

# Code Template for ACM-ICPC

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November 2, 2018

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# 1 1 数据结构

## 1.1 二叉搜索树

### 1.1.1 1 二叉树

// 通过中序遍历和后序遍历建立二叉树

//<https://vjudge.net/problem/UVA-548>

```
#include<bits/stdc++.h>

using namespace std;
const int maxn = 1e5+10;
const int INF = 1e8;
int in_order[maxn],post_order[maxn],l[maxn],r[maxn];
int n;
int read_order(int *a)
{
    string s;
    if(!getline(cin,s)) return false;
    stringstream ss(s);
    n = 0;
    int v;
    while(ss >> v)
        a[n++] = v;
    return n > 0;
}
int build_tree(int L1,int R1,int L2,int R2)
{
    if(L1 > R1)
        return 0;
    int root = post_order[R2];
    int p = L1;
    while(in_order[p] != root)
        p++;
    int cnt = p-L1;
    l[root] = build_tree(L1,p-1,L2,L2+cnt-1);
    r[root] = build_tree(p+1,R1,L2+cnt,R2-1);
    return root;
}
int best,bestsum;
void dfs(int a,int b)
{
    if(!l[a] && !r[a])
    {
        b += a;
        if(bestsum > b||(bestsum == b&&best > a))
        {
            best = a;
            bestsum = b;
        }
    }
    if(l[a]) dfs(l[a],b+a);
    if(r[a]) dfs(r[a],b+a);
}
```

```
}
```

```
int main(void)
{
    while(read_order(in_order))
    {
        read_order(post_order);
        build_tree(0,n-1,0,n-1);
        // cout<<0<<endl;
        bestsum = INF;
        dfs(post_order[n-1],0);
        cout<<best<<endl;
    }

    return 0;
}
```

### 1.1.2 2 treap

```
// UVA LA 5031
```

```
/*
```

给定  $n$  个节点  $m$  条边的无向图，每个节点都有一个整数权值。

$D X$  删除  $ID$  为  $x$  的边

$Q X K$  计算与节点  $X$  连通的节点中权值第  $k$  大的数

$C X K$  把节点  $X$  的权值改为  $V$

```
*/
```

```
#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
#define Pb push_back
#define FI first
#define SE second
#define rep(i,a,n) for (int i=a;i<n;i++)
#define per(i,a,n) for (int i=n-1;i>=a;i--)
#define IOS ios::sync_with_stdio(false)
#define DEBUG cout<<endl<<"DEBUG"<<endl;
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 0x7FFFFFFF;
const LL INFF = 0x7FFFFFFFFFFFFFFF;
const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-6;
const LL mod = 1e9 + 7;
LL qpow(LL a,LL b){LL s=1;while(b>0){if(b&1)s=s*a%mod;a=a*a%mod;b>>=1;}return s;}
LL gcd(LL a,LL b) {return b?gcd(b,a%b):a;}
```

```

int dr[2][4] = {1,-1,0,0,0,0,-1,1};
typedef pair<int,int> P;
struct Node{
    Node *ch[2]; // 左右子树
    int r; // 随机优先值
    int v; // 值
    int s; // 节点总数

    Node(int v):v(v){ch[0] = ch[1] = NULL; r = rand(); s = 1;}
    int cmp(int x) {
        if(x==v) return -1;
        return x < v?0:1;
    }

    void maintain(){
        s = 1;
        if(ch[0] != NULL) s += ch[0]->s;
        if(ch[1] != NULL) s += ch[1]->s;
    }
};

void rotate(Node * &o,int d){
    Node *k = o->ch[d^1]; o->ch[d^1] = k->ch[d]; k->ch[d] = o;
    o->maintain();k->maintain(); o = k;
}

void insert(Node * &o,int x){
    if(o ==NULL) o = new Node(x);
    else{
        int d = (x < o->v?0:1);
        insert(o->ch[d],x);
        if(o->ch[d]->r > o->r) rotate(o,d^1);
    }
    o->maintain();
}

void remove(Node * &o,int x){
    int d = o->cmp(x);
    // int ret = 0;
    if(d == -1){
        Node *u = o;
        if(o->ch[0] != NULL && o->ch[1] != NULL){
            int d2 = (o->ch[0]->r > o->ch[1]->r?1:0);
            rotate(o,d2); remove(o->ch[d2],x);
        }
        else{
            if(o->ch[0] == NULL) o = o->ch[1];
            else o = o->ch[0];
            delete u;
        }
    } else
        remove(o->ch[d],x);
    if(o != NULL) o->maintain();
}

```

```

const int maxc = 5e5+10;
struct Command{
    char type;
    int x,p;
} commands[maxc];

const int maxn = 2e4+10;
const int maxm = 6e4+10;
int n,m,weight[maxn],from[maxm],to[maxm],removed[maxm];
// 并查集相关
int pa[maxn];
int findset(int x){ return pa[x] != x?pa[x] = findset(pa[x]) : x;}
// 名次数相关
Node *root[maxn]; // Treap;
int kth(Node *o,int k){
    if(o == NULL || k <= 0 || k > o->s) return 0;
    int s = (o->ch[1] == NULL?0:o->ch[1]->s);
    if(k == s+1) return o->v;
    else if(k <= s) return kth(o->ch[1],k);
    else return kth(o->ch[0],k-s-1);
}
void mergeto(Node* &src,Node * &dest){
    if(src->ch[0] != NULL) mergeto(src->ch[0],dest);
    if(src->ch[1] != NULL) mergeto(src->ch[1],dest);
    insert(dest,src->v);
    delete src;
    src = NULL;
}
void removetree(Node *&x){
    if(x->ch[0] != NULL) removetree(x->ch[0]);
    if(x->ch[1] != NULL) removetree(x->ch[1]);
    delete x;
    x = NULL;
}

void add_edge(int x){
    int u = findset(from[x]), v = findset(to[x]);
    if(u != v){
        if(root[u]-> s < root[v] -> s){ pa[u] = v; mergeto(root[u],root[v]);}
        else {pa[v] = u; mergeto(root[v],root[u]);}
    }
}

int query_cnt;
long long query_tot;
void query(int x,int k){
    query_cnt++;
    query_tot += kth(root[findset(x)],k);
}

void change_weight(int x,int v){
    int u = findset(x);

```

```

        remove(root[u],weight[x]);
        insert(root[u],v);
        weight[x] = v;
    }

int main(void){
    int kase = 0;
    while(scanf("%d%d",&n,&m) == 2&& n){
        rep(i,1,n+1) scanf("%d",&weight[i]);
        rep(i,1,m+1) scanf("%d%d",&from[i],&to[i]);
        me(removed);
        int c = 0;
        for(;;){
            char type;
            int x,p = 0,v = 0;
            scanf(" %c",&type);
            if(type == 'E') break;
            scanf("%d",&x);
            if(type == 'D') removed[x] = 1;
            if(type == 'Q') scanf("%d",&p);
            if(type == 'C') {
                scanf("%d",&v);
                p = weight[x];
                weight[x] = v;
            }
            commands[c++] = (Command){type,x,p};
        }
        rep(i,1,n+1) {
            pa[i] = i; if(root[i] != NULL) removetree(root[i]);
            root[i] = new Node(weight[i]);
        }
        rep(i,1,m+1) if(!removed[i]) add_edge(i);
        // 反向操作
        query_tot = query_cnt = 0;
        per(i,0,c){
            if(commands[i].type == 'D') add_edge(commands[i].x);
            if(commands[i].type == 'Q') query(commands[i].x,commands[i].p);
            if(commands[i].type == 'C') change_weight(commands[i].x,commands[i].p);
        }
        printf("Case %d: %.6lf\n", ++kase, query_tot / (double)query_cnt);
    }
}

```

### 1.1.3 3 伸展树

```

/*
UVA 11922
序列反转 (a,b)

*/
#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))

```



```

#define lowbit(x) (x&(-x))
#define Pb push_back
#define FI first
#define SE second
#define rep(i,a,n) for (int i=a;i<n;i++)
#define per(i,a,n) for (int i=n-1;i>=a;i--)
#define IOS ios::sync_with_stdio(false)
#define DEBUG cout<<endl<<"DEBUG"<<endl;
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 0x7FFFFFFF;
const LL INFF =0x7FFFFFFFFFFFFFFF;
const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-6;
const LL mod = 1e9 + 7;
LL qpow(LL a,LL b){LL s=1;while(b>0){if(b&1)s=s*a%mod;a=a*a%mod;b>>=1;}return s;}
LL gcd(LL a,LL b) {return b?gcd(b,a%b):a;}
int dr[2][4] = {1,-1,0,0,0,0,-1,1};
typedef pair<int,int> P;
struct Node{
    Node *ch[2];
    int s;
    int flip;
    int v;
    int cmp(int k) const {
        int d = k-ch[0]->s;
        if(d == 1) return -1;
        return d <= 0?0:1;
    }
    void maintain(){
        s = ch[0]->s+ch[1]->s+1;
    }
    void pushdown(){
        if(flip){
            flip = 0;
            swap(ch[0],ch[1]);
            ch[0]->flip = !ch[0]->flip;
            ch[1]->flip = !ch[1]->flip;
        }
    }
};
Node *null = new Node();
void rotate(Node *&o,int d){
    Node *k = o->ch[d^1];
    o->ch[d^1] = k->ch[d];
    k->ch[d] = o;
    o->maintain(); k->maintain(); o = k;
}

void splay(Node * &o,int k){

```

```

    // cout<<1<<endl;
    o->pushdown();
    int d = o->cmp(k);
    if(d == 1) k -= o->ch[0]->s + 1;
    // DEBUG;
    if(d != -1){
        Node *p = o->ch[d];
        p->pushdown();
        int d2 = p->cmp(k);
        int k2 = (d2==0?k:k-p->ch[0]->s-1);
        // cout<<k2<<endl;
        if(d2 != -1){
            splay(p->ch[d2],k2);
            if(d == d2) rotate(o,d^1);
            else rotate(o->ch[d],d);
        }
        rotate(o,d^1);
    }
}

Node * Merge(Node *left,Node*right){
    splay(left,left->s);
    left->ch[1] = right;
    left->maintain();
    return left;
}

void split(Node *o,int k,Node * &left,Node *&right){
    splay(o,k);
    left = o;
    right = o->ch[1];
    o->ch[1] = null;
    left->maintain();
}

const int maxn = 1e5+10;
struct SplaySequence{
    int n;
    Node seq[maxn];
    Node *root;
    Node *build(int sz){
        if(!sz) return null;
        Node *L = build(sz/2);
        Node *o = &seq[++n];
        o->v = n;
        o->ch[0] = L;
        o->ch[1] = build(sz-sz/2-1);
        o->flip = o->s = 0;
        o->maintain();
        return o;
    }
    void init(int sz){
        n = 0;
        null->s = 0;
        root = build(sz);
    }
}

```

```

};
vector<int> ans;
void print(Node *o){
    if(o!=null){
        o->pushdown();
        print(o->ch[0]);
        ans.push_back(o->v);
        print(o->ch[1]);
    }
}
void debug(Node *o){
    if(o!=null){
        o->pushdown();
        debug(o->ch[0]);
        printf("%d ",o->v-1);
        debug(o->ch[1]);
    }
}
SplaySequence ss;
int main(void)
{
    int n,m;
    scanf("%d%d",&n,&m);
    // cout<<n<<" "<<m<<endl;
    ss.init(n+1);

    while(m--){
        int a,b;
        scanf("%d %d",&a,&b);
        // cout<<a<<" "<<b<<endl;
        Node *left,*mid,*right,*o;
        split(ss.root,a,left,o);
        // DEBUG;
        split(o,b-a+1,mid,right);
        mid->flip ^= 1;
        ss.root = Merge(Merge(left,right),mid);
    }
    print(ss.root);
    for(int i = 1; i < ans.size(); i++)
        printf("%d\n",ans[i]-1);
    return 0;
}

```

## 1.2 堆

// 堆的插入和删除操作

```

void Insert(int vv)
{
    int t = sz++;
    h[t] = vv;
    while(t > 1)
    {

```

```

        if(h[t] < h[t/2])
        {
            swap(h[t],h[t/2]);
            t /= 2;
        }
        else break;
    }
}

int Down(int i)
{
    int t;
    while(i * 2 <= n)
    {
        if(h[i] > h[2*i])
            t = 2*i;
        else
            t = i;
        if(i*2+1 <= n&&h[i*2+1] < h[t])
            t = i*2+1;
        if(i == t)
            break;
        swap(h[t],h[i]);
        i = t;
    }
}

```

## 1.3 字符串

### 1.3.1 1 Trie(前缀树)

```

const int maxnode = 4e5+100;
const int sigma_size = 26;
struct Trie
{
    int ch[maxnode][sigma_size];
    int val[maxnode];
    int sz;
    Trie()
    {
        sz = 1;
        memset(ch[0],0,sizeof(ch[0]));
    }
    int idx(char c)
    {
        return c-'a';
    }
    void init(void)
    {
        memset(ch,0,sizeof(ch));
        memset(val,0,sizeof(val));
    }
    void insert(char *s,int v)
    {
        int u = 0, n = strlen(s);
        for(int i = 0; i < n; ++i)

```

```

    {
        int c = idx(s[i]);
        if(!ch[u][c])
        {
            memset(ch[sz],0,sizeof(ch[sz]));
            val[sz] = 0;
            ch[u][c] = sz++;
        }
        u = ch[u][c];
    }
    val[u] = v;
}
int query(char *s,int t)
{
    int sum = 0;
    int u = 0,n = strlen(s);
    for(int i = 0; i < n; ++i)
    {
        int c = idx(s[i]);
        if(ch[u][c])
        {
            if(val[ch[u][c]])
                sum = (sum+ans[i+t+1]) % mod;
        }
        else
            return sum;
        u = ch[u][c];
    }
    return sum;
}

};

```

### 1.3.2 2 KMP

```

#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 0x7FFFFFFF;
const LL INFF =0x7FFFFFFFFFFFFFFF;
const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-6;
const LL mod = 20071027 ;
int f[1100];
char ch[100];
void getFail(char *P,int *f)
{
    int m = strlen(P);

```

```

    f[0] = 0, f[1] = 0;
    for(int i = 1; i < m; ++i)
    {
        int j = f[i];
        while(j && P[i] != P[j]) j = f[j];
        f[i+1] = P[i] == P[j] ? j + 1 : 0;
    }
}

void find(char * T, char * P, int* f)
{
    int n = strlen(T), m = strlen(P);
    getFail(P, f);
    int j = 0;
    for(int i = 0; i < n; ++i)
    {
        while(j && P[j] != T[i]) j = f[j];
        if(P[j] == T[i]) j++;
        if(j == m) printf("%d\n", i-m+1);
    }
}

int main(void)
{
    cin >> ch;
    getFail(ch, f);
    printf("%d", f[strlen(ch)-1]);

    return 0;
}

```

### 1.3.3 3 AC 自动机

```

const int SIGMA_SIZE = 26;
const int MAXNODE = 11000;
const int MAXS = 150 + 10;

struct AhoCorasickAutomata {
    int ch[MAXNODE][SIGMA_SIZE];
    int f[MAXNODE]; // fail 函数
    int val[MAXNODE]; // 每个字符串的结尾结点都有一个非 0 的 val
    int last[MAXNODE]; // 输出链表的下一个结点
    int sz;

    void init() {
        sz = 1;
        memset(ch[0], 0, sizeof(ch[0]));
    }

    // 字符 c 的编号
    int idx(char c) {
        return c - 'a';
    }
}

```

```

}

// 插入字符串。v 必须非 0
void insert(char *s, int v) {
    int u = 0, n = strlen(s);
    for(int i = 0; i < n; i++) {
        int c = idx(s[i]);
        if(!ch[u][c]) {
            memset(ch[sz], 0, sizeof(ch[sz]));
            val[sz] = 0;
            ch[u][c] = sz++;
        }
        u = ch[u][c];
    }
    val[u] = v;
}

// 递归打印以结点 j 结尾的所有字符串
void print(int j) {
    if(j) {
        print(last[j]);
    }
}

// 在 T 中找模板
int find(char* T) {
    int n = strlen(T);
    int j = 0; // 当前结点编号, 初始为根结点
    for(int i = 0; i < n; i++) { // 文本串当前指针
        int c = idx(T[i]);
        while(j && !ch[j][c]) j = f[j]; // 顺着细边走, 直到可以匹配
        j = ch[j][c];
        if(val[j]) print(j);
        else if(last[j]) print(last[j]); // 找到了!
    }
}

// 计算 fail 函数
void getFail() {
    queue<int> q;
    f[0] = 0;
    // 初始化队列
    for(int c = 0; c < SIGMA_SIZE; c++) {
        int u = ch[0][c];
        if(u) { f[u] = 0; q.push(u); last[u] = 0; }
    }
    // 按 BFS 顺序计算 fail
    while(!q.empty()) {
        int r = q.front(); q.pop();
        for(int c = 0; c < SIGMA_SIZE; c++) {
            int u = ch[r][c];
            if(!u) continue;
            q.push(u);
            int v = f[r];

```

```

        while(v && !ch[v][c]) v = f[v];
        f[u] = ch[v][c];
        last[u] = val[f[u]] ? f[u] : last[f[u]];
    }
}
};

```

#### 1.3.4 4 KMP-KMP 变形

[//https://www.nowcoder.com/acm/contest/119/E](https://www.nowcoder.com/acm/contest/119/E)

```

#include <bits/stdc++.h>
using namespace std;

const int N=200010;
int a[N],b[N];
int x[N],y[N],nxt[N];

void kmp_pre(int x[],int m,int nxt[])
{
    int i,j;
    j=nxt[0]=-1;
    i=0;
    while(i<m) {
        while(-1!=j && (x[i]!=x[j]&&x[j]!=-1))j=nxt[j];
        nxt[++i]=++j;
    }
}

int KMP_Count(int x[],int m,int y[],int n)
{
    // for (int i=0;i<n;i++) {
    //     printf("%d ",y[i]);
    // }
    // puts("");
    // for (int i=0;i<m;i++) {
    //     printf("%d ",x[i]);
    // }
    // puts("");
    int i,j;
    int ans=0;
    kmp_pre(x,m,nxt);
    i=j=0;
    while(i<n) {
        while(-1!=j && !(y[i]==x[j]||(x[j]==-1&&(y[i]==-1||j-y[i]<0)))) j=nxt[j];
        i++;
        j++;
        if(j>=m) {
            ans++;
            j=nxt[j];
        }
    }
}

```



```

    return ans;
}

int main()
{
    int n,m,k;
    scanf("%d%d",&n,&k);
    memset(x,-1,sizeof(x));
    memset(y,-1,sizeof(y));
    map<int,int> pre;
    for (int i=0;i<n;i++) {
        scanf("%d",&a[i]);
        auto pos=pre.find(a[i]);
        if (pos!=pre.end()) {
            y[i]=i-pos->second;
        }
        pre[a[i]]=i;
    }
    scanf("%d",&m);
    pre.clear();
    for (int i=0;i<m;i++) {
        scanf("%d",&b[i]);
        auto pos=pre.find(b[i]);
        if (pos!=pre.end()) {
            x[i]=i-pos->second;
        }
        pre[b[i]]=i;
    }
    printf("%d\n",KMP_Count(x,m,y,n));
    return 0;
}

```

### 1.3.5 5 字符串 hash

```

// 字符串 hash, 查找在字符串中至少出现 k 次的最长字符串
#include<cstdio>
#include<cstring>
#include<algorithm>
using namespace std;

const int maxn = 40000+10;
const int x = 123;
int n,m,pos;

unsigned long long H[maxn],xp[maxn];

unsigned long long Hash[maxn];
int Rank[maxn];

int cmp(const int &a,const int &b){
    return Hash[a] < Hash[b] || (Hash[a] == Hash[b] &&a < b );
}

int possible(int L){

```

```

    int c = 0;
    pos = -1;
    for(int i = 0; i < n-L+1; ++i){
        Rank[i] = i;
        Hash[i] = H[i]-H[i+L]*xp[L];
    }
    sort(Rank, Rank+n-L+1, cmp);
    for(int i = 0; i < n-L+1; ++i){
        if(i == 0 || Hash[Rank[i]] != Hash[Rank[i-1]]) c = 0;
        if(++c >= m) pos = max(pos, Rank[i]);
    }
    return pos >= 0;
}

char s[maxn];
int main(void)
{
    while((scanf("%d",&m)) == 1 && m){
        scanf("%s",s);
        n = strlen(s);
        H[n] = 0;
        for(int i = n-1; i >= 0; i--) H[i] = H[i+1]*x+(s[i]-'a');
        xp[0] = 1;
        for(int i = 1; i <= n; ++i) xp[i] = xp[i-1]*x;
        if(!possible(1)) printf("none\n");
        else{
            int L = 1, R = n;
            while(R >= L){
                int M = (R+L)/2;
                if(possible(M)) L = M+1;
                else R = M-1;
            }
            possible(R);
            printf("%d %d\n", R, pos);
        }
    }

    return 0;
}

```

### 1.3.6 6 后缀数组

```

const int maxn = 1e6 + 10;

struct SuffixArray {
    int s[maxn];      // 原始字符数组 (最后一个字符应必须是 0, 而前面的字符必须非 0)
    int sa[maxn];      // 后缀数组
    int rank[maxn];    // 名次数组. rank[0] 一定是 n-1, 即最后一个字符
    int height[maxn];  // height 数组
    int t[maxn], t2[maxn], c[maxn]; // 辅助数组
    int n;             // 字符个数

    void clear() { n = 0; memset(sa, 0, sizeof(sa)); }
}

