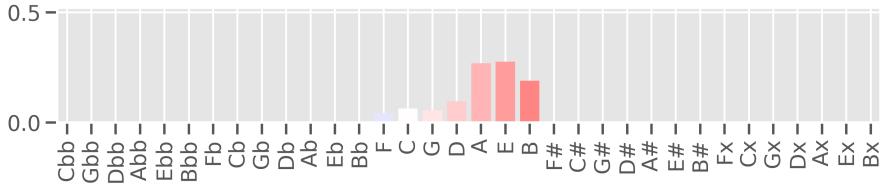


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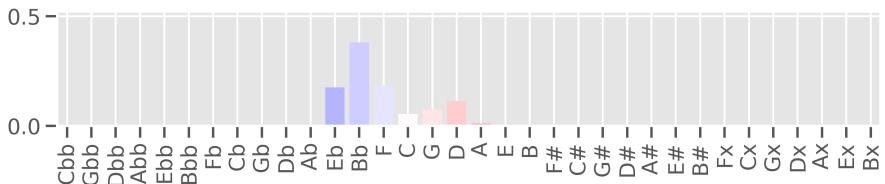
Figure 6: The note distributions for $K = 24$ topics (cont.).



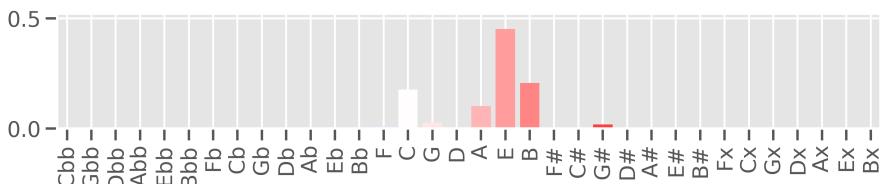
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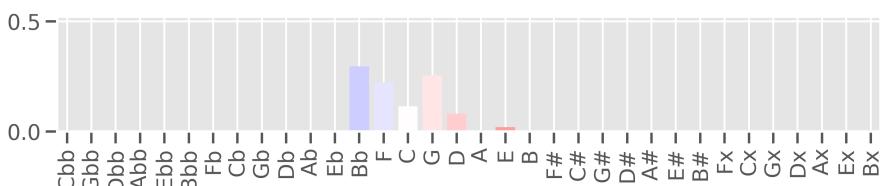
(n) Topic 14 of 24.



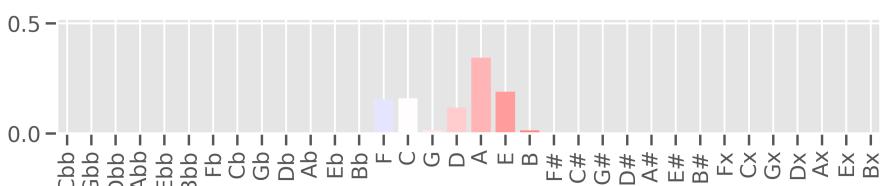
(o) Topic 15 of 24.



(p) Topic 16 of 24.

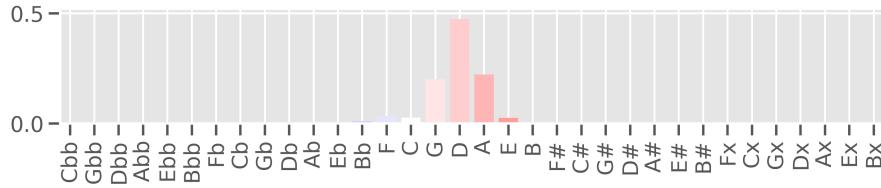


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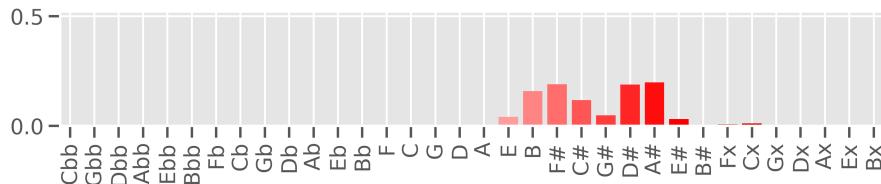


(r) Topic 18 of 24.

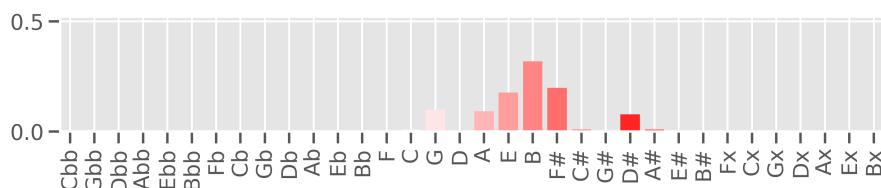
Figure 6: The note distributions for $K = 24$ topics (cont.).



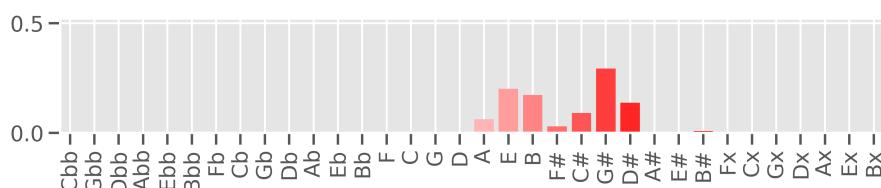
(s) Topic 19 of 24.



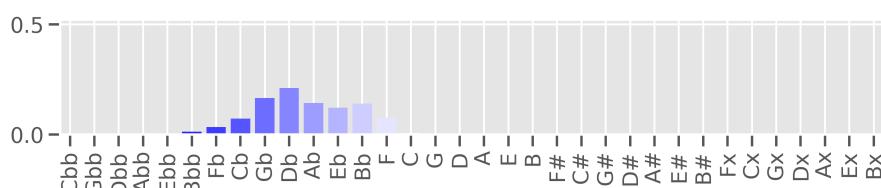
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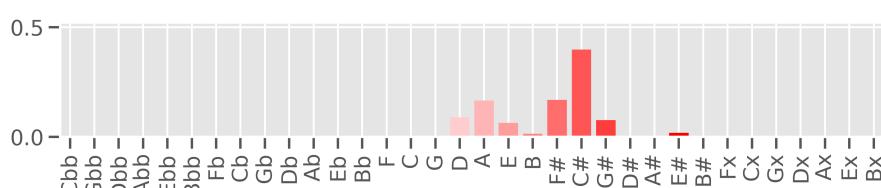
(u) Topic 21 of 24.



