```
(* Define the given values *)

n = 116; (* number of samples *)
X = 17; (* the observed value of x *)
```

1. What is the conditional distribution of X, the number of samples containing Giardia cysts, given θ?

Given θ, X is distributed according to binomial distribution with number of trials being n and θ being the success probability:

```
ConditionalDistribution[θ_] := BinomialDistribution[n, θ]
```

2. Before the experiment, the NIWA scientists elicited that the expected value of θ is 0.2 with a standard deviation of 0.16. Determine the parameters α and β of a Beta prior distribution for θ with this prior mean and standard deviation. (Round α and β to the nearest integer).

In[23]:=
```
Clear[α, β];
PriorDist := BetaDistribution[α, β];
(* Solve equation for the α & β and set the values for them into the scope,
rounding them *)
RoundSolution[a_ → p_] := a → Round[p] ;
Set @@@ (RoundSolution /@
    Solve[
      Mean[PriorDist] == 1 / 5 &&
       StandardDeviation[PriorDist] == 4 / 25
     ][[1]]);

StringForm["α = ``, β = ``", α, β]
```

Out[27]= $\alpha = 1, \beta = 4$

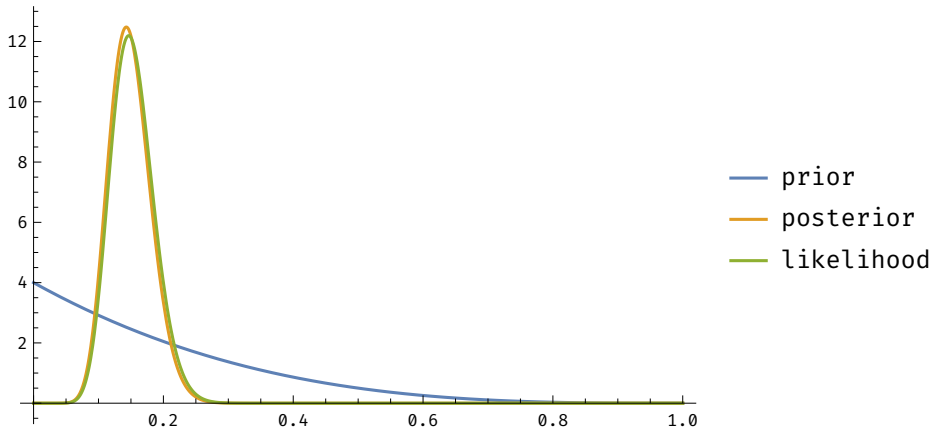
3. Find the posterior distribution of θ and summarize it by its posterior mean and standard deviation.

```
α′ = α + X;
β′ = β + n - X;

PosteriorDist = BetaDistribution[α′, β′];

StringForm["α′ = ``, β′ = ``", α′, β′]
```

$\alpha' = 18, \beta' = 103$

4. Plot the prior, posterior and normalized likelihood.

```
(* Function to normalize the likelihood to look like a PDF *)
NormLikelihood[f_] := With[{fac = ∫_0^1 f[θ] dθ}, f[#]/fac &]

Plot[{
  PDF[PriorDist][θ],
  PDF[PosteriorDist][θ],
  Evaluate[NormLikelihood[PDF[BinomialDistribution[n, #]][X] &][θ]],
 },
 {θ, 0, 1},
 PlotLegends → LineLegend[{"prior", "posterior", "likelihood"}],
 PlotRange → Full,
 LabelStyle → {FontFamily → "Fira Code Light"}
]
```



5. Find the posterior probability that $\theta < 0.1$

```
CDF[PosteriorDist, .1]
```

```
0.0530944
```

6. Find a central 95% posterior credible interval for θ

```
Lo := Quantile[PosteriorDist, .025]
Hi := Quantile[PosteriorDist, .975]
StringForm["The 95% credible interval is [``, ``]",
 NumberForm[Lo, {9, 3}], NumberForm[Hi, {9, 3}]]
```

```
The 95% credible interval is [0.091, 0.217]
```