```

```

// m 为最大字符值加 1。调用之前需设置好 s 和 n
void build_sa(int m) {
    int i, *x = t, *y = t2;
    for(i = 0; i < m; i++) c[i] = 0;
    for(i = 0; i < n; i++) c[x[i] = s[i]]++;
    for(i = 1; i < m; i++) c[i] += c[i-1];
    for(i = n-1; i >= 0; i--) sa[--c[x[i]]] = i;
    for(int k = 1; k <= n; k <= 1) {
        int p = 0;
        for(i = n-k; i < n; i++) y[p++] = i;
        for(i = 0; i < n; i++) if(sa[i] >= k) y[p++] = sa[i]-k;
        for(i = 0; i < m; i++) c[i] = 0;
        for(i = 0; i < n; i++) c[x[y[i]]]++;
        for(i = 0; i < m; i++) c[i] += c[i-1];
        for(i = n-1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
        swap(x, y);
        p = 1; x[sa[0]] = 0;
        for(i = 1; i < n; i++)
            x[sa[i]] = y[sa[i-1]]==y[sa[i]] && y[sa[i-1]+k]==y[sa[i]+k] ? p-1 : p++;
        if(p >= n) break;
        m = p;
    }
}

void build_height() {
    int i, j, k = 0;
    for(i = 0; i < n; i++) rank[sa[i]] = i;
    for(i = 0; i < n; i++) {
        if(k) k--;
        int j = sa[rank[i]-1];
        while(s[i+k] == s[j+k]) k++;
        height[rank[i]] = k;
    }
}
};

```

## 2 2 动态规划

### 2.1 1 单调队列.cpp

```

//https://ac.nowcoder.com/acm/contest/223/C
//C 区间区间区间
//$$ v_{l,r} = \max(a_i - a_j) \ (l \leq i, j \leq r)$$
//$$ \sum_{i=1}^n \sum_{j=1}^n v_{i,j}$$
const int maxn = 1e5+100;
int a[maxn];
int s[maxn]; // 单调栈
// 第一遍求在这个区间里面最大
int pre[maxn];
int nxt[maxn];
int main(void)
{
    int T,n;

```

```

cin>>T;
while(T--){
    scanf("%d",&n);
    for(int i = 1;i <= n; ++i){
        scanf("%d",&a[i]);
    }
    int t = 0;
    for(int i = 1;i <= n; ++i){
        pre[i] = nxt[i] = 0;
        while(t > 0&&a[i] > a[s[t]]) nxt[s[t]] = i,t--;
        pre[i] = s[t];
        s[++t] = i;
        // cout<<pre[i]<<" ";
    }
    while(t > 0)
        nxt[s[t]] = n+1,t--;
    LL ans = 0;
    for(int i = 1;i <= n; ++i){
        ans += 1ll*a[i]*(nxt[i]-i)*(i-pre[i]);
    }
    t = 0;
    for(int i = 1;i <= n; ++i){
        pre[i] = nxt[i] = 0;
        while(t > 0&&a[i] < a[s[t]]) nxt[s[t]] = i,t--;
        pre[i] = s[t];
        s[++t] = i;
    }
    while(t > 0)
        nxt[s[t]] = n+1,t--;
    for(int i = 1;i <= n; ++i){
        ans -= 1ll*a[i]*(nxt[i]-i)*(i-pre[i]);
    }
    printf("%lld\n",ans);
}

return 0;
}

```

## 2.2 1 最长上升子序列.cpp

//最长上升子序列 *The longest increasing sequence*

```

template <class It>
int n_lisLength(It begin,It end)
{
    typedef typename iterator_traits<It>::value_type T;
    T inf = 1<<30;
    vector<T> best(end-begin,inf);
    for(It i = begin; i != end; ++i)
        *lower_bound(best.begin(),best.end(),*i) = *i;
    return lower_bound(best.begin(),best.end(),inf) - best.begin();
}

```

## 2.3 1 树的重心

```
// Size[u] 代表以节点 u 为根的子树节点个数
// dp[u] 代表去除 u 节点后最大子树的节点个数
const int maxn = 2e4+100;
vector<int> G[maxn];
int dp[maxn];
int Size[maxn];
int n;
int ans;
void dfs(int u,int fa){
    dp[u] = Size[u] = 0;
    for(int i = 0;i < G[u].size(); ++i){
        if(fa==G[u][i])continue;
        dfs(G[u][i],u);
        // sum += tmp;
        Size[u] += Size[G[u][i]];
        dp[u] = max(dp[u],Size[G[u][i]]);
    }
    Size[u]++;
    dp[u] = max(n-Size[u],dp[u]);
    if(dp[u] < dp[ans]) ans = u;
}
int main(void)
{
    int T;
    cin>>T;
    while(T--){
        scanf("%d",&n);
        for(int i = 1;i <= n; ++i) G[i].clear();
        for(int i = 1;i <= n-1; ++i){
            int u,v;
            scanf("%d%d",&u,&v);
            G[u].push_back(v);
            G[v].push_back(u);
        }
        ans = 0;
        dp[0] = INF;
        dfs(1,-1);
        printf("%d %d\n",ans,dp[ans]);
    }
    return 0;
}
```

## 3 3 图论

### 3.1 DFS

#### 3.1.1 1. 无向图的割点和桥

SPF POJ - 1523

// 如果有割点，那么割点与子节点边就是割边

```
int dfs(int u,int fa){
    int lowu = pre[u] = ++dfs_clock;
    int child = 0;
```

```

for(int i = 0; i < G[u].size(); ++i){
    int v = G[u][i];
    if(!pre[v]){
        child++;
        int lowv = dfs(v,u);
        lowu = min(lowu,lowv);
        if(lowv >= pre[u]){
            iscut[u]++;
        }
    }
    else if(pre[v] < pre[u] && v != fa){
        lowu = min(lowu,pre[v]);
    }
}
if(fa < 0 && child == 1) iscut[u] = 0;
else if(fa < 0 && child >= 2) iscut[u] = child-1;
return low[u] = lowu;
}

```

如果要输出去掉割点之后的联通分量的个数，需要谈判根的情况

```

#include<iostream>
#include<cstdio>
#include<cctype>
#include<cstring>
#include<algorithm>
#include<vector>
#include<stack>
#include<map>
#include<queue>
#include<cmath>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
#define Pb push_back
#define FI first
#define SE second
#define rep(i,a,n) for (int i=a;i<n;i++)
#define per(i,a,n) for (int i=n-1;i>=a;i--)
#define IOS ios::sync_with_stdio(false)
#define DEBUG cout<<endl<<"DEBUG"<<endl;
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 0x7FFFFFFF;
const LL INFF = 0x7FFFFFFFFFFFFFFF;
const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-6;
const LL mod = 1e9 + 7;
LL qpow(LL a,LL b){LL s=1;while(b>0){if(b&1)s=s*a%mod;a=a*a%mod;b>>=1;}return s;}
LL gcd(LL a,LL b) {return b?gcd(b,a%b):a;}
int dr[2][4] = {1,-1,0,0,0,0,-1,1};
typedef pair<int,int> P;
const int maxn = 1000+100;

```

```

// const int maxm = 1e6+100
int pre[maxn];
int dfs_clock = 0;
vector<int> G[maxn];
int iscut[maxn];
int low[maxn];

void init(){
    dfs_clock = 1;
    rep(i,1,maxn) G[i].clear();
    me(iscut);
    me(low);
    me(pre);
}

int dfs(int u,int fa){
    int lowu = pre[u] = ++dfs_clock;
    int child = 0;
    for(int i = 0;i < G[u].size(); ++i){
        int v = G[u][i];
        if(!pre[v]){
            child++;
            int lowv = dfs(v,u);
            lowu = min(lowu,lowv);
            if(lowv >= pre[u]){
                iscut[u]++;
            }
        }
        else if(pre[v] < pre[u] && v != fa){
            lowu = min(lowu,pre[v]);
        }
    }
    if(fa < 0&&child == 1) iscut[u] = 0;
    else if(fa < 0&&child >= 2) iscut[u] = child-1;
    return low[u] = lowu;
}

// #define Debug
int main(void)
{
    #ifdef Debug
    freopen("input.txt","r",stdin);
    freopen("output.txt","w+",stdout);
    #endif
    int kase = 0;
    while(1){
        init();
        int u,v;
        int t = 0;
        while(scanf("%d",&u)==1&&u != 0){
            t++;
            scanf("%d",&v);
            G[u].Pb(v);
            G[v].Pb(u);
        }
        if(t==0)break;
    }
}

```

```

// rep(i,1,maxn) if(!G[i].empty()){

// dfs(i,-1);
// break;
// }
dfs(1,-1);
int num = 0;
rep(i,1,1001) if(iscut[i]) num++;

printf("Network #d\n",++kase);
if(num > 0)
{
    rep(i,1,1001) if(iscut[i]){
        printf(" SPF node %d leaves %d subnets\n",i,iscut[i]+1);
    }
}
else
    printf(" No SPF nodes\n");
if(kase) puts("");
}

return 0;
}

```

### 3.1.2 2. 无向图的双连通分量

// 无向图的点联通分量

```

const int maxn= 1000+10;
int pre[maxn],iscut[maxn],bccno[maxn],dfs_clock,bcc_cnt;
vector<int> G[maxn],bcc[maxn];

stack<Edge> S;
int dfs(int u,int fa){
    int lowu = pre[u] = ++dfs_clock;
    int child = 0;
    for(int i = 0;i < G[u].size(); ++i){
        int v = G[u][i];
        Edge e = (Edge) {u,v};
        if(!pre[v]){
            S.push(e);
            child++;
            int lowv = dfs(v,u);
            lowu = min(lowu,lowv);
            if(lowv >= pre[u]){
                iscut[u] = true;
                bcc_cnt++;
                bcc[bcc_cnt].clear();
                for(;;){
                    Edge x = S.top(); S.pop();
                    if(bccno[x.u] != bcc_cnt) {bcc[bcc_cnt].push_back(x.u); bccno[x.u] = bcc_cnt;}
                    if(bccno[x.v] != bcc_cnt) {bcc[bcc_cnt].push_back(x.v); bccno[x.v] = bcc_cnt;}
                    if(x.u == u&&v == v) break;
                }
            }
        }
    }
    return lowu;
}

```



```

        }
    }
}
else if(pre[v] < pre[u]&&v != fa){
    S.push(e);lowu = min(pre[v],lowu);
}
}
if(fa < 0&& child == 1) iscut[u] = 0;
return lowu;
}
void find_bcc(int n){
    memset(pre,0,sizeof(pre));
    memset(iscut,0,sizeof(iscut));
    memset(bccno,0,sizeof(bccno));
    dfs_clock = bcc_cnt = 0;
    for(int i = 0;i < n; ++i) if(!pre[i]) dfs(i,-1);
}

```

//无向图的边-双联通分量

// 第一边 *dfs* 求出所有的割边, 然后第二边 *dfs* 求出所有边一双联通分量 (不经过割边)

### 3.1.3 3 有向图的强联通分量

// *tarjan* 算法

const int maxn = 2e4+100;

```

vector<int> G[maxn];
int pre[maxn],lowlink[maxn],sccno[maxn],dfs_clock,scc_cnt;
stack<int> S;
void dfs(int u){
    pre[u] = lowlink[u] = ++dfs_clock;
    S.push(u);
    for(int i = 0;i < G[u].size(); ++i){
        int v = G[u][i];
        if(!pre[v]){
            dfs(v);
            lowlink[u] = min(lowlink[u],lowlink[v]);
        }
        else if(!sccno[v]){
            lowlink[u] = min(lowlink[u],pre[v]);
        }
    }
    if(lowlink[u] == pre[u]){
        scc_cnt++;
        for(;;){
            int x = S.top(); S.pop();
            sccno[x] = scc_cnt;
            if(x == u) break;
        }
    }
}
}

```

```

void find_scc(int n){
    dfs_clock= scc_cnt = 0;
    me(sccno),me(pre);
    rep(i,0,n) if(!pre[i]) dfs(i);
}
// kosaraju

const int maxn = 2e4+100;
vector<int> G[maxn],G2[maxn];
vector<int> S;
int vis[maxn],sccno[maxn],scc_cnt;
void dfs1(int u){
    if(vis[u]) return ;
    vis[u] = 1;
    for(int i = 0;i < G[u].size(); ++i) dfs1(G[u][i]);
    S.push_back(u);
}
void dfs2(int u){
    if(sccno[u]) return ;
    sccno[u] = scc_cnt;
    for(int i = 0;i < G2[u].size(); ++i) dfs2(G2[u][i]);
}
void find_scc(int n){
    scc_cnt = 0;
    S.clear();
    memset(sccno,0,sizeof(sccno));
    memset(vis,0,sizeof(vis));
    for(int i = 0;i < n; ++i) dfs1(i);
    for(int i = n-1;i >= 0;--i){
        if(!sccno[S[i]]) {
            scc_cnt++;
            dfs2(S[i]);
        }
    }
}

```

### 3.1.4 4 2-sat 问题

//  $O(n*m)$  复杂度不确定

```

const int maxn = 2000 + 10;

struct TwoSAT {
    int n;
    vector<int> G[maxn*2];
    bool mark[maxn*2];
    int S[maxn*2], c;

    bool dfs(int x) {
        if (mark[x^1]) return false;
        if (mark[x]) return true;
        mark[x] = true;
    }
}

```

```

    S[c++] = x;
    for (int i = 0; i < G[x].size(); i++)
        if (!dfs(G[x][i])) return false;
    return true;
}

void init(int n) {
    this->n = n;
    for (int i = 0; i < n*2; i++) G[i].clear();
    memset(mark, 0, sizeof(mark));
}

// x = xval or y = yval
void add_clause(int x, int xval, int y, int yval) {
    x = x * 2 + xval;
    y = y * 2 + yval;
    G[x].push_back(y^1); // G[0].Pb(1)
    G[y].push_back(x^1); // G[1].Pb(0);
}

bool solve() {
    for(int i = 0; i < n*2; i += 2)
        if(!mark[i] && !mark[i+1]) {
            c = 0;
            if(!dfs(i)) {
                while(c > 0) mark[S[--c]] = false;
                if(!dfs(i+1)) return false;
            }
        }
    return true;
}
};

```

## 3.2 LCA

### 3.2.1 1 DFS+RMQ

```

#include<cstdio>
#include<cstring>
#include<vector>
#include<cmath>
#include<iostream>
using namespace std;

const int maxn = 40000+100;
const int maxlogv = 17;
struct Edge{
    int to,weight;
    Edge(int t,int w):to(t),weight(w){};
};
vector<Edge> G[maxn];

int id[maxn],dis[maxn];
int vs[maxn*2],depth[maxn*2];
int dp[maxn*2][maxlogv];

```

```

void dfs(int node,int fa,int d,int &k){
    id[node] = k;
    vs[k] = node;
    depth[k++] = d;
    // dis[node] = distance;
    for(int i = 0;i < G[node].size(); ++i){
        Edge &t = G[node][i];
        if(t.to == fa) continue;
        dis[t.to] = dis[node]+t.weight;
        dfs(t.to,node,d+1,k);
    }
    vs[k] = node;
    depth[k++] = d;
}

void init_rmq(int n){
    for(int i = 0;i < n ; ++i) dp[i][0] = i;
    for(int j = 1;(1<<j) <= n; ++j){
        for(int i = 0;i + (1<<j)-1 < n; ++i){
            if(depth[dp[i][j-1]] < depth[dp[i+(1<<(j-1))][j-1]])
                dp[i][j] = dp[i][j-1];
            else
                dp[i][j] = dp[i+(1<<(j-1))][j-1];
        }
    }
}

int query(int l,int r){
    int k = 0;
    while(((1<<(k+1)) <= r-l+1) k++);
    if(depth[dp[l][k]] < depth[dp[r-(1<<k)+1][k]])
        return dp[l][k];
    else
        return dp[r-(1<<k)+1][k];
}

int lca(int u,int v){
    return vs[query(min(id[u],id[v]),max(id[u],id[v]))];
}

void init(int n){
    int k = 0;
    dfs(0,-1,0,k);
    init_rmq(2*n-1);
}

int main(void){
    int n,m,q;
    while(~scanf("%d%d",&n,&m)){
        for(int i = 0;i < n; ++i) G[i].clear();
        int u,v,w;
        for(int i = 0;i < m; ++i){
            scanf("%d%d%d",&u,&v,&w);
            u--,v--;
            G[u].push_back(Edge(v,w));
            G[v].push_back(Edge(u,w));
        }
    }
}

```

```

    }
    init(n);
    scanf("%d",&q);
    while(q--){
        int u,v;
        scanf("%d %d",&u,&v);
        u--,v--;
        int f = lca(u,v);
        printf("%d\n",dis[u]+dis[v]-2*dis[f]);
    }
}

return 0;
}

```

### 3.2.2 2 倍增算法

// POJ1330  
// LCA 的倍增算法

```

#include<vector>
#include<cstdio>
#include<cstring>
using namespace std;

const int maxn = 1e4+100;
const int maxlogv = 14;
vector<int> G[maxn];
int root;

int parent[maxlogv][maxn];
int depth[maxn];

void dfs(int v,int p,int d){
    parent[0][v] = p;
    depth[v] = d;
    for(int i = 0;i < G[v].size(); ++i){
        if(G[v][i] != p){
            dfs(G[v][i],v,d+1);
        }
    }
}

void init(int V){
    dfs(root,-1,0);
    for(int k = 0;k+1 < maxlogv; ++k){
        for(int v = 0; v < V; ++v){
            if(parent[k][v] < 0) parent[k+1][v] = -1;
            else parent[k+1][v] = parent[k][parent[k][v]];
        }
    }
}

int lca(int u,int v){
    if(depth[u] > depth[v]) swap(u,v);

```

```

        for(int k = 0; k < maxlogv; ++k){
            if(((depth[v] - depth[u]) >> k) & 1){
                v = parent[k][v];
            }
        }
        if(u == v) return u;
        for(int k = maxlogv-1; k >= 0; --k){
            if(parent[k][u] != parent[k][v]){
                u = parent[k][u];
                v = parent[k][v];
            }
        }
        return parent[0][u];
    }
    bool OUT[maxn];
    int main(void)
    {
        int T;
        scanf("%d",&T);
        while(T--){
            int n;
            for(int i = 0; i < n; ++i) G[i].clear();
            memset(OUT, 0, sizeof(OUT));
            scanf("%d",&n);
            for(int i = 1; i < n; ++i) {
                int u, v;
                scanf("%d %d",&u, &v);
                u--, v--;
                G[u].push_back(v);
            }
            OUT[v] = 1;
            for(int i = 0; i < n; ++i) if(!OUT[i]){
                root = i;
                break;
            }
            init(n);
            int u, v;
            scanf("%d %d",&u, &v);
            u--, v--;
            printf("%d\n", lca(u, v) + 1);
        }

        return 0;
    }

```

### 3.3 Maxflow

#### 3.3.1 1 Dinic

```

// dinic
#include <cstdio> // C 语言 io
#include <cstring> // 以下是 c 语言常用头文件
#include <cmath>

```

```

#include <cstdlib>
#include <ctime>
#include <cctype>
#include <cstring>
#include <cmath>
#include <iostream> //c++ IO
#include <sstream>
#include <string>
#include <list> //c++ 常用容器
#include <vector>
#include <set>
#include <map>
#include <queue>
#include <stack>
#include <algorithm> //c++ 泛型的一些函数
#include <functional> //用来提供一些模版
#define fo0(i,n) for(int i = 0; i < n; ++i)
#define fo1(i,n) for(int i = 1; i <= n; ++i)
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 0x7FFFFFFF;
const LL INFF = 0x7FFFFFFFFFFFFFFF;
const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-6;
const LL mod = 1e9 + 7;
const int LEN = 20000+1000;
const int maxn = 1e8;
struct Edge{
    int from,to,cap,flow;
    Edge(int u,int v,int w,int f): from(u),to(v),cap(w),flow(f){}
};
struct Dinic{
    int n,m,s,t;
    vector<Edge> edges;
    vector<int> G[LEN];
    int a[LEN];
    int vis[LEN];
    int d[LEN];
    int cur[LEN]; //好吧就是点，代表该点在一次求增广的过程中搜索到了那条边，意思就是从这条边往下肯定搜索不到结
    void init(int n)
    {
        this->n = n;
        for(int i = 0; i < n; ++i)
            G[i].clear();
        edges.clear();
    }
    void Add(int u,int v,int w)
    {

```

```

        edges.push_back(Edge(u,v,w,0));
        edges.push_back(Edge(v,u,0,0));
        m = edges.size();
        G[u].push_back(m-2);
        G[v].push_back(m-1);
    }
    bool Bfs(void)//分层
    {
        me(d);
        me(vis);
        d[s] = 0;
        vis[s] = 1;

        queue<int> Q;
        Q.push(s);
        while(!Q.empty())
        {
            int q = Q.front();Q.pop();

            for(size_t i = 0;i < G[q].size();++i)
            {
                Edge &tmp = edges[G[q][i]];
                if(!vis[tmp.to]&&tmp.cap>tmp.flow)
                {
                    vis[tmp.to] = 1;
                    d[tmp.to] = d[q] + 1;
                    Q.push(tmp.to);
                }
            }
        }
        return vis[t];
    }
    int Dfs(int node,int a)
    {
        if(node == t||a == 0)
            return a;
        int flow = 0,f;
        for(int &i = cur[node];i < G[node].size();++i)
        {
            Edge &tmp = edges[G[node][i]];
            if(d[tmp.to]==d[node]+1&&(f=Dfs(tmp.to,min(a,tmp.cap-tmp.flow)))>0)
            {
                flow += f;
                tmp.flow += f;
                edges[G[node][i]^1].flow -= f;
                a -= f;
                if(a==0)
                    break;
            }
        }
        return flow;
    }
    int MaxFlow(int s,int t)