(v) Topic 22 of 24.



(w) Topic 23 of 24.



(x) Topic 24 of 24.

Figure 6: The note distributions for $K = 24$ topics (cont.).

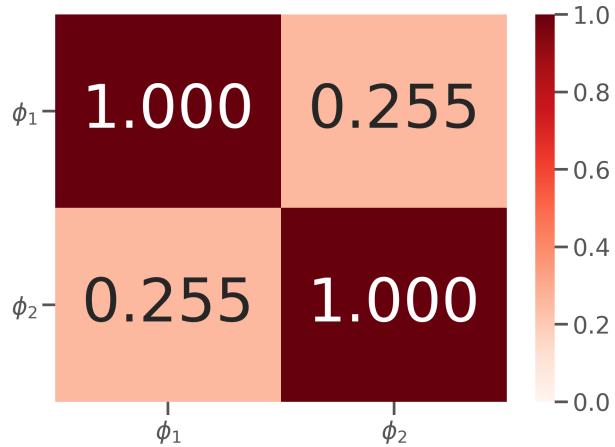


Figure 7: Jensen-Shannon similarities for $K = 2$ topics.

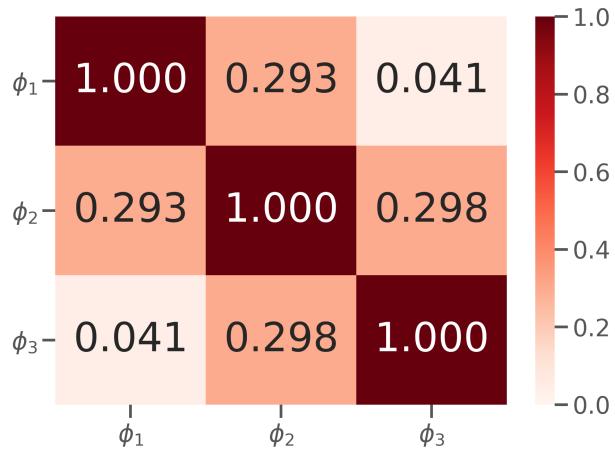


Figure 8: Jensen-Shannon similarities for $K = 3$ topics.

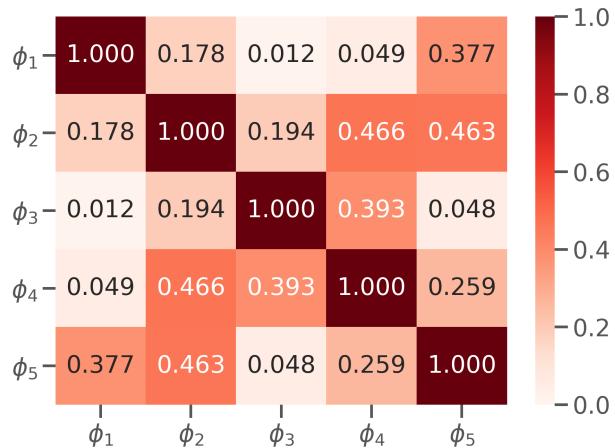


Figure 9: Jensen-Shannon similarities for $K = 5$ topics.

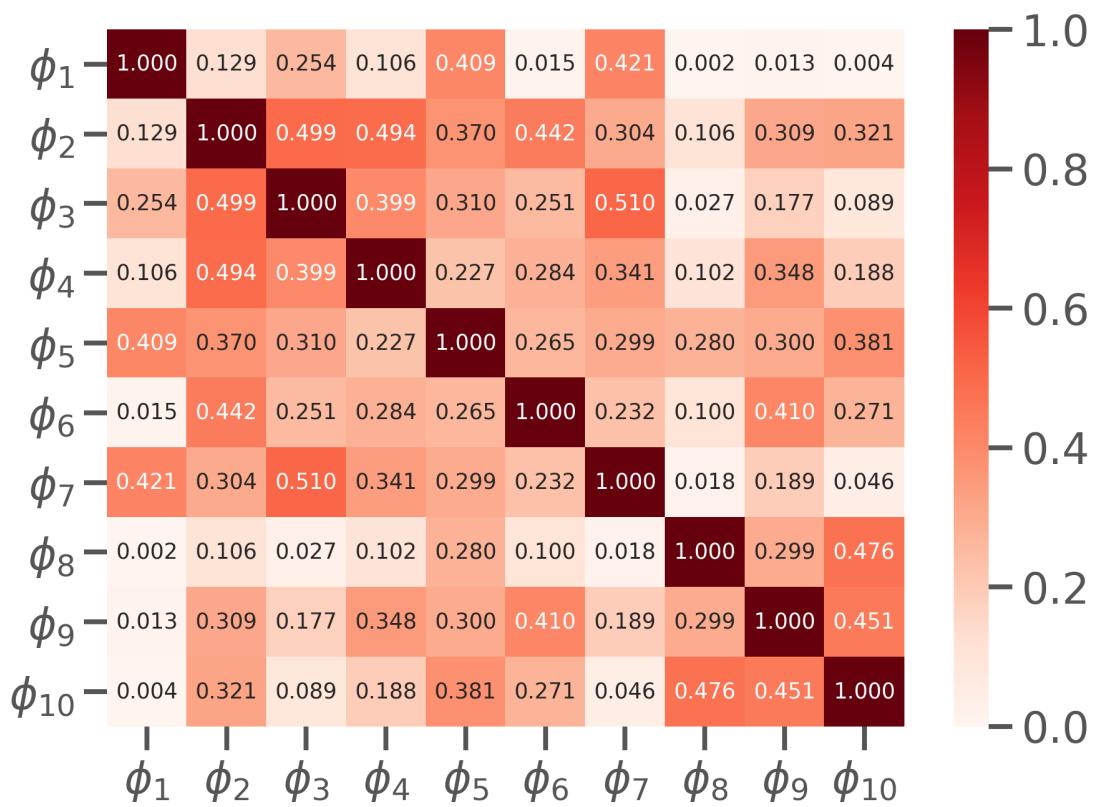


Figure 10: Jensen-Shannon similarities for $K = 10$ topics.

