EXERCISE WEEK 1:
PRACTICAL COURSE
MODELING, SIMULATION, OPTIMIZATION

DANIËL VELDMAN

Due date: Sunday May 1 2021, 24:00h (midnight)

Send your solutions to daniel.veldman@math.fau.de. Put all your files in one .zip-file and name it `Exercises_Week1_<your name>.zip`

The solutions to the exercises will be published on StudOn on Monday May 2. We cannot consider submissions after the solutions have been uploaded.

Consider the steady-state temperature distribution in the aluminum rod in Figure 1 with a length of $L = 0.3$ [m], a cross sectional area of $A_{\mathrm{cs}} = 0.01$ [m^2], and a thermal conductivity of $k = 237$ [W/m/K]. Along the length of the rod, a constant heat load $Q(x) = Q_0 \exp(-(x - \frac{1}{2}L)^2/a^2)$ [W/m] is applied. The parameters for the heat load are $Q_0 = 100$ [W/m] and $a = 0.1$ [m].

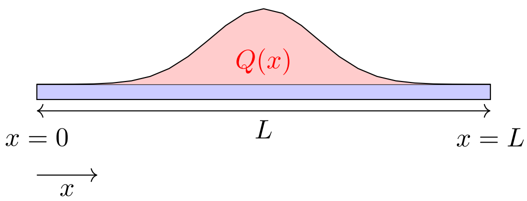

FIGURE 1. The considered aluminum rod

The temperature increase w.r.t. a reference temperature of $T_0 = 293$ [K] is $T(x)$. At the left end of the rod, the temperature is fixed at the reference temperature T_0, i.e. $T(0) = 0$. At the right end of the rod, the (outgoing) heat flow is proportional to the temperature increase, i.e. $A_{\mathrm{cs}} q(L) = hT(L)$ [W], where $h = 3$ [W/K] is the cooling coefficient and the outgoing heat flux is $q(L) = -k\frac{\mathrm{d}T}{\mathrm{d}x}(L)$.

a. (3pts) Write down the boundary value problem for the temperature increase in the rod $T(x)$.
   Hint: check the units!
b. (3pts) Make a finite difference discretization with $N = 11$ grid points of the boundary value problem for $T(x)$.
   Hint: You can use the file `Week1_exerciseb.m` as a starting point.
c. (4pts) Compute the ℓ^∞-error in the finite difference discretization with $N = 11$, 31, 101, 301, and 1001 grid points. Plot the relative ℓ^∞-error vs. grid spacing Δx. Use a logarithmic scale for both axes. Is the error $O(\Delta x)$ or $O(\Delta x^2)$? Is this also the rate you expect to find?
   Hint: You can use the file `Week1_exercisec.m` as a starting point.
   Hint: To obtain a reference solution, you can solve the boundary value problem analytically or use a finite difference approximation on a fine grid.