```



```

{
    this->s = s;
    this->t = t;
    int flow = 0;
    while(Bfs())
    {
        me(cur);
        flow += Dfs(s,maxn);
    }
    return flow;
}

};
Dinic dinic;
int main()
{
    int N,M,S,T;
    while(cin>>N>>M)
    {
        S =1, T = N;
        dinic.init(N);
        int u,v,w;
        for(int i = 0;i < M;++i)
        {
            scanf("%d %d %d",&u,&v,&w);
            dinic.Add(u,v,w);
        }
        int ans = 0;
        ans = dinic.MaxFlow(S,T);
        printf("%d\n",ans);
    }

    return 0;
}

```

### 3.3.2 2 ISAP

```

// 点的下标从零开始，注意初始化
#include<cstdio>
#include<cstring>
#include<queue>
#include<vector>
#include<algorithm>
using namespace std;

const int maxn = 10000 + 10;
const int INF = 1000000000;

struct Edge {

```

```

    int from, to, cap, flow;
};

bool operator < (const Edge& a, const Edge& b) {
    return a.from < b.from || (a.from == b.from && a.to < b.to);
}

struct ISAP {
    int n, m, s, t;
    vector<Edge> edges;
    vector<int> G[maxn];    // 邻接表, G[i][j] 表示结点 i 的第 j 条边在 e 数组中的序号
    bool vis[maxn];        // BFS 使用
    int d[maxn];           // 从起点到 i 的距离
    int cur[maxn];         // 当前弧指针
    int p[maxn];           // 可增广路上的上一条弧
    int num[maxn];         // 距离标号计数

    void AddEdge(int from, int to, int cap) {
        edges.push_back((Edge){from, to, cap, 0});
        edges.push_back((Edge){to, from, 0, 0});
        m = edges.size();
        G[from].push_back(m-2);
        G[to].push_back(m-1);
    }

    bool BFS() {
        memset(vis, 0, sizeof(vis));
        queue<int> Q;
        Q.push(t);
        vis[t] = 1;
        d[t] = 0;
        while(!Q.empty()) {
            int x = Q.front(); Q.pop();
            for(int i = 0; i < G[x].size(); i++) {
                Edge& e = edges[G[x][i]^1];
                if(!vis[e.from] && e.cap > e.flow) {
                    vis[e.from] = 1;
                    d[e.from] = d[x] + 1;
                    Q.push(e.from);
                }
            }
        }
        return vis[s];
    }

    void init(int n) {
        this->n = n;
        for(int i = 0; i < n; i++) G[i].clear();
        edges.clear();
    }

    int Augment() {
        int x = t, a = INF;

```

```

while(x != s) {
    Edge& e = edges[p[x]];
    a = min(a, e.cap-e.flow);
    x = edges[p[x]].from;
}
x = t;
while(x != s) {
    edges[p[x]].flow += a;
    edges[p[x]^1].flow -= a;
    x = edges[p[x]].from;
}
return a;
}

int Maxflow(int s, int t) {
    this->s = s; this->t = t;
    int flow = 0;
    BFS();
    memset(num, 0, sizeof(num));
    for(int i = 0; i < n; i++) num[d[i]]++;
    int x = s;
    memset(cur, 0, sizeof(cur));
    while(d[s] < n) {
        if(x == t) {
            flow += Augment();

            x = s;
        }
        int ok = 0;
        for(int i = cur[x]; i < G[x].size(); i++) {
            Edge& e = edges[G[x][i]];
            if(e.cap > e.flow && d[x] == d[e.to] + 1) { // Advance
                ok = 1;
                p[e.to] = G[x][i];
                cur[x] = i; // 注意
                x = e.to;
                break;
            }
        }
        if(!ok) { // Retreat
            int m = n-1; // 初值注意
            for(int i = 0; i < G[x].size(); i++) {
                Edge& e = edges[G[x][i]];
                if(e.cap > e.flow) m = min(m, d[e.to]);
            }
            if(--num[d[x]] == 0) break;
            num[d[x] = m+1]++;
            cur[x] = 0; // 注意
            if(x != s) x = edges[p[x]].from;
        }
    }
    return flow;
}
};

```

```

ISAP g;

int main() {

    int N,M;
    int S,T;
    scanf("%d %d",&N,&M);
    scanf("%d %d",&S,&T);
    int u,v,w;
    g.init(N);
    while(M--){
        scanf("%d %d %d",&u,&v,&w);
        u--,v--;
        g.AddEdge(u,v,w);
    }
    printf("%d",g.Maxflow(S-1,T-1));

    return 0;
}

```

### 3.3.3 3 MCMF

// 最小费用最大流, 下标从 1 开始

```

#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
#define Pb push_back
#define FI first
#define SE second
#define For(i,a,b) for(int i = a; i < b; ++i)
#define IOS ios::sync_with_stdio(false)
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 1e8;
const LL INFF = 0x7FFFFFFFFFFFFFFF;
const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-6;
const LL mod = 1e9 + 7;
LL qpow(LL a,LL b){LL s=1;while(b>0){if(b&1)s=s*a%mod;a=a*a%mod;b>>=1;}return s;}
LL gcd(LL a,LL b) {return b?gcd(b,a%b):a;}
int dr[2][4] = {1,-1,0,0,0,0,-1,1};
typedef pair<int,int> P;
struct Edge{
    int from,to,cap,flow,cost;
};
const int maxn = 5000+100;

```

```

struct MCMF{
    int n,m,s,t;
    vector<Edge> edges;
    vector<int> G[maxn];
    int inq[maxn];
    int d[maxn];
    int p[maxn];
    int a[maxn];
    void init(int n){
        this->n = n;
        for(int i = 0;i < n; ++i) G[i].clear();
        edges.clear();
    }
    void AddEdge(int from,int to,int cap,int cost){
        edges.push_back((Edge){from,to,cap,0,cost});
        edges.push_back((Edge){to,from,0,0,-cost});
        int m = edges.size();
        G[from].push_back(m-2);
        G[to].push_back(m-1);
    }
    bool BellmanFord(int s,int t,int &flow,int &cost){
        for(int i = 0;i < n; ++i) d[i] = INF;
        memset(inq,0,sizeof(inq));
        d[s] = 0,inq[s] = 1;p[s] = 0,a[s] = INF;

        queue<int> Q;
        Q.push(s);
        while(!Q.empty()){

            int u = Q.front(); Q.pop();
            inq[u] = 0;
            for(int i = 0;i < G[u].size(); ++i){
                Edge& e = edges[G[u][i]];
                if(e.cap > e.flow && d[e.to] > d[u]+e.cost){
                    d[e.to] = d[u]+e.cost;
                    p[e.to] = G[u][i];
                    a[e.to] = min(a[u],e.cap-e.flow);
                    if(!inq[e.to]) {
                        Q.push(e.to); inq[e.to] = 1;
                    }
                }
            }
        }

        if(d[t] == INF) return false;

        flow += a[t];
        cost += d[t]*a[t];
        int u = t;
        while(u != s){
            edges[p[u]].flow += a[t];
            edges[p[u]^1].flow -= a[t];
            u = edges[p[u]].from;
        }
    }
};

```

```

        }
        return true;
    }
    int Mincost(int s,int t,int &flow,int &cost){
        flow = 0,cost = 0;

        while(BellmanFord(s,t,flow,cost));
        return cost;
    }
};
MCMF mcmf;
int main(void)
{
    int n,m,s,t;
    scanf("%d %d %d %d",&n,&m,&s,&t);
    int u,v,w,c;
    mcmf.init(n+1);
    while(m--){
        scanf("%d %d %d %d",&u,&v,&w,&c);
        mcmf.AddEdge(u,v,w,c);
    }
    int flow,cost;
    flow = 0,cost = 0;
    mcmf.Mincost(s,t,flow,cost);
    printf("%d %d\n",flow,cost);

    return 0;
}

```

## 3.4 二分图

### 3.4.1 1 匈牙利算法

```

#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
#define Pb push_back
#define FI first
#define SE second
#define For(i,a,b) for(int i = a; i < b; ++i)
#define IOS ios::sync_with_stdio(false)
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 0x7FFFFFFF;
const LL INFF = 0x7FFFFFFFFFFFFFFF;
const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-6;
const LL mod = 1e9 + 7;
LL qpow(LL a,LL b){LL s=1;while(b>0){if(b&1)s=s*a%mod;a=a*a%mod;b>>=1;}return s;}

```

```

LL gcd(LL a,LL b) {return b?gcd(b,a/b):a;}
int dr[2][4] = {1,-1,0,0,0,0,-1,1};
typedef pair<int,int> P;
const int maxn = 1000+10;
vector<int> G[maxn];
int match[maxn];
bool used[maxn];
int N,M;
bool dfs(int v){
    used[v] = true;
    for(int i = 0;i < G[v].size(); ++i){
        if(used[u]) continue; used[u] = true;
        int u = G[v][i],w = match[u];
        if(w < 0 || !used[w]&&dfs(w)){
            match[v] = u;
            match[u] = v;
            return true;
        }
    }
    return false;
}
int main(void)
{
    scanf("%d %d",&N,&M);

    while(M--){
        int u,v;
        scanf("%d %d",&u,&v);
        G[u].Pb(v);
        G[v].Pb(u);
    }
    int ans = 0;
    memset(match,-1,sizeof(match));
    for(int i = 1;i <= N; ++i){
        if(match[i] < 0){
            memset(used,0,sizeof(used));
            if(dfs(i)){
                ans++;
            }
        }
    }
    cout<<ans<<endl;
    return 0;
}

```

### 3.4.2 2 KM

```

const int maxn = 500+5;
struct KM{
    int n;
    vector<int> G[maxn];
    int W[maxn][maxn];
    int Lx[maxn];
    int Ly[maxn];

```

```

int Left[maxn];
bool S[maxn], T[maxn];
void init(int n){
    this->n = n;
    for(int i = 1; i <= n; ++i) G[i].clear();
    memset(W, 0, sizeof(W));
}

void AddEdge(int u, int v, int w){
    G[u].push_back(v);
    W[u][v] = w;
}

bool match(int u){
    S[u] = true;
    for(int i = 0; i < G[u].size(); ++i){
        int v = G[u][i];
        if(Lx[u] + Ly[v] == W[u][v] && !T[v]){
            T[v] = true;
            if(Left[v] == -1 || match(Left[v])){
                Left[v] = u;
                return true;
            }
        }
    }
    return false;
}

void update(){
    int a = INF;
    for(int u = 0; u < n; ++u)
        if(S[u])
            for(int i = 0; i < G[u].size(); ++i){
                int v = G[u][i];
                if(!T[v])
                    a = min(a, Lx[u] + Ly[v] - W[u][v]);
            }
    for(int i = 0; i < n; ++i){
        if(S[i]) Lx[i] -= a;
        if(T[i]) Ly[i] += a;
    }
}

void solve(){
    for(int i = 0; i < n; ++i){
        Lx[i] = *max_element(W[i], W[i] + n);
        Left[i] = -1;
        Ly[i] = 0;
    }
    for(int u = 0; u < n; ++u){
        for(;;){
            for(int i = 0; i < n; ++i) S[i] = T[i] = 0;
            if(match(u)) break;
            else update();
        }
    }
}

};

```



### 3.4.3 3 一般图最大匹配

```
#include<cstdio>
#include<algorithm>
#include<cmath>
#include<cstring>
#include<vector>
#define SF scanf
#define PF printf
#define MAXN 510
using namespace std;
int mk[MAXN],fa[MAXN],nxt[MAXN],q[MAXN],vis[MAXN],match[MAXN];
int fr,bk,t,n,m;
vector<int> a[MAXN];
int find(int x){
    if(fa[x]==x)
        return x;
    fa[x]=find(fa[x]);
    return fa[x];
}
int LCA(int x,int y){
    t++;
    while(1){
        if(x){
            x=find(x);
            if(vis[x]==t)
                return x;
            vis[x]=t;
            if(match[x])
                x=nxt[match[x]];
            else
                x=0;
        }
        swap(x,y);
    }
}
void Union(int x,int y){
    if(find(x)!=find(y))
        fa[fa[x]]=fa[y];
}
void gr(int a,int p){
    while(a!=p){
        int b=match[a];
        int c=nxt[b];
        if(find(c)!=p)
            nxt[c]=b;
        if(mk[b]==2){
            q[++bk]=b;
            mk[b]=1;
        }
        Union(a,b);
        Union(b,c);
        a=c;
    }
}
```

```

}
void aug(int S){
    for(int i=1;i<=n;i++){
        mk[i]=nxt[i]=0;
        fa[i]=i;
    }
    mk[S]=1;
    fr=bk=0;
    q[fr]=S;
    while(fr<=bk){
        int x=q[fr++];
        for(int i=0;i<a[x].size();i++){
            int y=a[x][i];
            if(match[x]==y)
                continue;
            else if(find(x)==find(y))
                continue;
            else if(mk[y]==2)
                continue;
            else if(mk[y]==1){
                int r=LCA(x,y);
                if(find(x)!=r)
                    nxt[x]=y;
                if(find(y)!=r)
                    nxt[y]=x;
                gr(x,r);
                gr(y,r);
            }
            else if(!match[y]){
                nxt[y]=x;
                for(int u=y;u;){
                    int v=nxt[u];
                    int mv=match[v];
                    match[u]=v;
                    match[v]=u;
                    u=mv;
                }
                return;
            }
            else{
                nxt[y]=x;
                mk[y]=2;
                q[++bk]=match[y];
                mk[match[y]]=1;
            }
        }
    }
}

}

int main(){
    SF("%d%d",&n,&m);
    int u,v;
    for(int i=1;i<=m;i++){
        SF("%d%d",&u,&v);
        a[u].push_back(v);
    }
}

```

```

        a[v].push_back(u);
    }
    for(int i=1;i<=n;i++)
        if(!match[i])
            aug(i);
    int sum=0;
    for(int i=1;i<=n;i++)
        if(match[i])
            sum++;
    PF("%d\n",sum/2);
    for(int i=1;i<=n;i++)
        PF("%d ",match[i]);
}

```

### 3.5 最小生成树

#### 3.5.1 1 Krustal 卡鲁斯卡尔算法

```

/*
复杂度  $E \cdot \log(E)$ , 适用于稀疏图
https://vjudge.net/problem/HDU-1863
*/

#include<bits/stdc++.h>

using namespace std;

const int maxn = 100+100;
struct Edge//边
{
    int from,to,cost;
    bool operator< ( const Edge & a)
    {
        return cost < a.cost;
    }
};
Edge edge[maxn];
int F[maxn];
int Find(int x)//并查集算法
{
    return x == F[x] ? x:F[x] = Find(F[x]);
}
int main(void)
{
    int N,M;
    while(cin>>N>>M&&N)// N 代表的是道路数量, M 代表村庄的数量
    {
        for(int i = 0; i <= M; ++i)
            F[i] = i;
        for(int i = 0; i < N; ++i)
        {
            Edge &t = edge[i];
            scanf("%d %d %d",&t.from,&t.to,&t.cost);
        }
    }
}

```

```

sort(edge,edge+N); // 对边进行排序
int sum = 0;
int num = M;
for(int i = 0; i < N ; ++i) // 一个个将边加进去
{
    Edge t = edge[i];
    if(Find(t.from) == Find(t.to))
        continue;
    F[Find(t.from)] = F[Find(t.to)];
    sum += t.cost;
    num--;
}
if(num == 1)
    cout<<sum<<endl;
else
    cout<<"?"<<endl;
}

return 0;
}

```

### 3.5.2 2 prim 算法

```

/*
prim 算法是进行加点，使用于稠密图，可以选择用堆或者不用
不用堆  $O(V^2)$ ;
用堆  $O(E * \log(V))$ ;
https://vjudge.net/problem/HDU-1863
*/

typedef pair<int,int> P;
const int LEN = 2e6+100;
int Away[LEN]; //记录从当前已选结点到 j 节点的路径的最小值
bool vis[LEN];
int N,M; //N 道路数目, M 村庄个数
vector<vector<P>> > vec(LEN);
int main()
{
    cin>>M>>N;

    int from,to,weight;
    while(N--)
    {
        scanf("%d %d %d",&from,&to,&weight);
        vec[from].push_back(P(weight,to));
        vec[to].push_back(P(weight,from));
    } // 添加边

    for(int i = 2; i <= M; ++i)
        Away[i] = INF; //初始化 Away 数组
    Away[1] = 0;
}

```

```

    int Left = M;
    int All_cost = 0;
    priority_queue<P,vector<P>,greater<P> > q; // 小顶堆
    q.push(P(0,1));
    while(!q.empty() && Left>0)
    {
        P tmp = q.top();q.pop();
        int To = tmp.second;
        if(vis[To])
            continue;
        vis[To] = 1;
        Left--;
        All_cost += tmp.first;
        for(int i = 0; i < vec[To].size(); ++i) // 更新 Away 数组
        {
            P &t = vec[To][i];
            if(!vis[t.second] && Away[t.second] > t.first)
            {
                Away[t.second] = t.first;
                q.push(t);
            }
        }
    }

    cout<<All_cost<<endl;

    return 0;
}

```

### 3.5.3 3 最小限制生成树

```

// 限制某一点的度数不能超过 K
#include<cstring>
#include<map>
#include<cstdio>
#include<iostream>
#include<algorithm>
#include<set>
using namespace std;
#define me(ar) memset(ar,0,sizeof(ar))
const int INF = 1e8;
//.....
const int LEN = 30;
int K;
int n,m;
struct Edge
{
    int x,y;
    int weight;
    bool operator <(const Edge &a) const
    {
        return weight < a.weight;
    }
}

```

```

    }
} edge[LEN*LEN+10]; //邻接表存边, Kruskal 算法要用
int dis[LEN][LEN]; //邻接矩阵
int sign[LEN][LEN]; //记录那些边已经在生成树里面了
int vis[LEN]; //记录是否相连
int F[LEN]; //并查集所用
int Father[LEN]; //由 i 到 i+1 度限制生成树需要用动态规划求解, 用来状态转移
int Best[LEN]; //Best[i] 指的是由当前节点到 park 这些边中最长边是多少
int Find(int x) //并查集所用 Find 函数
{
    return x == F[x]?x:F[x] = Find(F[x]);
}
void Dfs(int x) //Dfs 动态规划记忆化搜索
{
    // vis[x] = 1;
    for(int i = 1; i <= n; ++i )
    {
        if(sign[i][x]&!vis[i]) //如果有边相连并且下一个节点没有被访问
        {
            if(x==0)
                Best[i] = -INF; //与 park 直接相连的边不能删除

            else
                Best[i] = max(Best[x], dis[x][i]); //状态转移方程
            Father[i] = x;
            vis[i] = 1;
            Dfs(i);
        }
    }
}
void init(){
    for(int i = 0; i < LEN; ++i)
        F[i] = i;
    me(sign); //初始化标记数组
    me(vis);
    //初始化邻接矩阵
    for(int i = 0; i < LEN; ++i)
        for(int j = 0; j < LEN; ++j)
            dis[i][j] = INF;
}
int main(void)
{
    while(cin>>m)
    {
        //初始化并查集数组
        init();
        n = 0; //用来记录共有多少个节点
        // set<string> se;
        map<string, int> ma; //将地点编号
        ma["Park"] = 0; //将 park 加入节点
        string s1, s2;
        int a, b;
        int weight = 0;
        for(int i = 0; i < m; ++i)
    }
}

```

```

{
    cin>>s1>>s2>>weight;
    if(s1 == "Park" || ma[s1] != 0)
        a = ma[s1]; //如果节点已编号，则直接使用
    else
        a = ma[s1] = ++n; //如果没有编号，编号
    if(s2 == "Park" || ma[s2] != 0)
        b = ma[s2];
    else
        b = ma[s2] = ++n;
    dis[a][b] = dis[b][a] = weight;
    edge[i].x = a;
    edge[i].y = b;
    edge[i].weight = weight;
}
//求最小生成树
int ans = 0; //kruskal 算法求最小生成树
sort(edge, edge+m);
for(int i = 0; i < m; ++i)
{
    int x = edge[i].x;
    int y = edge[i].y;
    weight = edge[i].weight;
    if(x==0 || y==0) //去除掉 park 这个点
        continue;
    int xx = Find(x);
    int yy = Find(y);
    if(xx != yy)
    {
        F[xx] = F[yy];
        ans += weight;
        sign[x][y] = sign[y][x] = 1;
    }
}

cin>>K; //最小 k 度生成树
int Min[LEN]; //用来记录每一个最小生成树到 park 点的最小路径
for(int i = 0; i < LEN; ++i)
    Min[i] = INF; //初始化
int index[LEN]; //用来记录最小路径的点
for(int i = 1; i <= n; ++i)
{
    if(dis[i][0] < Min[Find(i)])
    {
        Min[Find(i)] = dis[i][0];
        index[Find(i)] = i;
    }
}

////
cout<<se.size()<<endl;
int m = 0; //用来记录除去 park 点即 0 点之后共有多少个连通分量
for(int i = 1; i <= n; ++i)
{
    if(Min[i] != INF)

```

```

        {
            ans += Min[i];
            sign[index[i]][0] = sign[0][index[i]] = 1; //将这个最小路径的点与 park 相连
            m++;
        }
    }
    int MMin = ans;
    for(int i = m + 1; i <= K; ++i) //从 m+1 到 K 求最小 i 度生成树
    {
        me(vis);
        vis[0] = 1;
        Dfs(0);
        int select = -1; //select 用来记录选择哪个与 park 点相连是最小的
        int sum = INF;
        for(int i = 1; i <= n; ++i)
        {
            if(!sign[0][i] && dis[0][i] != INF)
            {
                if(dis[i][0] - Best[i] < sum)
                {
                    select = i;
                    sum = dis[i][0] - Best[i];
                }
            }
        }
        if(select == -1) //如果找不到，就跳出循环
            break;
        ans += sum;
        sign[select][0] = sign[0][select] = 1;
        MMin = min(MMin, ans);
        for(int i = select; i != 0; i = Father[i])
        {
            if(dis[Father[i]][i] == Best[select])
            {
                sign[i][Father[i]] = sign[Father[i]][i] = 0;
                break;
            }
        }
    }
    cout << ans << endl;

    }
    printf("Total miles driven: %d\n", MMin);
    // cout << MMin << endl;
}
return 0;
}