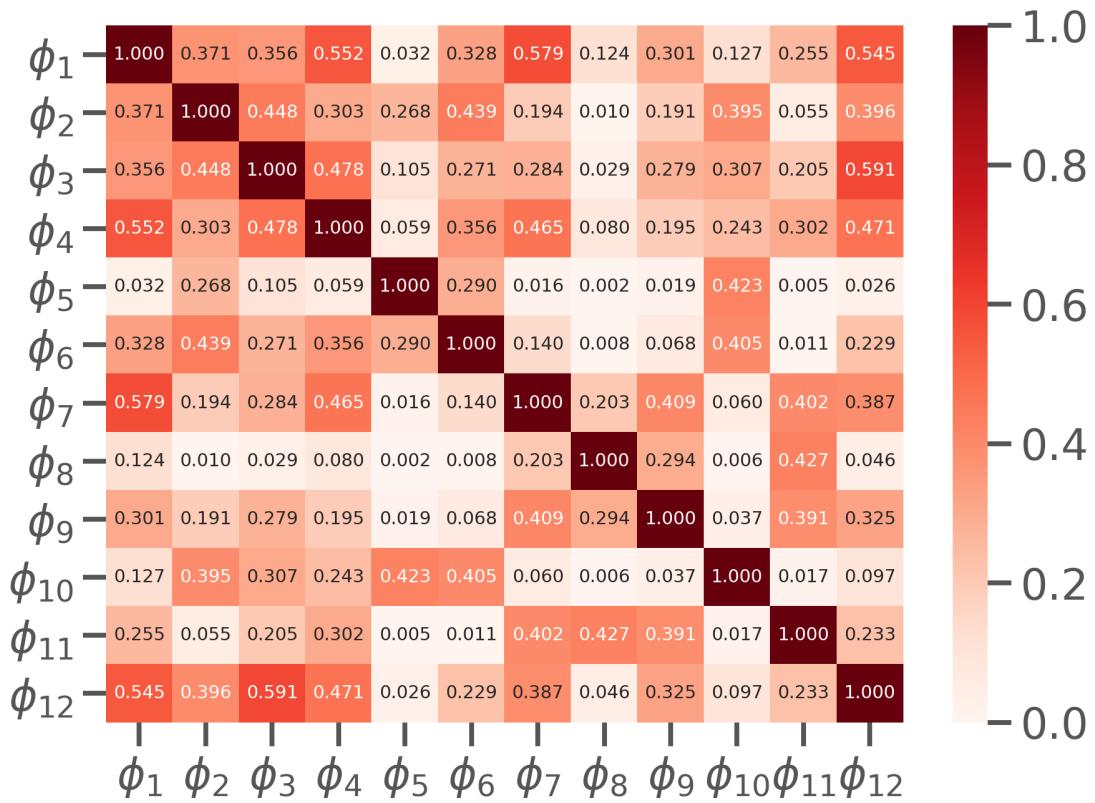


Figure 11: Jensen-Shannon similarities for $K = 12$ topics.

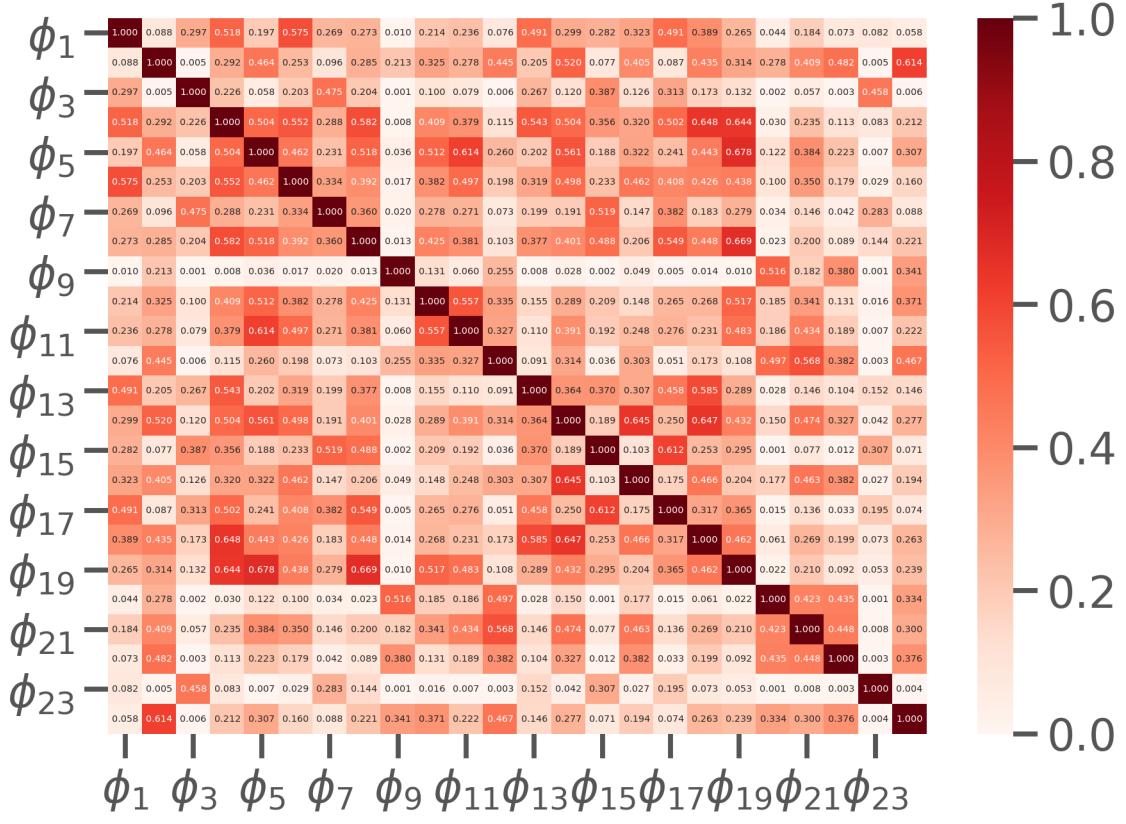


Figure 12: Jensen-Shannon similarities for $K = 24$ topics.

