EXERCISE WEEK 2:
PRACTICAL COURSE
MODELING, SIMULATION, OPTIMIZATION

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Due date: Sunday May 8 2021, 24:00h (midnight)

Send your solutions to daniel.veldman@math.fau.de. Put all your files in one .zip-file and name it Exercises_Week2_<your name>.zip
The solutions to the exercises will be published on StudOn on Monday May 9. We cannot consider submissions after the solutions have been uploaded.

Consider the temperature distribution $T(t,x)$ in the steel rod in Figure 1 with a length of $L = 0.3$ [m], a cross sectional area of $A_{\mathrm{cs}} = 0.01$ [m^2], a thermal conductivity of $k = 57.7$ [W/m/K], a heat capacity $c = 448$ [J/kg/K], and a mass density $\rho = 7840$ [kg/m^3]. Along the length of the rod, a constant heat load $Q(x) = Q_0 \exp(-(x - \frac{1}{2}L)^2/a^2)$ [W/m] is applied. The parameters for the heat load are $Q_0 = 100$ [W/m] and $a = 0.1$ [m]. At time $t = 0$, the temperature distribution is $T(0,x) = x^2/L^2$.

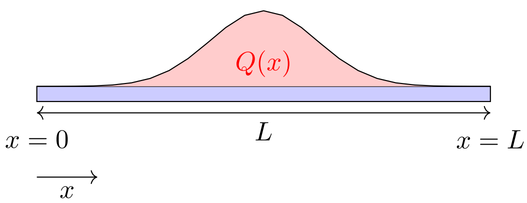

FIGURE 1. The considered aluminum rod

At the left end of the rod, the temperature is fixed at the reference temperature T_0, i.e. $T(t,0) = 0$. At the right end of the rod, the (outgoing) heat flow is proportional to the temperature increase, i.e. $A_{\mathrm{cs}} q(L) = hT(L)$ [W], where $h = 3$ [W/K] is the cooling coefficient and the outgoing heat flux is $q(L) = -k\frac{\mathrm{d}T}{\mathrm{d}x}(L)$.

a. (2pts) Write down the initial value problem for the temperature increase in the rod $T(t,x)$.
Hint: check the units!
Hint: You need to specify 3 things: the PDE, the boundary conditions, and the initial condition.
b. (3pts) Set up the spatial discretization by finite differences with $N = 51$ grid points for this problem. Use the explicit formulation in which the values for the ghost points have been eliminated. Use the obtained matrix to compute (an approximation of) the steady-state temperature field $T_{\mathrm{ss}}(x)$.
Hint: You can use the file `Week2_exercisebc.m` as a starting point.
c. (3pts) Approximate the solution $T(t,x)$ on the time interval $[0, T_{\mathrm{sim}}]$ with $T_{\mathrm{sim}} = 1$ minute $= 60$ s. Use $N_T = 101$ grid points in time and $N = 51$

grid points in space. Compute your solution with three different time discretization schemes: Forward Euler, Crank-Nicolson, and Backward Euler. Are all three schemes stable? Does the solution approach the steady state temperature field you computed in part b.?
Hint: you can use `Week2_exercisebc` as a starting point.

d. (2pts) Now increase the length of the time interval to $T_{\text{sim}} = 2$ hour $= 3600$ s in your code from part c. Keep the number of grid points in time fixed to $N_T = 101$. Are all three schemes stable? Does the solution approach the steady state temperature field you computed in part b.? Are your observations in agreement with the theory?

e. (Optional, 2 bonus pts) Take again $T_{\text{sim}} = 1$ minute $= 60$ s. Plot the error in the solutions obtained with the three considered time discretization schemes for $N_T = 12 \cdot 2^i + 1$ with $i \in \{1, 2, 3, 4, 5, 6, 7\}$ against the time step Δt. What is the convergence rate you observe for each scheme?
Hint: You can use the file `Week2_exercisee.m` as a starting point.
Hint: Use a reference solution on a time grid with $N_T = 3073 = 12 \cdot 2^8 + 1$. Which discretization do you use to compute the reference solution?