```

#### 3.5.4 4 次小生成树

```

#include<iostream>
#include<cstdio>
#include<cstring>
#include<string>
#include<algorithm>

```



```

#include<cmath>
#include<vector>
#include<queue>
#define ll long long
using namespace std;

int getint()
{
    int i=0,f=1;char c;
    for(c=getchar();(c<'0' || c>'9')&&c!='-';c=getchar());
    if(c=='-')f=-1,c=getchar();
    for(;c>='0'&&c<='9';c=getchar())i=(i<<3)+(i<<1)+c-'0';
    return i*f;
}

const int N=100005,M=300005;
struct node
{
    int x,y,w;
    inline friend bool operator < (const node &a,const node &b)
    {
        return a.w<b.w;
    }
}bian[M];
int n,m;
int id[N],fa[N][20],mx1[N][20],mx2[N][20],dep[N];
int tot,first[N],nxt[N<<1],to[N<<1],w[N<<1];
ll totlen,ans;
bool chs[M];

void add(int x,int y,int z)
{
    nxt[++tot]=first[x],first[x]=tot,to[tot]=y,w[tot]=z;
}

int find(int x)
{
    return id[x]==x?x:id[x]=find(id[x]);
}

void kruskal()
{
    for(int i=1;i<=n;i++)id[i]=i;
    sort(bian+1,bian+m+1);
    int cnt=0;
    for(int i=1;i<=m;i++)
    {
        int x=find(bian[i].x),y=find(bian[i].y);
        if(x!=y)
        {
            cnt++;
            totlen+=bian[i].w;
            chs[i]=true;
            add(bian[i].x,bian[i].y,bian[i].w);
        }
    }
}

```

```

        add(bian[i].y,bian[i].x,bian[i].w);
        id[y]=x;
        if(cnt==n-1)break;
    }
}

void dfs(int u)
{
    for(int i=1;i<20;i++)fa[u][i]=fa[fa[u][i-1]][i-1];
    for(int i=1;i<20;i++)mx1[u][i]=max(mx1[u][i-1],mx1[fa[u][i-1]][i-1]);
    for(int i=1;i<20;i++)
    {
        mx2[u][i]=max(mx2[u][i-1],mx2[fa[u][i-1]][i-1]);
        if(mx1[u][i-1]<mx1[fa[u][i-1]][i-1]&&mx2[u][i]<mx1[u][i-1])
            mx2[u][i]=mx1[u][i-1];
        if(mx1[u][i-1]>mx1[fa[u][i-1]][i-1]&&mx1[fa[u][i-1]][i-1]>mx2[u][i])
            mx2[u][i]=mx1[fa[u][i-1]][i-1];
    }
    for(int e=first[u];e;e=nxt[e])
    {
        int v=to[e];
        if(v==fa[u][0])continue;
        fa[v][0]=u;mx1[v][0]=w[e];
        dep[v]=dep[u]+1;
        dfs(v);
    }
}

int Find(int x,int y,int len)
{
    int Mx1=0,Mx2=0;
    if(dep[x]<dep[y])swap(x,y);
    int delta=dep[x]-dep[y];
    for(int i=19;i>=0;i--)
        if(delta&(1<<i))
        {
            if(Mx1>mx1[x][i]&&mx1[x][i]>Mx2)Mx2=mx1[x][i];
            if(Mx1<mx1[x][i])Mx2=max(Mx1,mx2[x][i]),Mx1=mx1[x][i];
            x=fa[x][i];
        }
    if(x==y)return Mx1==len?Mx2:Mx1;
    for(int i=19;i>=0;i--)
        if(fa[x][i]!=fa[y][i])
        {
            if(Mx1>mx1[x][i]&&mx1[x][i]>Mx2)Mx2=mx1[x][i];
            if(Mx1<mx1[x][i])Mx2=max(Mx1,mx2[x][i]),Mx1=mx1[x][i];
            x=fa[x][i];
            if(Mx1>mx1[y][i]&&mx1[y][i]>Mx2)Mx2=mx1[y][i];
            if(Mx1<mx1[y][i])Mx2=max(Mx1,mx2[y][i]),Mx1=mx1[y][i];
            y=fa[y][i];
        }
    if(Mx1>mx1[x][0]&&mx1[x][0]>Mx2)Mx2=mx1[x][0];
    if(Mx1<mx1[x][0])Mx2=max(Mx1,mx2[x][0]),Mx1=mx1[x][0];

```

```

        x=fa[x][0];
        if(Mx1>mx1[y][0]&&mx1[y][0]>Mx2)Mx2=mx1[y][0];
        if(Mx1<mx1[y][0])Mx2=max(Mx1,mx2[y][0]),Mx1=mx1[y][0];
        y=fa[y][0];
        return Mx1==len?Mx2:Mx1;
    }

    void solve(int e)
    {
        int x=bian[e].x,y=bian[e].y,len=bian[e].w;
        int tmp=Find(x,y,len);
        ans=min(ans,totlen-tmp+len);
    }

    int main()
    {
        //freopen("lx.in", "r", stdin);
        n=getint(),m=getint();
        for(int i=1;i<=m;i++)
        {
            bian[i].x=getint();
            bian[i].y=getint();
            bian[i].w=getint();
        }
        kruskal();
        dfs(1);
        ans=1e18;
        for(int i=1;i<=m;i++)
            if(!chs[i])solve(i);
        printf("%lld",ans);
    }

```

## 3.6 最短路

### 3.6.1 1 Dijkstra

```

#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
#define Pb push_back
#define FI first
#define SE second
#define For(i,a,b) for(int i = a; i < b; ++i)
#define IOS ios::sync_with_stdio(false)
using namespace std;
typedef long long LL;
//typedef unsigned long long ULL;
//const int prime = 999983;
//const int INF = 0x7FFFFFFF;
//const LL INFF =0x7FFFFFFFFFFFFFFF;
//const double pi = acos(-1.0);
//const double inf = 1e18;
//const double eps = 1e-6;
//const LL mod = 1e9 + 7;

```

```

//LL qpow(LL a,LL b){LL s=1;while(b>0){if(b&1)s=s*a%mod;a=a*a%mod;b>>=1;}return s;}
//LL gcd(LL a,LL b) {return b?gcd(b,a%b):a;}
//int dr[2][4] = {1,-1,0,0,0,0,-1,1};
//typedef pair<int,int> P;
struct Dijkstra{
    #define maxn 1234
    #define INF 123456789
    int n,m;
    int s,t;

    int dis[maxn],M[maxn][maxn];
    bool vis[maxn];
    void init(){
        scanf("%d %d %d %d",&n,&m,&s,&t);
        int u,v,c;
        for(int i = 1;i <= n; ++i)
            for(int j = 1;j <= n; ++j)
                if(i != j)
                    M[i][j] = INF;
        for(int i = 0;i < m; ++i){
            scanf("%d %d %d",&u,&v,&c);
            M[u][v] = M[v][u] = min(M[u][v],c);
        }
    }
    void solve(){
        memset(vis,0,sizeof(vis));
        fill(dis+1,dis+n+1,INF);
        dis[s] = 0;
        for(int i = 1;i <= n; ++i){
            int x,Min = INF;
            for(int j = 1;j <= n; ++j){
                if(!vis[j]&&dis[j] <= Min)
                    Min = dis[j];
            }
            vis[x] = 1;

            for(int j = 1;j <= n; ++j){
                if(!vis[j]&&dis[j] > dis[x]+M[x][j])
                    dis[j] = dis[x]+M[x][j];
            }
        }
        printf("%d\n",dis[t]);
    }
};
Dijkstra Dij;
int main(void)
{
    Dij.init();
    Dij.solve();

    return 0;
}
// 加了堆优化的 dij

```

```

#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
#define Pb push_back
#define FI first
#define SE second
#define For(i,a,b) for(int i = a; i < b; ++i)
#define IOS ios::sync_with_stdio(false)
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;

int dr[2][4] = {1,-1,0,0,0,0,-1,1};
typedef pair<int,int> P;
struct Edge{
    int u,v,d;
    Edge(int uu,int vv,int dd):u(uu),v(vv),d(dd){
    }
};
struct Dijstra{
    #define maxn 123456
    #define INF 123456789
    int N,M,S,T;

    typedef pair<int,int> P;
    vector<Edge> edges;
    vector<int> G[maxn];
    bool done[maxn];
    int d[maxn];
    int p[maxn];
    void init(){
        for(int i = 1;i <= N; ++i) G[i].clear();
        edges.clear();
        scanf("%d %d %d %d",&N,&M,&S,&T);
        // cout<<N<<M<<S<<T<<endl;
        int u,v,w;
        for(int i = 1;i <= M; ++i){
            scanf("%d %d %d",&u,&v,&w);
            AddEdge(u,v,w);
            AddEdge(v,u,w);
        }

    }
    void AddEdge(int u,int v,int d){
        edges.push_back(Edge(u,v,d));
        int m = edges.size();
        G[u].push_back(m-1);
    }
    void solve(){
        priority_queue<P,vector<P>,greater<P>> Q;
        for(int i = 1;i <= N; ++i) d[i] = INF;
    }
};

```

```

        d[S] = 0;
        memset(done,0,sizeof(done));
        Q.push(P(0,S));
        while(!Q.empty()){
            P x = Q.top(); Q.pop();
            int u = x.second;
            if(done[u]) continue;
            done[u] = true;
            for(int i = 0;i < G[u].size(); ++i){
                Edge &e = edges[G[u][i]];
                if(!done[e.v] && d[e.v] > d[u]+e.d){
                    d[e.v] = d[u]+e.d;
                    p[e.v] = G[u][i];
                    Q.push(P(d[e.v],e.v));
                }
            }
        }

        printf("%d\n",d[T]);
    }
};

Dijkstra Dij;
int main(void)
{
    Dij.init();
    Dij.solve();

    return 0;
}

```

### 3.6.2 2 Bellman-ford

```

#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
#define Pb push_back
#define FI first
#define SE second
#define For(i,a,b) for(int i = a; i < b; ++i)
#define IOS ios::sync_with_stdio(false)
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 0x7FFFFFFF;
const LL INFF = 0x7FFFFFFFFFFFFFFF;
const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-6;
const LL mod = 1e9 + 7;
LL qpow(LL a,LL b) {
    LL s=1;
    while(b>0) {

```

```

        if(b&1)
            s=s*a%mod;
        a=a*a%mod;
        b>>=1;
    }
    return s;
}
LL gcd(LL a,LL b) {
    return b?gcd(b,a%b):a;
}
int dr[2][4] = {1,-1,0,0,0,0,-1,1};
typedef pair<int,int> P;
struct Edge{
    int from,to,dist;
    Edge(int u,int v,int d):from(u),to(v),dist(d){
    }
};
struct Bellman_ford {
    #define maxn 1234567
    bool inq[maxn]; // 用来记录入队次数
    int cnt[maxn], d[maxn], p[maxn];
    // cnt 来记录入队次数, 大于 n 就退出, d 用来记录最短距离, p 用来记录路径
    int n,m;
    int s,t;
    vector<Edge> edges;
    vector<int> G[maxn];
    void AddEdge(int from,int to,int dist){
        edges.push_back(Edge(from,to,dist));
        edges.push_back(Edge(to,from,dist));
        int m = edges.size();
        G[from].push_back(m-2);
        G[to].push_back(m-1);
    }
    void init(){
        scanf("%d %d %d %d",&n,&m,&s,&t);
        int u,v,c;
        for(int i = 0;i < m; ++i){
            scanf("%d %d %d",&u,&v,&c);
            AddEdge(u,v,c);
        }
        /// cout<<"test"<<endl;
    }
    bool bellman_ford() {
        queue<int> Q;
        memset(inq,0,sizeof(inq));
        memset(cnt,0,sizeof(cnt));
        for(int i = 1; i <= n; ++i)
            d[i] = INF;
        d[s] = 0;
        inq[s] = true;
        Q.push(s);

        while(!Q.empty()) {

```

```

        int u = Q.front();
        Q.pop();
        inq[u] = false;
        for(int i = 0; i < G[u].size(); ++i) {
            Edge &e = edges[G[u][i]];
            if(d[u] < INF && d[e.to] > d[u]+e.dist) {
                d[e.to] = d[u]+e.dist;
                p[e.to] = G[u][i];
                if(!inq[e.to]) {
                    Q.push(e.to);
                    inq[e.to] = true;
                    if(++cnt[e.to] > n)
                        return false;
                }
            }
        }
    }
    printf("%d\n",d[t]);
}

};
Bellman_ford bell;
int main(void) {
    bell.init();
    bell.bellman_ford();

    return 0;
}

```

### 3.6.3 Floyd

```

// https://hihocoder.com/problemset/problem/1089?sid=1348128
#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
#define Pb push_back
#define FI first
#define SE second
#define For(i,a,b) for(int i = a; i < b; ++i)
#define IOS ios::sync_with_stdio(false)
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 0x7FFFFFFF;
const LL INFF = 0x7FFFFFFFFFFFFFFF;
const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-6;
const LL mod = 1e9 + 7;
LL qpow(LL a,LL b){LL s=1;while(b>0){if(b&1)s=s*a%mod;a=a%mod;b>>=1;}return s;}
LL gcd(LL a,LL b){return b?gcd(b,a%b):a;}
int dr[2][4] = {1,-1,0,0,0,0,-1,1};

```



```

typedef pair<int,int> P;
struct Floyd{
    // 复杂度  $O(n^3)$ 
    #define maxn 300
    int d[maxn][maxn];
    int n,m;
    void init(void){
        scanf("%d %d",&n,&m);
        for(int i = 1;i <= n ;++i)
            for(int j = 1;j <= n; ++j)
                if(i != j)
                    d[i][j] = INF;
        int u,v,c;
        for(int i = 0;i < m; ++i){
            scanf("%d %d %d",&u,&v,&c);
            d[u][v] = d[v][u] = min(d[v][u],c);
        }
    }
    void floyd(void){
        for(int k = 1; k <= n; ++k)
            for(int i = 1;i <= n ;++i)
                for(int j = 1;j <= n; ++j)
                    if(d[i][k] < INF&&d[j][k] < INF)
                        d[i][j] = min(d[i][j],d[i][k]+d[j][k]);
    }
    void print(void){
        for(int i = 1;i <= n; ++i){
            for(int j = 1;j <= n; ++j)
                printf("%d%c",d[i][j]," \n"[j==n]);
        }
    }
};

Floyd floyd;
int main(void)
{
    floyd.init();
    floyd.floyd();
    floyd.print();

    return 0;
}

```

## 4 4 数学

### 4.1 3 FWT 模板.cpp

```

// 异或
void FWT(int *a,int N,int opt){
    const int inv2 = qpow(2,mod-2);
    // j 是区间开始点, i 是区间距离, k 是具体位置, j+k, i+j+k 就是在 a 数组中的坐标
    for(int i = 1;i < N; i <= 1){
        for(int p = i<<1,j = 0;j < N; j += p){

```

```

        for(int k = 0; k < i; ++k){
            int X = a[j+k], Y = a[i+j+k];
            a[j+k] = (X+Y)%mod;
            a[i+j+k] = (X+mod-Y)%mod;
            if(opt == -1) a[j+k] = 1ll*a[j+k]*inv2%mod, a[i+j+k] = 1ll*a[i+j+k]*inv2%mod;
        }
    }
}

```

或

```

if(opt == 1) F[i+j+k] = (F[i+j+k]+F[j+k]) %mod;
else F[i+j+k] = (F[i+j+k+mod-F[j+k]]) %mod;

```

和

```

if(opt == 1) F[j+k] = (F[j+k]+F[i+j+k]) %mod;
else F[j+k] = (F[j+k] +mod-F[i+j+k])%mod;

```

## 4.2 BM.cpp

//O(n<sup>2</sup>) n 是传入的数  
 //输入的 n 是第几个数

```

#include<bits/stdc++.h>
using namespace std;
#define rep(i,a,n) for (int i=a;i<n;i++)
#define per(i,a,n) for (int i=n-1;i>=a;i--)
#define pb push_back
#define mp make_pair
#define all(x) (x).begin(),(x).end()
#define fi first
#define se second
#define SZ(x) ((int)(x).size())
typedef vector<int> VI;
typedef long long ll;
typedef pair<int,int> PII;
const ll mod=1000000007;
ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b;b>>=1){if(b&1)res=res*a%mod;a=a*a%mod;}return res;}
ll _,n;
namespace linear_seq{
    const int N=10010;
    ll res[N],base[N],_c[N],_md[N];
    vector<ll> Md;
    void mul(ll *a,ll *b,int k)
    {
        rep(i,0,k+k) _c[i]=0;
        rep(i,0,k) if (a[i]) rep(j,0,k) _c[i+j]=(_c[i+j]+a[i]*b[j])%mod;
        for (int i=k+k-1;i>=k;i--) if (_c[i])
            rep(j,0,SZ(Md)) _c[i-k+Md[j]]=( _c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
        rep(i,0,k) a[i]=_c[i];
    }
    int solve(ll n,VI a,VI b)

```

```

{
    ll ans=0,pnt=0;
    int k=SZ(a);
    assert(SZ(a)==SZ(b));
    rep(i,0,k) _md[k-1-i]=-a[i];_md[k]=1;
    Md.clear();
    rep(i,0,k) if (_md[i]!=0) Md.push_back(i);
    rep(i,0,k) res[i]=base[i]=0;
    res[0]=1;
    while ((1ll<<pnt)<=n) pnt++;
    for (int p=pnt;p>=0;p--)
    {
        mul(res,res,k);
        if ((n>>p)&1)
        {
            for (int i=k-1;i>=0;i--) res[i+1]=res[i];res[0]=0;
            rep(j,0,SZ(Md)) res[Md[j]]=(res[Md[j]]-res[k]*_md[Md[j]])%mod;
        }
    }
    rep(i,0,k) ans=(ans+res[i]*b[i])%mod;
    if (ans<0) ans+=mod;
    return ans;
}
VI BM(VI s) {
    VI C(1,1),B(1,1);
    int L=0,m=1,b=1;
    rep(n,0,SZ(s)) {
        ll d=0;
        rep(i,0,L+1) d=(d+(1ll)C[i]*s[n-i])%mod;
        if (d==0) ++m;
        else if (2*L<=n) {
            VI T=C;
            ll c=mod-d*powmod(b,mod-2)%mod;
            while (SZ(C)<SZ(B)+m) C.pb(0);
            rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
            L=n+1-L; B=T; b=d; m=1;
        } else {
            ll c=mod-d*powmod(b,mod-2)%mod;
            while (SZ(C)<SZ(B)+m) C.pb(0);
            rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
            ++m;
        }
    }
    return C;
}
int gao(VI a,ll n){
    VI c=BM(a);
    c.erase(c.begin());
    rep(i,0,SZ(c)) c[i]=(mod-c[i])%mod;
    return solve(n,c,VI(a.begin(),a.begin()+SZ(c)));
}
};
int main()
{

```

```

int t;
scanf("%d",&t);
while(t--)
{
    scanf("%lld",&n);
    vector<int>v {2,3,4,5,7,9,12,15,19,24,31,40,52,67,86,110,141,181,233,300,386,496,637};
    // n = v.size();
    // v.push_back({2,3,4,5,7,9,12,15,19,24,31,40,52,67,86,110,141,181,233,300,386,496,637});
    printf("%lld\n",linear_seq::gao(v,n-1)%mod);
}
}

```

## 4.3 FFT

### 4.3.1 FFT

```

const double PI = acos(-1.0);
struct Complex
{
    double r,i;
    Complex(double _r = 0,double _i = 0){
        r = _r; i = _i;
    }
    Complex operator +(const Complex &b) {
        return Complex(r+b.r,i+b.i);
    }
    Complex operator -(const Complex &b) {
        return Complex(r-b.r,i-b.i);
    }
    Complex operator *(const Complex &b){
        return Complex(r*b.r-i*b.i,r*b.i+i*b.r);
    }
};

void FFT(Complex y[],int n ,int on)
{
    for(int i = 0, j = 0; i < n; i++) {
        if(j > i) swap(y[i], y[j]);
        int k = n;
        while(j & (k >= 1)) j ^= ~k;
        j |= k;
    }
    for(int h = 2;h <= n;h <= 1){
        Complex wn(cos(-on*2*PI/h),sin(-on*2*PI/h));
        for(int j = 0;j < n;j += h){
            Complex w(1,0);
            for(int k = j;k < j+h/2;k++){
                Complex u = y[k];
                Complex t = w*y[k+h/2];
                y[k] = u+t;
                y[k+h/2] = u-t;
                w = w*wn;
            }
        }
    }
}

```

```

    if(on == -1)
        for(int i = 0;i < n;i++)
            y[i].r /= n;
}

```

#### 4.3.2 kuangbin

```

#include <stdio.h>
#include <iostream>
#include <string.h>
#include <algorithm>
#include <math.h>
using namespace std;

const double PI = acos(-1.0);
struct complex
{
    double r,i;
    complex(double _r = 0,double _i = 0)
    {
        r = _r; i = _i;
    }
    complex operator +(const complex &b)
    {
        return complex(r+b.r,i+b.i);
    }
    complex operator -(const complex &b)
    {
        return complex(r-b.r,i-b.i);
    }
    complex operator *(const complex &b)
    {
        return complex(r*b.r-i*b.i,r*b.i+i*b.r);
    }
};

void change(complex y[],int len)
{
    int i,j,k;
    for(i = 1, j = len/2;i < len-1;i++)
    {
        if(i < j)swap(y[i],y[j]);
        k = len/2;
        while( j >= k)
        {
            j -= k;
            k /= 2;
        }
        if(j < k)j += k;
    }
}

void fft(complex y[],int len,int on)
{
    change(y,len);
    for(int h = 2;h <= len;h <= 1)