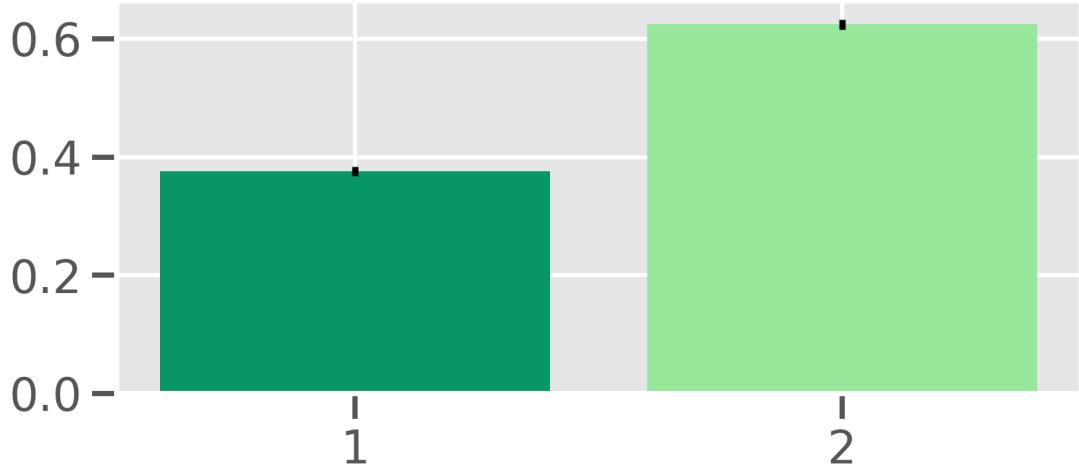


Figure 13: Average distribution of topics for all documents in the XML corpus for $K = 2$ topics. Error bars show the standard error of the mean.

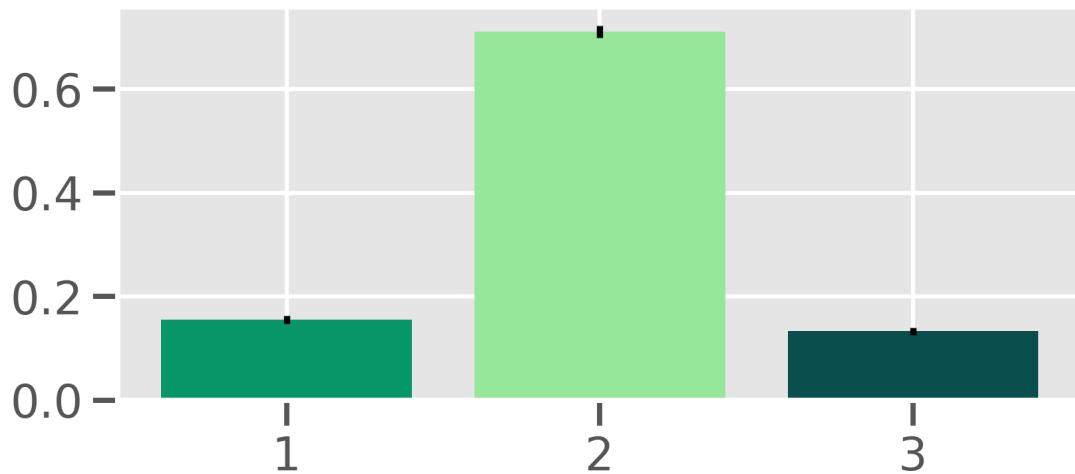


Figure 14: Average distribution of topics for all documents in the XML corpus for $K = 3$ topics. Error bars show the standard error of the mean.

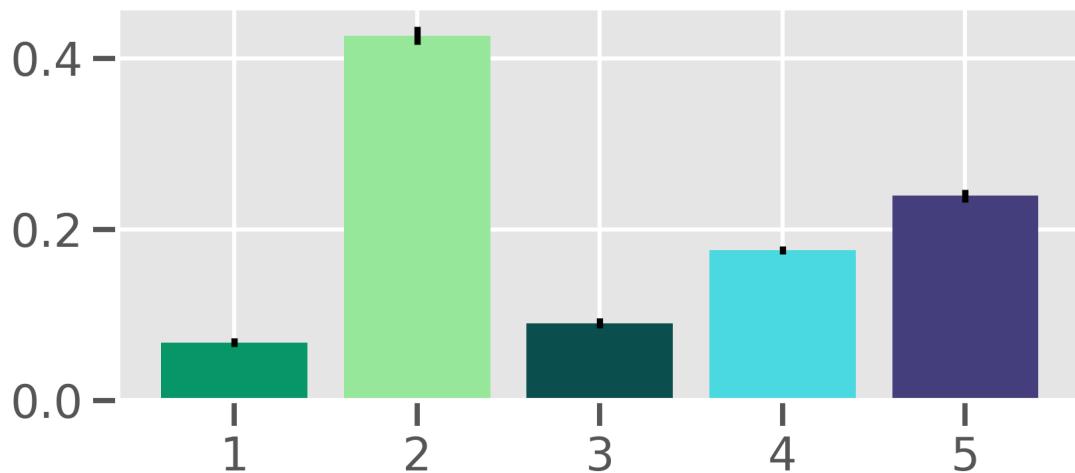


Figure 15: Average distribution of topics for all documents in the XML corpus for $K = 5$ topics. Error bars show the standard error of the mean.

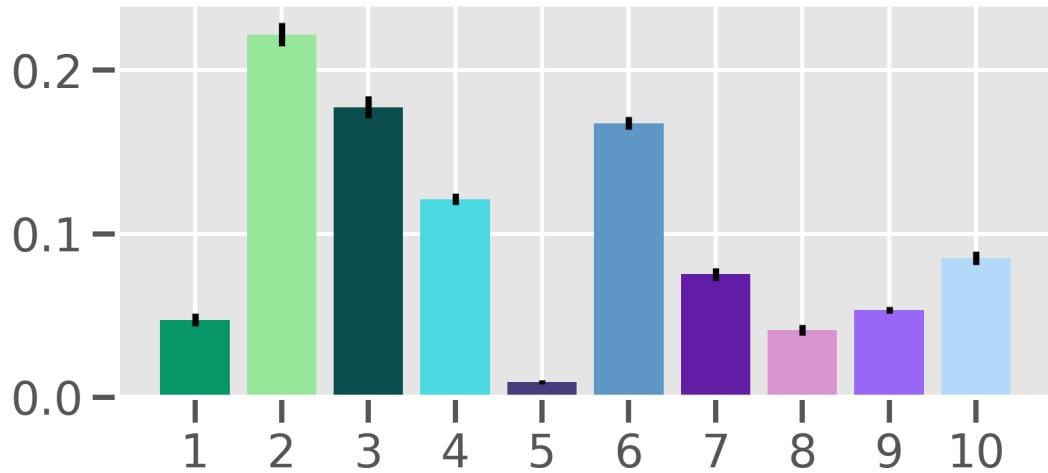


Figure 16: Average distribution of topics for all documents in the XML corpus for $K = 10$ topics. Error bars show the standard error of the mean.

