EXERCISE WEEK 4:
PRACTICAL COURSE
MODELING, SIMULATION, OPTIMIZATION

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Due date: Sunday May 22 2021, 24:00h (midnight)

Send your solutions to daniel.veldman@math.fau.de. Put all your files in one .zip-file and name it `Exercises_Week4_<your name>.zip`

The solutions to the exercises will be published on StudOn on Monday May 23. We cannot consider submissions after the solutions have been uploaded.

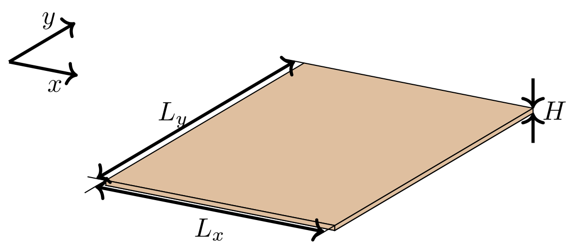

FIGURE 1. The considered copper plate

We consider the steady-state temperature distribution in the copper plate in Figure 1. The plate is $L_x = 0.5$ [m] by $L_y = 0.75$ [m], has a thickness of $H = 0.01$ [m], and a thermal conductivity of $k = 400$ [W/m/K]. Because the plate is thin, the steady state temperature increase T only depends on the in-plane coordinates x and y, i.e. $T(x, y)$. The plate is subjected to a heat load

$$Q(x, y) = Q_0 \exp\left(-\frac{(x - x_0)^2 + (y - y_0)^2}{a^2}\right),$$

where the center of the heat load is at $(x_0, y_0) = 0.3(L_x, L_y)$, the width parameter is $a = 0.05$ [m], and the intensity is $Q_0 = 100$ [W/m^2].

The boundary conditions at the four edges are given by

$$H\mathbf{q} \cdot \mathbf{n} = hT,$$

where the heat flux $\mathbf{q} = [q_x, q_y]^\top$ is given by Fourier's law of heat conduction, $\mathbf{n} = [n_x, n_y]^\top$ is the outward pointing unit normal to the edge, and the thermal conductance between the boundary and the environment is $h = 5$ [W/m/K].

a. (3pts) Write the weak form of the boundary value problem for the steady-state temperature increase $T(x, y)$.
b. (5pts) Discretize the problem formulated at a. using linear rectangular finite elements. Use $M_x = 32$ elements in the x-direction and $M_y = 42$ elements in the y-direction.
Hint: Follow the approach from `Week6_lecture.pdf`.

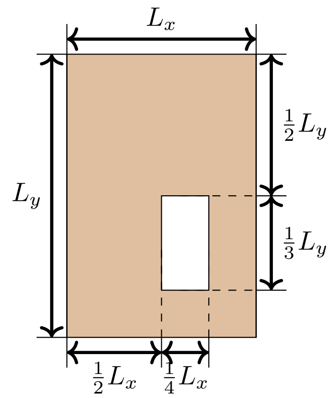

FIGURE 2. The location of the hole in the plate

Hint: You can use the file `Week6_exerciseb` a starting point.
Hint: To compute the load vector, you can use the approximation

$$\int_{x_{i-1}}^{x_i} \int_{y_{j-1}}^{y_j} (\mathbf{N}(x,y))^\top Q(x,y)\ \mathrm{d}y\ \mathrm{d}x$$
$$\approx Q\left(\frac{x_i + x_{i+1}}{2}, \frac{y_{j-1}+y_j}{2}\right) \int_{x_{i-1}}^{x_i} \int_{y_{j-1}}^{y_j} (\mathbf{N}(x,y))^\top\ \mathrm{d}y\ \mathrm{d}x.$$

c. (2pts) Modify the code from part b. in one of the following ways. (You can receive 2 bonus points when you do both)
   i. Replace the linear shape functions inside each element by quadratic shape functions.
      Hint: The numbering of the nodes is much easier when you use the quadratic shape functions that have 9 nodes inside each element instead of the serendipity elements with 8 nodes inside each element.
   ii. Create a hole in the plate at the location indicated in Figure 2. For simplicity, you can consider zero Neumann boundary conditions on the boundary of the hole (the boundary conditions on the other edges do not change).
      Hint: Note that the boundaries of the hole coincide with the element boundaries in the considered mesh.
      Hint: In the most elegant solution, you do not assign node numbers to nodes inside the hole.