```

```

{
    complex wn(cos(-on*2*PI/h),sin(-on*2*PI/h));
    for(int j = 0;j < len;j += h)
    {
        complex w(1,0);
        for(int k = j;k < j+h/2;k++)
        {
            complex u = y[k];
            complex t = w*y[k+h/2];
            y[k] = u+t;
            y[k+h/2] = u-t;
            w = w*wn;
        }
    }
}
if(on == -1)
    for(int i = 0;i < len;i++)
        y[i].r /= len;
}

const int MAXN = 400040;
complex x1[MAXN];
int a[MAXN/4];
long long num[MAXN];//100000*100000 会超 int
long long sum[MAXN];

int main()
{
    int T;
    int n;
    scanf("%d",&T);
    while(T--)
    {
        scanf("%d",&n);
        memset(num,0,sizeof(num));
        for(int i = 0;i < n;i++)
        {
            scanf("%d",&a[i]);
            num[a[i]]++;
        }
        sort(a,a+n);
        int len1 = a[n-1]+1;
        int len = 1;
        while( len < 2*len1 )len <= 1;
        for(int i = 0;i < len1;i++)
            x1[i] = complex(num[i],0);
        for(int i = len1;i < len;i++)
            x1[i] = complex(0,0);
        fft(x1,len,1);
        for(int i = 0;i < len;i++)
            x1[i] = x1[i]*x1[i];
        fft(x1,len,-1);
        for(int i = 0;i < len;i++)
            num[i] = (long long)(x1[i].r+0.5);
    }
}

```

```

    len = 2*a[n-1];
    //减掉取两个相同的组合
    for(int i = 0;i < n;i++)
        num[a[i]+a[i]]--;
    //选择的无序, 除以 2
    for(int i = 1;i <= len;i++)
    {
        num[i]/=2;
    }
    sum[0] = 0;
    for(int i = 1;i <= len;i++)
        sum[i] = sum[i-1]+num[i];
    long long cnt = 0;
    for(int i = 0;i < n;i++)
    {
        cnt += sum[len]-sum[a[i]];
        //减掉一个取大, 一个取小的
        cnt -= (long long)(n-1-i)*i;
        //减掉一个取本身, 另外一个取其它
        cnt -= (n-1);
        //减掉大于它的取两个的组合
        cnt -= (long long)(n-1-i)*(n-i-2)/2;
    }
    //总数
    long long tot = (long long)n*(n-1)*(n-2)/6;
    printf("%.7lf\n", (double)cnt/tot);
}
return 0;
}

```

### 4.3.3 lrj

```

#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 0x7FFFFFFF;
const LL INFF = 0x7FFFFFFFFFFFFFFF;
//const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-6;
const LL mod = 1e9 + 7;
int dr[2][4] = {1,-1,0,0,0,0,-1,1};
// UVA12298 Super Poker II
// Rujia Liu

const long double PI = acos(0.0) * 2.0;

typedef complex<double> CD;

```

```

// Cooley-Tukey 的 FFT 算法，迭代实现。inverse = false 时计算逆 FFT
inline void FFT(vector<CD> &a, bool inverse) {
    int n = a.size();
    // 原地快速 bit reversal
    for(int i = 0, j = 0; i < n; i++) {
        if(j > i) swap(a[i], a[j]);
        int k = n;
        while(j & (k >>= 1)) j &= ~k;
        j |= k;
    }

    double pi = inverse ? -PI : PI;
    for(int step = 1; step < n; step <= 1) {
        // 把每相邻两个 “step 点 DFT” 通过一系列蝴蝶操作合并为一个 “2*step 点 DFT”
        double alpha = pi / step;
        // 为求高效，我们并不是依次执行各个完整的 DFT 合并，而是枚举下标 k
        // 对于一个下标 k，执行所有 DFT 合并中该下标对应的蝴蝶操作，即通过 E[k] 和 O[k] 计算 X[k]
        // 蝴蝶操作参考：http://en.wikipedia.org/wiki/Butterfly\_diagram
        for(int k = 0; k < step; k++) {
            // 计算 omega^k。这个方法效率低，但如果用每次乘 omega 的方法递推会有精度问题。
            // 有更快更精确的递推方法，为了清晰起见这里略去
            CD omegak = exp(CD(0, alpha*k));
            for(int Ek = k; Ek < n; Ek += step << 1) { // Ek 是某次 DFT 合并中 E[k] 在原始序列中的下标
                int Ok = Ek + step; // Ok 是该 DFT 合并中 O[k] 在原始序列中的下标
                CD t = omegak * a[Ok]; // 蝴蝶操作：x1 * omega^k
                a[Ok] = a[Ek] - t; // 蝴蝶操作：y1 = x0 - t
                a[Ek] += t; // 蝴蝶操作：y0 = x0 + t
            }
        }
    }

    if(inverse)
        for(int i = 0; i < n; i++) a[i] /= n;
}

// 用 FFT 实现的快速多项式乘法
inline vector<double> operator * (const vector<double>& v1, const vector<double>& v2) {
    int s1 = v1.size(), s2 = v2.size(), S = 2;
    while(S < s1 + s2) S <= 1;
    vector<CD> a(S,0), b(S,0); // 把 FFT 的输入长度补成 2 的幂，不小于 v1 和 v2 的长度之和
    for(int i = 0; i < s1; i++) a[i] = v1[i];
    FFT(a, false);
    for(int i = 0; i < s2; i++) b[i] = v2[i];
    FFT(b, false);
    for(int i = 0; i < S; i++) a[i] *= b[i];
    FFT(a, true);
    vector<double> res(s1 + s2 - 1);
    for(int i = 0; i < s1 + s2 - 1; i++) res[i] = a[i].real(); // 虚部均为 0
    return res;
}

```



## 4.4 template

// 适用范围, 求  $n$  次多项式第  $x$  项的值

```
namespace polysum {
    #define rep(i,a,n) for (int i=a;i<n;i++)
    #define per(i,a,n) for (int i=n-1;i>=a;i--)
    const int D=1e6+10;
    ll a[D],f[D],g[D],p[D],p1[D],p2[D],b[D],h[D][2],c[D];
    ll powmod(ll a,ll b){ll res=1;a%=mod;assert(b>=0);for(;b;b>>=1){if(b&1)res=res*a%mod;a=a*a%mod;}return res;}
    //.....
    // 已知  $a_i$  的  $d$  次多项式, 求第  $n$  项
    ll calcn(int d,ll *a,ll n) { //  $a[0].. a[d]$   $a[n]$ 
        if (n<=d) return a[n];
        p1[0]=p2[0]=1;
        rep(i,0,d+1) {
            ll t=(n-i+mod)%mod;
            p1[i+1]=p1[i]*t%mod;
        }
        rep(i,0,d+1) {
            ll t=(n-d+i+mod)%mod;
            p2[i+1]=p2[i]*t%mod;
        }
        ll ans=0;
        rep(i,0,d+1) {
            ll t=g[i]*g[d-i]%mod*p1[i]%mod*p2[d-i]%mod*a[i]%mod;
            if ((d-i)&1) ans=(ans-t+mod)%mod;
            else ans=(ans+t)%mod;
        }
        return ans;
    }
    // 初始化, 初始化的时候记得将  $D$  的值
    void init(int M) {
        f[0]=f[1]=g[0]=g[1]=1;
        rep(i,2,M+5) f[i]=f[i-1]*i%mod;
        g[M+4]=powmod(f[M+4],mod-2);
        per(i,1,M+4) g[i]=g[i+1]*(i+1)%mod;
    }
    // 已知  $a_i$ , 并且知道  $a_i$  是  $m$  次多项式
    ll polysum(ll m,ll *a,ll n) { //  $a[0].. a[m]$   $\sum_{i=0}^n a[i]$ 
        ll b[D];
        ll b[D];
        for(int i=0;i<=m;i++) b[i]=a[i];
        b[m+1]=calcn(m,b,m+1);
        rep(i,1,m+2) b[i]=(b[i-1]+b[i])%mod;
        return calcn(m+1,b,n); //  $m$  次多项式的和是  $m+1$  次多项式
    }

    ll qpolysum(ll R,ll n,ll *a,ll m) {
        //  $a[0].. a[m]$   $\sum_{i=0}^{n-1} a[i]*R^i$ 
        if (R==1) return polysum(n,a,m);
        a[m+1]=calcn(m,a,m+1);
        ll r=powmod(R,mod-2),p3=0,p4=0,c,ans;
    }
```

```

    h[0][0]=0;h[0][1]=1;
    rep(i,1,m+2) {
        h[i][0]=(h[i-1][0]+a[i-1])*r%mod;
        h[i][1]=h[i-1][1]*r%mod;
    }
    rep(i,0,m+2) {
        ll t=g[i]*g[m+1-i]%mod;
        if (i&1) p3=((p3-h[i][0]*t)%mod+mod)%mod,p4=((p4-h[i][1]*t)%mod+mod)%mod;
        else p3=(p3+h[i][0]*t)%mod,p4=(p4+h[i][1]*t)%mod;
    }
    c=powmod(p4,mod-2)*(mod-p3)%mod;
    rep(i,0,m+2) h[i][0]=(h[i][0]+h[i][1]*c)%mod;
    rep(i,0,m+2) C[i]=h[i][0];
    ans=(calcn(m,C,n)*powmod(R,n)-c)%mod;
    if (ans<0) ans+=mod;
    return ans;
}
} // polysum::init();

```

## 4.5 博弈

### 4.5.1 2. 威佐夫博弈

// 威佐夫博弈  
 // 两对石子，只能选择在一堆或者两堆石子里面取相同石子  
 // 打表发现规律，第  $k$  个必败点， $a_k = b_k + k$   
 //  $a_k = (1+\sqrt{5})/2 * k$ ，判断就是直接下面的式子了

```

int main(void)
{
    int a,b;
    while(cin>>a>>b){
        if(a > b)
            swap(a,b);
        int c = floor((b-a)*((1.0+sqrt(5.0))/2.0));
        if(a == c)
            cout<<0<<endl;
        else
            cout<<1<<endl;
    }
    return 0;
}

```

### 4.5.2 3 Nim 积

/\* 在一个二维平面中，有  $n$  个灯亮着并告诉你坐标，  
 每回合需要找到一个矩形，这个矩形  $xy$  坐标最大的那个角落的点必须是亮着的灯，  
 然后我们把四个角落的灯状态反转，不能操作为败

```

*/
#include<set>
#include<map>
#include<stack>
#include<cmath>
#include<queue>
#include<vector>
#include<cstdio>

```

```

#include<cstring>
#include<iostream>
#include<algorithm>
typedef long long ll;
const int maxn = 1e6 + 10;
const int seed = 131;
const ll MOD = 1e9 + 7;
const int INF = 0x3f3f3f3f;
using namespace std;
int m[2][2] = {0, 0, 0, 1};
int Nim_Mul_Power(int x, int y){
    if(x < 2) return m[x][y];
    int a = 0;
    for(; ; a++){
        if(x >= (1 << (1 << a)) && x < (1 << (1 << (a + 1))))
            break;
    }
    int m = 1 << (1 << a);
    int p = x / m, s = y / m, t = y % m;
    int d1 = Nim_Mul_Power(p, s);
    int d2 = Nim_Mul_Power(p, t);
    return (m * (d1 ^ d2)) ^ Nim_Mul_Power(m / 2, d1);
}
int Nim_Mul(int x, int y){
    if(x < y) return Nim_Mul(y, x);
    if(x < 2) return m[x][y];
    int a = 0;
    for(; ; a++){
        if(x >= (1 << (1 << a)) && x < (1 << (1 << (a + 1))))
            break;
    }
    int m = 1 << (1 << a);
    int p = x / m, q = x % m, s = y / m, t = y % m;
    int c1 = Nim_Mul(p, s), c2 = Nim_Mul(p, t) ^ Nim_Mul(q, s), c3 = Nim_Mul(q, t);
    return (m * (c1 ^ c2)) ^ c3 ^ Nim_Mul_Power(m / 2, c1);
}
int main(){
    int T;
    scanf("%d", &T);
    int ans;
    while(T--){
        ans = 0;
        int n, x, y;
        scanf("%d", &n);
        while(n--){
            scanf("%d%d", &x, &y);
            ans ^= Nim_Mul(x, y);
        }
        if(ans)
            printf("Have a try, lxhgww.\n");
        else
            printf("Don't waste your time.\n");
    }
    return 0;
}

```

```
}
```

#### 4.5.3 4 K 倍动态减法

/\*  
有  $n$  个石子, 先手第一次最多取  $n-1$  个, 之后如果前一个人取  $m$  个,  
则下一个人可以取 1 到  $k*m$  个, 取完最后一个为胜,  
问先手是否会胜, 如果会胜输出第一次取几个。

```
*/  
const int maxn = 2e6+100;  
int a[maxn], b[maxn];  
int main(void)  
{  
    int T;  
    cin >> T;  
    for(int kase = 1; kase <= T; ++kase){  
        int n, k;  
        cin >> n >> k;  
        a[0] = 1, b[0] = 1;  
        int i = 0, j = 0;  
        while(a[i] < n){  
            i++;  
            a[i] = b[i-1] + 1;  
            if(a[j+1] * k < a[i]) j++;  
            if(a[j] * k < a[i]) b[i] = b[j] + a[i];  
            else b[i] = a[i];  
        }  
        printf("Case %d: ", kase);  
        if(a[i] == n) {  
            puts("lose");  
            continue;  
        }  
        // i--;  
        while(i >= 0){  
            if(n - a[i] > 0)  
                n -= a[i];  
            if(n == a[i]) break;  
            i--;  
        }  
        printf("%d\n", n);  
    }  
    return 0;  
}
```

#### 4.5.4 5 海盗分金问题

/\*  
*A Puzzle for Pirates HDU - 1538*  
\*/

```
int solve(int n, int m, int q){  
    if(n <= 2*m+2){  
        if(q == n){
```

```

        return m-(n-1)/2;
    }
    else{
        if(q % 2== n%2) return 1;
        else return 0;
    }
}
else{
    if(q <= 2*m+2) return 0;
    if(n == q)
    {
        LL t = 2*m+2;
        while(t < n)
            t = 2*(t-m);
        if(t == n) return 0;
        else return -1;
    }
    else{
        LL t = 2*m+2;
        while(t < q)
            t = 2*(t-m);
        if(t <= n) return 0;
        else return -1;
    }
}
}
}
int main(void)
{
    int T;
    cin>>T;
    while(T--){
        LL n,m,q;
        cin>>n>>m>>q;
        LL ans = solve(n,m,q);

        if(ans == -1) puts("Thrown");
        else printf("%lld\n",ans);
    }

    return 0;
}

```

#### 4.5.5 6 Green Hackbush

//  $N$  个点,  $M$  条边

```

#include<bits/stdc++.h>
using namespace std;
#define min(x,y) ((x)<(y))? (x):(y)

int Cases,N,M;
vector< list<int> > G,G2;
vector<int> GV;

```

```

vector<int> visited,from,time_disc,time_up;
int DFSTime;

void DFS_Visit(int v){
    int edges_to_parent=0;
    visited[v]=1; time_disc[v]=time_up[v]++DFSTime;
    for (list<int>::iterator start=G[v].begin();start!=G[v].end();start++) {
        if (!visited[*start]) { from[*start]=v; DFS_Visit(*start); time_up[v]=min(time_up[v],time_up[*start]); }
        else {
            if ((*start)!=from[v]) { time_up[v]=min(time_up[v],time_disc[*start]); }
            else {
                if (edges_to_parent) { time_up[v]=min(time_up[v],time_disc[*start]); }
                edges_to_parent++;
            }
        }
    }
}

void FindBridges(void){
    time_disc.clear(); time_up.clear(); visited.clear(); from.clear();
    visited.resize(N+3,0); time_disc.resize(N+3,0); time_up.resize(N+3,0); from.resize(N+3,0);
    from[1]=1; DFSTime=0;
    DFS_Visit(1);
}

int IsBridge(int v_lo, int v_high) {
    if (v_high!=from[v_lo]) return 0;
    return ( time_disc[v_lo]==time_up[v_lo] );
}

void ContractGraph(void){
    vector<int> color(N+3,0);
    int colors=1;
    color[1]=1;

    list<int> Q;
    Q.clear(); Q.push_back(1);
    while (!Q.empty()) {
        int where=Q.front(); Q.pop_front();
        for (list<int>::iterator it=G[where].begin(); it!=G[where].end(); it++) if (!color[*it]) {
            if (IsBridge(*it,where)) color[*it]++colors; else color[*it]=color[where];
            visited[*it]=1; Q.push_back(*it);
        }
    }

    G2.clear(); G2.resize(N+3);
    for (int i=1;i<=N;i++)
        for (list<int>::iterator it=G[i].begin(); it!=G[i].end(); it++)
            G2[color[i]].push_back(color[*it]);
}

int GrundyValue(int v){
    int loops=0,gv=0;

```

```

    if (GV[v] != -1) return GV[v]; GV[v] = 1000000000;

    for (list<int>::iterator start = G2[v].begin(); start != G2[v].end(); start++) {
        if ((*start) == v) loops++; else if (GV[*start] != 1000000000) gv^=(1+GrundyValue(*start));
    }
    loops /= 2; if (loops % 2) gv ^= 1;
    return GV[v] = gv;
}

int main(void) {
    int v1, v2;
    // freopen("input.txt", "r", stdin);
    // freopen("out.txt", "w+", stdout);
    cin >> Cases;
    while (Cases--) {
        // read graph dimensions
        cin >> N >> M;
        // read the graph
        G.clear(); G.resize(N+3);
        for (int i = 0; i < M; i++) { cin >> v1 >> v2; G[v1].push_back(v2); G[v2].push_back(v1); }
        // collapse all circuits in the graph
        FindBridges();
        ContractGraph();
        // compute the SG value
        GV.clear(); for (int i = 0; i <= N; i++) GV.push_back(-1);
        int result = GrundyValue(1);
        if (result) cout << "Alice\n"; else cout << "Bob\n"; // cout << result << "\n";

        //cout << result << "\n";
    }
    return 0;
}

typedef pair<int, int> P;
vector<P> edges;
// 边连通分量
const int maxn = 1000+100;
// const int maxm = 1e6+100
int pre[maxn];
int dfs_clock = 0;
vector<int> G[maxn];
vector<int> G2[maxn];
bool Is[maxn];
int low[maxn];

void init() {
    dfs_clock = 1;
    rep(i, 1, maxn) G[i].clear(), G2[i].clear();
    me(low);
    me(pre);
    me(Is);
}

int dfs1(int u, int fa) {

```

```

    int lowu = pre[u] = ++dfs_clock;
    int child = 0;
    for(int i = 0; i < (int)G[u].size(); ++i){
        int v = edges[G[u][i]].second;
        if(!pre[v]){
            child++;
            int lowv = dfs1(v,u);
            lowu = min(lowu,lowv);
            if(lowv >= pre[u]){
                // iscut[u]++;
                Is[G[u][i]] = 1;
            }
        }
        else if(pre[v] < pre[u] && v != fa){
            lowu = min(lowu,pre[v]);
        }
    }

    return low[u] = lowu;
}
// #define Debug

int belong[maxn];
int num[maxn];

void dfs(int u,int be){
    belong[u] = be;
    for(int i = 0; i < (int)G[u].size(); ++i){
        if(Is[G[u][i]])
            continue;
        int v = edges[G[u][i]].second;
        if(!belong[v])
            dfs(v,be);
    }
}

int SG(int u,int fa){
    int t = 0;
    for(int i = 0; i < (int)G2[u].size(); ++i){
        int v = G2[u][i];
        if(v==fa) continue;
        t ^= (SG(v,u)+1);
    }
    if(num[u]&1) t ^= 1;
    return t;
}

int main(void)
{
    int n,m,k;
    while(cin>>n){
        int sum = 0;
        while(n--){
            init();
            edges.clear();
            me(belong);

```



```

me(num);
scanf("%d%d",&m,&k);
rep(i,0,k){
    int u,v;
    scanf("%d%d",&u,&v);
    edges.push_back(P(u,v));
    edges.push_back(P(v,u));
    G[u].push_back(edges.size()-2);
    G[v].push_back(edges.size()-1);
}
dfs1(1,-1);

int tot = 0;
rep(i,1,m+1)
    if(!belong[i])
        dfs(i,++tot);
// dfs(m+1,)
for(int i = 0;i < (int)edges.size(); i += 2){
    int x = belong[edges[i].first];
    int y = belong[edges[i].second];
    if(x != y)
        G2[x].Pb(y),G2[y].Pb(x);
    else
        num[x]++;
}

// cout<<SG(1,-1)<<endl;
sum ^= SG(1,-1);
}
if(sum)
    puts("Sally");
else
    puts("Harry");
}
return 0;
}