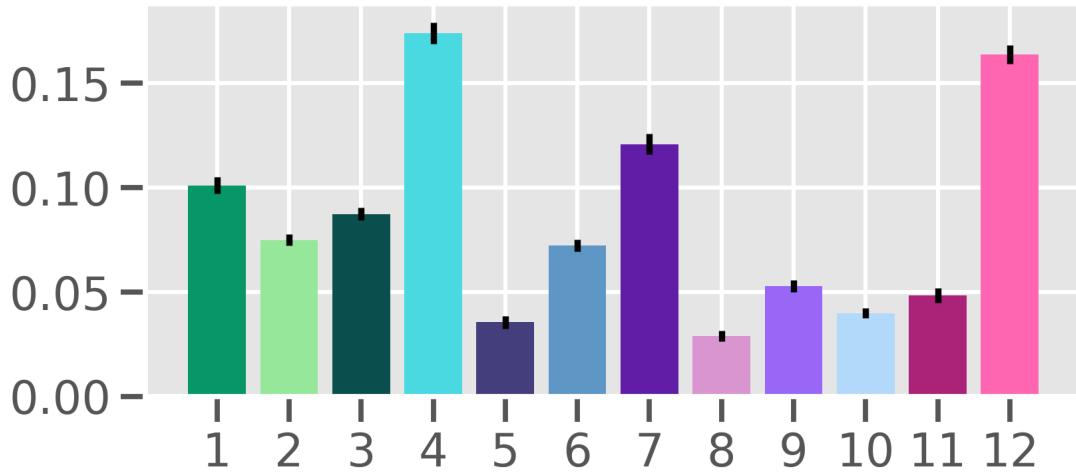


Figure 17: Average distribution of topics for all documents in the XML corpus for $K = 12$ topics. Error bars show the standard error of the mean.

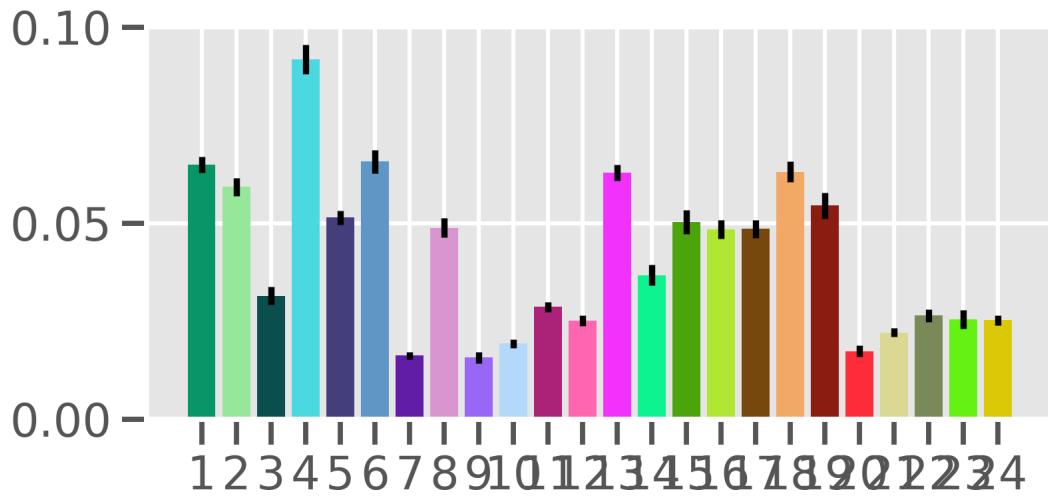


Figure 18: Average distribution of topics for all documents in the XML corpus for $K = 24$ topics. Error bars show the standard error of the mean.

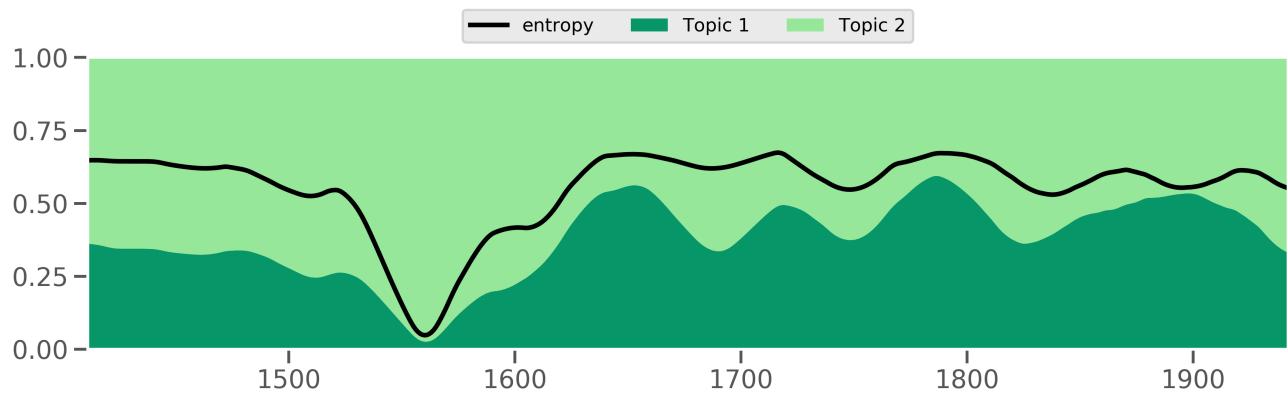


Figure 19: Topic evolution for $K = 2$ topics.

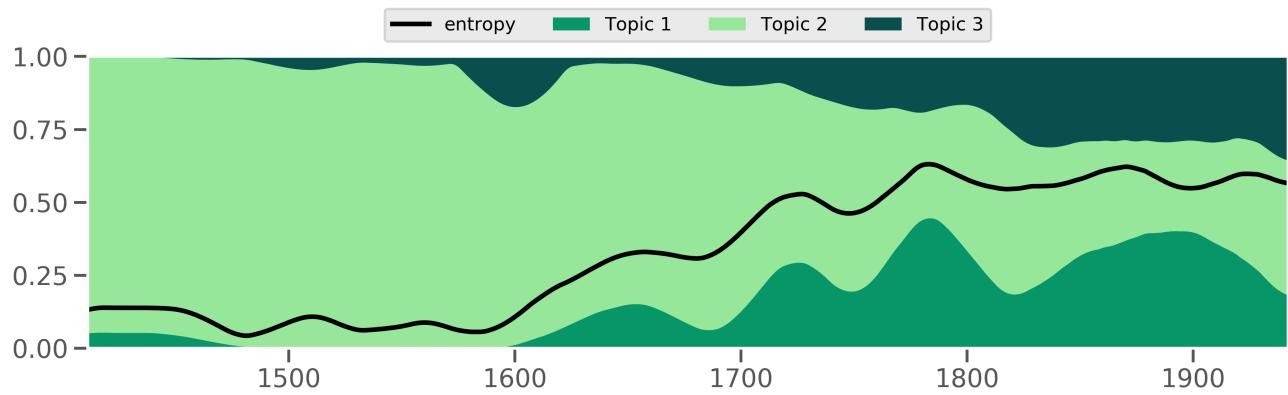


Figure 20: Topic evolution for $K = 3$ topics.

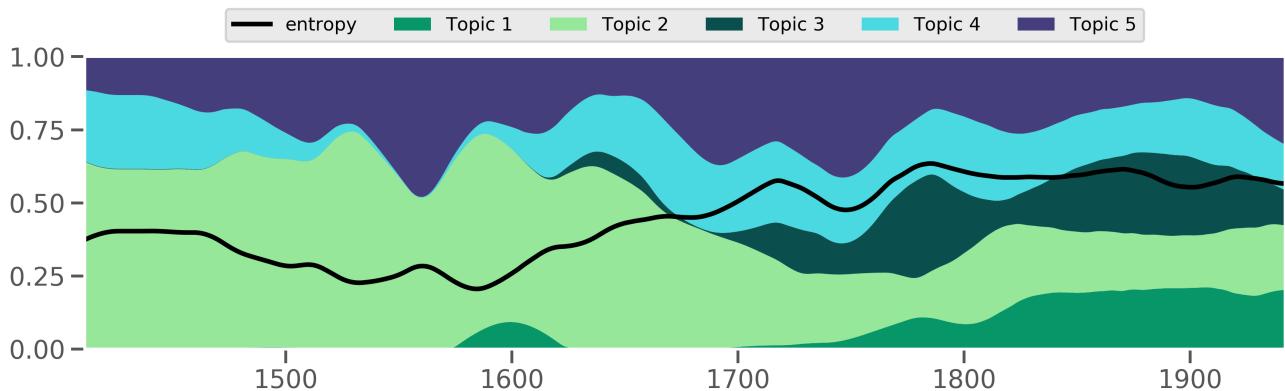


Figure 21: Topic evolution for $K = 5$ topics.

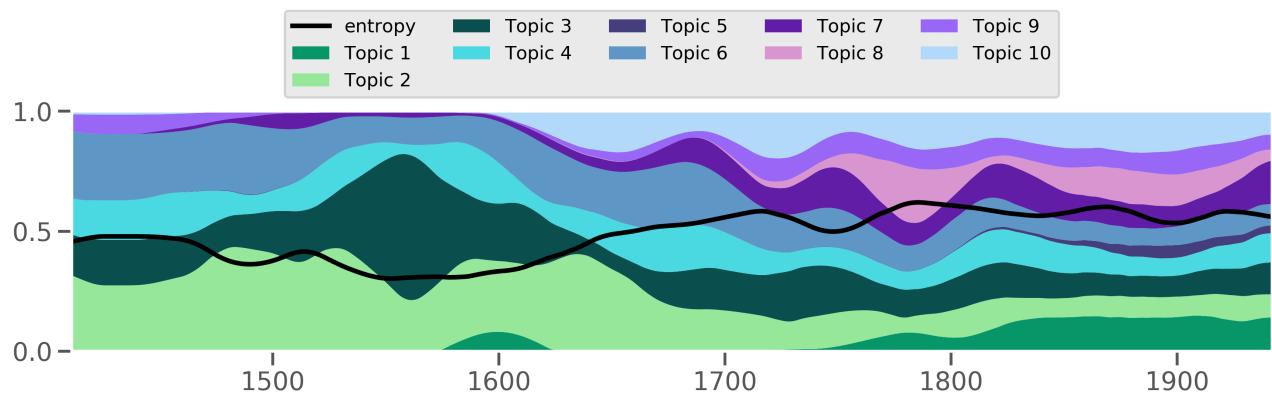


Figure 22: Topic evolution for $K = 10$ topics.

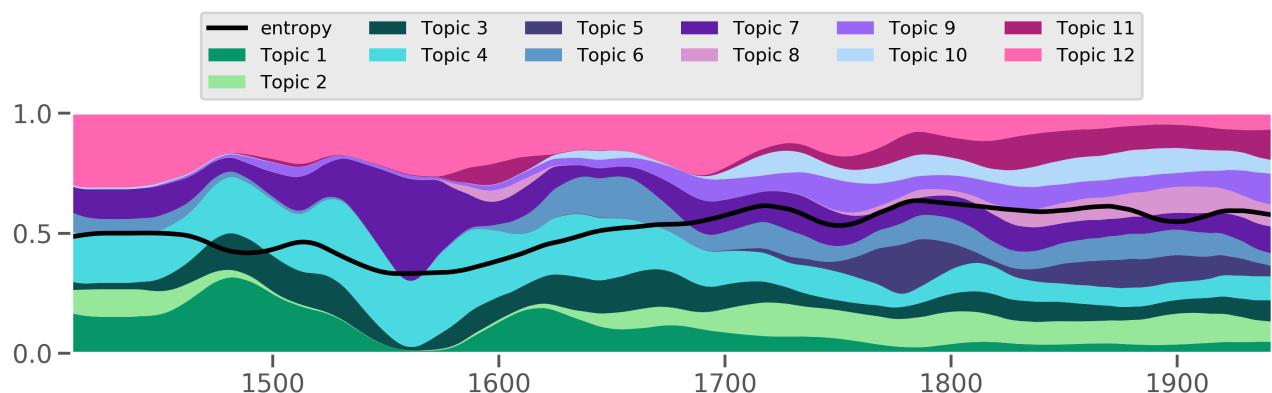


Figure 23: Topic evolution for $K = 12$ topics.