```

#### 4.5.6 7 反 nim 博弈

/\*

先手必胜当且仅当：

(1) 所有堆的石子数都为 1 且游戏的 SG 值为 0；

(2) 有些堆的石子数大于 1 且游戏的 SG 值不为 0。

对于任意一个 *Anti-SG* 游戏，如果我们规定当局面中所有的单一游戏的 SG 值为 0 时，游戏结束，则先手必胜当且仅当：

(1) 游戏的 SG 函数不为 0 且游戏中某个单一游戏的 SG 函数大于 1；

(2) 游戏的 SG 函数为 0 且游戏中没有单一游戏的 SG 函数大于 1。

*Every-SG* 游戏规定，对于还没有结束的单一游戏，游戏者必须

对该游戏进行一步决策；

*Every-SG* 游戏的其他规则与普通 SG 游戏相同

对于 *Every-SG* 游戏先手必胜当且仅当单一游戏中最大的 *step* 为奇数。

\*/

#### 4.5.7 8 超自然数

```
//[POJ-2931]
// 超自然数求解不平等博弈问题
char ar[100];
bool b[100];
LL surreal(int n){
    LL k = 1;
    k <= 52;
    for(int i = 0; i < n; ++i){
        scanf("%s", ar);
        if(ar[0] == 'W')
            b[i] = 1;
        else
            b[i] = 0;
    }
    LL x = 0, i = 0;
    while(i < n && b[i] == b[0]){
        if(b[i]) x += k;
        else x -= k;
        i++;
    }
    k >= 1;
    while(i < n){
        if(b[i])
            x += k;
        else
            x -= k;
        i++;
        k >= 1;
    }
    return x;
}
int main(void)
{
    int T;
    cin >> T;
    while(T--){
        int n;
        char br[100];
        scanf("%s %d: ", br, &n);

        LL ans1 = 0, ans2 = 0;
        int a[3];
        rep(i, 0, 3) scanf("%d", &a[i]);
        rep(i, 0, 3) ans1 += surreal(a[i]);
        rep(i, 0, 3) scanf("%d", &a[i]);
        rep(i, 0, 3) ans2 += surreal(a[i]);
        // cout << ans1 << " " << ans2 << endl;
        printf("%s %d: ", br, n);
        if(ans1 >= ans2)
            puts("Yes");
        else
            puts("No");
    }
}
```

```

    }

    return 0;
}

```

## 4.6 数论

### 4.6.1 1 加法

```

string add(string a,string b)
{
    string c;
    int len1=a.length();
    int len2=b.length();
    int len=max(len1,len2);
    for(int i=len1;i<len;i++)
        a="0"+a;
    for(int i=len2;i<len;i++)
        b="0"+b;
    int ok=0;
    for(int i=len-1;i>=0;i--)
    {
        char temp=a[i]+b[i]-'0'+ok;
        if(temp>'9')
        {
            ok=1;
            temp-=10;
        }
        else ok=0;
        c=temp+c;
    }
    if(ok) c="1"+c;
    return c;
}

```

### 4.6.2 1 逆元

// 欧几里得扩展

```

long long ex_gcd(long long a,long long b,long long &x,long long &y)
{
    if(b == 0)
    {
        x = 1;
        y = 0;
        return a;
    }
    long long m = ex_gcd(b,a%b,y,x);
    y -= a/b * x;
    return m;
}

int main()
{
    long long a,b,x,y;
    cin>>a>>b; //求 a 关于 b 的逆元
    if(ex_gcd(a,b,x,y)==1)

```

```

        cout<<(x%b+b)%b<<endl;
    else
        cout<<"None"<<endl;
    return 0;
}
// 费马小定理求逆元
qpow(a,p-2,p);
// 逆元打表

int inv[10000];
int p;
cin>>p;
inv[1] = 1;
for(int i = 2;i < p; ++i)
{
    inv[i] = (p - p/i*inv[p%i]%p)%p;
}
for(int i = 1;i < p; ++i)
    cout<<inv[i]<<" ";
cout<<endl;
for(int i = 1;i < p; ++i)
    cout<<i * inv[i] % p<<" ";

// 快速阶乘逆元

const int maxn = 1e5+10;
long long fac[maxn],invfac[maxn];
void init(int n){
    fac[0] = 1;
    for(int i = 1;i <= n; ++i) fac[i] = fac[i-1]*i%mod;
    invfac[n] = qpow(fac[n],mod-2);
    for(int i = n-1;i >= 0; --i) invfac[i] = invfac[i+1]*(i+1)%mod;
}

```

#### 4.6.3 2 减法

```

string sub(string a,string b)
{
    string c;
    bool ok=0;
    int len1=a.length();
    int len2=b.length();
    int len=max(len1,len2);
    for(int i=len1;i<len;i++)
        a="0"+a;
    for(int i=len2;i<len;i++)
        b="0"+b;
    if(a<b)
    {
        string temp=a;
        a=b;
        b=temp;
        ok=1;
    }
}

```

```

for(int i=len-1;i>=0;i--)
{
    if(a[i]<b[i])
    {
        a[i-1]-=1;
        a[i]+=10;
    }
    char temp=a[i]-b[i]+'0';
    c=temp+c;
}
int pos=0;
while(c[pos]=='0' && pos<len) pos++;
if(pos==len) return "0";
if(ok) return "-" + c.substr(pos);
return c.substr(pos);
}

```

#### 4.6.4 3 乘法

```

string mul(string a,int b)
{
    string c;
    char s;
    int len=a.length();
    int ok=0;
    for(int i=len-1;i>=0;i--)
    {
        int temp=(a[i]-'0')*b+ok;
        ok=temp/10;
        s=temp%10+'0';
        c=s+c;
    }
    while(ok)
    {
        s=ok%10+'0';
        c=s+c;
        ok/=10;
    }
    return c;
}

```

#### 4.6.5 4 除法

```

string div(string a,int b)
{
    string c;
    int len=a.length();
    int ans=0;
    char s;
    for(int i=0;i<len;i++)
    {
        ans=ans*10+a[i]-'0';
        s=ans/b+'0';
        ans%=b;
        c+=s;
    }
}

```

```

    }
    int pos=0;
    while(pos<len && c[pos]!='0') pos++;
    if(pos==len) return "0";
    return c.substr(pos);
}

```

#### 4.6.6 5. 蒙哥马利快速模

```

#include <bits/stdc++.h>
using namespace std;
#define rep(i,a,n) for (int i=a;i<n;i++)
#define per(i,a,n) for (int i=n-1;i>=a;i--)
#define pb push_back
#define mp make_pair
#define all(x) (x).begin(),(x).end()
#define fi first
#define se second
#define SZ(x) ((int)(x).size())
typedef vector<int> VI;
typedef long long ll;
typedef pair<int,int> PII;
const ll mod=1000000007;
ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b>=1){if(b&1)res=res*a%mod;a=a*a%mod;}return res;}
ll gcd(ll a,ll b) { return b?gcd(b,a%b):a;}
// head

typedef unsigned long long u64;
typedef __int128_t i128;
typedef __uint128_t u128;
int _,k;
u64 A0,A1,M0,M1,C,M;

struct Mod64 {
    Mod64():n_(0) {}
    Mod64(u64 n):n_(init(n)) {}
    static u64 init(u64 w) { return reduce(u128(w) * r2); }
    static void set_mod(u64 m) {
        mod=m; assert(mod&1);
        inv=m; rep(i,0,5) inv*=2-inv*m;
        r2=-u128(m)%m;
    }
    static u64 reduce(u128 x) {
        u64 y=u64(x>>64)-u64((u128(u64(x)*inv)*mod)>>64);
        return ll(y)<0?y+mod:y;
    }
    Mod64& operator += (Mod64 rhs) { n_+=rhs.n_-mod; if (ll(n_)<0) n_+=mod; return *this; }
    Mod64 operator + (Mod64 rhs) const { return Mod64(*this)+=rhs; }
    Mod64& operator -= (Mod64 rhs) { n_-=rhs.n_; if (ll(n_)<0) n_+=mod; return *this; }
    Mod64 operator - (Mod64 rhs) const { return Mod64(*this)-=rhs; }
    Mod64& operator *= (Mod64 rhs) { n_ = reduce(u128(n_)*rhs.n_); return *this; }
    Mod64 operator * (Mod64 rhs) const { return Mod64(*this)*=rhs; }
    u64 get() const { return reduce(n_); }
    static u64 mod,inv,r2;

```



```

    ~~~
    线性筛  $O(n)$ 
    ~~~
    const int maxn = 1e6+100;
    bool check[maxn];
    int phi[maxn], Prime[maxn];
    void init(int MAXN){
        int N = maxn-1;
        memset(check, false, sizeof(check));
        phi[1] = 1;
        int tot = 0;
        for(int i = 2; i <= N; ++i){
            if(!check[i]){
                Prime[tot++] = i;
                phi[i] = i-1;
            }
            for(int j = 0; j < tot; ++j){
                if(i*Prime[j] > N) break;
                check[i*Prime[j]] = true;
                if(i%Prime[j] == 0){
                    phi[i*Prime[j]] = phi[i]*Prime[j];
                    break;
                }
                else{
                    phi[i*Prime[j]] = phi[i]*(Prime[j]-1);
                }
            }
        }
    }
    ~~~

```

#### 4.6.8 lucas , 组合数

```

LL qpow(LL a, LL b, LL m){
    LL ans = 1;
    a %= m;
    while(b > 0){
        if(b&1)
            ans = ans*a%m;
        a = a*a%m;
        b >>= 1;
    }
    return ans;
}

LL C(LL n, LL m, LL p){
    if(m > n) return 0;
    LL tmp1 = 1, tmp2 = 1;
    m = min(n-m, m);
    for(LL i = 1; i <= m; ++i){
        tmp1 = tmp1*(n-m+i)%p;
        tmp2 = tmp2*i%p;
    }
    return tmp1*qpow(tmp2, p-2, p)%p;
}

```



```

}
LL lucas(LL n, LL m, LL p){
    if(m == 0)
        return 1;
    return lucas(n/p,m/p,p)*C(n%p,m%p,p)%p;
}

```

#### 4.6.9 miller-rabin-Pollard-rho

// 可以对一个  $2^{63}$  的素数进行判断。

可以分解比较大的数的因子。

```

#include<stdio.h>
#include<string.h>
#include<iostream>
#include<math.h>
#include<stdlib.h>
#include<time.h>
using namespace std;

typedef long long LL;
#define maxn 10000

LL factor[maxn];
int tot;
const int S=20;
LL muti_mod(LL a,LL b,LL c){    //返回 (a*b) mod c, a,b,c<2^63
    a%=c;
    b%=c;
    LL ret=0;
    while (b){
        if (b&1){
            ret+=a;
            if (ret>=c) ret-=c;
        }
        a<<=1;
        if (a>=c) a-=c;
        b>>=1;
    }
    return ret;
}

LL pow_mod(LL x,LL n,LL mod){    //返回 x^n mod c , 非递归版
    if (n==1) return x%mod;
    int bit[90],k=0;
    while (n){
        bit[k++]=n&1;
        n>>=1;
    }
    LL ret=1;
    for (k=k-1;k>=0;k--){
        ret=muti_mod(ret,ret,mod);

```

```

        if (bit[k]==1) ret=muti_mod(ret,x,mod);
    }
    return ret;
}

bool check(LL a,LL n,LL x,LL t){ //以 a 为基,  $n-1=x*2^t$ , 检验 n 是不是合数
    LL ret=pow_mod(a,x,n),last=ret;
    for (int i=1;i<=t;i++){
        ret=muti_mod(ret,ret,n);
        if (ret==1 && last!=1 && last!=n-1) return 1;
        last=ret;
    }
    if (ret!=1) return 1;
    return 0;
}

bool Miller_Rabin(LL n){
    LL x=n-1,t=0;
    while ((x&1)==0) x>>=1,t++;
    bool flag=1;
    if (t>=1 && (x&1)==1){
        for (int k=0;k<S;k++){
            LL a=rand()%(n-1)+1;
            if (check(a,n,x,t)) {flag=1;break;}
            flag=0;
        }
    }
    if (!flag || n==2) return 0;
    return 1;
}

LL gcd(LL a,LL b){
    if (a==0) return 1;
    if (a<0) return gcd(-a,b);
    while (b){
        LL t=a%b; a=b; b=t;
    }
    return a;
}

LL Pollard_rho(LL x,LL c){
    LL i=1,x0=rand()%x,y=x0,k=2;
    while (1){
        i++;
        x0=(muti_mod(x0,x0,x)+c)%x;
        LL d=gcd(y-x0,x);
        if (d!=1 && d!=x){
            return d;
        }
        if (y==x0) return x;
        if (i==k){
            y=x0;
            k+=k;
        }
    }
}

```

```

    }
}

void findfac(LL n){          //递归进行质因数分解 N
    if (!Miller_Rabin(n)){
        factor[tot++] = n;
        return;
    }
    LL p=n;
    while (p>=n) p=Pollard_rho(p,rand() % (n-1) +1);
    findfac(p);
    findfac(n/p);
}

int main()
{
    // srand(time(NULL)); //POJ 上 G++ 要去掉这句话
    int T;
    scanf("%d",&T);
    long long n;
    while(T--){
        scanf("%I64d",&n);
        if (!Miller_Rabin(n)) {printf("Prime\n"); continue; }
        tot = 0;
        findfac(n);
        long long ans=factor[0];
        for(int i=1;i<tot;i++)
            if(factor[i]<ans)ans=factor[i];
        printf("%I64d\n",ans);
    }
    return 0;
}

```

#### 4.6.10 快速数论变换

```

const int mod = 998244353;
LL qpow(LL a,LL b){LL s=1;while(b>0){if(b&1)s=s*a%mod;a=a*a%mod;b>>=1;}return s;}
const int g = 3; //原根
LL quick_mod(LL a,LL b)
{
    LL ans=1;
    for(;b;b/=2)
    {
        if(b&1)
            ans=ans*a%mod;
        a=a*a%mod;
    }
    return ans;
}

int rev(int x,int r) //蝴蝶操作
{
    int ans=0;
    for(int i=0; i<r; i++)

```

```

    {
        if(x&&(1<<i))
        {
            ans+=1<<(r-i-1);
        }
    }
    return ans;
}
void NTT(int n, LL A[],int on) // 长度为 N (2 的次数)
{
    int r=0;
    for(;; r++)
    {
        if((1<<r)==n)
            break;
    }
    for(int i=0; i<n; i++)
    {
        int tmp=rev(i,r);
        if(i<tmp)
            swap(A[i],A[tmp]);
    }
    for(int s=1; s<=r; s++)
    {
        int m=1<<s;
        LL wn=quick_mod(g,(mod-1)/m);
        for(int k=0; k<n; k+=m)
        {
            LL w=1;
            for(int j=0; j<m/2; j++)
            {
                LL t,u;
                t=w*(A[k+j+m/2]%mod)%mod;
                u=A[k+j]%mod;
                A[k+j]=(u+t)%mod;
                A[k+j+m/2]=(u-t)%mod+mod)%mod;
                w=w*wn%mod;
            }
        }
    }
    if(on== -1)
    {
        for(int i=1; i<n/2; i++)
            swap(A[i],A[n-i]);
        LL inv=quick_mod(n,mod-2);
        for(int i=0; i<n; i++)
            A[i]=A[i]%mod*inv%mod;
    }
}

```

#### 4.6.11 欧拉筛和埃氏筛

```
void Era_s(void){
    check[1] = 1;
    tot = 1;
    for(int i = 2; i < maxn; ++i){
        if(!check[i]){
            Prime[tot++] = i;
            for(int j = i+i; j < maxn; ++j) check[j] = 1;
        }
    }
}

void Euler_s(void){
    check[1] = 1;
    tot = 1;
    int n = 1e6;
    for(int i = 2; i <= n; ++i){
        if(!check[i]) Prime[tot++] = i;
        for(int j = 1; j < tot; ++j){
            if(i*Prime[j] > n) break;
            check[i*Prime[j]] = 1;
            if(i % Prime[j] == 0) break;
        }
    }
}
```

#### 4.6.12 素性检测

```
#include <bits/stdc++.h>

using namespace std;
//typedef long long LL;
const int LEN = 1e6+1;
bool vis[LEN];
//int prime[LEN];
int Prime[LEN];
int cnt = 1;
typedef unsigned long long LL;

LL modular_multi(LL x, LL y, LL mo) {
    LL t;
    x %= mo;
    for(t=0; y; x=(x<<1)%mo, y>>=1)
        if (y&1)
            t=(t+x)%mo;
    return t;
}

LL modular_exp(LL num, LL t, LL mo) {
    LL ret=1, temp=num%mo;
    for(; t>>=1, temp=modular_multi(temp, temp, mo))
        if (t&1)
            ret=modular_multi(ret, temp, mo);
    return ret;
}
```

```

bool miller_rabin(LL n) {
    if (n==2 || n==7 || n==61)
        return true;
    if (n==1 || (n&1)==0)
        return false;
    int t=0,num[3]={2,7,61}; //2,7,61 对 unsigned int 内的所有数够用了, 最小不能判断的数为 4 759 123 1
    LL a,x,y,u=n-1;
    while((u&1)==0)
        t++,u>>=1;
    for(int i=0;i<3;i++) {
        a=num[i];
        x=modular_exp(a,u,n);
        for(int j=0;j<t;j++) {
            y=modular_multi(x,x,n);
            if (y==1&&x!=1&&x!=n-1)
                return false;
        }
        //其中用到定理, 如果对模 n 存在 1 的非平凡平方根, 则 n 是合数。
        //如果一个数 x 满足方程  $x^2 \equiv 1 \pmod{n}$ , 但 x 不等于对模 n 来说 1 的两个‘平凡’平方根: 1 或 -1, 则
        x=y;
        if (x!=1) //根据费马小定理, 若 n 是素数, 有  $a^{n-1} \equiv 1 \pmod{n}$ . 因此 n 不可能是素数
            return false;
    }
    return true;
}

void init(void)
{
    int n = LEN - 1;
    for(int i = 2; i <= n; ++i)
    {
        if(!vis[i])
        {
            Prime[cnt++] = i;
            for(LL j = (LL)i * i; j <= n; j += i)
                vis[j] = 1;
        }
    }
}

bool isPrime(LL n)
{
    if(n < 1e6)
    {
        for(LL i = 1; i < cnt&&Prime[i] < n; ++i)
        {
            if(n % Prime[i] == 0)
                return false;
        }
        return true;
    }
    else
        return miller_rabin(n);
}

```

```

int main(void)
{
    init();

    int T;
    cin>>T;
    while(T-->0)
    {
        LL n;
        cin>>n;
        if(isPrime(n))
            cout<<"Yes"<<endl;
        else
            cout<<"No"<<endl;
    }

    return 0;
}

```

#### 4.6.13 素数筛

~~~

Eratosthenes筛法 (埃拉托斯特尼筛法)

```

const int maxn = 1e6+10;
bool check[maxn];
int Prime[maxn];
int tot = 1;
void Eratosthenes(void){
    const int n = maxn -1;
    memset(check,0,sizeof(check));
    for(int i = 2;i < n; ++i){
        if(!check[i]){
            Prime[tot++] = i;
            for(int j = i+i;j < n;j += i) check[j] = 1;
        }
    }
}

```

~~~

欧拉筛

~~~

```

const int maxn = 1e6+10;
bool check[maxn];
int Prime[maxn];
int tot = 1;
void Euler_shai(void){
    int n = maxn-1;
    memset(check,0,sizeof(check));
    for(int i = 2;i <= n; ++i){
        if(!check[i]){
            Prime[tot++] = i;
        }

        for(int j = 1;j < tot; ++j){
            if(i*Prime[j] > n) break;
            check[i*Prime[j]] = 1 ;
        }
    }
}

```

```

        }
        if(i % Prime[j]==0) break;
    }
}

```

```

```

```

#### 4.6.14 逆元打表

```

int inv[10000];
int p;
cin>>p;
inv[1] = 1;
for(int i = 2;i < p; ++i)
{
    inv[i] = (p - p/i*inv[p%i]%p)%p;
}
for(int i = 1;i < p; ++i)
    cout<<inv[i]<<" ";
cout<<endl;
for(int i = 1;i < p; ++i)
    cout<<i * inv[i] % p<<" ";

```

### 4.7 矩阵快速幂.cpp

// 注意修改 maxn 的值, 要不然容易 T

```

const int maxn = 100;
int n;
struct Matrix{
    int n,m;
    Matrix(int nn = 1,int mm = 1):n(nn),m(mm){ memset(a,0,sizeof(a));};
    long long a[maxn][maxn];
};
// void print(const Matrix &a)
// {
//     for(int i = 1;i <= a.n; ++i,cout<<endl)
//         for(int j= 1;j <= a.m; ++j)
//             cout<<a.a[i][j]<<" ";
// }
Matrix operator*(Matrix a,Matrix b)
{
    Matrix c(a.n,b.m);
    for(int i = 1;i <= a.n; ++i)
    {
        for(int j = 1;j <= b.m; ++j)
        {
            for(int k = 1;k <= a.m; ++k)
            {
                c.a[i][j] += a.a[i][k] * b.a[k][j];
                c.a[i][j] %= mod;
            }
        }
    }
}

```



```
//      print(c);
return c;
}
```

## 4.8 自适应辛普森积分.cpp

```
double F(double x)
{
    //Simpson 公式用到的函数
}
double simpson(double a, double b)//三点 Simpson 法, 这里要求 F 是一个全局函数
{
    double c = a + (b - a) / 2;
    return (F(a) + 4 * F(c) + F(b)) * (b - a) / 6;
}
double asr(double a, double b, double eps, double A)//自适应 Simpson 公式 (递归过程)。已知整个区间 [a,b]
{
    double c = a + (b - a) / 2;
    double L = simpson(a, c), R = simpson(c, b);
    if (fabs(L + R - A) <= 15 * eps) return L + R + (L + R - A) / 15.0;
    return asr(a, c, eps / 2, L) + asr(c, b, eps / 2, R);
}
double asr(double a, double b, double eps)//自适应 Simpson 公式 (主过程)
{
    return asr(a, b, eps, simpson(a, b));
}
```