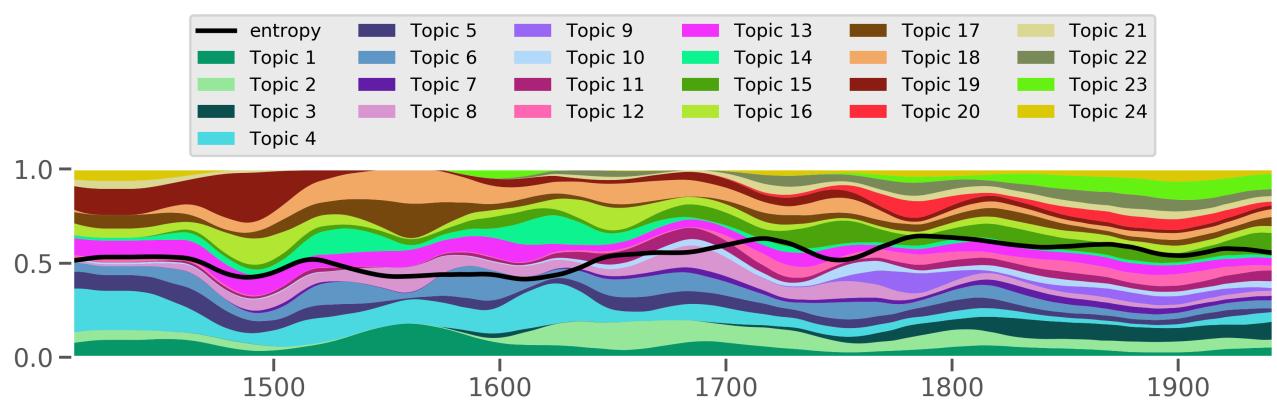


Figure 24: Topic evolution for $K = 24$ topics.

2 Tables

Table 1: Tonal pitch-class distribution for $K = 2$ topics.

	ϕ_1	ϕ_2
F $\flat\flat$	0.000	0.000
C $\flat\flat$	0.000	0.000
G $\flat\flat$	0.000	0.000
D $\flat\flat$	0.000	0.000
A $\flat\flat$	0.000	0.000
E $\flat\flat$	0.000	0.001
B $\flat\flat$	0.000	0.002
F \flat	0.000	0.004
C \flat	0.000	0.010
G \flat	0.000	0.021
D \flat	0.000	0.038
A \flat	0.000	0.061
E \flat	0.000	0.087
B \flat	0.000	0.115
F	0.003	0.141
C	0.018	0.151
G	0.054	0.135
D	0.096	0.105
A	0.125	0.070
E	0.149	0.039
B	0.146	0.015
F \sharp	0.122	0.004
C \sharp	0.100	0.001
G \sharp	0.073	0.000
D \sharp	0.050	0.000
A \sharp	0.031	0.000
E \sharp	0.017	0.000
B \sharp	0.009	0.000
F $\sharp\sharp$	0.005	0.000
C $\sharp\sharp$	0.003	0.000
G $\sharp\sharp$	0.001	0.000
D $\sharp\sharp$	0.000	0.000
A $\sharp\sharp$	0.000	0.000
E $\sharp\sharp$	0.000	0.000
B $\sharp\sharp$	0.000	0.000

Table 2: Tonal pitch-class distribution for $K = 3$ topics.

	ϕ_1	ϕ_2	ϕ_3
F $\flat\flat$	0.000	0.000	0.000
C $\flat\flat$	0.000	0.000	0.000
G $\flat\flat$	0.000	0.000	0.000
D $\flat\flat$	0.000	0.000	0.000
A $\flat\flat$	0.000	0.000	0.000
E $\flat\flat$	0.000	0.000	0.001
B $\flat\flat$	0.000	0.000	0.003
F \flat	0.000	0.000	0.009
C \flat	0.000	0.000	0.021
G \flat	0.000	0.000	0.045
D \flat	0.000	0.000	0.082
A \flat	0.000	0.000	0.130
E \flat	0.000	0.002	0.184
B \flat	0.000	0.038	0.181
F	0.000	0.099	0.135
C	0.000	0.144	0.106
G	0.008	0.177	0.067
D	0.042	0.184	0.026
A	0.084	0.154	0.006
E	0.129	0.111	0.002
B	0.154	0.063	0.000
F \sharp	0.161	0.020	0.000
C \sharp	0.141	0.007	0.000
G \sharp	0.108	0.002	0.000
D \sharp	0.075	0.000	0.000
A \sharp	0.046	0.000	0.000
E \sharp	0.025	0.000	0.000
B \sharp	0.013	0.000	0.000
F $\sharp\sharp$	0.007	0.000	0.000
C $\sharp\sharp$	0.004	0.000	0.000
G $\sharp\sharp$	0.001	0.000	0.000
D $\sharp\sharp$	0.000	0.000	0.000
A $\sharp\sharp$	0.000	0.000	0.000
E $\sharp\sharp$	0.000	0.000	0.000
B $\sharp\sharp$	0.000	0.000	0.000

Table 3: Tonal pitch-class distribution for $K = 5$ topics.

	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
F $\flat\flat$	0.000	0.000	0.000	0.000	0.000
C $\flat\flat$	0.000	0.000	0.000	0.000	0.000
G $\flat\flat$	0.000	0.000	0.000	0.000	0.000
D $\flat\flat$	0.000	0.000	0.000	0.000	0.000
A $\flat\flat$	0.001	0.000	0.000	0.000	0.000
E $\flat\flat$	0.002	0.000	0.000	0.000	0.000
B $\flat\flat$	0.005	0.000	0.000	0.000	0.000
F \flat	0.014	0.000	0.000	0.000	0.000
C \flat	0.032	0.000	0.000	0.000	0.000
G \flat	0.070	0.000	0.000	0.000	0.000
D \flat	0.126	0.000	0.000	0.000	0.000
A \flat	0.197	0.000	0.000	0.000	0.002
E \flat	0.193	0.000	0.000	0.000	0.077
B \flat	0.128	0.000	0.000	0.000	0.203
F	0.106	0.102	0.000	0.000	0.179
C	0.085	0.206	0.000	0.002	0.137
G	0.032	0.167	0.000	0.094	0.170
D	0.004	0.109	0.005	0.213	0.155
A	0.001	0.145	0.038	0.201	0.068
E	0.001	0.175	0.108	0.129	0.005
B	0.002	0.091	0.145	0.122	0.000
F \sharp	0.001	0.001	0.146	0.140	0.002
C \sharp	0.000	0.000	0.147	0.084	0.000
G \sharp	0.000	0.004	0.149	0.015	0.000
D \sharp	0.000	0.000	0.113	0.000	0.000
A \sharp	0.000	0.000	0.070	0.000	0.000
E \sharp	0.000	0.000	0.039	0.000	0.000
B \sharp	0.000	0.000	0.020	0.000	0.000
F $\sharp\sharp$	0.000	0.000	0.011	0.000	0.000
C $\sharp\sharp$	0.000	0.000	0.006	0.000	0.000
G $\sharp\sharp$	0.000	0.000	0.002	0.000	0.000
D $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000
A $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000
E $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000
B $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000

Table 4: Tonal pitch-class distribution for $K = 10$ topics.

	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}
F $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
G $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A $\flat\flat$	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
E $\flat\flat$	0.002	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.000
B $\flat\flat$	0.004	0.000	0.000	0.000	0.014	0.000	0.000	0.000	0.000	0.000
F \flat	0.019	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.000
C \flat	0.044	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.000
G \flat	0.085	0.000	0.000	0.000	0.062	0.000	0.000	0.000	0.000	0.000
D \flat	0.148	0.000	0.000	0.000	0.144	0.000	0.000	0.000	0.000	0.000
A \flat	0.212	0.000	0.000	0.001	0.081	0.000	0.058	0.000	0.000	0.000
E \flat	0.193	0.000	0.000	0.000	0.020	0.000	0.224	0.000	0.000	0.000
B \flat	0.124	0.000	0.192	0.000	0.051	0.000	0.228	0.000	0.000	0.000
F	0.096	0.136	0.314	0.034	0.087	0.000	0.096	0.000	0.000	0.000
C	0.062	0.203	0.209	0.257	0.044	0.000	0.084	0.000	0.001	0.000
G	0.006	0.078	0.123	0.373	0.009	0.106	0.145	0.000	0.096	0.000
D	0.001	0.090	0.062	0.129	0.030	0.368	0.132	0.000	0.182	0.006
A	0.000	0.183	0.073	0.001	0.101	0.361	0.027	0.007	0.086	0.116
E	0.001	0.231	0.028	0.051	0.099	0.124	0.000	0.042	0.040	0.248
B	0.000	0.075	0.000	0.134	0.038	0.004	0.002	0.075	0.155	0.236
F \sharp	0.000	0.000	0.000	0.021	0.046	0.001	0.004	0.124	0.308	0.127
C \sharp	0.000	0.000	0.000	0.000	0.077	0.031	0.000	0.168	0.130	0.099
G \sharp	0.000	0.005	0.000	0.000	0.057	0.005	0.000	0.167	0.000	0.106
D \sharp	0.000	0.000	0.000	0.000	0.017	0.000	0.000	0.150	0.000	0.051
A \sharp	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.119	0.002	0.012
E \sharp	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.073	0.000	0.000
B \sharp	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.038	0.000	0.000
F $\sharp\sharp$	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.021	0.000	0.000
C $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.011	0.000	0.000
G $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.000	0.000
D $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
A $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
E $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
B $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 5: Tonal pitch-class distribution for $K = 12$ topics.

	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}	ϕ_{12}
F $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
G $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
E $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.000
B $\flat\flat$	0.000	0.000	0.000	0.000	0.000	0.000	0.011	0.000	0.000	0.000	0.000	0.000
F \flat	0.000	0.000	0.000	0.000	0.000	0.000	0.031	0.000	0.000	0.000	0.000	0.000
C \flat	0.000	0.000	0.000	0.000	0.000	0.000	0.069	0.000	0.000	0.000	0.000	0.000
G \flat	0.000	0.000	0.000	0.000	0.000	0.000	0.151	0.001	0.000	0.000	0.000	0.000
D \flat	0.000	0.000	0.000	0.000	0.000	0.000	0.207	0.000	0.000	0.000	0.053	0.000
A \flat	0.000	0.000	0.000	0.000	0.000	0.000	0.154	0.004	0.000	0.214	0.000	0.000
E \flat	0.000	0.000	0.000	0.000	0.000	0.000	0.146	0.275	0.000	0.161	0.000	0.000
B \flat	0.043	0.000	0.000	0.000	0.000	0.000	0.232	0.134	0.323	0.000	0.068	0.000
F	0.195	0.000	0.003	0.133	0.000	0.000	0.336	0.075	0.044	0.000	0.166	0.021
C	0.079	0.000	0.119	0.278	0.000	0.000	0.179	0.012	0.026	0.001	0.224	0.135
G	0.046	0.070	0.326	0.085	0.000	0.000	0.067	0.001	0.152	0.005	0.108	0.319
D	0.214	0.220	0.169	0.038	0.001	0.049	0.075	0.002	0.155	0.000	0.002	0.317
A	0.314	0.169	0.039	0.143	0.001	0.236	0.088	0.001	0.009	0.027	0.000	0.160
E	0.101	0.069	0.083	0.249	0.015	0.291	0.021	0.001	0.000	0.185	0.002	0.040
B	0.002	0.100	0.224	0.072	0.053	0.101	0.000	0.000	0.004	0.328	0.001	0.006
F \sharp	0.000	0.237	0.038	0.000	0.133	0.008	0.000	0.000	0.007	0.211	0.000	0.002
C \sharp	0.005	0.125	0.000	0.000	0.188	0.154	0.002	0.000	0.000	0.028	0.000	0.000
G \sharp	0.000	0.000	0.000	0.001	0.176	0.153	0.000	0.000	0.000	0.056	0.000	0.000
D \sharp	0.000	0.000	0.000	0.000	0.146	0.005	0.000	0.000	0.000	0.119	0.000	0.000
A \sharp	0.000	0.006	0.000	0.000	0.116	0.000	0.000	0.000	0.000	0.041	0.000	0.000
E \sharp	0.000	0.005	0.000	0.000	0.082	0.000	0.000	0.000	0.000	0.000	0.000	0.000
B \sharp	0.000	0.000	0.000	0.000	0.046	0.001	0.000	0.000	0.000	0.000	0.000	0.000
F $\sharp\sharp$	0.000	0.000	0.000	0.000	0.025	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C $\sharp\sharp$	0.000	0.000	0.000	0.013	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
G $\sharp\sharp$	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D $\sharp\sharp$	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
E $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
B $\sharp\sharp$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 6: Tonal pitch-class distribution for $K = 24$ topics.