## 5 5 几何

### 5.1 2D

#### 5.1.1 PSLG

```
typedef vector<Point> Polygon;
double PolygonArea(Polygon poly)
{
    double area = 0;
    int n = poly.size();
    for(int i = 1; i < n-1; i++)
        area += Cross(poly[i]-poly[0], poly[(i+1)%n]-poly[0]);
    return area/2;
}

struct Edge
{
    int from, to; // 起点, 终点, 左边的面编号
    double ang;
    Edge(int f, int t, double a):from(f),to(t),ang(a) {}
};

const int maxn = 10000 + 10; // 最大边数

// 平面直线图 (PSGL) 实现
struct PSLG
```

```

{
    int n, m, face_cnt; //face_cnt 面数
    double x[maxn], y[maxn];
    vector<Edge> edges; //储存边
    vector<int> G[maxn]; //指向边
    int vis[maxn*2]; // 每条边是否已经访问过
    int left[maxn*2]; // 左面的编号
    int prev[maxn*2]; // 相同起点的上一条边（即顺时针旋转碰到的下一条边）的编号

    vector<Polygon> faces; //faces 储存面
    double area[maxn]; // 每个 polygon 的面积

    void init(int n)
    {
        this->n = n;
        for(int i = 0; i < n; i++)
            G[i].clear();
        edges.clear();
        faces.clear();
    }

    // 有向线段 from->to 的极角
    double getAngle(int from, int to)
    {
        return atan2(y[to]-y[from], x[to]-x[from]);
    }

    void AddEdge(int from, int to)
    {
        edges.push_back((Edge){ from, to, getAngle(from, to)});
        edges.push_back((Edge){ to, from, getAngle(to, from)});
        m = edges.size();
        G[from].push_back(m-2);
        G[to].push_back(m-1);
    }

    // 找出 faces 并计算面积
    void Build()
    {
        for(int u = 0; u < n; u++)
        {
            // 给从 u 出发的各条边按极角排序
            int d = G[u].size();
            for(int i = 0; i < d; i++)
                for(int j = i+1; j < d; j++) // 这里偷个懒，假设从每个点出发的线段不会太多
                    if(edges[G[u][i]].ang > edges[G[u][j]].ang)
                        swap(G[u][i], G[u][j]);
            for(int i = 0; i < d; i++)
                prev[G[u][(i+1)%d]] = G[u][i];
        }

        memset(vis, 0, sizeof(vis));
        face_cnt = 0;
        for(int u = 0; u < n; u++)

```

```

for(int i = 0; i < G[u].size(); i++)
{
    int e = G[u][i];
    if(!vis[e])    // 逆时针找圈
    {
        face_cnt++;
        Polygon poly;
        for(;;)
        {
            vis[e] = 1;
            left[e] = face_cnt;
            int from = edges[e].from;
            poly.push_back(Point(x[from], y[from]));
            e = prev[e^1];
            if(e == G[u][i])
                break;
            assert(vis[e] == 0);
        }
        faces.push_back(poly);
    }
}

for(int i = 0; i < faces.size(); i++)
{
    area[i] = PolygonArea(faces[i]);
}
}
};

```

### 5.1.2 二维几何模板

```

#include <bits/stdc++.h>
#define mem(ar,num) memset(ar,num,sizeof(ar))
#define me(ar) memset(ar,0,sizeof(ar))
#define lowbit(x) (x&(-x))
#define forn(i,n) for(int i = 0;i < n; ++i)
using namespace std;
typedef long long LL;
typedef unsigned long long ULL;
const int prime = 999983;
const int INF = 0x7FFFFFFF;
const LL INFF =0x7FFFFFFFFFFFFFFF;
const double pi = acos(-1.0);
const double inf = 1e18;
const double eps = 1e-10;
const LL mod = 1e9 + 7;
struct Point
{
    double x,y;

    Point(double x = 0,double y = 0):x(x),y(y) {}
};
typedef Point Vector;

```

```

Vector operator + (Vector A,Vector B)
{
    return Vector(A.x + B.x,A.y + B.y);
}
Vector operator - (Vector A,Vector B)
{
    return Vector(A.x-B.x,A.y-B.y);
}
Vector operator / (Vector A,double p)
{
    return Vector(A.x/p,A.y/p);
}
Vector operator * (Vector A,double p)
{
    return Vector(A.x*p,A.y*p);
}
double angle(Vector v)//求向量的角度从 0 到 2*pi
{
    return atan2(v.y,v.x);
}
int dcmp(double x)
{
    if(fabs(x)<eps)
        return 0;
    else
        return x < 0?-1:1;
}
bool operator < (const Point &a,const Point &b)
{
    if(dcmp(a.x-b.x)==0)
        return a.y<b.y;
    else
        return a.x<b.x;
}

bool operator == (const Point &a,const Point &b)
{
    return !dcmp(a.x-b.x)&&!dcmp(a.y-b.y);
}
double Dot(Vector A,Vector B)
{
    return A.x*B.x+A.y*B.y;
}
double Length(Vector A)
{
    return sqrt(A.x*A.x+A.y*A.y);
}
double Angle(Vector A,Vector B)
{
    return acos(Dot(A,B)/Length(A)/Length(B));
}
double Cross(Vector A,Vector B)
{

```

```

    return A.x*B.y - A.y*B.x;
}
double Area2(Point A,Point B,Point C)
{
    return Cross(B-A,C-A);
}
Vector Rotate(Vector A,double rad)
{
    return Vector (A.x*cos(rad)-A.y*sin(rad),A.x*sin(rad)+A.y*cos(rad));
}
Vector Normal(Vector A)//单位法线
{
    double L = Length(A);
    return Vector(-A.y/L,A.x/L);
}
//调用前确保直线有唯一交点, 当且仅当 Cross(v,w) 非 0
Point Get_Line_Intersection(Point P,Vector v,Point Q,Vector w)
{
    Vector u = P - Q;
    double t = Cross(w,u)/Cross(v,w);
    return P+v*t;
}
double Distance_To_Line(Point P,Point A,Point B)//点到直线的距离
{
    Vector v1 = B-A,v2 = P-A;
    return fabs(Cross(v1,v2)/Length(v1));
}
double Distance_To_Segment(Point P,Point A,Point B)
{
    if(A==B)
        return Length(P-A);
    Vector v1 = B-A,v2 = P-A,v3 = P-B;
    if(dcmp(Dot(v1,v2))<0)
        return Length(v1);
    else if(dcmp(Dot(v1,v3))>0)
        return Length(v3);
    else
        return fabs(Cross(v1,v2))/Length(v1);
}
Point Get_Line_Projection(Point P,Point A,Point B)//求投影点
{
    Vector v = B - A;
    return A + v*(Dot(v,P-A)/Dot(v,v));
}
//线段相交判定 相交不在线段的端点
bool Segment_Proper_Intersection(Point a1,Point a2,Point b1,Point b2)
{
    double c1 = Cross(a2-a1,b1-a1),c2 = Cross(a2-a1,b2-a1),
           c3 = Cross(b2-b1,a2-b1),c4 = Cross(b2-b1,a1-b1);
    return dcmp(c1)*dcmp(c2)<0&&dcmp(c3)*dcmp(c4)<0;
}
//判断点是否在线段上 (不包括端点)
bool Onsegment(Point p,Point a1,Point a2)
{

```

```

    return dcmp(Cross(a1-p,a2-p))==0&&dcmp(Dot(a1-p,a2-p))<0;
}

```

### 5.1.3 二维凸包

//计算凸包, 输入点数组  $p$ , 个数为  $p$ , 输出点数组为  $ch$ 。函数返回凸包顶点数

//输入不能有重复节点

//如果精度要求搞需要用  $dcmp$  判断

//如果不希望在边上右点, 需要将  $\leq$  改为  $<$

```

int ConvexHull(Point *p,int n ,Point *ch)
{
    sort(p,p+n);
    int m = 0;
    for(int i = 0;i < n; ++i)
    {
        while(m>1&& Cross(ch[m-1]-ch[m-2],p[i]-ch[m-2])<=0) m--;
        ch[m++] = p[i];
    }
    int k = m;
    for(int i = n-2; i >= 0; --i)
    {
        while(m > k&& Cross(ch[m-1]-ch[m-2],p[i]-ch[m-2]) <= 0) m--;
        ch[m++] = p[i];
    }
    if(n > 1) m--;
    return m;
}

```

### 5.1.4 判断点是否在多边形内

```

typedef vector<Point> Polygon;
int isPointInPolygon(Point p,Polygon poly)
{
    int n = poly.size();
    int wn = 0;
    for(int i = 0;i < n; ++i)
    {
        if(Onsegment(p,poly[i],poly[(i+1)%n])) return -1;
        int k = dcmp(Cross(poly[(i+1)%n]-poly[i],p-poly[i]));
        int d1 = dcmp(poly[i].y-p.y);
        int d2 = dcmp(poly[(i+1)%n].y-p.y);
        if(k>0&&d1 <= 0&&d2 > 0) wn ++;
        if(k<0&&d2 <= 0&&d1 > 0) wn --;
    }
    if(wn != 0) return 1;
    return 0;
}

```

### 5.1.5 圆与多边形相交的面积

```

#include <iostream>
#include <cstdio>
#include <string>
#include <cmath>

```

```

#include <iomanip>
#include <ctime>
#include <climits>
#include <cstdlib>
#include <cstring>
#include <algorithm>
#include <queue>
#include <vector>
#include <set>
#include <map>
using namespace std;
typedef unsigned int UI;
typedef long long LL;
typedef unsigned long long ULL;
typedef long double LD;
const double pi = acos(-1.0);
const double e = exp(1.0);
const double eps = 1e-8;
const int maxn = 400;
double x, y, h;
double vx, vy;
double R;
int n;
struct point
{
    double x, y;
    point(double _x=0.0, double _y=0.0)
        : x(_x), y(_y) {}
    point operator - (const point & p)
    {
        return point(x-p.x, y-p.y);
    }
    double sqrx()
    {
        return sqrt(x*x+y*y);
    }
} p[maxn];

double xmult(point & p1, point & p2, point & p0);
double distancex(point & p1, point & p2);
point intersection(point u1, point u2, point v1, point v2);
void intersection_line_circle(point c, double r, point l1, point l2, point & p1, point & p2);
point ptoseg(point p, point l1, point l2);
double distp(point & a, point & b);
double Direct_Triangle_Circle_Area(point a, point b, point o, double r);

double xmult(point & p1, point & p2, point & p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p1.y-p0.y)*(p2.x-p0.x);
}

double distancex(point & p1, point & p2)
{

```

```

    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y));
}

point intersection(point u1, point u2, point v1, point v2)
{
    point ret = u1;
    double t = ((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
               / ((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
    ret.x += (u2.x-u1.x)*t;
    ret.y += (u2.y-u1.y)*t;
    return ret;
}

void intersection_line_circle(point c, double r, point l1, point l2, point & p1, point & p2)
{
    point p = c;
    double t;
    p.x += l1.y-l2.y;
    p.y += l2.x-l1.x;
    p = intersection(p, c, l1, l2);
    t = sqrt(r*r-distancex(p, c)*distancex(p, c))/distancex(l1, l2);
    p1.x = p.x+(l2.x-l1.x)*t;
    p1.y = p.y+(l2.y-l1.y)*t;
    p2.x = p.x-(l2.x-l1.x)*t;
    p2.y = p.y-(l2.y-l1.y)*t;
}

point ptoseg(point p, point l1, point l2)
{
    point t = p;
    t.x += l1.y-l2.y;
    t.y += l2.x-l1.x;
    if (xmult(l1, t, p)*xmult(l2, t, p)>eps)
        return distancex(p, l1)<distancex(p, l2) ? l1 : l2;
    return intersection(p, t, l1, l2);
}

double distp(point & a, point & b)
{
    return (a.x-b.x)*(a.x-b.x)+(a.y-b.y)*(a.y-b.y);
}

double Direct_Triangle_Circle_Area(point a, point b, point o, double r)
{
    double sign = 1.0;
    a = a-o;
    b = b-o;
    o = point(0.0, 0.0);
    if (fabs(xmult(a, b, o)) < eps)
        return 0.0;
    if (distp(a, o) > distp(b, o))
    {
        swap(a, b);
        sign = -1.0;
    }
}

```



```

}
if (distp(a, o) < r*r+eps)
{
    if (distp(b, o) < r*r+eps)
        return xmult(a, b, o)/2.0*sign;
    point p1, p2;
    intersection_line_circle(o, r, a, b, p1, p2);
    if (distancex(p1, b) > distancex(p2, b))
        swap(p1, p2);
    double ret1 = fabs(xmult(a, p1, o));
    double ret2 = acos((p1.x*b.x+p1.y*b.y)/p1.sqr() / b.sqr())*r*r;
    double ret = (ret1+ret2)/2.0;
    if (xmult(a, b, o)<eps && sign>0.0 || xmult(a, b, o)>eps && sign<0.0)
        ret = -ret;
    return ret;
}
point ins = ptoseg(o, a, b);
if (distp(o, ins)>r*r-eps)
{
    double ret = acos((a.x*b.x+a.y*b.y)/a.sqr() / b.sqr())*r*r/2.0;
    if (xmult(a, b, o)<eps && sign>0.0 || xmult(a, b, o)>eps && sign<0.0)
        ret = -ret;
    return ret;
}
point p1, p2;
intersection_line_circle(o, r, a, b, p1, p2);
double cm = r/(distancex(o, a)-r);
point m = point((o.x+cm*a.x)/(1+cm), (o.y+cm*a.y)/(1+cm));
double cn = r/(distancex(o, b)-r);
point n = point((o.x+cn*b.x)/(1+cn), (o.y+cn*b.y)/(1+cn));
double ret1 = acos((m.x*n.x+m.y*n.y)/m.sqr() / n.sqr())*r*r;
double ret2 = acos((p1.x*p2.x+p1.y*p2.y)/p1.sqr() / p2.sqr())*r*r-fabs(xmult(p1, p2, o));
double ret = (ret1-ret2)/2.0;
if (xmult(a, b, o)<eps && sign>0.0 || xmult(a, b, o)>eps && sign<0.0)
    ret = -ret;
return ret;
}
double Inter(double x,double y,double R,int n,point *area){
    area[n] = area[0];
    point temp = point(x, y);
    double sum = 0;
    for (int i=0; i<n-1; i++)
        sum += Direct_Triangle_Circle_Area(area[i], area[i+1], temp, R);

    sum += Direct_Triangle_Circle_Area(area[n-1], area[0], temp, R);
    return fabs(sum);
}
double Cross(point A,point B)
{
    return A.x*B.y - A.y*B.x;
}
int N,M;
double PolygonArea (point * p,int n)
{

```

```

    double area = 0;
    for(int i = 1; i < n - 1; ++i)
    {
        area += Cross(p[i]-p[0],p[i+1]-p[0]);
    }
    return fabs(area/2);
}

int dcmp(double x)
{
    if(fabs(x)<eps)
        return 0;
    else
        return x < 0?-1:1;
}

double S ;
double xi,yi,P,Q;
bool check(double R){
    //      cout<<xi<<" "<<yi<<" "<<P<<" "<<Q<<endl;
    //      printf("r = %lf Intersect = %lf\n",R,Inter(xi,yi,R,N,p) );
    //      printf("%lf\n", (1-P/Q)*S);
    return dcmp(Inter(xi,yi,R,N,p) - (1-P/Q)*S) > 0;
}

int main()
{
    cin>>N;
    for(int i=0;i< N;i++)
    {
        scanf("%lf%lf",&p[i].x,&p[i].y);
    }

    S= PolygonArea(p,N);
    //cout<<S<<endl;
    cin>>M;
    for(int i = 0;i < M; ++i){

        scanf("%lf %lf %lf %lf",&xi,&yi,&P,&Q);

        double l = 0,r = 1e6;
        for(int j = 0;j < 100; ++j){
            double mid = l+(r-l)/2;
            if(check(mid))
                r = mid;
            else
                l = mid;
            // printf("%lf %lf\n",l,r);
        }
        printf("%.8lf\n",r);
    }

    return 0;
}

```

### 5.1.6 求圆与直线的交点

```
int getLineCircleIntersection(Point A, Point B, Point C, double r, double& t1, double& t2, vector<Point>
// 初始方程:  $(A.x + t(B.x - A.x) - C.x)^2 + (A.y + t(B.y - A.y) - C.y)^2 = r^2$ 
// 整理得:  $(at + b)^2 + (ct + d)^2 = r^2$ 
double a = B.x - A.x;
double b = A.x - C.x;
double c = B.y - A.y;
double d = A.y - C.y;
// 展开得:  $(a^2 + c^2)t^2 + 2(ab + cd)t + b^2 + d^2 - r^2 = 0$ , 即  $et^2 + ft + g = 0$ 
double e = a * a + c * c;
double f = 2 * (a * b + c * d);
double g = b * b + d * d - r * r;
double delta = f * f - 4 * e * g; // 判别式
if(dcmp(delta) < 0) return 0; // 相离
if(dcmp(delta) == 0){ // 相切
    t1 = t2 = -f / (2 * e);
    sol.push_back(A+(B-A)*t1);
    return 1;
}
t1 = (-f - sqrt(delta)) / (2 * e);
t2 = (-f + sqrt(delta)) / (2 * e);
sol.push_back(A+(B-A)*t1);
sol.push_back(A+(B-A)*t2);
return 2;
}
```

## 5.2 3D

### 5.2.1 三维几何的基本操作

```
#include <bits/stdc++.h>

using namespace std;
struct Point3
{
    double x,y,z;
    Point3(double x = 0, double y = 0, double z = 0):x(x),y(y),z(z) {}
};
typedef Point3 Vector3;

Vector3 operator +(Vector3 v1, Vector3 v2)
{
    return Vector3(v1.x+v2.x, v1.y+v2.y, v1.z+v2.z);
}
Vector3 operator -(Vector3 v1, Vector3 v2)
{
    return Vector3(v1.x-v2.x, v1.y-v2.y, v1.z-v2.z);
}
Vector3 operator *(Vector3 v, double c)
{
    return Vector3(v.x*c, v.y*c, v.z*c);
}
Vector3 operator /(Vector3 v, double c)
{
    return Vector3(v.x/c, v.y/c, v.z/c);
}
```

```

    return Vector3(v.x/c,v.y/c,v.z/c);
}
double Dot(Vector3 A,Vector3 B)
{
    return A.x*B.x+A.y*B.y+A.z*B.z;
}
double Length(Vector3 A)
{
    return sqrt(Dot(A,A));
}
double Angle(Vector3 A,Vector3 B)
{
    return acos(Dot(A,B)/(2*Length(A)*Length(B)));
}
double DistanceToPlane(const Point3 &p,const Point3 &p0,const Vector3& n)
{
    return fabs(Dot(p-p0,n))/Length(n);
}
Point3 GetPlaneProjection(const Point3&p,const Point3&p0,const Vector3&n)
{
    return p-n*Dot(p-p0,n);
}
//直线 p1-p2 到平面 p0-n 的交点。 假定交点唯一存在
Point3 LinePlaneIntersection(Point3 p1,Point3 p2,Point3 p0,Vector3 n)
{
    Vector3 v= p2 - p1;
    //    /*if(dcmp(Dot(v,n))==0)
    //    {
    //        if(dcmp(Dot(p1-p0,n))==0)
    //            直线在平面上
    //        else
    //            直线与平面平行
    //    }
    //    */
    double t = Dot(n,p0-p1)/Dot(n,p2-p1);
    return p1 + v*t;
}

```

### 5.2.2 三维几何的模版

```

#include <bits/stdc++.h>
const double eps = 1e-6;
using namespace std;

struct Point3
{
    double x,y,z;
    Point3(double x = 0,double y = 0,double z = 0):x(x),y(y),z(z) {}
};
typedef Point3 Vector3;
int dcmp(double d)
{
    if(fabs(d)< eps)
        return 0;
}

```

```

        else
            return d < 0?-1:1;
    }
    Vector3 operator +(Vector3 v1,Vector3 v2)
    {
        return Vector3(v1.x+v2.x,v1.y+v2.y,v1.z+v2.z);
    }
    Vector3 operator -(Vector3 v1,Vector3 v2)
    {
        return Vector3(v1.x-v2.x,v1.y-v2.y,v1.z-v2.z);
    }
    Vector3 operator *(Vector3 v,double c)
    {
        return Vector3(v.x*c,v.y*c,v.z*c);
    }
    Vector3 operator /(Vector3 v,double c)
    {
        return Vector3(v.x/c,v.y/c,v.z/c);
    }
    bool operator ==(Point3 A,Point3 B)
    {
        return !dcmp(A.x-B.x)&&!dcmp(A.y-B.y)&&!dcmp(A.z-B.z);
    }
    double Dot(Vector3 A,Vector3 B)
    {
        return A.x*B.x+A.y*B.y+A.z*B.z;
    }
    double Length(Vector3 A)
    {
        return sqrt(Dot(A,A));
    }
    double Angle(Vector3 A,Vector3 B)//求两向量的夹角
    {
        return acos(Dot(A,B)/(2*Length(A)*Length(B)));
    }
    double DistanceToplan(const Point3 &p,const Point3 &p0,const Vector3& n)//
    {
        return fabs(Dot(p-p0,n))/Length(n);
    }
    Point3 GetPlaneProjection(const Point3&p,const Point3&p0,const Vector3&n)
    {
        return p-n*Dot(p-p0,n);
    }
    //直线 p1-p2 到平面 p0-n 的交点。 假定交点唯一存在
    Point3 LinePlaneIntetsection(Point3 p1,Point3 p2,Point3 p0,Vector3 n)
    {
        Vector3 v= p2 - p1;
        //      /*if(dcmp(Dot(v,n))==0)
        //      {
        //          if(dcmp(Dot(p1-p0,n))==0)
        //              直线在平面上
        //          else
        //              直线与平面平行
        //      }

```

```

//    */
double t = Dot(n,p0-p1)/Dot(n,p2-p1);
return p1 + v*t;
}
Point3 LinePlaneIntetsection(Point3 p1,Point3 p2,double A,double B,double C,double D)
{
    Vector3 v = p2-p1;
    double t = (A*p1.x+B*p1.y+C*p1.z+D)/(A*(p1.x-p2.x)+B*(p1.y-p2.y)+C*(p1.z-p2.z));
    return p1 + v*t;
}
Vector3 Cross(Vector3 A,Vector3 B)
{
    return Vector3(A.y*B.z-A.z*B.y,A.z*B.x-A.x*B.z,A.x*B.y-A.y*B.x);
}
double Area2(Point3 A,Point3 B,Point3 C)
{
    return Length(Cross(B-A,C-A));
}
////已知平面的三点, 求出点法式
//Vector3 Solven(Point3 A,Point3 B,Point3 C)
//{
//    return Cross(B-A,C-A);
//}
//判断一个点是否在三角形内, 可以用面积法
bool PointInTri(Point3 P,Point3 A,Point3 B,Point3 C)
{
    double area1 = Area2(P,A,B);
    double area2 = Area2(P,A,C);
    double area3 = Area2(P,B,C);
    double area4 = Area2(A,B,C);
    return dcmp(area1+area2+area3-area4)==0;
}
//判断线段是否与三角形相交
bool TriSegIntersection(Point3 P0,Point3 P1,Point3 P2,Point3 A,Point3 B,Point3 &P)
{
    Vector3 n = Cross(P1-P0,P2-P0);

    if(dcmp(Dot(n,B-A))==0)
        return false;

    double t = Dot(n,P0-A)/Dot(n,B-A);
    if(dcmp(t) < 0 || dcmp(t-1) > 0)
        return false;
    P = A + (B-A) * t;
    return PointInTri(P,P0,P1,P2);
}
double DitantceToLine(Point3 P,Point3 A,Point3 B)
{
    return Length(Cross(A-P,B-P))/Length(A-B);
}
double DistanceToSegment(Point3 P,Point3 A,Point3 B)
{
    if(A==B) return Length(P-A);
    Vector3 v1 = B - A, v2 = P - A,v3 = P-B;

```

```

    if(dcmp(Dot(v1,v2)) == 0) return Length(v2);
    if(dcmp(Dot(v1,v3)) > 0) return Length(v3);
    return Length(Cross(v1,v2))/Length(v1);
}
double Volume6(Point3 A,Point3 B,Point3 C,Point3 D)
{
    return Dot(D-A,Cross(B-A,C-A));
}
//
int main(void)
{
    Point3 A(0,0,0),B(0,100,0),C(100,0,0),D(25,25,0);
    cout<<PointInTri(D,A,B,C)<<endl;
    return 0;
}

```

### 5.2.3 三维凸包

```

struct Face{
    int v[3];
    Vector3 normal(Vector *P)
    {
        return Cross(P[v[1]]-P[v[0]],P[v[2]]-P[v[0]]);
    }
    int cansee(Point *P,int i)const
    {
        return Dot(P[i]-P[v[0]],normal(P)) > 0?1 : 0;
    }
};
vector <Face> CH3D(Point3* P,int n)
{
    vector <Face> cur;
    cur.push_back((Face){0,1,2});
    cur.push_back((Face){2,1,0});
    for(int i = 3;i < n; ++i)
    {
        vector<Face> next;
        //计算每条边“左面”的可见性
        for(int j= 0;j < cur.size(); ++j)
        {
            Face &f = cur[j];
            int res = f.cansee(P,i);
            if(!res) next.push_back(f);
            for(int k = 0;k < 3; ++k)
                vis[f.v[k]][f.v[(k+1)%3]] = res;
        }
        for(int j = 0;j < cur.size(); ++j)
        {
            for(int k = 0;k < 3; ++k)
            {
                int a = cur[j].v[k],b = cur[j].v[(k+1)%3];
                if(vis[a][b] != vis[b][a]&&vis[a][b])//(a,b) 是分界线, 左边对 P[i] 可见
                    next.push_back((Face){a,b,i});
            }
        }
    }
}

```

```

    }
    }
    cnr = next;
}
return cur;
}
double rand01() {return rand() / (double) RAND_MAX;} //0-1 的随机数
double randeps() {return (rand01()-0.5) * eps;}
Point3 add_noise(Point3 p)
{
    return Point3(p.x + randeps(),p.y+randeps(),p.z+randeps());
}

//.....
struct Face{
    int v[3];
    Vector3 normal(Vector *P)
    {
        return Cross(P[v[1]]-P[v[0]],P[v[2]]-P[v[0]]);
    }
    int cansee(Point *P,int i)const
    {
        return Dot(P[i]-P[v[0]],normal(P)) > 0?1 : 0;
    }
};
vector <Face> CH3D(Point3* P,int n)
{
    vector <Face> cur;
    cur.push_back((Face){0,1,2});
    cur.push_back((Face){2,1,0});
    for(int i = 3;i < n; ++i)
    {
        vector<Face> next;
        //计算每条边“左面”的可见性
        for(int j= 0;j < cur.size(); ++j)
        {
            Face &f = cur[j];
            int res = f.cansee(P,i);
            if(!res) next.push_back(f);
            for(int k = 0;k < 3; ++k)
                vis[f.v[k]][f.v[(k+1)%3]] = res;
        }
        for(int j = 0;j < cur.size(); ++j)
        {
            for(int k = 0;k < 3; ++k)
            {
                int a = cur[j].v[k],b = cur[j].v[(k+1)%3];
                if(vis[a][b] != vis[b][a]&&vis[a][b])//(a,b) 是分界线, 左边对 P[i] 可见
                    next.push_back((Face){a,b,i});
            }
        }
        cnr = next;
    }
    return cur;
}

```



```

}
double rand01() {return rand() / (double) RAND_MAX;} //0-1 的随机数
double randeps() {return (rand01()-0.5) * eps;}
Point3 add_noise(Point3 p)
{
    return Point3(p.x + randeps(),p.y+randeps(),p.z+randeps());
}

```

#### 5.2.4 维度转换为三维坐标

```

// 经纬度转换为球坐标
double torad(double deg)
{
    return deg/180*acos(-1);
}
void get_coordinate(double R,double lat,double lng,double &x,double &y,double &z)
{
    lat = torad(lat);
    lng = torad(lng);
    x = R*cos(lat)*cos(lng);
    y = R*cos(lat)*sin(lng);
    z = R*sin(lat);
}

```

## 6 6 其它

### 6.1 IO

#### 6.1.1 fread

```

namespace io {
    const int L = 1 << 20 | 1;
    char ibuf[L], *iS, *iT, c, obuf[L], *oS = obuf, *oT = obuf + L - 1, qu[55]; int f, qr;
    #ifdef whzzt
        #define gc() getchar()
    #else
        #define gc() (iS == iT ? (iT = (iS = ibuf) + fread (ibuf, 1, L, stdin), iS == iT ? EOF : *iS++) :
    #endif
    template <class I>
    inline void gi (I &x) {
        for (f = 1, c = gc(); c < '0' || c > '9'; c = gc()) if (c == '-') f = -1;
        for (x = 0; c <= '9' && c >= '0'; c = gc()) x = x * 10 + (c & 15); x *= f;
    }
    inline void flush () {
        fwrite (obuf, 1, oS - obuf, stdout);
    }
    inline void putc (char x) {
        *oS++ = x;
        if (oS == oT) flush (), oS = obuf;
    }
    template <class I>
    void print (I x) {
        if (!x) putc ('0'); if (x < 0) putc ('-'), x = -x;
        while (x) qu[++qr] = x % 10 + '0', x /= 10;
    }
}

```

```

        while (qr) putc (qu[qr --]);
    }
    struct io_ff { ~io_ff() { flush(); } } _io_ff_;
}
using io :: gi;
using io :: putc;
using io :: print;

```

### 6.1.2 fread2

```

namespace IO{
#define BUF_SIZE 100000
#define OUT_SIZE 100000
#define ll long long
//fread->read

bool IOerror=0;
inline char nc(){
    static char buf[BUF_SIZE],*p1=buf+BUF_SIZE,*pend=buf+BUF_SIZE;
    if (p1==pend){
        p1=buf; pend=buf+fread(buf,1,BUF_SIZE,stdin);
        if (pend==p1){IOerror=1;return -1;}
        //{printf("IO error!\n");system("pause");for (;;);exit(0);}
    }
    return *p1++;
}

inline bool blank(char ch){return ch==' '||ch=='\n'||ch=='\r'||ch=='\t';}
inline void read(int &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (sign)x=-x;
}

inline void read(ll &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (sign)x=-x;
}

inline void read(double &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (ch=='.'){
        double tmp=1; ch=nc();
        for (;ch>='0'&&ch<='9';ch=nc())tmp/=10.0,x+=tmp*(ch-'0');
    }
    if (sign)x=-x;
}

```

```

}
inline void read(char *s){
    char ch=nc();
    for (;blank(ch);ch=nc());
    if (I0error)return;
    for (;!blank(ch)&&!I0error;ch=nc())*s++=ch;
    *s=0;
}
inline void read(char &c){
    for (c=nc();blank(c);c=nc());
    if (I0error){c=-1;return;}
}
//fwrite->write
struct Ostream_fwrite{
    char *buf,*p1,*pend;
    Ostream_fwrite(){buf=new char[BUF_SIZE];p1=buf;pend=buf+BUF_SIZE;}
    void out(char ch){
        if (p1==pend){
            fwrite(buf,1,BUF_SIZE,stdout);p1=buf;
        }
        *p1++=ch;
    }
    void print(int x){
        static char s[15],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1);
    }
    void println(int x){
        static char s[15],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1); out('\n');
    }
    void print(ll x){
        static char s[25],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1);
    }
    void println(ll x){
        static char s[25],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1); out('\n');
    }
    void print(double x,int y){
        static ll mul[]={1,10,100,1000,10000,100000,1000000,10000000,100000000,1000000000,
            10000000000,100000000000LL,1000000000000LL,10000000000000LL,100000000000000LL,
            1000000000000000LL,10000000000000000LL,100000000000000000LL,1000000000000000000LL};
        if (x<-1e-12)out('-'),x=-x;x*=mul[y];
        ll x1=(ll)floor(x); if (x-floor(x)>=0.5)++x1;
        ll x2=x1/mul[y],x3=x1-x2*mul[y]; print(x2);
        if (y>0){out('.'); for (size_t i=1;i<y&& x3*mul[i]<mul[y];out('0'),++i); print(x3);}
    }
}

```

```

    }
    void println(double x,int y){print(x,y);out('\n');}
    void print(char *s){while (*s)out(*s++);}
    void println(char *s){while (*s)out(*s++);out('\n');}
    void flush(){if (p1!=buf){fwrite(buf,1,p1-buf,stdout);p1=buf;}}
    ~Ostream_fwrite(){flush();}
}Ostream;
inline void print(int x){Ostream.print(x);}
inline void println(int x){Ostream.println(x);}
inline void print(char x){Ostream.out(x);}
inline void println(char x){Ostream.out(x);Ostream.out('\n');}
inline void print(ll x){Ostream.print(x);}
inline void println(ll x){Ostream.println(x);}
inline void print(double x,int y){Ostream.print(x,y);}
inline void println(double x,int y){Ostream.println(x,y);}
inline void print(char *s){Ostream.print(s);}
inline void println(char *s){Ostream.println(s);}
inline void println(){Ostream.out('\n');}
inline void flush(){Ostream.flush();}
#undef ll
#undef OUT_SIZE
#undef BUF_SIZE
};

```

### 6.1.3 保留小数

```

#include <bits/stdc++.h>
using namespace std;
const double pi = acos(-1.0);
int main(void)
{
    for(int i = 0;i < 5; ++i)
        printf("%.*f\n",i,pi);
    for(int i = 0;i < 5; ++i)
        cout<<setiosflags(ios::fixed)<<setprecision(i)<<pi<<endl;
    return 0;
}

```

### 6.1.4 读取整数

```

//读取正负整数
inline int input(void)
{
    int num = 0;
    char c;
    int flag = 0;
    while((c = getchar()) < '0' || c > '9') flag = c=='-' ? 1:flag;
    while(c >= '0' && c <= '9')
        num = num * 10 + c - '0',c = getchar();
    if(flag) num = -num;
    return num;
}

```

## 6.2 c++ 中处理 2 进制的一些函数.cpp

Built-in Function: `int __builtin_ffs (unsigned int x)`

Returns one plus the index of the least significant 1-bit of x, or if x is zero, returns zero.  
返回右起第一个 '1' 的位置。

Built-in Function: `int __builtin_clz (unsigned int x)`

Returns the number of leading 0-bits in x, starting at the most significant bit position. If x is 0, the number of leading 0-bits is undefined.  
返回左起第一个 '1' 之前 0 的个数。

Built-in Function: `int __builtin_ctz (unsigned int x)`

Returns the number of trailing 0-bits in x, starting at the least significant bit position. If x is 0, the number of trailing 0-bits is undefined.  
返回右起第一个 '1' 之后的 0 的个数。

Built-in Function: `int __builtin_popcount (unsigned int x)`

Returns the number of 1-bits in x.  
返回 '1' 的个数。

Built-in Function: `int __builtin_parity (unsigned int x)`

Returns the parity of x, i.e. the number of 1-bits in x modulo 2.  
返回 '1' 的个数的奇偶性。

Built-in Function: `int __builtin_ffsl (unsigned long)`

Similar to `__builtin_ffs`, except the argument type is `unsigned long`.

Built-in Function: `int __builtin_clzl (unsigned long)`

Similar to `__builtin_clz`, except the argument type is `unsigned long`.

Built-in Function: `int __builtin_ctzl (unsigned long)`

Similar to `__builtin_ctz`, except the argument type is `unsigned long`.

Built-in Function: `int __builtin_popcountl (unsigned long)`

Similar to `__builtin_popcount`, except the argument type is `unsigned long`.

Built-in Function: `int __builtin_parityl (unsigned long)`

Similar to `__builtin_parity`, except the argument type is `unsigned long`.

Built-in Function: `int __builtin_ffsll (unsigned long long)`

Similar to `__builtin_ffs`, except the argument type is `unsigned long long`.

Built-in Function: `int __builtin_clzll (unsigned long long)`

Similar to `__builtin_clz`, except the argument type is `unsigned long long`.

❑ Built-in Function: `int __builtin_ctzll (unsigned long long)`

Similar to `__builtin_ctz`, except the argument type is `unsigned long long`.

❑ Built-in Function: `int __builtin_popcountll (unsigned long long)`

Similar to `__builtin_popcount`, except the argument type is `unsigned long long`.

❑ Built-in Function: `int __builtin_parityll (unsigned long long)`

Similar to `__builtin_parity`, except the argument type is `unsigned long long`.

## 6.3 测量程序的运行时间.cpp

```
clock_t start,end;
start = clock();
end = clock();
dur = double(end - start);
printf("Use Time: %f\n", (dur/CLOCKS_PER_SEC));
```

# 7 7 模拟

## 7.1 1 日期.cpp

1 计算日期差

```
#include <stdio.h>
#include <stdlib.h>

bool isLeapYear(int year)
{
    return ((year%4==0 && year%100!=0) || year%400==0);
}
// 以公元 1 年 1 月 1 日为基准, 计算经过的日期
int getDays(int year, int month, int day)
{
    int m[] = {0,31,28,31,30,31,30,31,31,30,31,30,31};
    if(isLeapYear(year))
        m[2]++;
    int result = 0;
    for(int i = 1; i < year; i++)
    {
        result += 365;
        if(isLeapYear(i))
            result ++;
    }
    for(int i = 1; i < month; i++)
    {
        result += m[i];
    }
    result += day;

    return result;
```

```

}
int dayDis (int year1, int month1, int day1,
            int year2, int month2, int day2)
{
    return abs(getDays(year2, month2, day2) - getDays(year1, month1, day1));
}

int main(void)
{
    printf("%d\n",dayDis(2012, 9, 1, 2018, 3, 25));

    return 0;
}
2 计算某一天星期几
int cal1(int y,int m,int d)
{
    if(m==1||m==2)
        m+=12,y--;
    int w=(d+2*m+3*(m+1)/5+y+y/4-y/100+y/400)%7;
    return ++w;
}
int cal2(int y,int m,int d)
{
    if(m==1||m==2)
        m+=12,y--;
    int c=y/100,ty=y%100;
    int w=ty+ty/4+c/4-2*c+26*(m+1)/10+d-1;
    return w%7==0?7:(w+7)%7;
}
3 计算从2000 01 01 到9999 12 31 之间任意日期之间日期表示有多少个9
#include<bits/stdc++.h>

using namespace std;

int year,month,day;
int a1,b1,c1,a2,b2,c2;

const int maxn = 1e4+100;
int a[maxn];
int c[maxn]; // 代表当前年所有的 9
// int mon[30] = {0,2,2,2,}
int run(int y){
    return y%400 == 0 || (y%4==0&& y%100!=0);
}
int wanyue(int t,int y){
    if(t == 2) return 2+run(y);
    if(t == 9) return 3+30;
    return 3;
}
int wanyear(int t){
    int num = 0;
    int tt = t;
    while(tt > 0){

```

3 计算从2000 01 01 到9999 12 31 之间任意日期之间日期表示有多少个9

```

        if(tt % 10 == 9) num++;
        tt /= 10;
    }
    a[t] = num;
    int tmp = run(t);
    return num*(365+tmp)+65+tmp;
}
int mo[20] = {0,31,28,31,30,31,30,31,31,30,31,30,31};
int Howmuchday(int y,int t){
    if(t==2){
        return run(y)+28;
    }
    return mo[t];
}
int subday(int a,int b){
    int sum = 0;
    for(int i = a;i <= b; ++i)
        if(i%10 == 9)
            sum++;
    return sum;
}
int numsubday(int a,int b){
    return b-a+1;
}

int numsubday(int y,int b1,int c1,int b2,int c2){
    int num = 0;
    if(b1 == b2)
        return numsubday(c1,c2);
    for(int i = b1+1;i < b2; ++i)
        num += mo[i]+(i==2&&run(y));
    num += numsubday(c1,Howmuchday(y,b1));
    num += numsubday(1,c2);
    return num;
}
int FF(int t){
    int num = 0;
    int tt = t;
    while(tt > 0){
        if(tt % 10 == 9) num++;
        tt /= 10;
    }
    return num;
}
int submonth(int y,int b1,int c1,int b2,int c2){
    if(b1 == b2)
        return subday(c1,c2)+(c2-c1+1)*FF(b1);
    int sum = 0;
    for(int i = b1+1;i < b2; ++i)
        sum += wanyue(i,y);

    sum += subday(c1,Howmuchday(y,b1))+FF(b1)*(Howmuchday(y,b1)-c1+1);
    // cout<<sum<<endl;
    sum += subday(1,c2)+FF(b2)*(c2);
}

```



```

    return sum;
}

int subyear(int a1,int b1,int c1,int a2,int b2,int c2){
    if(a1 == a2)
        return numsubday(a1,b1,c1,b2,c2)*a[a1] + submonth(a1,b1,c1,b2,c2);
    int ans = 0;
    ans += c[a2-1]-c[a1];
    ans += numsubday(a1,b1,c1,12,31)*a[a1];
    ans += numsubday(a2,1,1,b2,c2)*a[a2];
    return ans + submonth(a1,b1,c1,12,31)+submonth(a2,1,1,b2,c2);
}

int main(void){

    for(int i = 2000;i < maxn; ++i){
        c[i] = wanyear(i);
        c[i] += c[i-1];
    }
    int T;
    cin>>T;
    while(T--){
        scanf("%d%d%d %d%d%d",&a1,&b1,&c1,&a2,&b2,&c2);
        int ans = subyear(a1,b1,c1,a2,b2,c2);
        printf("%d\n",ans);
    }
    return 0;
}
// 同上
#include <stdio.h>
#include <string.h>

int sum[10005][15][35],pre[10005][15][35];
int mon[15] = {0,31,28,31,30,31,30,31,31,30,31,30,31};

int leap(int x)
{
    if (x % 400 == 0) return 1;
    if (x % 100 == 0) return 0;
    if (x % 4 == 0) return 1;

    return 0;
}

int check(int y,int m,int d)
{
    int num = 0;

    while (y)
    {
        y % 10 == 9 ? ++num : num += 0;
        y /= 10;
    }
}

```

```

while (m)
{
    m % 10 == 9 ? ++num : num += 0;
    m /= 10;
}

while (d)
{
    d % 10 == 9 ? ++num : num += 0;
    d /= 10;
}

return num;
}

void init(int y1,int m1,int d1,int y2,int m2,int d2)
{
    int tmp = 0;

    while (y1 != y2 || m1 != m2 || d1 != d2)
    {
        mon[2] = leap(y1) + 28;

        pre[y1][m1][d1] = tmp; //tmp 是到前一个日期显示的 9 的数量。

        tmp += check(y1,m1,d1);

        sum[y1][m1][d1] = tmp; //现在的日期显示的 9 的数量

        if (++d1 > mon[m1])
        {
            d1 = 1;

            if (++m1 > 12)
            {
                m1 = 1;
                mon[2] = 28 + leap(++y1);
            }
        }
    }
}

int main()
{
    int t;

    scanf("%d",&t);

    init(2000,1,1,10000,1,1);

    while (t--)
    {

```

```

    int y1,m1,d1,y2,m2,d2;

    scanf("%d%d%d%d%d", &y1, &m1, &d1, &y2, &m2, &d2);

    printf("%d\n", sum[y2][m2][d2] - pre[y1][m1][d1]); //结束日期减去开始日期之前的那天, 因为开始日期也
}

return 0;
}

```